Kinetic Instabilities in Magnetized, Collisionless Plasmas

Bale et al 2009 Wind
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\[ \frac{T_{\perp p}}{T_{\parallel p}} \]

\[ \frac{1}{\beta_{\parallel p}} \]

\[ \gamma_{\text{max}} / \Omega_p \]

\[ \frac{1}{C_{5 thief}} \]

\[ \frac{1}{C_{X thief}} \]

\[ \frac{1}{C_{6 thief}} \]

\[ \frac{1}{C_{D thief}} \]

\[ \frac{1}{C_{B thief}} \]

\[ \frac{1}{C_{0 thief}} \]

\[ \frac{1}{C_{B thief}} \]

\[ \frac{1}{C_{Y thief}} \]

\[ \frac{1}{C{4 thief}} \]

\[ \frac{1}{C_{0 thief}} \]

\[ \frac{1}{C_{B thief}} \]

\[ \frac{1}{C_{Y thief}} \]

\[ \frac{1}{C_{0 thief}} \]

\[ \frac{1}{C_{B thief}} \]

\[ \frac{1}{C_{Y thief}} \]

\[ \frac{1}{C_{0 thief}} \]
The Solar Wind Frequently Departures from LTE

Particle Velocity Distributions $f_s(v)$

Due to its hot, diffuse nature, collisions are unable to enforce a Maxwellian distribution in the solar wind:

$$\nu_{a,b} \approx 4\pi \frac{q_a^2 q_b^2 n_b \ln \Lambda}{m_a^2 w_b^3} \propto nT^{-3/2}$$

Typical structures include:
- $T_\perp \neq T_\parallel$
- $T_i \neq T_j$
- relative drifts
- agyrotropy
Kasper et al, 2017: Wind

’Collisionality’ (e.g. $N_c = \nu_{a,b} \frac{R}{V_{sw}}$) organizes anisotropies, disequalibria, and relative drift speeds of components.
Extracting “Energy” from non-Maxwellian Distributions

Particle Velocity Distributions $f_s(v)$

Fluid Firehose Instability:
Treumann & Baumjohann, 1997

Cyclotron Resonant Instability:
Verscharen, Klein, & Maruca, under review

Marsch et al, 2012: Helios
“Simple Models” for Stability Thresholds

Verscharen, Klein, & Maruca  Wind

\[
\frac{T_{\perp,p}}{T_{\parallel,p}} = 1 + \frac{a}{(\beta_{\parallel,p} - \beta_0)^b}
\]

Hellinger et al. 2006

Chen et al 2016  Wind

\[
\Lambda_{\text{firehose}} = \frac{\beta_{\parallel} - \beta_{\perp}}{2} + \frac{\sum_s \rho_s |\Delta \mathbf{v}_s|^2}{\rho v_A^2} > 1
\]

\[
\Lambda_{\text{mirror}} = \sum_s \beta_{\perp s} \left( \frac{\beta_{\perp s}}{\beta_{\parallel s}} - 1 \right) + \frac{\left( \sum_s q_s n_s \frac{\beta_{\perp s}}{\beta_{\parallel s}} \right)^2}{2 \sum_s (q_s n_s)^2} > 1
\]

Kunz et al 2015

Such models do not account for other energy sources (e.g. other species anisotropies, relatively drifting components)
Given the wave vector equation
\[ \mathbf{n} \times (\mathbf{n} \times \mathbf{E}) + \mathbf{\epsilon} \cdot \mathbf{E} = \mathbf{D} \cdot \mathbf{E} = 0 \]
normal modes \((\omega, \gamma)\) arise for \(|\mathbf{D}| = 0\).

We model \(|\mathbf{D}|\) in the solar wind using:
- a collection of \(N_s\) bi-Maxwellian distributions
- drifting relative to one another
- with a background magnetic field

Normal modes are a function of \(6N_s - 1\) parameters:
- wavevector \(k_{\perp}\rho_R, k_{\parallel}\rho_R\)
- 'global' plasma parameters \(\beta_{\parallel,R}\) and \(v_{t,\parallel,R}/c\)
- the ratios \(T_{\perp}/T_{\parallel,s}, T_{\parallel,R}/T_{\parallel,s}, n_s/n_R, m_s/m_R, q_s/q_R,\) and \(V_{\text{drift,}\parallel,s}/v_{AR}\) for each population \(s\)
Given the wave vector equation
\[ \mathbf{n} \times (\mathbf{n} \times \mathbf{E}) + \epsilon \cdot \mathbf{E} = \mathbf{D} \cdot \mathbf{E} = 0 \]
unstable modes are solutions \(|\mathbf{D}| = 0\) in complex frequency space \((\omega, \gamma)\) that have a positive damping rate \(\gamma > 0\).

Klein & Howes 2015 Hot linear dispersion relation from Plasma in a Linear Uniform Magnetized Environment (PLUME)

Stability sensitively depends on the bulk parameters of each species, which cover a wide range of values in the solar wind, making implementing a manual scan time consuming.
The Nyquist Instability Criterion (Nyquist 1932)

Instead of searching for solutions of $|D(\omega, \gamma > 0)| = 0$, we evaluate the contour integral for the number of unstable modes: (PLUMAGE; Klein et al. 2017 JGR).

$$W_n = \frac{1}{2\pi i} \oint \frac{d\omega}{|D(\omega, \gamma)|}$$

$$W_n = 0$$

$$W_n = 1$$

“In another hundred and twenty days the building of the Integral will be completed. The great historic hour is near, when the first Integral will rise into the limitless space of the universe.”

—We Yevgeny Zamyatin
We can test for arbitrarily fast growing modes

Instead of using $\gamma = 0$ to define the contour, we calculate $W_n$ for any growth rate $\gamma_{\text{min}}$. (This requires the insertion of a branch cut).

This procedure for a range of wavevectors produces $\gamma_{\text{max}}(k)$.
A Statistical Data Set from the Solar Wind

- We select a random set of Wind spectra, the first nominal spectra a day from 309 days in 2016 & 2017.
- For each spectrum, a nonlinear-least-squares Bi-Maxwellian fit is performed for up to three ion components:
  - **proton core** $n_p, T_{\perp p}, T_{\parallel p}$,
  - **proton beam** $n_b, T_{\perp b}, T_{\parallel b}, \Delta v_{pb}$,
  - **$\alpha$ population** $n_\alpha, T_{\perp \alpha}, T_{\parallel \alpha}, \Delta v_{p\alpha}$,
and combined with $|B|$ to produce the associated dimensionless parameters. (Klein et al 2018)

- For Each Spectra, we calculate $W_n(k\rho_p)$ on a grid covering $(k_{\perp}, k_{\parallel})\rho_p \in [10^{-2}, 10^1]$.
- The maximum growth rates of unstable spectra are found within $\gamma_{\text{min}}/\Omega_p = [10^{-4}, 1]$. 
Occurrence of Ion-Driven Instabilities

<table>
<thead>
<tr>
<th></th>
<th># Spectra</th>
<th># Unstable</th>
<th>Mirror</th>
<th>CGL FH</th>
<th>Kinetic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>309</td>
<td>166</td>
<td>14</td>
<td>1</td>
<td>151</td>
</tr>
<tr>
<td>p, b, &amp; α</td>
<td>189</td>
<td>130</td>
<td>12</td>
<td>0</td>
<td>118</td>
</tr>
<tr>
<td>p &amp; α</td>
<td>114</td>
<td>33</td>
<td>2</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>p &amp; b</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>p</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- 54% of spectra are unstable
- The majority of the instabilities are kinetic, i.e. $k_\perp \rho_p < k_\parallel \rho_p \lesssim 1$
- Instabilities preferentially arise when a proton beam is resolved

Klein et al 2018
The fluid instabilities have the largest growth rates

Unstable intervals w/o proton beams are less virulent

10% of the spectra have $\gamma$ comparable to the cascade time at $k_\perp \rho_p = 1$.
A significant fraction of spectra in the ‘stable’ region support growing modes. (Klein et al 2018)
Are there actually waves here?
Distance from Sun (au)

Verscharen, Klein, & Maruca, under review
Seeking Insight into Radial Variations.

Applying PLUMAGE to \( \sim 40,000 \) measurements from Helios (Stansby et al 2018), we find \( \sim 80\% \) are unstable.

Stability does not seem to primarily depend on proximity to the Sun or \( v_{sw} \), but rather on \( N_C, T_\perp/T_\parallel \), and \( \Delta v_{\alpha,p} \).
**Arbitrary Linear Plasma Solver (ALPS, Verscharen et al 2018, JPP)** solves the full hot-plasma dispersion relation for a set of plasma populations with arbitrary velocity distributions defined on a grid in momentum space.

\[
\frac{T_\perp}{T_\parallel} = 1.40 \\
\beta_{\parallel,p} = 10.0 \\
k_\perp d_p = 0.40 \\
k_\parallel d_p = 0.20
\]

\[
f(v_\perp, v_\parallel) = f_{\text{Bi}}(v_\perp, v_\parallel) \left(1 - A \exp\left[-\frac{v_\parallel^2}{(0.6v_\parallel)^2}\right]\right)
\]
Non-thermal structure is ubiquitous and provides several sources of free energy to drive instabilities.
Future Concepts to Explore Space Plasma Processes

To-be-proposed missions comprised of many spacecraft, such as HelioSwarm with inter-spacecraft separations spanning large and small scales, will enable more detailed studies of energy transport, including unstable growth and its effect on the background turbulence.
Instabilities Limit the Solar Wind’s Evolution

Matteini et al 2007

These correlations may mask underlying dependencies.

(Hellinger & Travnicek 2014)

Bale et al 2009
Parametric Dependence of Ion-Driven Instabilities

\[ \Delta X \equiv \frac{\bar{X}_{\text{unstable}} - \bar{X}_{\text{stable}}}{\bar{X}}. \]

<table>
<thead>
<tr>
<th></th>
<th>( \beta_{\parallel p} )</th>
<th>( 10^4 v_{tp}/c )</th>
<th>( T_{\perp p}/T_{\parallel p} )</th>
<th>( T_{\perp \alpha}/T_{\parallel \alpha} )</th>
<th>( T_{\perp b}/T_{\parallel b} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>0.60</td>
<td>1.07</td>
<td>1.57</td>
<td>0.96</td>
<td>1.48</td>
</tr>
<tr>
<td>Stable</td>
<td>0.50</td>
<td>0.91</td>
<td>1.12</td>
<td>1.03</td>
<td>1.39</td>
</tr>
<tr>
<td>Unstable</td>
<td>0.68</td>
<td>1.21</td>
<td>1.96</td>
<td>0.90</td>
<td>1.52</td>
</tr>
<tr>
<td>( \Delta X_{p,\alpha,b}(%) )</td>
<td>19.12</td>
<td>13.46</td>
<td>50.59</td>
<td>-21.06</td>
<td>8.45</td>
</tr>
<tr>
<td>( \Delta X_{p,\alpha}(%) )</td>
<td>132.53</td>
<td>57.59</td>
<td>-26.77</td>
<td>14.16</td>
<td>—</td>
</tr>
</tbody>
</table>

|                | \( T_{\parallel \alpha}/T_{\parallel p} \) | \( T_{\parallel b}/T_{\parallel p} \) | \( n_{\alpha}/n_p \) | \( n_{b}/n_p \) | \( |v_{\alpha}|/v_A \) | \( |v_{b}|/v_A \) |
|----------------|----------------------------------|----------------------------------|----------------------|-----------------|-----------------|-----------------|
| Total          | 10.89                            | 2.72                             | 0.04                 | 0.43            | 0.31            | 0.84            |
| Stable         | 5.24                             | 2.35                             | 0.04                 | 0.41            | 0.16            | 0.73            |
| Unstable       | 15.74                            | 2.88                             | 0.05                 | 0.44            | 0.44            | 0.89            |
| \( \Delta X_{p,\alpha,b}(\%) \) | 64.27                           | 20.83                            | 2.61                 | 2.90            | 61.57           | 21.84           |
| \( \Delta X_{p,\alpha}(\%) \) | 26.46                           | —                                | 18.10                | —               | 77.44           | —               |
To Verify Our Algorithm

we calculate $W_n(k)$ for points in the $(\beta_{||p}, T_{\perp p}/T_{||p})$ plane using PLUME’s hot, magnetized plasma dispersion relation.
Instead of using the $\gamma = 0$ contour, we calculate $W_n$ for $\gamma_{\text{min}} = 10^{-2} \Omega_p$. 
Identified 6 intervals with observational signatures of parallel propagating instabilities in the magnetic power spectra.
How Robust are these Results?

10 % Monte Carlo variation of proton core & beam, He\textsuperscript{++}, and e\textsuperscript{−} parameters.

Events 4 & 6 are near unstable regions of parameter space

Klein et al 2017