The Sun as an astrophysical turbulence lab

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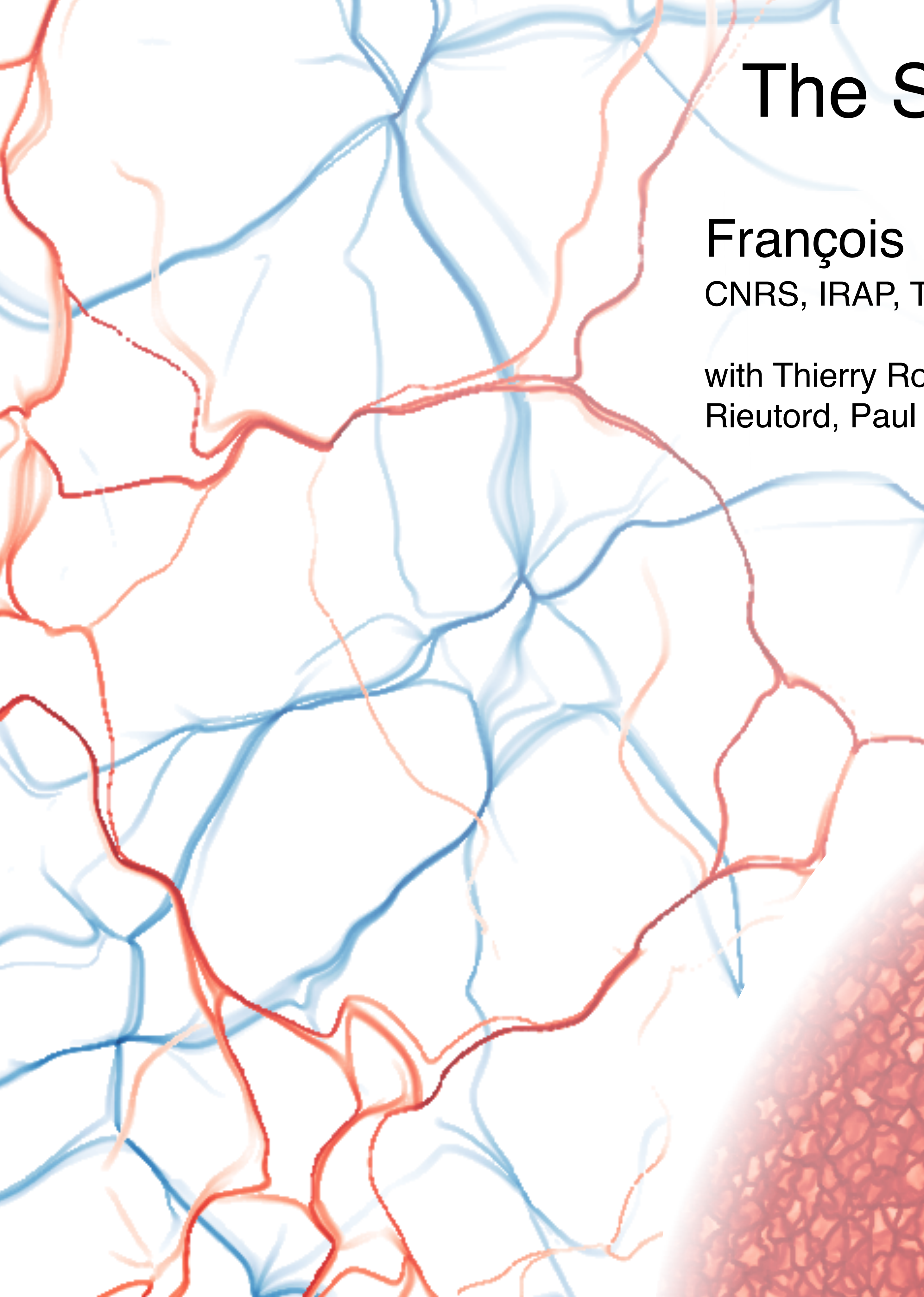
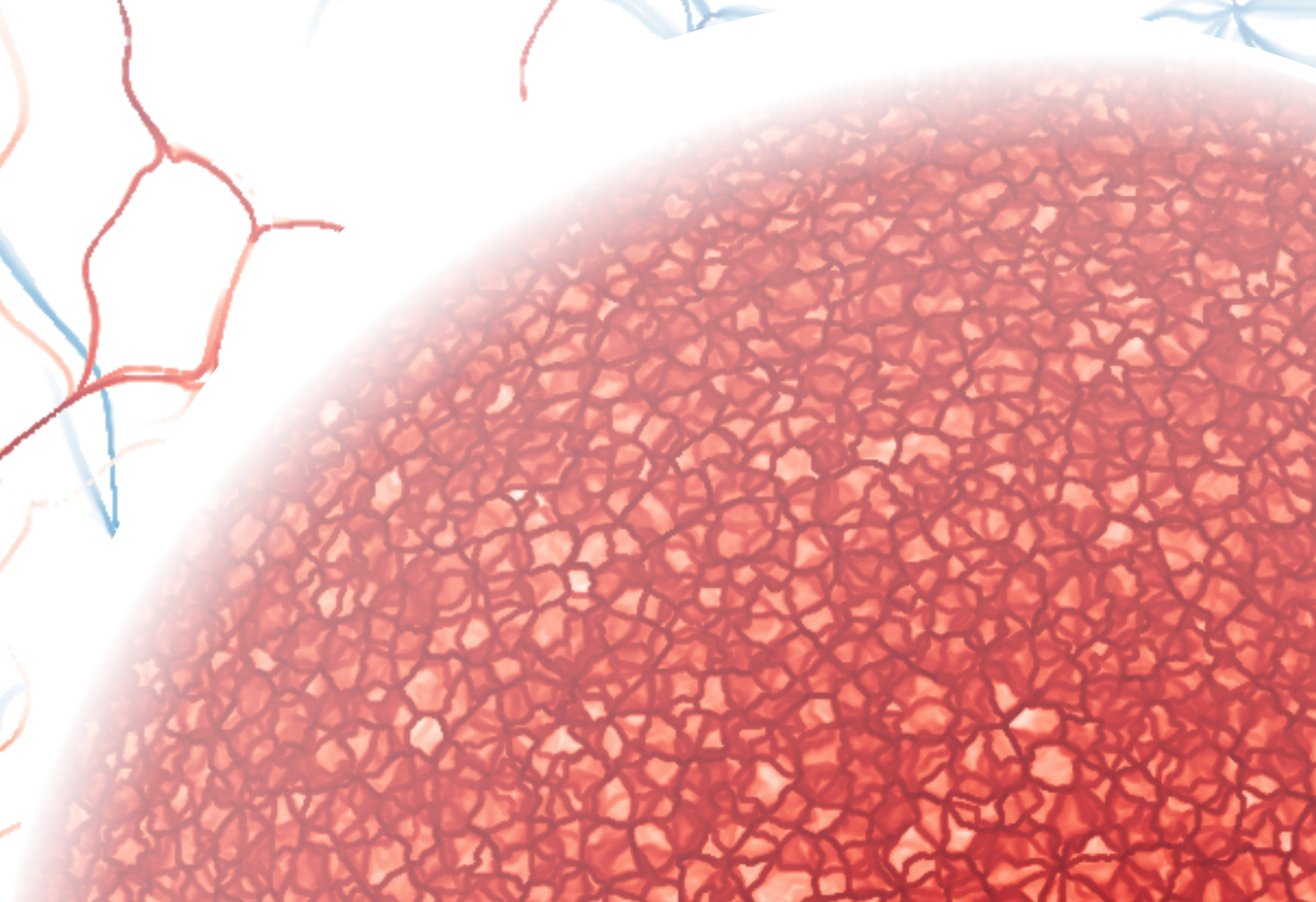
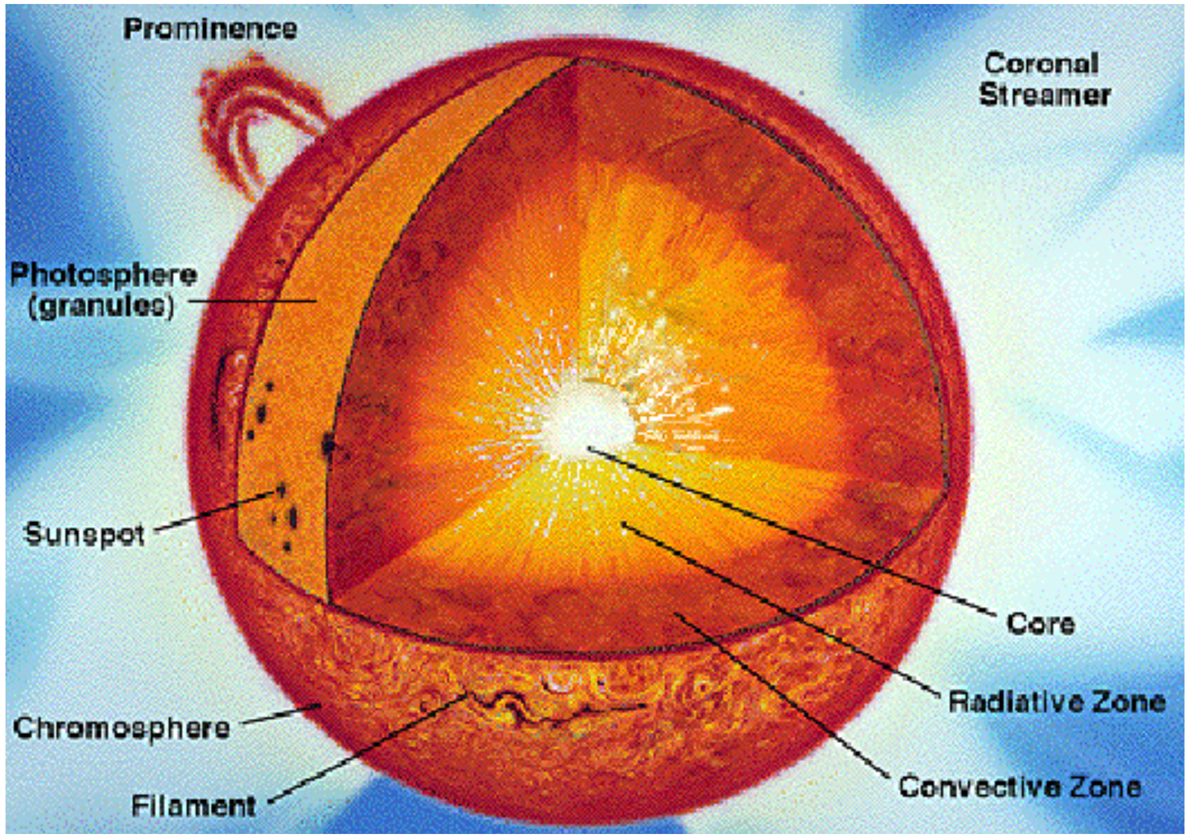




Motivations

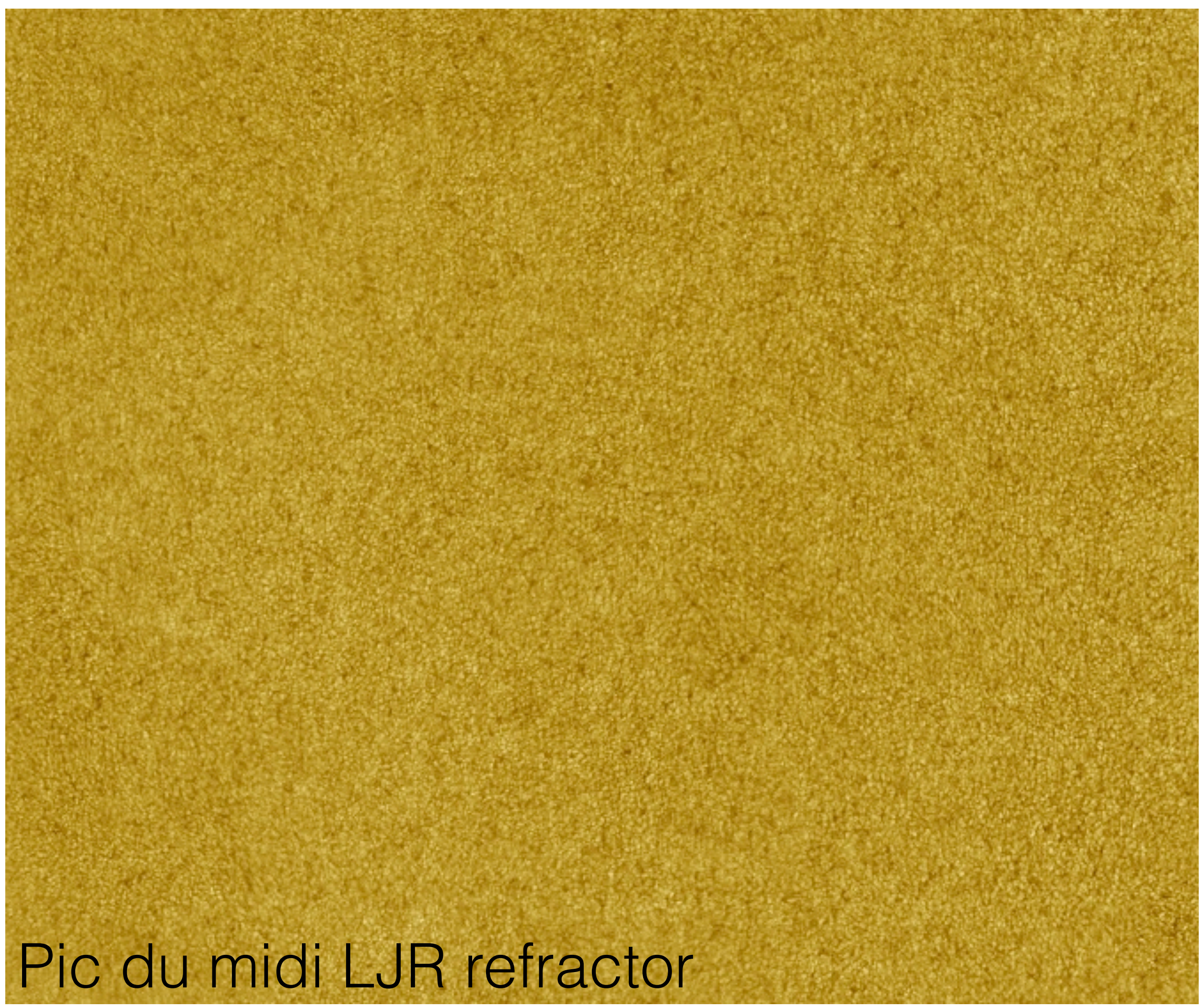
- The Sun is the closest astro system in a turbulent fluid MHD state
- Observable regions with different plasma β and B-field strengths
  - Solar convection zone (SCZ), corona
  - Active regions / quiet Sun

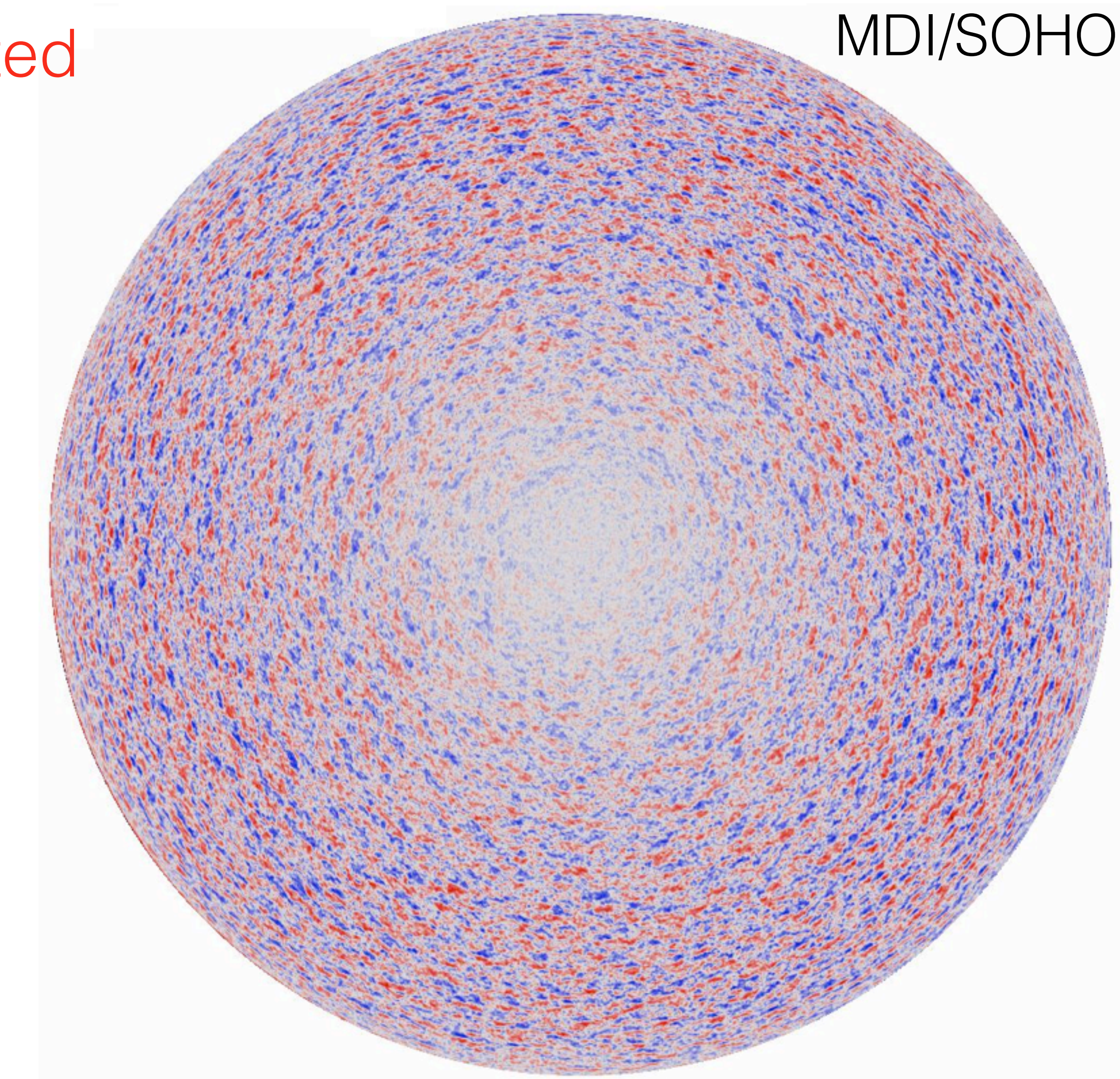


- Typical SCZ parameters
  - Re = $L_{inj}\, u_{rms} / \nu \sim 10^{10}$-$10^{12}$
  - Rm = $L_{inj}\, u_{rms} / \eta \sim 10^{6}$-$10^{10}$
  - Pm = $\nu / \eta \sim 10^{-6}$-10^{-2}
  - E = $\nu/(\Omega R^{2}) \sim 10^{-15}$
  - Ro $\sim u_{rms} / (L_{inj}\, \Omega)$ down to 10^{-1}
- Best available astrophysical fluid dynamics lab
  - Turbulent convection, transport/diffusion, large/small-scale dynamo, rotation

Solar supergranulation problem

- Detected since 1954 as a flow pattern, using Doppler imaging
- Light intensity imaging only shows smaller-scale granulation
  - Physical origin of SG long-debated

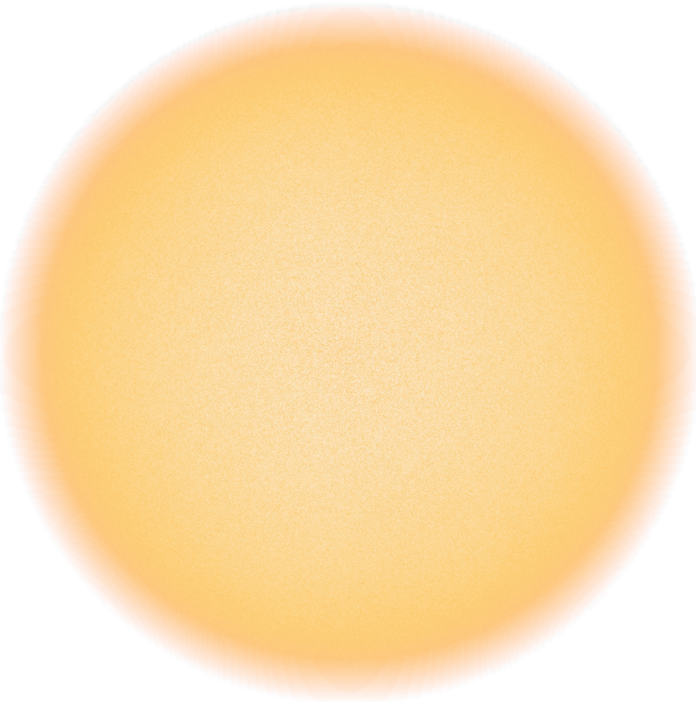


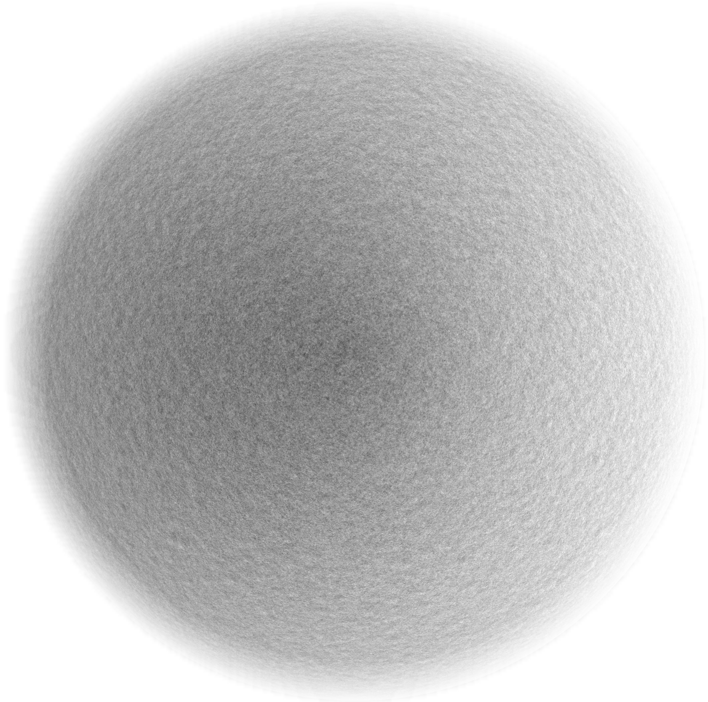


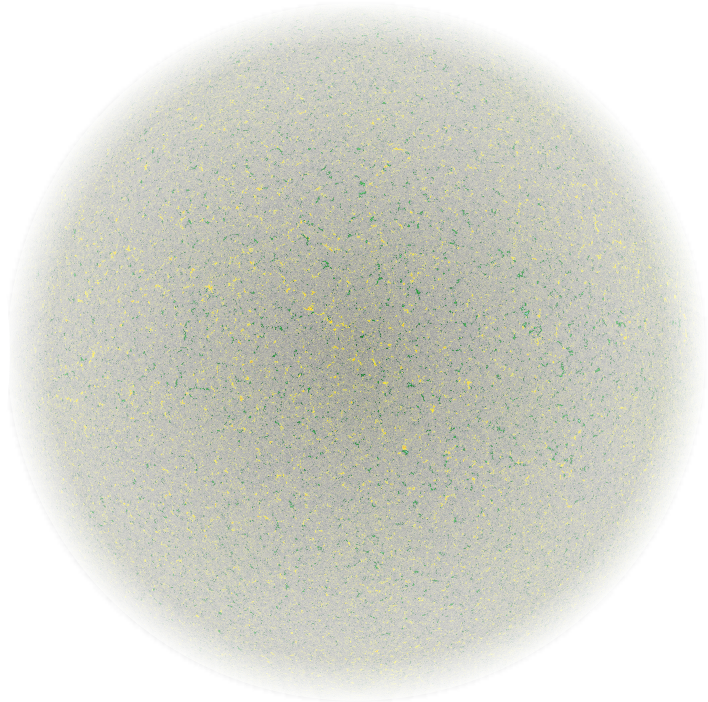
Rincon & Rieutord, Living Rev. Sol. Phys. 2018

The Solar Dynamics Observatory (SDO)

- SDO monitors the Sun 365d / 24h with three instruments
  - Helioseismic and Magnetic Imager (HMI) - Photosphere
  - AIA, EVE - Atmosphere, Corona
- HMI provides full-disk light intensity, Doppler and magnetic maps
  - 4096^2 pix: 45 seconds time-sampling with 350 km^2 resolution



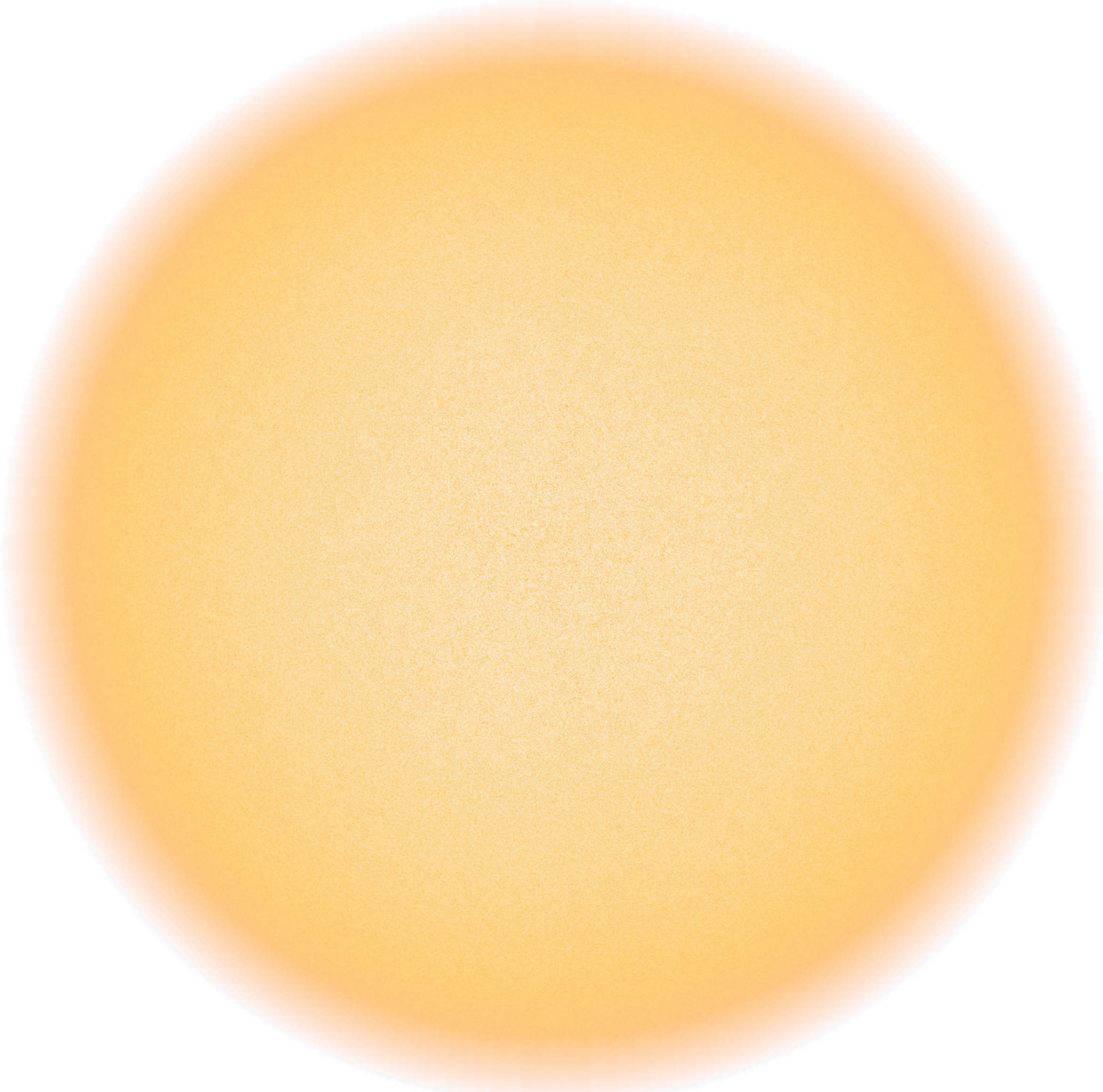


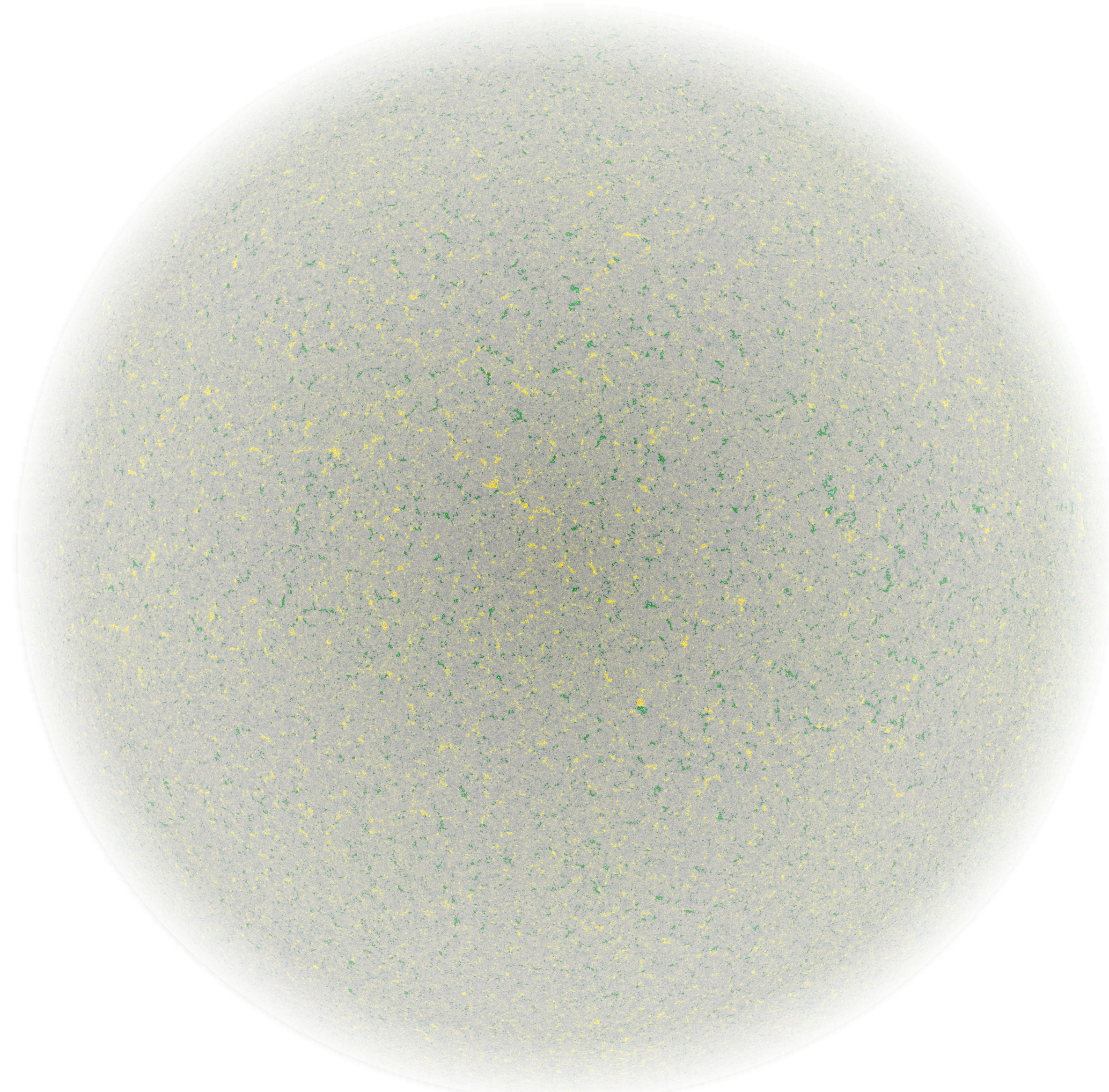


- All high-resolution data is public

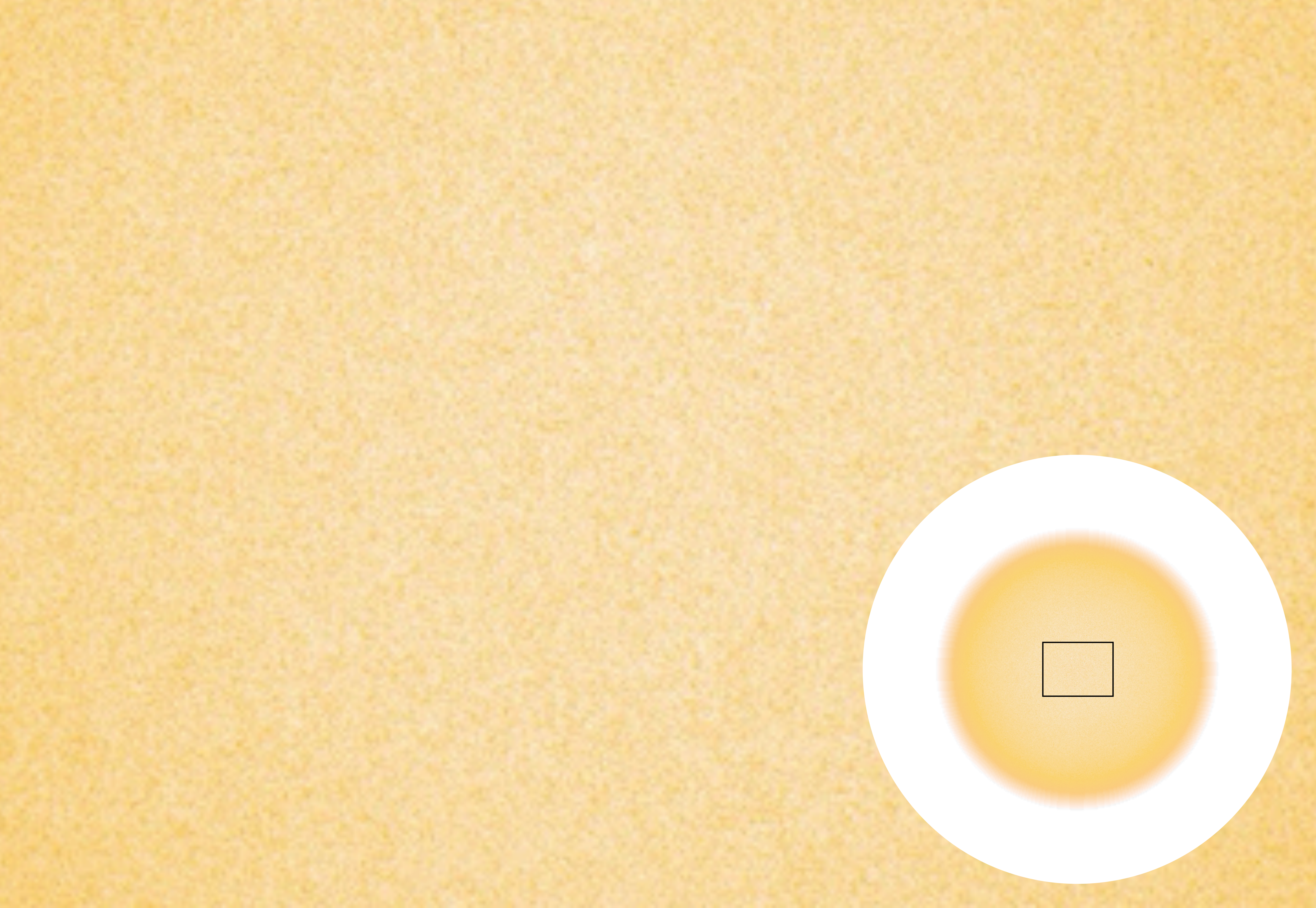
The quiet photosphere

- First goal: study the statistically steady turbulent surface flow
  - Use quiet observation periods with as few active regions as possible
    - Oct. 15th, 2010 (24h), Nov. 26 - Dec. 1 2018 (6 days uninterrupted)

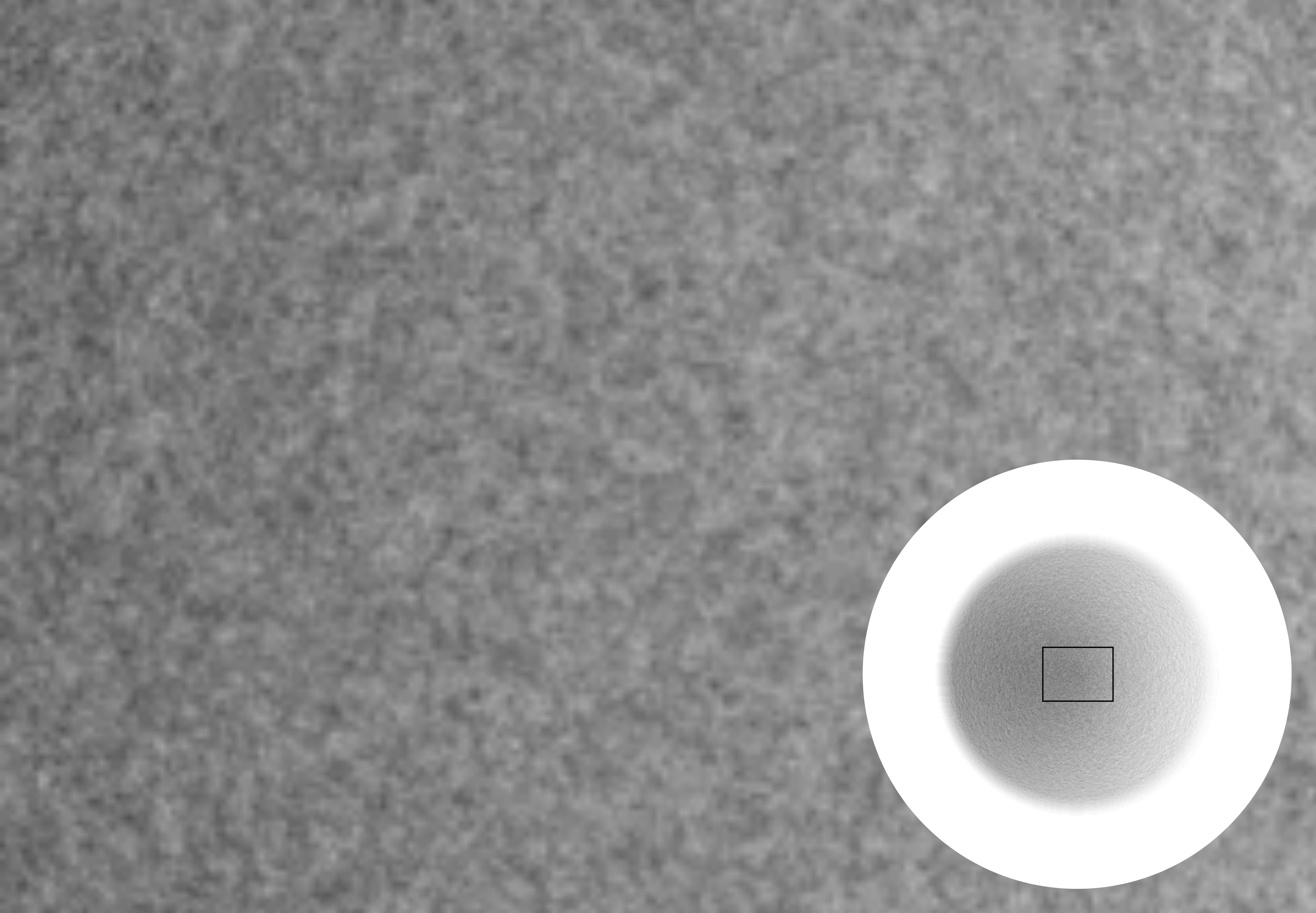


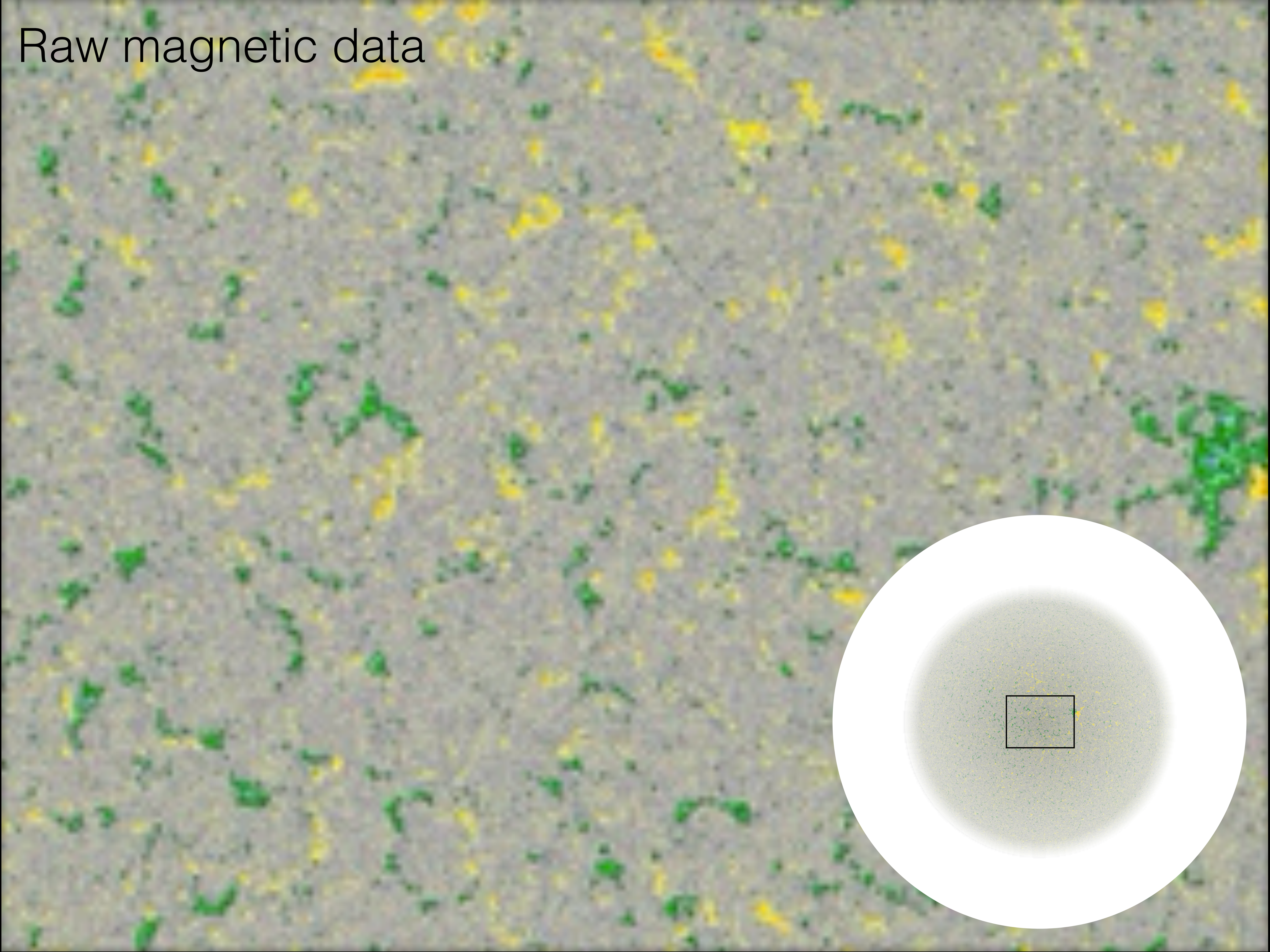


Raw white-light intensity data

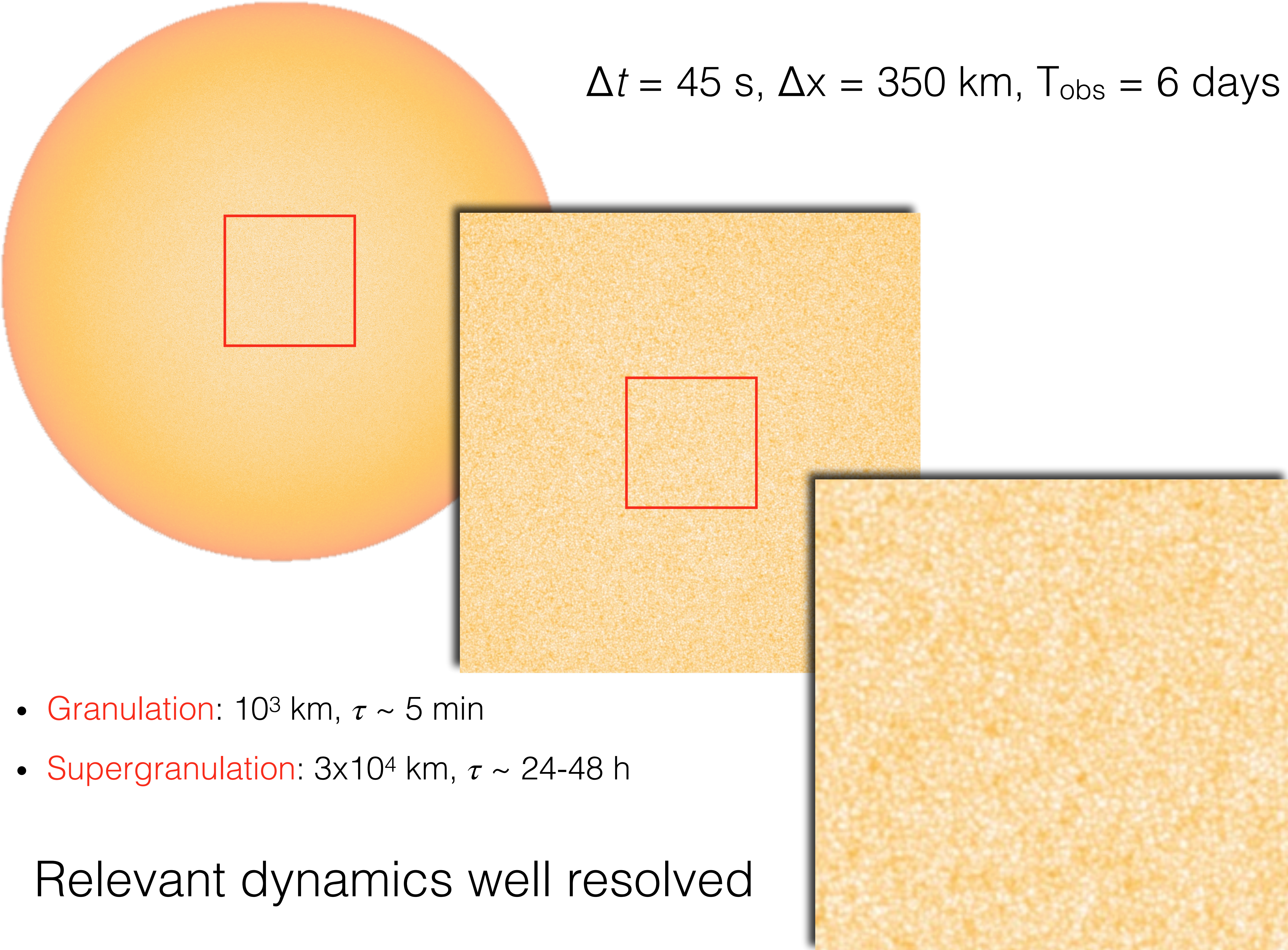


Raw Doppler data



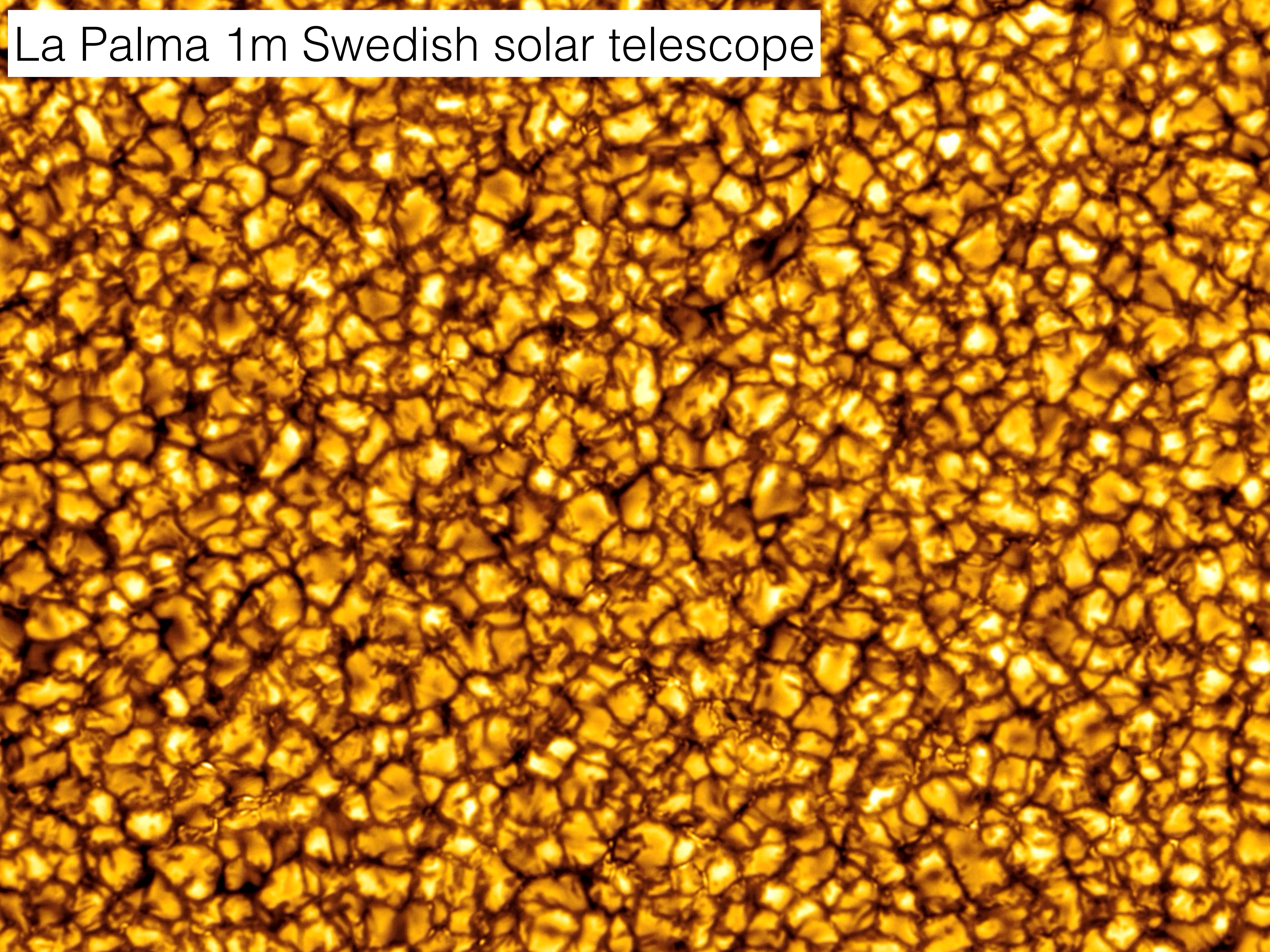

Raw magnetic data

Δt = 45 s, Δx = 350 km, T_{obs} = 6 days



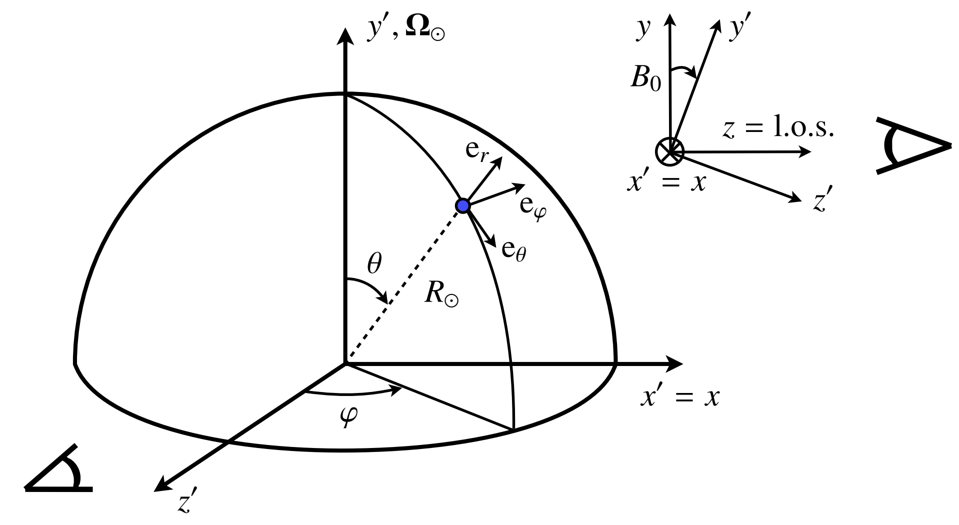
- Granulation: 10^3 km, $\tau \sim 5$ min
- Supergranulation: 3×10^4 km, $\tau \sim 24\text{-}48$ h

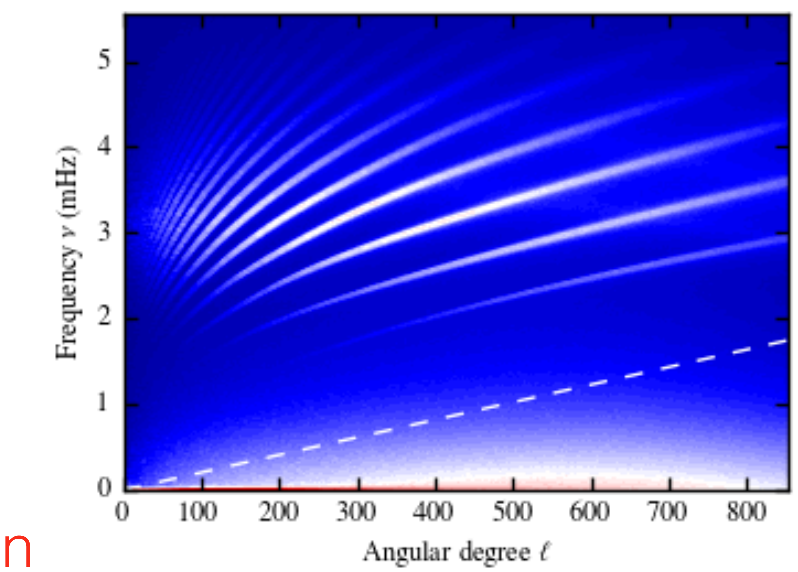
Relevant dynamics well resolved

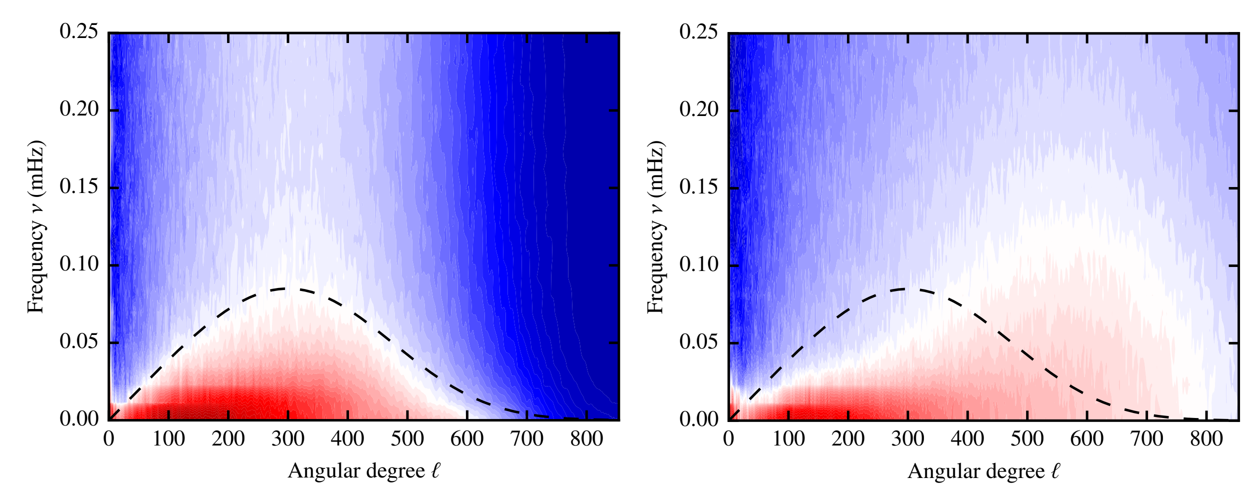

La Palma 1m Swedish solar telescope

Velocity-field inference

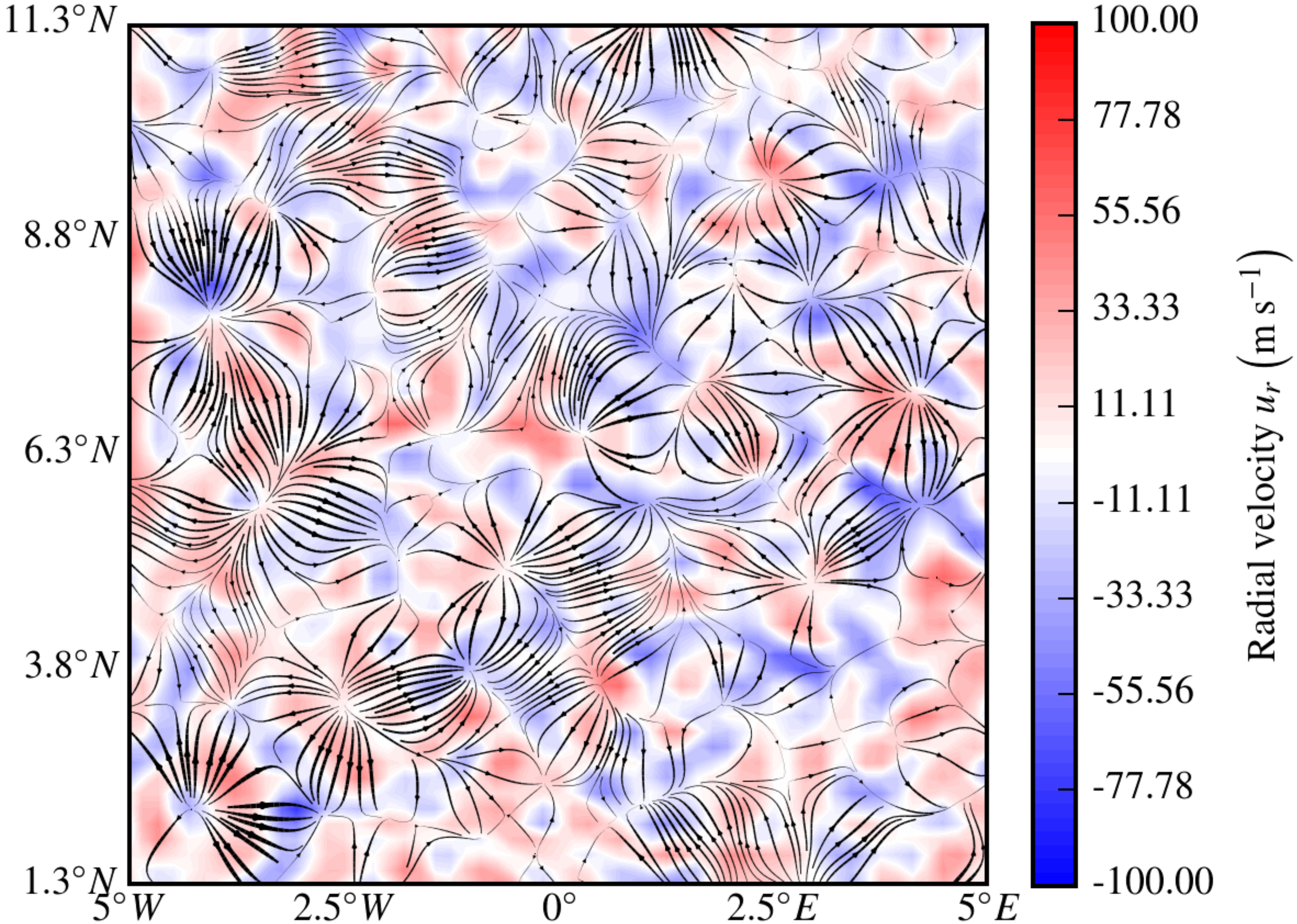
- In-plane eulerian velocity field (u_x, u_y)
  - Derived from Coherent Structure Tracking (CST) of granule motions
  - Δx = 2500 km, Δt = 15-30 min
- Out-of-plane Eulerian velocity field u_z
  - Derived from Doppler measurements
  - Raw resolution Δx = 350 km, Δt = 45 s
- Reduction of final data
  - Harmonize resolution Δx = 2500 km, Δt = 15-30 min
  - Project on Gauss-Legendre-Fourier grid
  - ℓ-ν filter (removal of 5-min p-modes)
  - Transform (u_x, u_y, u_z) to $(u_r, u_\theta, u_\varphi)$







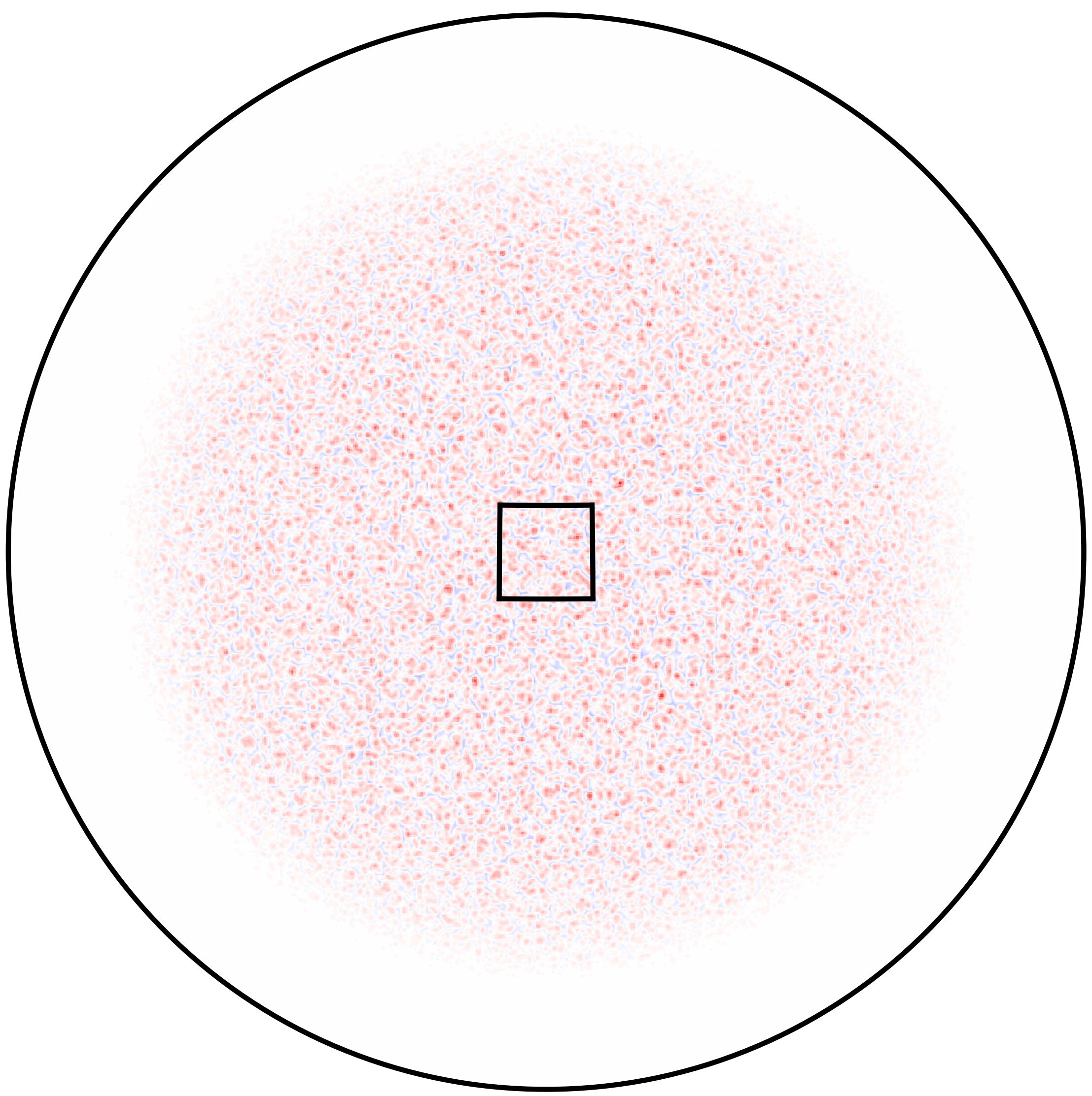
Local velocity field snapshot

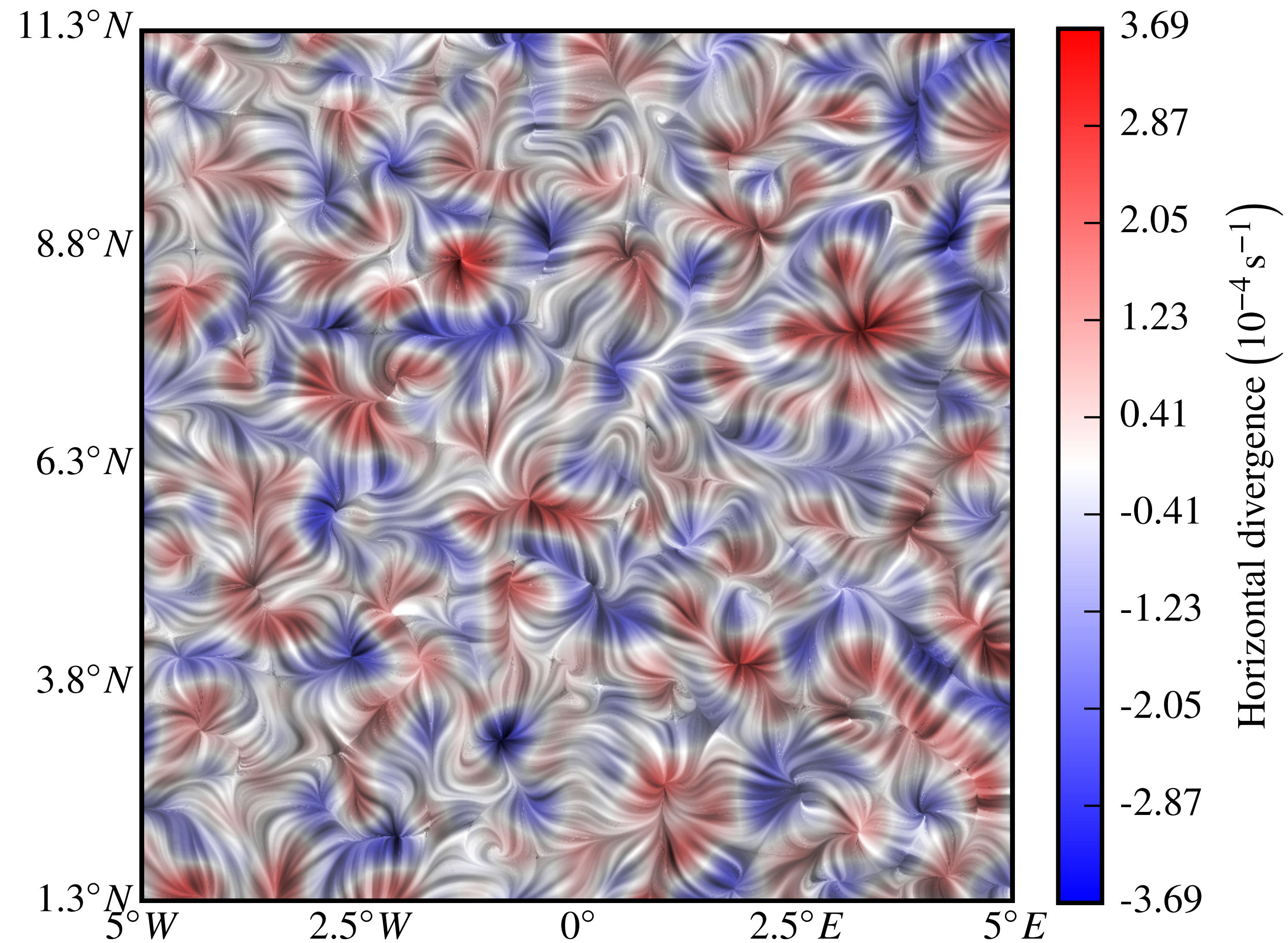


Supergranulation: 3×10^4 km, τ ~ 24-48 h, u_h ~ 400 m/s, u_v < 30 m/s

Spheroidal component

(horizontal divergences)

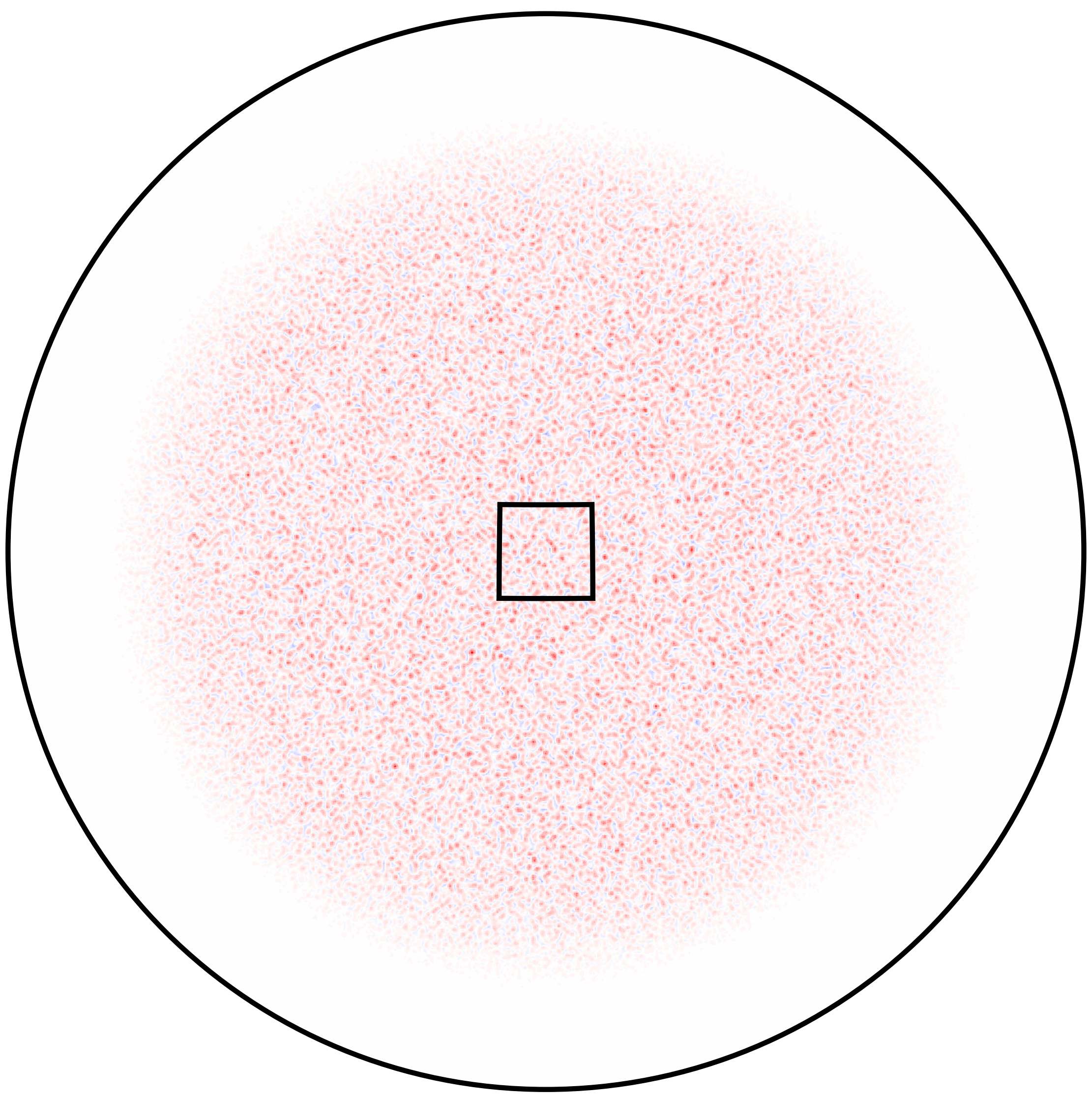


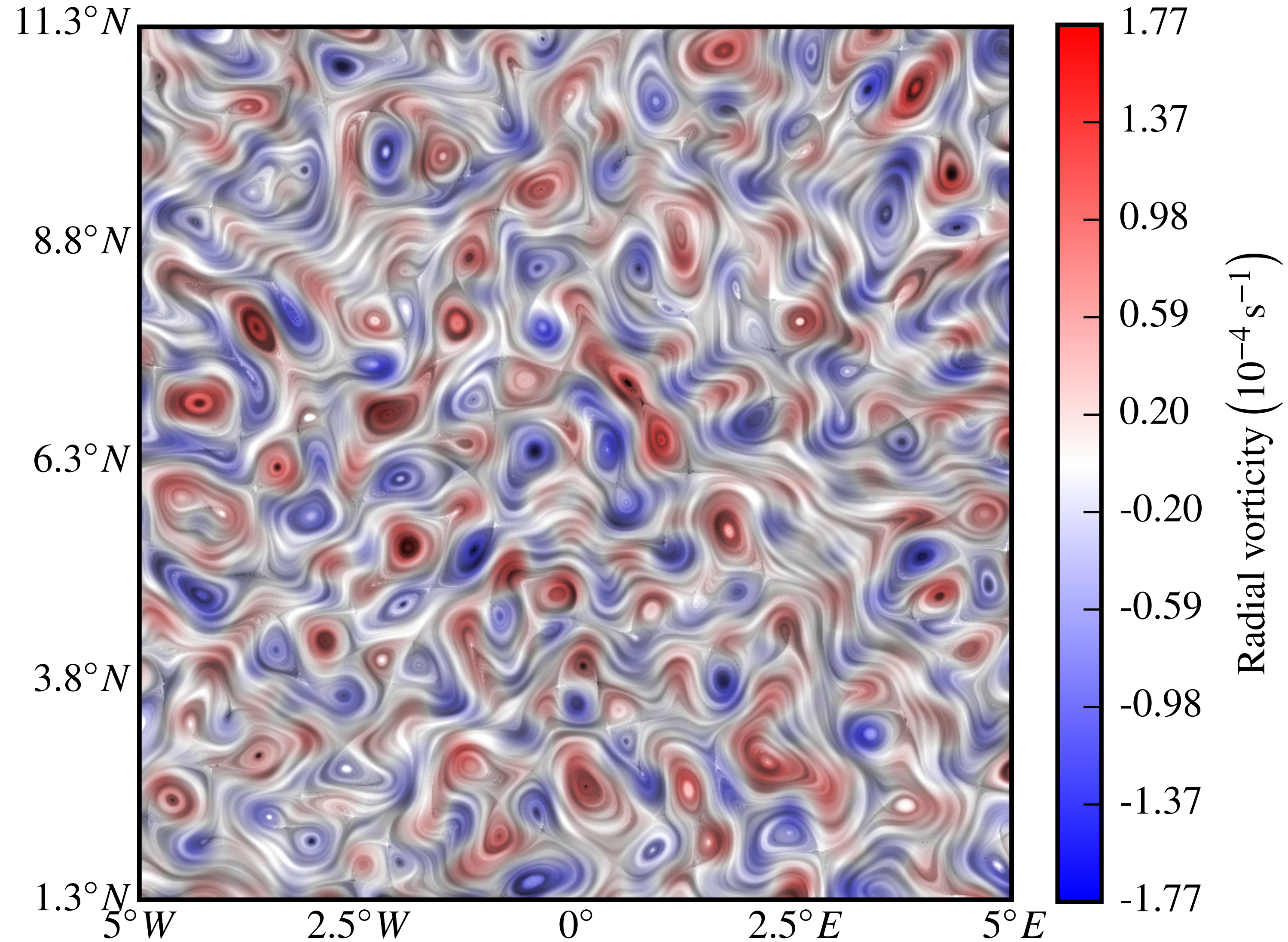


WPI Vienna, August 2019

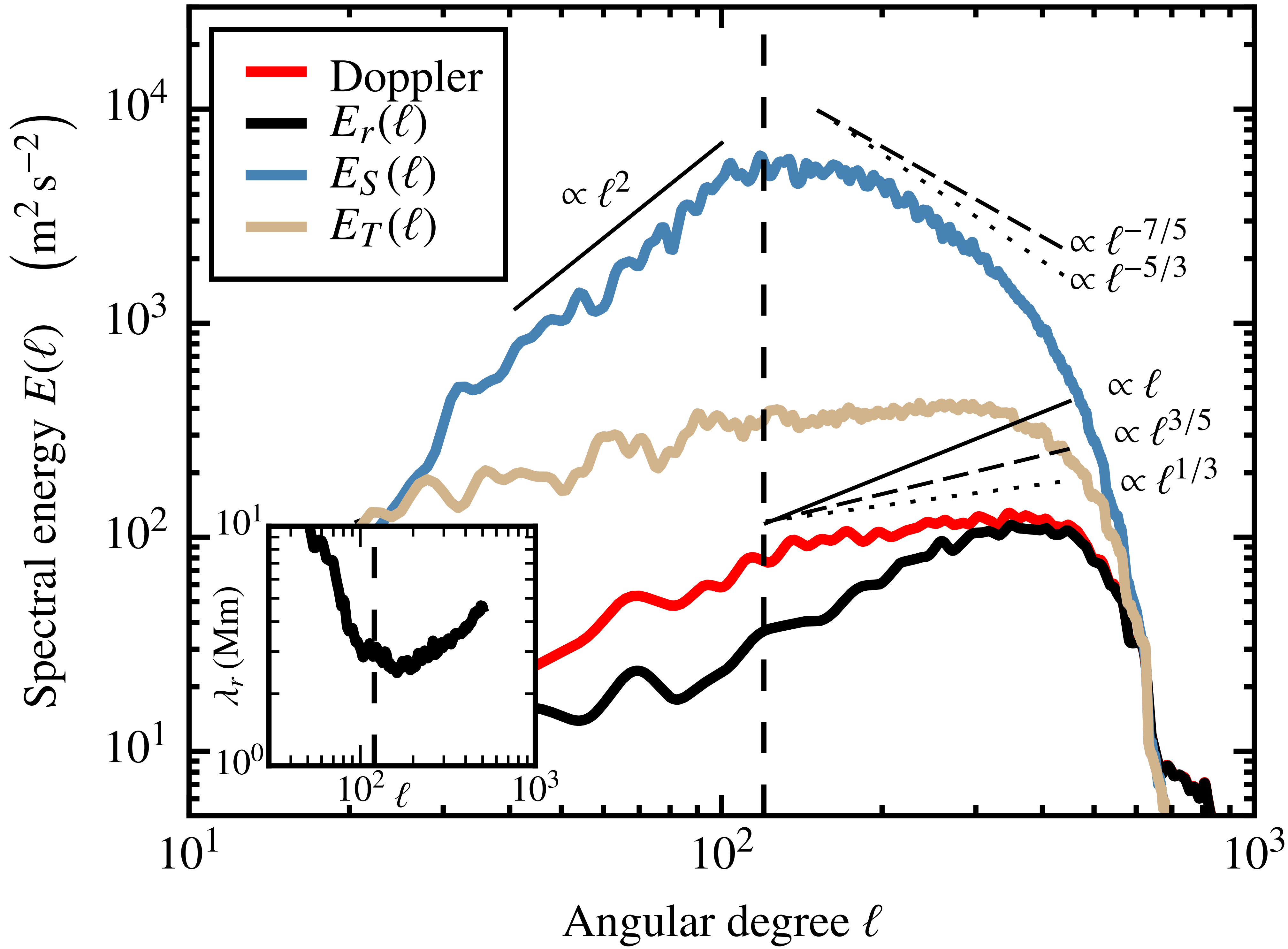
Toroidal component

(vertical vorticity)





Velocity spectrum



Rincon et al, A&A 2017

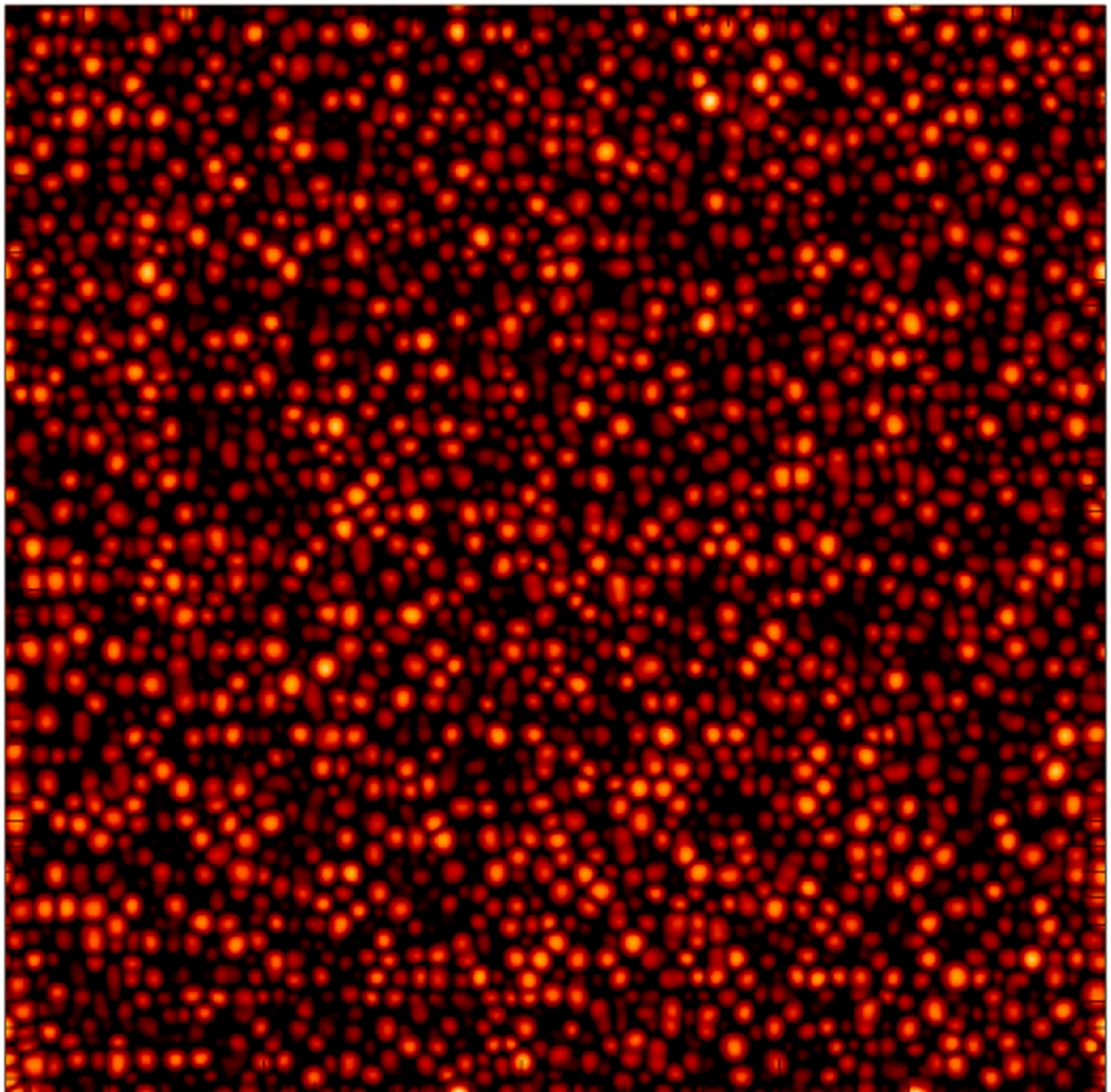
Main observational conclusions so far

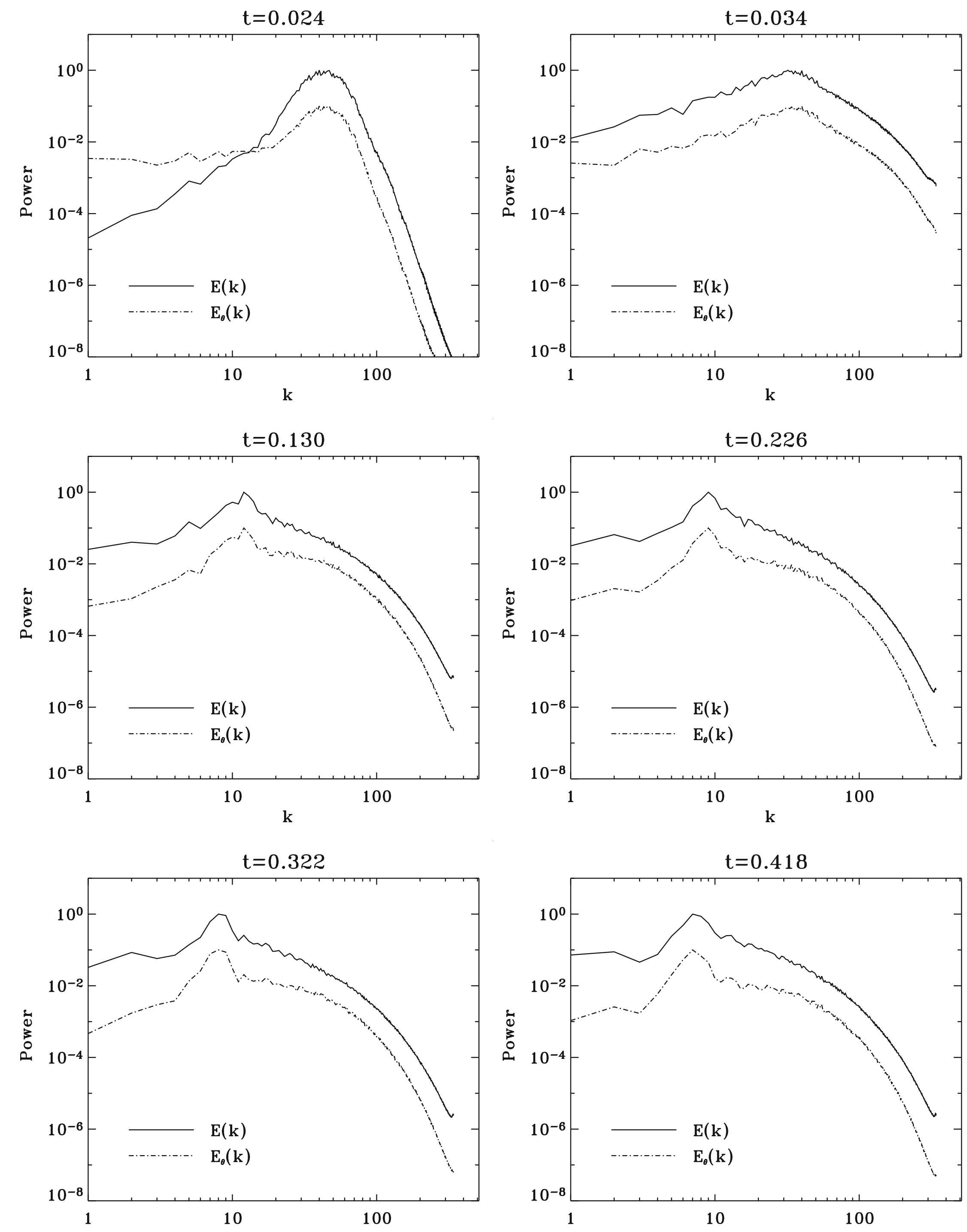
- Surface flows are dominated by horizontal divergences
  - Strong correlation with upflows
- On scales larger than a few 1000 kms, the flow is anisotropic
  - $u_h \sim 400$ m/s, $u_r < 30$ m/s at supergranulation scale
  - Typical vertical correlation scale is $H \sim 2000$-5000 km
- Spectral break suggests supergranulation is the largest driven scale at the surface

Can we make physical and theoretical sense of this ?

Nonlinear evolution of large-aspect ratio convection simulations

F. Rincon, 2004

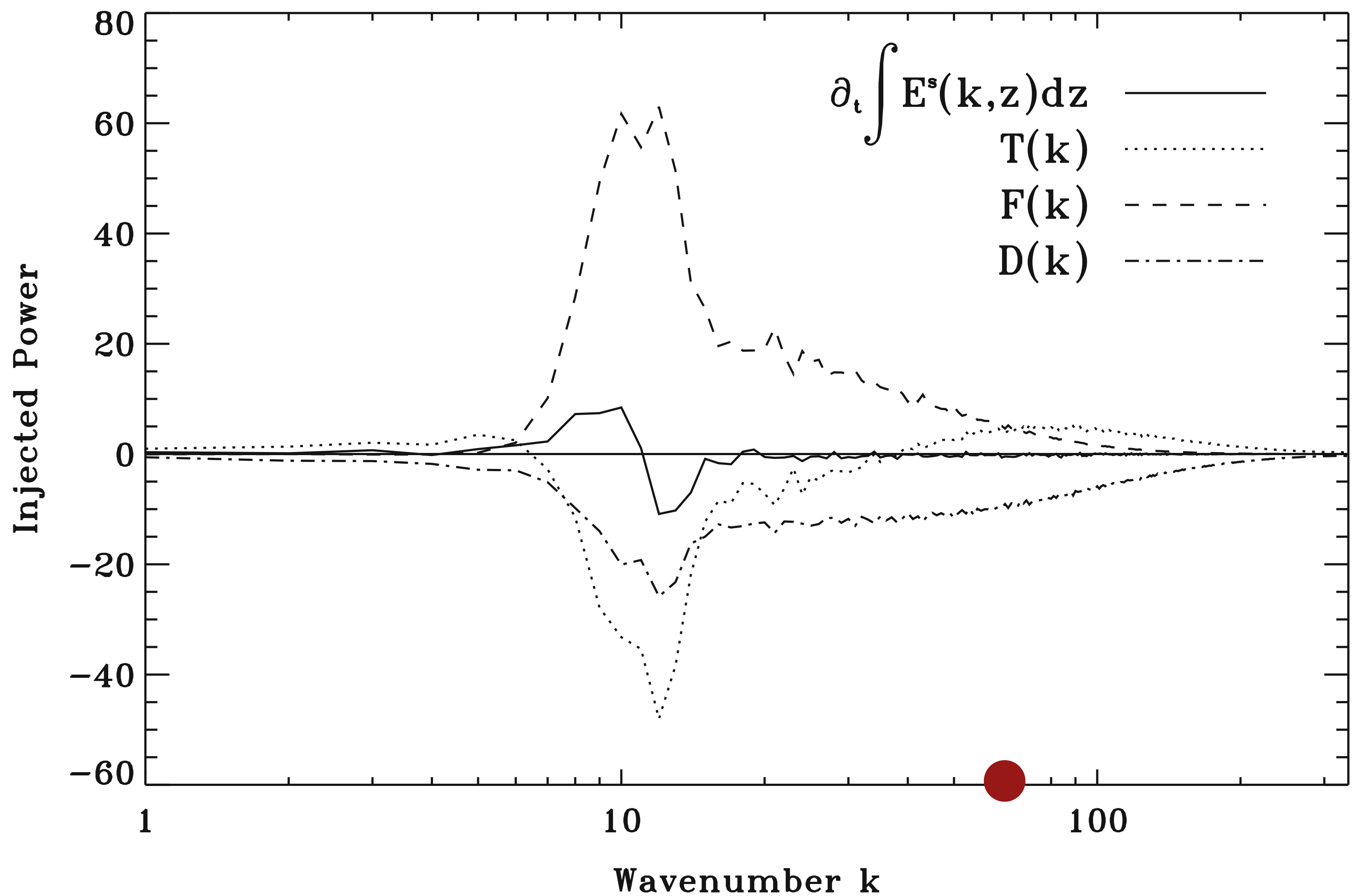




Energy budget/transfer

- Lin's equation

$$\frac{\partial}{\partial t}\int_0^H E(k_h,z)\mathrm{d}z = T(k_h) + F(k_h) + D(k_h)$$

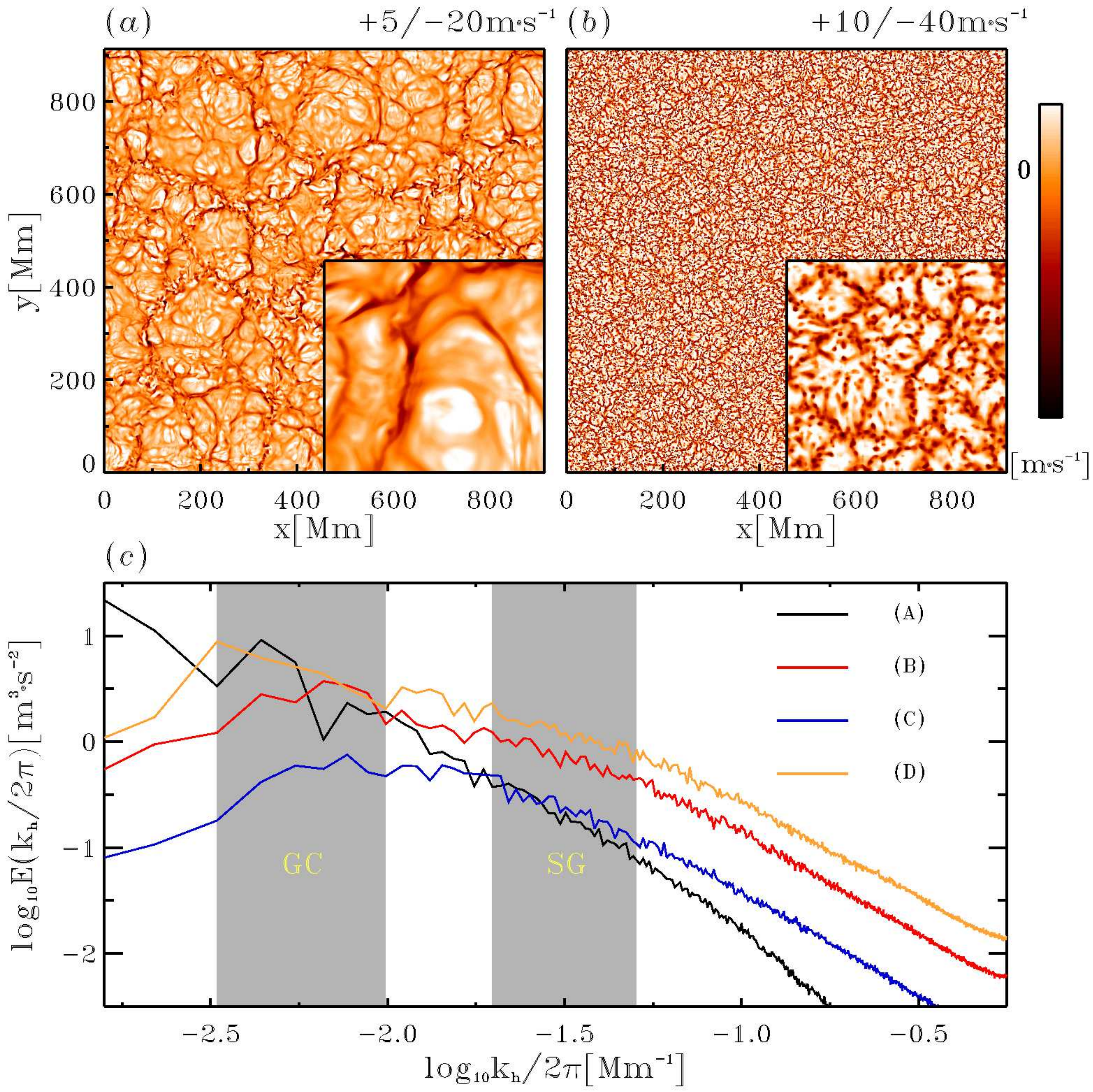


Nonlinear transfer
Buoyancy forcing
Viscous dissipation

Most unstable linear mode

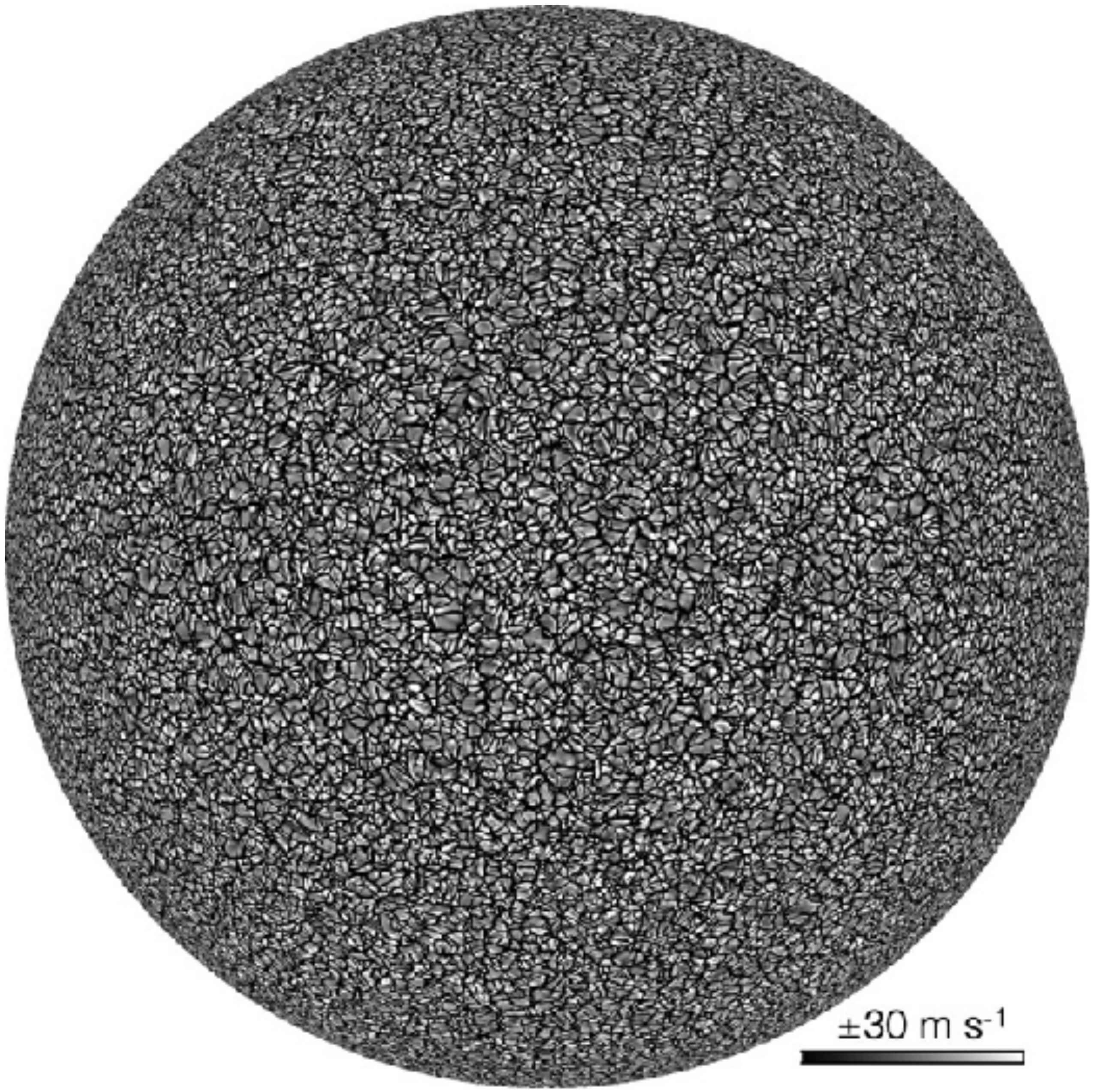
Rincon et al., A&A 2005

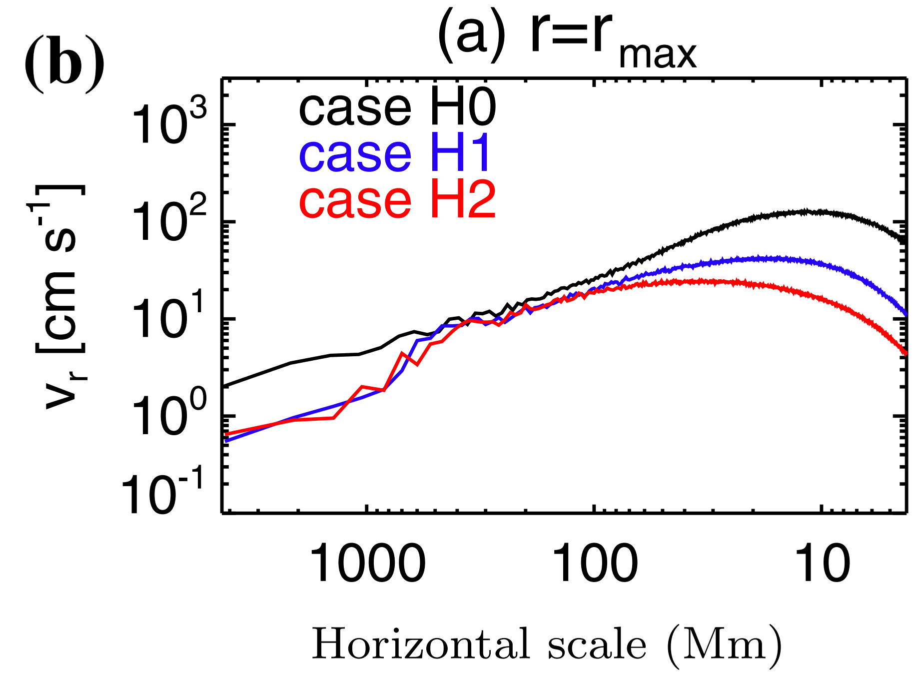
Dependence of injection scale on size of surface entropy jump

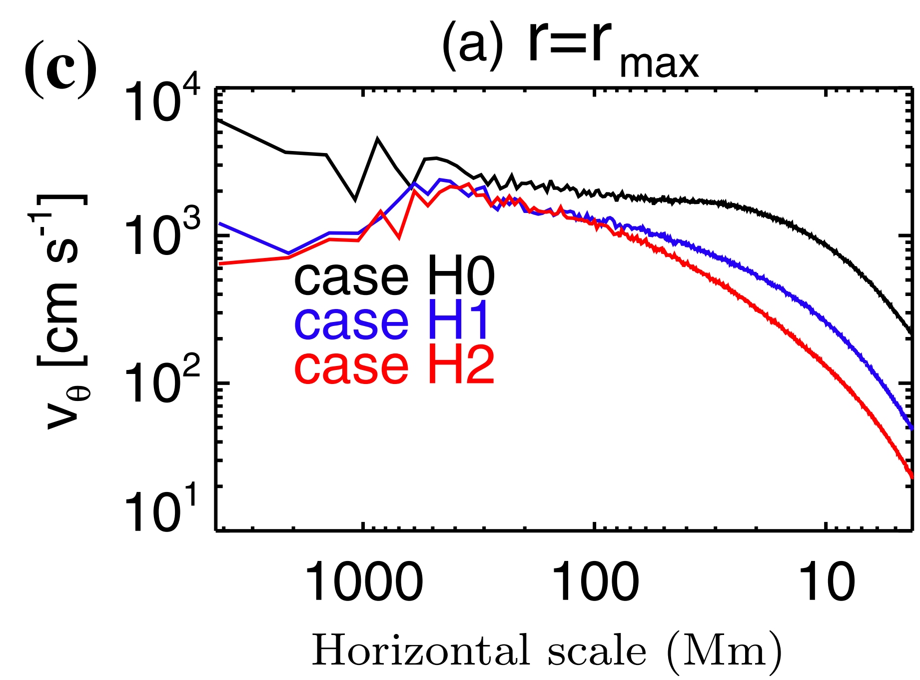


Cossette & Rast, ApJ 2016

Latest-generation global simulations







Hotta et al., ApJ 2014

Supergranulation: emerging picture

- Supergranulation is the largest buoyancy-driven scale at the photospheric level
  - Supported by both observational and numerical analysis
- It appears to be the outcome of a nonlinear self-organization of turbulent thermal convection
  - Much more complex than thought for decades: a lesson for AFD ?
- Not entirely understood
  - Scale-dependence on convective-driving intensity (stellar luminosity)
  - Lack of strong thermal signature (radiative granulation boundary layer blanket, weak thermal flux at SG scales ?)

Rincon & Rieutord, Living Rev. Sol. Phys. 2018

Turbulent convection phenomenology revisited

- Dynamical equations for fluctuations

$$\begin{aligned}
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} &= -\nabla p + \frac{\theta}{\Theta_0} g\,\mathbf{e}_z + \nu\Delta\mathbf{u} \\
\frac{\partial \theta}{\partial t} + \mathbf{u}\cdot\nabla\theta &= -u_z\nabla_z\Theta_0 + \kappa\Delta\theta \\
\nabla\cdot(\rho_0\mathbf{u}) &= 0
\end{aligned}$$

$$\Theta = \Theta_0 + \theta$$

$$\mathbf{g} = -g\,\mathbf{e}_z$$

- Derive evolution laws for statistics of fluctuation increments

$$\delta f(\mathbf{x},\mathbf{r}) = f(\mathbf{x}+\mathbf{r}) - f(\mathbf{x})$$

Isotropic, homogeneous theory

- Generalised Kolmogorov-Yaglom relations [Yakhot, PRL 1992]

$$\left\langle \delta u_r^3 \right\rangle = -\frac{4}{5}\varepsilon_u r + \frac{6}{r^4}\frac{g}{\Theta_0}\int_0^r y^4 \left\langle \delta\theta \delta u_z \right\rangle \mathrm{d}y + 6\nu \frac{\partial}{\partial r}\left\langle (\delta u_r)^2 \right\rangle$$

$$\left\langle (\delta\theta)^2 \delta u_r \right\rangle = -\frac{4}{3}\varepsilon_\theta r + \frac{2}{r^2}\int_0^r y^2 \left\langle \delta\theta \delta u_z \right\rangle \mathrm{d}y \frac{\partial \Theta_0}{\partial z} + 2\kappa \frac{\partial}{\partial r}\left\langle (\delta\theta)^2 \right\rangle$$

- Kolmogorov 41, passive scalar (constant fluxes):

$$\delta u \sim r^{1/3}, \delta\theta \sim r^{1/3} \qquad E(k) \sim k^{-5/3}, E_\theta(k) \sim k^{-5/3}$$

- Bolgiano-Obukhov 59 (inertia/buoyancy balance + constant thermal flux)

$$\delta u \sim r^{3/5}, \delta\theta \sim r^{1/5} \qquad E(k) \sim k^{-11/5}, E_\theta(k) \sim k^{-7/5}$$

Isotropic theory

- Bolgiano scale

$$L_B = \frac{\varepsilon_u^{5/4}\Theta_0^{3/2}}{\varepsilon_\theta^{3/4} g^{3/2}} \sim \frac{Nu^{1/2}H}{(Ra\,Pr)^{1/4}}$$

(within isotropic framework)

  - BO59 for $r > L_B$
  - K41 for $r < L_B$

- BO59 never observed in aspect ratio O(1) situations because L_B is always O(H) [Rincon et al., JFM 2006, Kumar et al., PRE 2014]

- At the solar surface, $L_B \sim H_\rho \sim$ 2000-5000 km
  - Transition takes place around granulation scale

- What happens for $k_h H < 1$? (i.e. in our observational scale-range)
  - Anisotropic generalization of BO59 needed

Tentative theory of large-scale, anisotropic turbulent convection

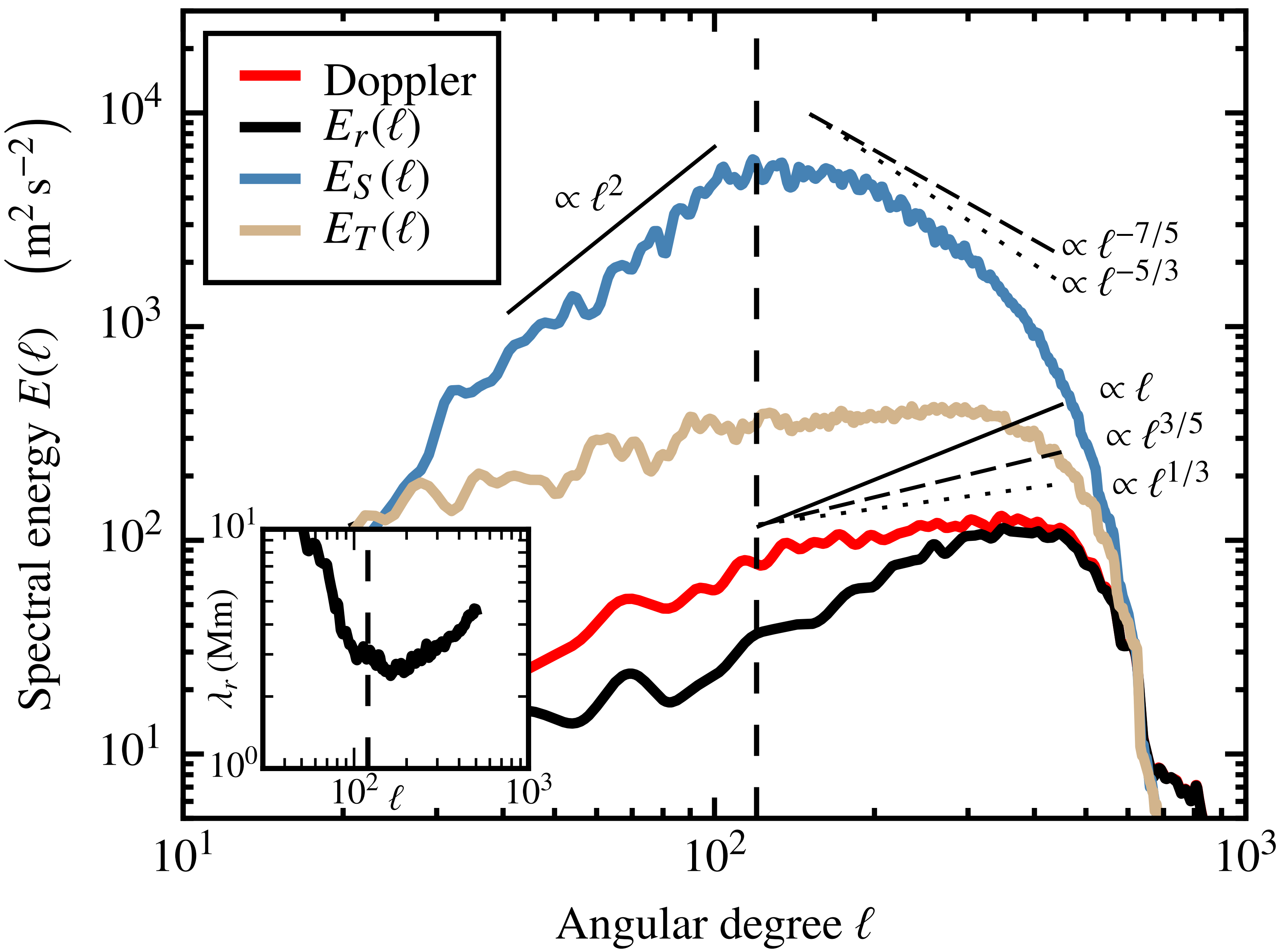
- Mass conservation $\dfrac{\delta u_h}{\lambda_h} \sim \dfrac{\delta u_z}{H}$ $\qquad \lambda_z \sim H$
- Constant flux of thermal fluctuations $\dfrac{\delta u_h\, \delta\theta^2}{\lambda_h} \sim \varepsilon_\theta \;= \text{const},$
- Dominant inertia/buoyancy balance $\dfrac{\delta u_h\, \delta u_z}{\lambda_h} \sim \left(\dfrac{H}{\lambda_h}\right)^2 g\,\dfrac{\delta\theta}{\Theta_0}$

$$
\begin{aligned}
\delta u_z &\sim \left(\frac{\varepsilon_\theta}{\Theta_0^2}\right)^{1/5} g^{2/5} H^{7/5} \lambda_h^{-4/5} \\
\delta u_h &\sim \left(\frac{\varepsilon_\theta}{\Theta_0^2}\right)^{1/5} g^{2/5} H^{2/5} \lambda_h^{1/5} \\
\delta\theta/\Theta_0 &\sim \left(\frac{\varepsilon_\theta}{\Theta_0^2}\right)^{2/5} g^{-1/5} H^{-1/5} \lambda_h^{2/5}
\end{aligned}
$$

$$
\begin{aligned}
E_z(k_h) &\sim \left(\frac{\varepsilon_\theta}{\Theta_0^2}\right)^{2/5} g^{4/5} H^{14/5} k_h^{3/5} \\
E_h(k_h) &\sim \left(\frac{\varepsilon_\theta}{\Theta_0^2}\right)^{2/5} g^{4/5} H^{4/5} k_h^{-7/5} \\
E_\theta(k_h) &\sim \left(\frac{\varepsilon_\theta}{\Theta_0^2}\right)^{4/5} g^{2/5} H^{-2/5} k_h^{-9/5}
\end{aligned}
$$

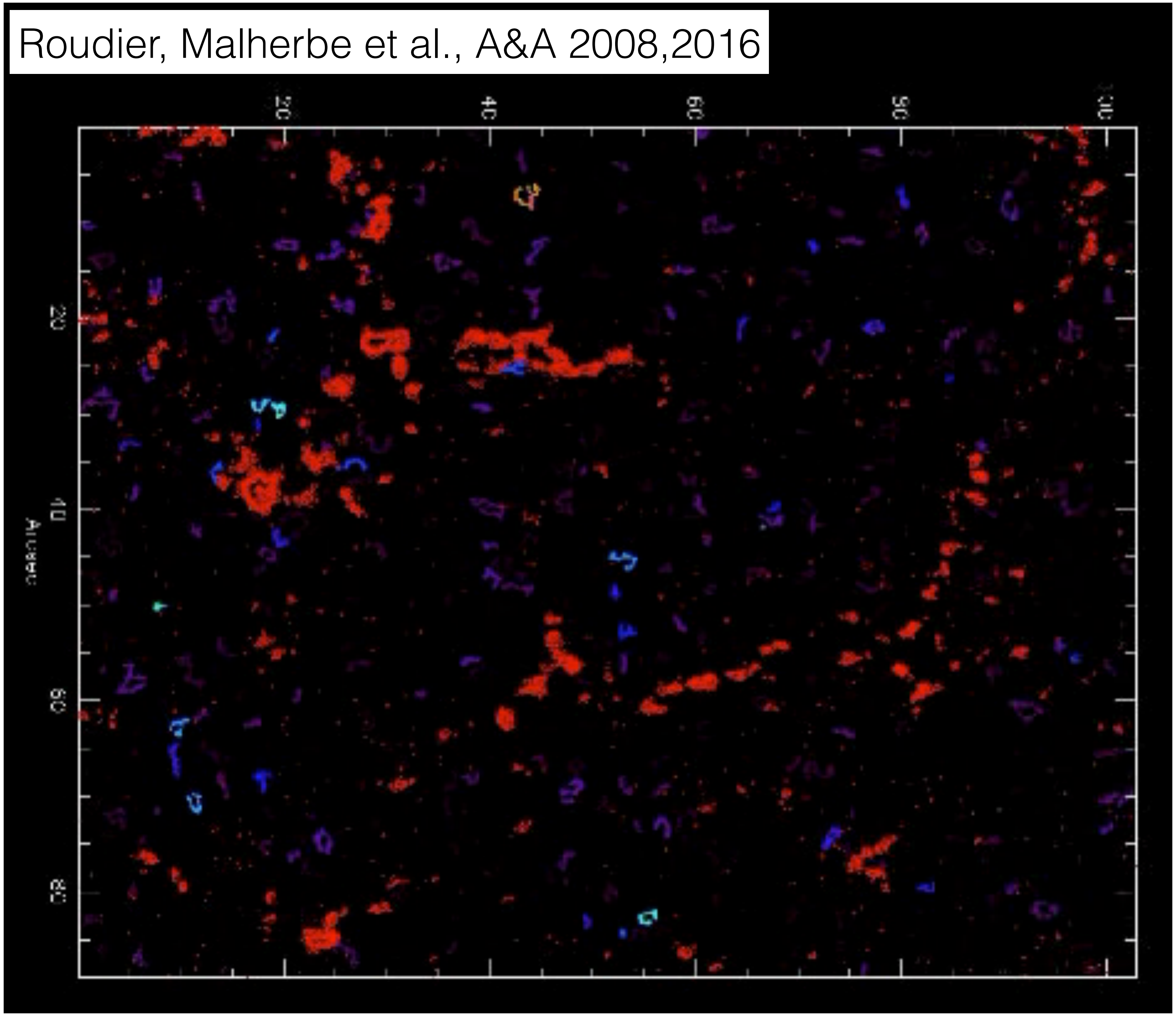
Theory vs. observations

$$E_z(k_h) \sim \left(\frac{\varepsilon_\theta}{\Theta_0^2}\right)^{2/5} g^{4/5} H^{14/5} k_h^{3/5} \qquad E_h(k_h) \sim \left(\frac{\varepsilon_\theta}{\Theta_0^2}\right)^{2/5} g^{4/5} H^{4/5} k_h^{-7/5}$$

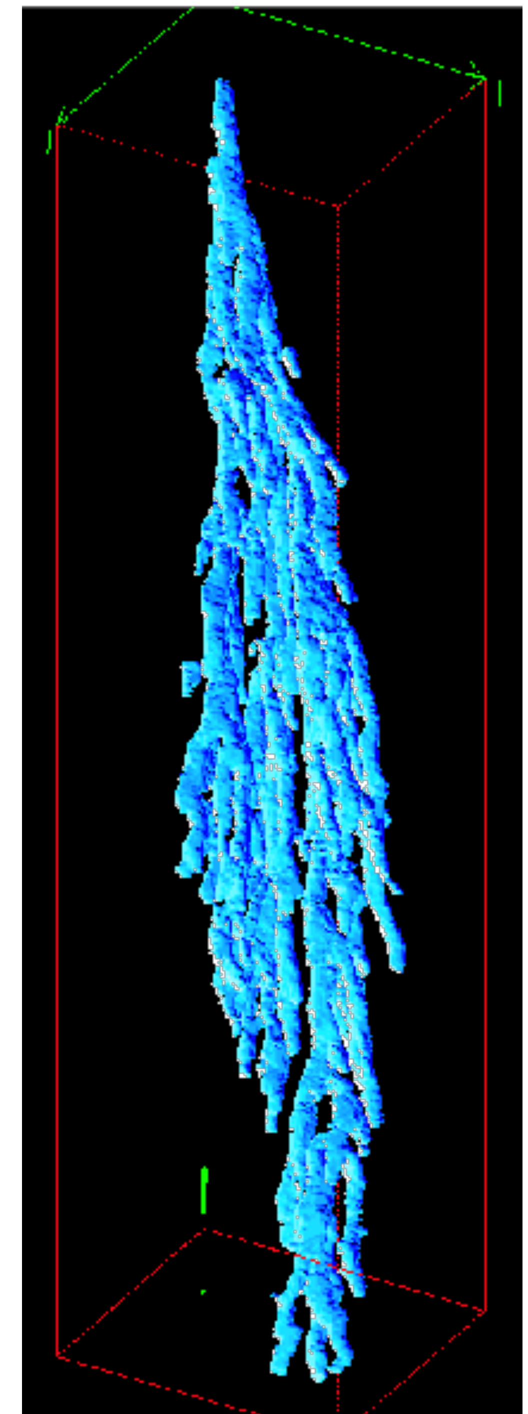


Rincon et al., A&A 2017

Signatures of self-similar buoyant dynamics: trees of fragmenting granules



Roudier et al., A&A 2003

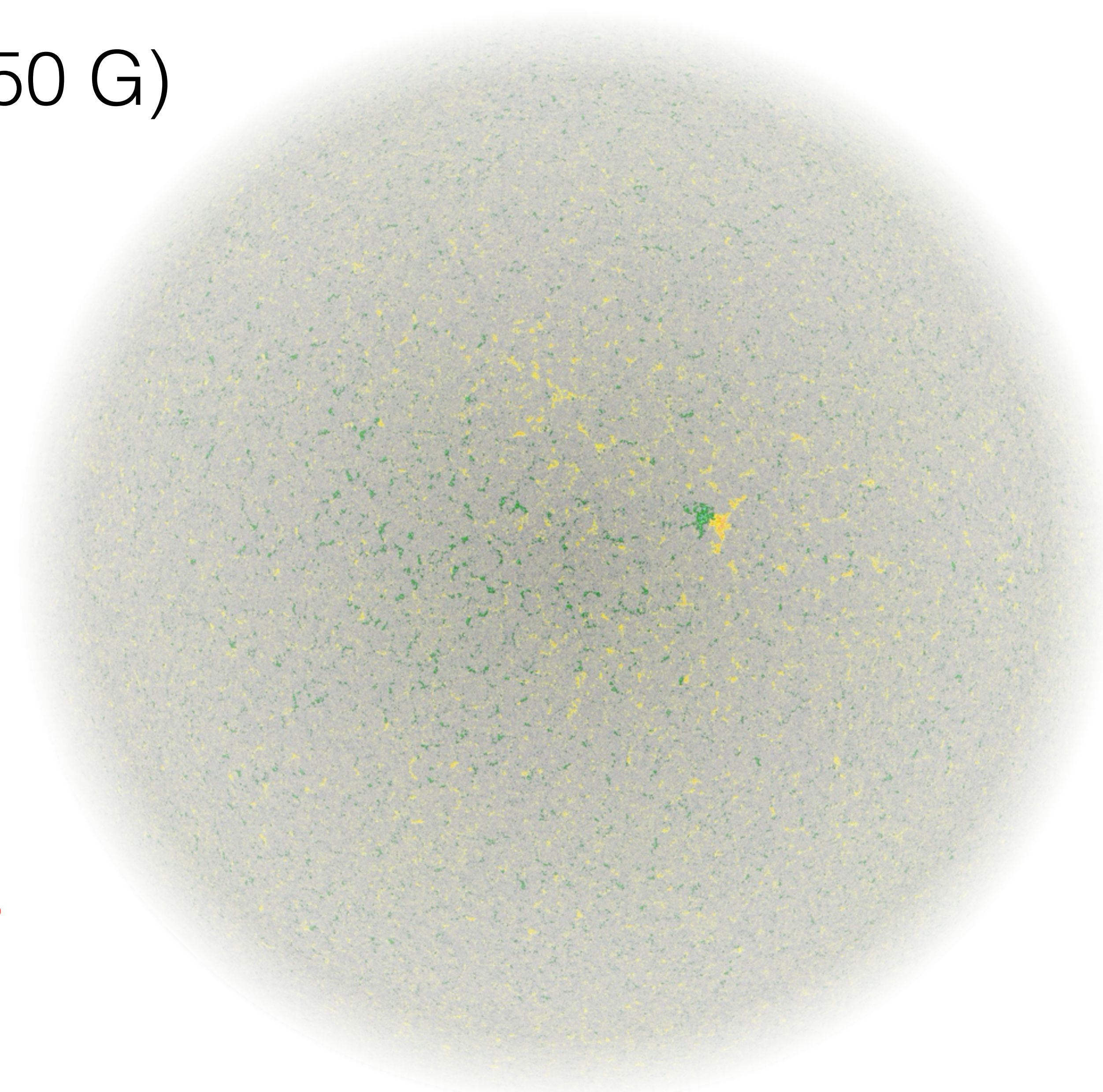


Time —>

Turbulent transport/diffusion

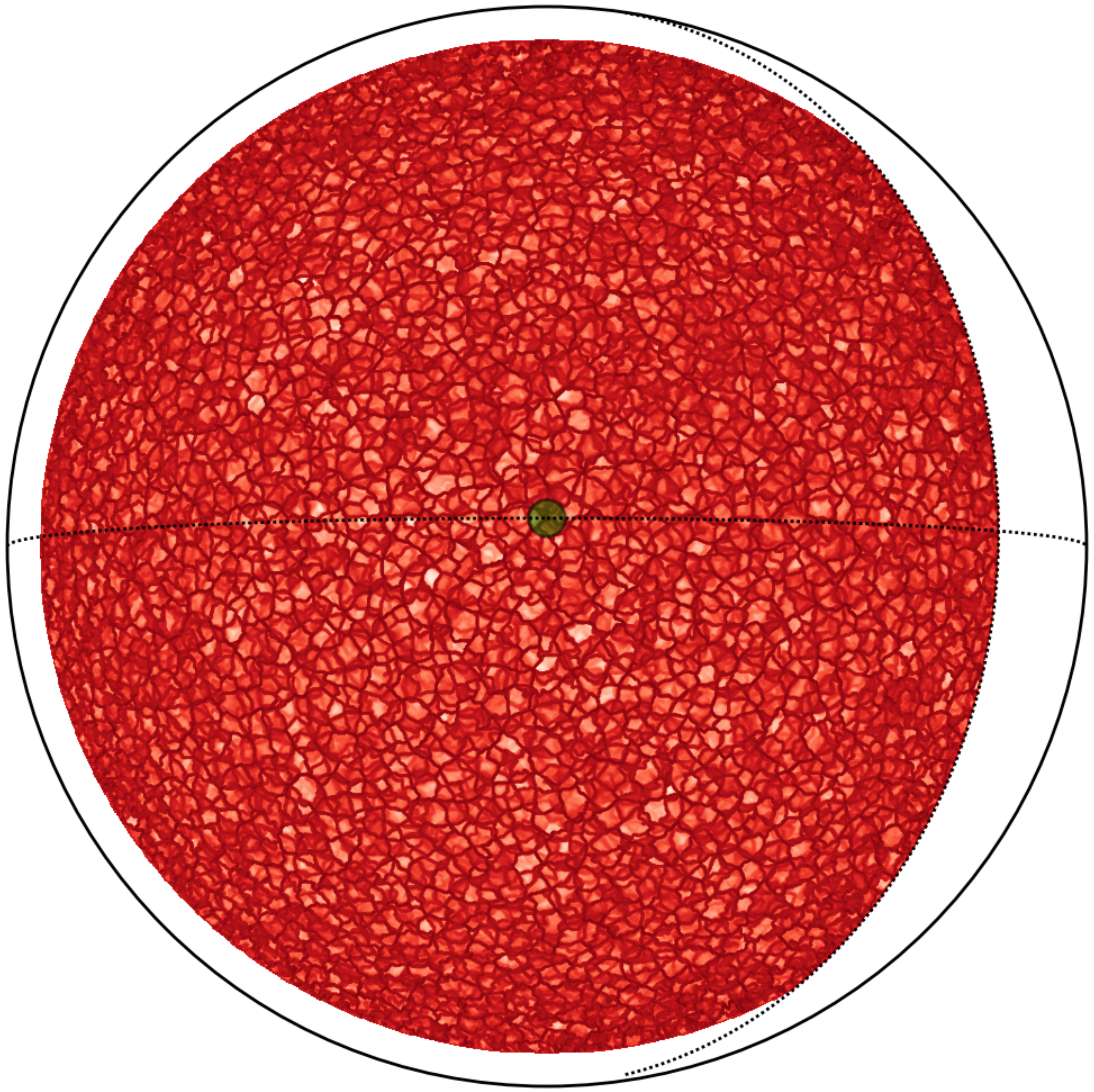
Work in progress with Peter Haynes and Paul Barrère

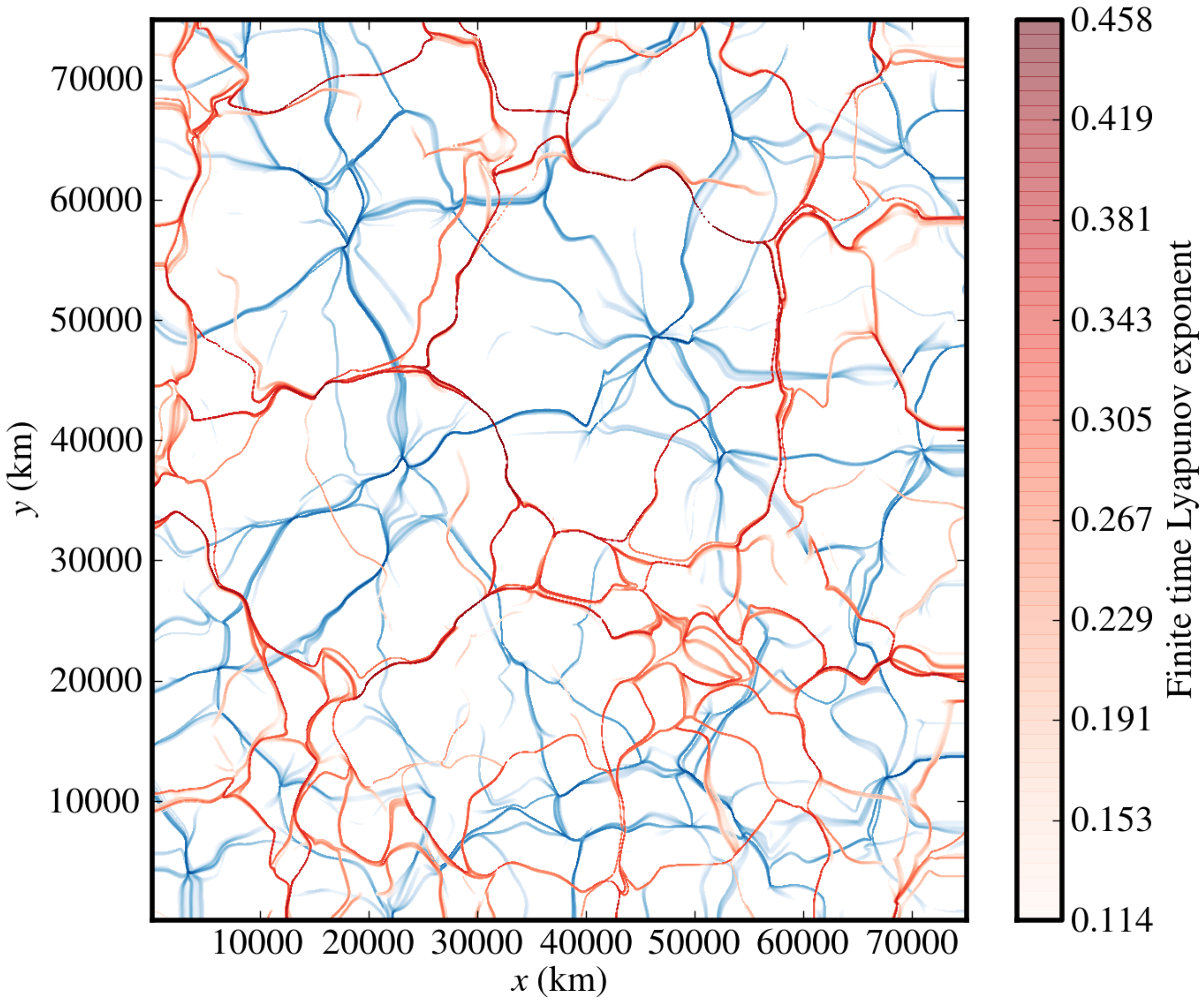
- How does turbulent convection transport magnetic fields ?
  - Turbulent diffusion is a key part of the global solar dynamo process
- *B* in the quiet photosphere is O(50 G)
  - Generated by small-scale dynamo
- Use observationally-derived flows to characterise turbulent magnetic diffusion
  - Assume that small-scale field is essentially passive
  - Study diffusion using passive tracers



Lagrangian Coherent Structures

- Finite-Time Lyapunov Exponent field derived from separation of grid of evolving passive tracers

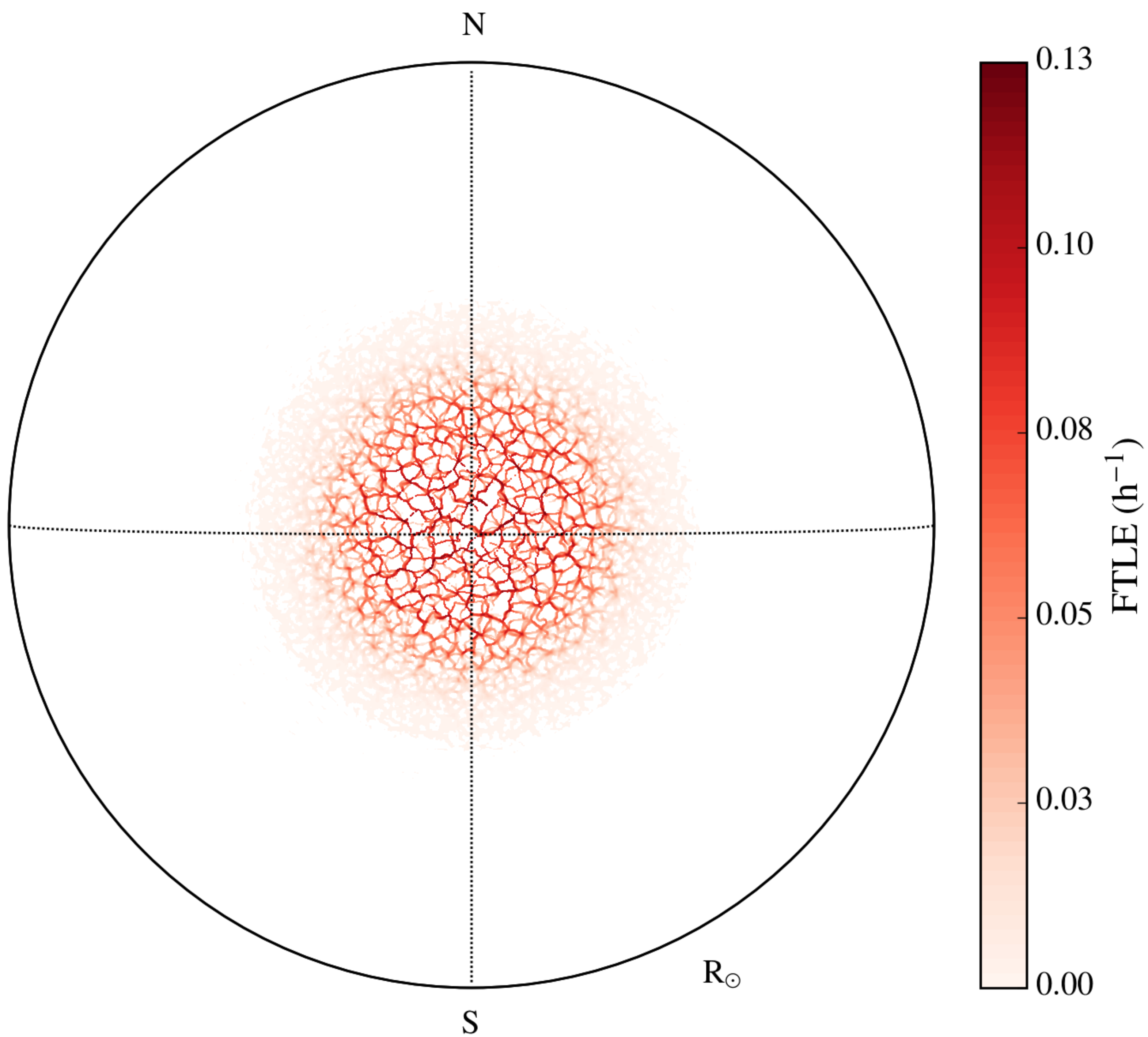




24h positive and negative-time FTLEs

6-days LCS

- $D_\lambda \sim L^2_{\lambda,\mathrm{integral}}\ \lambda_{\mathrm{rms}} \sim 10^6\ \mathrm{km^2\ h^{-1}} \sim 3 \times 10^8\ \mathrm{m^2\ s^{-1}}$



Eulerian approach

- Measure mean square separation of tracers on times longer than typical turbulence correlation time

- Look for power-law scalings

$$\langle \Delta x^2 \rangle \sim D\, t_0 \left(\frac{t}{t_0}\right)^\gamma$$

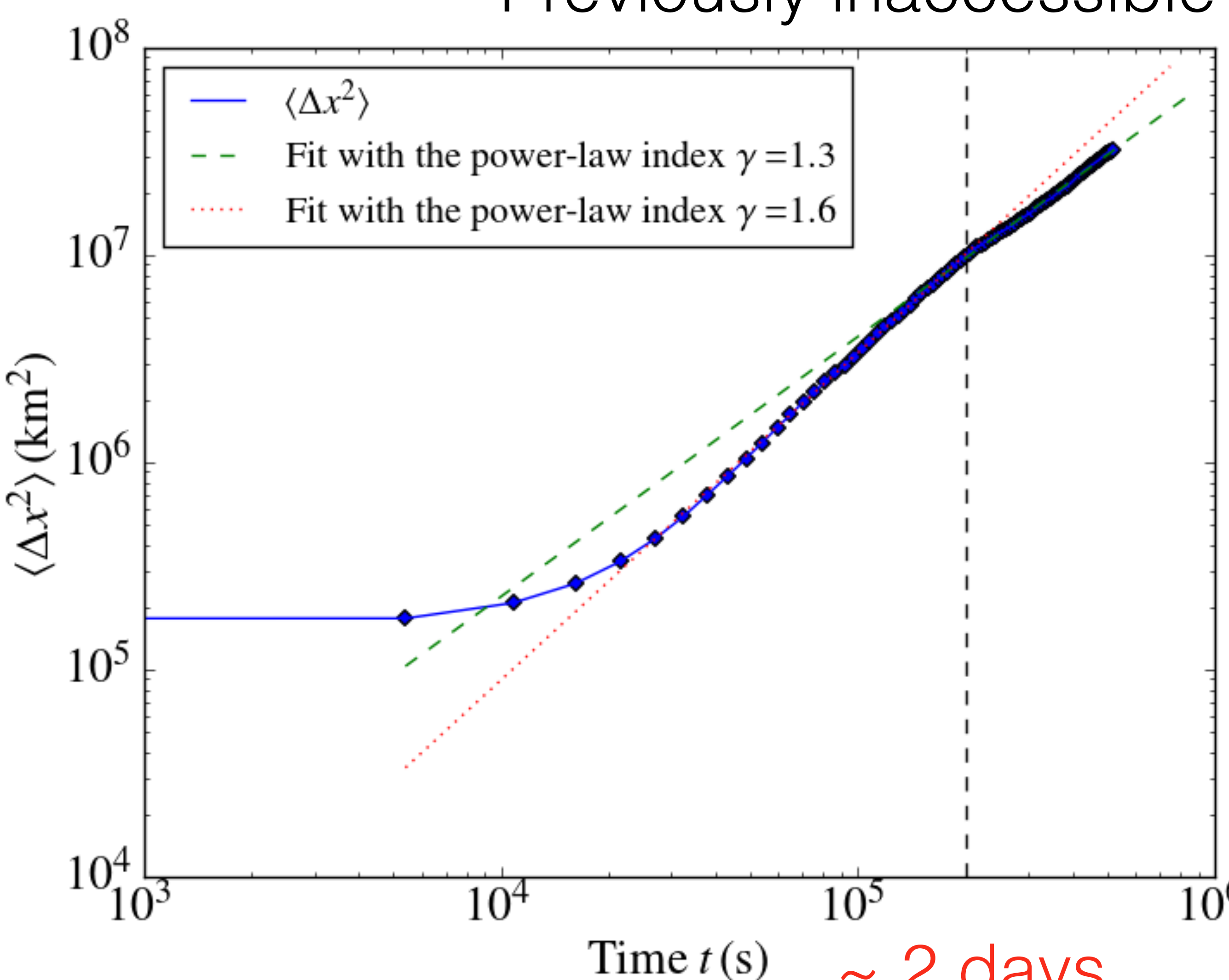
- Two super-diffusion regimes

$D = 2 \times 10^5$ km^2 h^{-1}

$\sim 5 \times 10^7$ m^2 s^{-1}

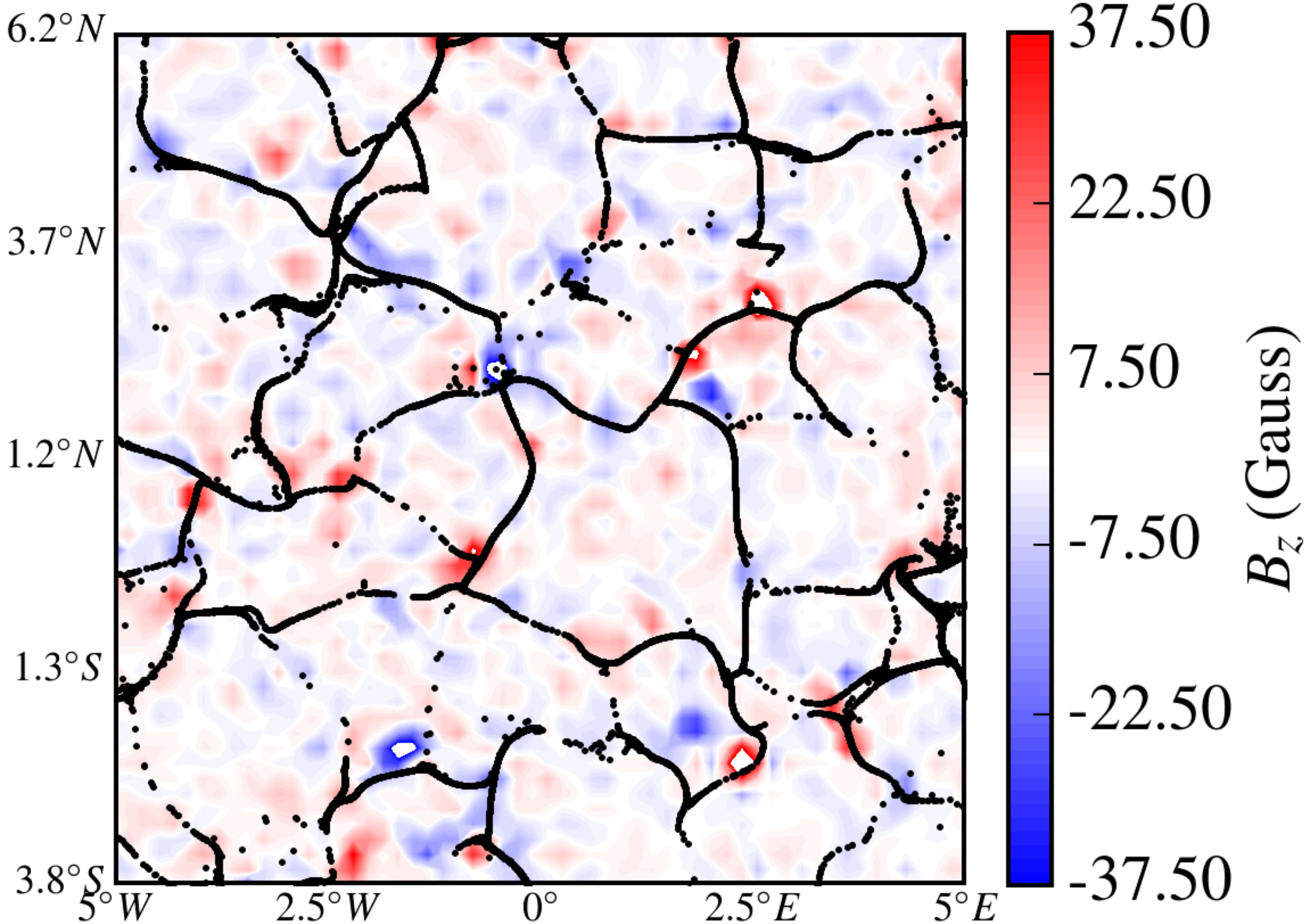
- Transition time t_0 consistent with SG correlation time

Previously inaccessible



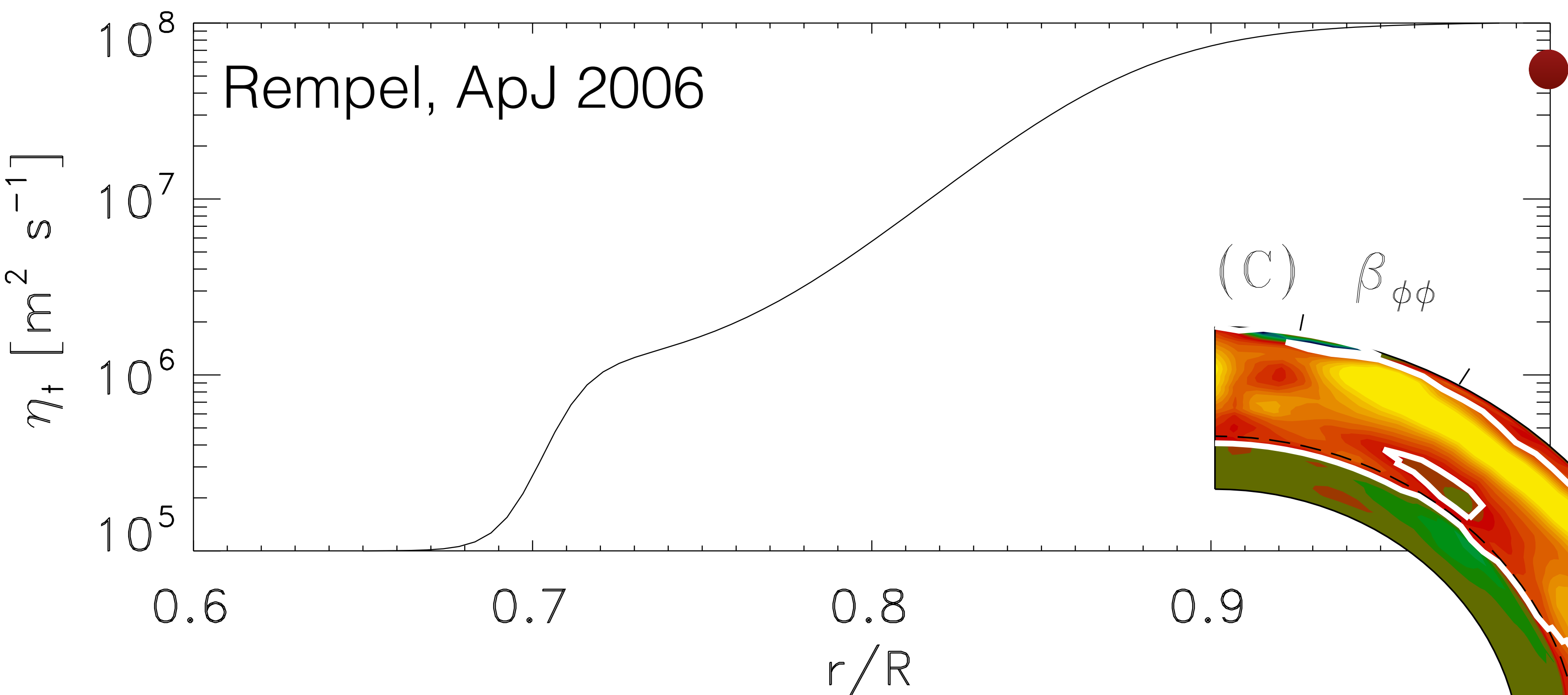
~ 2 days

Consistency check: B is essentially passive

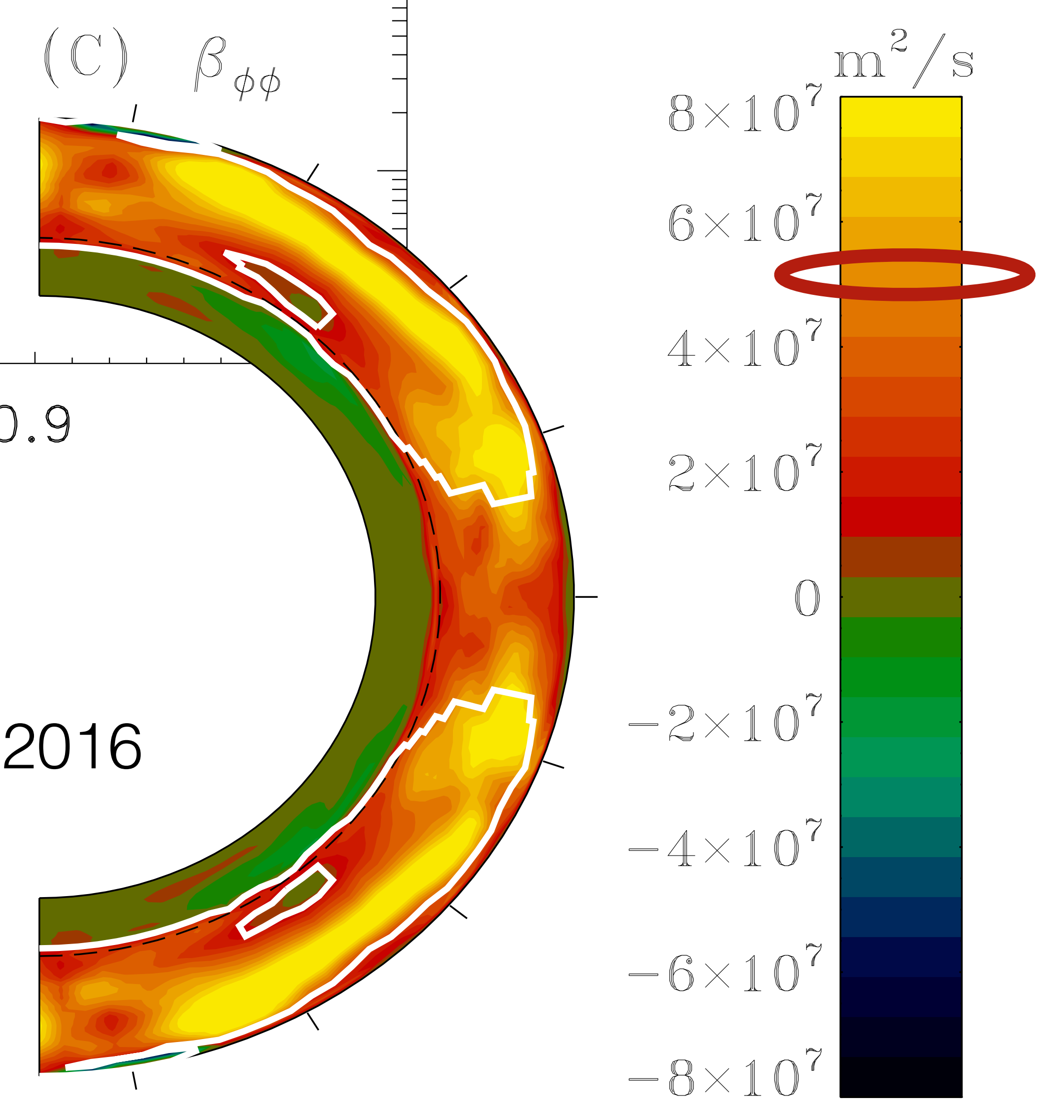


Consistency of dynamo models

- Mean-field flux-transport solar dynamo model



- Global solar dynamo simulations

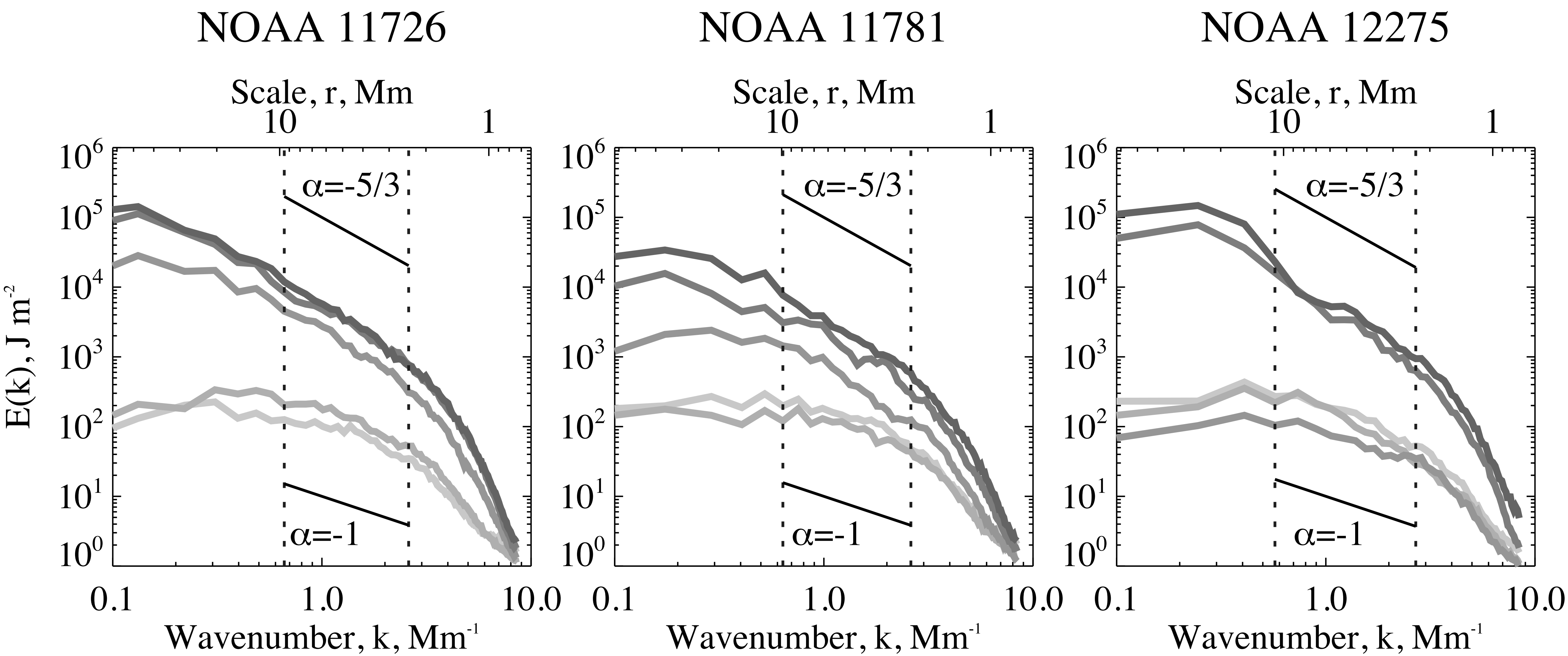


Simard et al., ApJ 2016

Conclusions

- Unprecedented characterisation of strongly driven astrophysical fluid turbulence
  - Determination of full 3D velocity field in a plane, over almost two scale decades
  - High-resolution in time and space, up to global scales
  - Followed over several typical turnover times
- Observations, numerics paint a complicated nonlinear dynamical picture
  - Significant implications for the understanding of solar convection (supergranulation)
  - Motivates the development of new turbulence phenomenology
  - Promising preliminary results on turbulent transport
- More discovery/understanding potential
  - Dynamo/MHD: α-effect, MHD turbulence, low Pm small-scale dynamo ?
  - Implications for stellar physics [and exoplanet detection, spectral noise problem]
  - Relevance to other astrophysical transport/turbulence problems (galaxies, disks, ICM)

Magnetic power spectra



Kutsenko et al., Sol. Phys. 2019