Kinetic Alfvén Turbulence: An Update

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Acknowledgements

Recent collaborators:

Alfred Mallet (Berkeley), Chris Chen (QMUL), Silvio Cerri (Princeton), Luca Franci (QMUL), Ravi Samtaney (KAUST), Kai Schneider (I2M-CNRS), Frank Jenko (IPP Garching),

OSIRIS code [developed and distributed by the OSIRIS Consortium (UCLA & IST, Portugal)]:

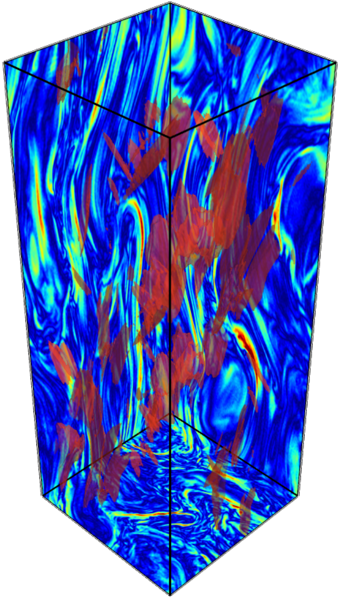
- 3D, fully kinetic, fully explicit, & relativistic PIC code

⇒ **Here**: Utilized for space/astro 3D kinetic plasma turbulence simulations

Outline

1. Polarization alignment in kinetic Alfvén wave (KAW) turbulence?
   (a) generalized spectral field ratios
   (b) some exact (localized) wave solutions of the electron reduced MHD eqs.
   (c) intermittency + alignment (simulated & observed)

2. 3D local anisotropy of KAW turbulence
   (a) the "statistical eddies" of sub-ion range turbulence
   (b) anisotropy scalings
   (c) comparison with other kinetic simulations



3D kinetic turbulence data: overview

Driven 3D fully kinetic simulation:

- $\beta_i \approx \beta_e \approx 0.5$, $m_i/m_e = 100$, $L_\perp \approx 19d_i$ $L_\perp/L_\| \approx 0.4$, non-relativistic regime
- spatial resolution $928^2 \times 1920$, about 0.5 trillion particles in total

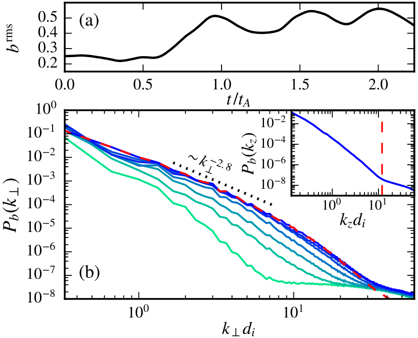
SW measurements:

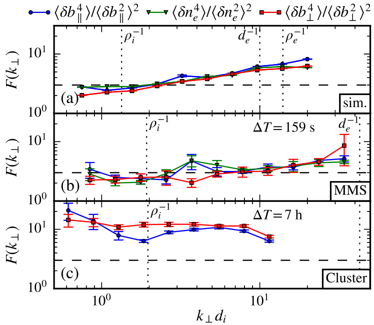
- 7 h interval from Cluster ($\mathbf{B}$ data) [Chen *et al.* (2015)] ($\beta_i \approx 0.3$, $\beta_e \approx 0.6$)
- 159 s interval from MMS ($\mathbf{B}$ & n_e data) [Gershman *et al.* (2018)] ($\beta_i \approx 0.3$, $\beta_e \approx 0.03$)

3D hybrid-kinetic simulations (kindly provided by main authors):

- Cerri, Servidio & Califano, ApJ (2017),
- Arzamasskiy *et al.*, ApJ (2019).

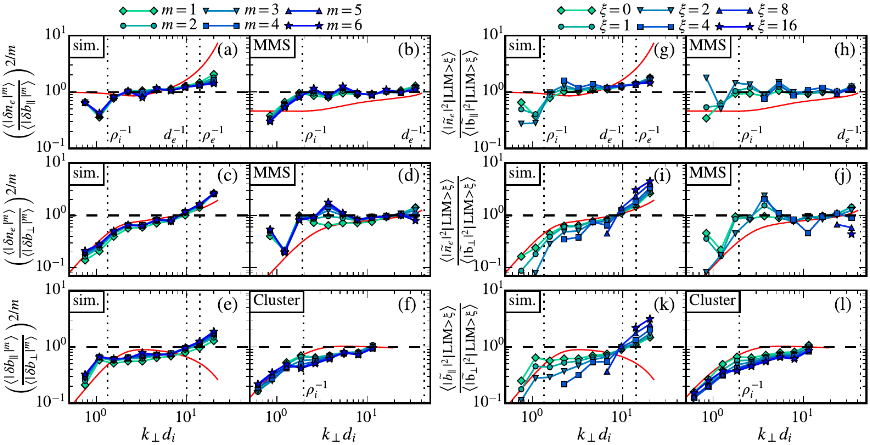
Note: In *all* simulations considered here, sub-ion range is limited to $kd_i \lesssim 10$!





Generalized spectral field ratios

- ratios of spectral amplitudes of $\delta b_\perp$, $\delta b_\parallel$, δn_e can be used to detect KAW polarization in a turbulent plasma
- we introduced "generalized ratios" to probe the statistical polarizations within the large-amplitude, localized turbulent structures (see Groselj *et al.*, PRX (accepted), arXiv:1806.05741)



$\Rightarrow$ Large-amplitude structures (often considered as non-wavelike) preserve a linear wave footprint

OK, but why?...

Why do large-amplitude (nonlinear) structures carry a wave signature?

- Nonlinear time = inversely proportional to fluct. amplitude $\Rightarrow$ Naively one might think that intense structures evolve faster than linear waves
- But, linear time scale $\propto \ell_{||}$ and max. $\ell_{||}$ is limited from above by causality (implying critical balance) so linear time scale keeps up
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Kinetic Alfvén turbulence may be approximately described with the reduced electron MHD eqs. [Schekochihin *et al.* (2009)]:

$$\partial_t \psi = -\partial_z n_e - \hat{\mathbf{z}} \cdot (\nabla_\perp \psi \times \nabla_\perp n_e), \tag{1}$$

$$\partial_t n_e = \partial_z \nabla_\perp^2 \psi + \hat{\mathbf{z}} \cdot (\nabla_\perp \psi \times \nabla_\perp \nabla_\perp^2 \psi). \tag{2}$$

How "robust" are the linear KAW solutions?

- A combination of co-propagating KAWs (with $\psi_{\mathbf{k}} = \pm k_\perp n_{e,\mathbf{k}}$) with a *fixed magnitude* of $k_\perp$ is an *exact* solution [Schekochihin *et al.* (2009)]
- Is that all? No!

Some exact wave solutions of the ERMHD eqs. 1/2

To find exact wave solutions we require that the nonlinear Poisson brackets vanish:

- Satisfied whenever the contours of ψ, n_e, & $\nabla_\perp^2 \psi$ are aligned in every $\perp$ plane (nonlinearity cancels geometrically)
- The alignment between the $\perp$ contours of ψ and $\nabla_\perp^2 \psi$ restricts the geometry of the solutions

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2 types of such exact wave solutions exist. Their $\perp$ profiles are either:

1. circularly symmetric ($n_e = n_e(r_\perp, z), \psi = \psi(r_\perp, z)$), or
2. one-dimensional (fixed *orientation* of $\mathbf{k}_\perp$)

$\Rightarrow$ n_e & ψ here need not satisfy a fixed-phase relation so solutions may be composed of counter-propagating KAWs

$\Rightarrow$ there is no fixed $k_\perp$ constraint so KAW packets can have a *localized* $\perp$ envelope (at some $t = t_{\text{ref}}$)

Some exact wave solutions of the ERMHD eqs. 2/2

Implications for sub-ion scale turbulence:

- Simulations find that sub-ion scale structures are either elongated sheets or circular tubes (e.g., Boldyrev & Perez, ApJL (2012); Meyrand & Galtier, PRL (2013); Kobayashi *et al.*, ApJ (2017))
- The *idealized* geometric versions of these two (1D sheets or circularly symmetric, field-aligned tubes) are exact wave solutions (for $k_{\|} \neq 0$) of ERMHD
- Due to wandering of field lines in turbulent flows, exact alignment cannot be reached [Boldyrev, PRL (2006)] but even if structures *resemble* the ideal solutions the nonlinearity is locally depleted and nonlinear time slows down

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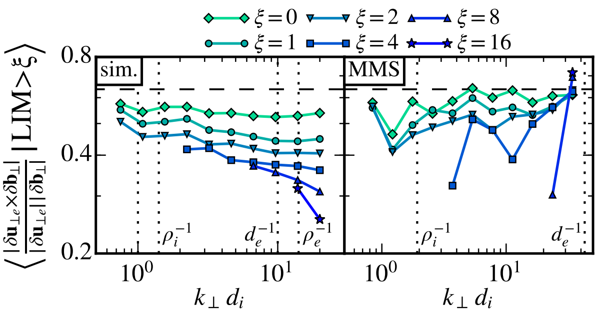
Is this reasonable?
Consider the scale-dependent alignment between the $\perp$ *electron* fluid velocity ($\propto \hat{\mathbf{z}} \times \nabla_\perp n_e$) & magnetic field ($\propto \hat{\mathbf{z}} \times \nabla_\perp \psi$):

$$\sin\theta \equiv |\delta\mathbf{u}_{\perp e} \times \delta\mathbf{b}_\perp| / |\delta\mathbf{u}_{\perp e}||\delta\mathbf{b}_\perp| \tag{3}$$

Compute $\sin\theta$ conditionally averaged on the (normalized) local KAW spectral energy density:

$$\left\langle \sin\theta(k_\perp) \middle| \mathrm{LIM} = \frac{\mathcal{E}_{KAW}(k_\perp, \mathbf{r})}{\langle \mathcal{E}_{KAW}(k_\perp, \mathbf{r}) \rangle_{\mathbf{r}}} > \xi \right\rangle_{\mathbf{r}} \tag{4}$$

Intermittent polarization alignment in KAW turbulence



- high-amplitude structures are indeed more aligned, similar to what was found in MHD (e.g., Beresnyak & Lazarian, ApJ (2006); Mallet *et al.*, MNRAS (2016))
- trend is seen in 3D fully kinetic simulation & in MMS data but is weaker in the latter case (note that MMS interval is only weakly intermittent)

Are KAW eddies 3D anisotropic?

- Introduce 3D conditional structure function in the *local* frame:

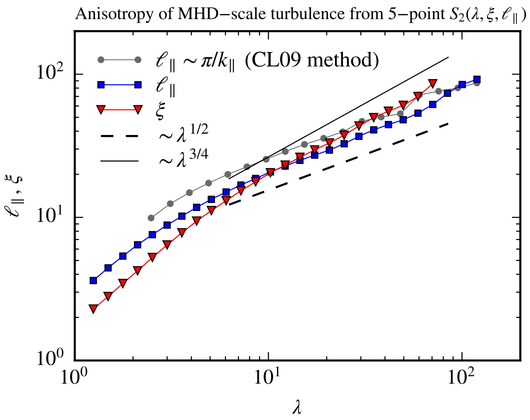
$$S_m(r,\theta,\phi) = \left\langle |\Delta f(\mathbf{r}_0,\mathbf{r})|^m \,\middle|\, r,\theta,\phi \right\rangle_{\mathbf{r}_0}, \tag{5}$$

  where $\cos\theta = \hat{\mathbf{r}} \cdot \hat{\mathbf{B}}_{\mathrm{loc}}$ & $\cos\phi = \hat{\mathbf{r}}_\perp \cdot \delta\hat{\mathbf{b}}_\perp$

- 3 natural directions: the parallel direction, $\ell_\parallel$ $(\theta = 0)$, fluctuation direction, ξ $(\theta = 90^\circ, \phi = 0)$, "perpendicular" direction, λ $(\theta = 90^\circ, \phi = 90^\circ)$
- Δf is the field increment. For steep spectra, increments with more than 2 points are needed to measure the true scaling.
- For 5-point increments: $\Delta f(\mathbf{r}_0,\mathbf{r}) = [f(\mathbf{r}_0 + 2\mathbf{r}) - 4f(\mathbf{r}_0+\mathbf{r}) + 6f(\mathbf{r}_0) - 4f(\mathbf{r}_0 - \mathbf{r}) + f(\mathbf{r}_0 - 2\mathbf{r})]/\sqrt{35}$.
- Alternatively, the spectral method of Cho & Lazarian (2009; CL09) may be used to estimate the $k_\parallel(k_\perp)$ scaling:

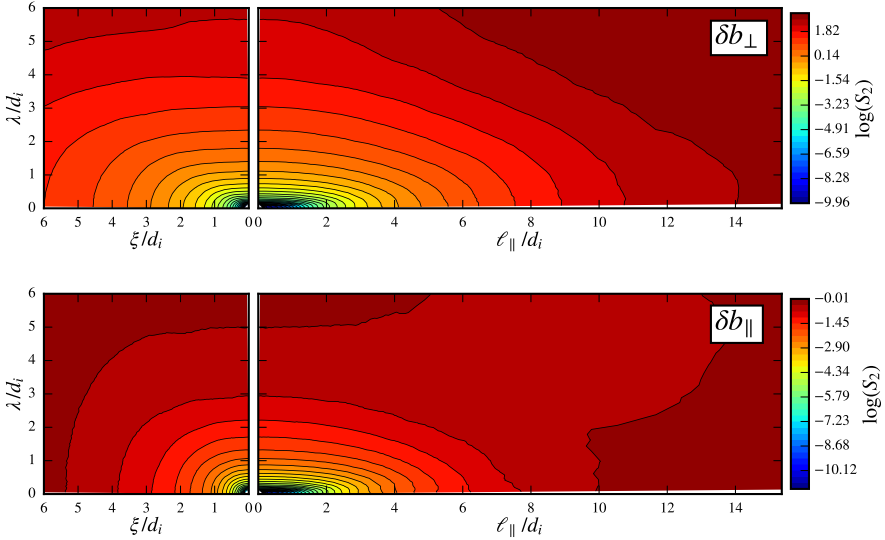
$$k_\parallel(k_\perp) \approx \frac{\langle |\mathbf{B}_{0k_\perp} \cdot \nabla \delta\mathbf{b}_{k_\perp}|^2 \rangle^{1/2}}{\langle |\delta\mathbf{b}_{k_\perp}|^2\rangle^{1/2} \langle |\mathbf{B}_{0k_\perp}|^2\rangle^{1/2}} \tag{6}$$

Consistency check: (driven) MHD-scale turbulence



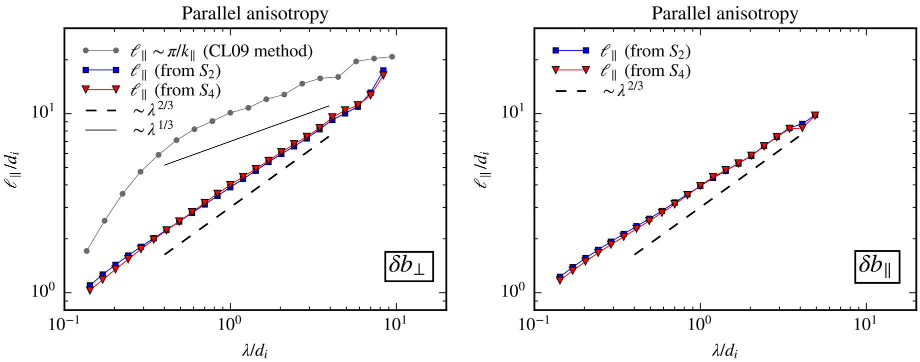
- reasonable agreement with predictions for (dynamically aligned) MHD turbulence [Boldyrev, PRL (2006)]: $\ell_\parallel \sim \lambda^{1/2}$, $\xi \sim \lambda^{3/4}$
- spectral method (CL09) scaling in agreement with scalings from 5-point structure function

The 3D statistical eddies of KAW turbulence



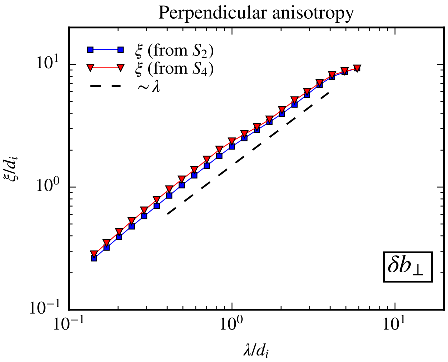
- Eddies *do* become more elongated along $\ell_\parallel$ (in this simulation!) with decreasing scale, but there is hardly any anisotropy in $\perp$ local plane

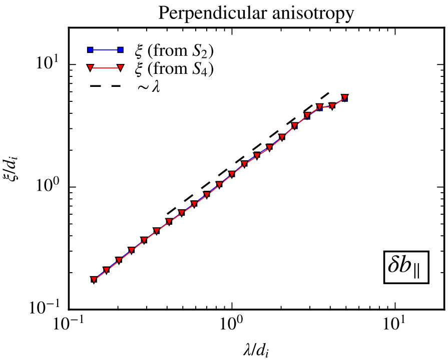
Parallel anisotropy scalings



- Structure function method gives weaker (but still scale-dependent) anisotropy than spectral method (consistent with what was found in the original Cho & Lazarian (2009) paper)
- Scalings deduced from isocontours of S_2 and S_4 are very similar $\Rightarrow$ structures of different intensity have almost identical parallel aspect ratios

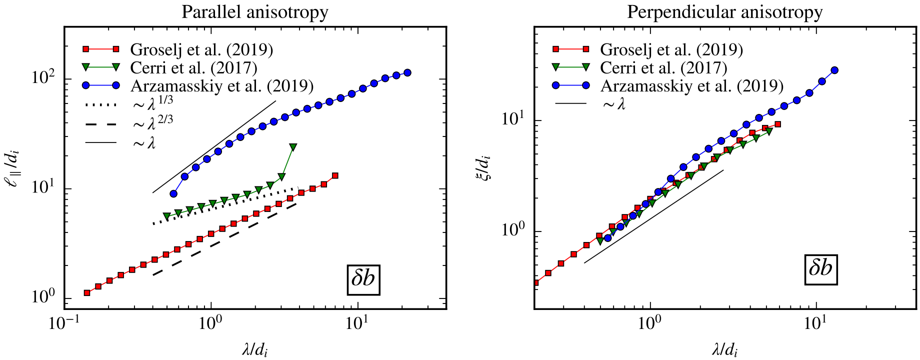
Perpendicular anisotropy scalings





- It seems that the sub-ion scale statistical eddies do *not* get more sheetlike with decreasing scale
- Structures of different intensities have also similar $\perp$ aspect ratios

Comparison with other 3D kinetic simulations



- The eddies from Arzamasskiy *et al.*, ApJ (2019) have *fixed* parallel aspect ratio at kinetic scales and become slightly *less* sheetlike with decreasing scale (both things happen at approximately the same λ)

Summary

- Kinetic-scale structures may be linked to exact wave solutions of ERMHD via local nonlinearity depletion (supported by some simulation and observation data)
- The sub-ion scale eddies do *not* get more sheetlike with decreasing scale
- There is work to be done regarding the $l_{\|}(\lambda)$ scaling. Most 3D kin. sims (I have analyzed also other data) indicate scale-dependent anisotropy, but data from Arzamasskiy *et al.*, ApJ (2019) do not show a scale-dependent anisotropy (solution: kinetic-scale tearing of sheetlike eddies??)