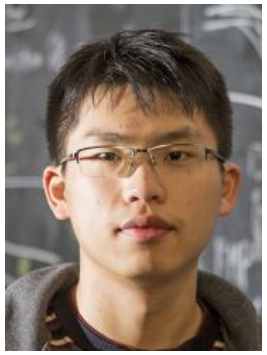
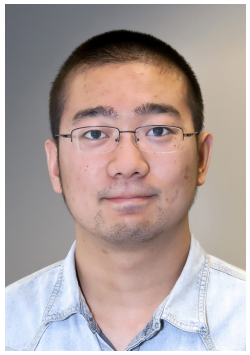
Cross-scale interactions and the Dimits shift in reduced models of drift-wave turbulence

Ilya Y. Dodin

Princeton Plasma Physics Laboratory, Princeton University


in collaboration with:

Hongxuan Zhu (Princeton)

Yao Zhou (PPPL)

Daniel E. Ruiz (Sandia)

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idodin@princeton.edu

Outline

- Part 1: Cross-scale interactions (CSI)
  - Quasilinear Hasegawa–Mima model
  - Quasistatic equation of state, basic scalings (new)
  - Zonal-flow merging as the cause of the CSI (new)
  - Comparison of the quasilinear and nonlinear models (new)

- Part 2: Stability of zonal flows and nonlinear suppression of DW turbulence
  - Tertiary instability in the Hasegawa–Mima model
  - Tertiary instability in the Terry–Horton model (new)
  - Dimits shift in the Terry–Horton model (new)
  - Dimits shift in the Hasegawa–Wakatani model (in progress)

(new) = updates since June

idodin@princeton.edu

The main results are yet to be published, but see the background...

- I. Y. Dodin, H. Zhu, Y. Zhou, and D. E. Ruiz, *Modeling drift-wave turbulence as quantumlike plasma*, in Proceedings of the 46th EPS Conference on Plasma Physics (Milan, Italy, 2019).†
- H. Zhu, Y. Zhou, and I. Y. Dodin, *Nonlinear saturation and oscillations of collisionless zonal flows*, New J. Phys. 21, 063009 (2019).
- Y. Zhou, H. Zhu, and I. Y Dodin, *Formation of solitary zonal structures via the modulational instability of drift waves*, Plasma Phys. Control. Fusion 61, 075003 (2019).
- D. E. Ruiz, M. E. Glinsky, and I. Y. Dodin, *Wave kinetic equation for inhomogeneous drift-wave turbulence beyond the quasilinear approximation*, J. Plasma Phys. 85, 905850101 (2019).
- H. Zhu, Y. Zhou, and I. Y. Dodin, *On the Rayleigh–Kuo criterion for the tertiary instability of zonal flows*, Phys. Plasmas 25, 082121 (2018).
- H. Zhu, Y. Zhou, and I. Y. Dodin, *On the structure of the drifton phase space and its relation to the Rayleigh–Kuo criterion of the zonal-flow stability*, Phys. Plasmas 25, 072121 (2018).
- H. Zhu, Y. Zhou, D. E. Ruiz, and I. Y. Dodin, *Wave kinetics of drift-wave turbulence and zonal flows beyond the ray approximation*, Phys. Rev. E 97, 053210 (2018).
- D. E. Ruiz, J. B. Parker, E. L. Shi, and I. Y. Dodin, *Zonal-flow dynamics from a phase-space perspective*, Phys. Plasmas 23, 122304 (2016).

† http://www.princeton.edu/~idodin/papers/EPS2019.pdf

Background and goal

- Gyrokinetic simulations show that low-k ITG turbulence can suppress high-k ETG turbulence. For example, see Maeyama *et al.*, PRL (2015):

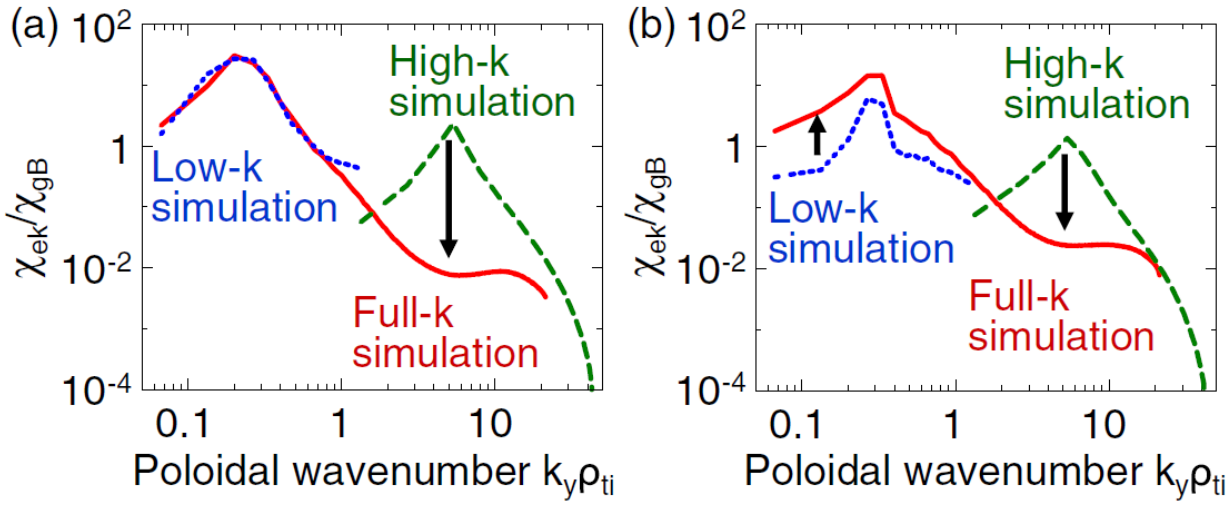


FIG. 2 (color online). Poloidal wave number spectrum of the time-averaged electron energy diffusivity χ_{ek} for (a) electrostatic ($\beta = 0.04\%$) and (b) electromagnetic ($\beta = 2.0\%$) cases. The solid (red), dotted (blue), and dashed (green) lines plot χ_{ek} as obtained from the full-k, low-k ($k_y\rho_{ti} < 1.3$), and high-k ($k_y\rho_{ti} > 1.3$) simulations, respectively.

Our goal is to explain this effect within the simplest meaningful model.

Maeyama *et al.* (2017); Howard *et al.* (2016); Maeyama *et al.* (2015) . . .

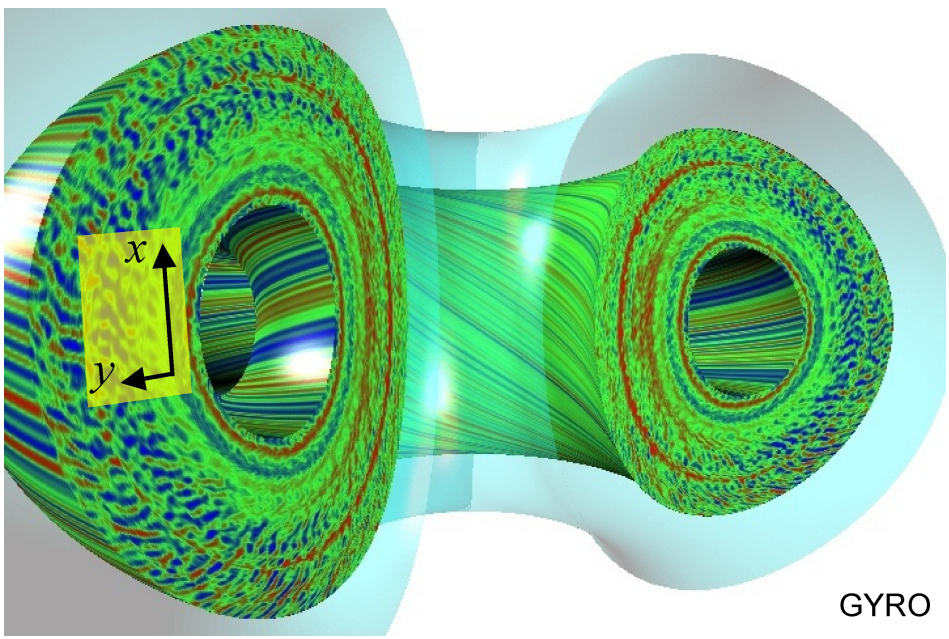
As a starting point, consider the Hasegawa–Mima model.

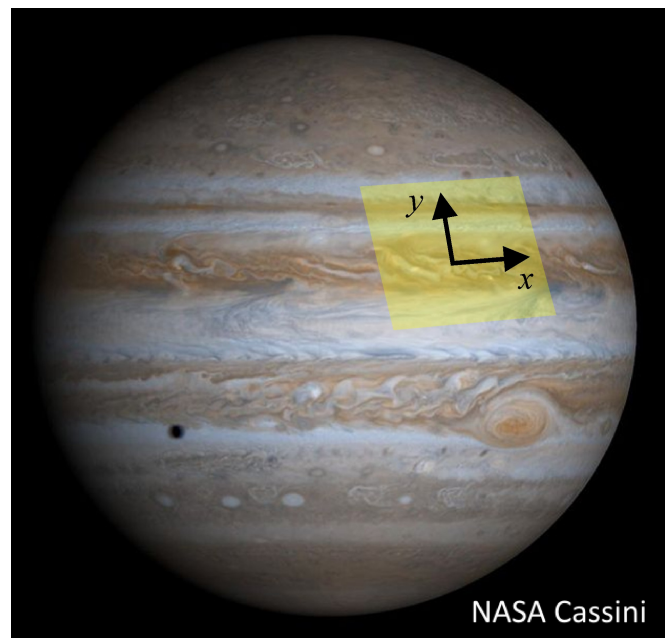
Basic physics of DW turbulence is often studied within the Hasegawa–Mima model:

$$\partial_t w + \{\varphi, w\} + \beta \partial_x \varphi = 0, \quad w = (\nabla^2 - \widehat{a})\varphi, \quad \beta \sim \partial_y N$$

Electrons respond adiabatically to drift waves ($k_\parallel \neq 0$) and do not respond to zonal flows (ZFs), which are spontaneously-generated banded shear flows with $k_\parallel = 0$.

$$w_{\mathrm{dw}} = (\nabla^2 - 1)\varphi_{\mathrm{dw}}, \quad w_{\mathrm{zf}} = \nabla^2 \varphi_{\mathrm{zf}}$$





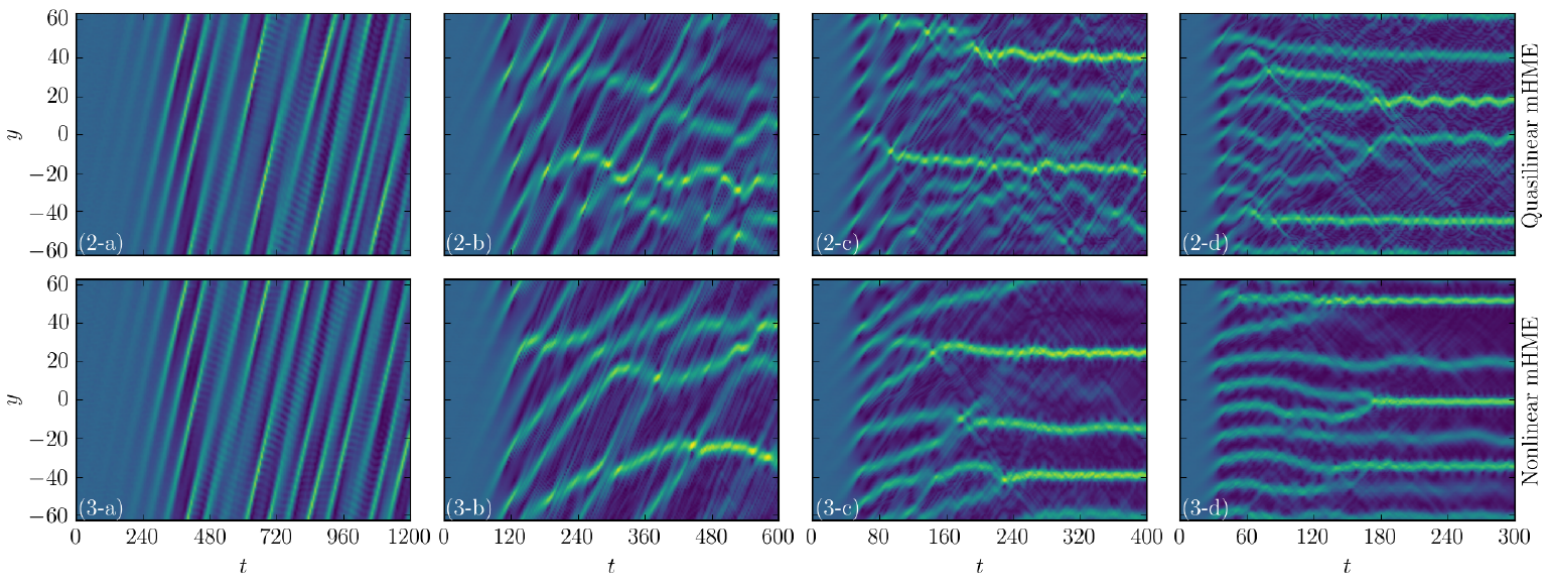
Hammett *et al.* (1993); Krommes and Kim (2000) . . .

We further reduce the model using the quasilinear approximation.

- The quasilinear approximation is sufficiently accurate to capture basic effects.

$$\textit{average:} \quad \partial_t U + \partial_y \overline{\tilde{v}_x \tilde{v}_y} = 0, \quad \tilde{\mathbf{v}} = \hat{\mathbf{z}} \times \nabla \tilde{\varphi}, \quad \tilde{w} = (\nabla^2 - 1)\tilde{\varphi}$$

$$\textit{fluctuations:} \quad \partial_t \tilde{w} + U \partial_x \tilde{w} + [\beta - (\partial_y^2 U)]\partial_x \tilde{\varphi} = \underbrace{\overline{\tilde{\mathbf{v}} \cdot \nabla \tilde{w}} - \tilde{\mathbf{v}} \cdot \nabla \tilde{w}}_{\text{neglected (QL model)}}$$



- The equation for $\tilde{w}$ can be expressed as a Schrödinger equation for "driftons":

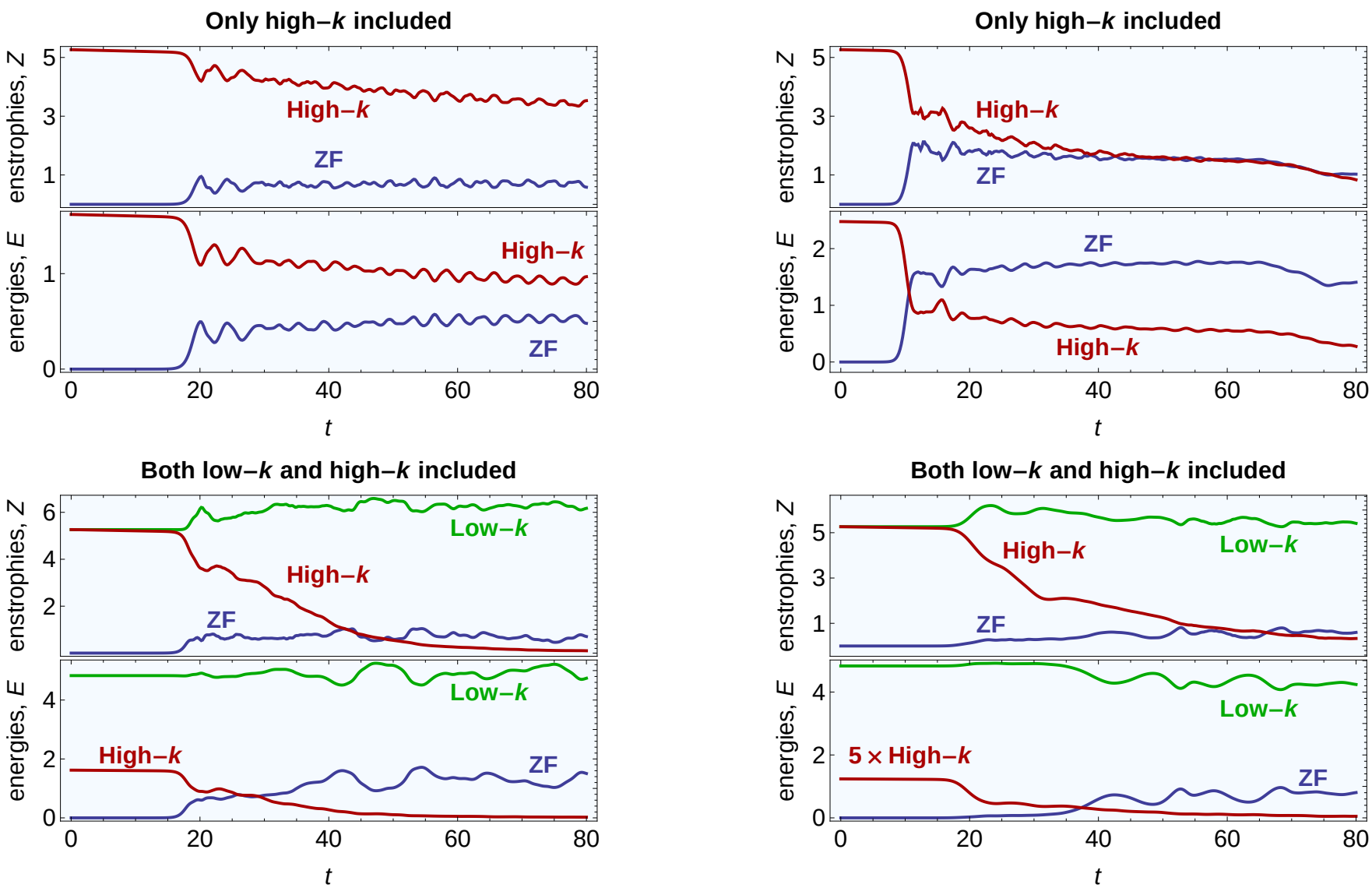
$$i\partial_t \tilde{w} = \hat{\mathcal{H}}\tilde{w} + \cancel{NL}, \quad \hat{\mathcal{H}} = \hat{k}_x \hat{U} - \hat{k}_x(\beta - \hat{U}'')(1 + \hat{k}_\perp^2)^{-1}, \quad \hat{\mathbf{k}} \doteq -i\nabla$$

Ruiz *et al.* (2016); Zhou *et al.* (2019)

The quasilinear HM model captures cross-scale interactions.

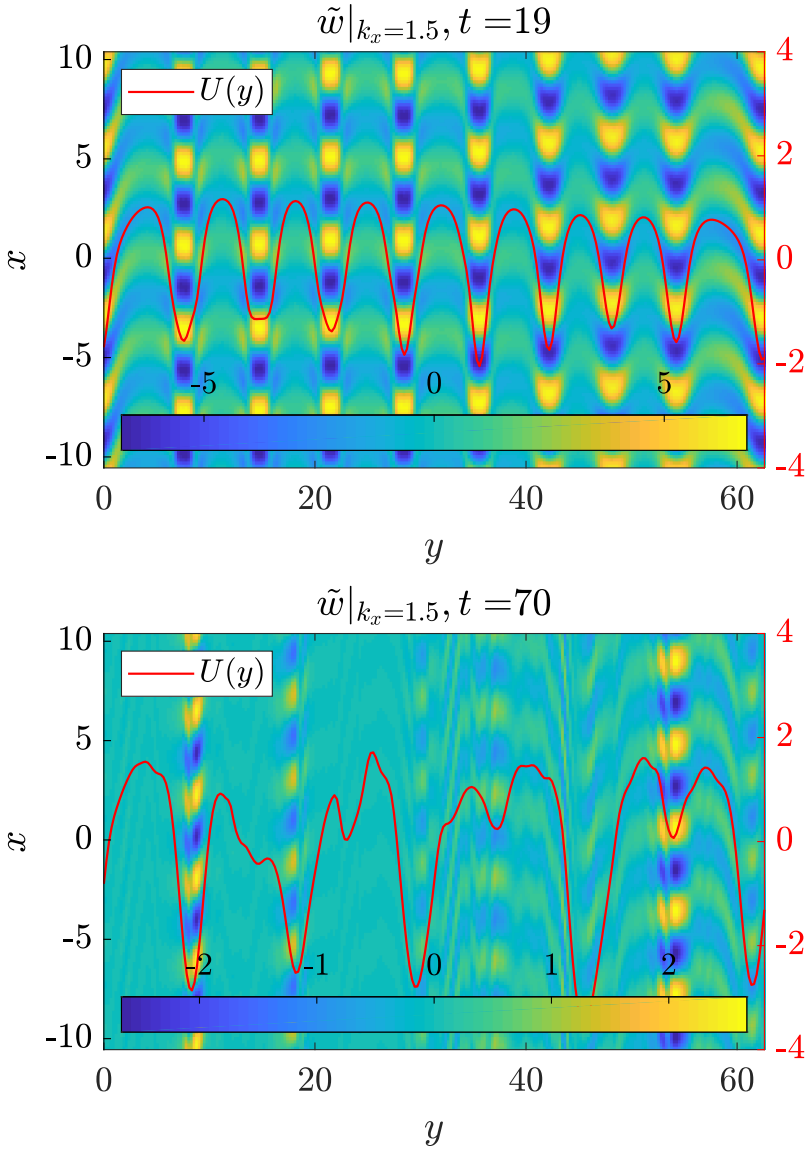
$k_{x2}/k_{x1} = 5$

$k_{x2}/k_{x1} = 15$



$Z_{\rm dw} \doteq \frac{1}{2}\int {\rm d}^2x\, \tilde{w}^2, \quad Z_{\rm zf} \doteq \frac{1}{2}\int {\rm d}y\, (U')^2, \quad E_{\rm dw} = -\frac{1}{2}\int {\rm d}^2x\, \tilde{w}\tilde{\varphi}, \quad E_{\rm zf} \doteq \frac{1}{2}\int {\rm d}y\, U^2$

The main claim



- Simulations show that ZFs exhibit substantial merging in multi-scale turbulence.

  Claim: this merging is the cause of the high-k turbulence demise and a generic property of multi-scale turbulence.

- To show this, some theory will be needed:
  - general wave-kinetic theory,
  - topology of the drifton phase-space,
  - approximate closure for U,
  - conditions for ZF merging.

- We will also argue that the physics beyond the quasilinear approximation is not very different.

A statistical theory is constructed by analogy with that in QM.

- The Wigner function $W(t,y,\mathbf{k}) \doteq \int \mathrm{d}^2 s\, e^{-i\mathbf{k}\cdot\mathbf{s}} \langle \tilde{w}(t,\mathbf{x}+\mathbf{s}/2)\tilde{w}(t,\mathbf{x}-\mathbf{s}/2)\rangle$ (i.e., the spectrum of the two-point correlator, or "quasiprobability distribution") satisfies

$$\boxed{\frac{\partial W}{\partial t} = \{\{\mathcal{H}_H, W\}\} + [\![\mathcal{H}_A, W]\!], \quad \frac{\partial U}{\partial t} = \frac{\partial}{\partial y}\int \frac{\mathrm{d}^2 k}{(2\pi)^2}\frac{1}{1+k_\perp^2} \star k_x k_y W \star \frac{1}{1+k_\perp^2}}$$

$$\mathcal{H}_H = k_x U - \frac{\beta k_x}{1+k_\perp^2} + \frac{1}{2}\left[\!\!\left[U'', \frac{k_x}{1+k_\perp^2}\right]\!\!\right], \quad \mathcal{H}_A = \frac{1}{2}\left\{\!\!\left\{U'', \frac{k_x}{1+k_\perp^2}\right\}\!\!\right\}$$

- Geometrical-optics limit: improved wave kinetic equation (iWKE) **with new terms**:

$$\widehat{\mathcal{L}} = O(\partial_x \partial_k) \sim (kL_{\mathrm{zf}})^{-1} \ll 1, \quad Ae^{i\widehat{\mathcal{L}}/2}B = 1 + {}^{i}\!/_{2}\{A,B\} + \ldots$$

$$\boxed{\frac{\partial W}{\partial t} = \{\mathcal{H}_H, W\} + \boldsymbol{2\mathcal{H}_A W}, \quad \frac{\partial U}{\partial t} = \frac{\partial}{\partial y}\int \frac{\mathrm{d}^2 k}{(2\pi)^2}\frac{k_x k_y W}{(1+k_\perp^2)^2}}$$

$$\mathcal{H}_H \approx k_x U - k_x(\beta - \boldsymbol{U''})/(1+k_\perp^2), \quad \mathcal{H}_A \approx -\boldsymbol{U'''} k_x k_y/(1+k_\perp^2)^2$$

$\widehat{\mathcal{L}} = \{\cdot,\cdot\}$ | $\{\{A,B\}\} = 2A\sin(\widehat{\mathcal{L}}/2)B$ | $[\![A,B]\!] = 2A\cos(\widehat{\mathcal{L}}/2)B$ | $A \star B = Ae^{i\widehat{\mathcal{L}}/2}B$

Ruiz *et al.* (2016); Parker (2016); cf. Smolyakov and Diamond (1999); Krommes and Kim (2000) . . .

Quasistatic approximation in the limit $U'' \ll \beta$

- Let us rewrite the iWKE in the following form using the group velocity v_g:

$$\frac{\partial W}{\partial t} + \frac{\partial}{\partial y}(W v_g) = \frac{\partial}{\partial k_y}\left(W \frac{\partial \mathcal{H}_H}{\partial y}\right) - \frac{U'''}{\beta - U''} W v_g, \quad v_g = \frac{2k_x k_y}{(1+k_\perp^2)^2}(\beta - U'')$$

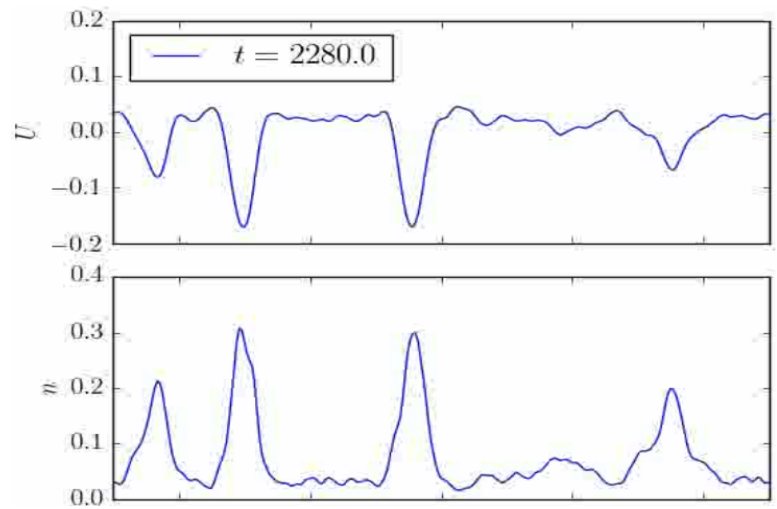
- By integrating the iWKE over $\mathbf{k}$, one obtains an equation for the drifton density. The **term on the right** can be neglected compared to $\partial_y J$ when $U'' \ll \beta$.

$$\partial_t N + \partial_y J = -\boldsymbol{J U'''/(\beta - U'')}, \quad N \doteq \int W \, \mathrm{d}^2 k, \quad J \doteq \int W v_g \, \mathrm{d}^2 k$$

- In this "quasistatic" limit, U becomes a local function of N ("equation of state"):

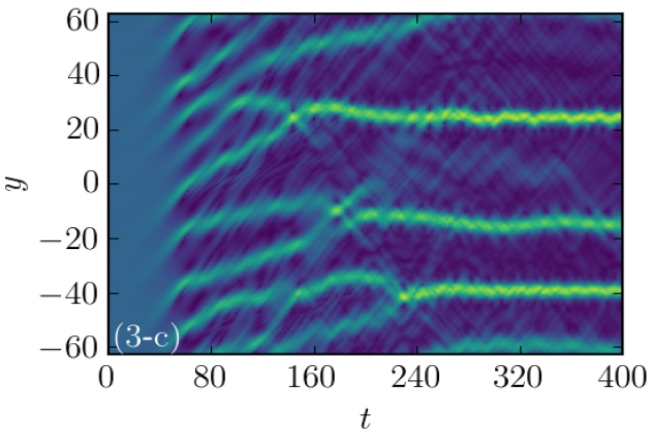
$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial y}\left[\frac{J}{2(\beta - U'')}\right] \approx \frac{\partial_y J}{2\beta} \approx -\frac{\partial_t N}{2\beta}$$

$$\boxed{U \approx -\frac{N}{2\beta} + \text{const}}$$



For quasimonochromatic turbulence as a special case, we discussed this in Zhou *et al.* (2019).

Even at small U, the ZF stability depends on the ZF wavenumber q.

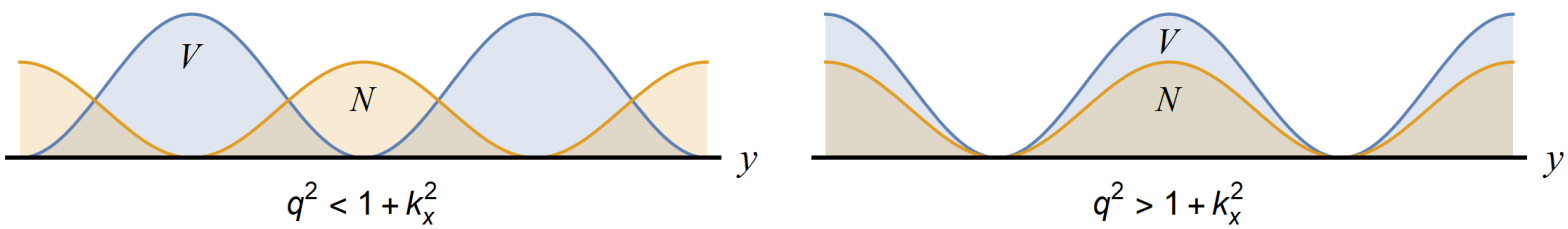


- The iWKE is only marginally applicable to ZF formation but can explain it qualitatively.

$$\mathcal{H} = \frac{k_x(-\beta + U'')}{1 + k_x^2 + k_y^2} + k_x U, \quad U \approx -\frac{N}{2\beta} + \text{const}^*$$

$$k_y^2 \ll 1 + k_x^2, \quad q^2 \doteq -U''/U, \quad k_x = \text{const}$$

$$\boxed{\mathcal{H} \approx \frac{k_x}{\beta}\left(\frac{k_y^2}{2m} + V\right) + \text{const}, \quad \frac{1}{m} \doteq \frac{2\beta^2}{(1+k_x^2)^2}, \quad V \doteq \left(\frac{\boldsymbol{q^2}}{\boldsymbol{1 + k_x^2}} - \boldsymbol{1}\right)\frac{N}{2}}$$



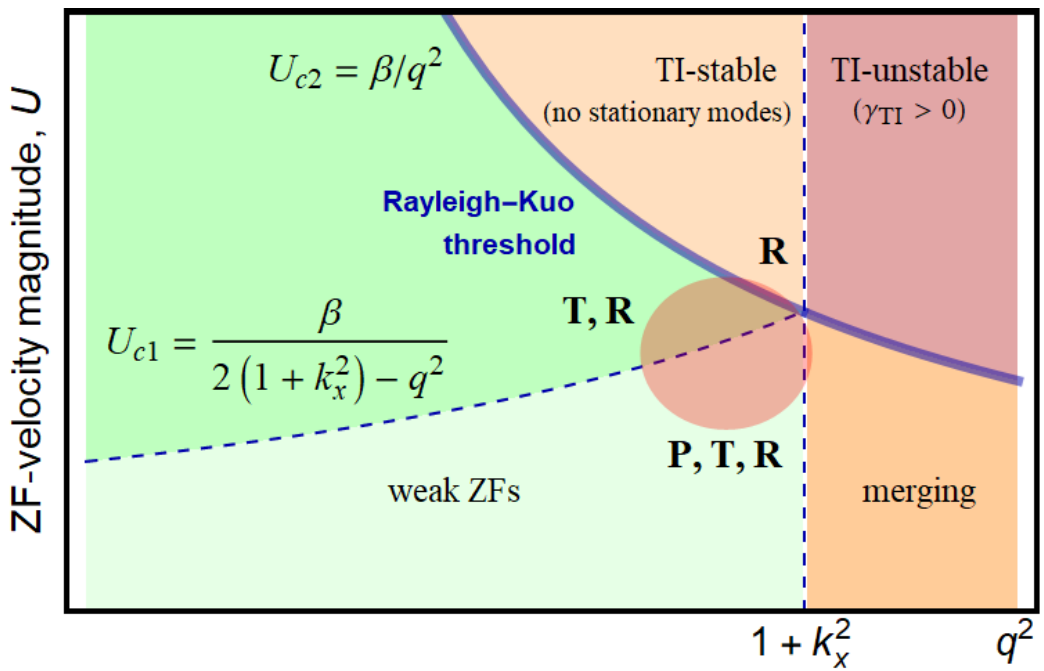
- If $q^2 < 1 + k_x^2$, driftons reside near minima of V, so the system is stable.
- If $q^2 > 1 + k_x^2$, driftons reside near maxima of V. The system can lower the energy by bifurcating to a lower-q state, so it is unstable to ZF merging.

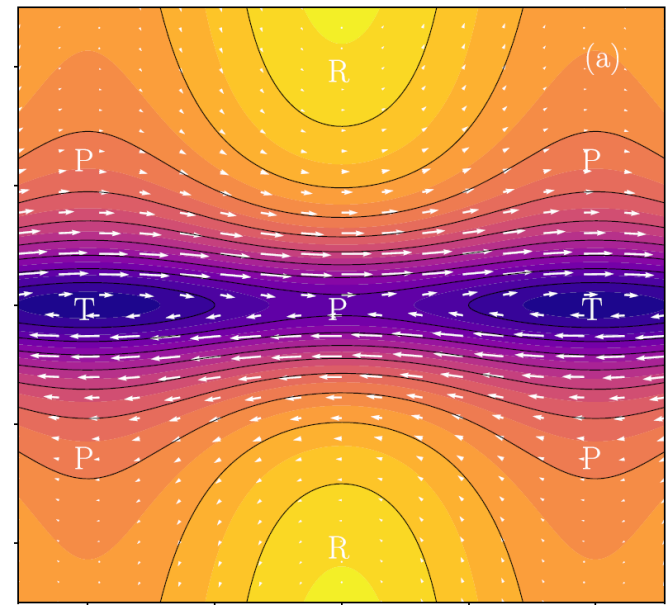
* Here, we assume $U'' \lesssim \beta$. Unlike in single-scale turbulence, this does not rule out $q \gtrsim k_x$.

ZFs have many regimes but not all of them are realized naturally.

- In single-scale turbulence, q corresponds to the maximum of $\gamma_{\rm secondary}$. Then, there are passing and/or trapped trajectories, and many driftons survive.

$$q \sim \min\left\{k_x^2\sqrt{N}/\beta, \sqrt{1+k_x^2}\right\}, \quad U \lesssim U_{c1}$$





- Low-k waves cause ZFs to merge down to $q^2 \sim 1 + k_x^2$. High-k waves become runaways and dissipate. Low-k waves remain passing because $U_{c1} = U_{c1}(k_x)$.

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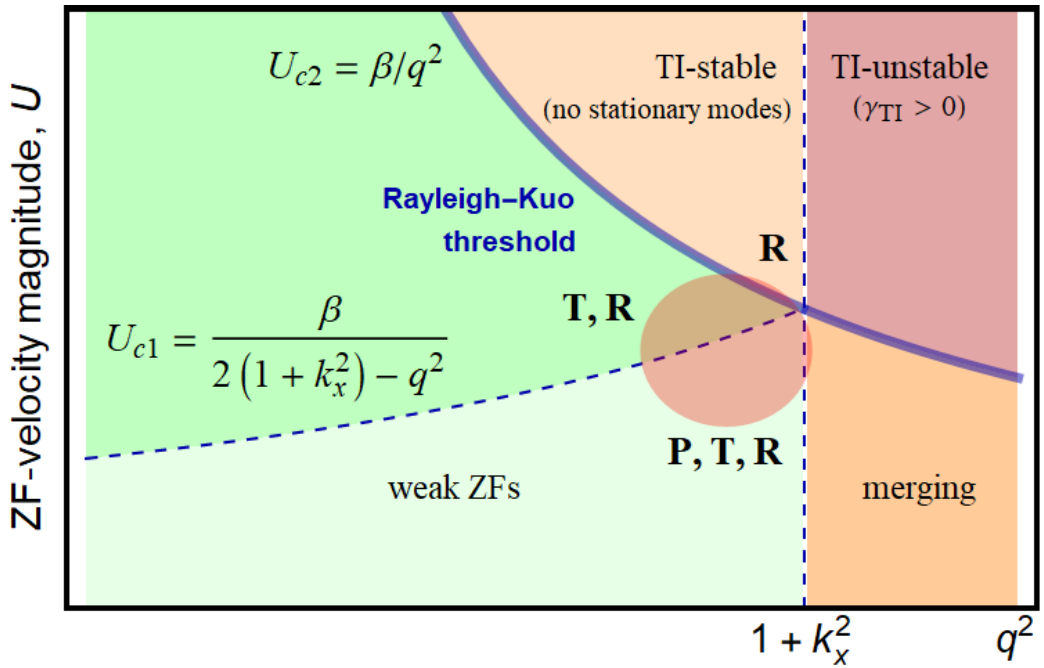

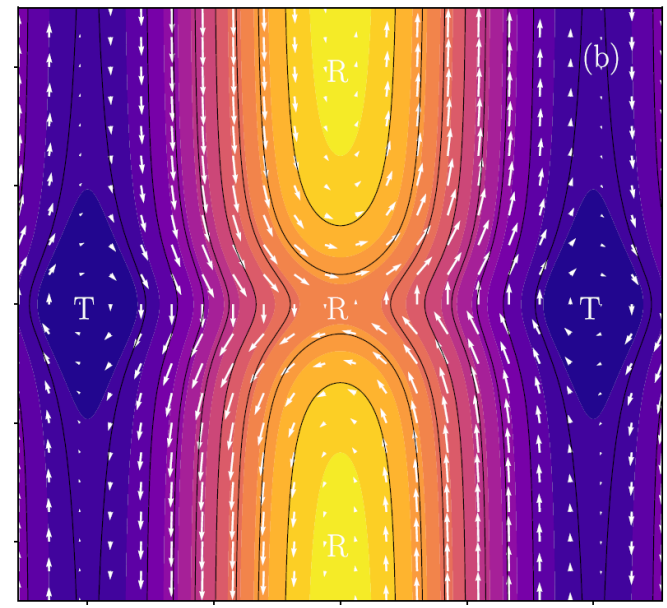

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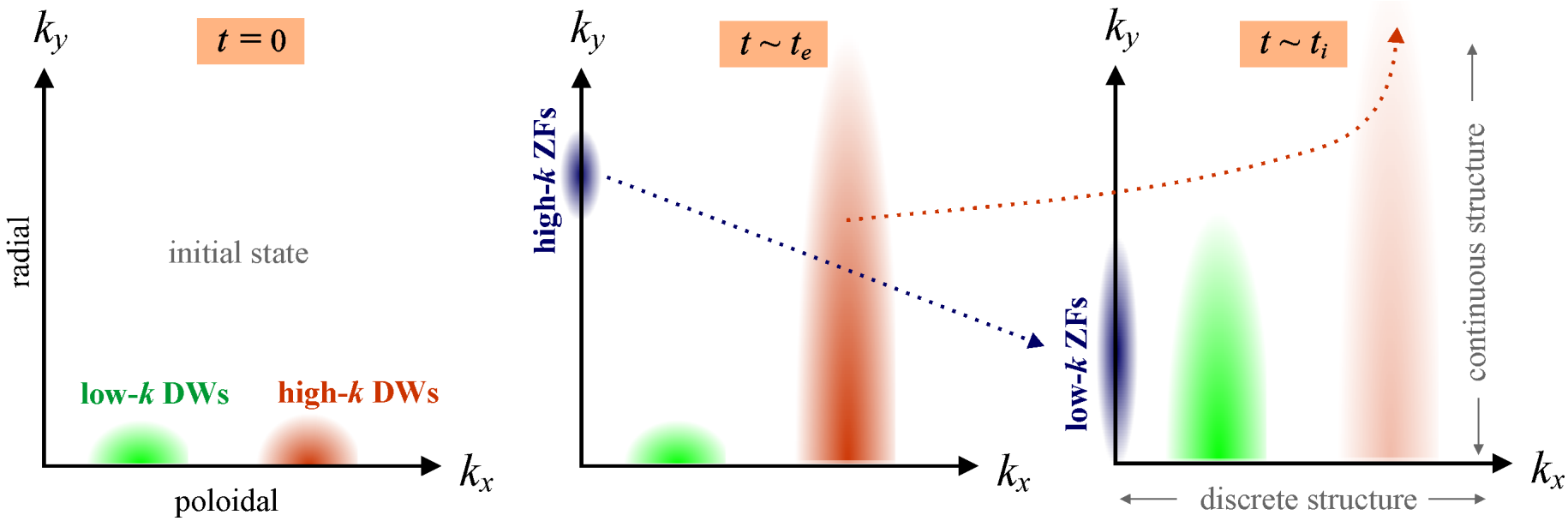
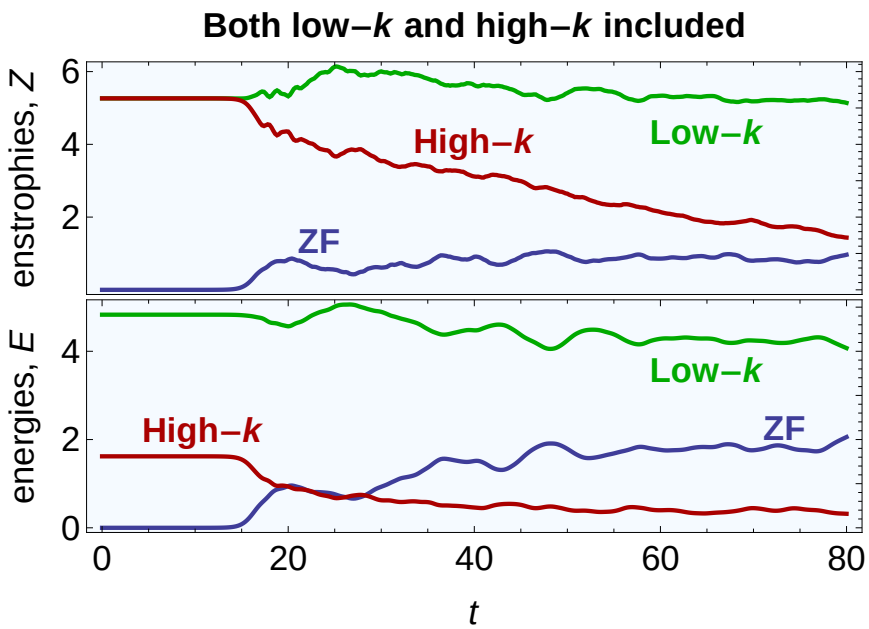
Cross-scale interactions in a nutshell

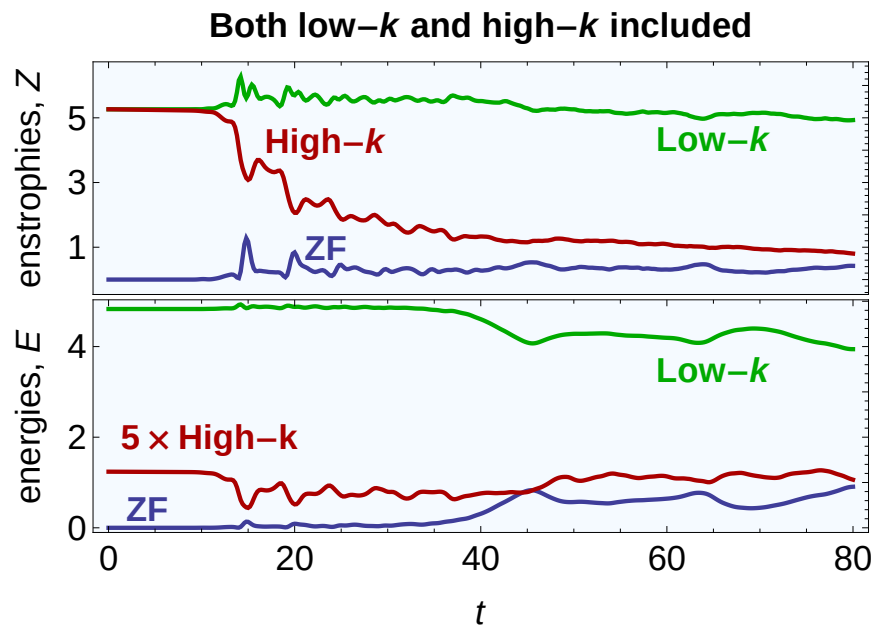
- The scale separation persists in the poloidal-k space.
- In the radial-k space, the scales are determined by the ZFs, which start at the electron scale and then merge down to the ion scale.
- During the ZF merging, most high-k driftons become runaways and dissipate.



Nonlinear HM simulations demonstrate similar behavior.



$k_{x2}/k_{x1} = 5$



$k_{x2}/k_{x1} = 15$

- Drifton collisions serve as an additional channel though which high-k driftons become runaways. Other than that, the effect remains the same.

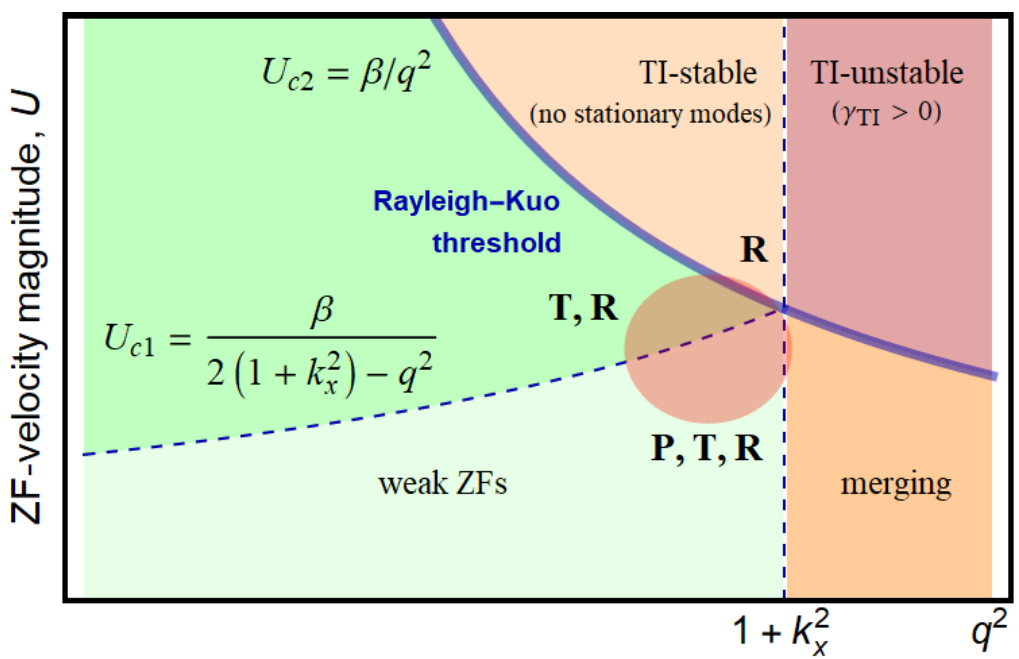
Summary: the demise of high-k turbulence via cross-scale interactions is robustly explained as a phase-space effect in the Hasegawa–Mima model.

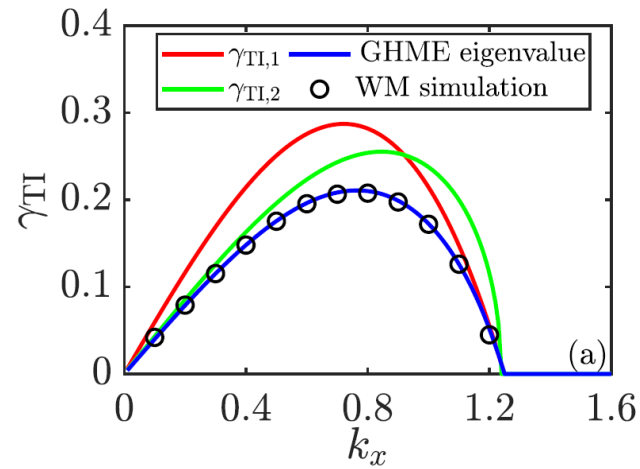
Tertiary instability in the conservative Hasegawa–Mima model

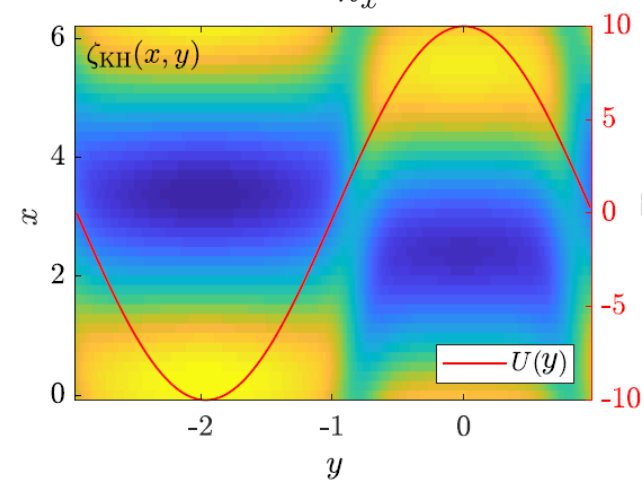
- In the Hasegawa–Mima model, the drifton Hamiltonian is pseudo-Hermitian, resulting in an instability of the Kelvin–Helmholtz type. (But is it relevant?)

$$i\partial_t \tilde{w} = \widehat{\mathcal{H}}\tilde{w}, \quad \widehat{\mathcal{H}} = k_x \widehat{U} - k_x(\beta - \widehat{U}'')(1 + k_x^2 + \widehat{k}_y^2)^{-1}$$

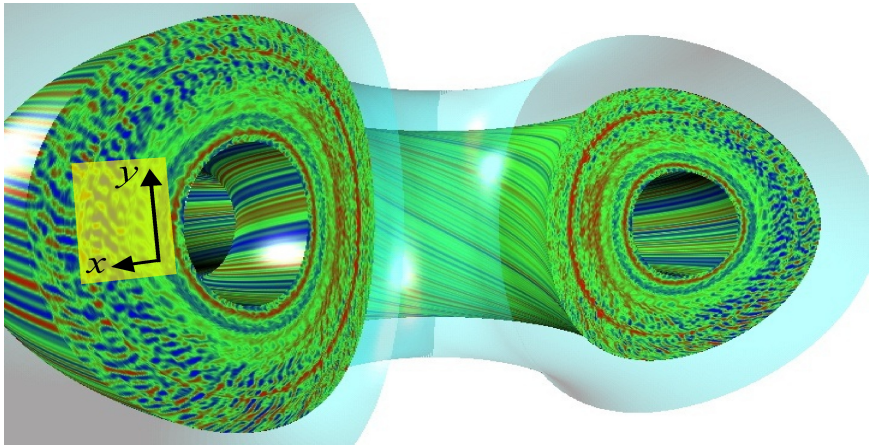
$$\gamma_{\mathrm{TI}} = |k_x U_0| \left(1 - \frac{1 + k_x^2}{q^2}\right) \sqrt{1 - \frac{\beta^2}{U_0^2 q^4}}$$







The dissipative TI is different. Consider the Terry–Horton model...



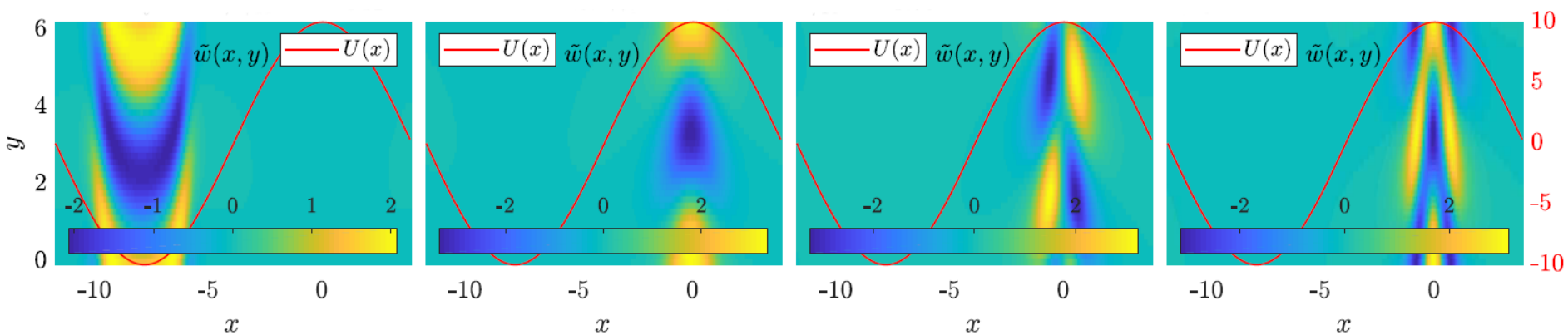
$$\partial_t w + \{\varphi, w\} = \beta \partial_y \varphi - \boldsymbol{\widehat{D}} w$$

$$w = (\nabla^2 - \widehat{\alpha} + \boldsymbol{i\widehat{\delta}})\varphi$$

$$\underbrace{\widehat{\delta} = \delta(\widehat{k}_y)}_{\text{primary instability}} \quad , \quad \underbrace{\widehat{D} = 1 - \kappa \nabla^2}_{\text{friction \& viscosity}}$$

- The "Hasegawa–Mima" TI mode becomes localized + other localized modes appear.

$$\boxed{\widehat{\mathcal{H}} = k_y \widehat{U} + k_y(\beta + \widehat{U}'')[1 + \widehat{k}_x^2 + k_y^2 - \boldsymbol{i\delta(k_y)}]^{-1} - \boldsymbol{i\widehat{D}}}$$

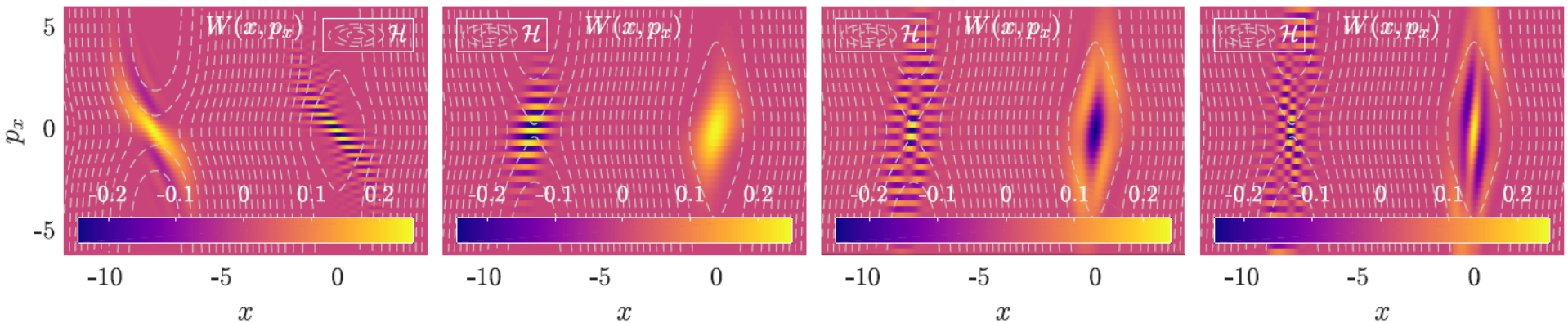


For this modified Terry–Horton model, see St-Onge (2017). The function $\delta(k_y)$ can be anything.

TI modes satisfy the equation of a quantum harmonic oscillator.

- The largest growth rates belong to the lowest-order modes. Those are localized in (x, k_x), so the drifton Hamiltonian can be approximated with its Taylor expansion:

$$\partial_t W = \{\{\mathcal{H}_H, W\}\} + [\![\mathcal{H}_A, W]\!] \quad \Rightarrow \quad \textbf{truncate } \boldsymbol{\mathcal{H}} \quad \Rightarrow \quad \widehat{\mathcal{H}} \approx c_0 + c_1 \widehat{x}^2 + c_2 \widehat{k}_x^2$$



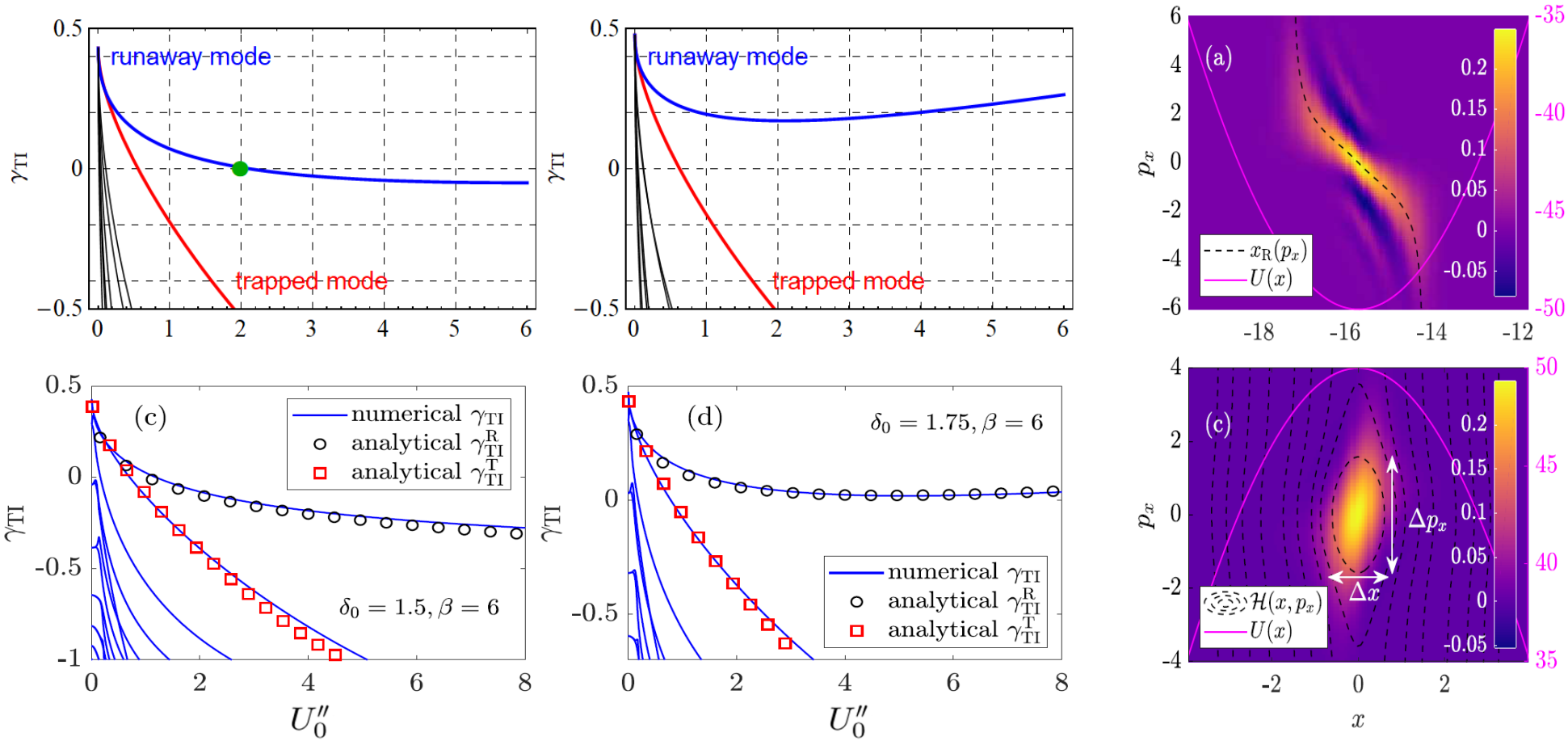
- This yields an equation of a quantum harmonic oscillator with complex coefficients:

$$\left(-\boldsymbol{\vartheta}^2 \frac{\mathrm{d}^2}{\mathrm{d}x^2} + x^2\right) \tilde{w} = \varepsilon \tilde{w} \quad \Rightarrow \quad \tilde{w}_n \sim H_n\left(\frac{x}{\sqrt{\vartheta}}\right) e^{-x^2/2\vartheta}, \quad \varepsilon_n = (2n+1)\vartheta$$

$$\vartheta \doteq -\frac{i\sqrt{2(1+\beta/U_0'')}}{1+k_y^2 - i\delta}, \quad \varepsilon \doteq \frac{2}{k_y U_0''}\left[\omega_{\mathrm{TI}} - k_y U_0 + iD_0 - \frac{k_y(\beta + U_0'')}{1+k_y^2 - i\delta}\right]$$

The growth rate is obtained explicitly and agrees with simulations.

$$\gamma_{\mathrm{TI}} = -D_0 + \mathrm{Im}\left[\frac{k_y(\beta + U_0'') - ik_y U_0''\sqrt{(1+\beta/U_0'')/2}}{1 + k_y^2 - i\delta}\right]$$

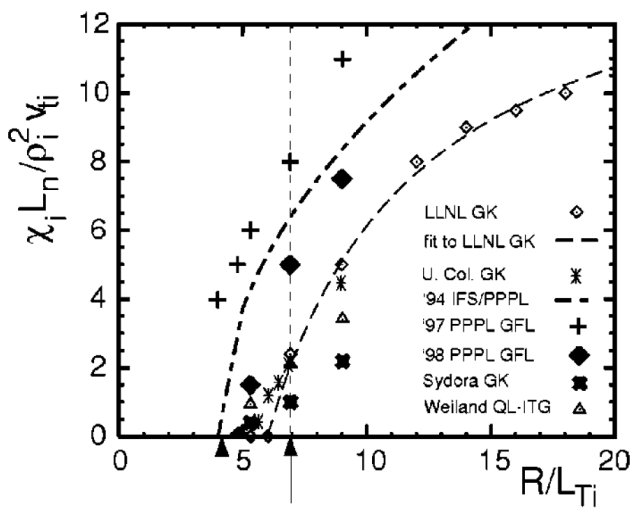


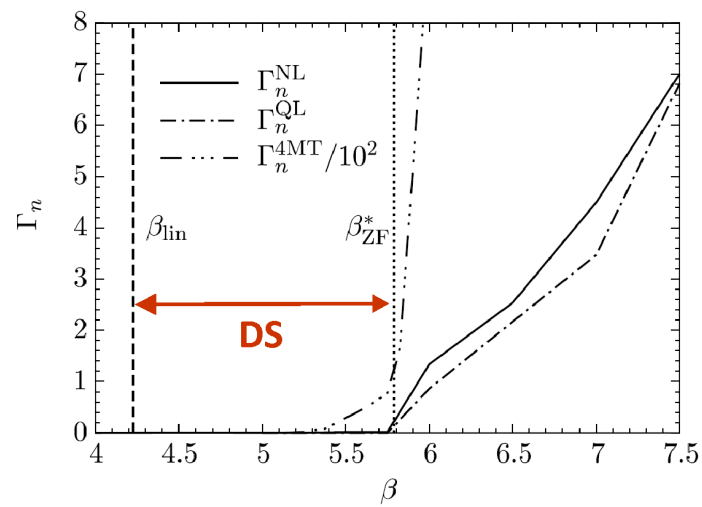
Upper: analytic formula. Lower: numerical simulations + an alternative theory with a fitting parameter (not shown).

Dimits shift

$$\gamma_{\mathrm{TI}} = -D_0 + \mathrm{Im}\left[\frac{k_y(\beta + \boldsymbol{U_0''}) - ik_y\boldsymbol{U_0''}\sqrt{(1+\beta/\boldsymbol{U_0''})/2}}{1+k_y^2 - i\delta}\right] \equiv \gamma_{\mathrm{primary}}^{(\mathrm{linear})} + \Delta\gamma(\boldsymbol{U_0''})$$

- The tertiary instability can be viewed as the primary instability modified by ZFs.
  - If $\gamma_{\mathrm{TI}} < 0$, turbulence is suppressed; ZFs survive, assuming $\hat{D}$ acts only on DWs.
  - If $\gamma_{\mathrm{TI}} > 0$, the system ends up in a turbulent state.
  - Due to $\Delta\gamma$, the transition to the turbulent state occurs at plasma parameters different from those without ZFs. This is called the **Dimits shift**.



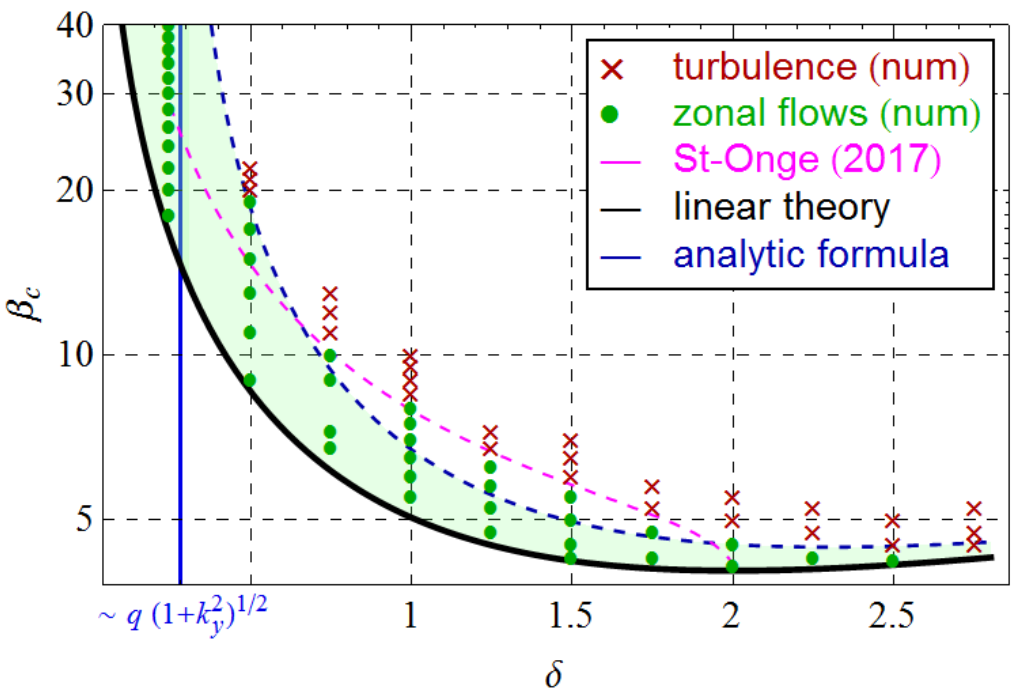


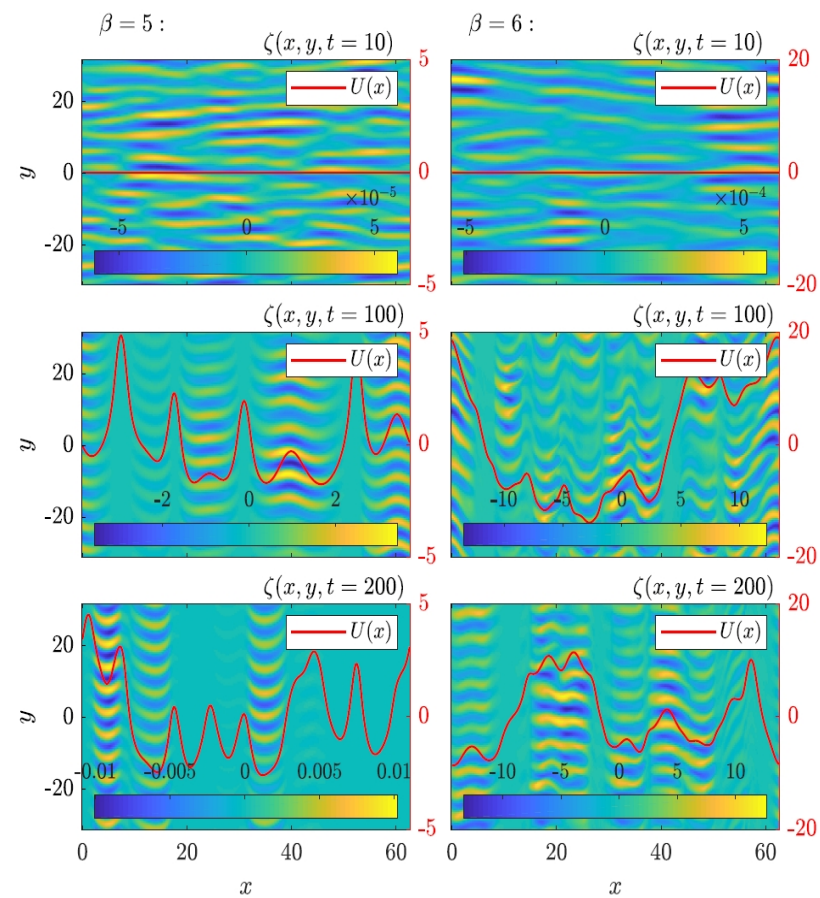
The figures are taken from Dimits *et al.* (2000) and St-Onge (2017).

Our explicit formula for the Dimits shift agrees with simulations.

- We calculate the values of β that correspond to $\gamma_{\text{primary}}^{(\text{linear})} = 0$ and $\gamma_{\text{TI}} = 0$ using $U_0'' \sim q^2 U_{c1}$. The difference between these values is the Dimits shift (shaded).
- Compared with a related calculation by St-Onge (2017), our model is a better fit at both large and small δ. For example, it has no spurious cutoff at $\delta = 2$.

$$\beta_c \approx \frac{D[(1+k_y^2)^2 + \delta^2]/k_y}{\delta - (1+k_y^2)\sqrt{U_0''/2\beta}}, \qquad \frac{U_0''}{\beta} \sim \frac{q^2}{k_y^2 + 1}$$





St-Onge (2017) used four-mode truncation (a stretch, also not intuitive) and did not calculate the Dimits shift *per se*.

Modified Hasegawa–Wakatani system (work in progress)

- Next step: consider the Hasegawa–Wakatani model, with $w = \nabla^2\varphi - n$:

$$\partial_t w + \{\varphi, w\} = \beta\partial_y\varphi - \hat{D}w, \quad \partial_t n + \{\varphi, n\} = \alpha(\tilde{\varphi} - \tilde{n}) - \beta\partial_y\varphi - \hat{D}n$$

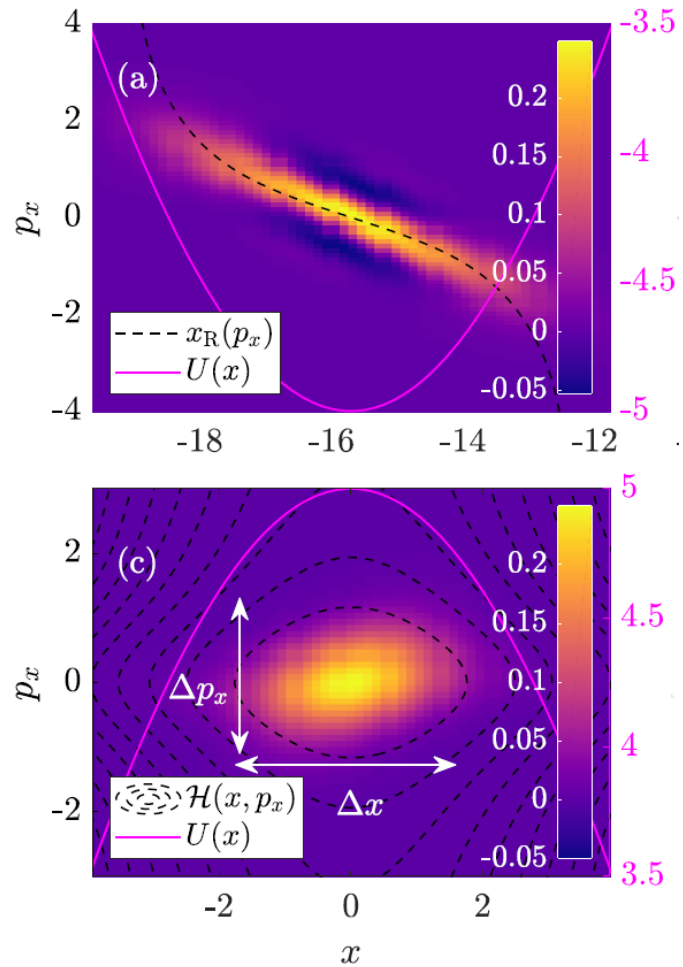
- The linear equation for tertiary modes is somewhat similar to that in the Terry–Horton model:

$$\omega\tilde{w} = \widehat{\mathcal{H}}\tilde{w}$$

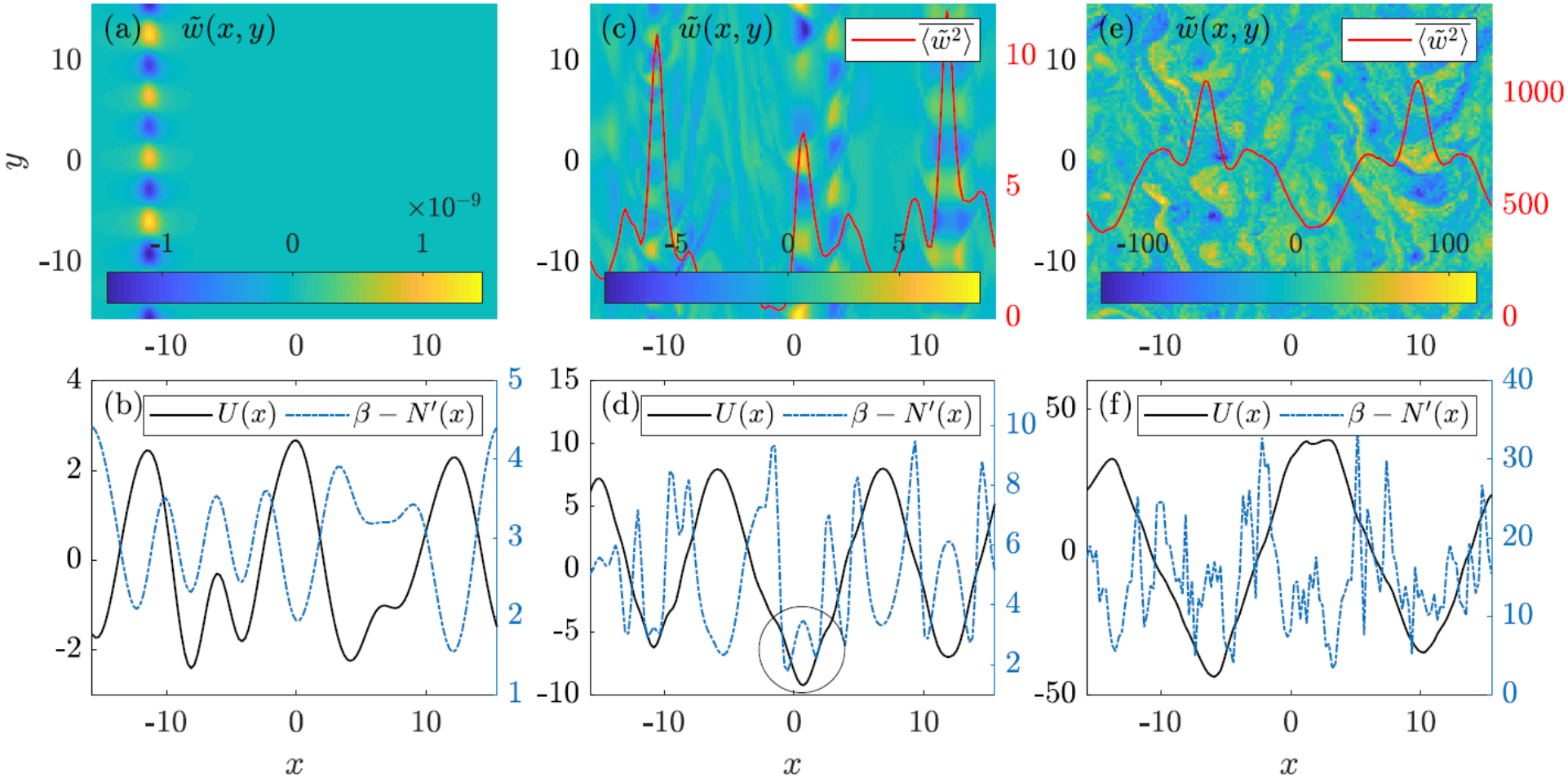
$$\widehat{\mathcal{H}} = k_y U + k_y(\beta + U'' - \boldsymbol{N'})\widehat{K}^{-2} - iD$$

$$\widehat{K}^2 \doteq \widehat{k}_x^2 + k_y^2 + \frac{k_y(\beta - \boldsymbol{N'}) + i\alpha}{\omega + iD - k_y U + i\alpha}$$

- The trapped and passing modes are still present.
- However, the dependence of the Hamiltonian on N' makes analytic predictions more difficult.



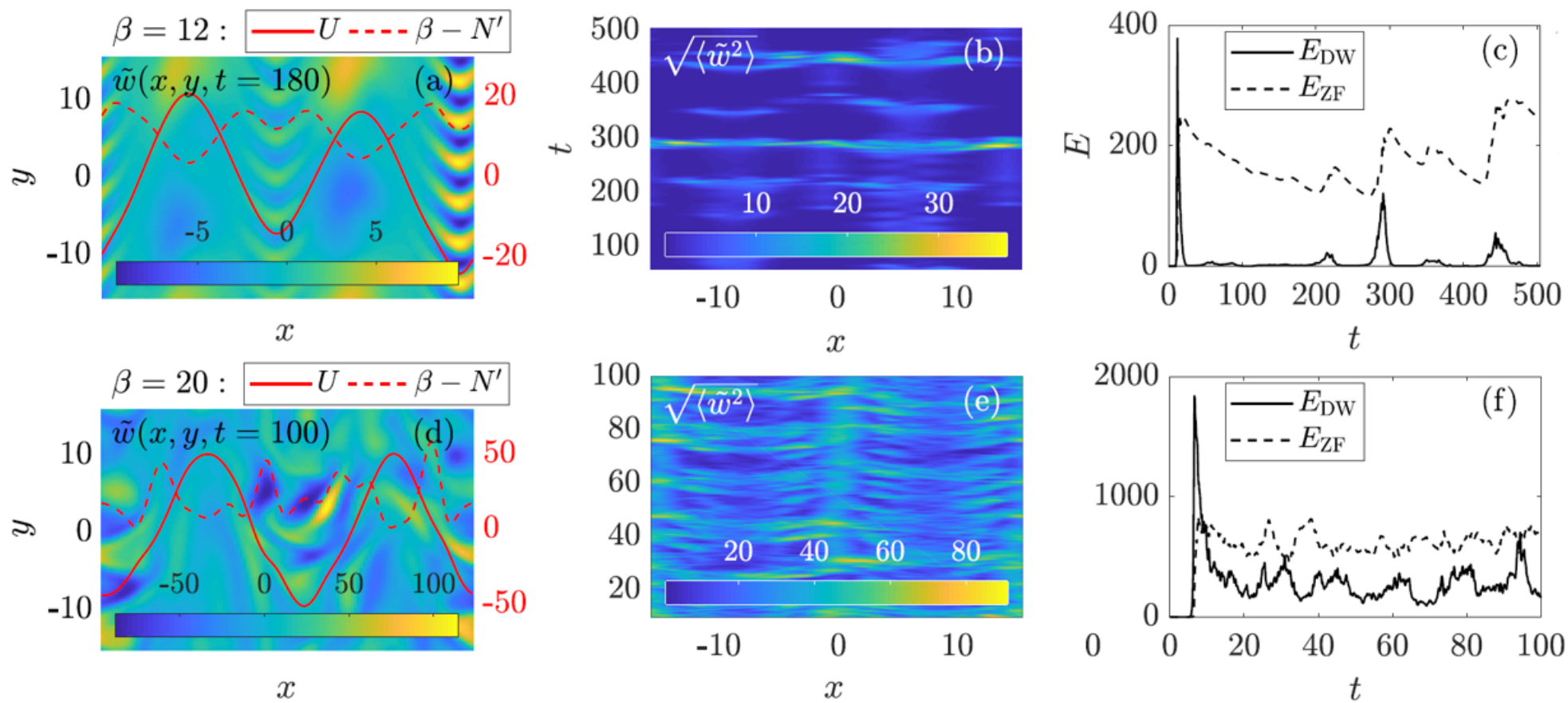
Collisionless ZFs: three different regimes are found numerically.



(a) Dimits regime, turbulence is suppressed; (ii) ZF-dominated regime, turbulence is localized, special structures; (iii) turbulence-dominated regime.

Collisional zonal flows

- When zonal flows are collisional, the complete suppression of turbulence is impossible. Instead, predator–prey oscillations are observed.
- A parabolic expansion of $\widehat{\mathcal{H}}$ leads to reasonable results, but more work is needed.



Summary

- Studying drift-wave turbulence in phase space requires:
  - looking beyond geometrical optics: do not neglect $\boldsymbol{\lambda/L}$ and $\boldsymbol{U''/\beta}$,
  - deriving WME/WKE from first principles: hand-waving leads to errors.
- Cross-scale interactions within the Hasegawa–Mima model:
  - Zonal flows tend to merge when there are driftons with $k_{\mathrm{poloidal}} \lesssim q$.
  - Zonal-flow merging gradually reduces the DW radial scale.
  - High-k_{poloidal} DWs are efficiently dissipated during this process.
- Tertiary instabilities and the Dimits shift:
  - Dissipation localizes the tertiary modes near the ZF-velocity extrema.
  - The growth rate of these modes can be made negative by $U'' \Rightarrow$ Dimits shift.
  - An analytic theory is developed within the Terry–Horton model.
  - The Hasegawa–Wakatani system is more subtle. *(in progress)*

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