Structure formation in ZF GK turbulence.

Arthur Peeters

With contributions from: F. Rath, F. Seiferling, A. Weikl

20 years back in time

- Nonlinear upshift of the turbulence threshold for the temperature gradient compared with linear theory.
- Connected with zonal flow generation that stabilize transport through ExB shear.

Heat conductivity described by
the Dimits fit¹
$$\chi_i = A\left(1 - \frac{R/L_{T|dim}}{R/L_T}\right)$$
extrapolated threshold:
 $R/L_{T|dim}$

A.M. Dimits, Phys. Plasmas 7, 969 (2000)



Indeed "short" simulations with "sufficient dissipation" yield this result

- Cyclone base case with adiabatic electrons.
- "Standard resolution" (16 points along the field line, 21 toroidal modes ($k~\rho_{max}=$ 1.4), 83 radial modes)
- Continuous transition to turbulent state.
- Dashed line is the Dimit's formula
- Note that there is a difference in threshold between s-alpha and circular geometry.



A.G. Peeters, Phys. Plasmas 23, 082517 (2016)

Dissipation on the zonal flow connected with parallel flow

- Assuming a Pfrisch Schlueter perturbuation $f = A \cos(2 \pi s)$
- The parallel dissipation can be estimated four the fourth order scheme to be $\gamma_d^4(N_S) = -D(\sqrt{3}/12 q)(2 \pi/N_S)^3$
- This is around 6 10⁻³ in normalized time units well below the growth rate of the ITG (0.18) but relevant for the flow development
- The damping due to dissipation is well above the collisional damping connected with Coulomb collisions

 $\gamma_C = -1 \ 10^{-3} \ R \ n_{19} T_k^{-2}$

Dissipation can be reduced by using either more grid points or a higher order scheme. Both have been investigated, with the higher order scheme being of sixth order and employed only for the zonal modes

$$\begin{split} v_{\parallel} \mathbf{b} \cdot \nabla f \\ = v_{\parallel i} F_i \frac{-f_{i-3} + 9f_{i-2} - 45f_{i-1} + 45f_{i+1} - 9f_{i+2} + f_{i+3}}{60\Delta s} \\ -v_d F_i \frac{f_{i-3} - 6f_{i-2} + 15f_{i-1} - 20f_i + 15f_{i+1} - 6f_{i+2} + f_{i+3}}{60\Delta s} \end{split}$$

If you lower the parallel dissipation

- The heat flux as function of the temperature gradients is qualitatively different
- There is a further upshift of around 30% (note: circular geometry $R/L_{TDim} = 4.7$)
- There is no gradual increase of the heat conductivity with the gradient but rather a finite heat flux at the threshold ("finite heat flux threshold")



Benchmark with GYRO

- I wasn't sure (very low dissipation many things can go wrong)
- GYRO simulations with many points along the orbit (low dissipation) yield the same result.

The cyclone base case $R/L_T = 6.9$ is nonlinearly stable!!!



Heat conduction coefficient obtained with GKW for Circular geometry (red) and s-alpha (geometry) black. Compared with GYRO (high number of points along The orbit) in blue

Low dissipation You have to wait a bit

Close to finite heat flux threshold $(R/L_T|_{finite} = 7.4)$ but above Dimits threshold $(R/L_T|_{dim} = 6.0)$



A.G. Peeters, Phys. Plasmas 23, 082517 (2016)

The physical reason – structure formation

Radial profiles of the $E \times B$ -shearing rate for $R/L_T = 7.2$ (red curve)



From: A.G. Peeters, Phys. Plasmas **23**, 082517 (2016)

Connected with staircases First observed with GYSELA G. Dif.-Pradalier, Phys Review E **82**, 025401 (2010)

Why the jump in heat flux at the finite heat flux threshold?

- At the "finite heat flux threshold" there is a sudden transition from a fully developed staircase to the more commonly observed sawtooth like shearing rate.
- There is then a relatively large radial domain with a small shearing rate.
- In this regions avalanches develop that then propagate through the regions of high shear

Figure: Top Shearing rate as a function of the radial coordinate. Middle (Bottom) heat conductivity (Temperature gradient) as a function of time (x-axis) and radius (y-axis)



What it means: one can not use a flux tube close to the nonlinear threshold

Since the heat flux is not continuous in the flux tube simulations, there are heat fluxes that can not be obtained \rightarrow Close to the threshold the flux tube can not predict the transport level.

We preform local heat flux driven simulations using a heat source and sink \rightarrow

Equations and implementation unchanged.





Comparison flux and gradient driven



flux-driven threshold is 25% upshifted compared to gradient-driven finite heat flux threshold ¹

▶ separation between low- and high flux region with critical heat flux $Q_{ic} \approx 10 Q_{GB}$

Comparison statistics and structure

3. and 4. order moments of the probability density function of the turbulent heat flux



- ▶ high flux region $Q_i \gtrsim 10 Q_{GB}$: GD and FD similar
- ▶ low flux region $Q_i \leq 10 Q_{GB}$: FD turbulence is intermittent

Intermittency



spatio temporal self-organisation

- ▶ persistent (~ $10^3 R/v_{thi}$) spatial meso scale staircases $50 \sim 70 \rho_i$
- ▶ intermittent heat avalanches^{1,2}
- ▶ Note: staircases state disappears above $Q_{ic} \approx 10 Q_{GB}$

ExB shear of staircases leads to upshift F. Rath Phys. Plasmas 23, 052309 (2016)



- ► characteristic staircase amplitude $\omega_{ExB} \approx 0.3 \sim 0.4 v_{thi}/R$
- ▶ shear stabilisation $|\omega_{ExB}| \approx \gamma(R/L_T) \rightarrow$ temperature gradient corrugations

 $E \times B$ -staircase \Rightarrow up-shifted flux-driven threshold

But what about collisions

²A. Weikl *et al.*, Phys. Plasmas **24**, 102317 (2017)

Collisions damp zonal flow $^1 \rightarrow \text{impact on } R/L_{T|\text{finite}}$?²



- ► $R/L_{T|finite}$ as well as finite heat conductivity decreases with ν_{ii}
- **but**: for ITER relevant $\nu_{ii} = 6 \times 10^{-4} v_{thi}/R$ no significant change in $R/L_{T|finite}$

 \Rightarrow finite heat flux threshold $R/L_{T|finite}$ persists for relevant collision frequencies!

What about kinetic electrons?

Additional effect :

- Instabilities become much more extended along the field lines
- The double parallel boundary conditions on the torus then play a role
- Due to the shear the radial wave vector increases when going once around.

$$\Delta k_{\psi} = k_{\zeta} \frac{\partial q}{\partial \psi} = k_{\zeta} \hat{s} \frac{q}{\epsilon},$$

• The interaction between the modes then leads to the generation of zonal flows with specific radial wave vectors



A. Weikl, Plasmas 25, 072305 (2018);

Shearing rate

- At not too small shear this leads to fine scale structure in the ExB shearing rate.
- There is (unpublished) evidence that this fine scale structure does not do much suppression of ITG turbulence (it certainly does not follow the Waltz rule)
- But it does seem to interrupt staircase formation: Not only can no staircase be observed in the ExB shearing rate, there is no clear effect visible in the heat conduction coefficient
- BUT the fine scale structures should be a finite rho* effect → they are expected to be much smaller in a reactor (in fact the simulations shown do not represent present day experiments very well.



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Conclusions

Behaviour closer to the threshold qualitatively and quantitatively different from the picture of the Dimits paper

- Turbulence does not increase gradually above the threshold, but rather has a finite value at the threshold → Finite heat flux threshold
- The upshift is much larger than the previous interpolation suggests
- Reason is structure formation in the zonal flow pattern, which in flux tube simulations can turn into fully developed staircases that lead to suppression of turbulence over the entire radial domain.
- These structure develop slowly and one needs to be careful not to have too much numerical dissipation.
- Collisions do not change this picture appreciately. It makes the threshold less sharp, but the difference is small for relevant collision frequencies.
- Kinetic electrons lead to the generation of small scale structures in the ExB shearing rate that are connected with the double period boundary conditions on the torus. These hinder the formation of staircases, but it is at present unclear if the small scale structure persist in a reactor, and staircase formation has been observed in some cases.