# The boundary of a magnetized plasma

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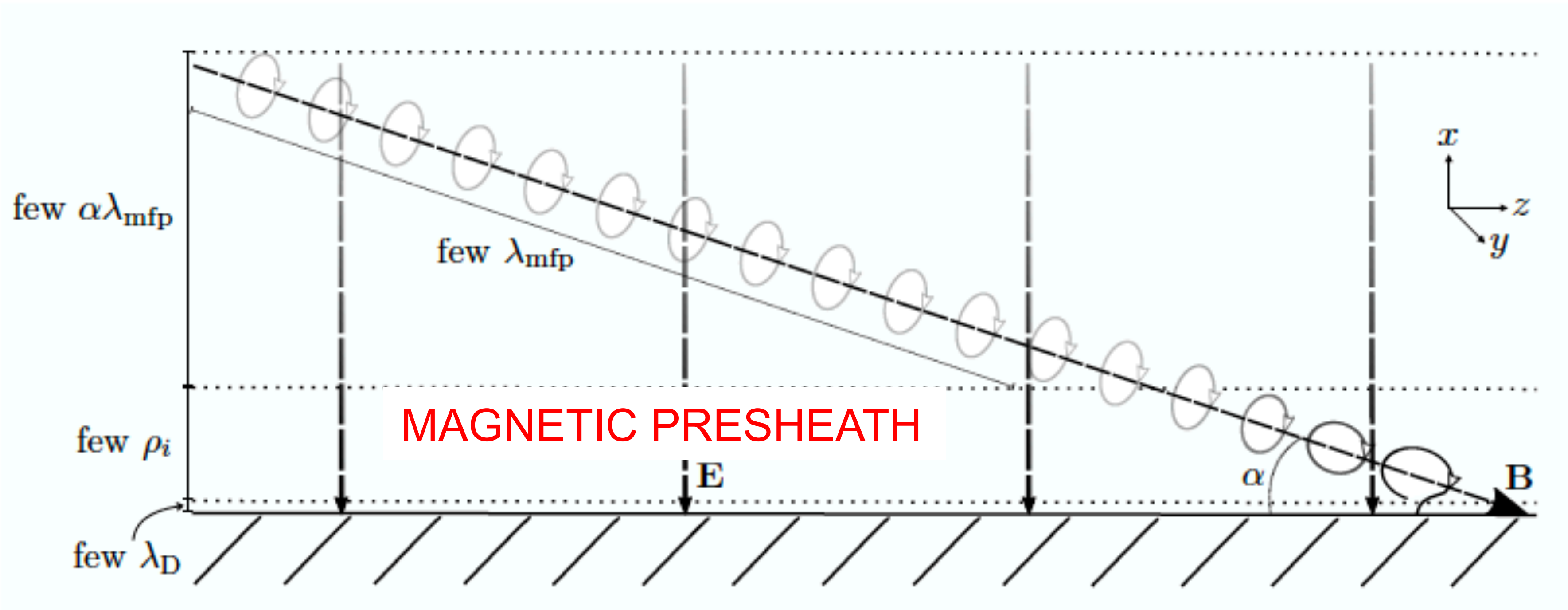
# Wall next to a magnetized plasma

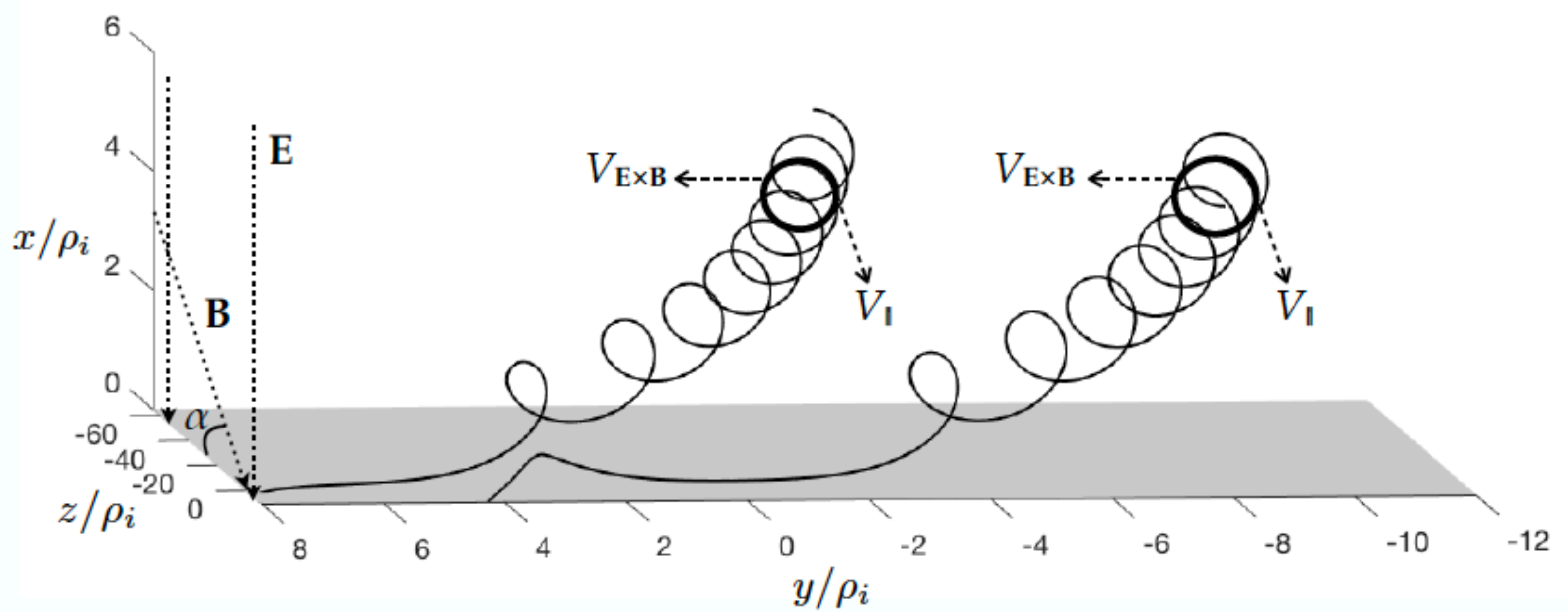
- Electrons absorbed by wall ⇒ $\mathbf{E}$ into the wall
- Three different boundary layers




- Solve layers to find boundary conditions for models away from the wall

# Magnetic presheath

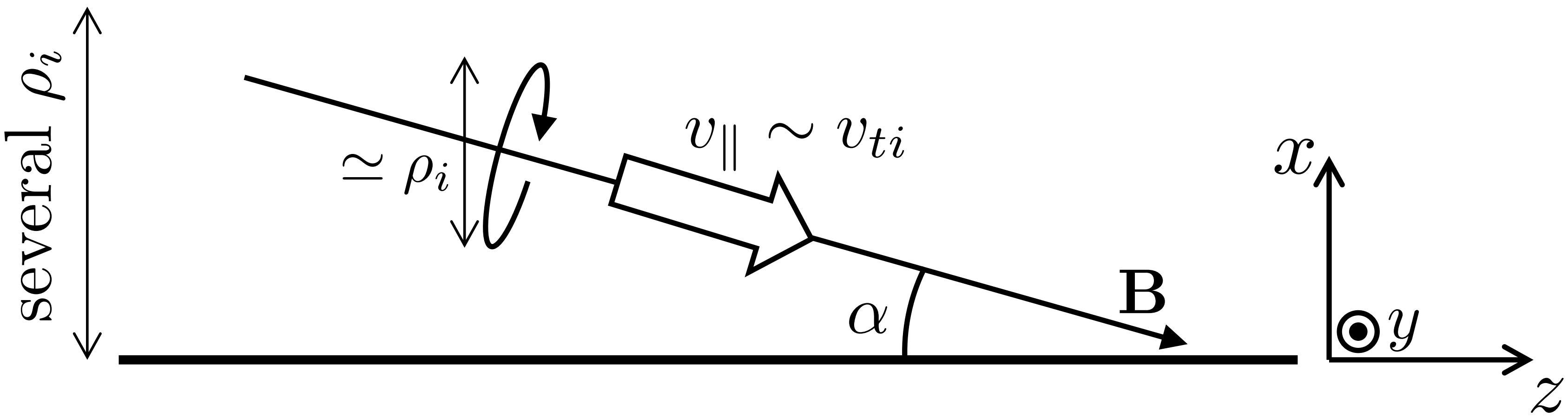
- Layer of thickness ~ $\rho_i$
- Large electric field disrupts gyro-orbits




- Solve layer for small angle $\alpha$
  - 0.05 rad (3°) now, but hope to get 0.02 rad (1°)

# Gyrokinetics for the presheath

- Slow approach of the orbit to the wall

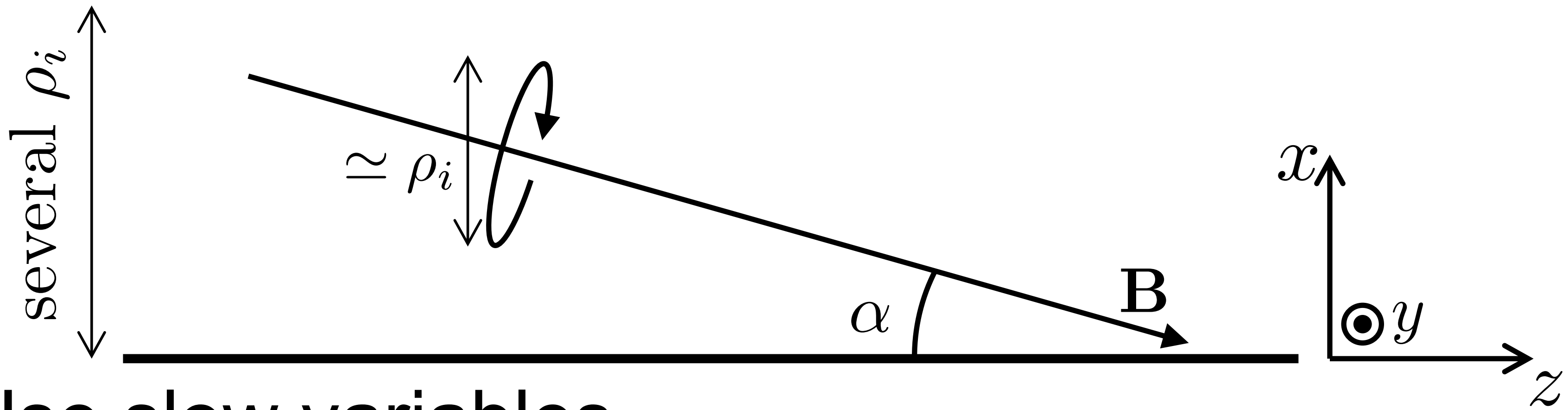



$$\text{gyro-orbit time} \sim \frac{1}{\Omega_i} \ll \text{parallel streaming time} \sim \frac{\rho_i/\alpha}{v_{ti}} \sim \frac{1}{\alpha\Omega_i}$$

- Separation of time scales

⇒ gyration + slowly varying non-circular orbit

# Gyrokinetics for the presheath




- Use slow variables
  - Orbit position

$$m_i \frac{\mathrm{d}v_y}{\mathrm{d}t} = -Zev_x B - Zev_z \alpha B \Rightarrow x = \underbrace{\bar{x}}_{\text{slow change}} - \frac{v_y}{\Omega_i}$$

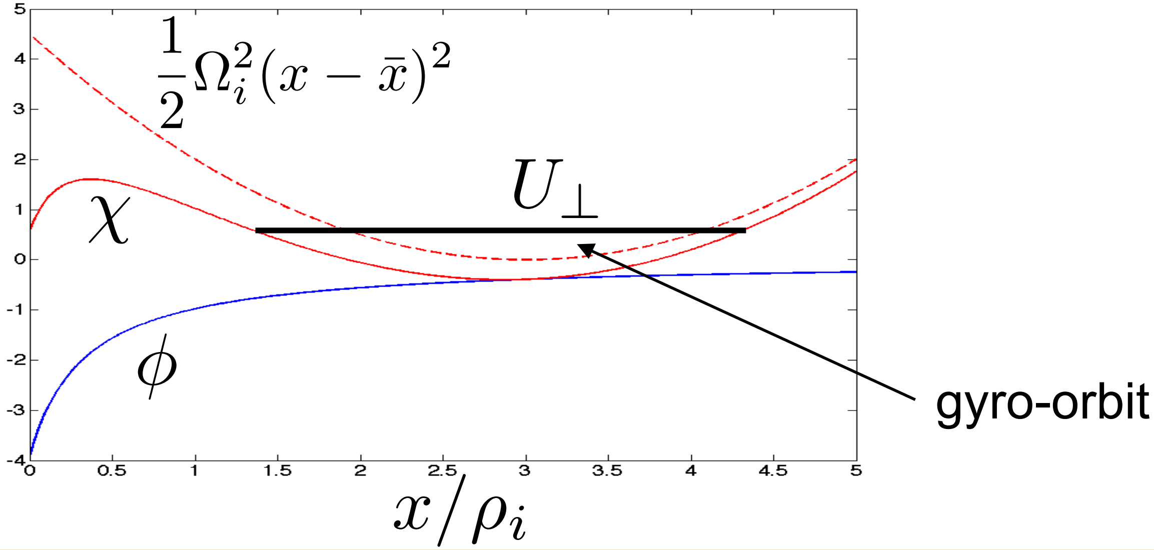
  - $\perp$ energy: $U_\perp = \frac{1}{2}(v_x^2 + v_y^2) + \frac{Ze\phi}{m_i} = \text{slow change}$
  - Total energy: $U = \frac{1}{2}(v_x^2 + v_y^2 + v_z^2) + \frac{Ze\phi}{m_i} = \text{constant}$

# Projection of the orbit ⊥ to wall

- Motion ⊥ to wall described by potential $\chi$

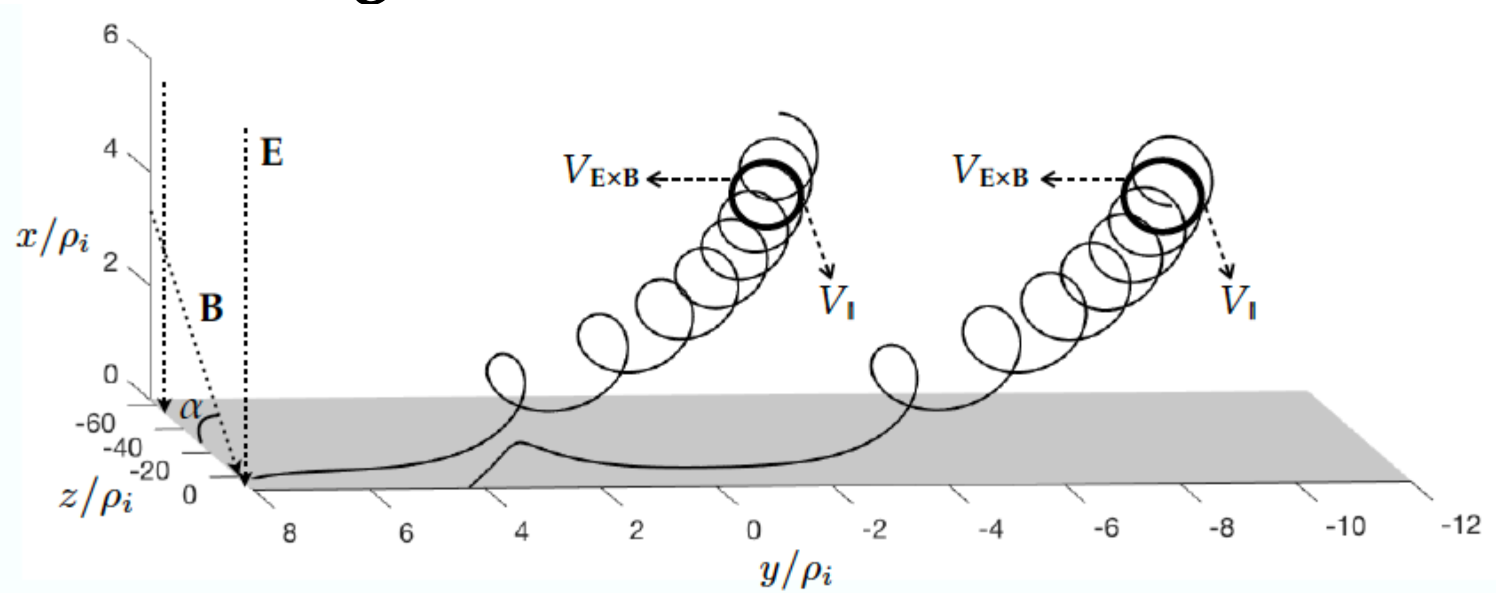
$$\frac{v_x^2}{2} = U_\perp - \frac{v_y^2}{2} - \frac{Ze\phi}{m_i} = U_\perp - \left[ \underbrace{\frac{1}{2}\Omega_i^2 (x-\bar{x})^2 + \frac{Ze\phi}{m_i}}_{\chi(x,\bar{x})} \right]$$

- Determine gyro-orbits using $\chi$
  - $\phi$ monotonically increases with $x$



# Final motion

- Orbits move towards wall along $\mathbf{B}$ ( $\dot{x} < 0$)
- $U_\perp$ decreases because $U$ = constant and $v_z \simeq v_{||}$ increases due to $\mathbf{E}$ acceleration
  - Obtain $U_\perp$ from adiabatic invariant $\mu$
- $\mathbf{E} \times \mathbf{B}$ drift in $y$-direction



# Solving quasineutrality

- Constant energy $U$ and magnetic moment $\mu$

$$n_i = \int f_i(\mu, U)\,\mathrm{d}^3v$$

Limits depend on $U$ and $\mu$

  - $f_i(\mu, U)$ is given by ions entering the presheath

- Find $\phi$ using quasineutrality

$$Zn_i[\phi] = n_{e\infty} \exp\left(\frac{e\phi}{T_e}\right)$$

  - Maxwell-Boltzmann electron response valid for electron repelling wall

$$\frac{v_{ti}}{v_{te}} \sim \sqrt{\frac{m_e}{m_i}}\sqrt{\frac{T_i}{T_e}} \ll \alpha \ll 1$$

# Far away from the wall

- Force balance along **B**

$$P_{\|} = \int f_i(\mu, U) m_i \underbrace{v_z^2}_{= 2\left(U - \mu B - \frac{v_E^2}{2} - \frac{Ze\phi}{m_i}\right)} \mathrm{d}^3 v + n_e T_e = \text{constant}$$

- $\mathbf{E} \times \mathbf{B}$ drift increases towards the wall

$$n_i m_i v_E^2 = 2 m_i \int f_i \left(U - \mu B - \frac{Ze\phi}{m_i}\right) \mathrm{d}^3 v + n_e T_e - P_{\|\infty} \geq 0$$

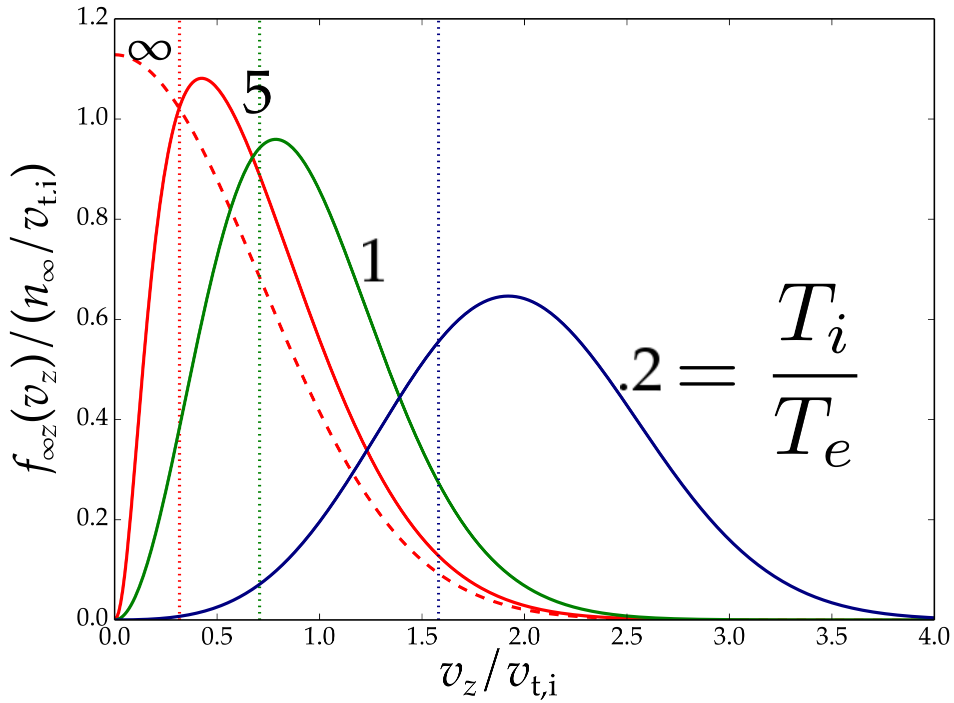
- Expand in $\phi << 1 \Rightarrow$ kinetic Chodura condition

$$\frac{e^2 \phi^2}{2} \left(\frac{n_{e\infty}}{T_e} - Z^2 \int \frac{f_i}{m_i v_{\|}^2} \mathrm{d}^3 v\right) \geq 0$$

  - Need enough || momentum to start presheath!

# Presheath entrance $f_i$

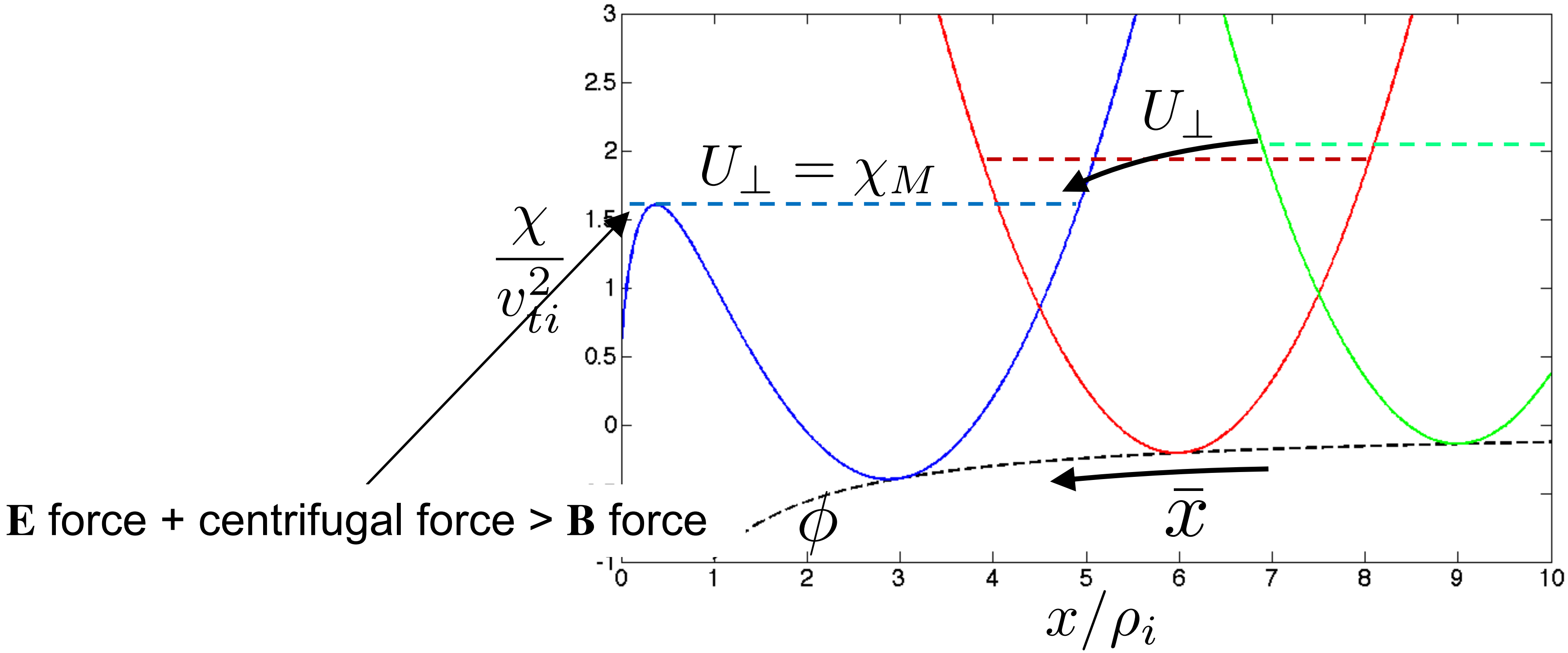
- So far, tried only $f_i$ that marginally satisfies kinetic Chodura condition




  - Number of particles with small $v_{||}$ determined by Chodura condition

# Near the wall

- Eventually, particles reach the wall

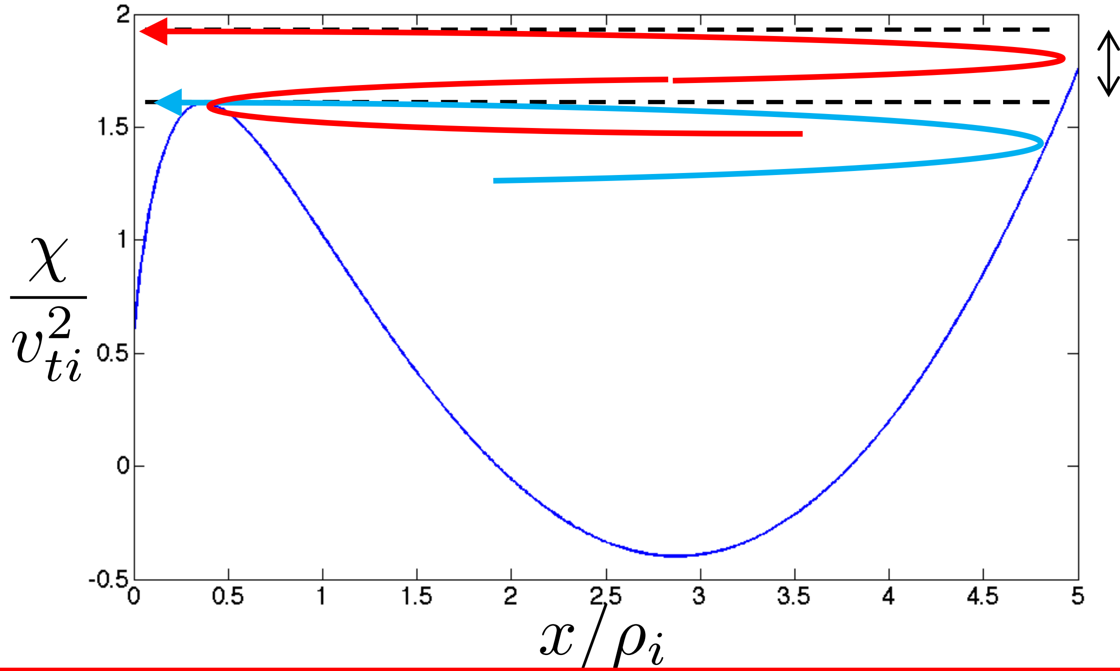



- Particles spend zero time in “open” orbit

⇒ density must be calculated to higher order

# "Open" orbit density

- Calculate the range of "open" orbits
  - Blue trajectory: $v_x = 0^-$ at $x_M$
  - Red trajectory: $v_x = 0^+$ at $x_M$

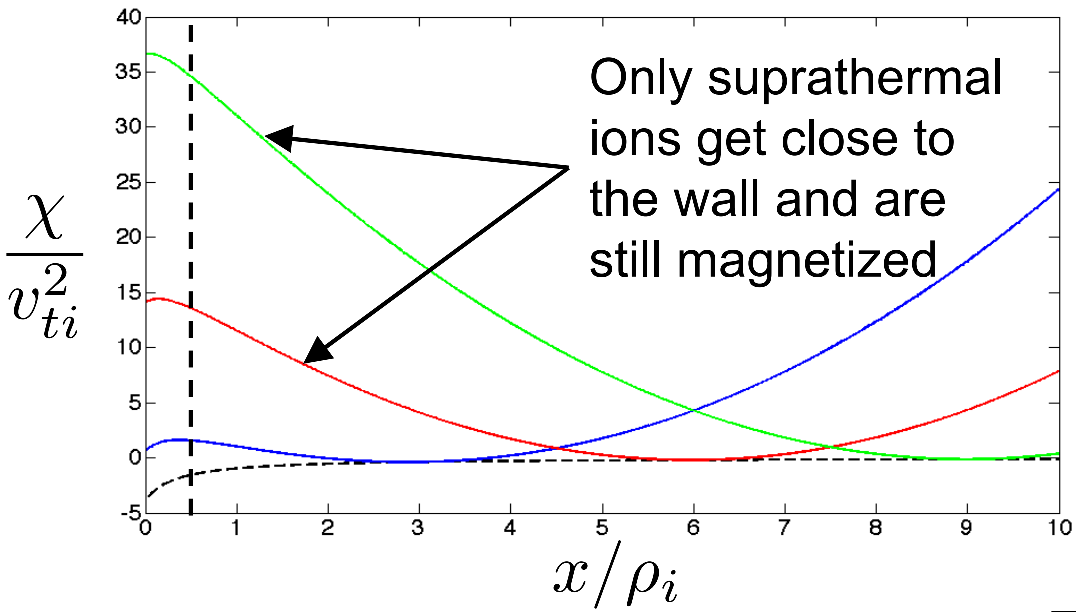



$$\Delta_M = \int \frac{\dot{U}_\perp - \dot{\chi}_M}{v_x}\,\mathrm{d}x \sim \alpha v_{ti}^2$$

$$\int f_i\,\mathrm{d}v_x \simeq f_i \Delta v_x, \quad \Delta v_x = \sqrt{2(U_\perp - \chi + \Delta_M)} - \sqrt{2(U_\perp - \chi)}$$

# Potential near the wall

- $\mathbf{E}$ always dominates in region near the wall



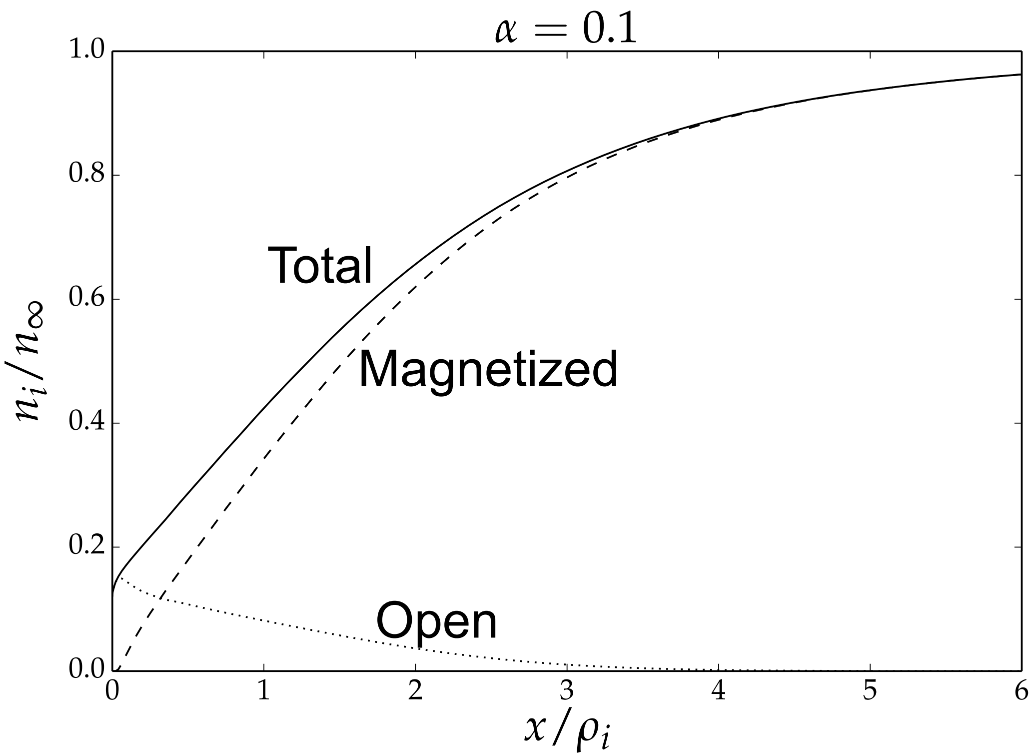

  - $\mathbf{E}$ diverges at the wall: $\phi - \phi_W \propto \sqrt{x}$

⇒ Debye sheath entrance

  - Bohm condition satisfied: $Z \int \frac{f_i}{v_x^2} \, \mathrm{d}^3 v = \frac{n_{eW} m_i}{Z T_e} = \frac{n_{eW}}{v_B^2}$

# Total density

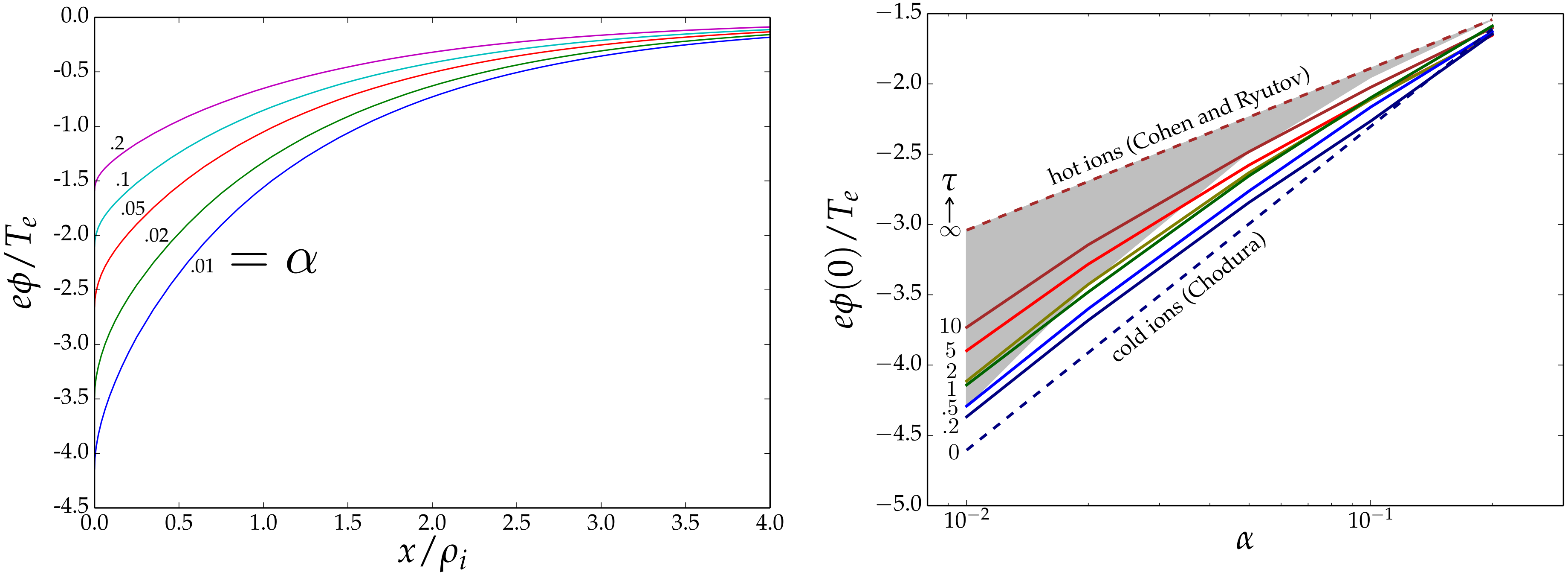
- Small crucial contribution from "open" orbits




  - Open orbit density small by $\alpha << 1$ because ions spend short time in this part of the orbit

# Potential profile in the presheath

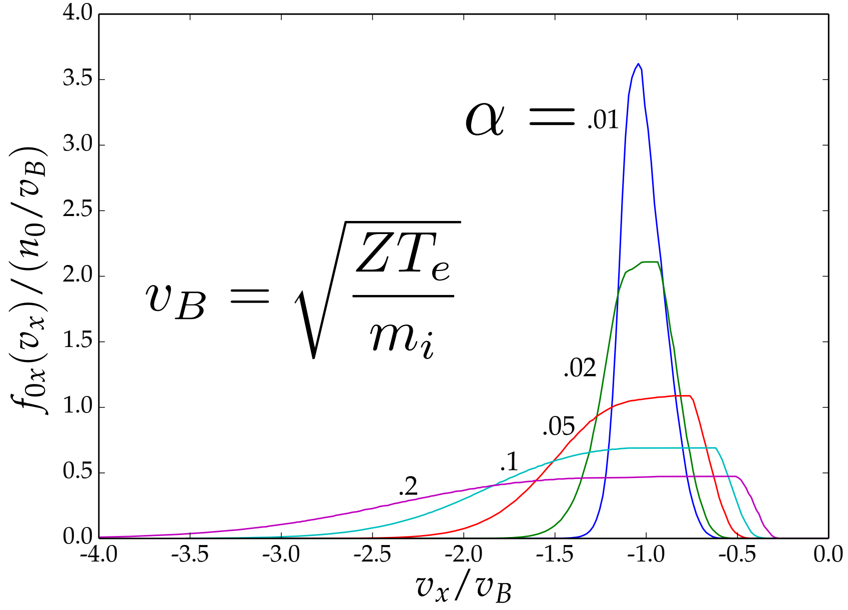
- Scans in $\alpha$ and $\tau = T_i/T_e$

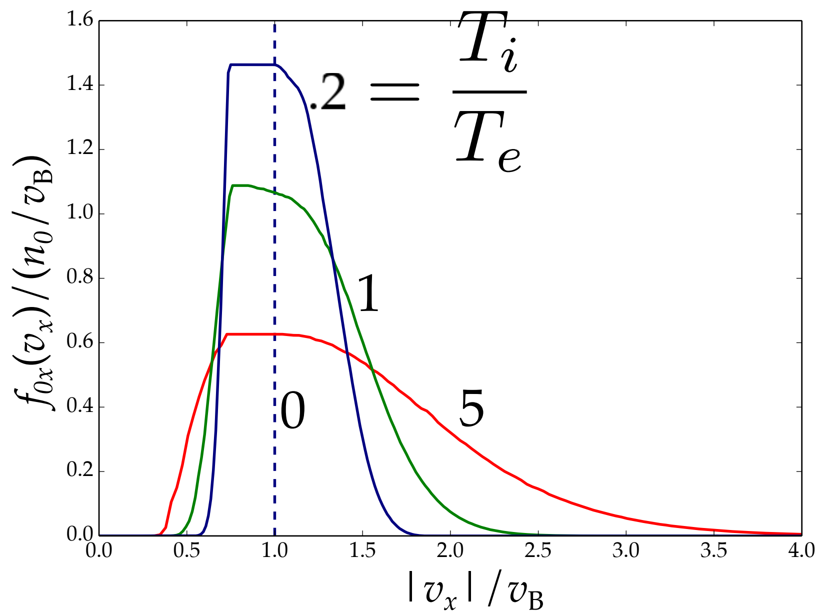



  - $e\phi/T_e \sim \ln \alpha$ because $e\phi/T_e \sim \ln\left(n_{i,\text{open}}/n_{i\infty}\right)$
  - $e\phi$ smaller for large $\tau$ because $e\phi \sim T_e$ and hence more magnetized particles reach the wall

# Distribution functions

- Particles are accelerated toward the wall




- Width of $f_i$ decreases for decreasing $\alpha$
  - Unexplained but ”open” orbit $\mathbf{\Delta} v_x \sim \alpha v_{ti}$ contributes
- Width of $f_i$ weakly dependent on $T_i$
  - Energetic ions are not accelerated much for $e\phi \sim T_e$

# Distribution functions

- $y$-direction $\mathbf{E} \times \mathbf{B}$ drift increases near the wall
- Ion acceleration along $\mathbf{B}$ ($\simeq \mathbf{z}$)