Transport + Global GK turbulence simulations in tokamaks

A multiple-timescale approach

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Outline

- Brief Aside on Astrophysical Fluid Dynamics

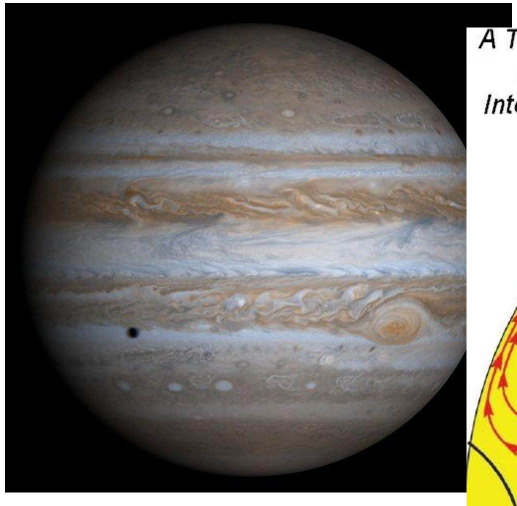
- Background/Context
  - Timescale separation and transport equations for tokamak core plasma
  - Simulations with GENE

- Numerical Method

- Adiabatic electron simulations
  - Evolution of ion pressure only
  - With realistic magnetic geometry

- Kinetic electron simulations
  - Evolution of ion pressure, electron pressure, and plasma density

Zonal flows (or not?) in rotating, MHD systems



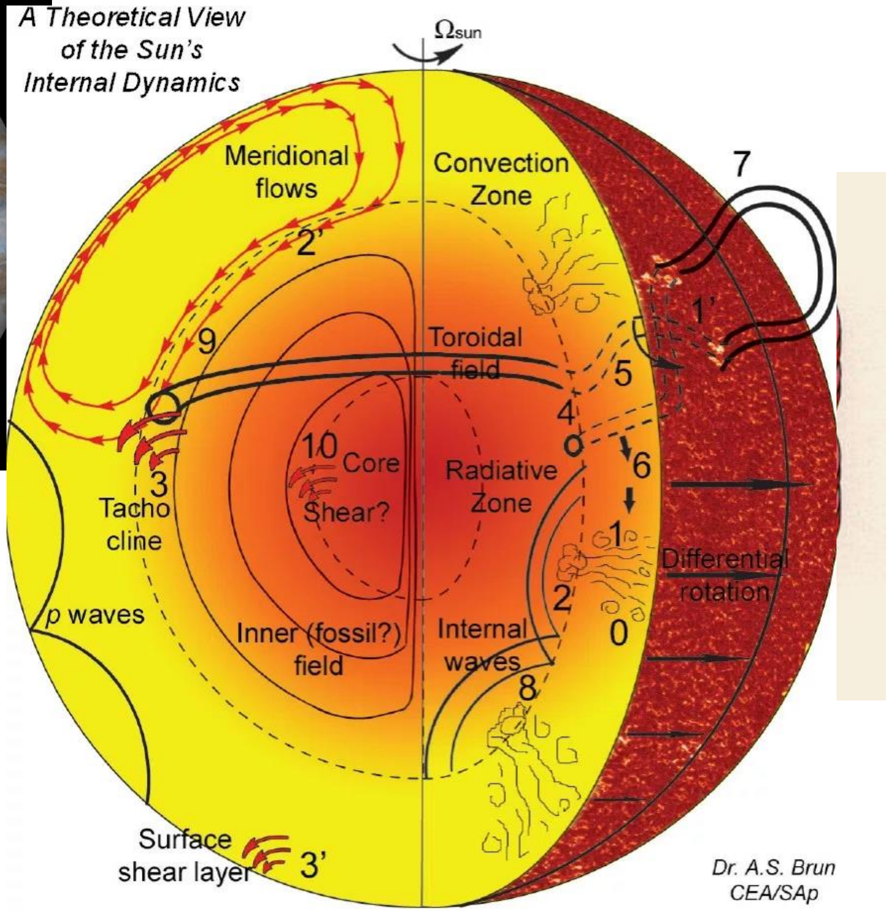
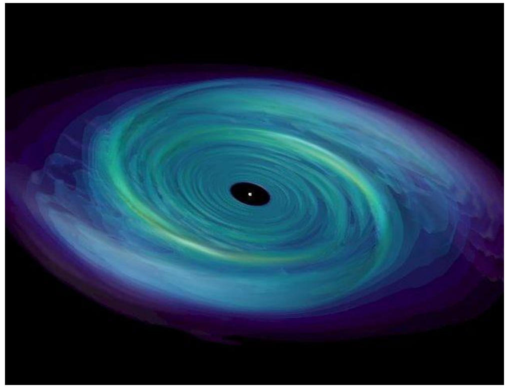


Figure Caption:
0: Turbulent convection (plumes); 1: Generation/self-induction of B field ("alpha-effect") or 1': Tilt of active region, source of B poloidal; 2: Turbulent pumping of B field in tachocline or 2': Transport of B field by meridional flows from CZ into tachocline (single or multi-cells flow?); 3: Field ordering into toroidal structures by large scale (radial and latitudinal) shear in tachocline ("omega-effect"); 3': Surface shear layer, Sub surface weather (SSW), surface dynamics of sun spot?; 4: Toroidal field becomes unstable to m=1 or 2 longitudinal instability (Parker's); 5: Rise (lift) + rotation (tilt) of twisted toroidal structures; 6: Recycling of weak field in CZ or; 7: Emergence of bipolar structures at the surface; 8: Internal waves propagating in RZ and possibly extracting angular momentum; 9: Interaction between dynamo induced field and inner (fossil?) field in the tachocline along with shear, turbulence, waves, etc...10: Instability of inner field (stable configuration?) + shearing via omega-effect at nuclear core edge? Is there a dynamo loop realized in RZ?

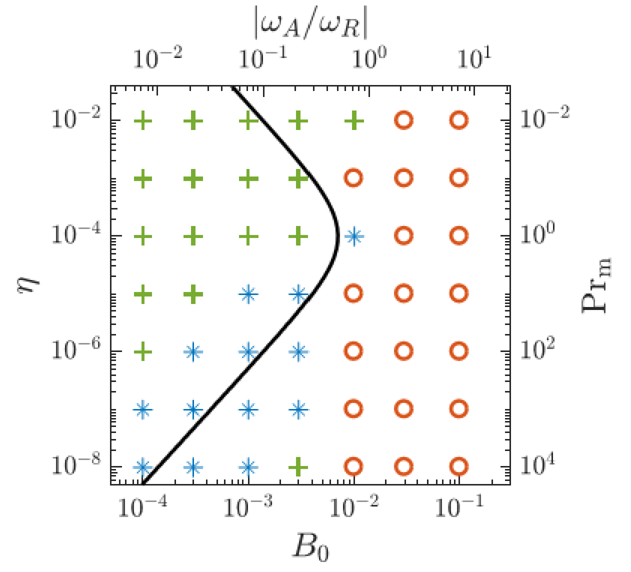


Zonal flows suppressed on a magnetized beta plane

- Suppose a uniform **azimuthal** field $\boldsymbol{B} = B_0 \widehat{\boldsymbol{\phi}}$
- Setting: 2D beta plane, force the vorticity variable
- Will azimuthal zonal flows grow?

Conclusion:

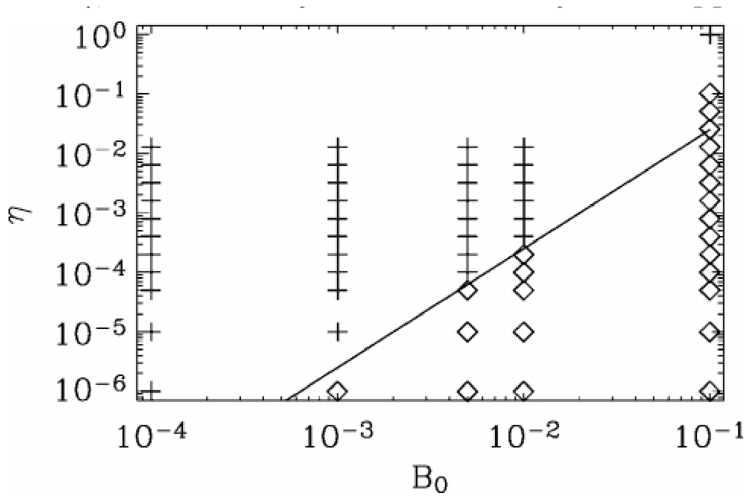
- Quasilinear analysis, but results agreed very well with DNS
- Zonal flows are definitely gone if $\frac{\omega_A}{\omega_R} \sim O(1)$
- But even for $\frac{\omega_A}{\omega_R} \ll 1$, zonal flows suppressed can be suppressed at large $\mathrm{Pr_m}$ (small resistivity η)
- Maxwell stress acts against the growth of zonal flows



Quasilinear analysis,

Constantinou & Parker, ApJ (2018)

arXiv 1805.09847



DNS,

S. Tobias et al, ApJ (2007)

Possibly applicable to:

- stellar interiors (solar tachocline)
- Gas giants (Jupiter's interior), exoplanets

What about other zonal modes? (magnetic field, pressure, etc.)

Outline: Global gyrokinetic turbulence and transport

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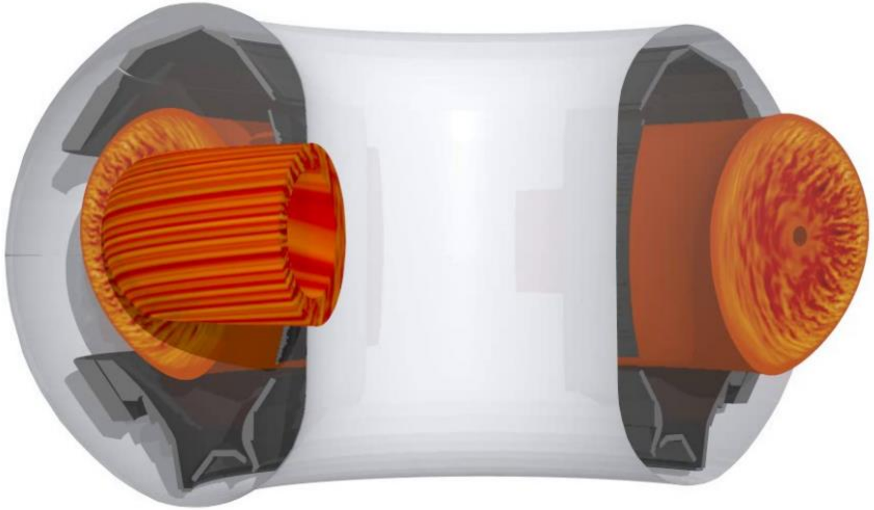
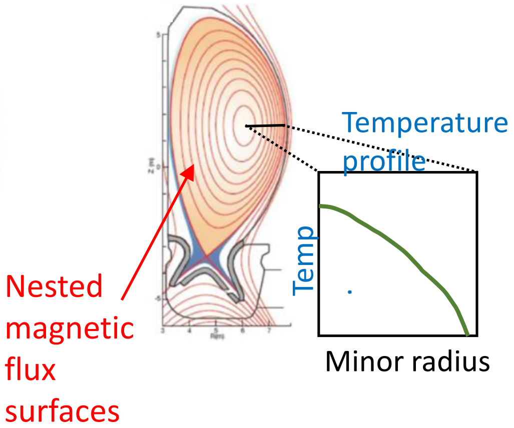
Global gyrokinetic turbulence + transport

Overarching goal: predict both profiles and turbulence in the core of a tokamak

Or: coupling a transport solver and turbulence simulation. **VERY PRACTICAL**



(Cross section of torus)



Turbulent fluxes in the core are small, resulting in long timescales for the evolution of macroscopic profiles, e.g., T(r)

Turbulence time ~ 10 μs
Energy confinement time ~ 1 s

Direct numerical integration capturing both turbulence and confinement time scales → computationally expensive!

Assuming a separation of timescales exists, how can we efficiently study the self-consistent evolution on the long timescale? How do we bridge the timescale gap?

Why global versus local simulation

- Nonlocal physical effects that you may not be able to get from local models
  - Internal transport barriers
  - Smallish tokamaks where ρ_* isn't so small

- Potentially less sensitive to issues of convergence due to local marginal stability

- Potentially computationally cheaper for similar runs because local simulations may need large radial box size to converge
  - But: less embarrassingly parallel

Multiscale gyrokinetics: rigorous derivation of transport & turbulence eqns

Sugama and Horton (1997, 1998)
Abel et al. (2013)

Ordering:

$$\frac{\omega}{\Omega} \sim \frac{\rho}{L} \sim \frac{k_\parallel}{k_\perp} \sim \frac{\delta f}{f} \sim \frac{|\delta \mathbf{B}|}{|\mathbf{B}|} \sim \frac{|\delta \mathbf{E}|}{|\mathbf{E}|} \sim \epsilon \qquad k_\perp \rho \sim O(1) \qquad \frac{1}{\omega \tau} \sim \epsilon^2$$

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m}[\mathbf{E} + \mathbf{v} \times \mathbf{B}] \cdot \frac{\partial f}{\partial \mathbf{v}} = C[f]$$

Transport Equations (slow timescale, 1D)

$$\frac{\partial n}{\partial t} + \frac{1}{V'} \frac{\partial}{\partial \psi}\left[V' \langle \mathbf{\Gamma} \cdot \nabla \psi \rangle\right] = S_n$$

$$\frac{3}{2}\frac{\partial p}{\partial t} + \frac{1}{V'} \frac{\partial}{\partial \psi}\left[V' \langle \mathbf{Q} \cdot \nabla \psi \rangle\right] = S_E$$

Gyrokinetic Equations (fast timescale, 5D)

$$\frac{\partial h}{\partial t} + \left(v_\parallel \hat{\mathbf{b}} + \mathbf{V}_D + \langle \mathbf{V}_\chi \rangle_{\mathbf{R}}\right) \cdot \frac{\partial h}{\partial \mathbf{R}} = \langle C_L[h] \rangle_{\mathbf{R}} + \frac{ZeF_0}{T} \frac{\partial \langle \chi \rangle_{\mathbf{R}}}{\partial t} - \frac{\partial F_0}{\partial \psi} \langle \mathbf{V}_\chi \rangle_{\mathbf{R}} \cdot \nabla \psi$$

(Some terms suppressed, for simplicity)

[This talk: global code in this formulation]

TRINITY – GS2 (Barnes et al., 2010) → GryfX → GX (Highcock et al., JPP 2018)
TGYRO – GYRO (Candy et al. 2009)

Numerical method for solving an implicitly-stepped transport equation

Key Elements (more detail on next few slides)

- **Represent turbulent flux as diffusive (+ possibly convective)**
- **Picard iteration with relaxation (no Newton steps) – No Jacobians or Jacobian-vector products**
- Computationally advantageous: A transport timestep may finish with a cost comparable to running a single standalone turbulence simulation
- Works with either local or global simulations

Shestakov, Cohen, Crotinger, LoDestro, Tarditi, Xu, JCP (2003)

Represent turbulent flux as diffusive/convective and use Picard iteration

Paradigm equation

$$\frac{n_m - n_{m-1}}{\Delta t} + \partial_x \Gamma_m = S_m$$

Nonlinear equation.
How to solve it?

Introduce a subscript l representing iteration: when solving for the mth timestep, let $n_{m,l}$ be the lth iterate. Represent the turbulent flux as diffusive:

$$\Gamma_{m,l} \rightarrow -D_{m,l-1}(\partial_x n_{m,l}) + c_{m,l-1} n_{m,l}$$

Picard iteration:

- Diffusion coefficient evaluated at previous iterate
- Gradient at current iterate

where

$$D_{m,l-1} \equiv -\theta \frac{\Gamma[n_{m,l-1}]}{\partial_x n_{m,l-1}}, \qquad c_{m,l-1} \equiv (1-\theta)\frac{\Gamma[n_{m,l-1}]}{n_{m,l-1}}$$

Flux computed in a separate turbulence simulation

Note: If the effective diffusion coefficient is negative or infinite, can use the convective piece

This gives a tractable, linear equation to solve for each iterate $n_{m,l}$:

$$\frac{n_{m,l} - n_{m-1}}{\Delta t} + \partial_x \left[-D_{m,l-1} \partial_x n_{m,l} + c_{m,l-1} n_{m,l} \right] = S_{m,l}$$

If it converges, it doesn't matter how you represented the turbulent flux: it's the (a) right answer

Contrast with Newton iteration

$$\frac{n_m - n_{m-1}}{\Delta t} + \partial_x \Gamma_m = S_m$$

A Newton-type of iteration would Taylor expand the flux:

$$\Gamma_{m,l} = \Gamma[n_{m,l}] \approx \Gamma[n_{m,l-1}] + \left.\frac{\delta \Gamma}{\delta n}\right|_{n_{m,l-1}} \cdot (n_{m,l} - n_{m,l-1})$$

This procedure requires calculation of Jacobian terms $\delta\Gamma/\delta n$. Two problems:

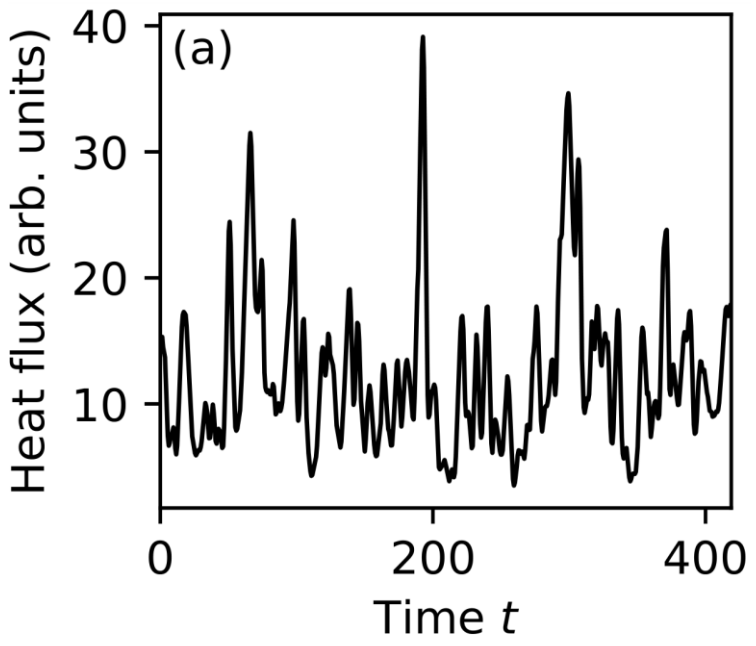
- **Computationally expensive** to calculate Jacobians or Jacobian-vector products – extra runs of turbulence simulations for each forward difference

- Fluxes are intrinsically noisy due to statistical fluctuations of turbulence simulations. **Errors are amplified** in the calculation of the Jacobian

- Newton-based method is used by TGYRO (J. Candy et al.) and TRINITY (M. Barnes et al.) for solving the transport equation with **local** gyrokinetic simulations.
- Not clear how to make a Newton-based method work for global gyrokinetic simulations, where turbulence can depend on the profiles everywhere (i.e., much more complicated calculation of Jacobians)

Transport solver & handling fluctuations from turbulence simulations

- Unlike a transport model such as TGLF, turbulence simulations have inherent fluctuations in the flux.
  - Repeated calculations $Q(T_i, T_i')$ give exactly the same answer every time from TGLF, but not from a turbulence simulation
- Can run turbulence simulation for very long time to gain better statistics (better estimates for the mean fluxes), but this is computationally expensive, so you want to get away with as little as you can
- What is the right statistical way to characterize the problem?



$$\langle q\rangle(x,t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} dt'\, q(x,t')$$

By the **central limit theorem for correlated sequences**, <q> approaches a normally distributed variable if T >> autocorrelation time

Want averaging time T to at least be a few autocorrelation times. (Actually, want T large enough so that the variance of $\langle q\rangle$ is acceptably small)

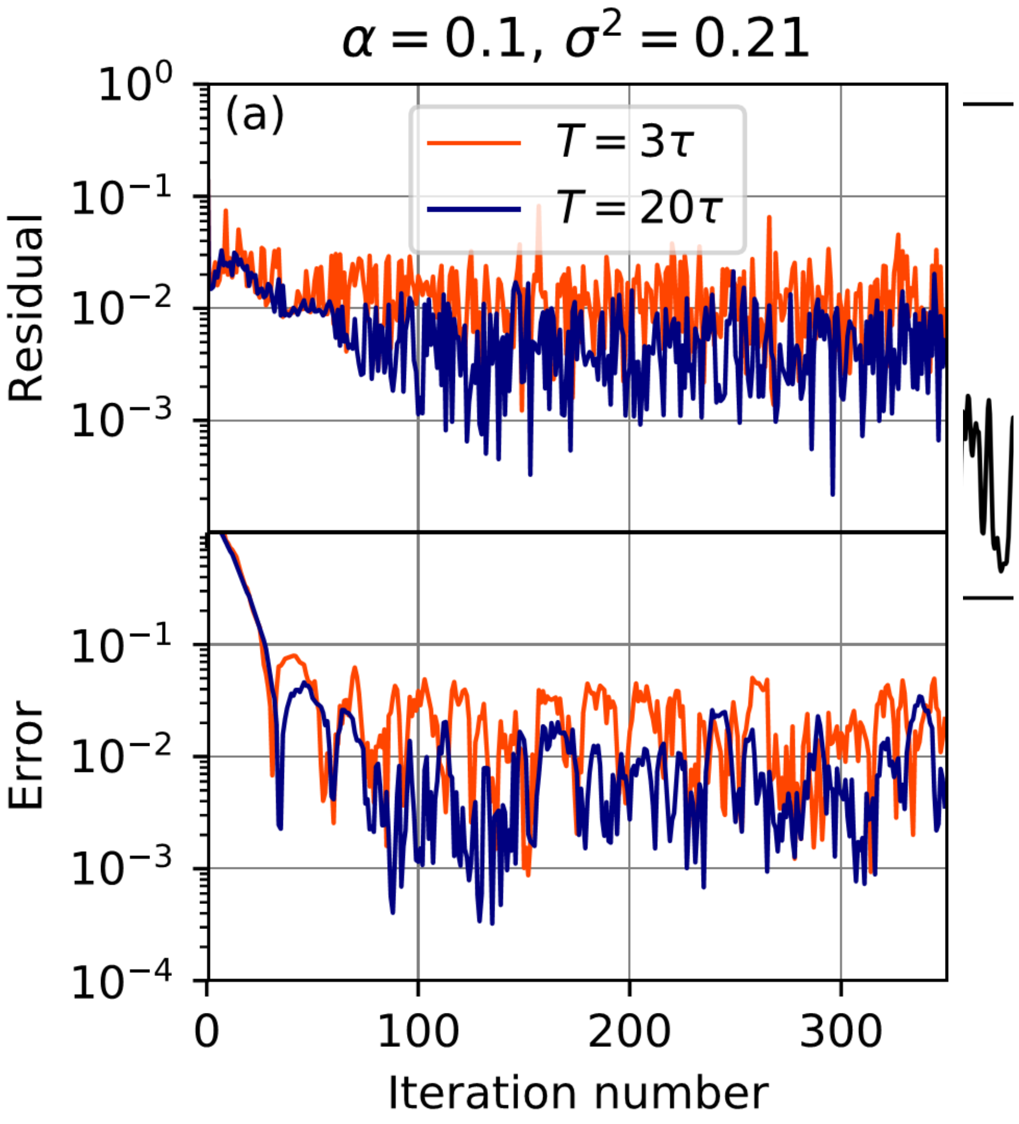
J.B. Parker et al, *plasma* (2018)

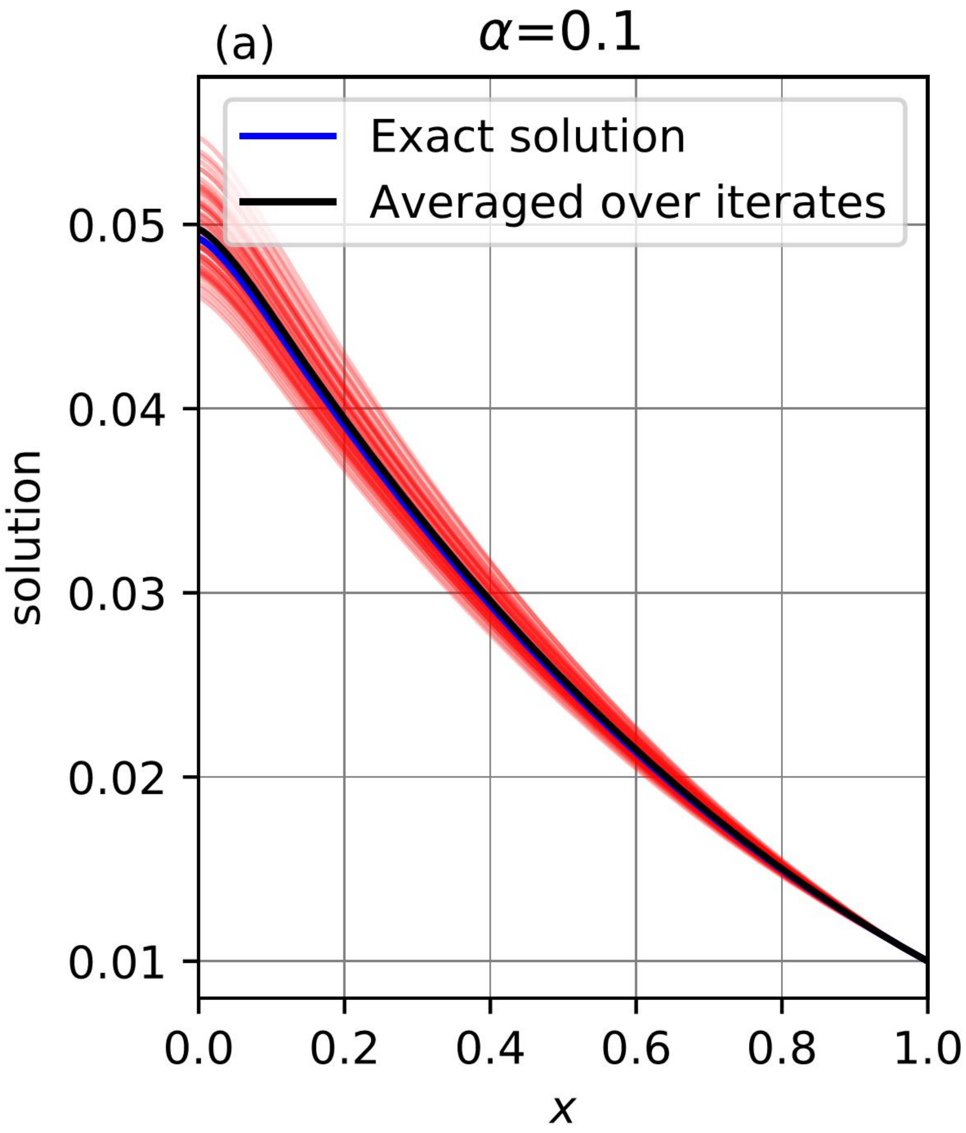
Silicon Valley motto: test often (fail fast)

- Use ARMA modeling
  from time-series ana
  random noise for qu
  purposes
  - Temporally corr
    spatially correla

- Test problem: take a
  where the flux is ana
  specified; sprinkle in
  How does converger

- **Quantify it**





J.B. Parker et al, *plasma* (2018)

Tango

- 1D transport solver; implements the just-described numerical method

- Written in Python, coupled with global GENE
  - `libtringene.F90` became `libtango.F90`

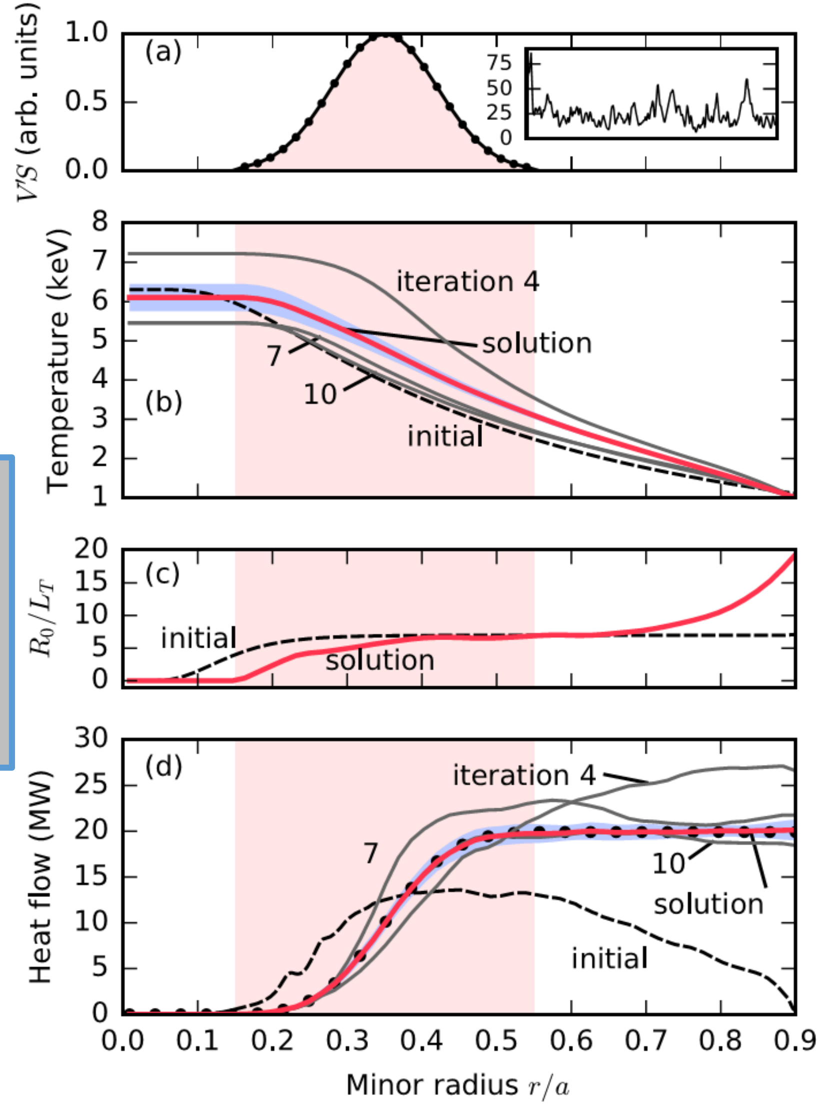
- github.com/LLNL/tango

Tango + global GENE simulations (adiabatic electron)

- <u>GENE parameters:</u>
  - Domain: $\frac{r}{a} \in [0.1, 0.9]$
  - a=1.0 m, R_0=3.0 m
  - Bref=2.5 T, m = 2m_i
  - **ρ_*= 1/292**
  - Circular geometry, adiabatic electrons, CBC-like
- <u>Tango parameters:</u>
  - Domain: $\frac{r}{a} \in [0, 0.9]$
  - Looking for a steady-state solution with an applied source
  - Relaxation EWMA parameter $\alpha = 0.3$ (fairly large)
  - 50 iterations
  - 50 R_0/v_{ti} per iteration
  - Evolving ion pressure only; density profile is prescribed and held fixed
  - Boundary conditions:
    - r=0: Neumann
    - r/a = 0.9: Dirichlet, fixed pressure/temperature
  - **Applied heat source 20 MW localized in 0.15 < r/a < 0.55**

2500 R_0/v_{ti}, ~35k core-hours

Now down to ~1000 R_0/v_{ti}



Parker et al., "Bringing Global Gyrokinetic Simulations to the Transport Timescale Using a Multiscale Approach", *NF* (2018)

Challenges: Tango + Kinetic Electron simulations

- Main change:
  - Now evolving ion pressure, electron pressure, and plasma density instead of just ion pressure
  - Implicit timestep. Is the numerical method still going to converge for long timesteps?
- How to generalize the numerical method for multiple evolving profiles, when cross-field transport channels may exist?
- Challenge: simulations are much slower than with adiabatic electrons, and the numerical method still needs to be "tested" and kept close eye on, because whether or not it converges (for long timesteps) is uncertain
- It is possible that TGLF may be a suitable stand-in for GENE, for testing the generalization of the numerical method
- Want to go to more realistic experimental geometry (e.g., TCV, DIII-D)

Simplest generalization of numerical method

$$\frac{\partial n}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial \psi}\left[V'\langle \mathbf{\Gamma} \cdot \nabla\psi\rangle\right] = S_n(\psi)$$

$$\frac{3}{2}\frac{\partial p_i}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial \psi}\left[V'\langle \mathbf{Q}_i \cdot \nabla\psi\rangle\right] = S_i(\psi)$$

$$\frac{3}{2}\frac{\partial p_e}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial \psi}\left[V'\langle \mathbf{Q}_e \cdot \nabla\psi\rangle\right] = S_e(\psi)$$

- Represent turbulent transport with "diagonal" transport coefficients
- Effective transport coefficients are still determined numerically from gyrokinetic simulation
- E.g.,

$$\Gamma \to -D_n \frac{\partial n}{\partial \psi} + c_n n, \qquad D_n = -\theta \frac{\Gamma_{GENE}}{\partial n/\partial \psi}, \quad c_n = (1-\theta)\frac{\Gamma_{GENE}}{n}$$

$$Q_i \to -D_i \frac{\partial p_i}{\partial \psi} + c_i p_i, \qquad D_i = -\theta \frac{Q_{i,GENE}}{\partial p_i/\partial \psi}, \quad c_i = (1-\theta)\frac{Q_{i,GENE}}{p_i}$$

Does it work?

Questions?

Auspices

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