

Nonlinear Reconnection in Magnetized Turbulence

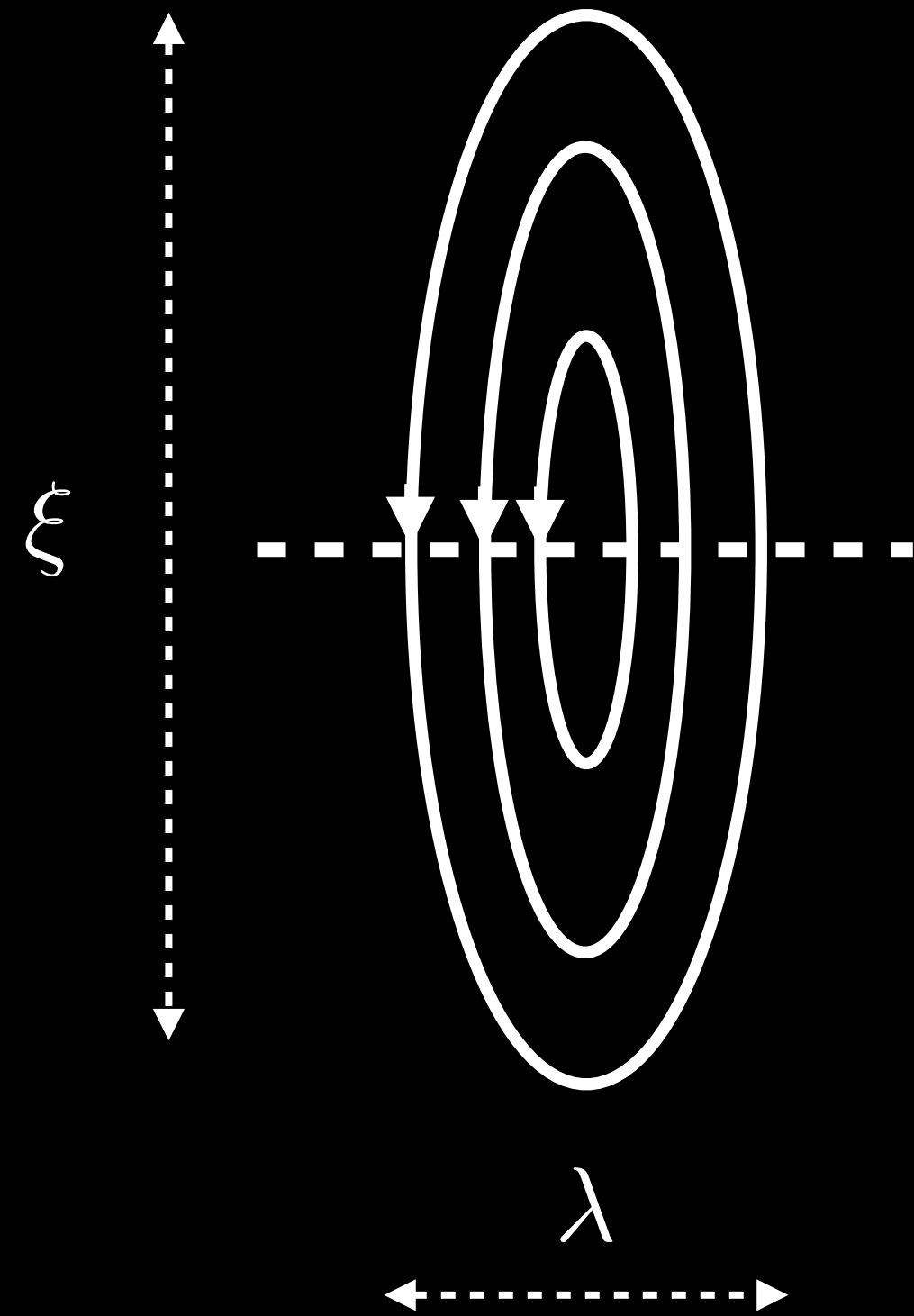
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- Work over the last ~ 3 years has suggested that the tearing mode can be triggered in turbulent plasmas, at scales in the inertial range.
- Tearing controls the dynamics of the eddy, setting a new eddy turn-over time, and thus changing the spectrum, eddy anisotropy, etc.
- Ensuing numerical simulations, and even a detailed analysis of solar wind data, seem to support these ideas.
- This work asks *whether the tearing onset in the eddies can lead to a deep nonlinear reconnecting stage in those eddies.*
- Answer is non-trivial because the tearing rate (and thus the eddy turn-over rate in the tearing-mediated range) *differs* from the reconnection rate.
- Significance is non-trivial: **it is only in the reconnecting stage that significant energy dissipation can occur.** The linear and early nonlinear stages of the tearing mode *dissipate negligible amounts of energy (reconnect negligible amounts of flux).*
- It turns out that full reconnection is indeed possible, but the parameter requirements for that to happen are *far from trivial* (and essentially impossible to simulate, at least in some cases of interest).
- Details in Loureiro & Boldyrev, arXiv:1907.09610

Dynamic Reconnection Onset

Matching the turbulence and reconnection timescales



- At what scale does the *eddy turnover time* become comparable to the *tearing mode growth time* in the eddy?

$$\gamma_t(\lambda)\tau_{nl}(\lambda) \sim 1$$

- This leads to the prediction of a *critical scale* below which reconnection is faster than turbulence:

$$\lambda_{cr}/L \sim S_L^{-4/7}$$

Loureiro & Boldyrev, PRL 2017

Boldyrev & Loureiro, ApJ 2017

Mallet *et al.*, MNRAS 2017

- Spectrum can be computed from enforcing constant energy flux: $\gamma_{nl} V_{A,\lambda}^2 = \epsilon$

where $\epsilon \sim V_{A,0}^3 / L_0$ is the constant rate of energy cascade over scales.

- We assume that when the tearing mode becomes nonlinear, it sets the timescale of the eddy:

$$\gamma_{nl} \sim \gamma_{\text{tear}}$$

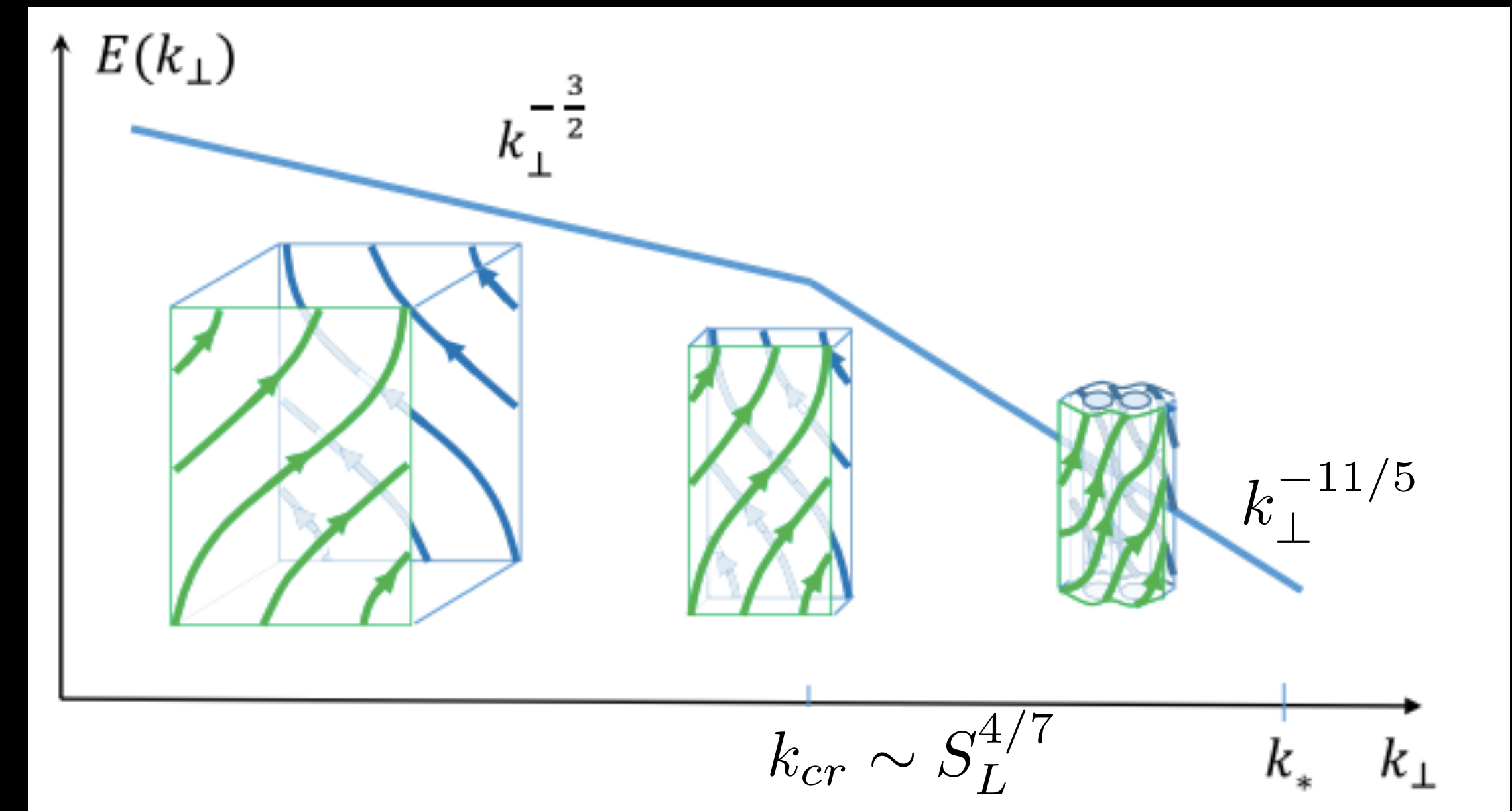
- Obtain:

$$E(k_{\perp}) dk_{\perp} \sim \epsilon^{4/5} \eta^{-2/5} k_{\perp}^{-11/5} dk_{\perp}$$

(or $E(k_{\perp}) \sim k_{\perp}^{-19/9}$)

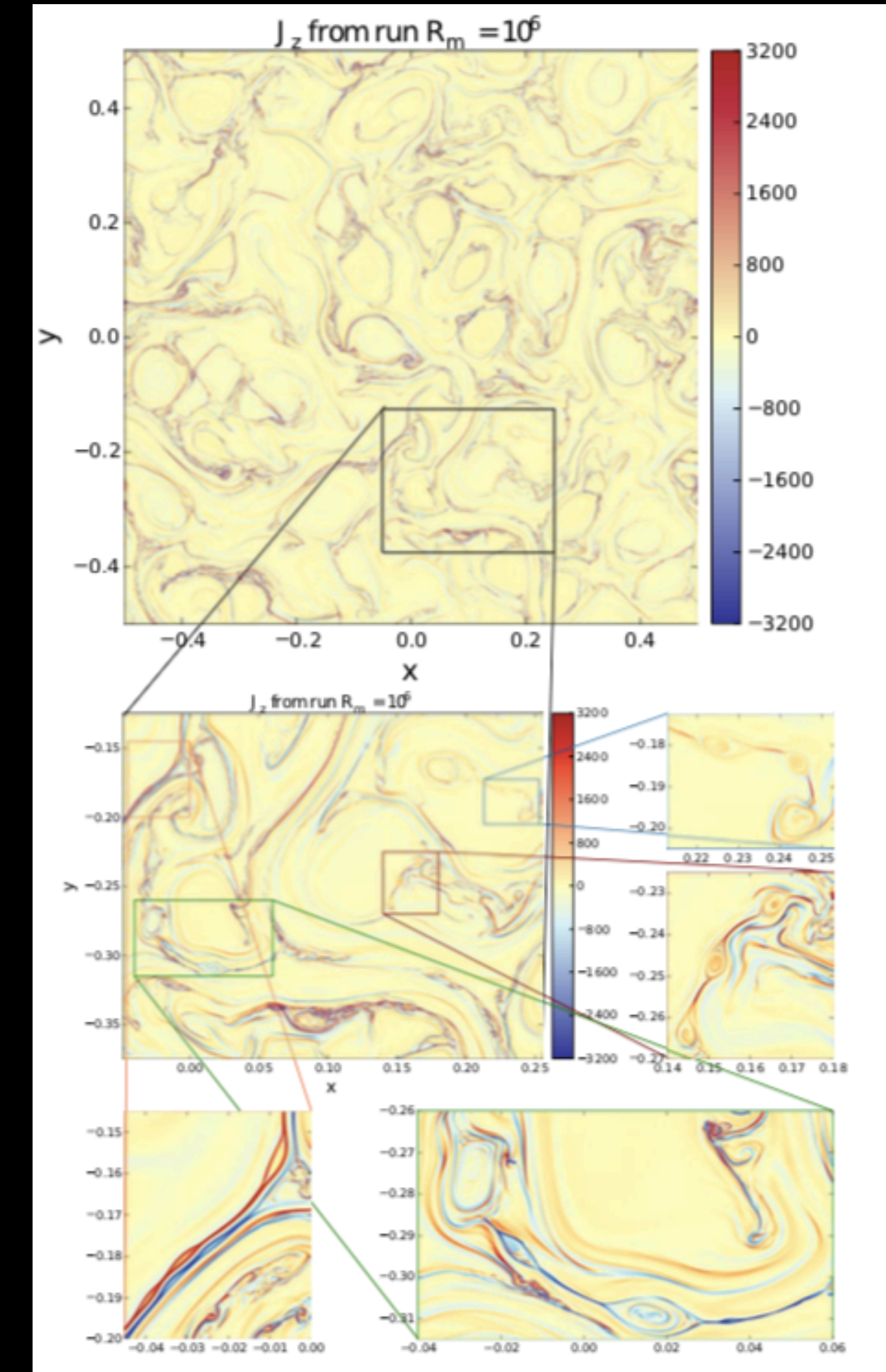
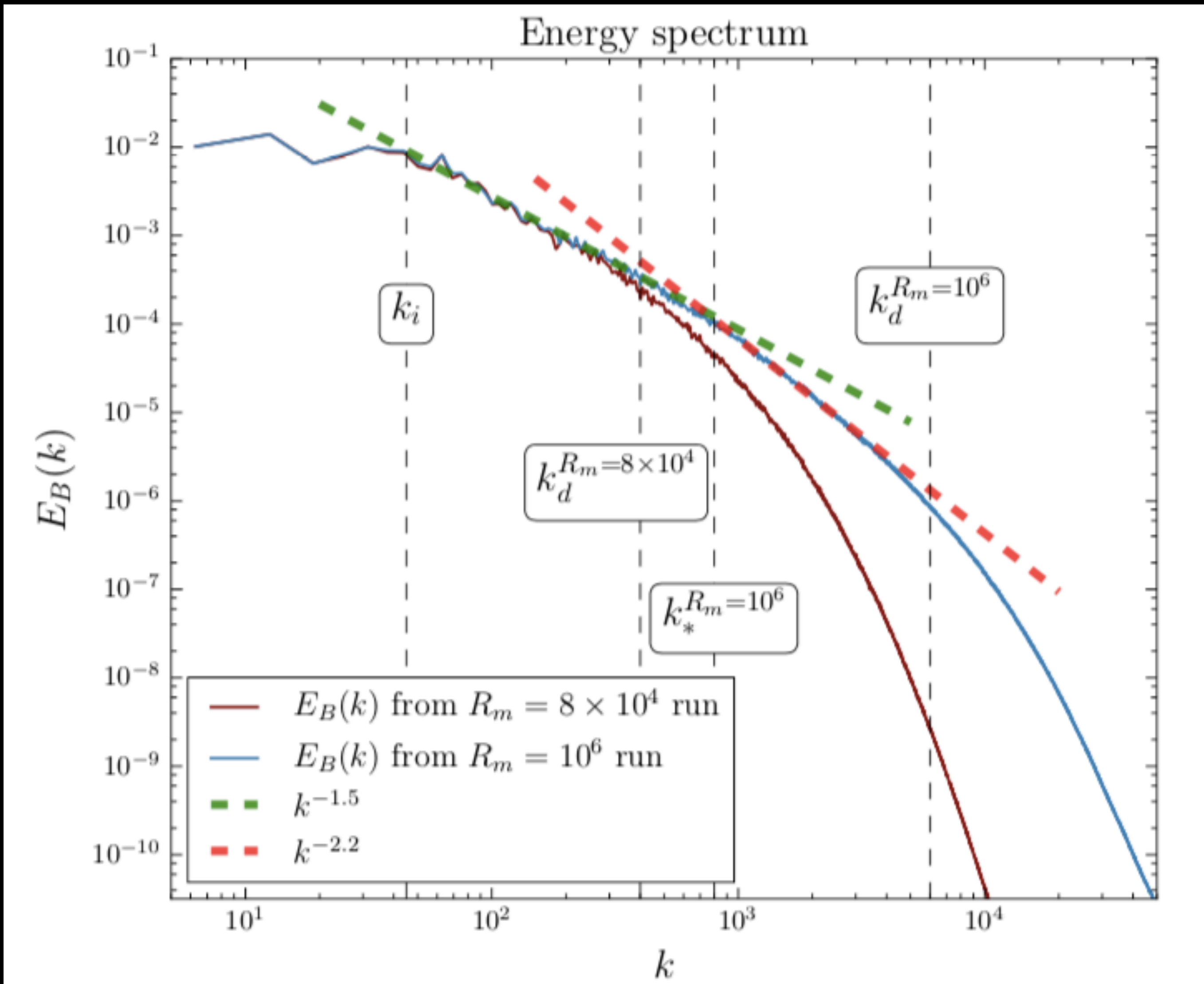
Boldyrev & Loureiro, ApJ 2017

Mallet *et al.*, MNRAS 2017



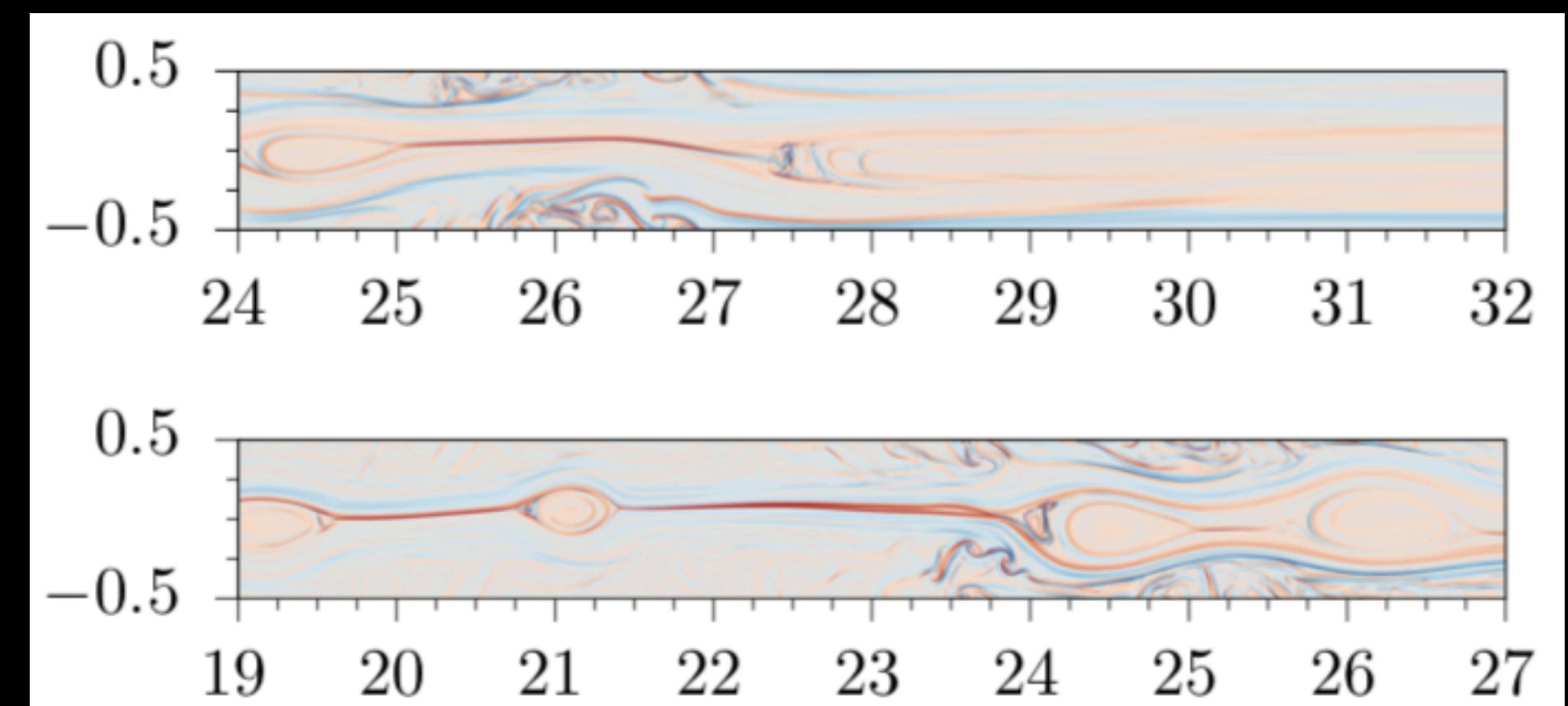
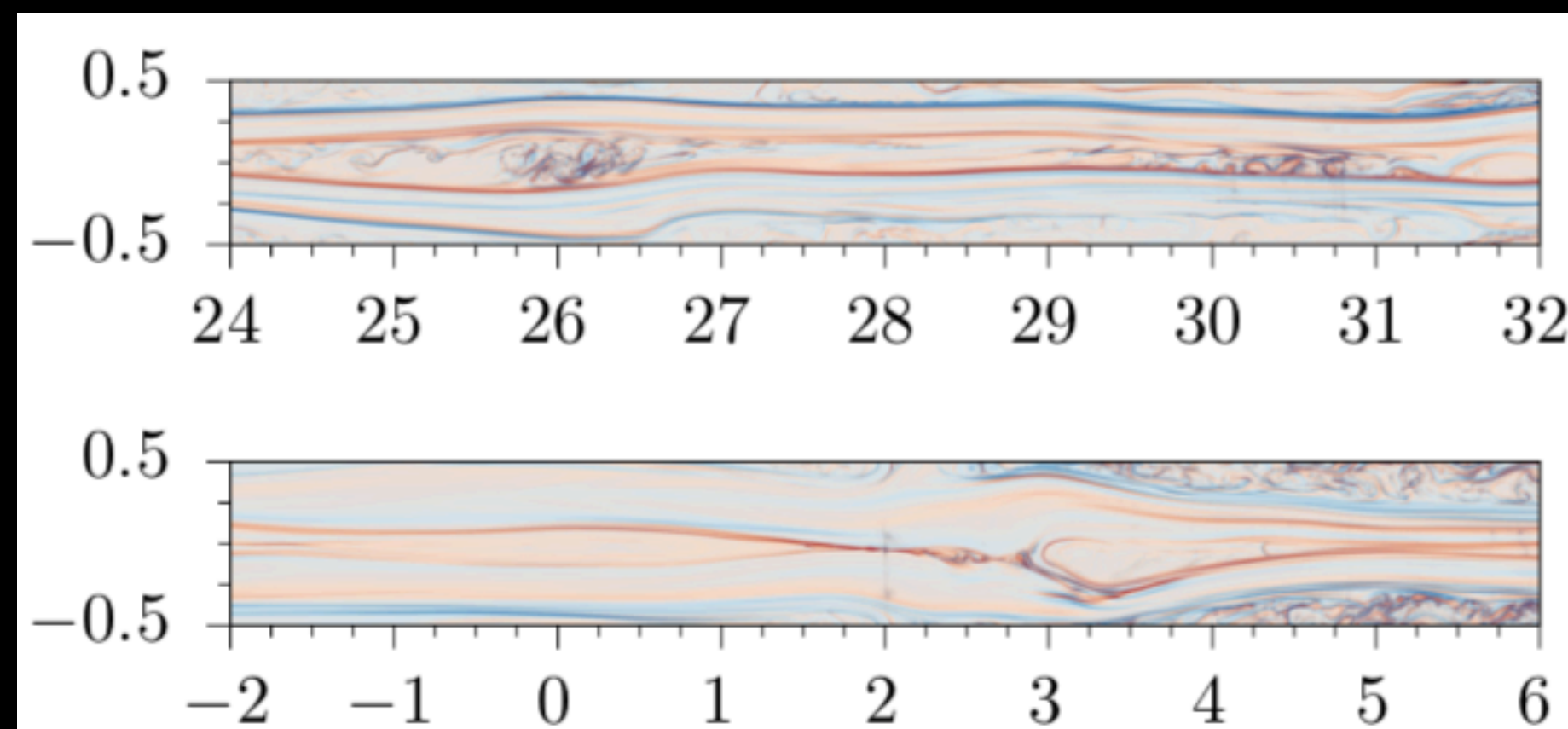
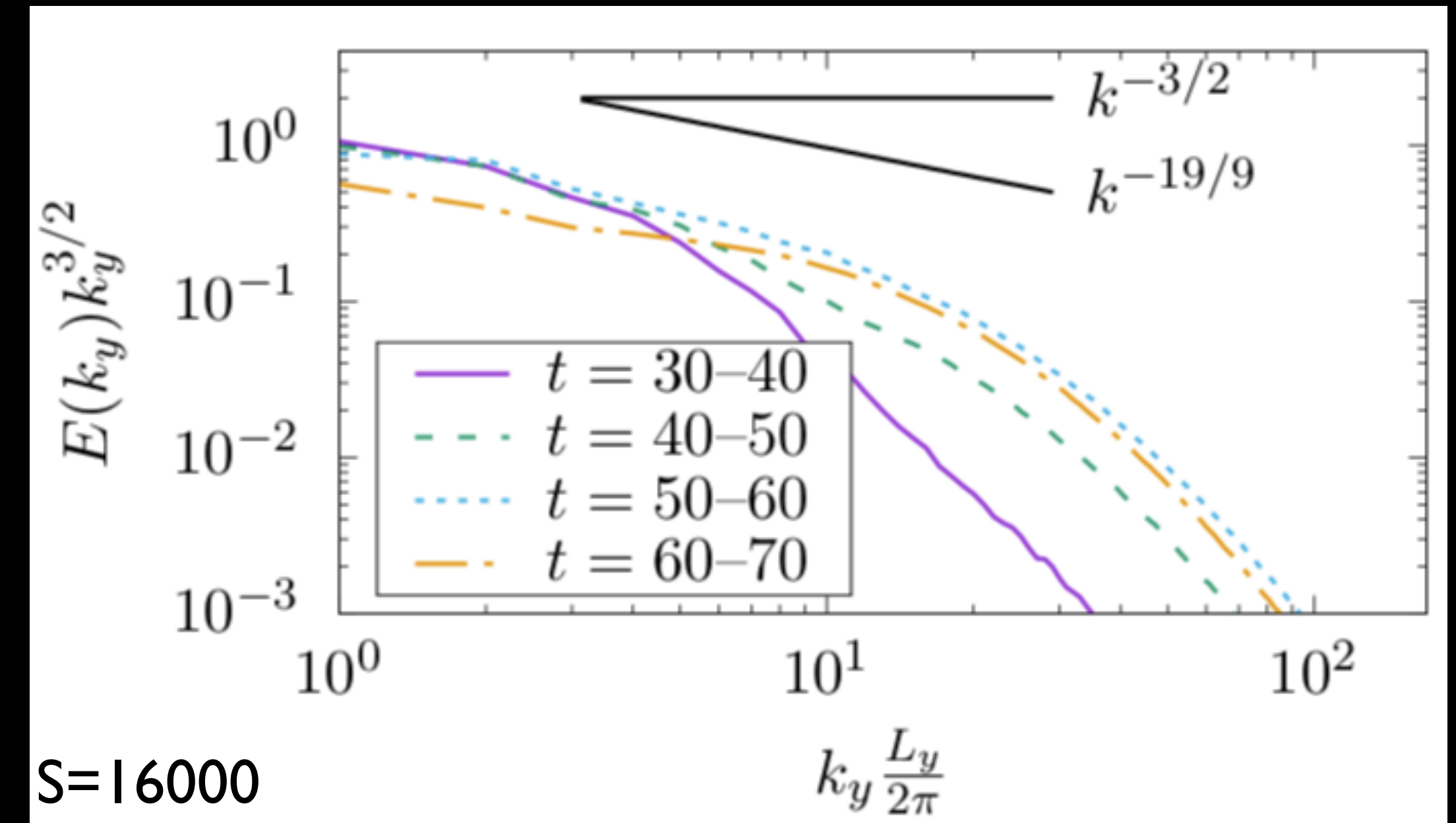
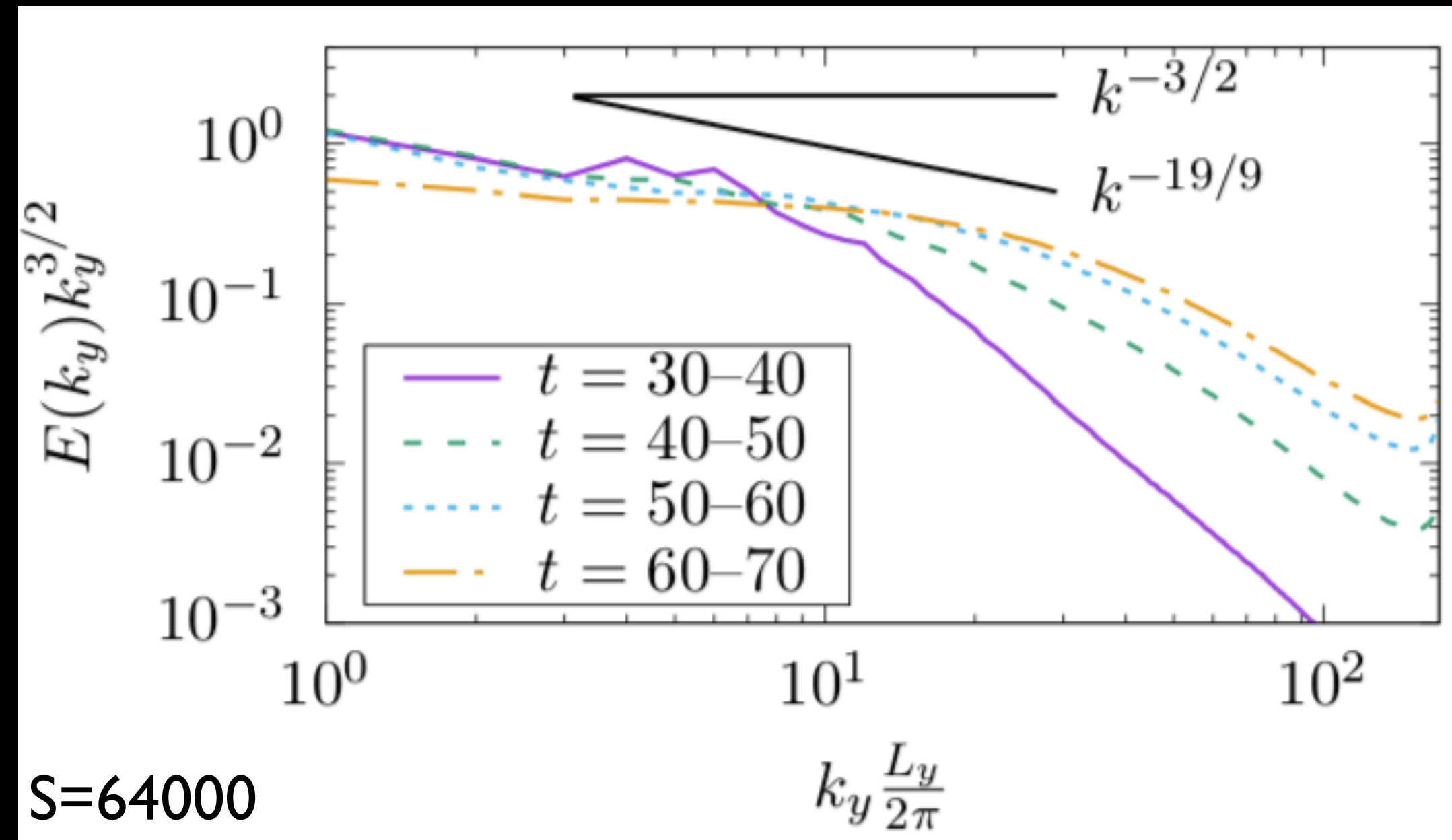
Numerical support for tearing in MHD turbulence

Simulations seem consistent with these predictions



Numerical support for tearing in MHD turbulence

Simulations seem consistent with these predictions



Extension to the kinetic reconnection regime



Collisionless reconnection in MHD-scale eddies

- In many realistic plasmas, collisions are so infrequent that reconnection in a MHD-scale eddy will trigger kinetic effects:

$$\lambda \gg \rho_i \gg \delta \sim d_e$$

- This can be handled with kinetic tearing mode theory (reconnection is caused by electron inertia, instead of collisions).
- Different cases can be analyzed, depending on electron beta.
- Invariably, obtain spectra that scale as

$$E(k_{\perp}) \propto k_{\perp}^{-3} \quad \text{or} \quad E(k_{\perp}) \propto k_{\perp}^{-8/3}$$

depending on what shape is assumed for the reconnecting magnetic field.

Loureiro & Boldyrev, ApJ '17. See also Mallet *et al.*, JPP '17.

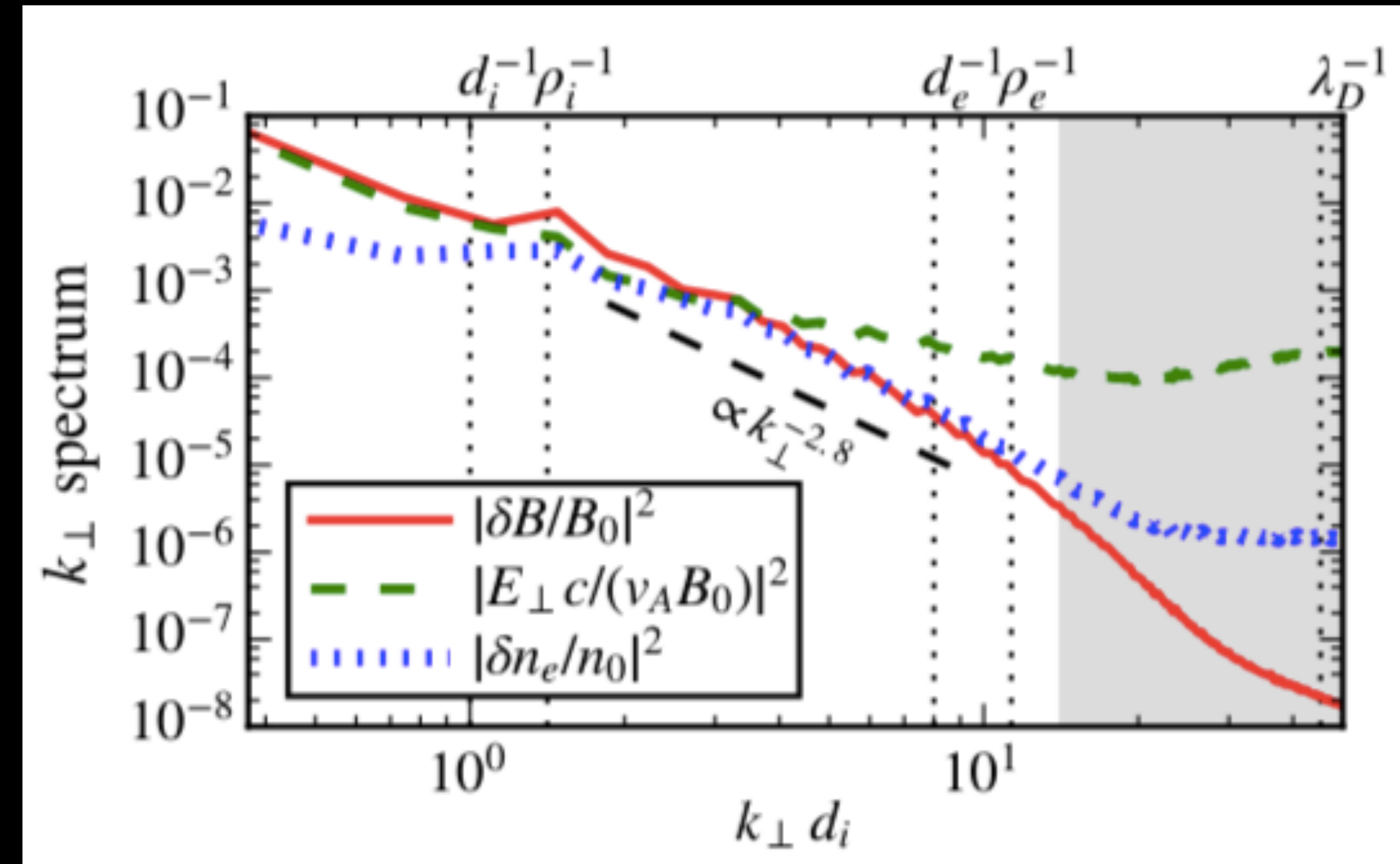
Reconnection in the kinetic turbulence range

Collisionless reconnection at sub-Larmor radius scales

- Can we extend these ideas to the kinetic turbulent range, i.e.,

$$\lambda \ll \max(\rho_i, \rho_s)$$

- Uncertain: no theory to describe the eddy aspect ratio, etc.
- Numerical simulations do suggest current sheet presence at these scales.
- Cannot estimate the critical scale for the transition to the reconnection range – this requires knowing what the eddies look like.



D. Grose et al., PRL '18

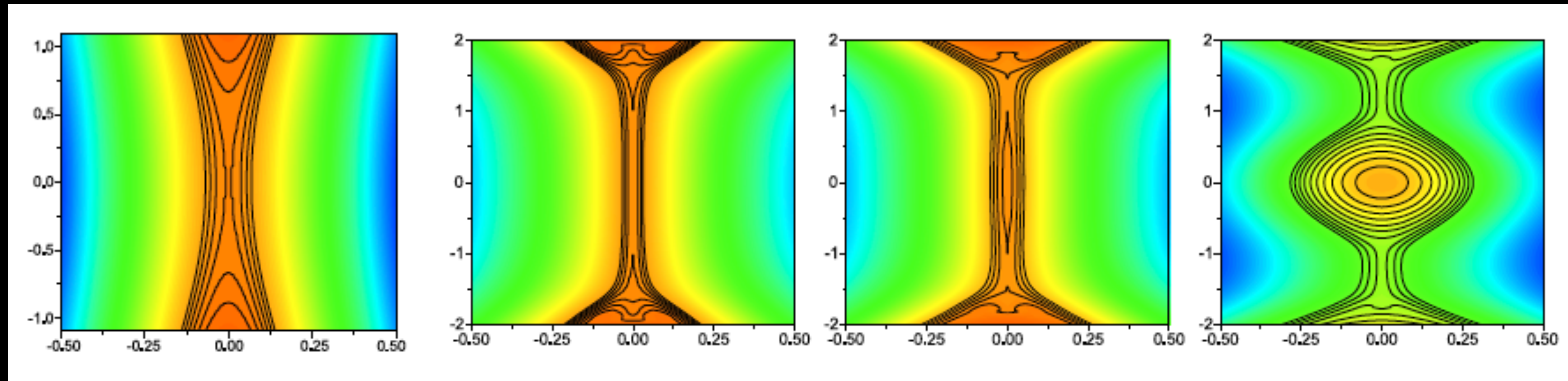
- But can estimate the spectrum given expression for the tearing mode growth rate at those scales. Again, we obtain:

$$E(k_{\perp}) \propto k_{\perp}^{-3} \quad \text{or} \quad E(k_{\perp}) \propto k_{\perp}^{-8/3}$$

Nonlinear Tearing-Mode Evolution

Recap of how MHD tearing proceeds in the absence of turbulence

- When the (most unstable) tearing mode becomes nonlinear, it continues to grow exponentially at the linear growth rate (the Waelbroeck collapse)
- *In the absence of background turbulence*, the collapse leads to the formation of a current sheet: of the Sweet-Parker (SP) kind, if $S < S_{cr} \sim 10^4$, or of the plasmoid-unstable kind, otherwise.
- If SP, the rate remains the same. If plasmoid-dominated, rate becomes $\sim S_{cr}^{-1/2} \sim 0.01$ (Uzdensky *et al.*, '10)

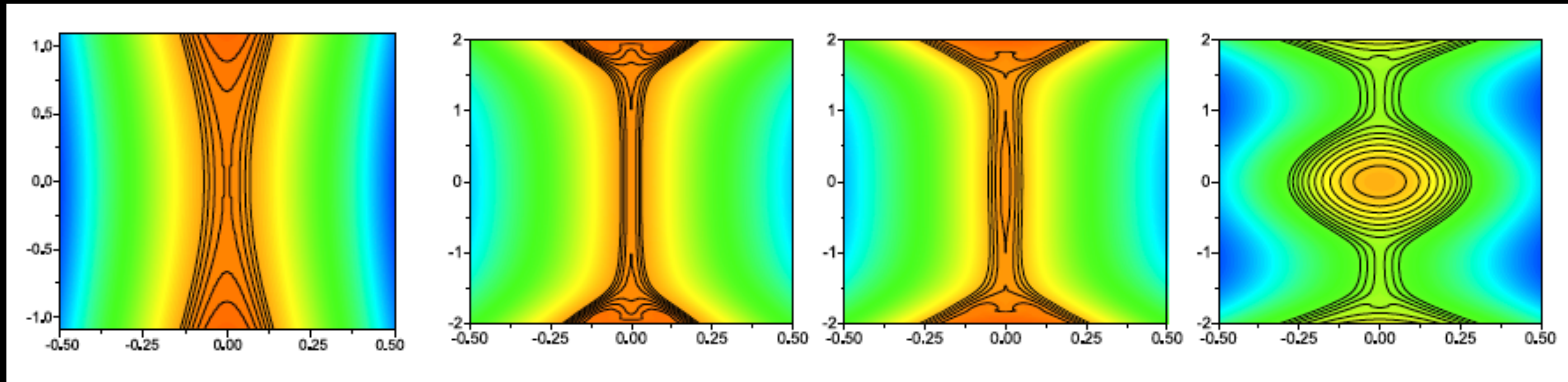


Loureiro *et al.* '05

- Collisionless tearing is similar, except rate becomes 0.1.

Nonlinear Tearing-Mode Evolution

Recap of how MHD tearing proceeds in the absence of turbulence



- What we usually call *reconnection* is the *advanced or the post-collapse stage*: more precisely, it is when the rate changes to 0.01 in the MHD plasmoid case, or 0.1 in the collisionless case.
- Crucially, **it is only in this (late nonlinear) stage that significant amounts of flux are reconnected, and significant (~50%) amounts of energy are dissipated/converted.**

Tearing vs. Reconnection: enter turbulence

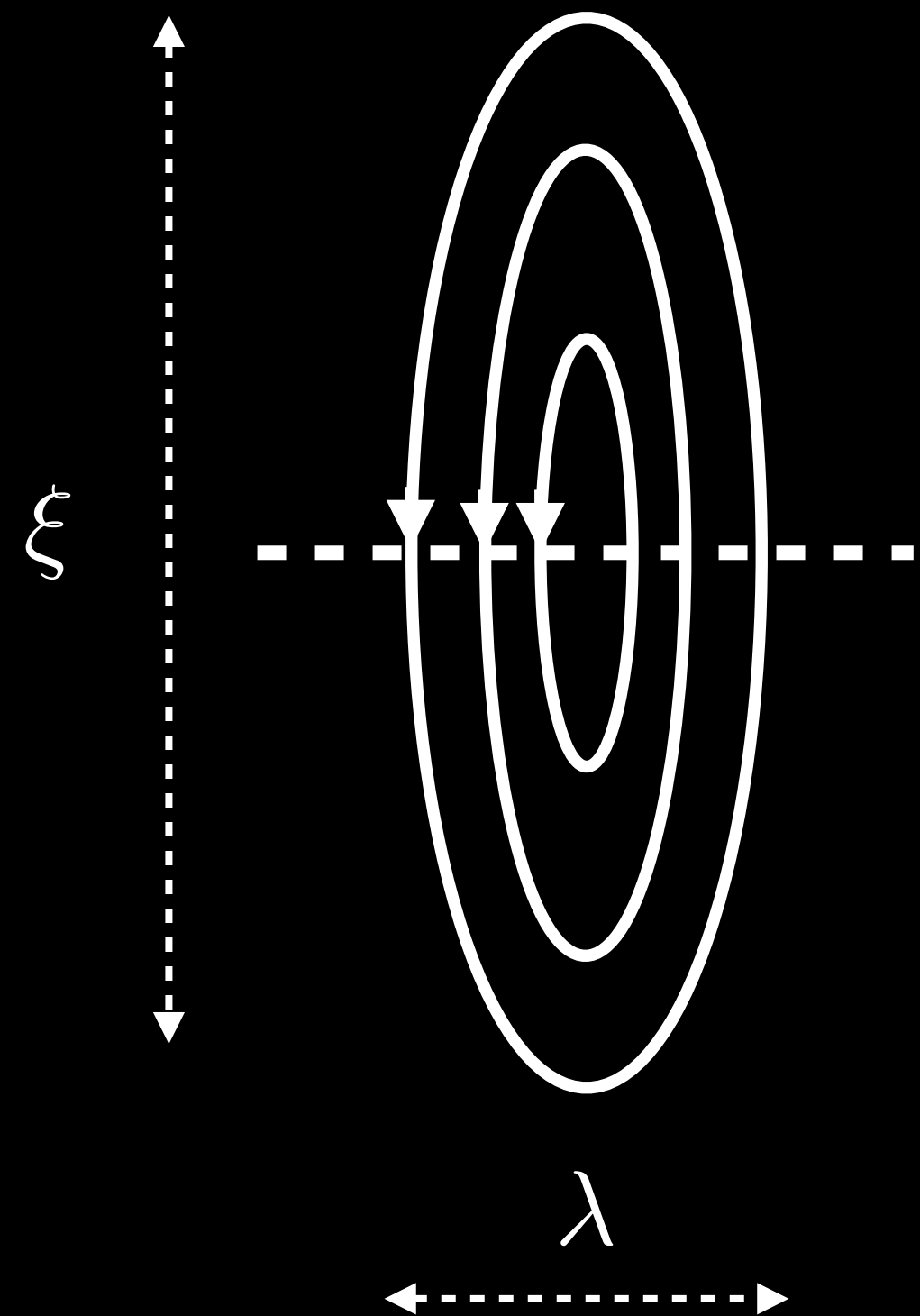


Tearing onset does not guarantee eddy will fully reconnect

- The X-point collapse is a **global** (i.e., at the scale of the eddy) nonlinear rearrangement (i.e., it's a loss of equilibrium); the eddy is forced (by the nonlinearities causing the collapse) to adjust its evolution rate to the tearing rate.
- This means that the collapse is happening on the same timescale as the background (the eddy) is evolving.
- So, how far the collapse proceeds cannot be estimated precisely (no timescale separation).
- All we can say is that there is a finite (order 1) probability that the collapse will last long enough to transition to the reconnection stage proper (and an order 1 probability that it won't)
- **If it does transition to the reconnection stage, will there be time to reconnect significant amounts of flux (and dissipate significant amounts of energy) in the lifetime of the eddy?**

Reconnecting the flux in an eddy

How long does it take to reconnect the flux in an eddy?



$$\psi_\lambda = B_\xi(\lambda)\lambda \quad \text{(reconnectable) flux contained in eddy.}$$

$$\tau_{rec} = \mathcal{R}^{-1} \tau_{A,\lambda} \quad \text{where} \quad \tau_{A,\lambda} = \lambda/v_{A,\lambda}$$

\mathcal{R} is the (dimensionless) reconnection rate.

$$\text{In MHD,} \quad \mathcal{R} \sim S_\xi^{-1/2} \quad \text{if} \quad S_\xi \lesssim S_{cr} \approx 10^4$$

$$\mathcal{R} \sim S_{cr}^{-1/2} \approx 0.01 \quad \text{if} \quad S_\xi > S_{cr}$$

$$\text{In collisionless plasmas,} \quad \mathcal{R} \sim 0.1$$

Can full eddy reconnection happen?



Criterion for reconnection to have time to occur

- A typical eddy at scales $\lambda \ll \lambda_{cr}$ exists for a time of order γ_t^{-1}
- It has a finite probability of reaching the deep nonlinear stage, whereupon it may transition to the reconnection regime.

- If

$$\gamma_t \tau_{rec} \ll 1$$

then the *reconnection time is much shorter than the eddy turnover time*, and it is thus expected that full reconnection will occur.

- *Otherwise, reconnection is slower*, and the eddy will cease to exist without significant reconnection having taken place.
- Now need to work out this condition in the MHD and in the collisionless regimes.

Reconnection in MHD turbulence



Criterion for reconnection to have time to occur

- Tearing becomes relevant below the scale $\lambda_{cr}/L \sim S_L^{-4/7}$
- The eddy turn over rate becomes the tearing mode rate $\gamma_t \sim \tau_{A,\lambda}^{-1} (\lambda v_{A,\lambda}/\eta)^{-1/2}$
where $v_{A,\lambda} \sim \varepsilon^{2/5} \eta^{-1/5} \lambda^{3/5}$ with $\varepsilon = V_{A,0}^3/L$
- So: $\gamma_t \tau_{rec} \ll 1 \implies \lambda/L \gg \mathcal{R}^{-5/4} S_L^{-3/4}$
- This is valid only if $\lambda_{cr} \gg \lambda \gg \lambda_{diss}$ where $\lambda_{diss} \sim S_L^{-3/4} L$
- The first inequality yields $S_L \gg \mathcal{R}^{-7} \sim S_{cr}^{7/2} \sim 10^{14}$

Reconnection in collisionless turbulence



Reconnection in fluid-scale eddies

- Collisionless plasma: electron inertia (not resistivity) breaks the frozen flux. Now $\mathcal{R} \sim 0.1$
- Consider first the case when the tearing onset happens above the ion kinetic scales: $\lambda_{cr} > \rho_i, \rho_s, d_i$
- Low beta case as an example.
- Tearing mode onset scale is $\lambda_{cr}/L \sim (d_e/L)^{4/9}(\rho_s/L)^{4/9}$. Below that scale we have $\gamma_t \sim v_{A,\lambda} d_e \rho_s / \lambda^3$

In this case, we find: $\gamma_t \tau_{rec} \ll 1 \Rightarrow \lambda \gg \mathcal{R}^{-1/2} (d_e \rho_s)^{1/2}$

Valid if $\lambda_{cr} \gg \lambda \gg \rho_s \Rightarrow \mathcal{R} \ll \frac{d_e}{\rho_s} \ll \mathcal{R}^9 \left(\frac{L}{\rho_s} \right)^2$

- In this case, we find: $\gamma_t \tau_{rec} \ll 1 \Rightarrow \lambda \gg \mathcal{R}^{-1/2} (d_e \rho_s)^{1/2}$
- Valid if $\lambda_{cr} \gg \lambda \gg \rho_s \Rightarrow \mathcal{R} \ll \frac{d_e}{\rho_s} \ll \mathcal{R}^9 \left(\frac{L}{\rho_s} \right)^2$
- The left inequality may not hold in the pristine SW: it requires $\beta_e \ll 2(m_e/m_i) \mathcal{R}^{-2} \approx 0.1$ which may be too low.
- But the solar corona should observe both of these conditions (and fall in this case of collisionless reconnection at fluid scales).
- The right inequality places spectacular demands on computer simulations...

- Now consider the case when tearing onset is at sub-ion scales. As an example, take a plasma where $\beta_i \sim 1 \gg \beta_e$ (so-called inertial kinetic-Alfvén turbulence).
- From Boldyrev & Loureiro, arXiv:1901.10096, we have $\gamma_t \sim \frac{v_{Ae,\lambda}}{\lambda} \left(\frac{d_e}{\lambda} \right)^2$
- Therefore, we find: $\gamma_t \tau_{rec} \ll 1 \Rightarrow \frac{\lambda}{d_e} \gg \mathcal{R}^{-1/2} \sim 3$

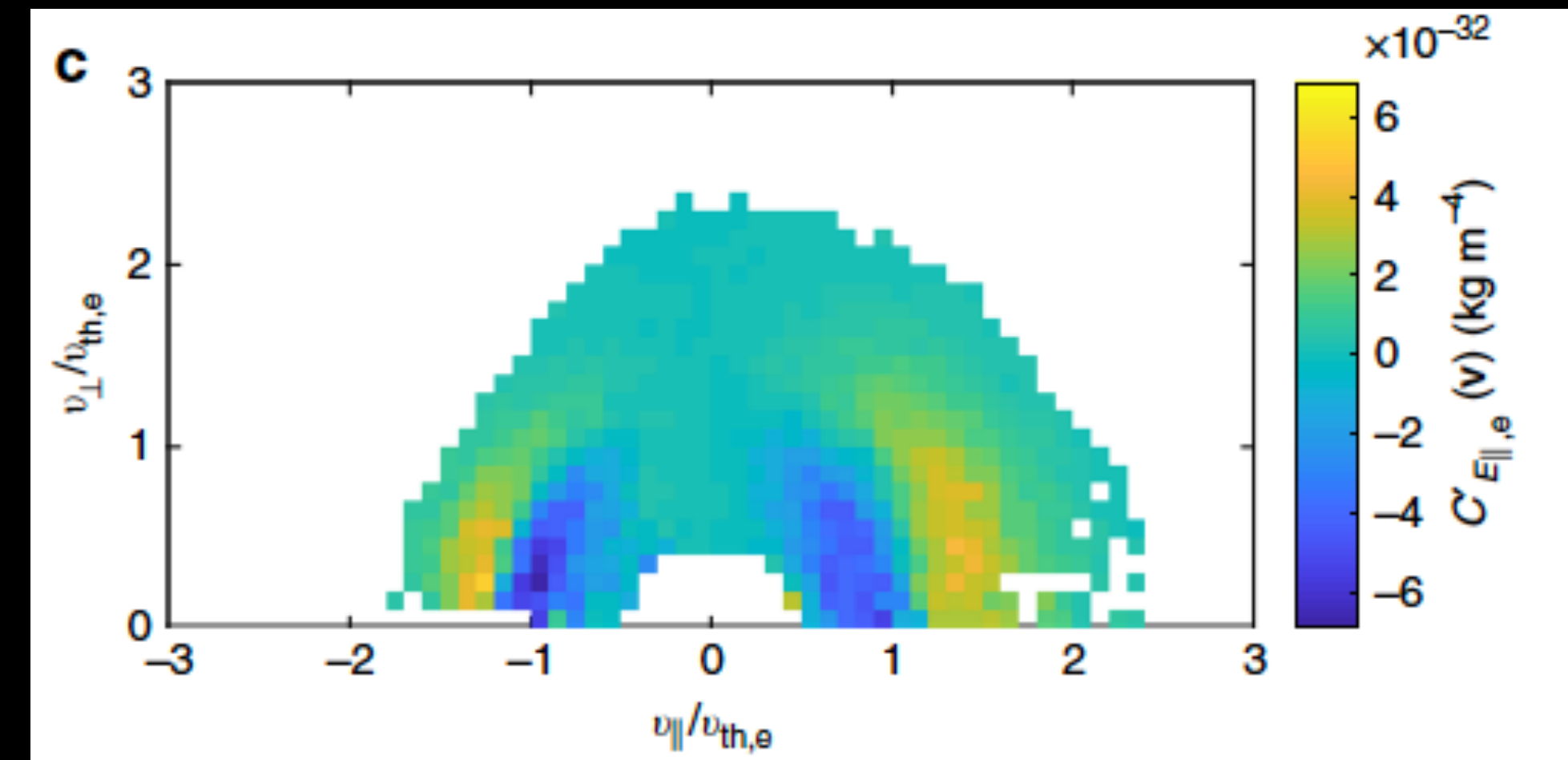
We don't know λ_{cr} for this case, so cannot compute the upper validity limit for this result.

The lower bound is that $\lambda/d_e \gg 1$, which is marginally satisfied.
- This is interestingly consistent with Phan's observations of 'electron-only' reconnection events in the Earth's magnetosheath: reports of reconnection on current sheets $\sim 4d_e$ wide.

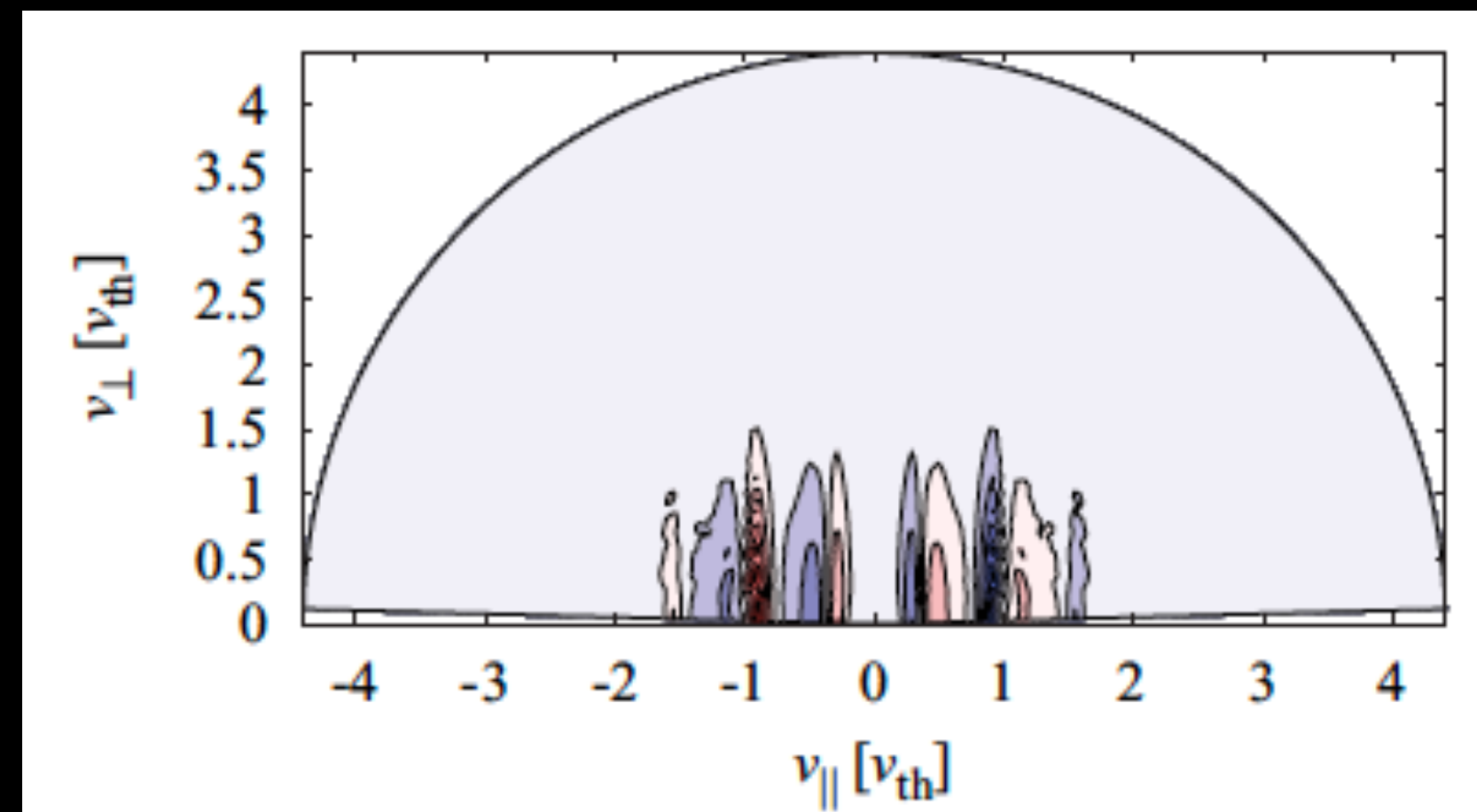
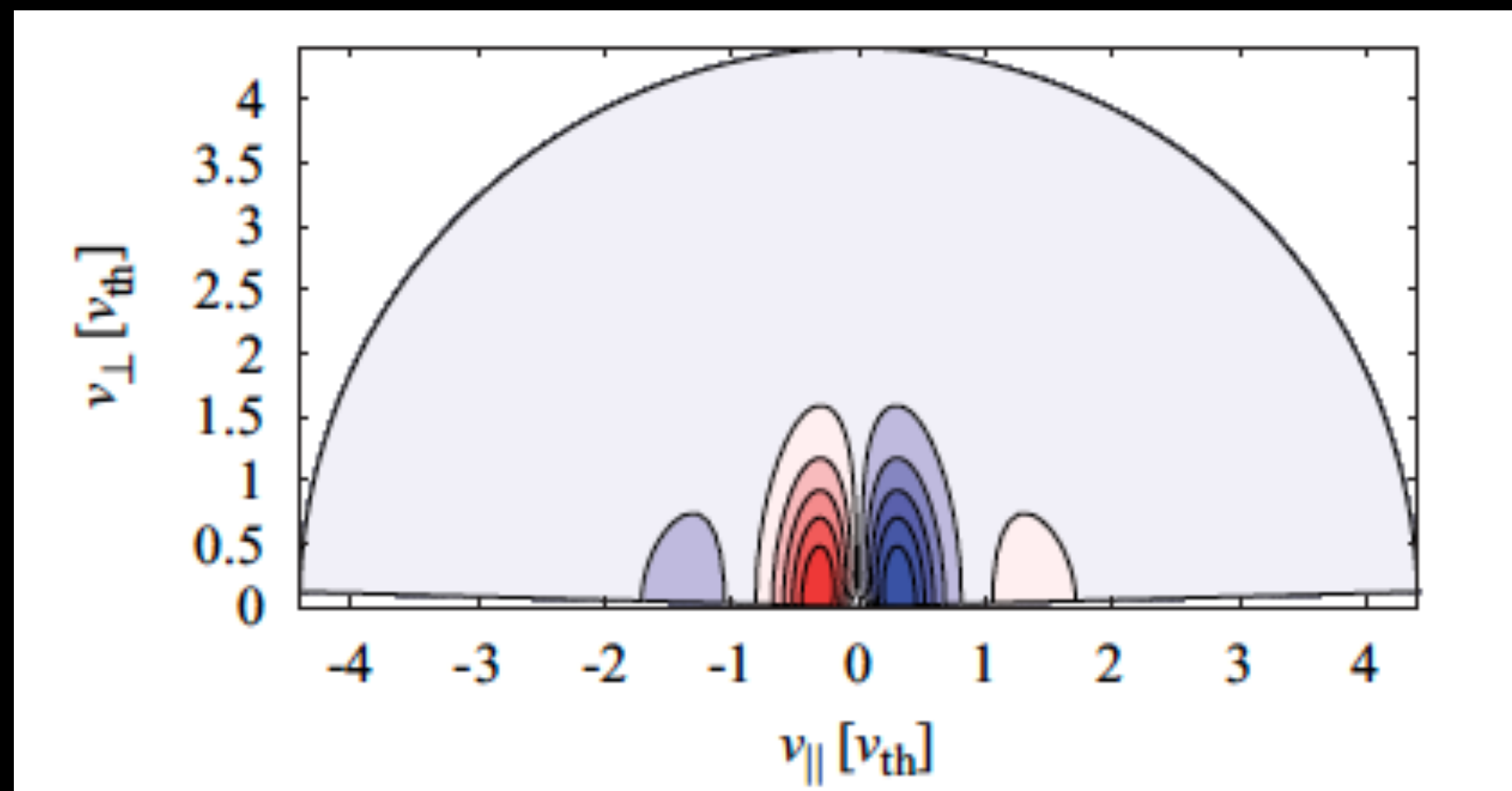
Reconnection at kinetic scales in collisionless turbulence

Consistence with observations

- Chen et al. (2019) claim that energy dissipation at kinetic scales in the magnetosheath is dominated by *linear electron Landau damping* (the energy dissipation rate via that channel being comparable to the energy cascade rate).
- Our analysis demonstrates that **full reconnection in sub-ion scale eddies is permitted** for typical magnetosheath parameters.
- Previous investigations of heating in (strong guide-field) collisionless reconnection (Loureiro et al. 2013; Numata & Loureiro 2015) show that when $\beta_e \ll 1$ linear electron Landau damping is by far the dominant energy dissipation channel.



Chen et al. 2019



Numata & Loureiro 2015

1. If current understanding of MHD turbulence is correct, tearing mode has to become important:

- Eddies become current sheets of progressively larger aspect ratios at small scales
- Therefore, they are progressively more unstable to the tearing mode
- Can compute the scale at which reconnection becomes important. This marks the onset of a new, sub-inertial range whose spectrum is $k_{\perp}^{-11/5}$ or $k_{\perp}^{-19/9}$
- These ideas can be extended to the kinetic regime. In all cases, we obtain spectra that scale as $k_{\perp}^{-8/3}$ or k_{\perp}^{-3} in good agreement with observations and simulations.

2. The onset of tearing does not automatically guarantee that the eddy will reconnect.

- Fundamentally, that's because the reconnection rate is different from the tearing rate; reconnection can only happen if it is *faster* than tearing.
- The conditions for reconnection to happen are *very demanding*
- *We may be significantly underestimating reconnection-driven energy dissipation/conversion in simulations*