Implications of the high magnetic field path to fusion energy for Alfvén eigenmode stability

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Much of the material in this talk can be found in the following paper:
E.A. Tolman, N.F. Loureiro, P. Rodrigues, J.W. Hughes, E.S. Marmar, *Dependence of alpha-particle-driven Alfvén eigenmode linear stability on device magnetic field strength and consequences for next-generation tokamaks*, Nuclear Fusion 59, 046020 (2019).

PSFC Plasma Science and Fusion Center
Massachusetts Institute of Technology

Outline

- MIT's high magnetic field path
- Brief introduction to alpha particles and Alfvén eigenmodes (AEs)
- Three implications of high field operation for AE behavior
- Conclusions and current work

Part 1: MIT's high magnetic field path

Acknowledgments for this section: Jerry Hughes, Jeff Freidberg, Martin Greenwald, Zach Hartwig, Alberto Loarte, Bob Mumgaard, Brian LaBombard, Dennis Whyte

Fusion power increases with field and size

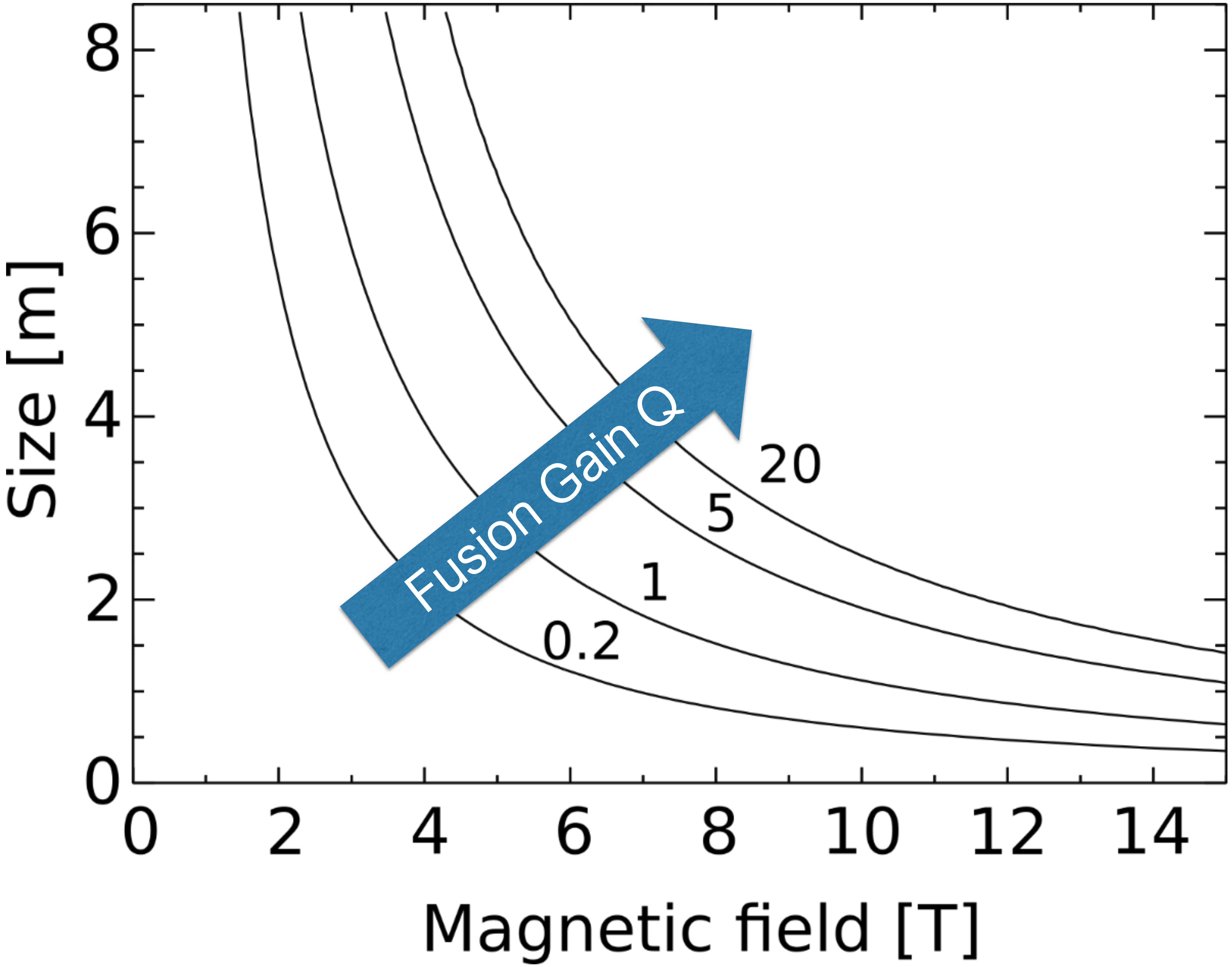
- Fusion power scales like:

$$P_f \sim \langle nT \rangle^2 V \sim \beta_N^2 \frac{I_p^2 B_0^2}{a^2} V \sim \beta_N^2 \frac{\epsilon^2 S(\kappa)}{q_\star^2} B_0^4 V$$

- Magnetic field B_0 and volume V are the main parameters which can be chosen when constructing a new tokamak

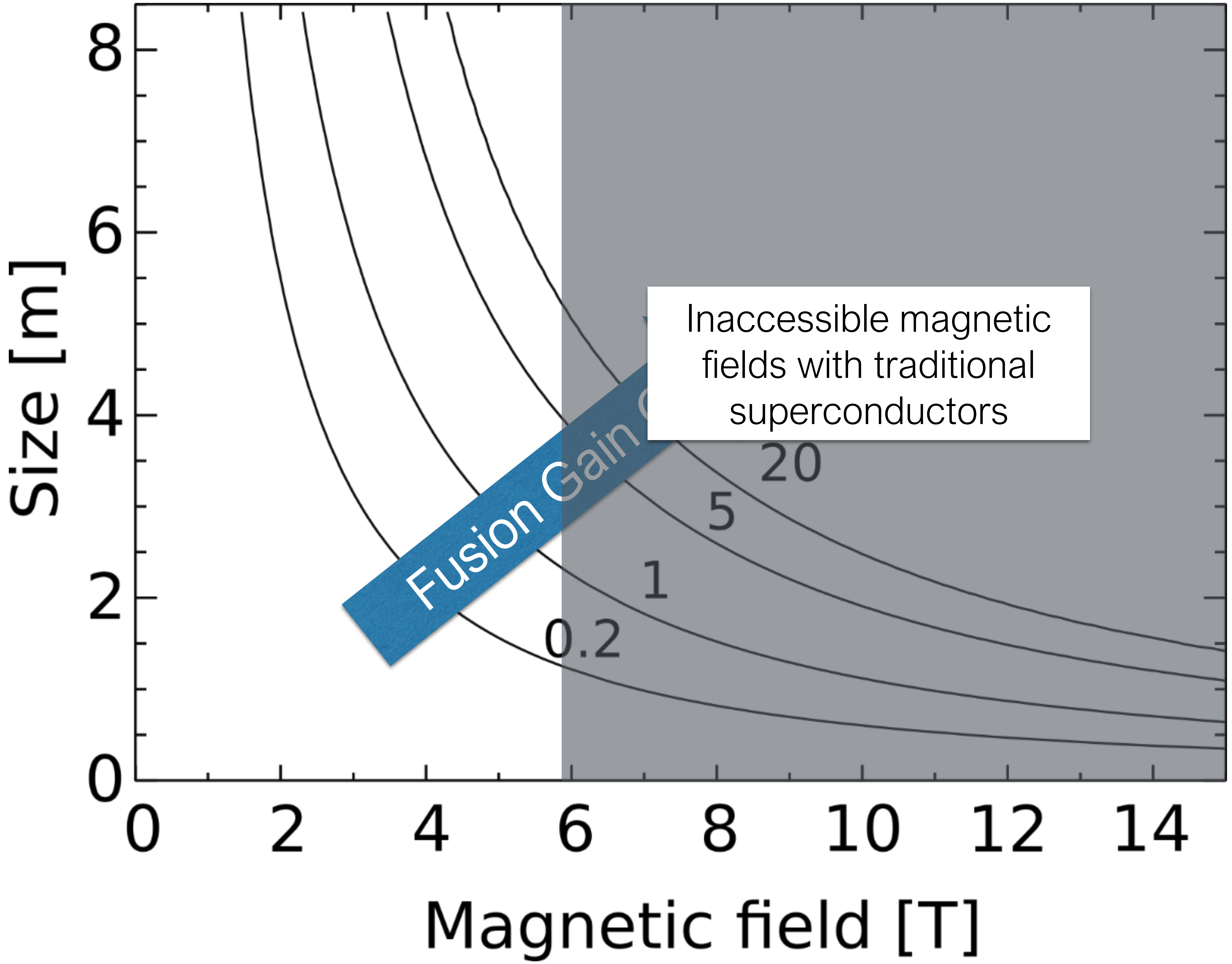
- Other parameters are limited by plasma physics

Q also increases with field, size



- Q calculated using ITER98y2 at fixed shaping, β_N , etc.

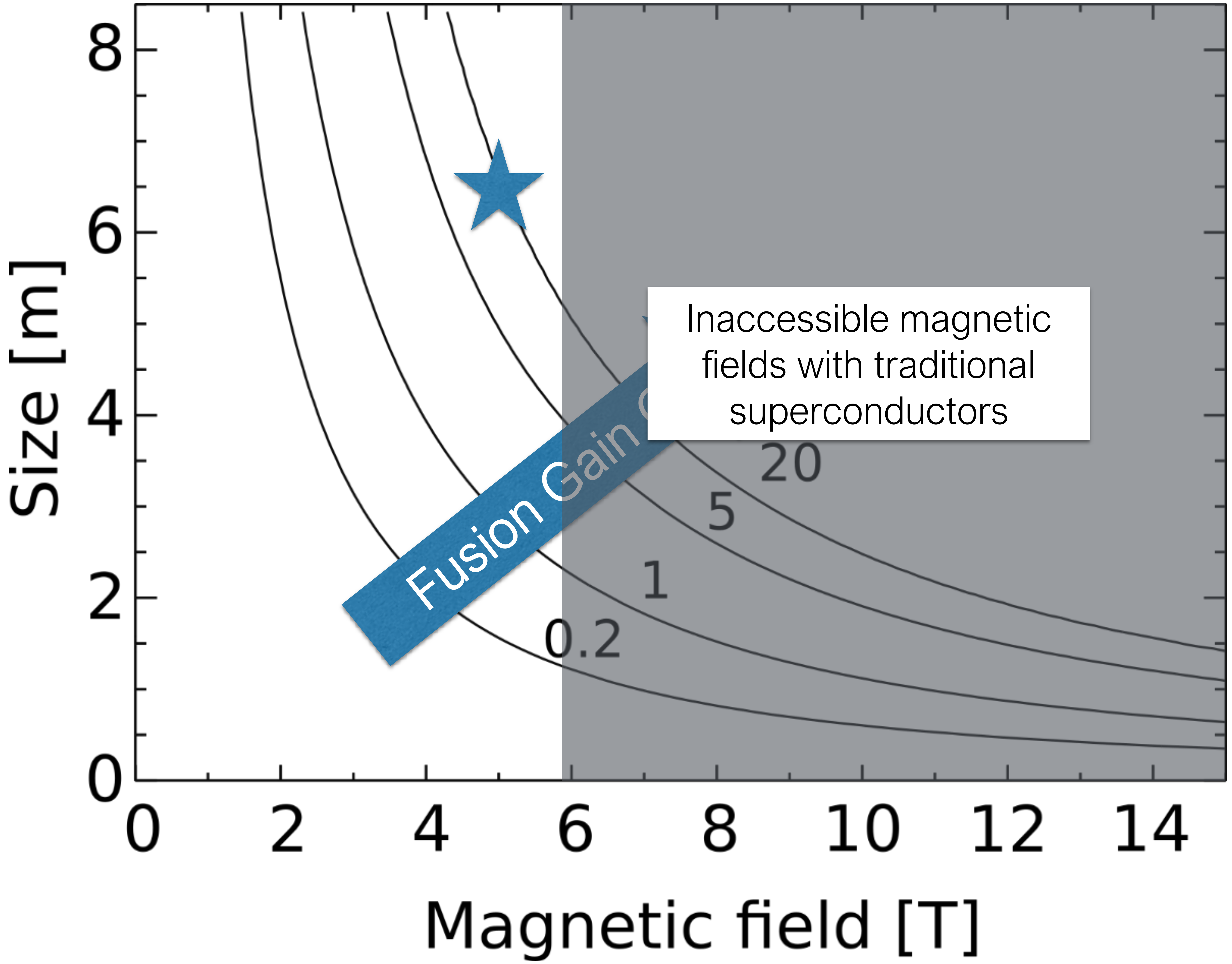
Historically, magnetic field technology has limited accessible space



$$P_f \sim \langle nT \rangle^2 V \sim \beta_N^2 \frac{\epsilon^2 S(\kappa)}{q_\star^2} B_0^4 V$$

- Fusion power must be increased by increasing volume or improving plasma physics

ITER seeks large Q by increasing tokamak size

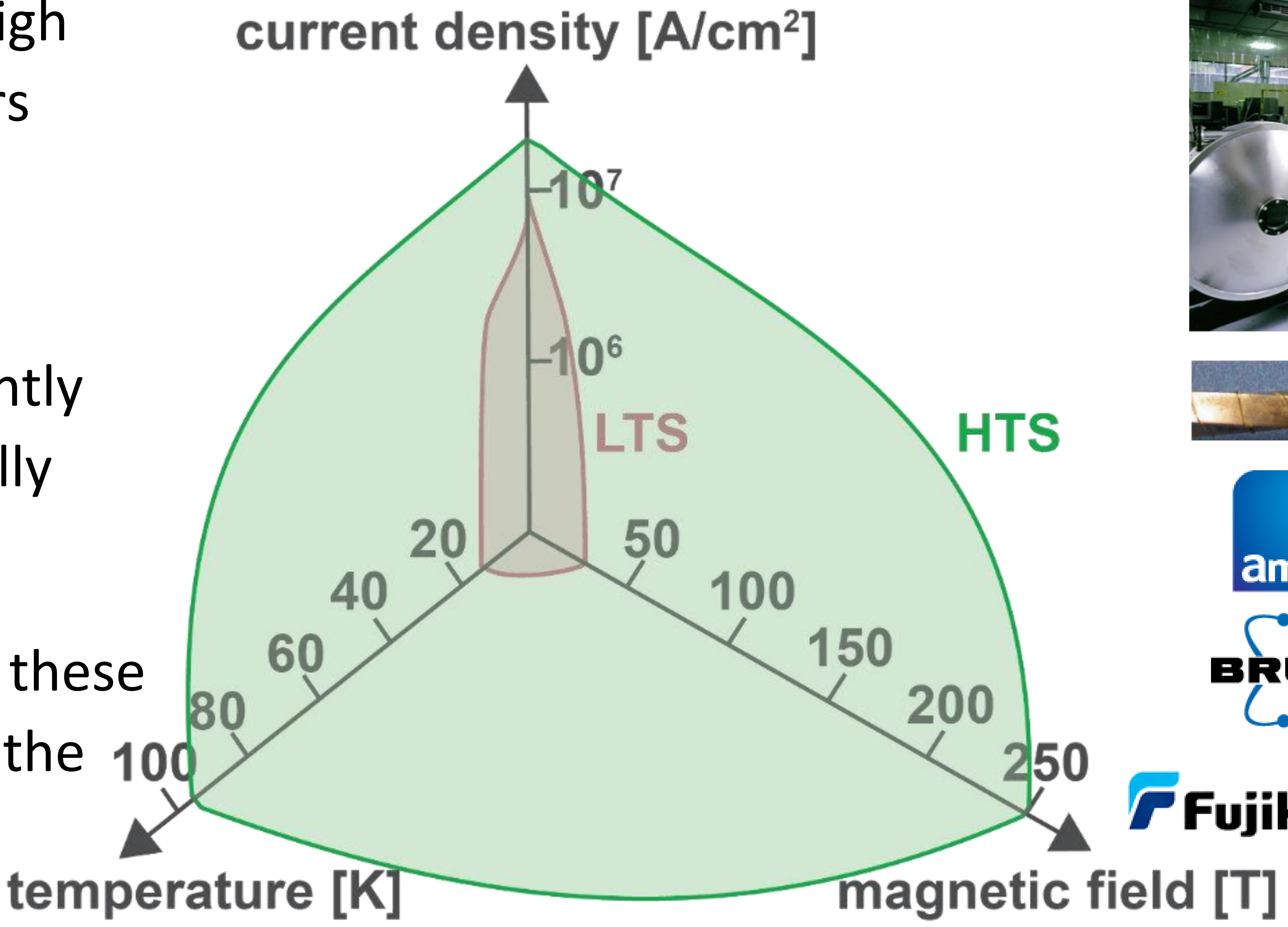


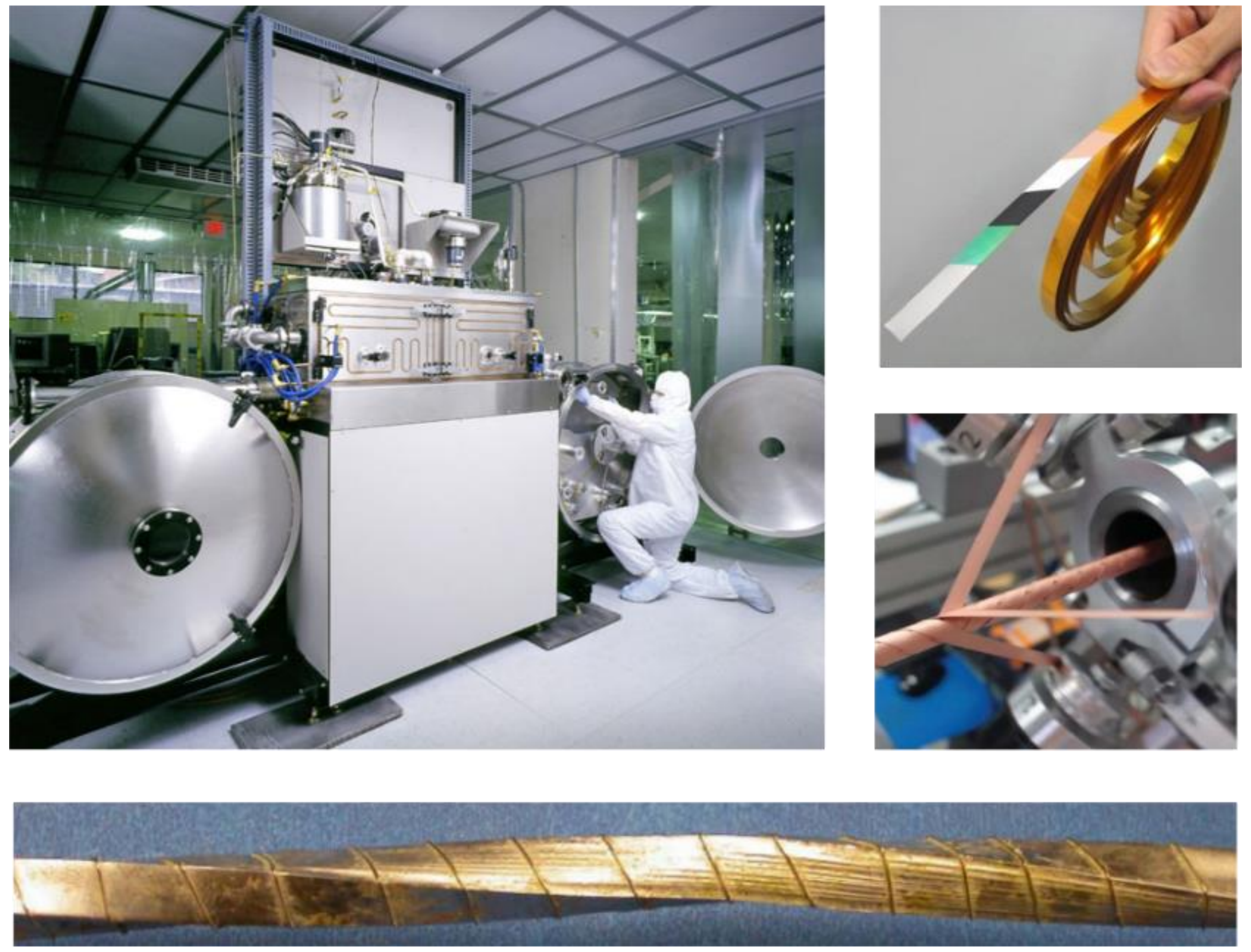
	ITER
Torus Radius [m]	6.2
Magnet Technology	LTS
Magnetic Field Strength [T]	5.3
P_f [MW]	500

- Large devices are expensive and slow, require many international partners
- Options for reducing size and accelerating development pathway merit exploration

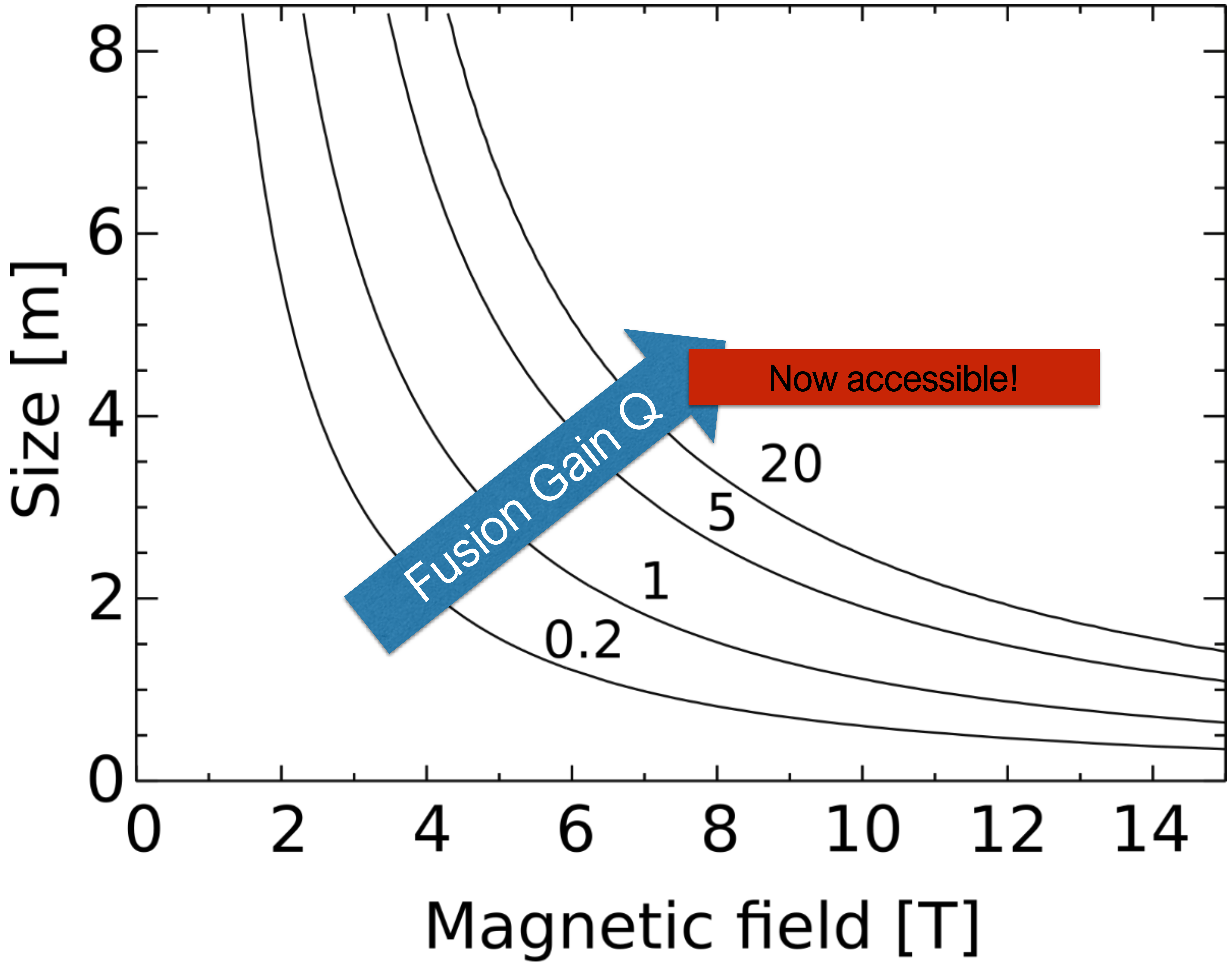
Recent developments enable more compact tokamaks

- High temperature, high field superconductors (HTS) have been developed
- HTS tapes have recently become an industrially produced product
- Magnets made from these tapes would expand the accessible fields for tokamaks





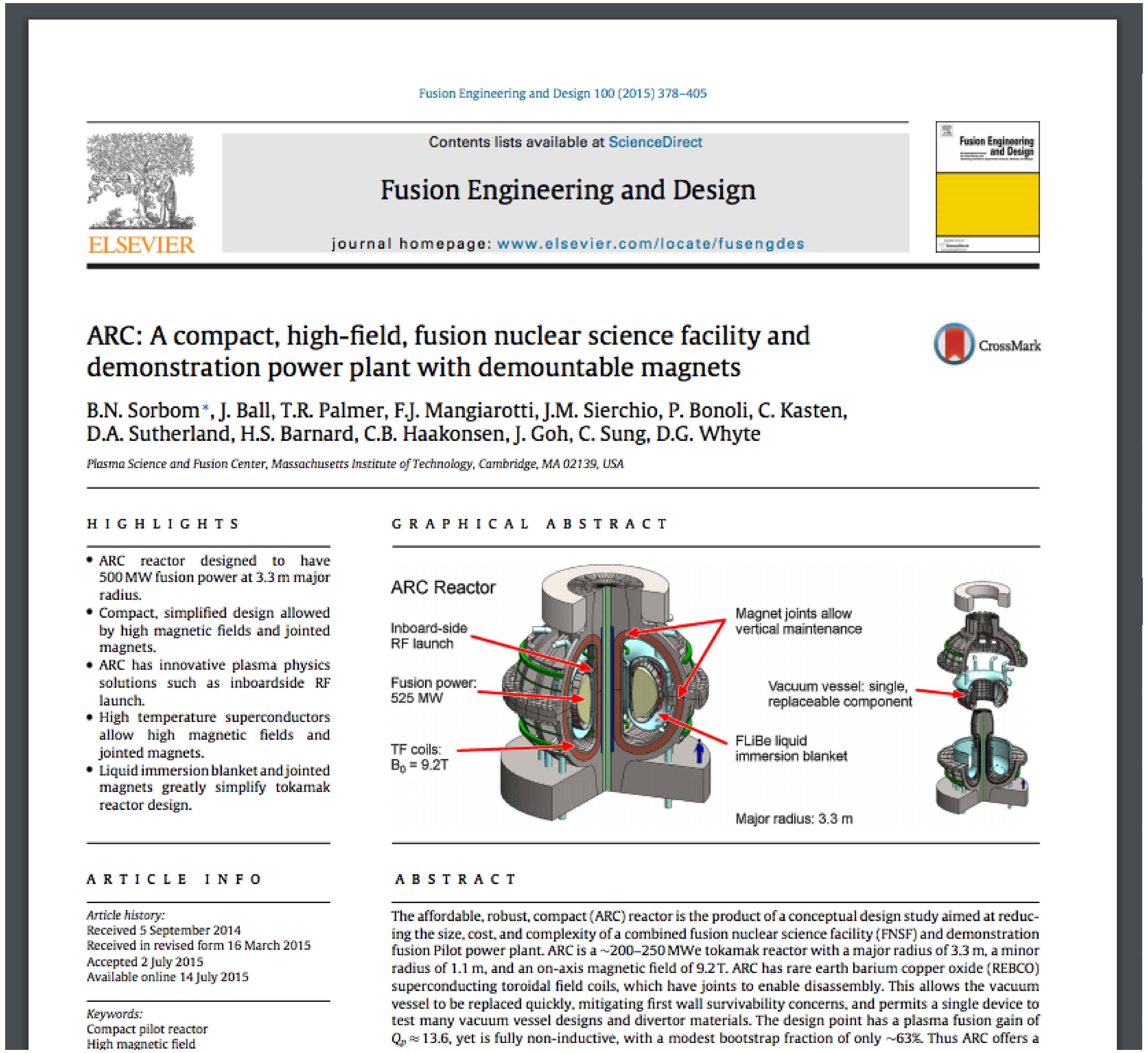
HTS expands accessible parameter space



- Smaller devices allow cheaper construction and smaller teams
- The high field, compact path is the focus of MIT's fusion program
- Has also received support from recent US National Academies of Science, Engineering, and Medicine strategic report

ARC study (2015) outlined the size reductions allowed by HTS

- Design study led by MIT students
- Conceptual design of demonstration fusion pilot power plant that obtains ITER-level performance in much smaller size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ARC: A compact, high-field, fusion nuclear science facility and demonstration power plant with demountable magnets

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HIGHLIGHTS

- ARC reactor designed to have 500 MW fusion power at 3.3 m major radius.
- Compact, simplified design allowed by high magnetic fields and jointed magnets.
- ARC has innovative plasma physics solutions such as inboardside RF launch.
- High temperature superconductors allow high magnetic fields and jointed magnets.
- Liquid immersion blanket and jointed magnets greatly simplify tokamak reactor design.

GRAPHICAL ABSTRACT

ARC Reactor

Inboard-side RF launch

Fusion power: 525 MW

TF coils: B_0 = 9.2T

Magnet joints allow vertical maintenance

Vacuum vessel: single, replaceable component

FLiBe liquid immersion blanket

Major radius: 3.3 m

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ABSTRACT

The affordable, robust, compact (ARC) reactor is the product of a conceptual design study aimed at reducing the size, cost, and complexity of a combined fusion nuclear science facility (FNSF) and demonstration fusion Pilot power plant. ARC is a ~200–250 MWe tokamak reactor with a major radius of 3.3 m, a minor radius of 1.1 m, and an on-axis magnetic field of 9.2 T. ARC has rare earth barium copper oxide (REBCO) superconducting toroidal field coils, which have joints to enable disassembly. This allows the vacuum vessel to be replaced quickly, mitigating first wall survivability concerns, and permits a single device to test many vacuum vessel designs and divertor materials. The design point has a plasma fusion gain of Q_p ≈ 13.6, yet is fully non-inductive, with a modest bootstrap fraction of only ~63%. Thus ARC offers a

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	ARC
Torus Radius [m]	3.2
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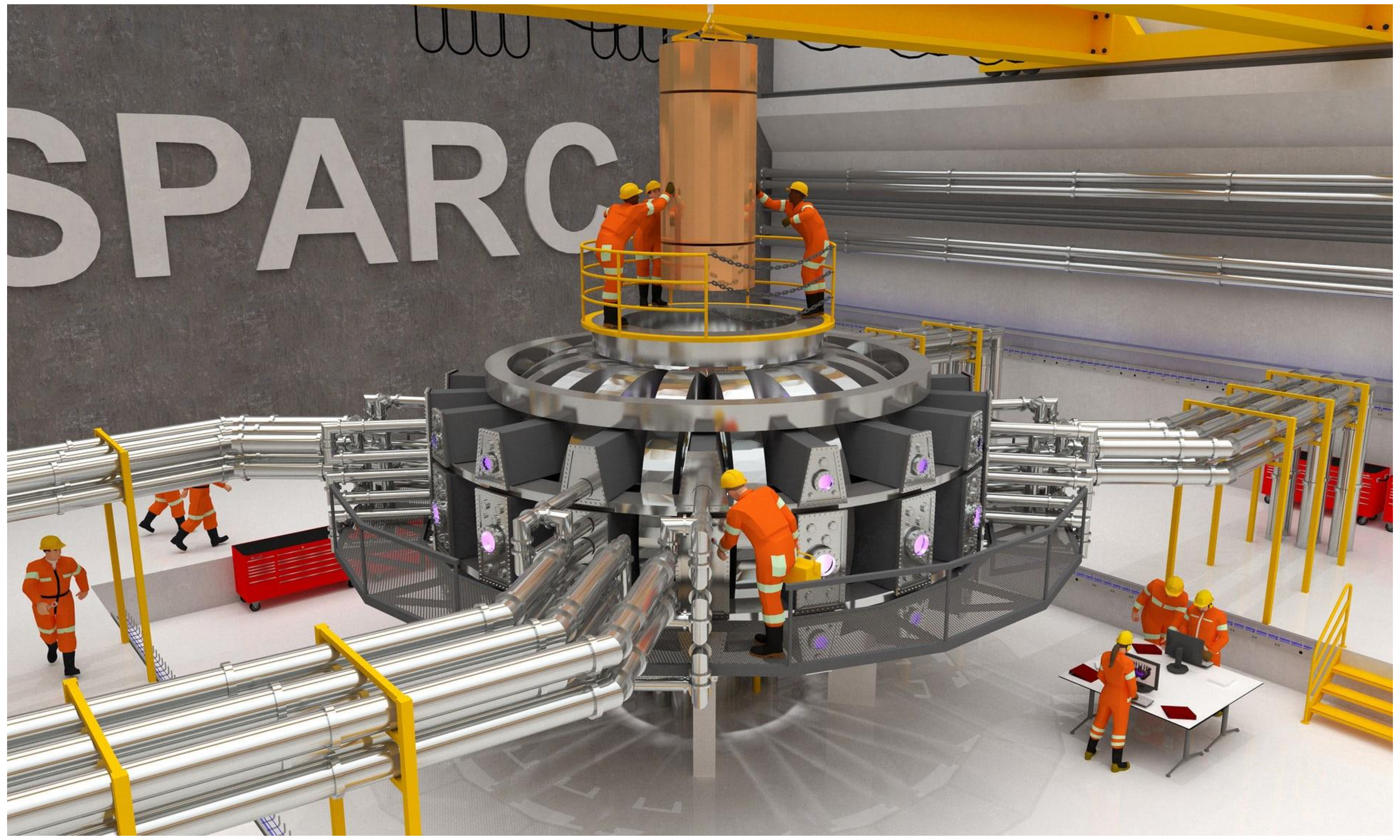
Magnetic Field Strength [T]	9.2
P_f [MW]	500

In 2018, PSFC started the SPARC project as first step towards a reactor



- ARC requires too big of an investment and is too big of a step to build immediately
- Instead, PSFC is working on the SPARC project:
  - $B_0 \sim 12$ T, $R_0 \sim 1.65$ m, $Q \geq 2$
- Project is a collaboration between MIT PSFC and the private, investor-backed Commonwealth Fusion Systems ($115 million Series A)
- First step of SPARC is magnet development: ongoing presently

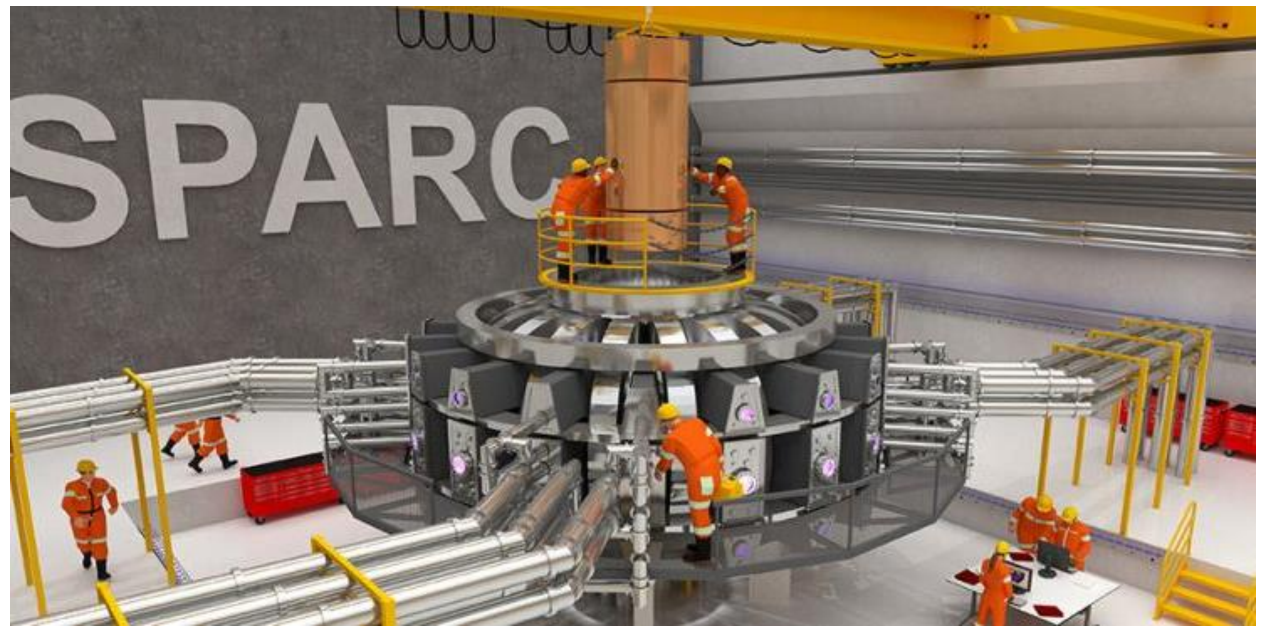
SPARC, ARC explore tokamak physics in different regime than ITER

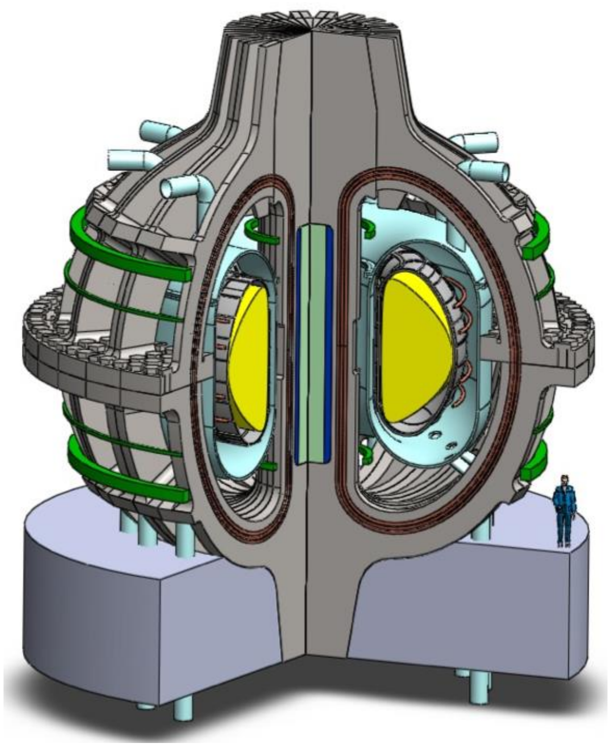
- SPARC and ARC will explore tokamak physics in a different parameter space than ITER and than current tokamaks

- This parameter space ***may*** present advantages for reactor operation

- This parameter space ***will*** allow increased, complementary understanding of tokamak physics

- Many interesting analytic questions can already be asked about general differences in behavior in high and low field devices





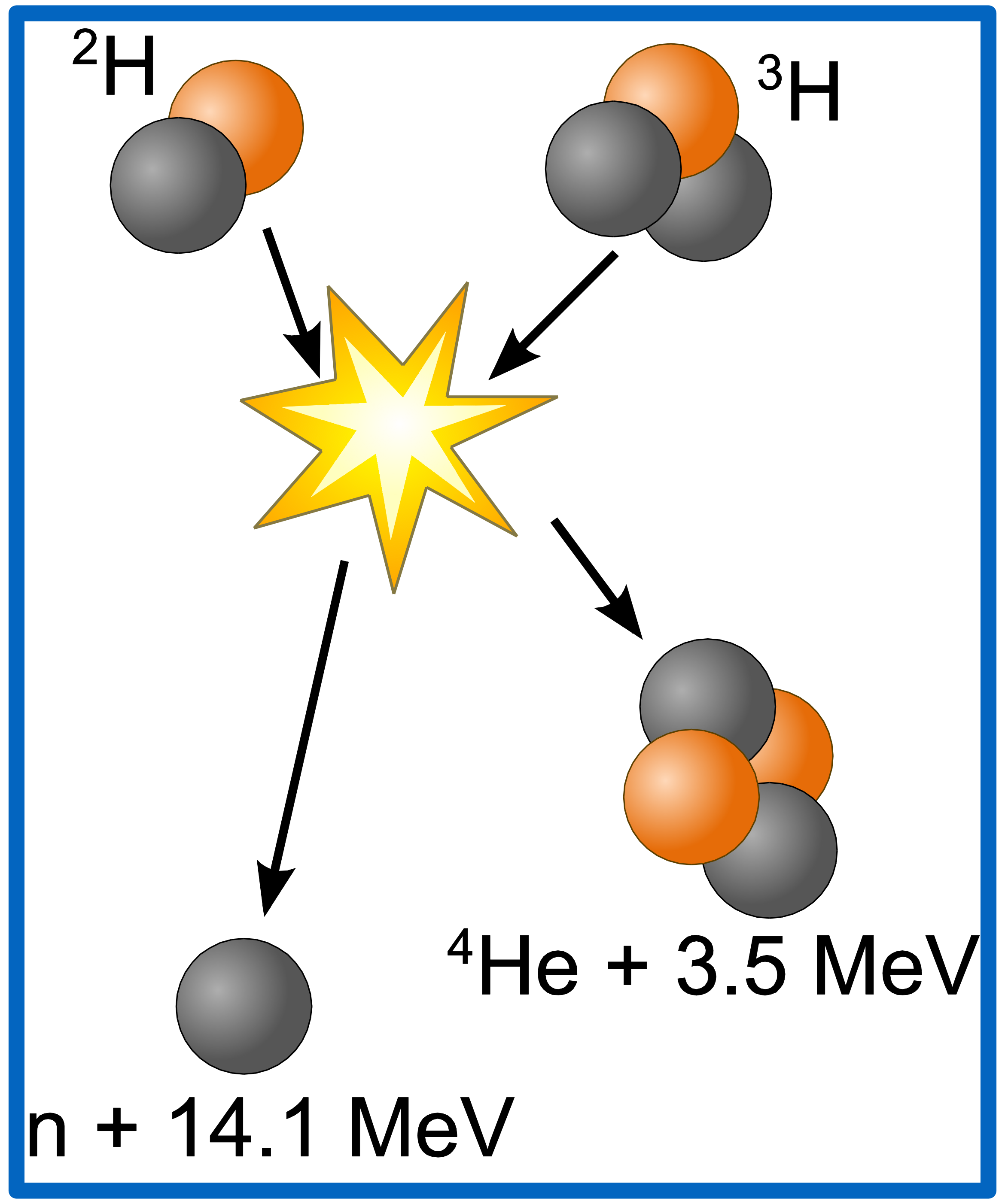
Quantity	ITER	SPARC strawman	ARC
Major radius	$R_0 = 6.2\ m$	$R_0 = 1.65\ m$	$R_0 = 3.3\ m$
On-axis electron density	$n_e(0) \sim 11 \times 10^{19}\ m^{-3}$	$n_e(0) \sim 40$ - $65 \times 10^{19}\ m^{-3}$	$n_e(0) \sim 18 \times 10^{19}\ m^{-3}$
On-axis magnetic field	$B_0 \sim 5.3$ T	$B_0 \sim 12$ T	$B_0 \sim 9.2$ T
Alfvén frequency	$v_{A0}/(2\pi R_0) \sim 1.8 \times 10^5\ Hz$	$v_{A0}/(2\pi R_0) \sim 6.3 - 8.0 \times 10^5\ Hz$	$v_{A0}/(2\pi R_0) \sim 4.6 \times 10^5\ Hz$

Alpha particles and Alfvén eigenmodes (AEs)

Next-generation tokamaks aim to produce a significant amount of alpha particles

- Next-generation tokamaks will produce a large “amount” of alpha particles

$n_e = 2n_D = 2n_T$	$T_i \sim T_e$	n_α	“T_α” [Calculated using average value of v^2 across slowing down distribution]	$n_\alpha T_\alpha/(2n_e T_e)$
$\sim 5 \times 10^{20}\ m^{-3}$	$\sim 20\ keV$	$\sim 10^{18}\ m^{-3}$	$\sim 580\ keV$	$\sim 15\%$



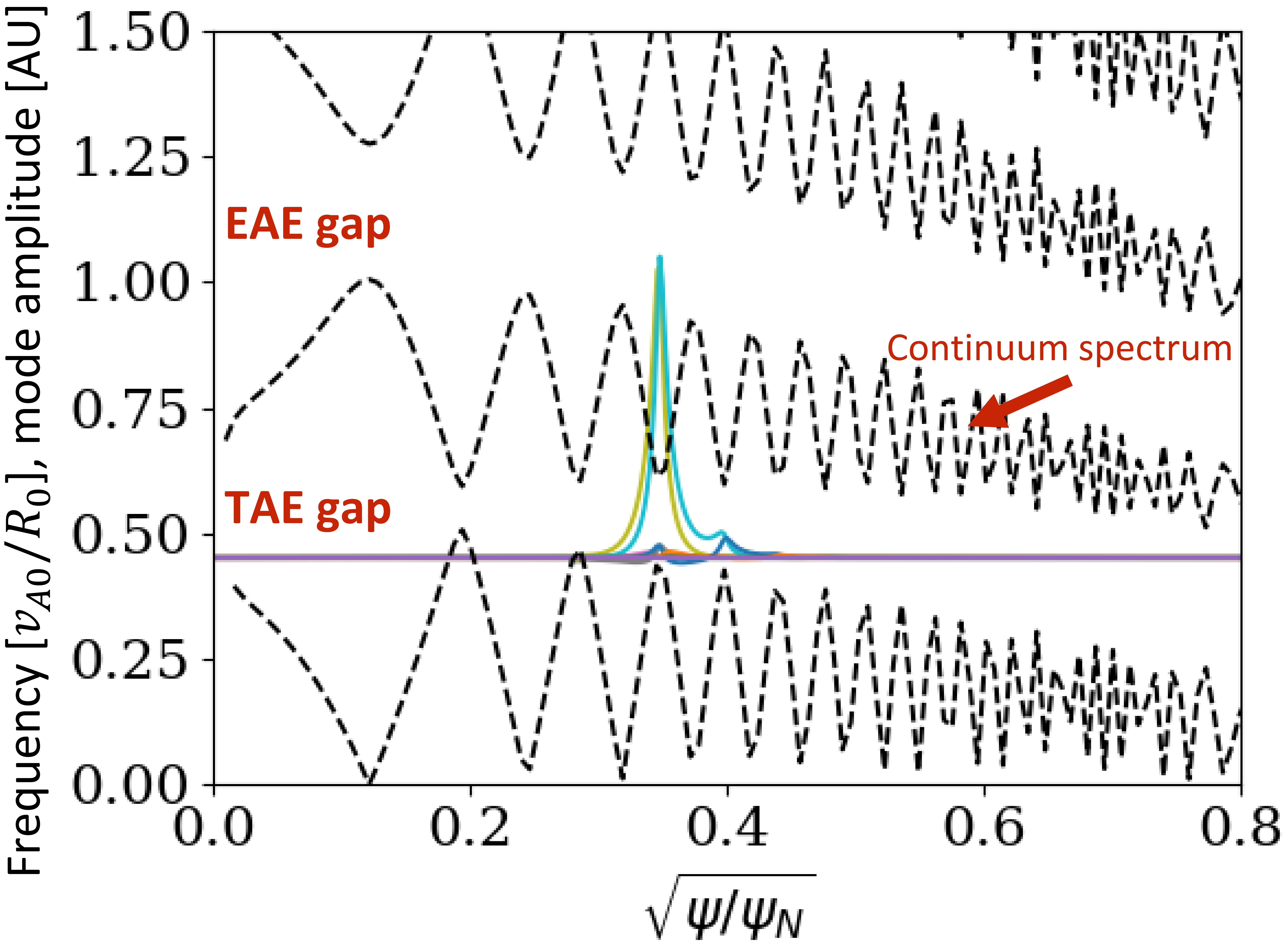
- Understanding alpha particle confinement and transport physics is important to performance goals
  - Alpha particle transport can modify where alpha heat is deposited and resulting background plasma profiles
  - Loss of alpha particles can degrade plasma performance and damage the device

- Alpha particle physics is similar to physics of energetic particles from ICRF and NBI but has differences as well

- Overall, alpha particle physics and related topics are perhaps the most interesting part of next-generation tokamaks

AEs are excited by energetic particles, can cause energetic particle loss

- One key part of alpha particle physics is alpha-AE interaction
- AEs are shear Alfvén waves that exist in tokamaks as discrete modes
- Energetic particles, including alphas, destabilize AEs
- AEs can cause transport of alphas to edge, degrading plasma performance and damaging the device



Growth rate determined by thermal species and energetic particles

- AE linear growth rate is determined by sum of drive and damping:

$$\frac{\gamma}{\omega} = \frac{\gamma_\alpha}{\omega} + \sum_j \frac{\gamma_j}{\omega}.$$

- Alpha growth is given approximately by [Betti and Freidberg, 1992]:

$$\frac{\gamma_\alpha}{\omega} \sim q_{AE}^2 \beta_\alpha F\left(\frac{v_{A0}}{v_{\alpha 0}}, \frac{1}{p_\alpha}\frac{dp_\alpha}{dr}, q_{AE}, \mathrm{n}, \mathrm{r}_{L\theta\alpha}\right)$$

- Ion damping is given approximately by [Betti and Freidberg, 1992]:

$$\frac{\gamma_j}{\omega} \sim -q_{AE}^2 \beta_j G\left(\frac{v_{A0}}{v_{thj}}\right)$$

Quantity	Definition
ω	mode frequency
$\gamma_\alpha, \gamma_j, \gamma$	alpha contribution, ion contribution, and overall growth rate
q_{AE}	safety factor at mode location
β_α, β_j	alpha, ion beta
$v_{A0}, v_{\alpha 0}, v_{thj}$	on-axis Alfvén speed, alpha birth speed, ion thermal speed
$\mathrm{n}, \mathrm{r}_{L\theta\alpha}$	toroidal mode number, poloidal alpha Larmor radius
F	growth from spatial gradient of alphas $\left(\frac{1}{\mathrm{p}_\alpha}\frac{\mathrm{dp}_\alpha}{\mathrm{dr}}\right)$ at resonant velocities
G	damping from energy gradient of ions at resonant velocities

Three implications of high field operation for AE behavior

Higher field affects typical plasma parameters

- At a given magnetic field, a tokamak can operate with a wide range of plasma parameters
- Overall trends can be understood by holding tokamak figures of merit constant

Tokamak figures of merit		
$\beta = \frac{p}{B_0^2/(2\mu_0)}$	$\beta_N = \frac{\beta a B_0}{I_p}$	$f_{GW} = \frac{\bar{n}_e}{I_p/\pi a^2}$

- Resulting trend in core plasma parameters is:

Trend with $\boldsymbol{B_0}$				
$p \sim B_0^2$	$I_p \sim B_0$	$n \sim B_0$	$v_A \sim \frac{B_0}{\sqrt{n}} \sim \sqrt{B_0}$	T: Increases; depends on p_α (see paper)

- Higher order/other trends in these quantities can also be discussed

For example: could consider decreasing f_{GW} with field or keeping temperature roughly constant (e.g. 15 keV) and decreasing β

High density in high field machines affects slowing down time, alpha source rate

- Alpha particle density scales like

$$n_\alpha \sim \frac{Rate\ of\ \alpha\ production}{Speed\ at\ which\ \alpha's\ slow\ down\ to\ ash} \sim \frac{n_D n_T \langle \sigma v \rangle}{\left(n_e / T_e^{1.5}\right)} \sim n_e T_e^{3.5}$$

- Neglecting resonance positions, ratio of AE drive to damping scales like:

$$\frac{Drive}{Damping} \sim \frac{\beta_\alpha}{\beta_j} \sim \frac{\frac{n_\alpha T_\alpha}{B^2}}{\frac{n_e T_e}{B^2}} \sim T_e^{2.5}$$

- High field devices have significant AE drive; to the extent that high *T* is caused by high field, this trend results from higher field
- High density helps to slow down alphas but also produces more alphas!

For economic viability, tokamaks should achieve a fixed value of fusion power density

- Economically, a reactor should achieve a fixed value of fusion power *density*, $p_f \left[\frac{MW}{m^3}\right] = E_f n_D n_T \langle \sigma v \rangle \sim \mathrm{n_e^2 T_e^2} \equiv p_{f,econ}$
- Recall: Neglecting resonance positions, ratio of AE drive to damping scales like:

$$\frac{Drive}{Damping} \sim \frac{\beta_\alpha}{\beta_j} \sim T_e^{2.5} \sim \frac{p_{f,econ}{}^{1.25}}{n_e^{2.5}}$$

- Higher magnetic field devices have higher currents at a given q, which allows higher densities while still obeying Greenwald limit
- Higher field devices thus incur less AE drive for a given power density

Equivalently, for a given AE drive/temperature (e.g. 15 keV), high field devices have a higher power density

Increasing field (or decreasing density) cuts off resonance

- Recall expression for alpha particle drive:

$$\frac{\gamma_\alpha}{\omega} \sim \ q_{AE}^2 \beta_\alpha \ F\left(\frac{v_{A0}}{v_{\alpha 0}}, \frac{1}{p_\alpha}\ \frac{dp_\alpha}{dr}, q_{AE}, \mathrm{n}, \mathrm{r}_{\mathrm{L}\theta\alpha}\right)$$

F: growth from spatial gradient of alphas at resonant velocities

- The most important TAE resonances are $v_{A0}, v_{A0}/3 \sim B_0/\sqrt{n_0}$

- The D-T fusion alpha particle birth velocity is $v_{\alpha 0} = 1.3\ \times 10^7\ m/s$

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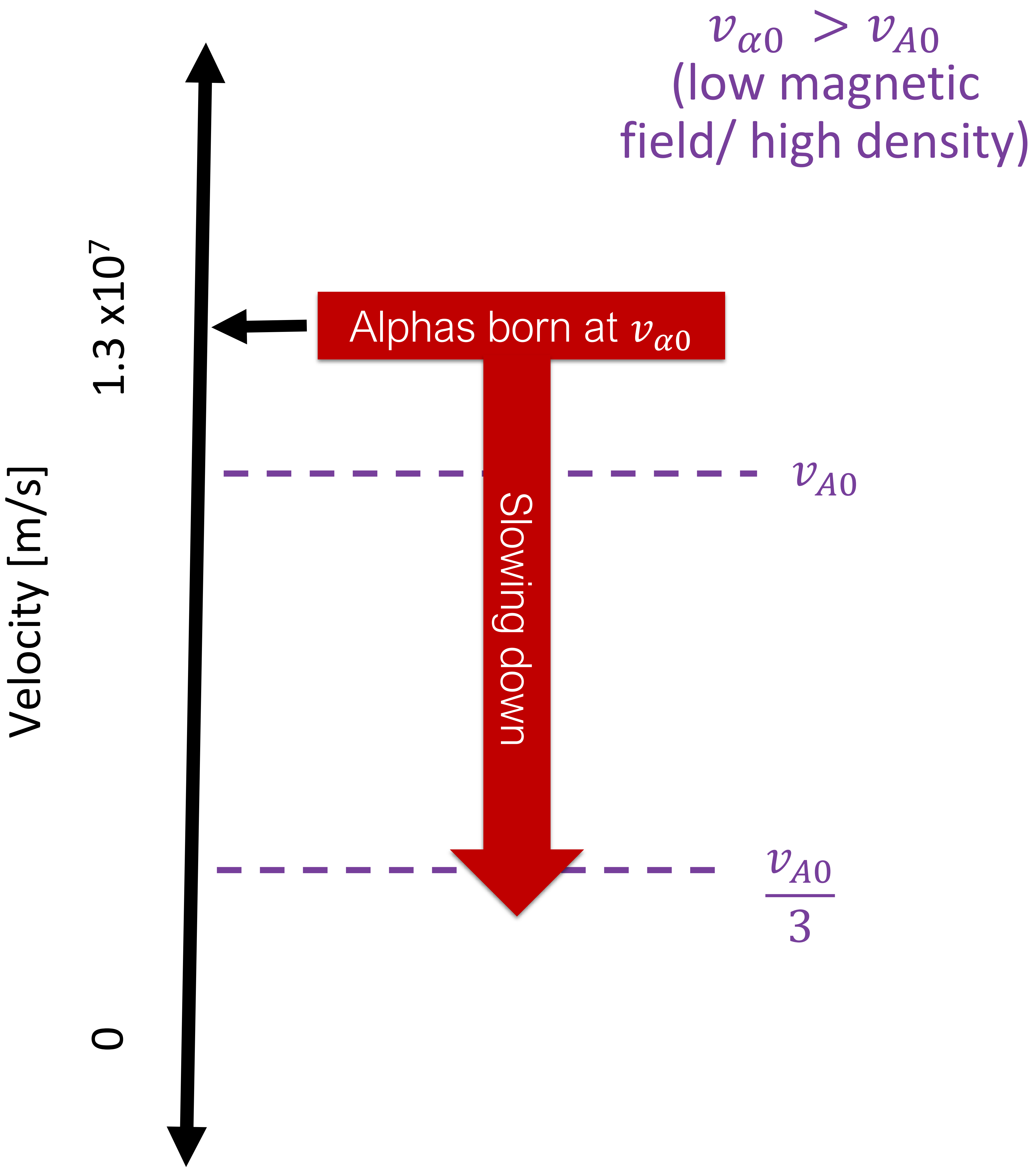

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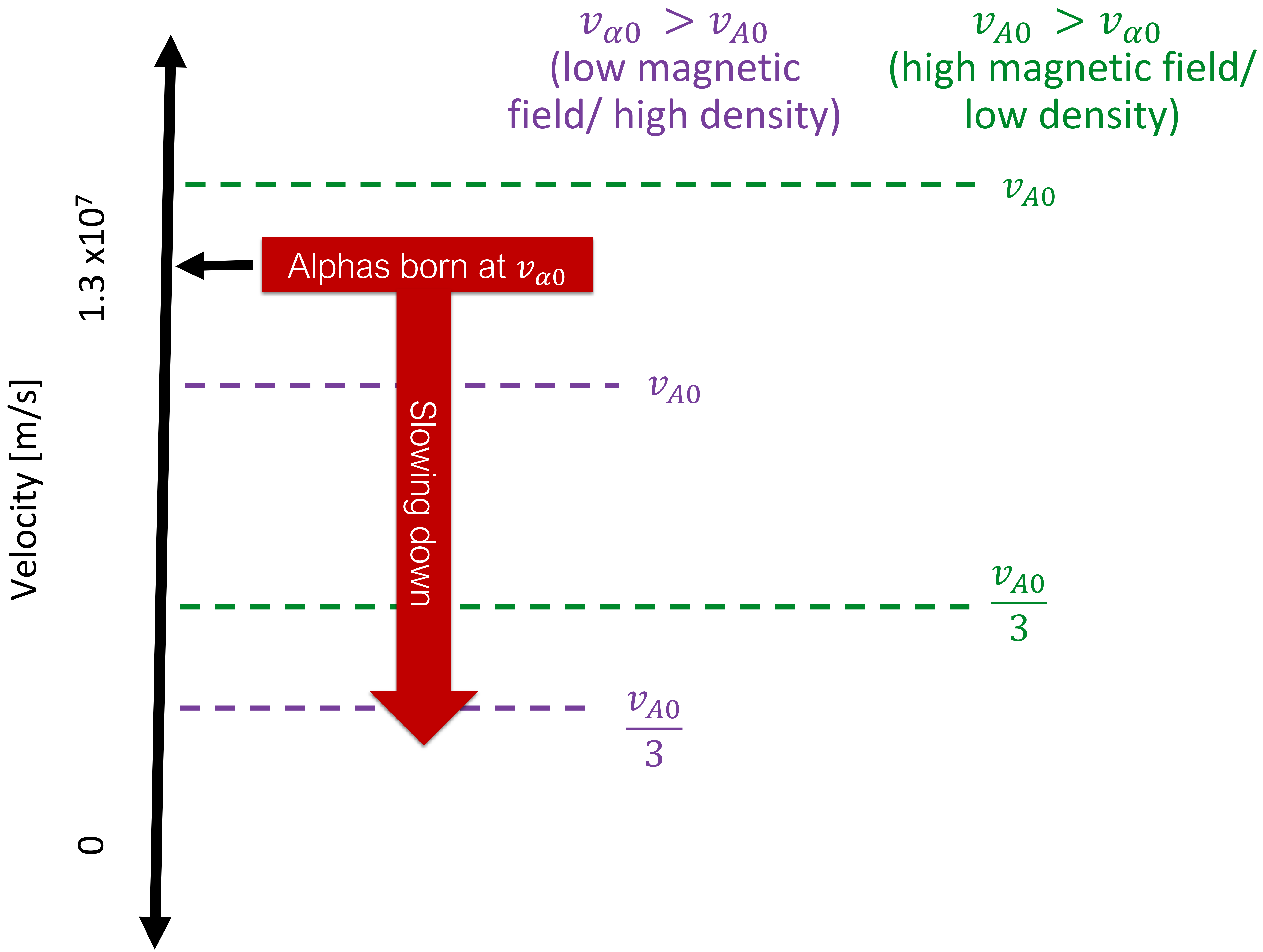
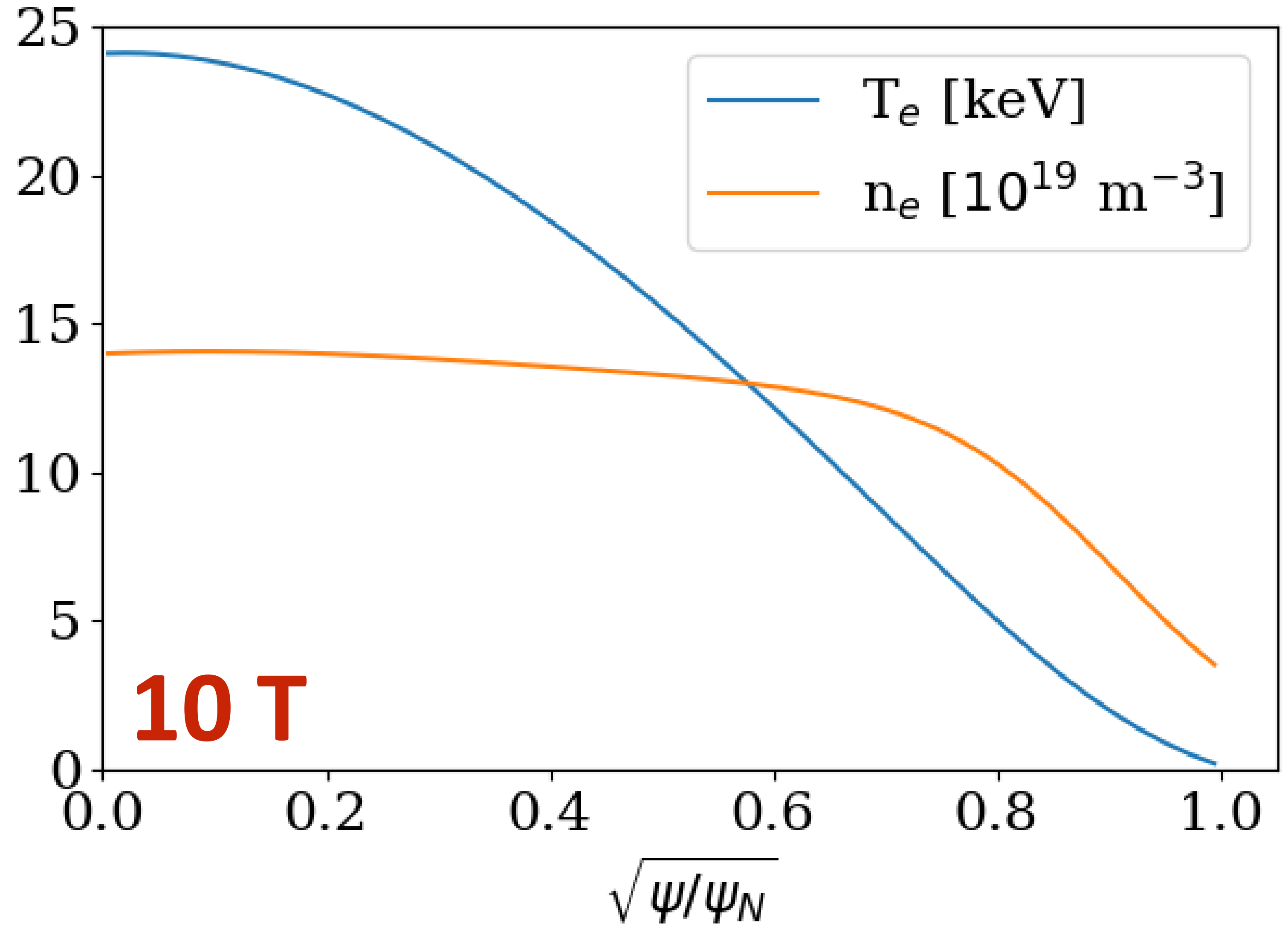


Computational study looks for resonance cutoff in plausible low f_{GW} SPARC-like shot

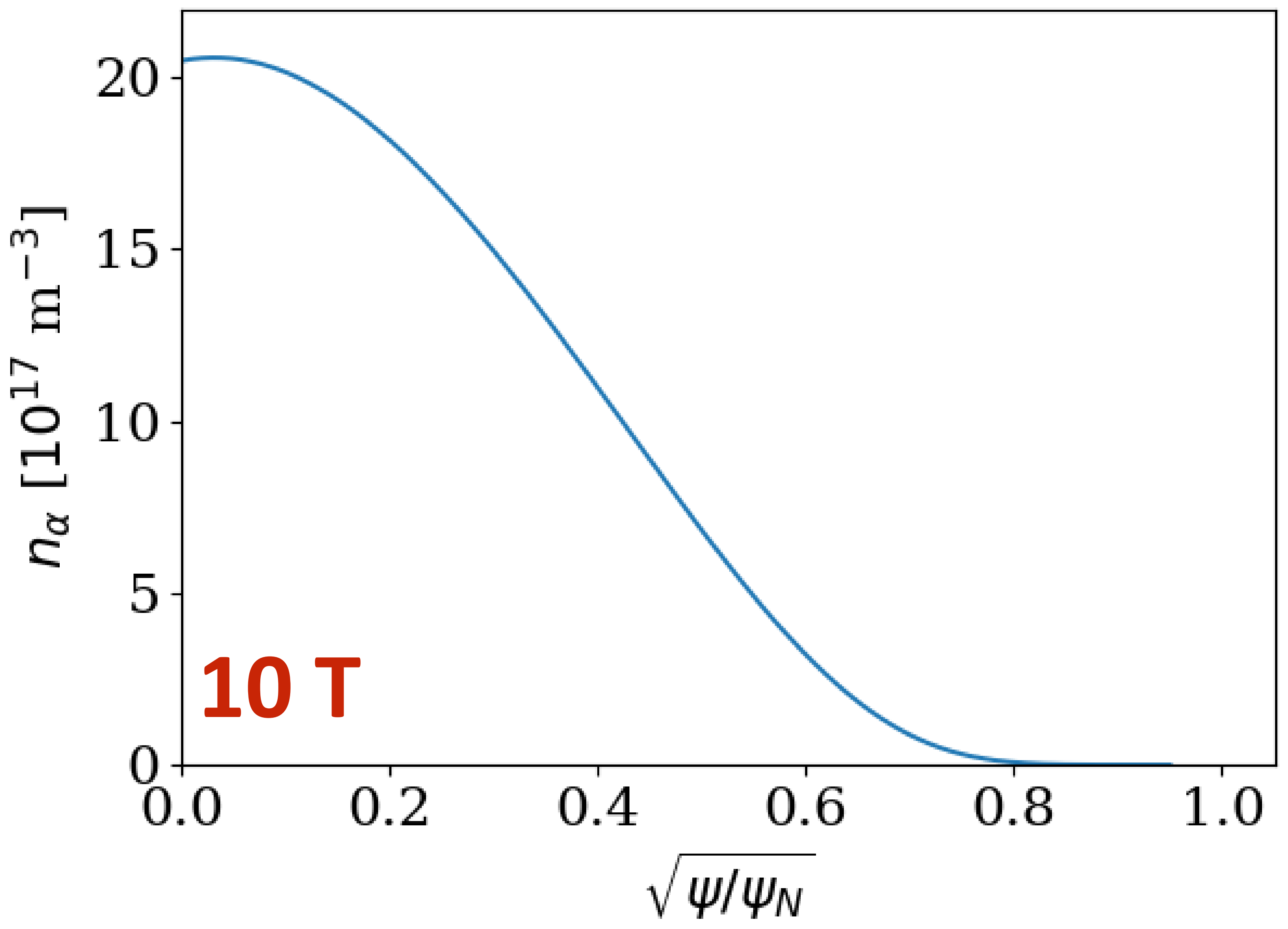
- Take $\beta, f_{GW}, \beta_N, q$ and profile shapes from a high-temperature, low-density C-Mod shot and create a model SPARC-sized D-T equilibrium

Figure of merit	Value in C-Mod shot and our scan
β	0.75%
f_{GW}	0.18
β_N	0.80
q_{95}	3.5



- Scan magnetic field in configuration while keeping $\beta, f_{GW}, \beta_N, q$ and profile shapes constant

- Temperature, density, and fusion power density P_f increase with field



- $n_e \sim B_0$; T_e increases with B_0 less than linearly due to increasing alpha pressure; $p_f \left[\frac{MW}{m^3}\right] \sim n_e^2 T_e^2$ increases with field

Computational study looks for resonance cutoff in plausible low f_{GW} SPARC-like shot

- Use suite of codes (HELENA, MISHKA, CASTOR-K) as implemented in *P. Rodrigues et al. NF 2015* to study AE behavior

HELENA (finite element method code that solves Grad-Shafranov) refines MHD equilibrium

→

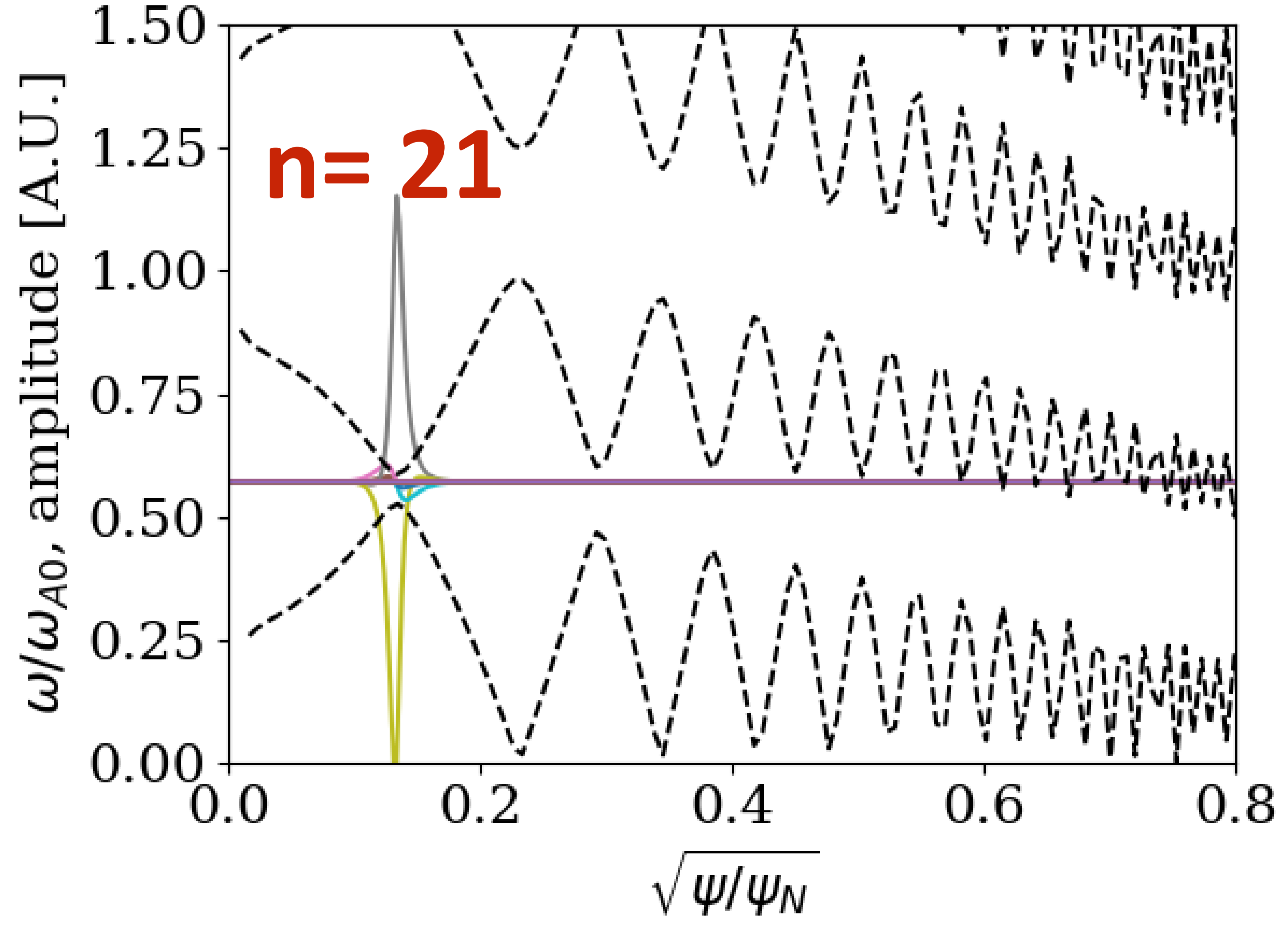
MISHKA (Ideal MHD stability code) finds AE frequencies and computes AE structures

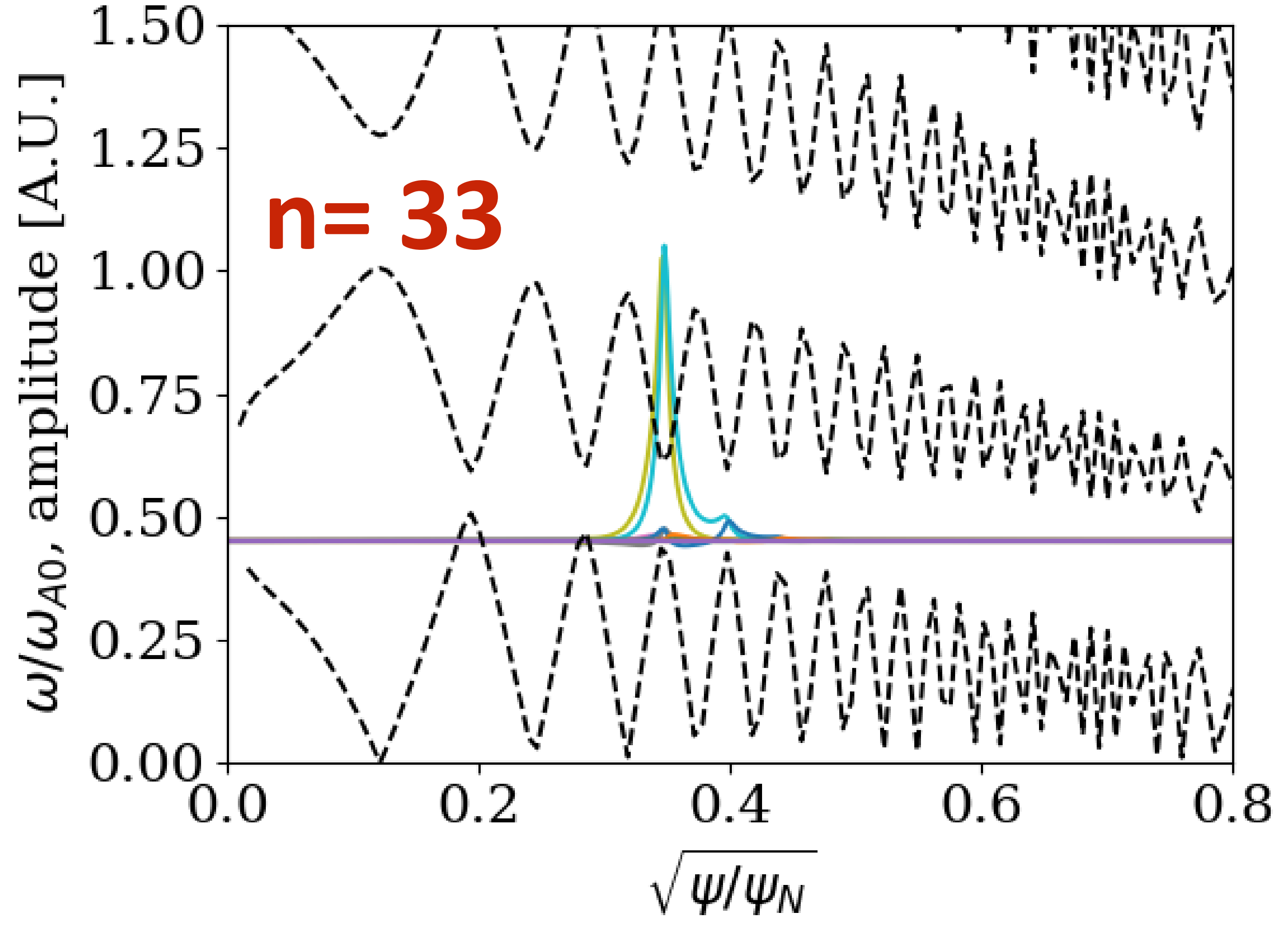
→

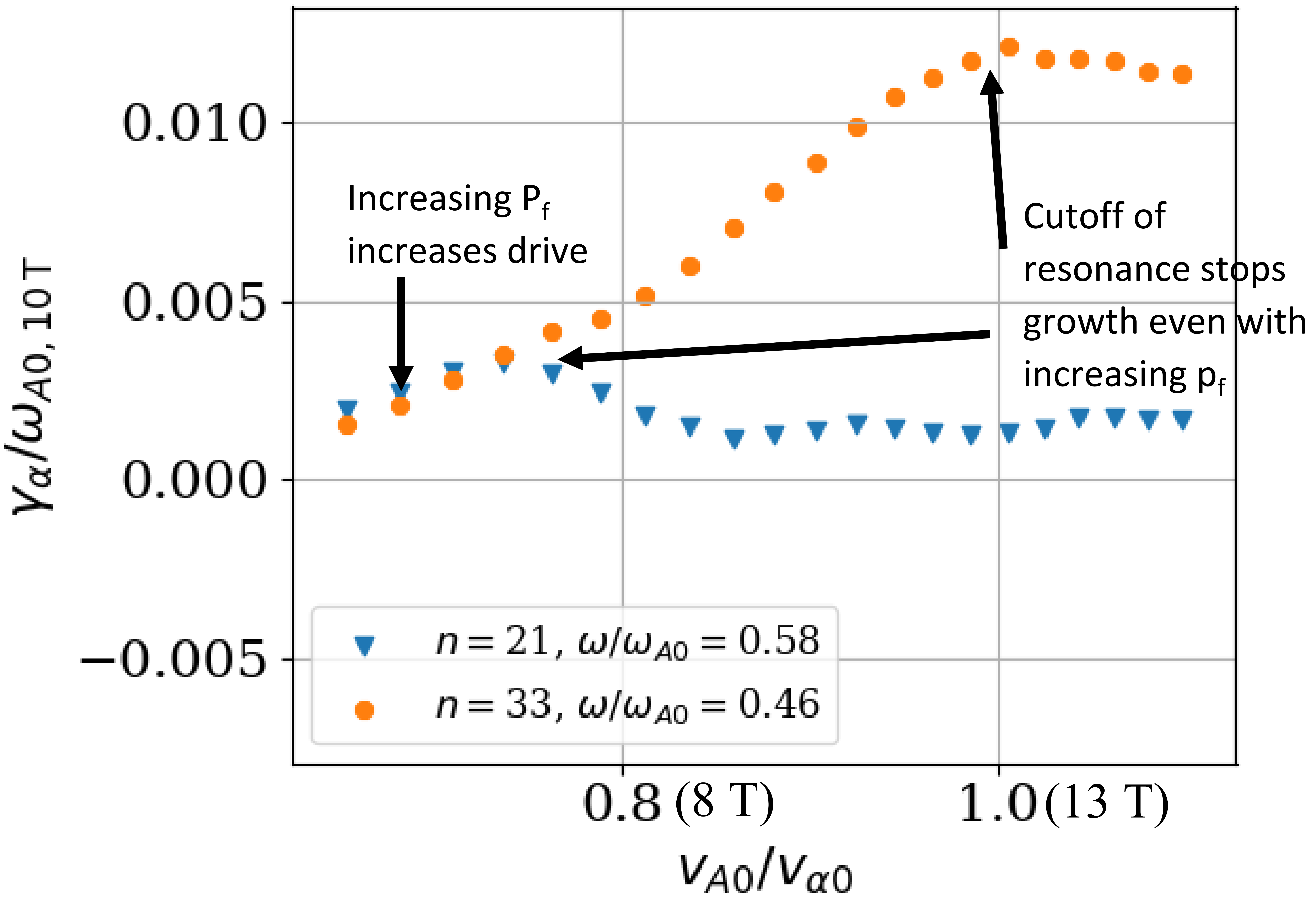
CASTOR-K (Drift-kinetic code) finds α-particle and contribution to AE linear growth.

- These code are used extensively in studies of ITER, JET
- We use them to find AEs and growth rates as magnetic field is scanned

Increased field causes increased power density, drive, then resonance cutoff







Large mode width and amplitude increase AE transport

- Transport of alphas by AEs is not fully understood
- However, two factors tend to increase AE transport: mode width and amplitude
- Larger width increases the amount of the minor radius over which the mode can move particles
    - Larger width modes (or multimode scenarios) increase transport
- Determinants of saturated amplitude are not fully understood
- In general, modes with a higher linear growth rate should saturate at a higher amplitude

Higher n modes have lower width

- The spectrum of modes with the potential to become excited has no B field dependence [see Tolman et al. NF 19]

- Which of these modes are unstable, and which are *most* unstable, is magnetic field dependent

- For mode $\overrightarrow{\xi_\perp} = e^{i(n\,\phi-\omega t)} \sum_m \overrightarrow{\xi_{\perp,m}}\,(r)e^{-im\theta}$, MHD eigenmode equation yields:

$$\frac{d}{dr}\left(\frac{\omega^2}{v_A^2} - k_{\parallel m}\right)\frac{d\overrightarrow{\xi_{\perp,m}}}{dr} - \frac{m^2}{r^2}\left(\frac{\omega^2}{v_A^2} - k_{\parallel m}\right)\overrightarrow{\xi_{\perp,m}} = 0$$

Quantity	Value/definition
$k_\parallel$	$\frac{1}{R}\left(n - \frac{m}{q(r)}\right)$
m	TAE: $nq + 1/2$ EAE: $nq + 1$
a	Minor radius

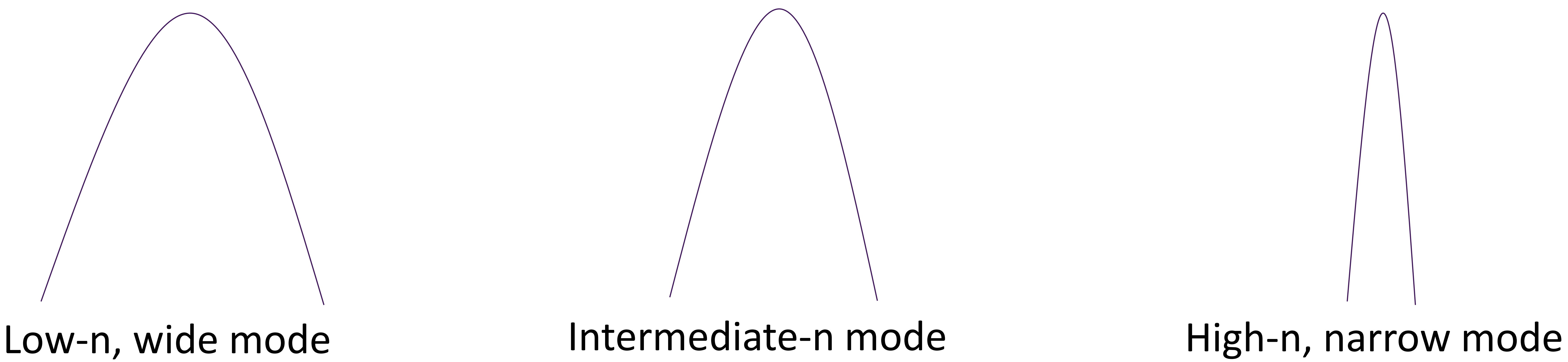
- Balancing terms in this equation gives mode width Δ_{mode}:

$$\Delta_{mode} \sim \frac{r}{m} \sim \frac{r}{nq} \sim \frac{a}{nq}$$

- Higher-n modes are narrower

High field and device size shift AE spectrum to higher n, lower width

- When orbit width effects neglected, AE drive increases with n [Betti and Freidberg Phys Fluids ’92, others]
- Orbit width effects modify this trend [Breizman and Sharapov PPCF ’95, others]



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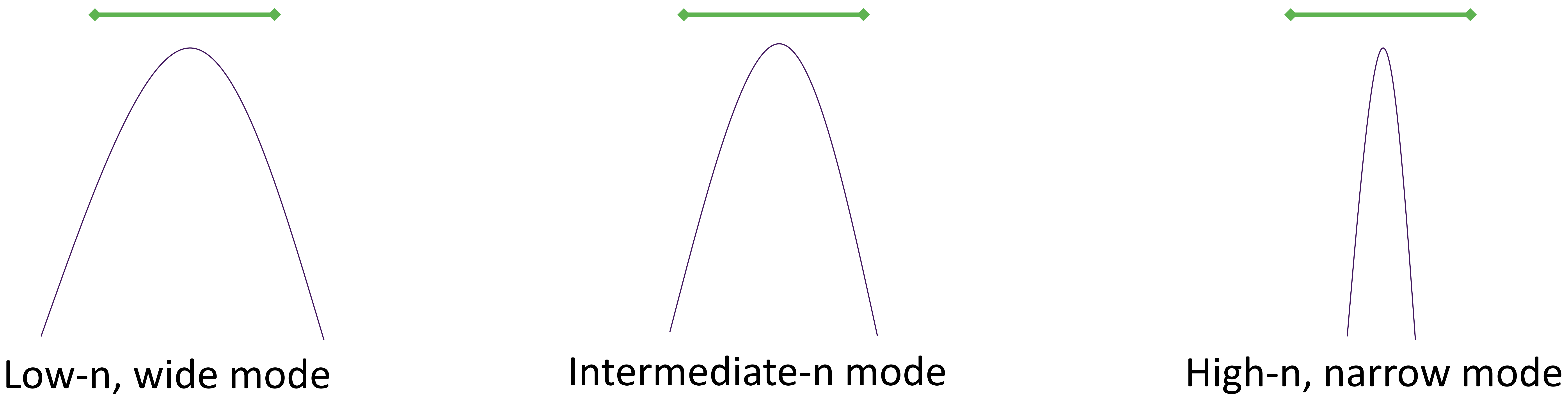

- Modes with width less than the alpha particle orbit width $\Delta_{\alpha,orbit}$ are ineffectively driven
- Most unstable modes are those for which $\Delta_{mode} \sim \frac{a}{nq} \sim \Delta_{\alpha,orbit} \sim \frac{q\,v}{\Omega_\alpha}$ $\Rightarrow$ $n_{max} \sim \frac{Z_\alpha e\, B_0 a}{m_\alpha\, q^2\, v}$
- Trend towards higher n has been seen experimentally [See Heidbrink PoP 2002]
- Higher field machines have higher mode numbers and narrower modes: reduces transport?

Conclusions and current work

Conclusions and current work

- The high field path to fusion energy is currently very active and offers interesting analytical questions

- High-performance high-B operation will affect AE stability physics

Current work

- I'm currently working with Peter Catto on developing a theory of alpha transport capable of treating both ripple and AE (hopefully preprint later this year!)
- Next, Peter will discuss a recent work of his about ripple transport
- In our work starts from a similar place, but must take a different course in order to properly include AE resonances (can't transit average, must include time dependence, etc.)
- Happy to talk this week!

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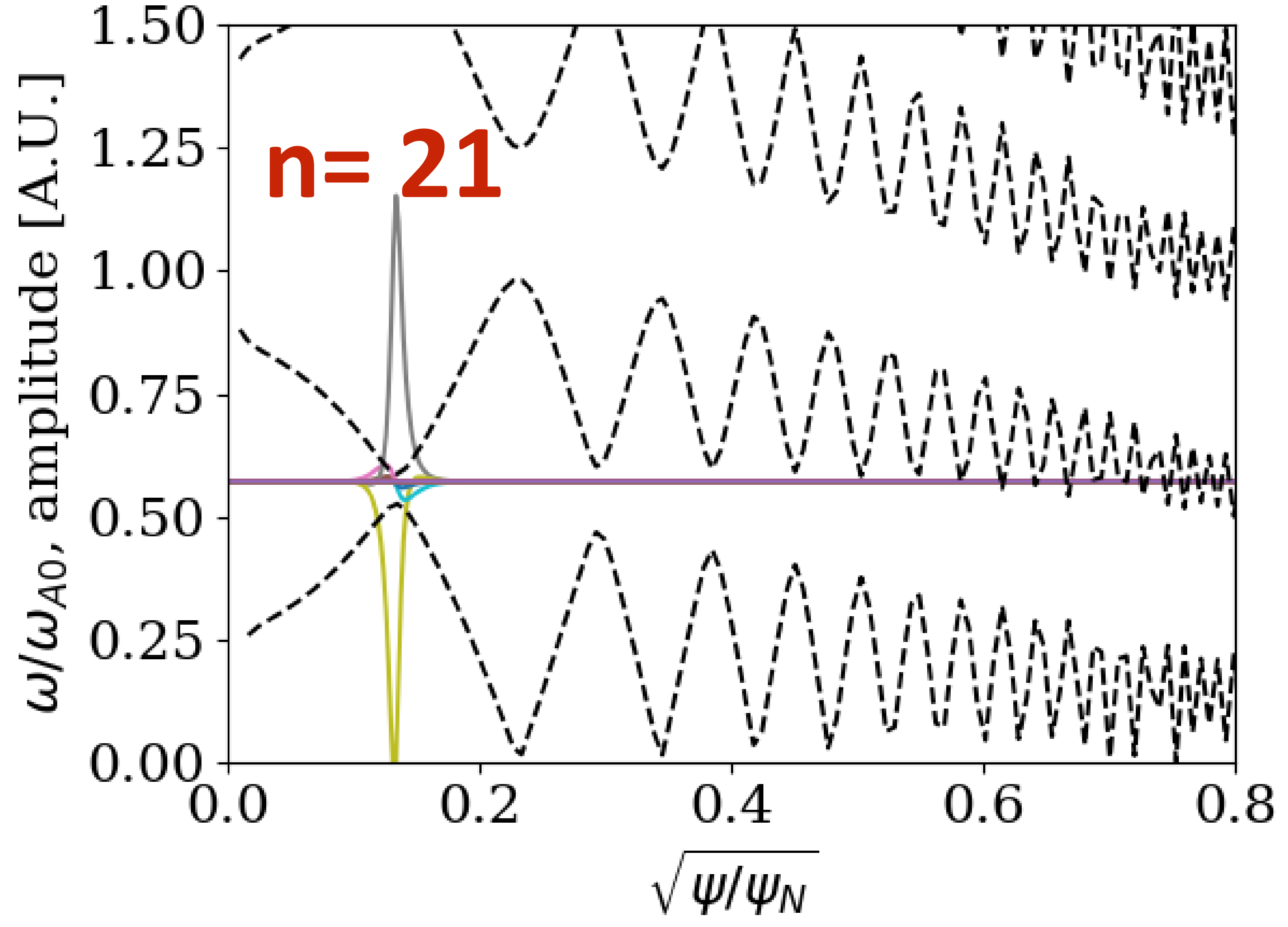
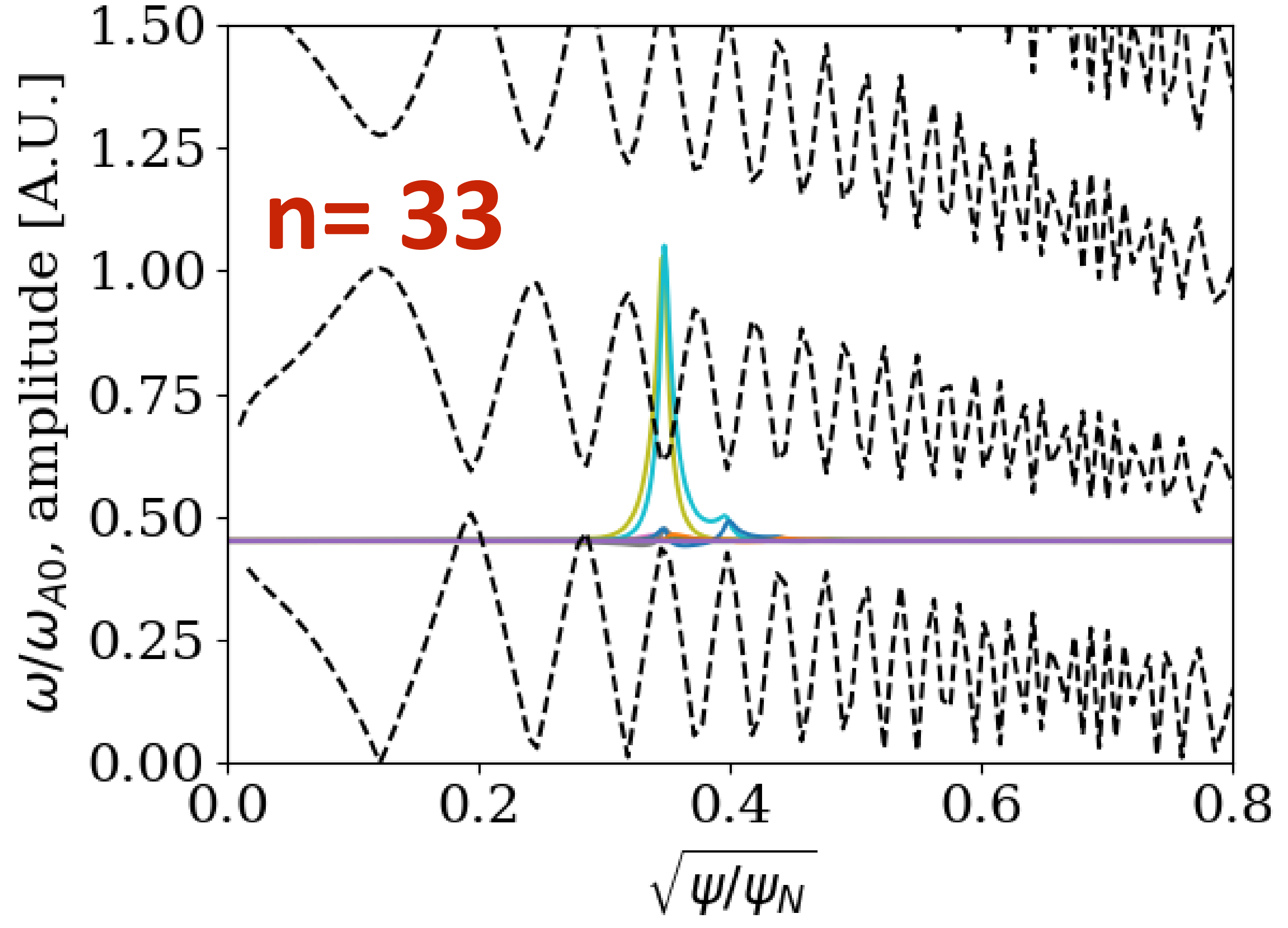
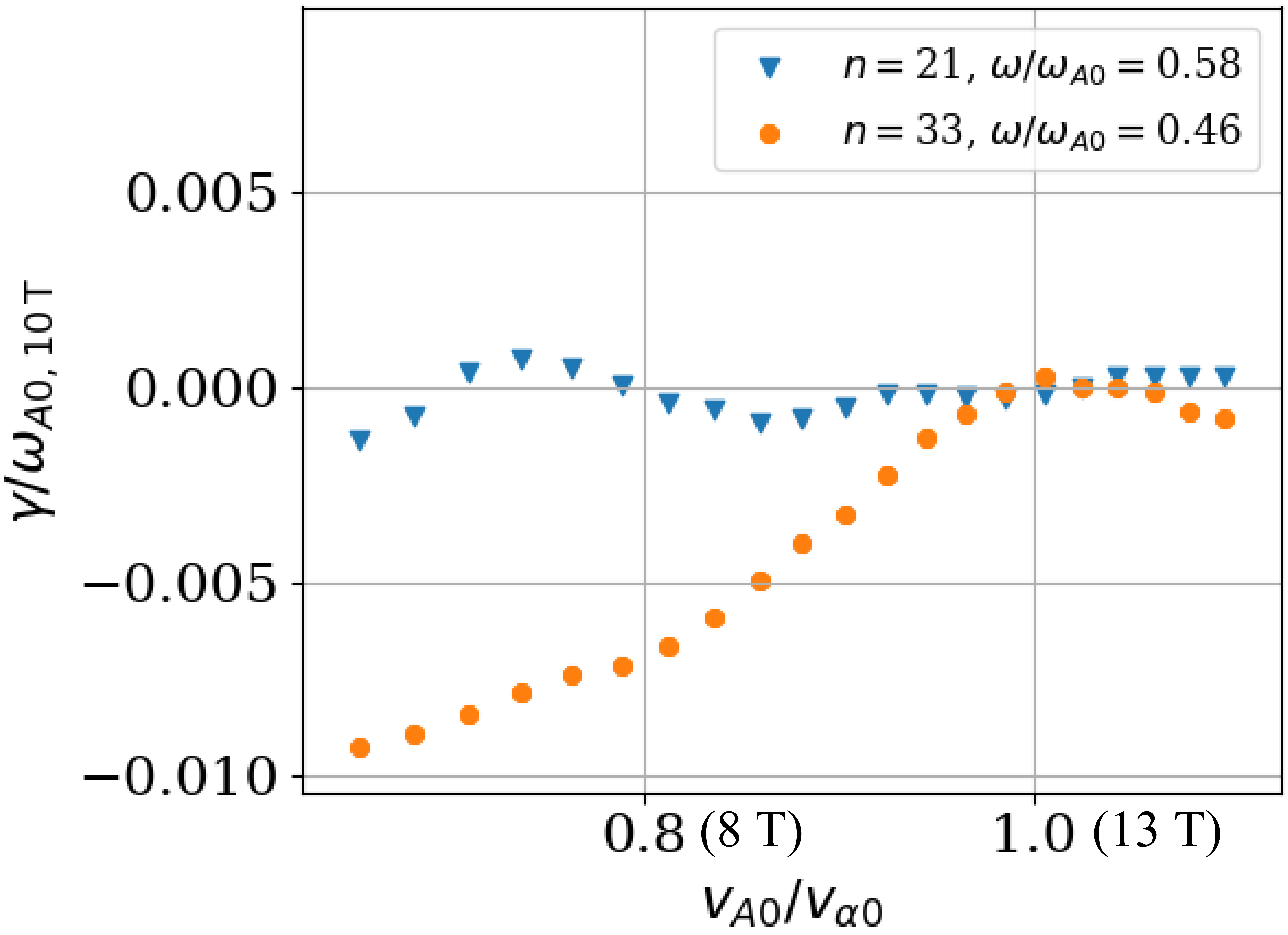
Backup slides

Cutoff of TAE resonances visible as AE switch from instability to stability







An Alfvén wave in toroidal geometry requires periodicity

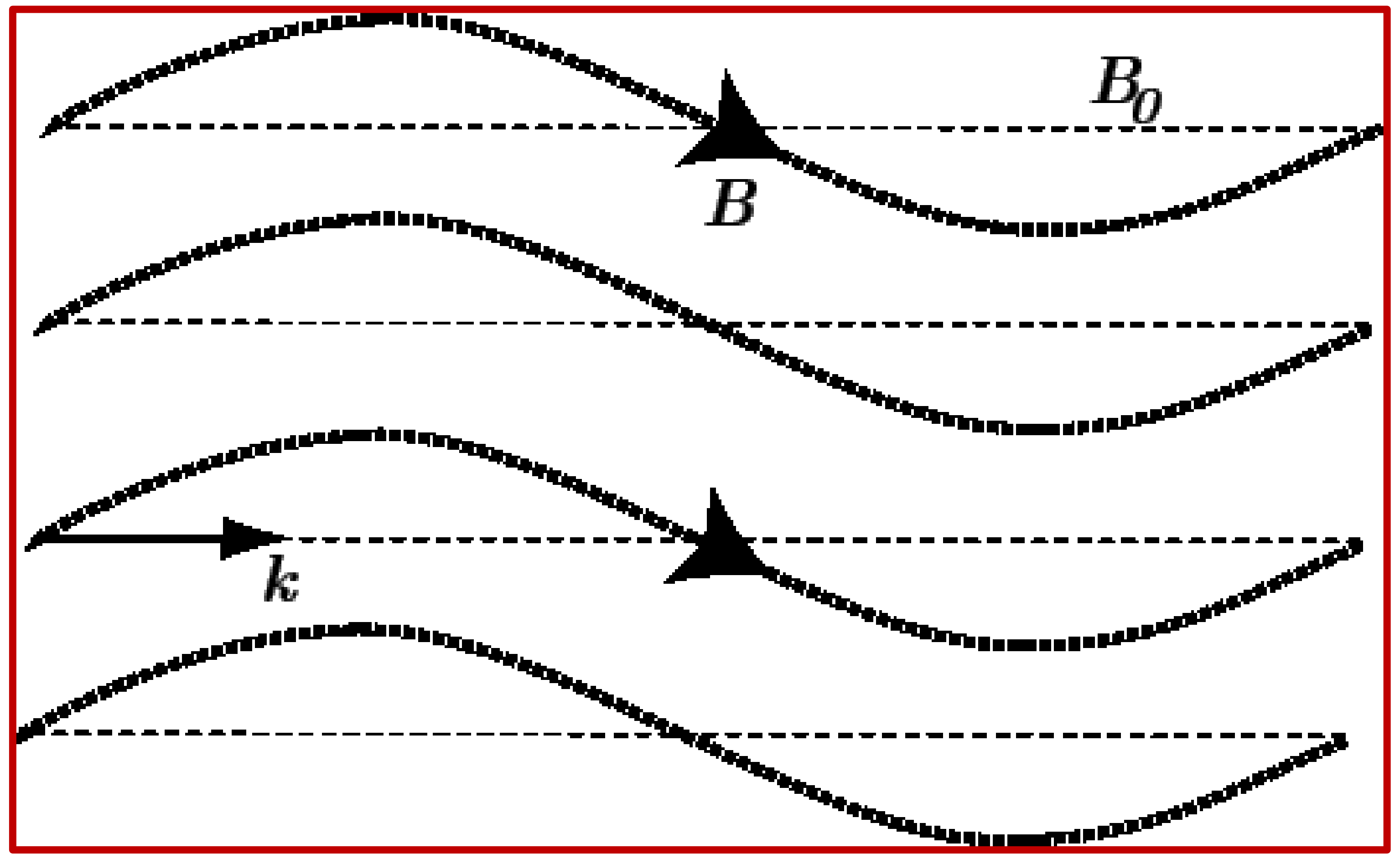
- Recall the shear Alfvén wave dispersion relation:

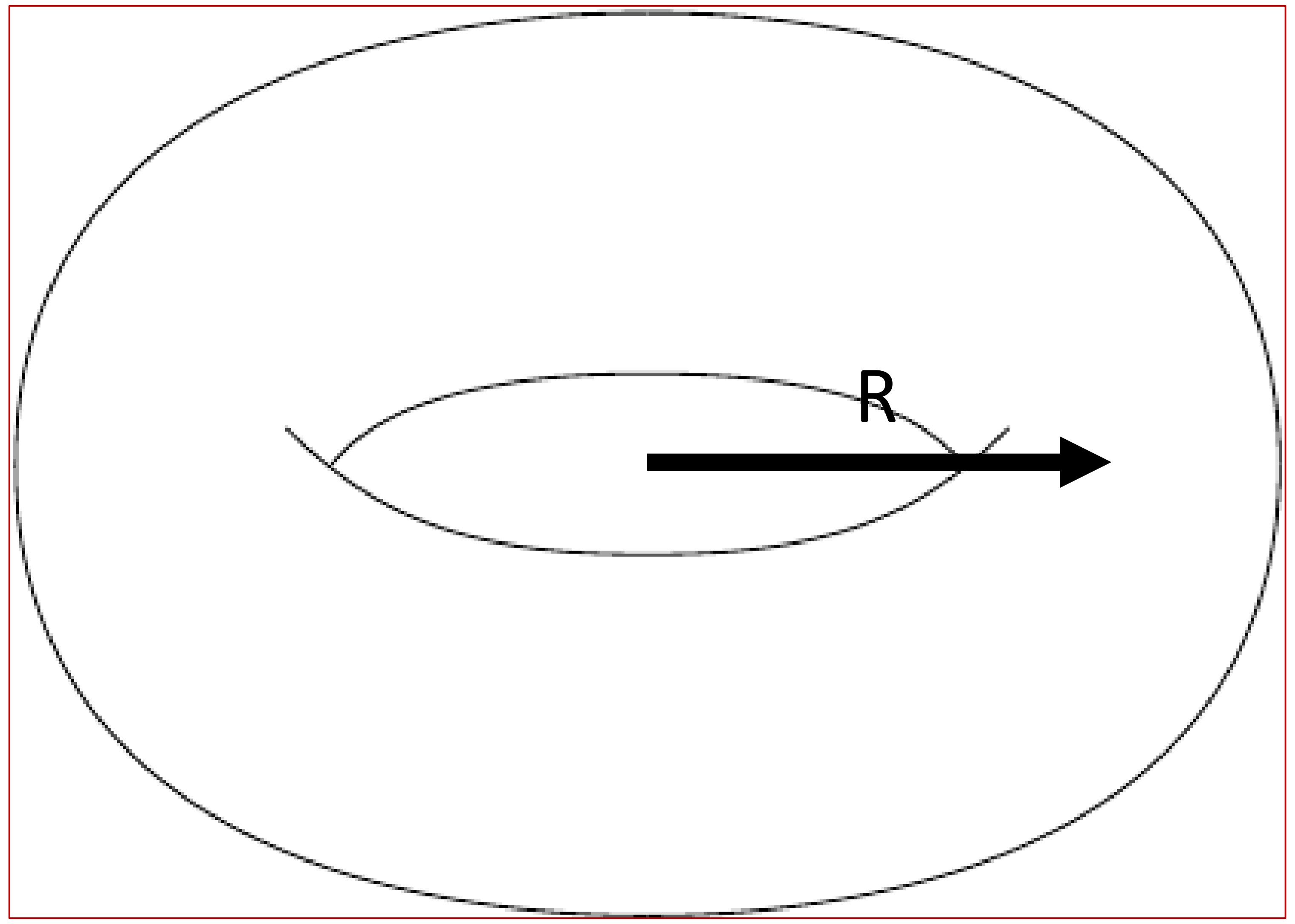
$$\omega = k_{\parallel} v_A,$$
$$v_A = \frac{B_0}{\sqrt{\mu_0 \Sigma n_i m_i}}$$

- In a tokamak this dispersion relation is modified to require periodicity:

$$k_{\parallel} = \frac{n - \frac{m}{q}}{R}$$

  - (Consider that a field line rotates azimuthally $1/q$ times in an axial periodicity length $2\,\pi R$)



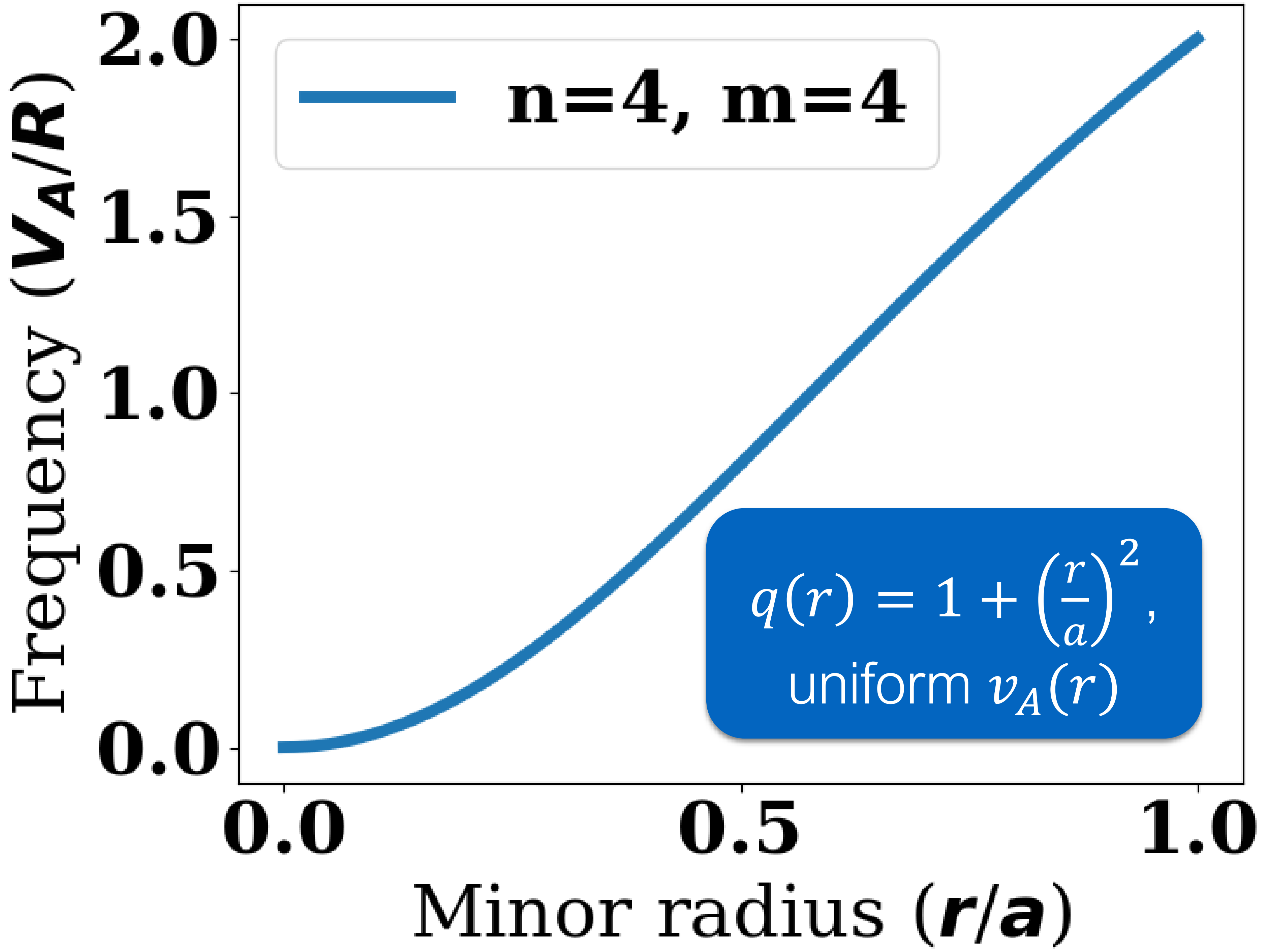


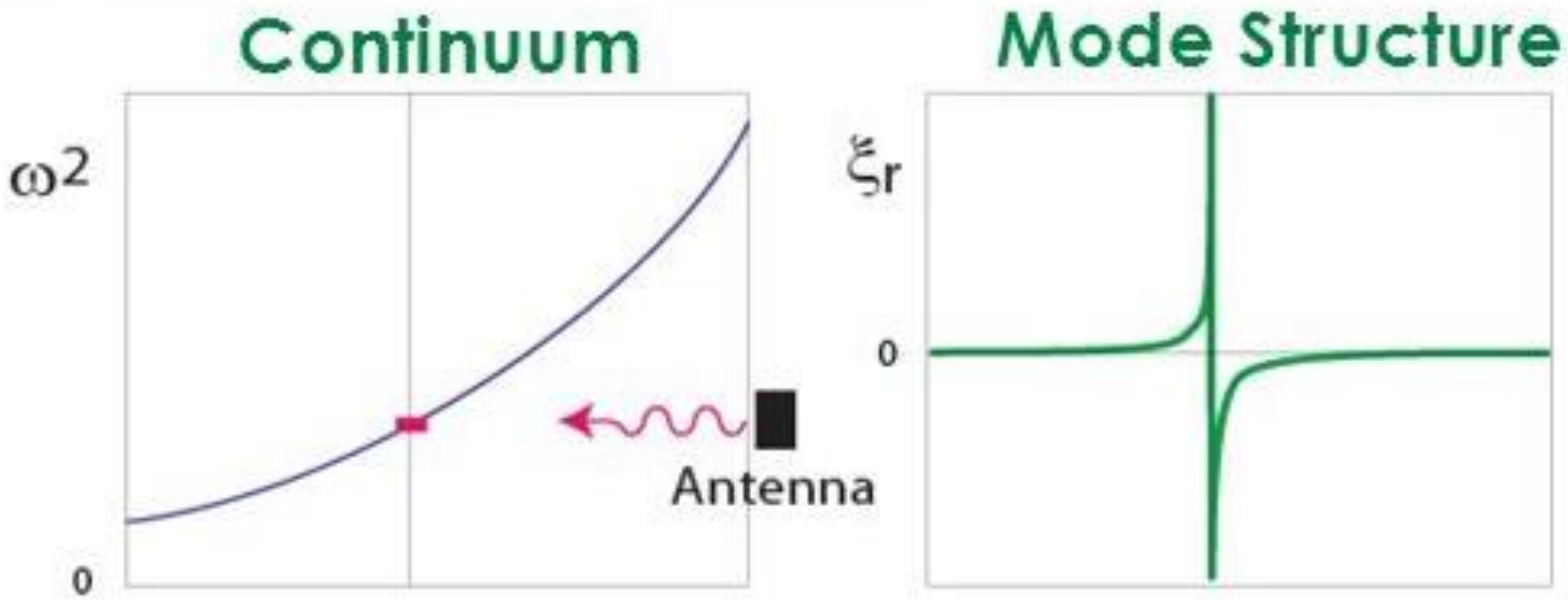
Position-dependent phase velocity causes continuum damping

- Dispersion relation becomes:

$$\omega(r) = \frac{n - \frac{m}{q(r)}}{R} v_A(r)$$

- A mode of a given frequency will be resonant at one point
- Its eigenfunction will have a singularity
- Such modes phase mix and are dissipated [See *Principles of Magnetohydrodynamics*, Goedbloed and Poedts, Chapter 11]
- But we've so far neglected an important effect…





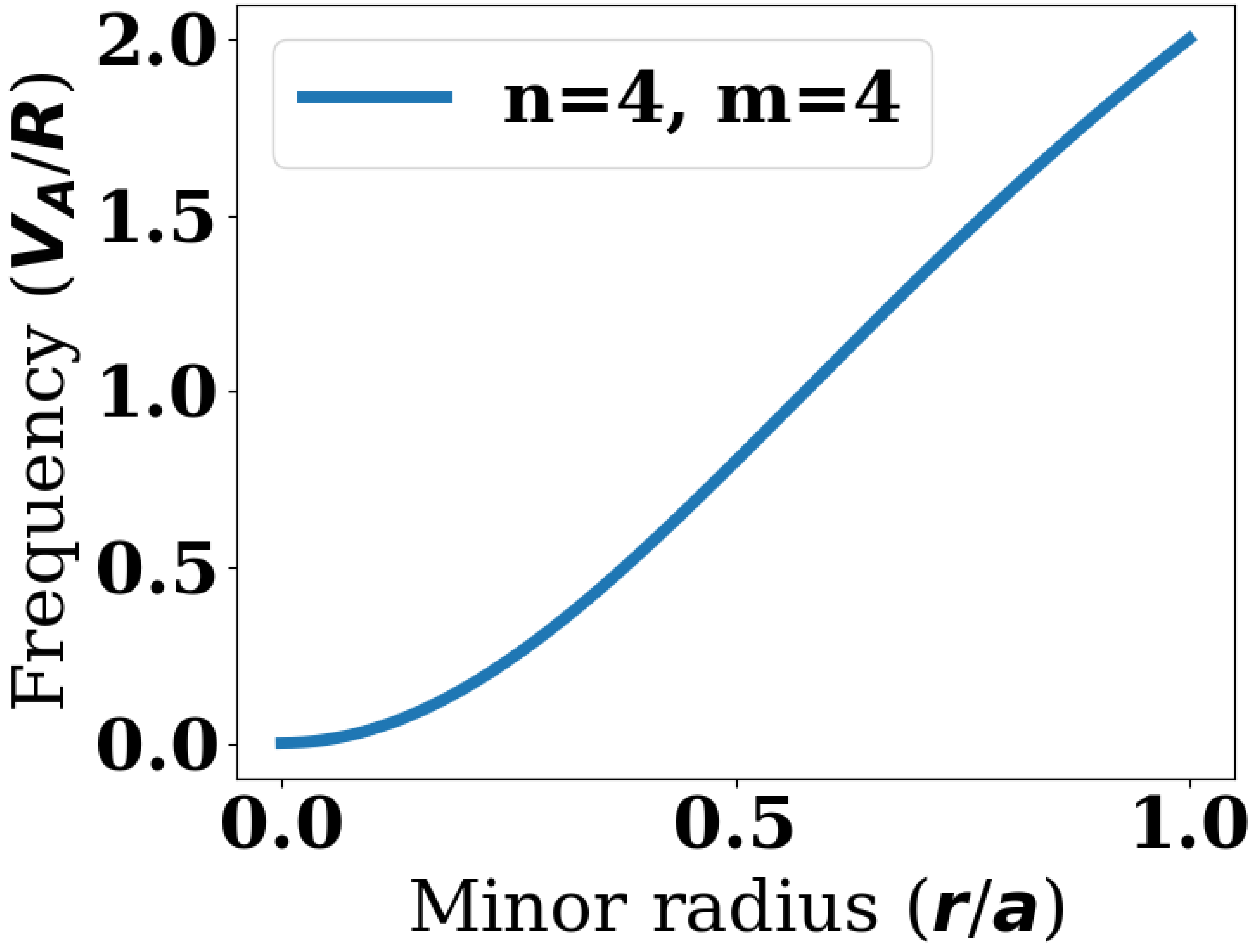
Toroidicity (and other effects) introduce frequency gaps

- Recall from physics (optics, condensed matter):
  - Periodic variation in phase velocity ω/k creates a **band gap** at the **Bragg frequency** $\omega = \frac{\pi\bar{v}}{\Delta z}$
  - ($\bar{v}$ = average phase velocity, Δz = length of periodicity)

- Tokamak $B \propto R^{-1}$, so Alfvén speed varies along field line with $\Delta z = q2\pi R$

- We have a gap at:

$$\omega = \frac{v_{A0}}{2\,q\,R}$$



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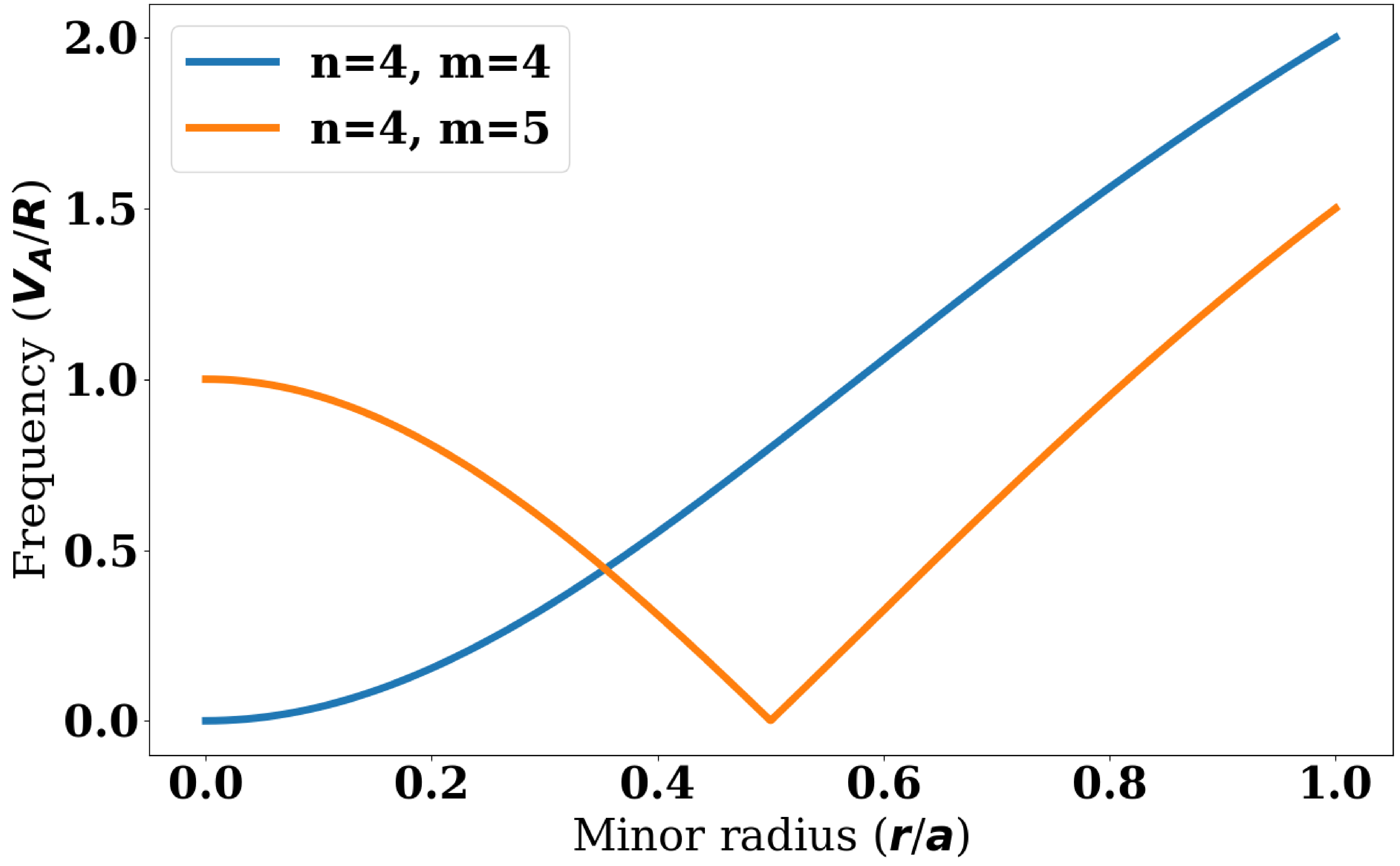
$$k_{\parallel,1} = \frac{n - \frac{m}{q(r)}}{R} = k_{\parallel,2} = -\left(\frac{n - \frac{(m+1)}{q(r)}}{R}\right) \Rightarrow q = \frac{m + \frac{1}{2}}{n},\ \omega = \frac{v_{A0}}{2\, q\, R}$$



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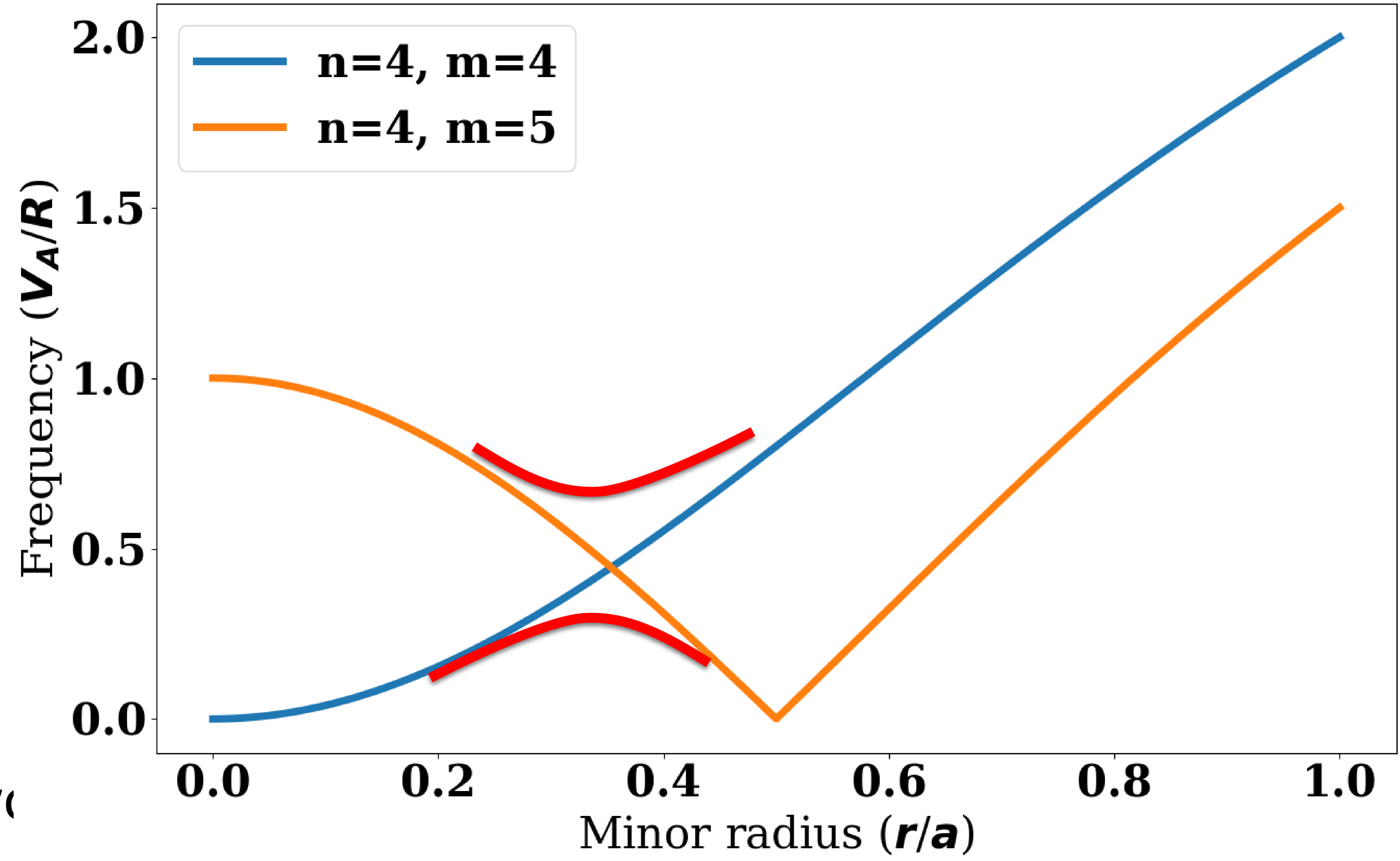

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Eigenmodes in the frequency gap are not continuum damped

- Similar gaps will be introduced by other effects
- The result is a continuum spectrum with a variety of gaps