

Two-temperature plasmas in relativistic kinetic turbulence

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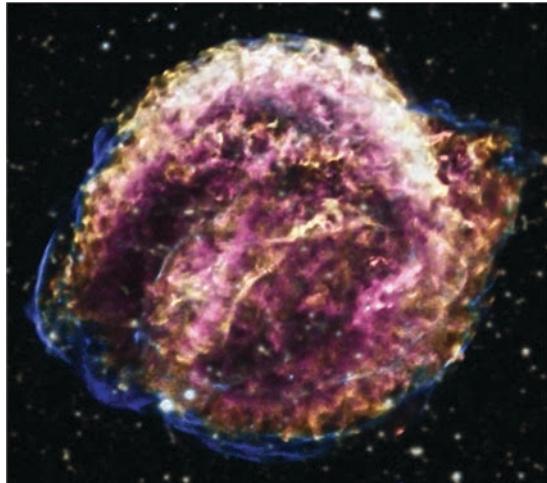
Vienna meeting, 8/6/2019



Outline

- I. Why relativistic kinetic turbulence?
- II. Pair plasmas
- III. Electron-ion plasmas
- IV. Radiative plasmas
- V. Conclusions

High-energy astrophysical turbulence

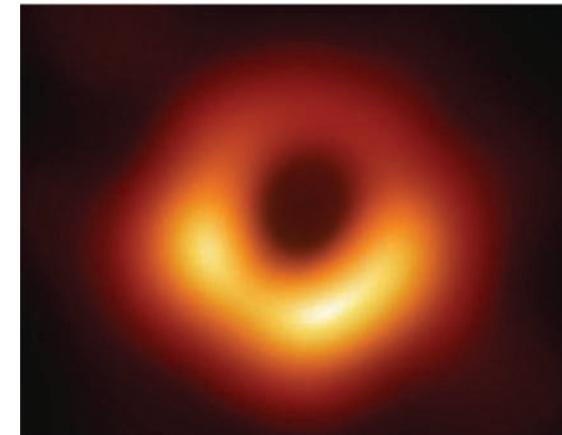


Kepler's supernova (SNR)

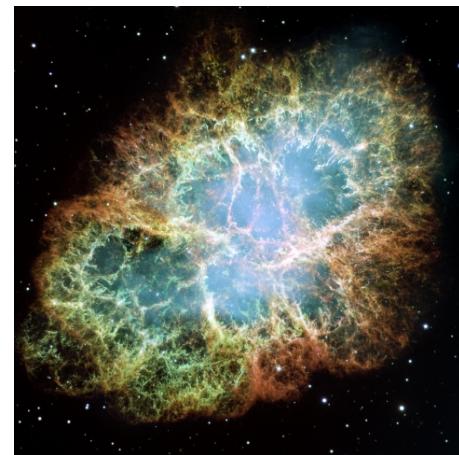
Turbulence is a **ubiquitous process** in high-energy astrophysics

Systems often comprise **relativistic, radiative, collisionless plasmas**

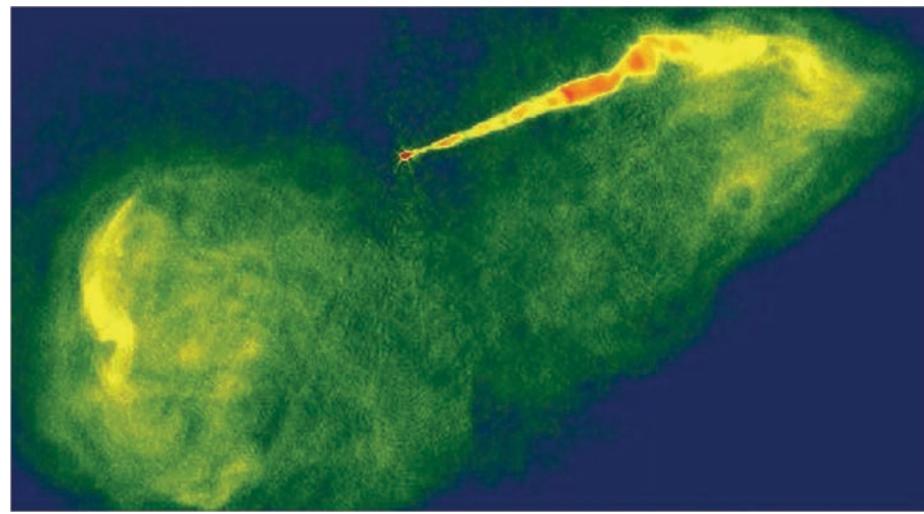
Small-scale turbulence important for understanding structure, spectra, etc.



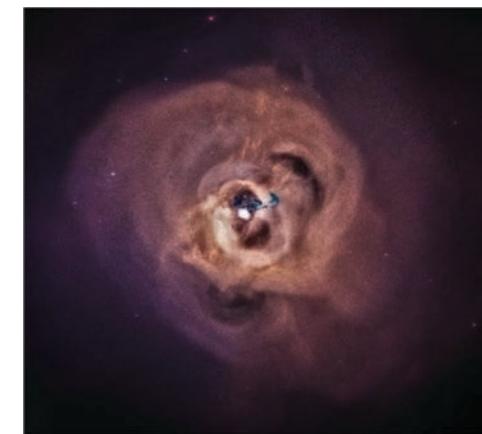
M87 (accretion flow)



Crab nebula (PWN)



M87 (AGN jet)



Perseus cluster (ICM)

Relativistic kinetic turbulence

Four motivations:

- Ubiquitous in high-energy astrophysics (AGN, GRB, PWN, XRB, etc.)
- Essentially unexplored regime of turbulence
- Ideal setting for studies of nonthermal particle energization
- Viable with first-principles particle-in-cell (PIC) simulations

Questions fall into two categories:

1. What are the statistical properties of the turbulence?
2. What are the kinetic properties of the particles?

Key questions about turbulent particle energization

- Is turbulent particle acceleration a viable and efficient source of nonthermal electrons or ions?
- Can collisionless dissipation of turbulence naturally establish a two-temperature plasma? What is the electron-ion heating ratio?
- How do results depend on plasma parameters? (e.g., magnetization, plasma beta, system size, temperature)
- What are mechanisms of heating and/or acceleration? (Fermi mechanism, gyroresonance, magnetic reconnection, shocks, etc.)
- How does radiative cooling influence the results?
- What are the observable radiative signatures from such a setting?

One motivation: hot accretion flows onto black holes (nonthermal radiative signatures, two-temperature plasma models)

Numerical simulations

- Externally driven turbulence with **3D PIC code Zeltron** (**Cerutti+ 2013**)
- Periodic cubic box (**no particle escape**)
- Initialize thermal plasma, apply large-scale driving (**TenBarge+ 2014**)
- Uniform background field $B_0 \sim \delta B_{\text{rms}}$
- **First consider relativistically hot pair plasma** (mathematically equivalent to ultra-relativistic electron-ion plasma)
- Two physical parameters (after fixing $T/m_e c^2 = 100$):
 - 1) Magnetization (ratio of magnetic energy to total particle energy):

$$\sigma \equiv \frac{B_{\text{rms}}^2}{4\pi n_0 \bar{\gamma} m_e c^2}$$

Alfvenic turbulence: $\frac{\delta v}{c} \sim \frac{v_A}{c} = \sqrt{\frac{\sigma}{\sigma + 4/3}}$

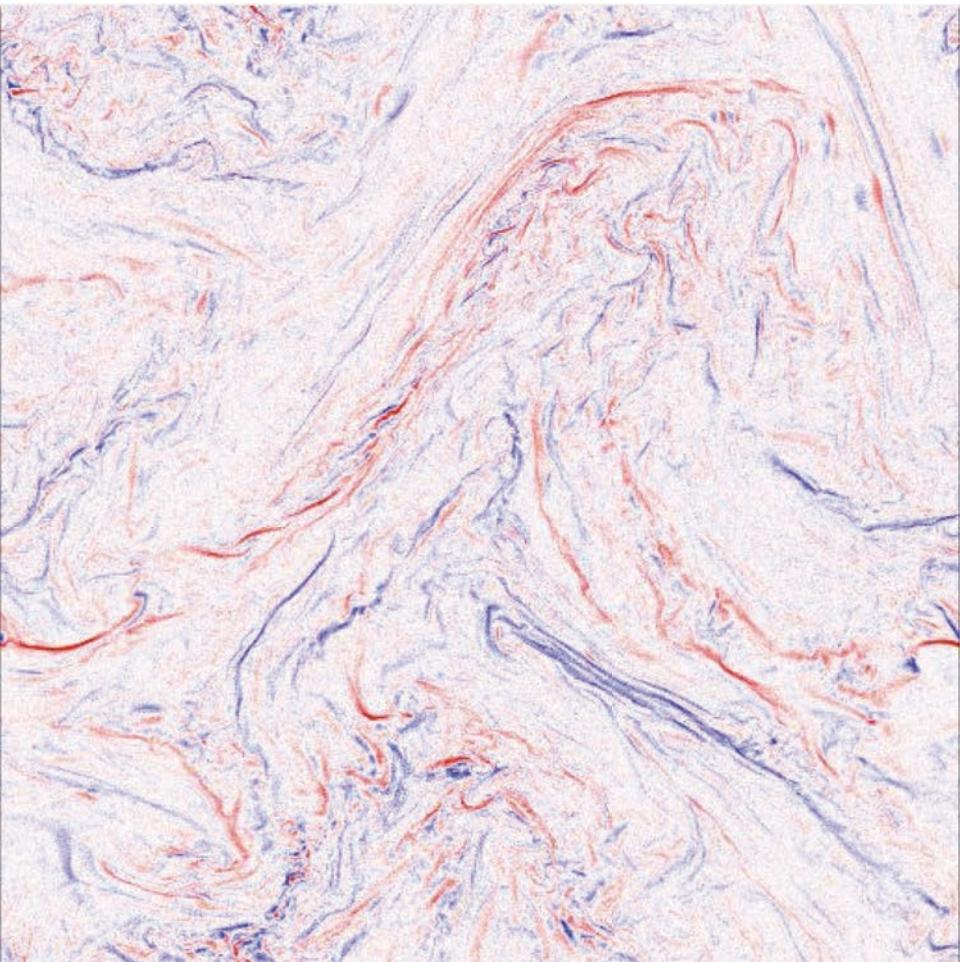
- 2) System size (ratio of driving scale to particle Larmor radius):

$$L/2\pi\rho_e \rightarrow 163$$

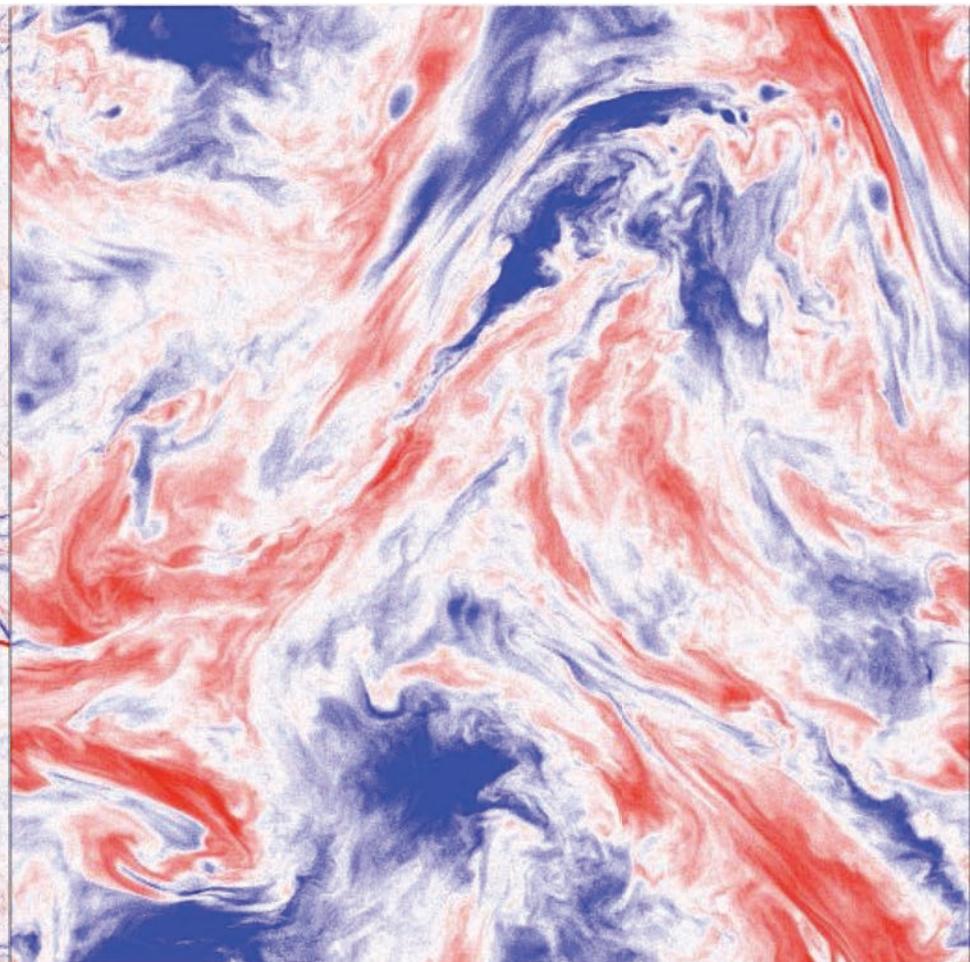
$$\rho_e = \frac{\bar{\gamma} m_e c^2}{e B_{\text{rms}}}$$

3D pair plasma turbulence - fixed-time fly-through

J_z

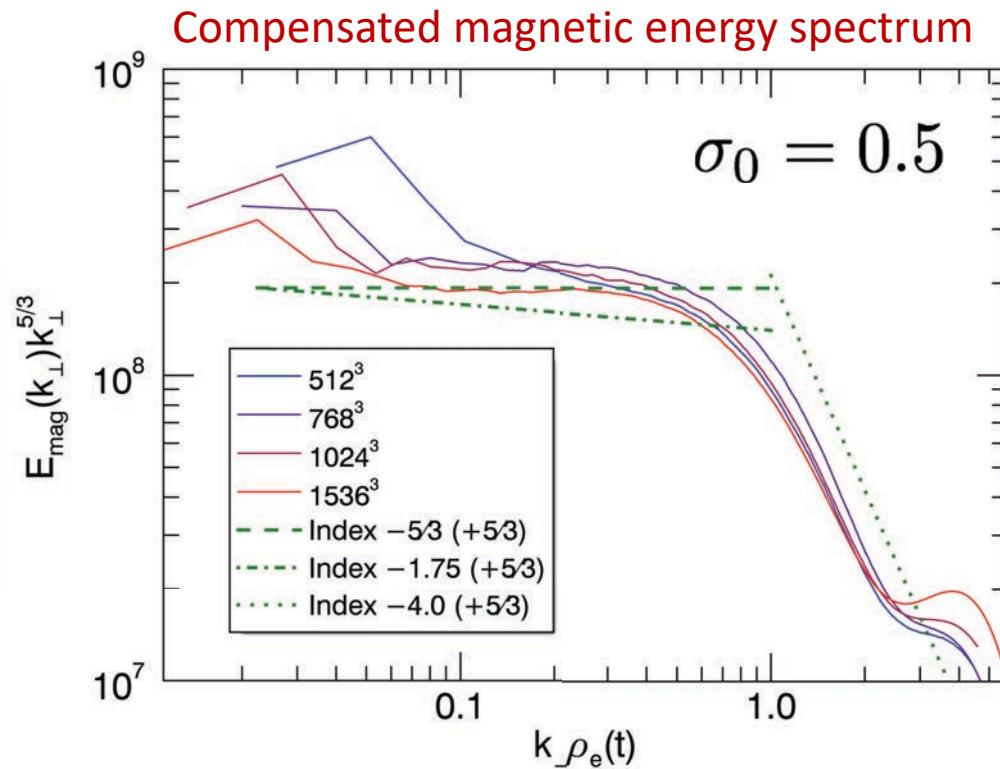
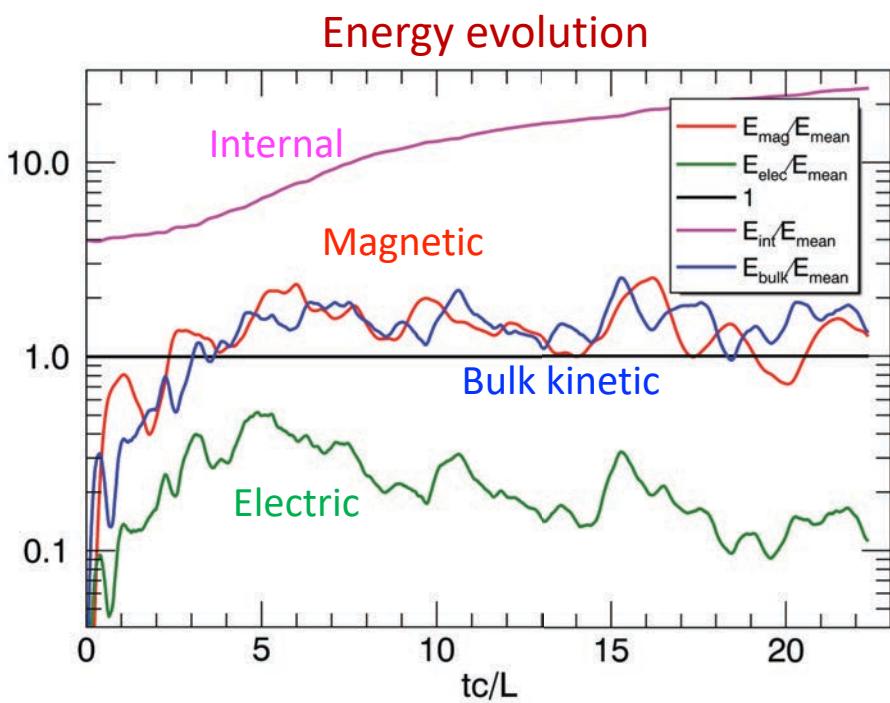


δn



$$\sigma_0 = 0.5$$

Turbulence statistics



Inertial range essentially converged for 768^3 and larger ($L/2\pi\rho_{e0} \gtrsim 80$)

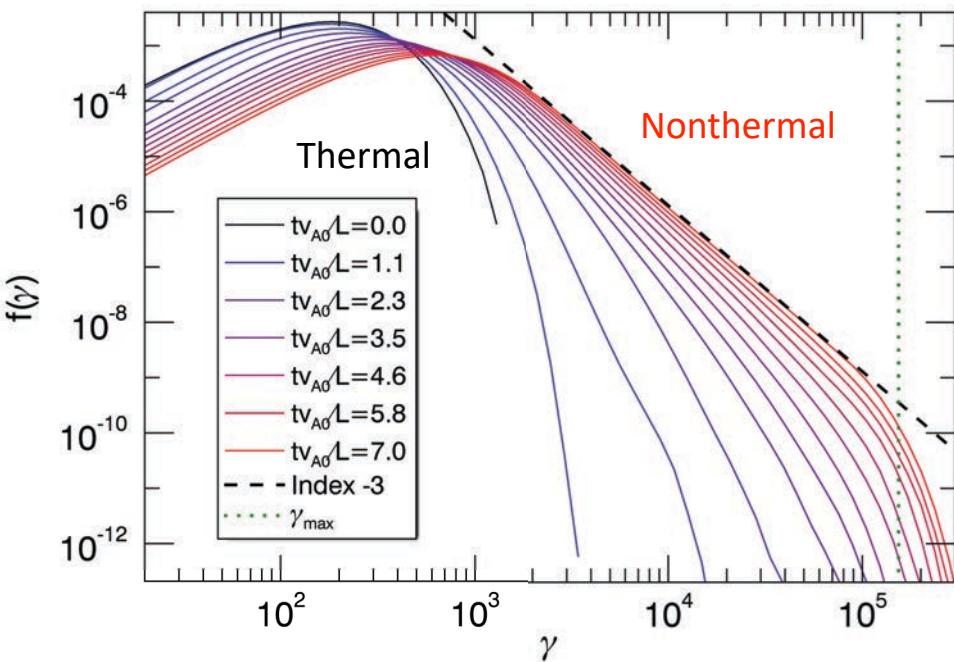
MHD range: $-5/3$ index (Goldreich & Sridhar 1995, Thompson & Blaes 1998)

Kinetic range: -4 index or steeper (kinetic cascade? Schekochihin+ 2009)

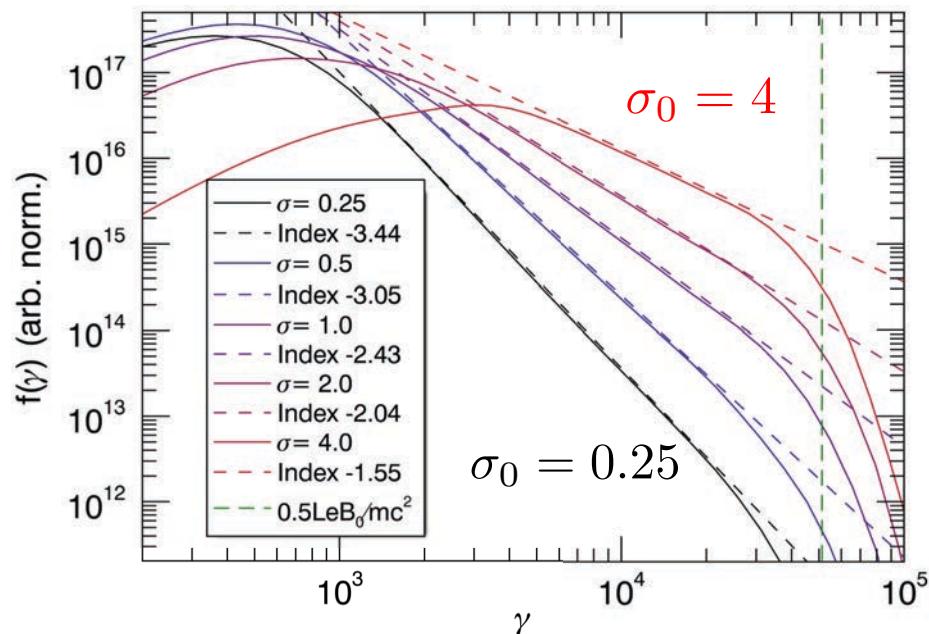
Complete turbulence statistics: VZ, Uzdensky, Werner & Begelman MNRAS 2018

Nonthermal particle acceleration

Energy distribution evolution (1536^3)



Magnetization scan

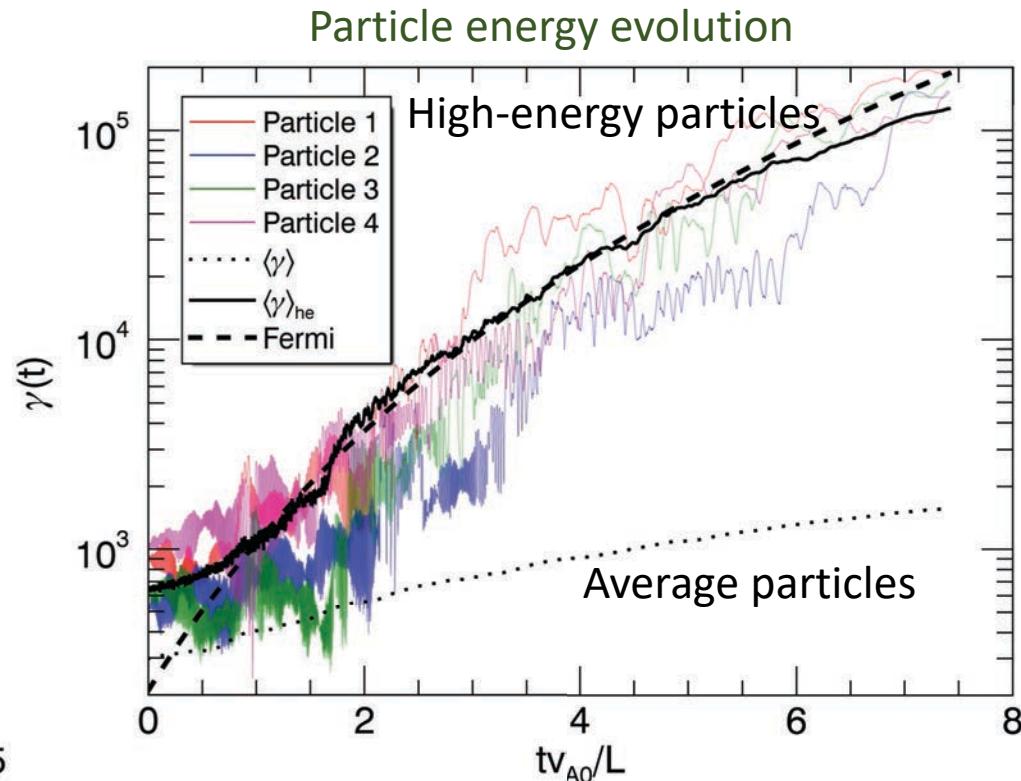
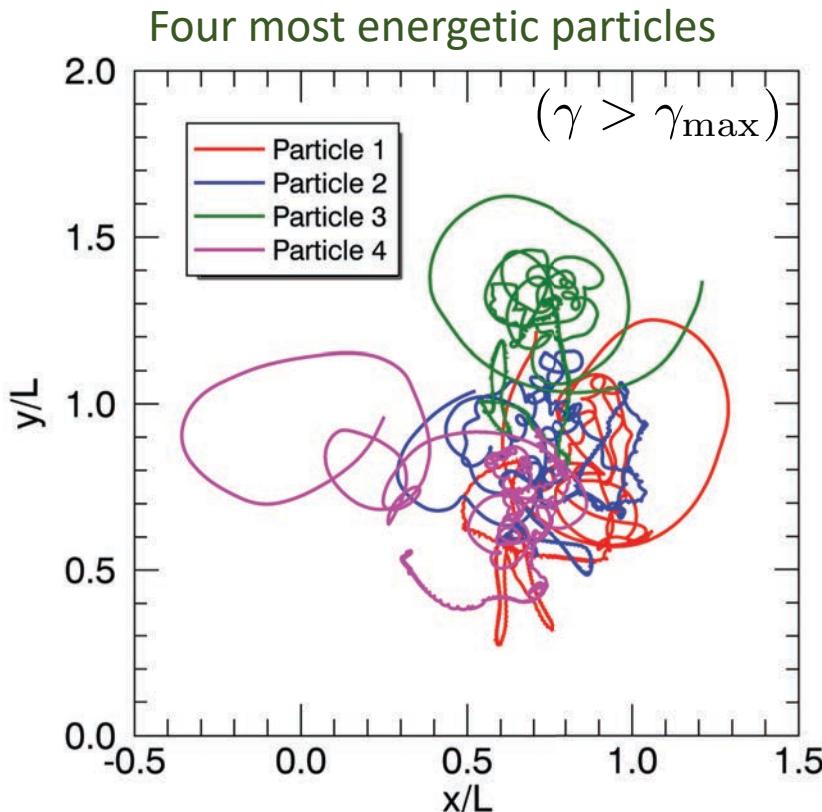


Power law tail: $f(\gamma) \sim \gamma^{-\alpha}$ ($\gamma = E/m_e c^2$)

Spans from mean energy $\langle \gamma \rangle$ to system-size limited energy $\gamma_{\max} = LeB/2mc^2$

Hardens with increasing magnetization, converges with system size (but time-dependent)

Acceleration process in detail



Approximate exponential energy gain, consistent with second-order Fermi acceleration or gyroresonant scattering (Fermi 1949; Schlickeiser 1989, Miller et al. 1990, Chandran 2000):

$$\frac{d\gamma}{dt} \sim \frac{\gamma}{\tau_{\text{acc}}(t)} \implies \gamma \sim \gamma_i \exp(t/\tau_{\text{acc}})$$

$$\tau_{\text{acc}} \propto \frac{Lc}{v_A^2(t)}$$

(Alfvenic scattering)
10

Particle acceleration is diffusive

Statistical evolution of tracked particles is described by Fokker-Planck equation:

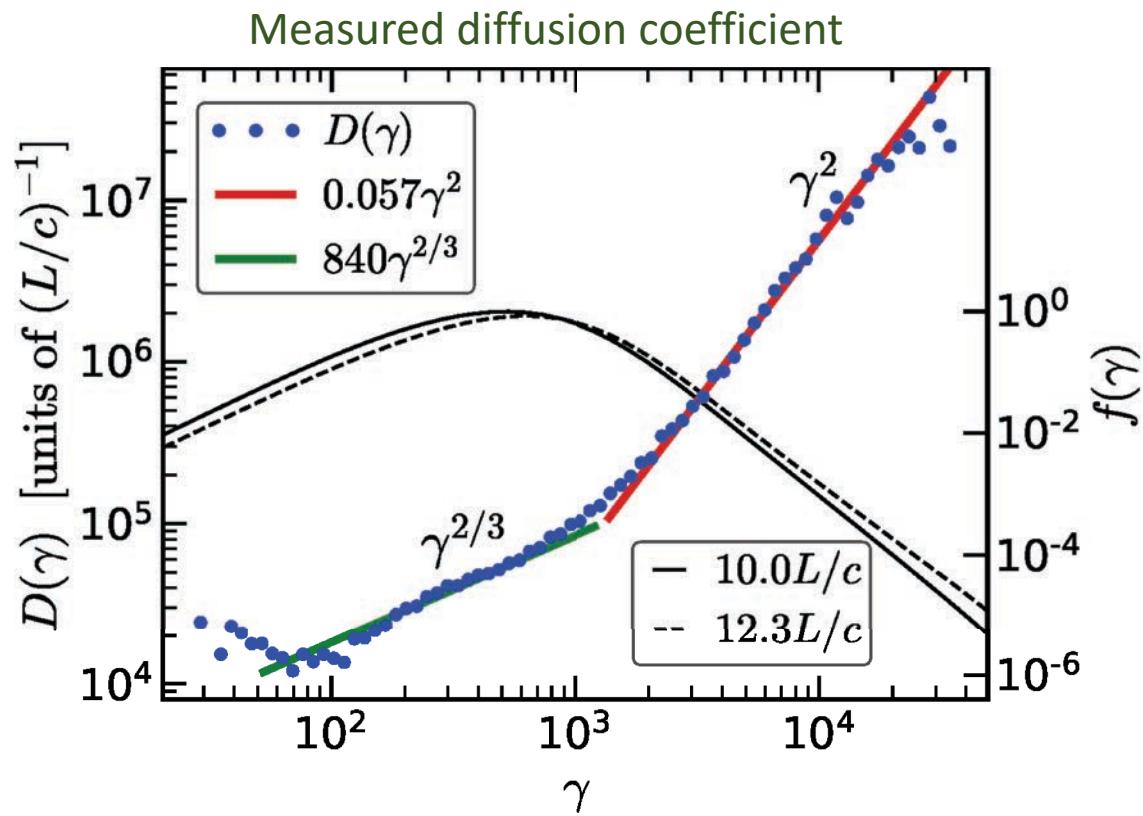
$$\partial_t f = \partial_\gamma(D\partial_\gamma f) - \partial_\gamma(Af) \quad (\text{advection-diffusion in energy space})$$

2nd order Fermi acceleration / gyroresonance by Alfvén waves (e.g., Blandford & Eichler 1987):

$$D(\gamma) \sim \frac{u_A^2}{3c\lambda_{\text{mfp}}} \gamma^2$$

Scattering by large-scale waves is sufficient to explain results:

$$\lambda_{\text{mfp}} \sim L$$



Wong, VZ, Uzdensky, Werner & Begelman, submitted [arXiv:1901.03439](https://arxiv.org/abs/1901.03439)

Electron-ion simulations

- Focus on temperatures such that **ions (protons) are sub-relativistic** and **electrons are ultra-relativistic** (“semirelativistic” case):

$$m_e c^2 < T < m_i c^2 \quad (0.5 \text{ MeV} \lesssim T \lesssim 1 \text{ GeV})$$

- Real mass ratio**, but electrons have large relativistic mass, reducing scale separation relative to ions
- Big parameter scan** with three physical parameters: ($T_{i0}/T_{e0} = 1$ fixed)

1) Temperature relative to ion rest mass: $\theta_i = T/m_i c^2$

Relativistic (pair plasma) regime: $\theta_i \gg 1 : \rho_i \sim \rho_e$

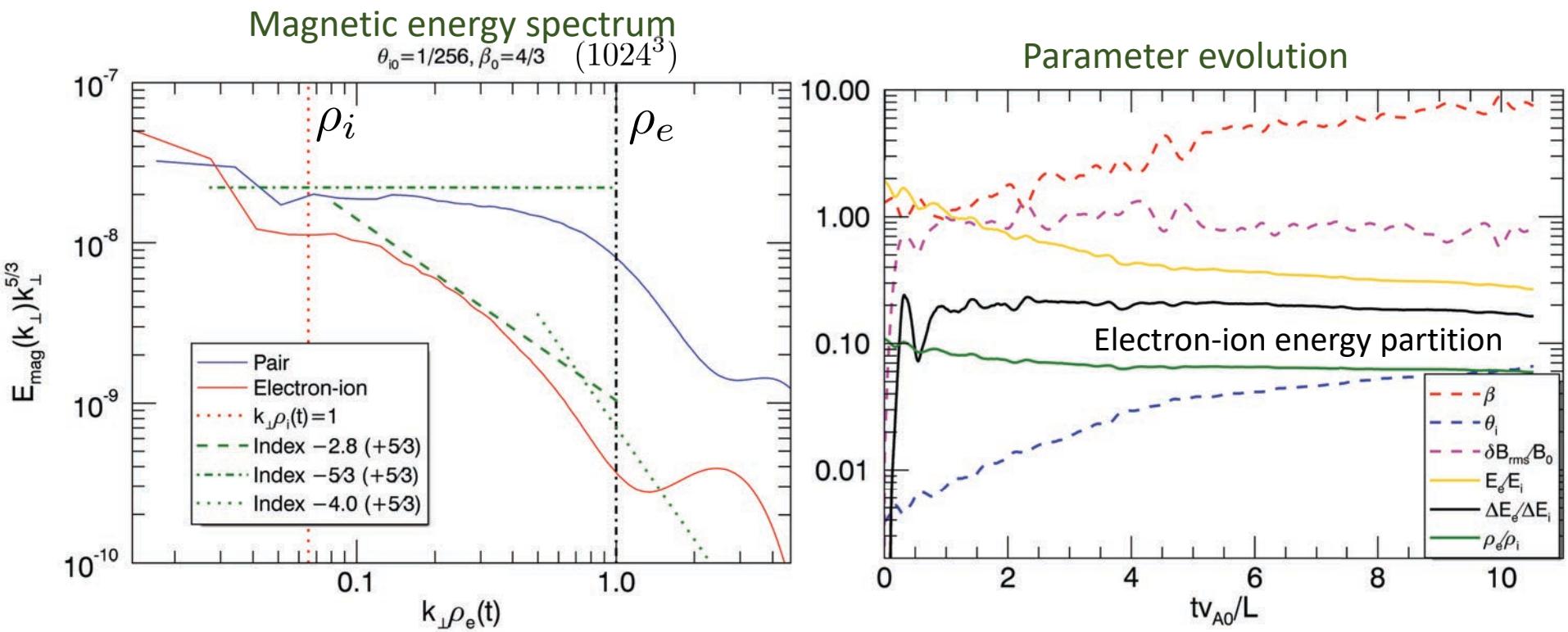
Semirelativistic regime: $10^{-3} \ll \theta_i \ll 1 : \rho_i \sim \theta_i^{-1/2} \rho_e$

2) Plasma beta: $\beta = \frac{16\pi n_0 T}{B_{\text{rms}}^2}$ (thermal pressure/magnetic pressure)

3) System size relative to ion Larmor radius: $L/2\pi\rho_i \quad \rho_s = \frac{\gamma_s m_s v_s c}{e B_{\text{rms}}}$

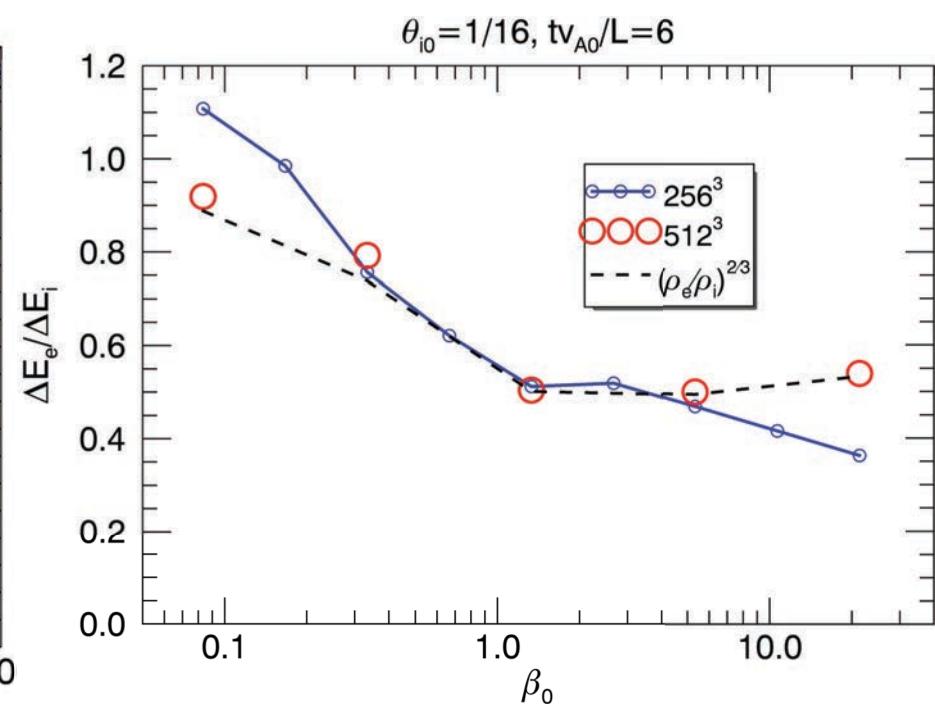
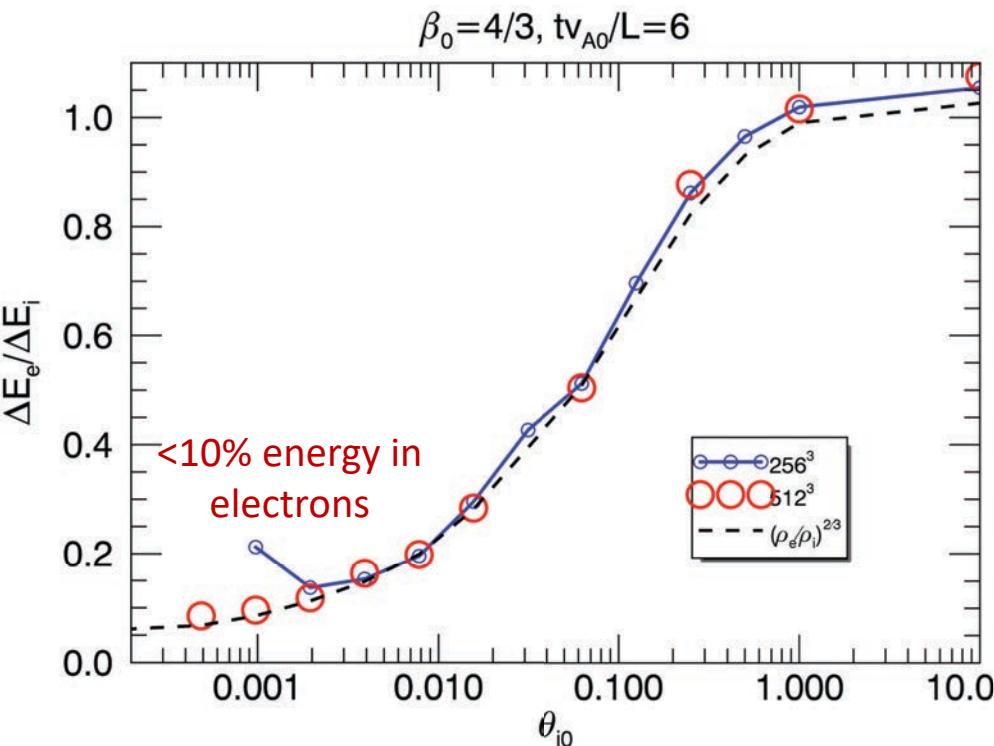
Electron-ion turbulence statistics

Spectrum steepens below ion Larmor radius: $E_{\text{mag}} \sim k_{\perp}^{-2.8}$?
(similar to non-relativistic case, e.g., solar wind kinetic Alfvén wave cascade)



Electron-ion energy partition

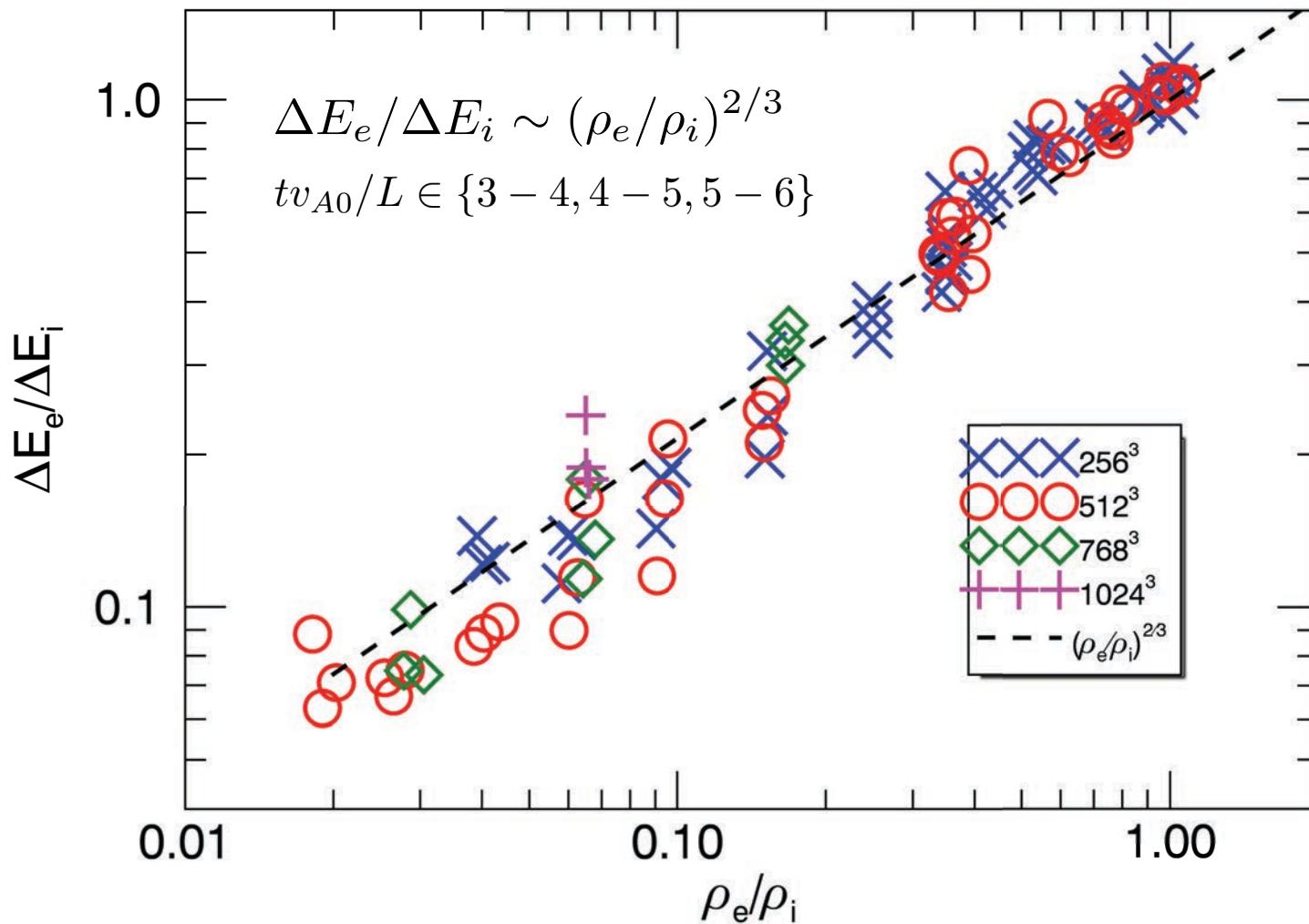
Preferential ion heating (up to 90% energy) in most of parameter space



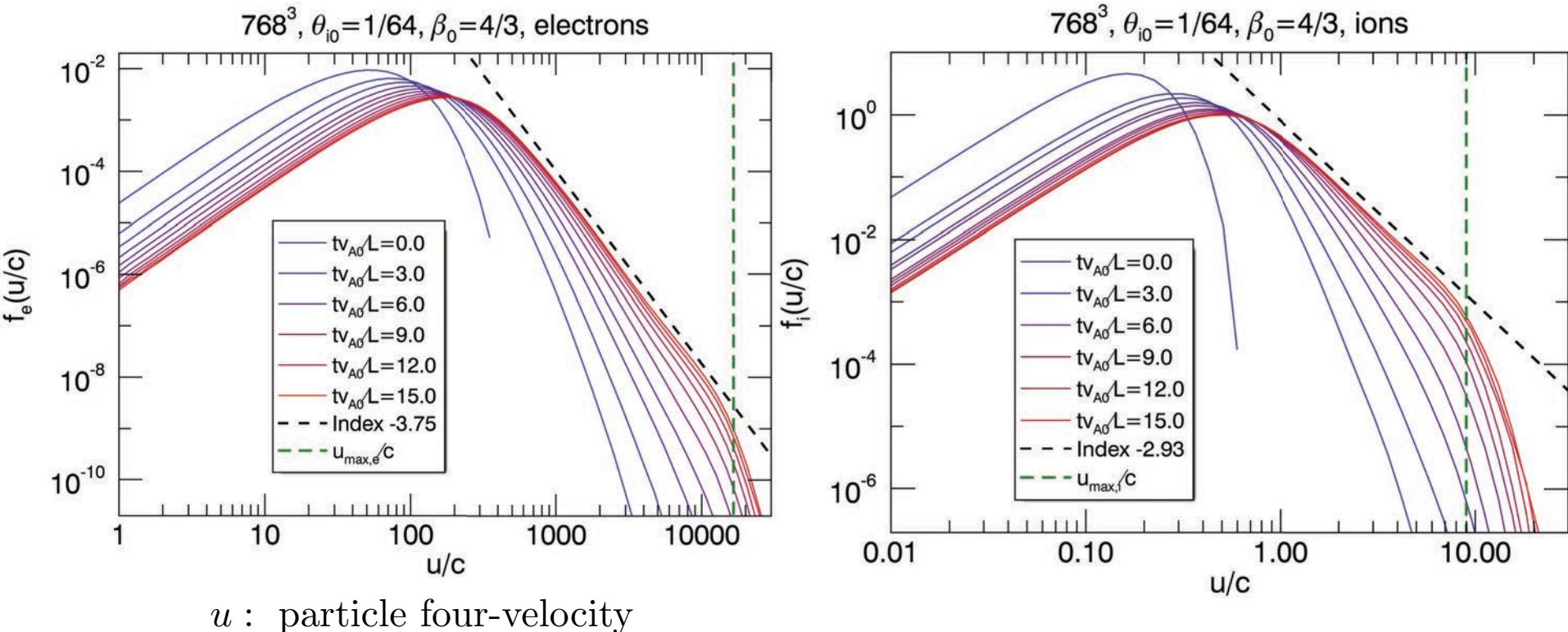
Broad parameter scan yields empirical fitting formula: $\Delta E_e / \Delta E_i \sim (\rho_e / \rho_i)^{2/3}$

VZ, Uzdensky, Werner & Begelman PRL 2019

“Spooky” empirical formula



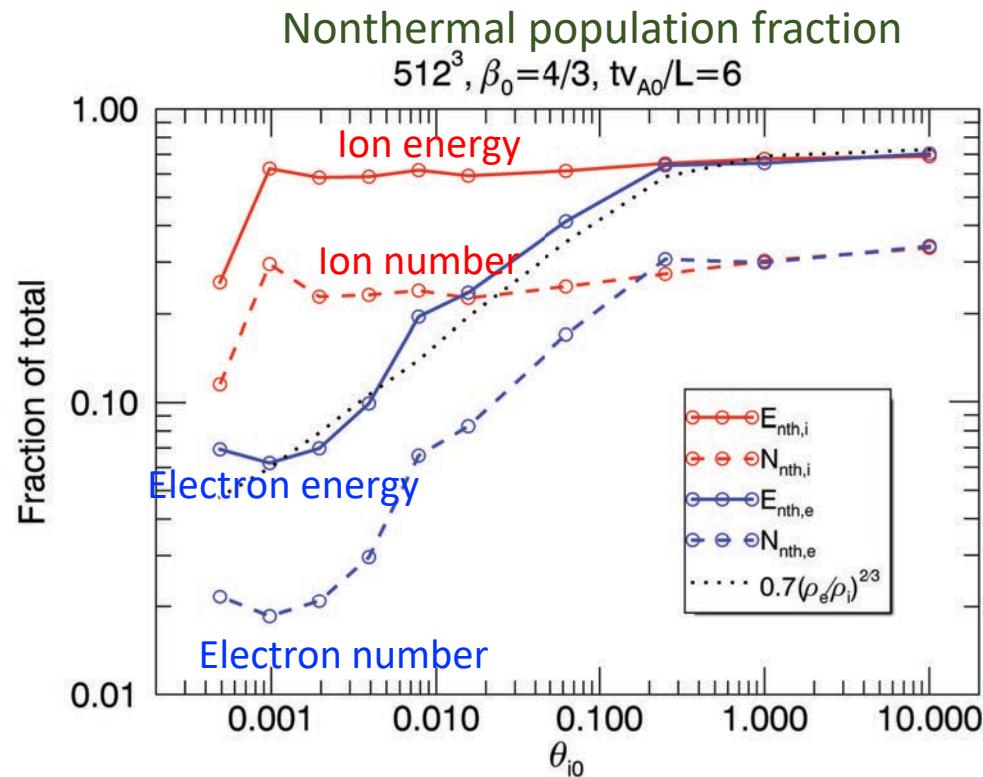
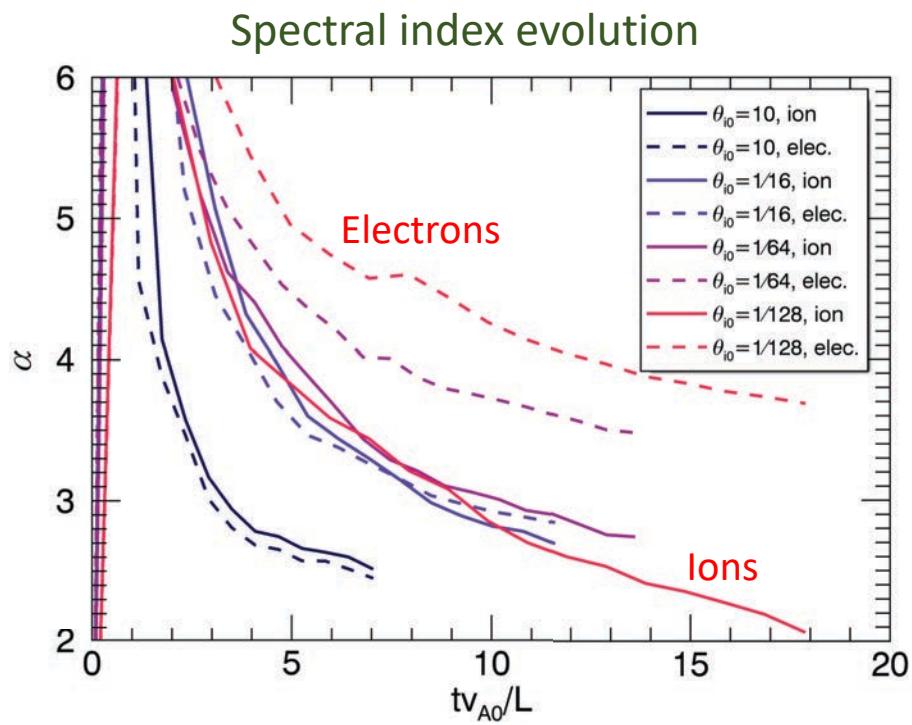
Nonthermal particle acceleration



Particle acceleration observed for both species

Implications for [cosmic ray acceleration](#), radiative signatures

Nonthermal electron and ion statistics



- Ions always accelerated efficiently
- Electrons less efficiently accelerated at low temperatures
- Fraction of nonthermal particles follows same trend as energy partition

Extreme two-temperature plasmas

- What further tests can be made on the empirical formula?

$$\Delta E_e / \Delta E_i \sim (\rho_e / \rho_i)^\alpha \quad \alpha \approx 2/3$$

- One option: vary initial temperature ratio, T_{i0}/T_{e0}
- Compare to solution as function of electron temperature T_e/T_{e0}
- Semi-relativistic regime: $E_e = 3T_e, E_i = (3/2)T_i, \rho_i \propto (m_i T_i)^{1/2}, \rho_e \propto T_e$

$$\frac{dT_i}{dT_e} = 2 \left(\frac{\rho_i}{\rho_e} \right)^\alpha \sim 2 \left(\frac{m_i T_i}{3T_e^2} \right)^{\alpha/2}$$

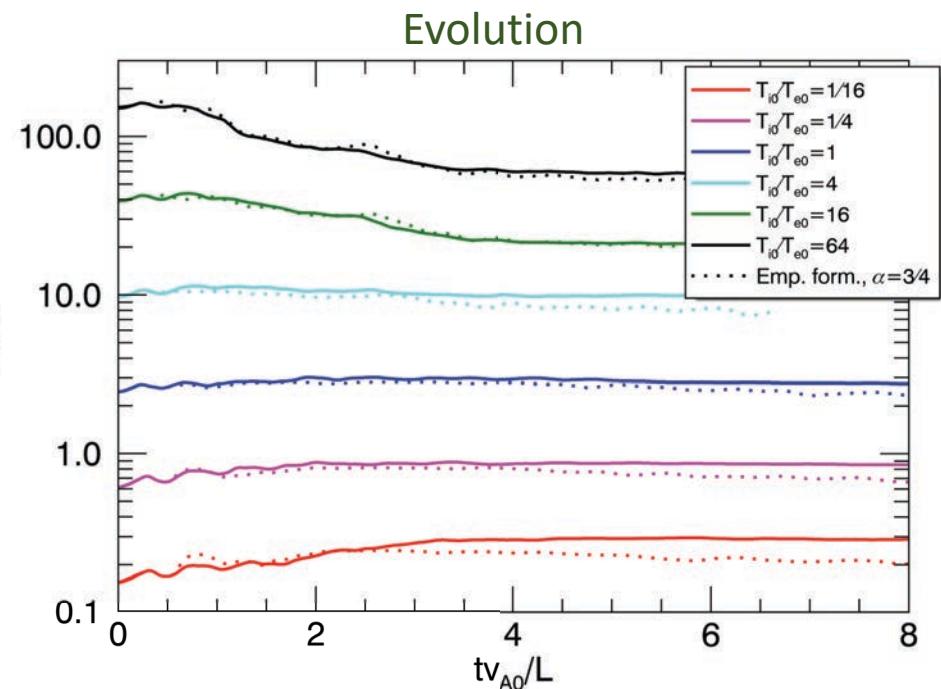
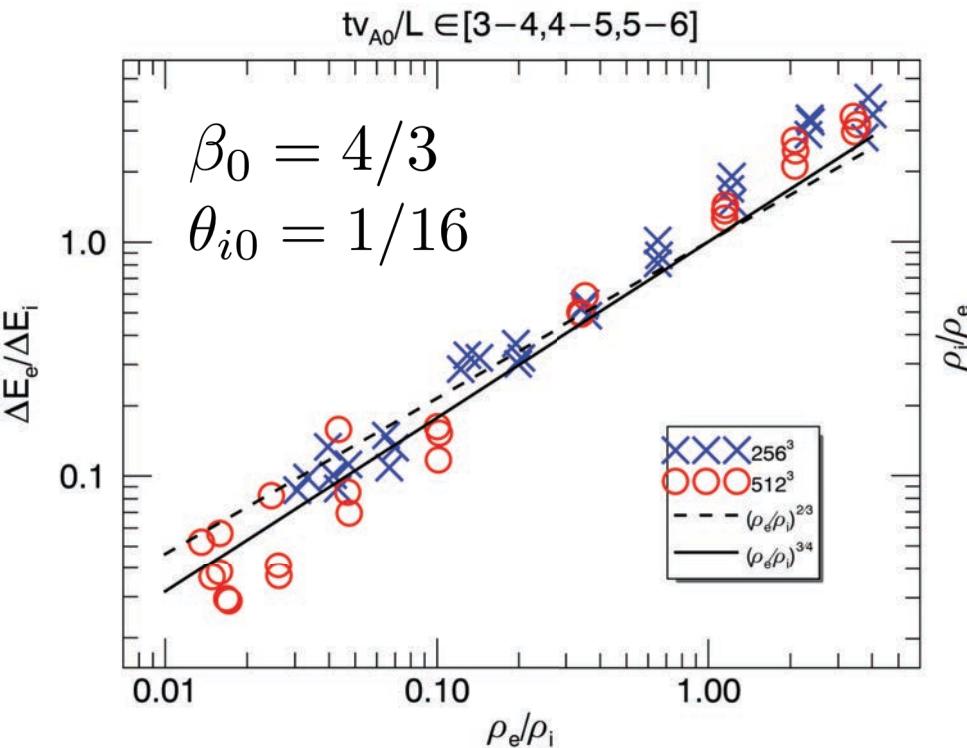
$$\implies \boxed{\frac{T_i}{T_e} = \frac{T_{i0}}{T_{e0}} \frac{T_{e0}}{T_e} \left[\frac{2 - \alpha}{(1 - \alpha)(3\theta_{i0})^{\alpha/2}} \left(\frac{T_{e0}}{T_{i0}} \right)^{1-\alpha} \left(\frac{T_e^{1-\alpha}}{T_{e0}^{1-\alpha}} - 1 \right) + 1 \right]^{1/(1-\alpha/2)}}$$

- Ultra-relativistic (pair plasma) regime: $E_s = 3T_s, \rho_s \propto T_s$

$$\frac{dT_i}{dT_e} = \left(\frac{\rho_i}{\rho_e} \right)^\alpha \sim \left(\frac{T_i}{T_e} \right)^\alpha$$

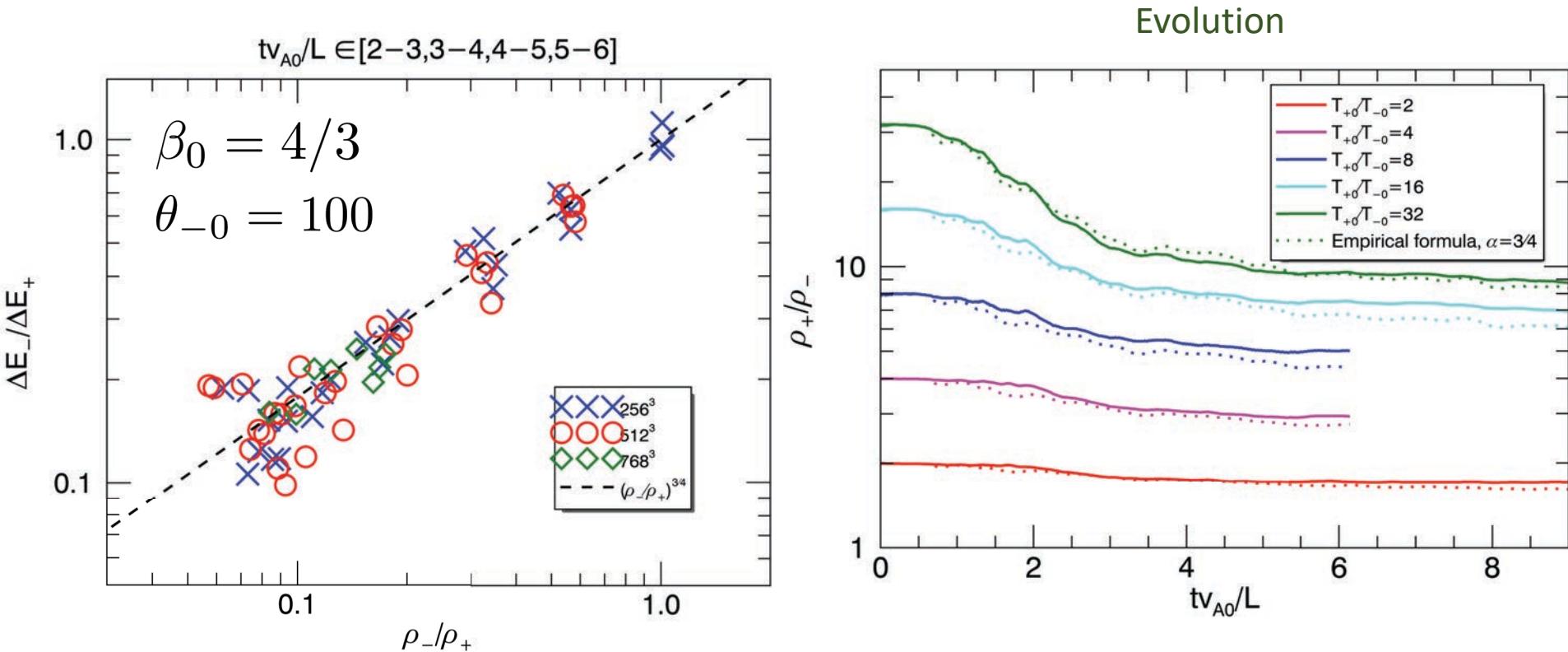
$$\implies \boxed{\frac{T_i}{T_e} = \left[\frac{(T_{i0}/T_{e0})^{1-\alpha} - 1}{(T_e/T_{e0})^{1-\alpha}} + 1 \right]^{1/(1-\alpha)}} \rightarrow 1 \quad \text{at late times}$$

Initial temperature ratio scan



- Initialize two-temperature state: $1/16 < T_{i0}/T_{e0} < 128$
- Empirical formula (with index 3/4) describes scaling and evolution, including cases where electrons are hotter than protons

Two-temperature “pair plasma” scan



- Initialize two-temperature ultra-relativistic pair plasma: $1 < T_{+0}/T_{-0} < 128$
- Empirical formula describes results, despite inherent particle symmetry
- Further numerical tests, diagnostics, theoretical illumination in progress

Radiative turbulence

- In many high-energy astrophysical systems, electrons/positrons are radiatively cooled (synchrotron, inverse Compton, etc.)
- In principle, rigorous statistical steady state may be obtained by balancing energy injection (from driving) with radiative cooling
- However, for collisionless electron-ion plasma, steady state requires an efficient thermal coupling mechanism between electrons and ions...

Key questions:

1. Do electrons and/or ions attain a steady state temperature?
2. Does radiative cooling influence nonthermal particle distributions?
3. What are observable radiative signatures? (spectra, beams)

Radiation implementation

- Implement external inverse Compton (IC) cooling by adding radiation reaction force to Lorentz force to relativistic electrons/positrons:

$$\mathbf{F}_{\text{IC}} = -\frac{4}{3}\sigma_T U_{\text{ph}} \gamma^2 \frac{\mathbf{v}}{c}$$

σ_T is Thomson cross section, U_{ph} is (external) photon energy density

- Assume optically thin medium (radiation escapes box)
- Radiative steady state if injected energy balanced by radiated energy:

$$\dot{\mathcal{E}}_{\text{inj}} \sim \frac{\mathcal{E}_{\text{turb}}}{\tau_{nl}} \sim \eta_{\text{inj}} \frac{B_0^2}{8\pi} \frac{v_A}{L}$$

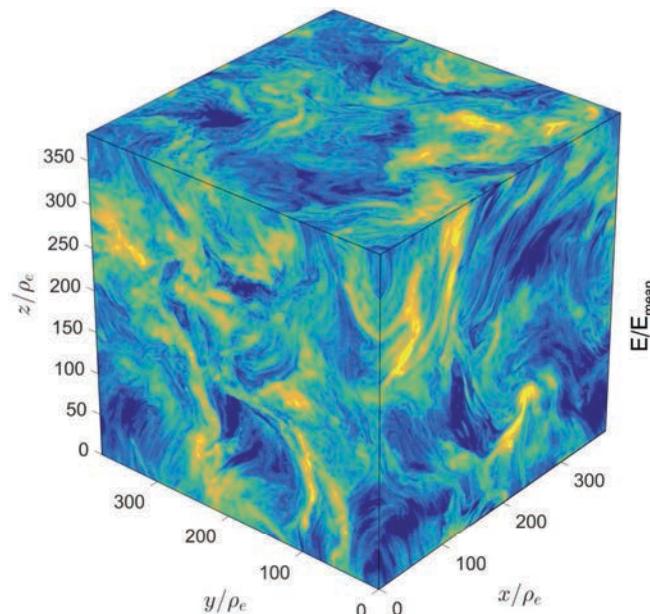
$$\dot{\mathcal{E}}_{\text{rad}} = \frac{4}{3} n_0 \sigma_T c U_{\text{ph}} \overline{\gamma_e^2}$$

$$\dot{\mathcal{E}}_{\text{rad}} \sim \dot{\mathcal{E}}_{\text{inj}} \implies \boxed{\theta_{ss} = \frac{\eta_{\text{inj}}}{16} \frac{m_e c^2}{\sigma_T U_{\text{ph}} L} \sigma \frac{v_A}{c}}$$

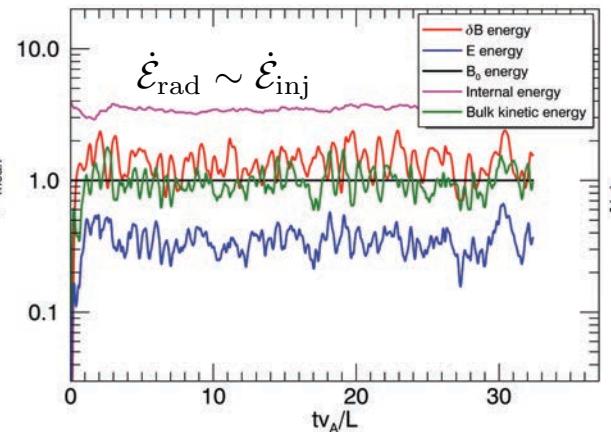
Expected electron
steady-state
temperature

- Note: empirical formula predicts no steady state for electrons or ions

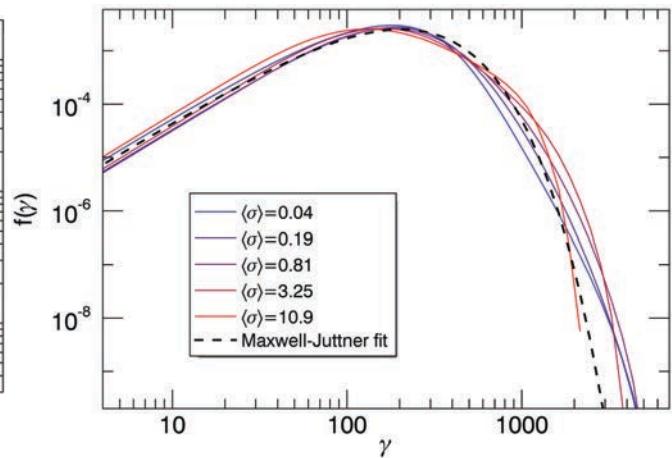
Brief note: radiative pair plasma



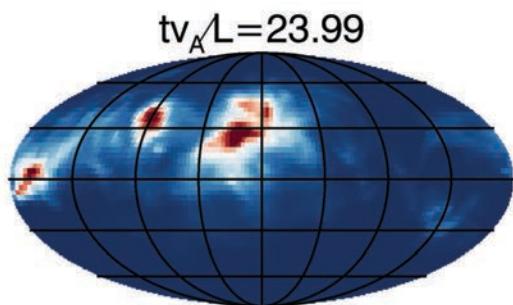
Statistical steady state



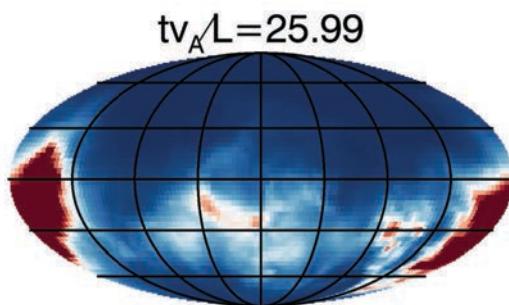
Thermal particle distributions



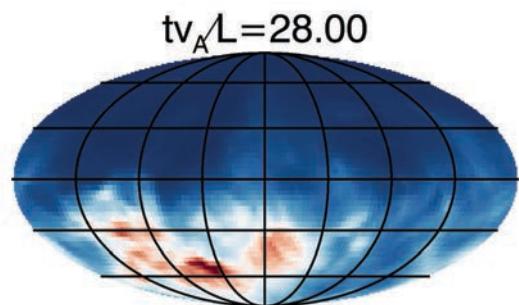
High-energy momentum anisotropy (kinetic beams)



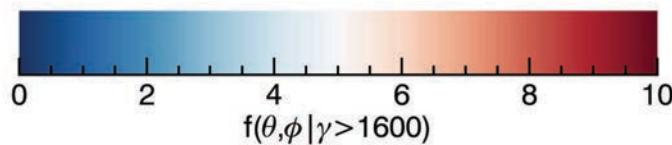
$tv_A L = 23.99$



$tv_A L = 25.99$

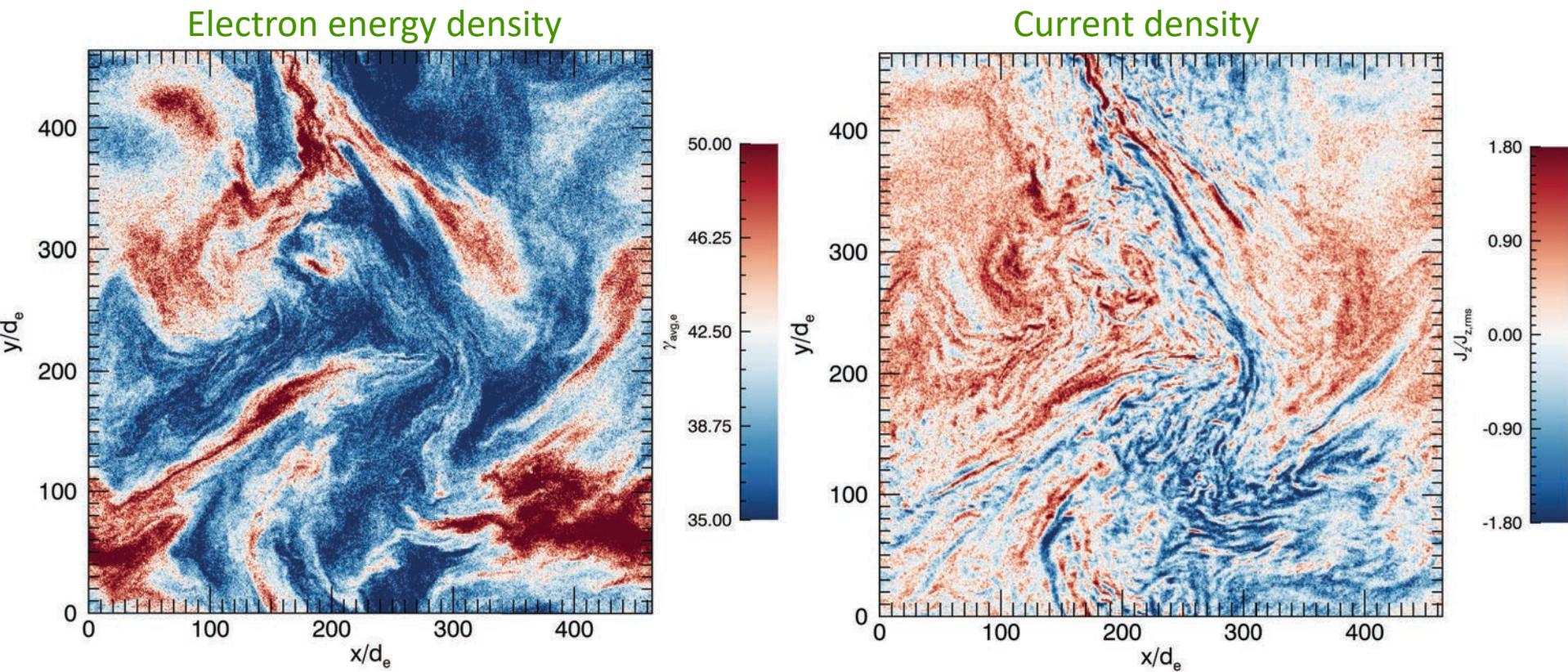


$tv_A L = 28.00$



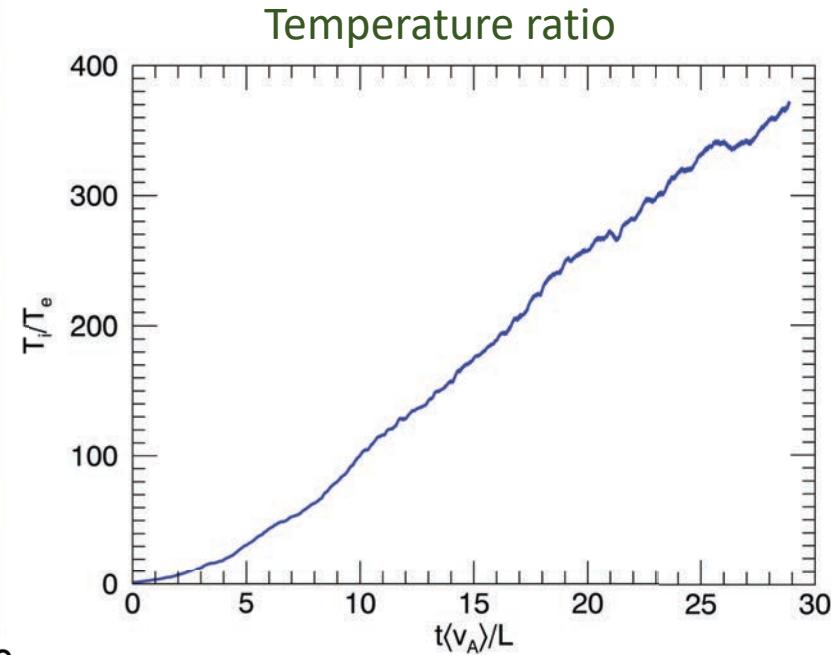
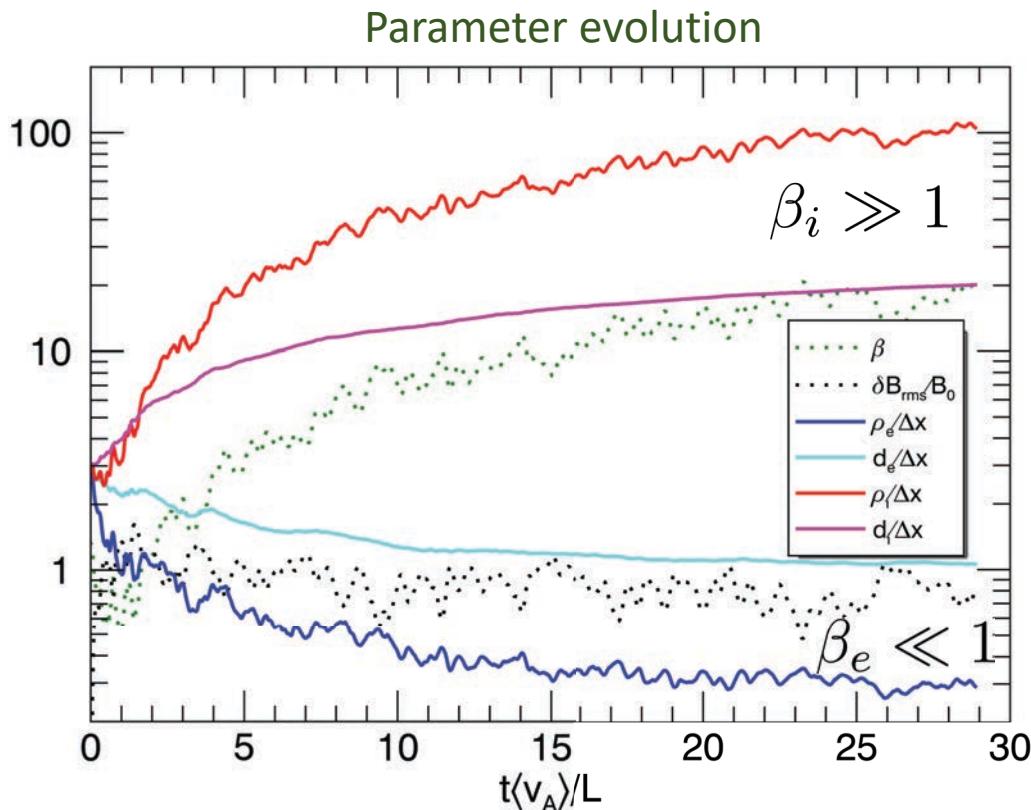
Radiative electron-ion setup

- Fiducial simulation: $\beta_0 = 4/3$ $L/2\pi\rho_{i0} = 27.2$ $\theta_{i0} = 100$
- Relativistically hot (i.e., pair plasma with non-radiative positrons)
- Photon energy density U_{ph} chosen near pair plasma steady state



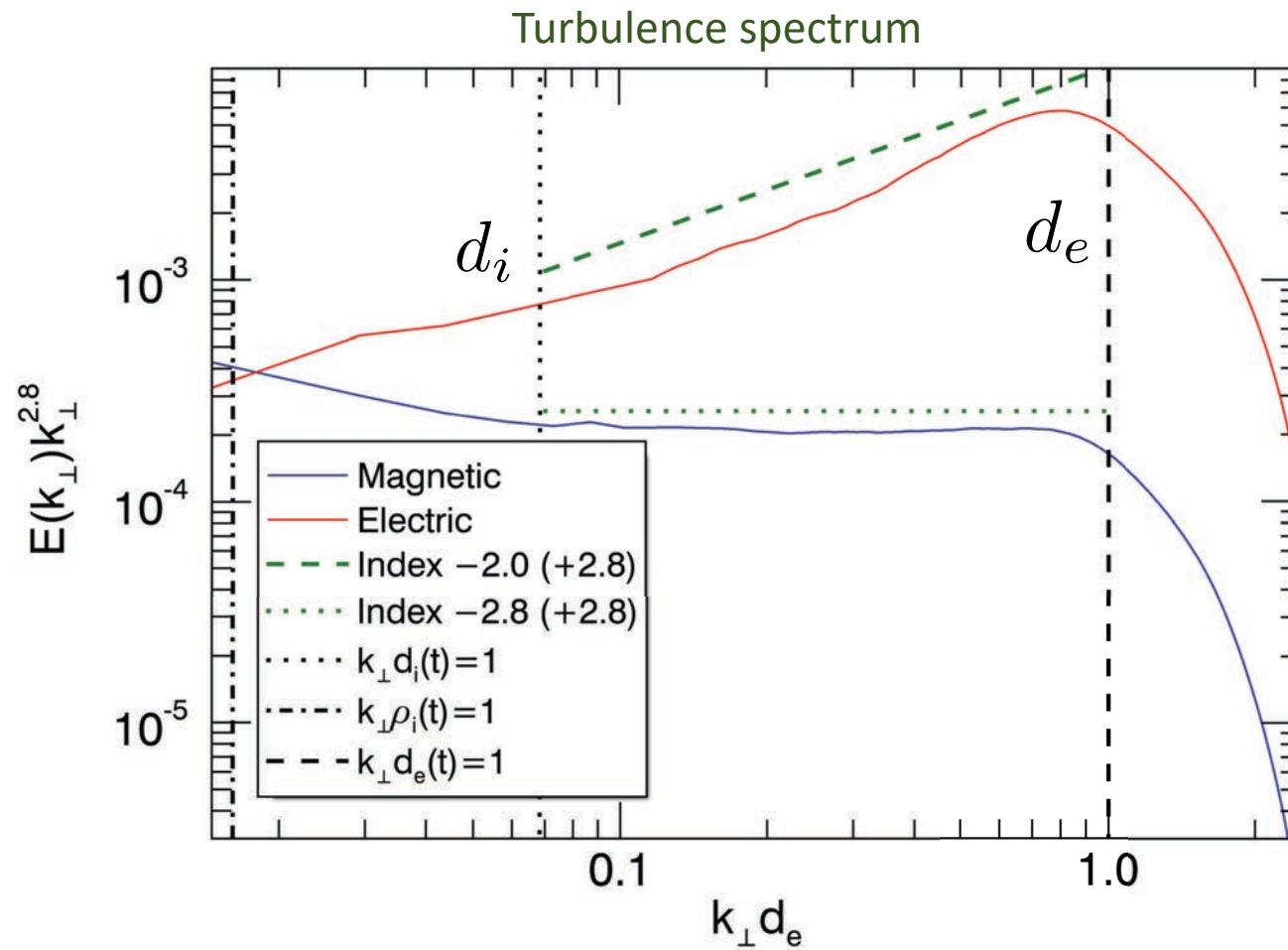
Unbounded growth of temperature ratio

- Electrons approach steady-state temperature, ions do not (**no coupling!**)
- Temperature ratio builds up to $T_i/T_e \gtrsim 300$
- 8% of injected energy is lost to radiation



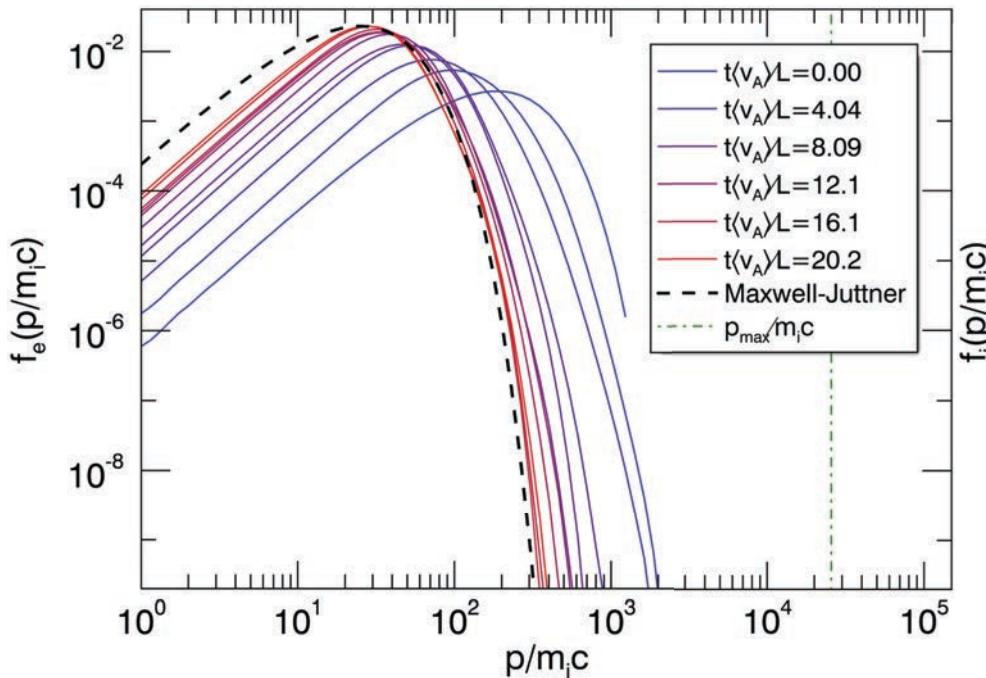
Turbulence statistics

Clean -2.8 power law in kinetic range! (kinetic Alfvén wave cascade?)

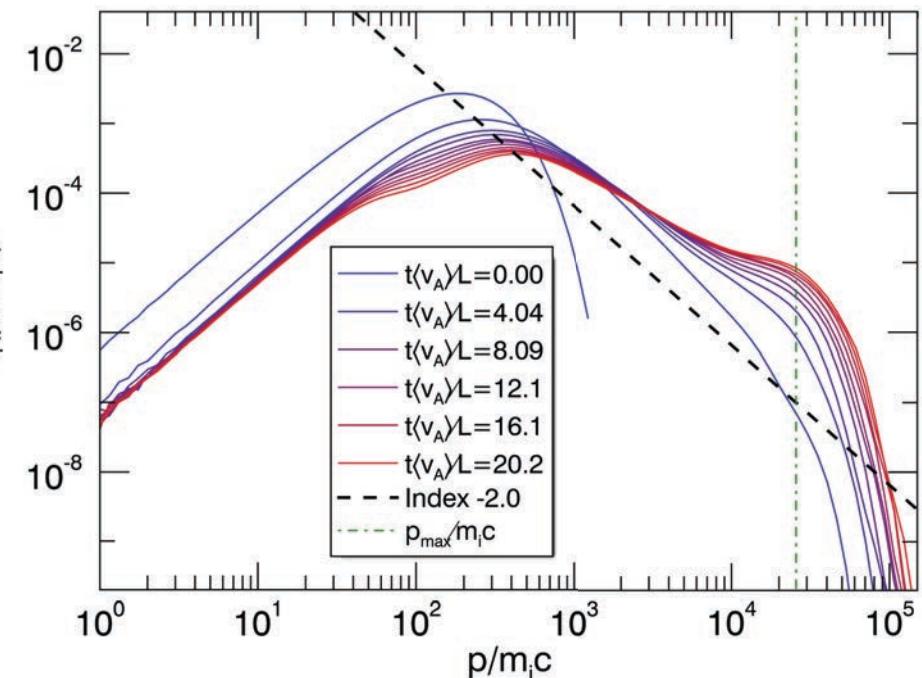


Electron-ion distributions

(Thermal) electron distribution



(Nonthermal) ion distribution



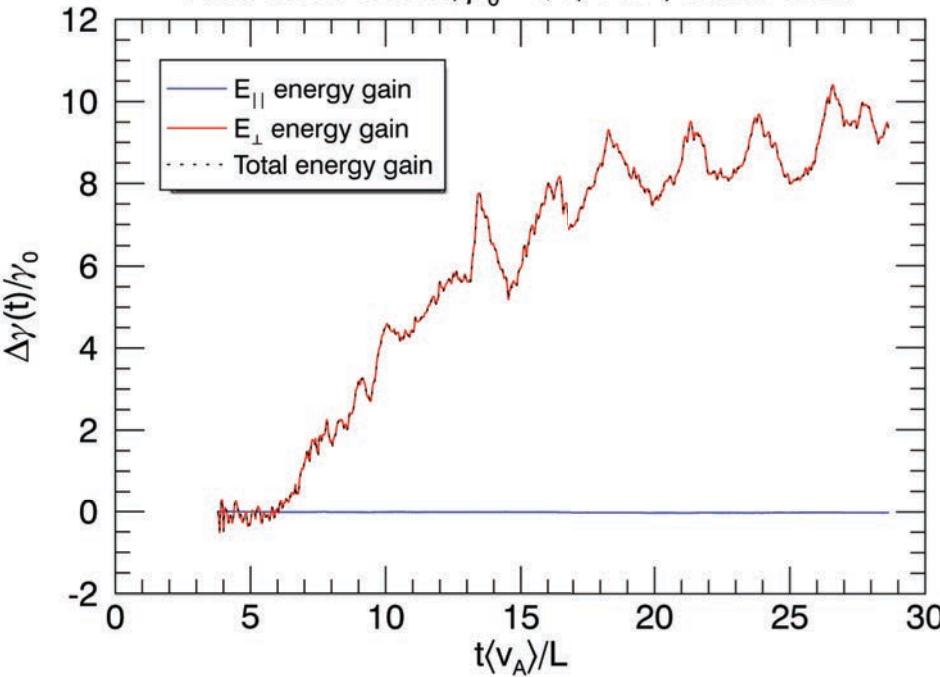
Electrons are thermalized by radiative cooling

Ions are efficiently accelerated to hard power law, up to system size limit

Tracked particle statistics

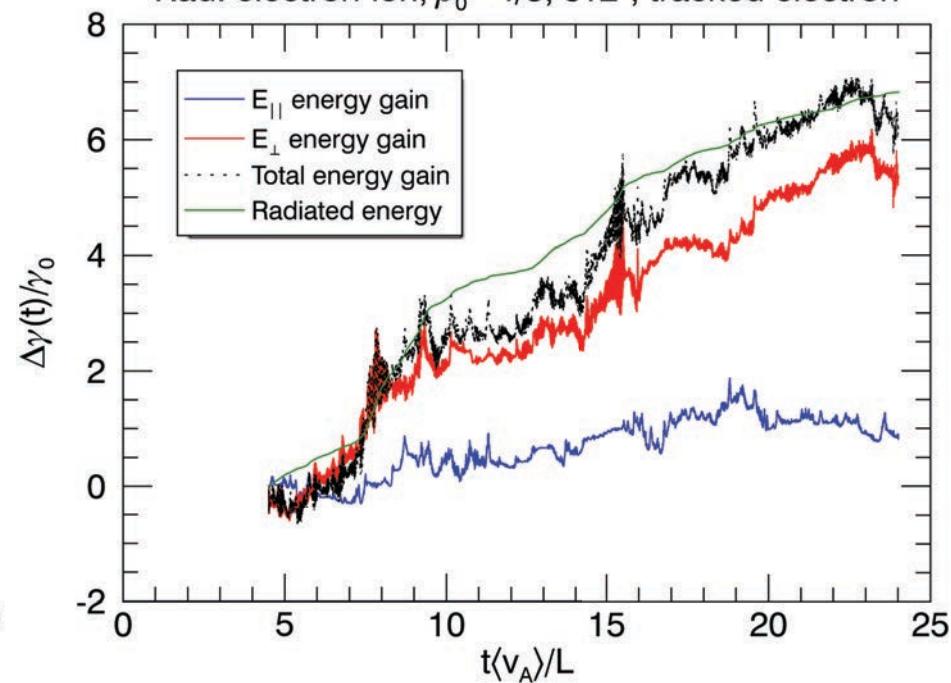
Random tracked ion

Rad. electron-ion, $\beta_0=4/3$, 512^3 , tracked ion



Random tracked electron

Rad. electron-ion, $\beta_0=4/3$, 512^3 , tracked electron

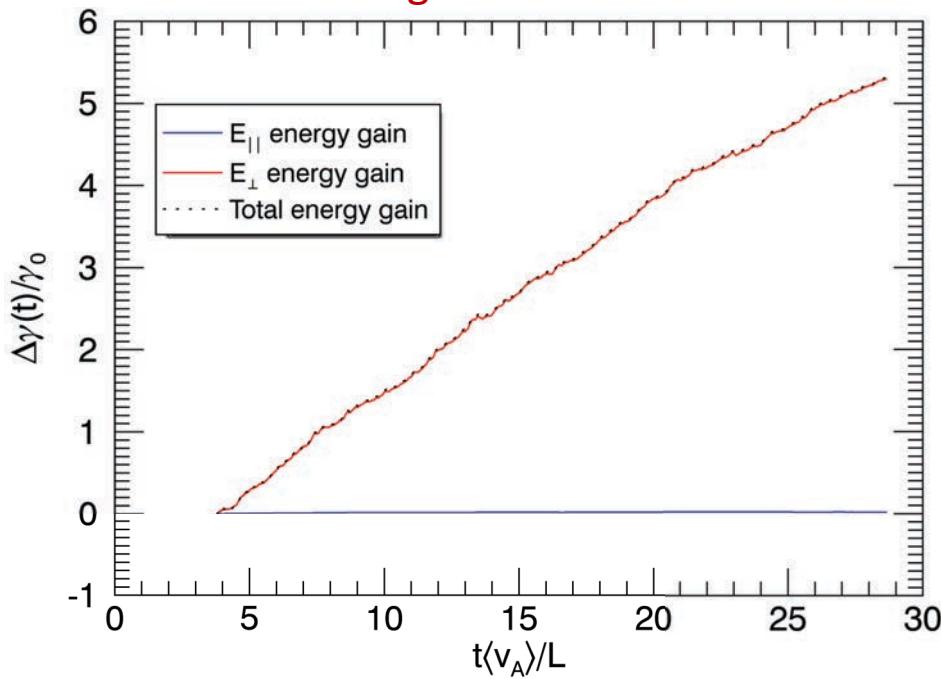


>99% of energy gain for ions is from perpendicular electric fields
Electrons gain from mixture of parallel and perpendicular fields

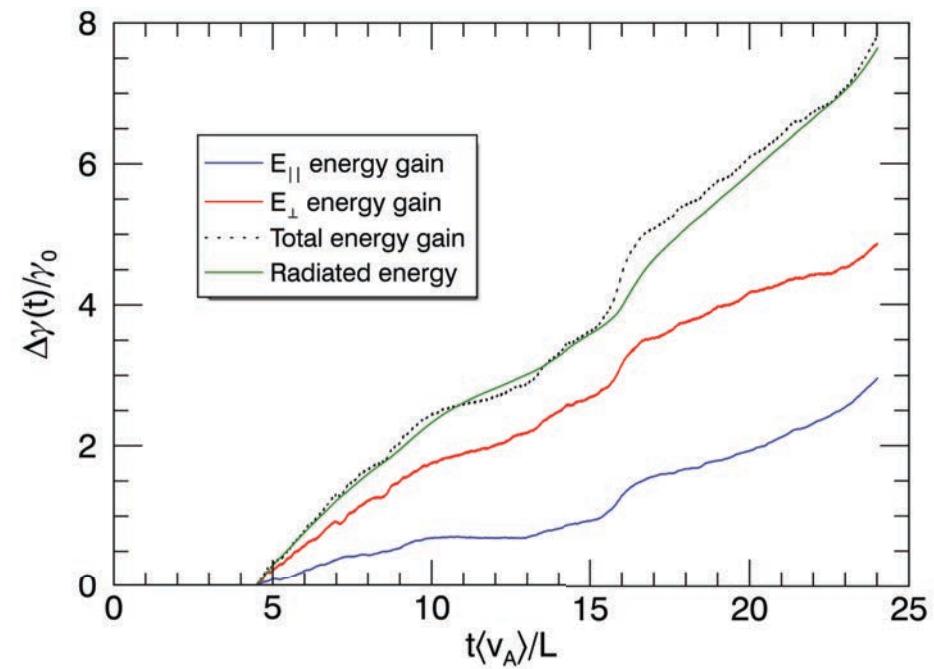
Consistent with diffusive acceleration of ions

Tracked particle statistics

Average of tracked ions



Average of tracked electrons



>99% of energy gain for ions is from perpendicular electric fields
Electrons gain from mixture of parallel and perpendicular fields

Conclusions

- Relativistic kinetic turbulence can be robustly investigated with 3D PIC
- Ions gain more energy and are more efficiently accelerated than electrons in most of explored parameter space
- Proposed an empirical formula that describes electron-ion energy partition as a function of particle gyroradii:
$$\Delta E_e / \Delta E_i \sim (\rho_e / \rho_i)^\alpha \quad \alpha \approx 2/3 - 3/4$$
- No indications of significant electron-ion thermal coupling in simulations with radiative cooling; however, electrons appear to attain a steady state temperature, departing from empirical formula
- Future directions: strong guide field, sub-relativistic regime, role of driving, theory, diagnostics