Roles of sub-ion-scale structures on cross-scale interactions in Tokamak plasma turbulence

Shinya Maeyama (Nagoya Univ.)

Collaborators: T.-H. Watanabe (Nagoya Univ.),
Y. Idomura (JAEA),
A. Ishizawa (Kyoto Univ.),
M. Nakata, M. Nunami (NIFS)

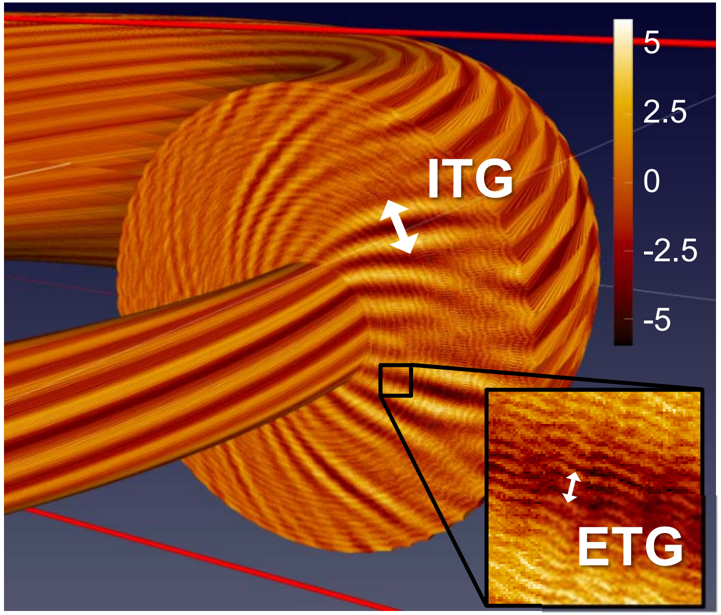
11th Plasma Kinetics Working Meeting, WPI, Vienna, Jul. 23 - Aug. 3, 2018

Outline

[20 min.] Cross-scale interactions between electron/ion-scale turbulence

- ✓ ITG/ETG turbulence [Maeyama'15PRL; '17NF]
- ✓ MTM/ETG turbulence [Maeyama'17PRL]

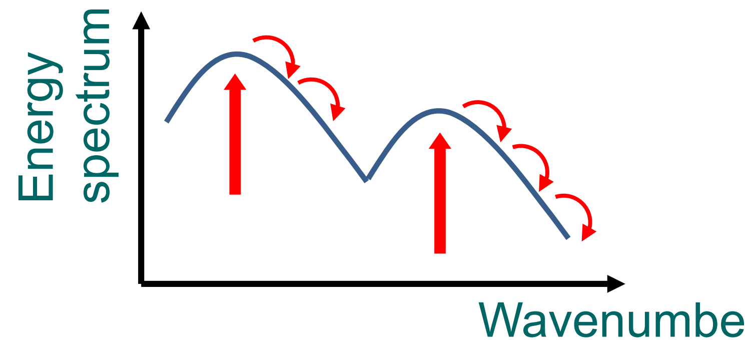
[10 min.] Ongoing work: extracting and modeling cross-scale interactions

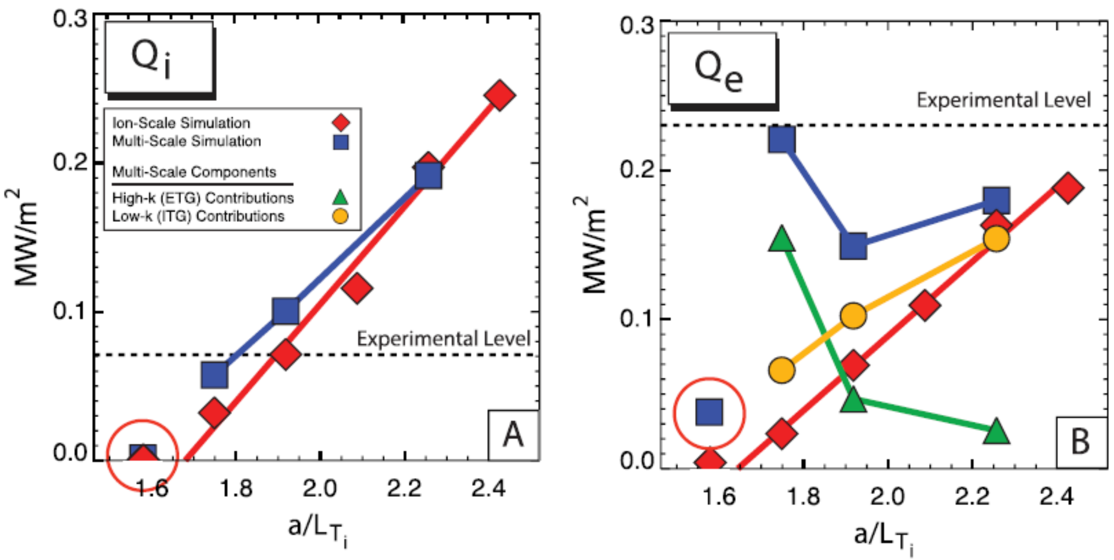


Background

Plasma turbulence is driven by a variety of instabilities, which can be nonlinearly coupled with. [Candy'07PPCF; Görler'08PRL; Maeyama'15PRL]

- Electron-scale modes $k_\theta \rho_{ti} \gg 1$
  ETG (Electron-temp.-grad. mode)
- Ion-scale modes $k_\theta \rho_{ti} \leq 1$
  ITG, KBM, TEM, MTM, …





Importance of cross-scale interactions is also reported in experimental comparison, and expected in ITER parameters.
[Howard'16PoP; Holland'17NF]

GKV – GyroKinetic Vlasov code [Watanabe'06NF]

http://www.p.phys.nagoya-u.ac.jp/gkv/

- δf gyrokinetics in a local flux-tube geometry.
- Solves time evolution of perturbed distributions $\tilde{f}_s$ and potentials $\tilde{\phi}$, $\tilde{A}_\parallel$ in 5D phase space $(x, y, z, v_\parallel, \mu)$.
- Spectral (x, y) and finite diff. $(z, v_\parallel, \mu)$ + Explicit time integration

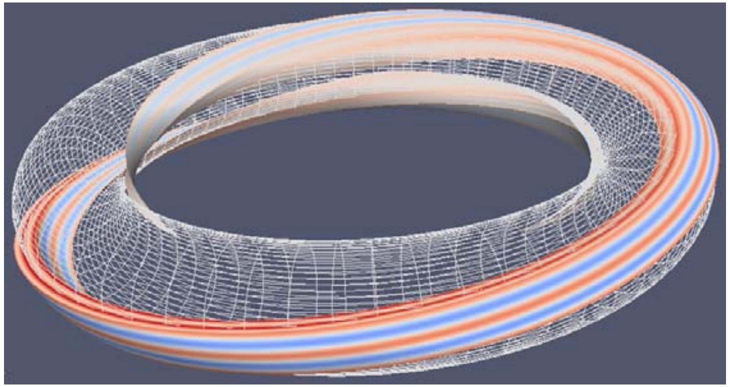
Delta-f gyrokinetic eqs.

$$\frac{\partial \tilde{f}_s}{\partial t} + \left(v_\parallel \frac{\boldsymbol{B} + \widetilde{\boldsymbol{B}}_\perp}{B} + \boldsymbol{v}_{sG} + \boldsymbol{v}_{sC} + \widetilde{\boldsymbol{v}}_E \right) \cdot \boldsymbol{\nabla} \tilde{f}_s + \frac{dv_\parallel}{dt} \frac{\partial \tilde{f}_s}{\partial v_\parallel} = S_s + C_s$$

$$\nabla_\perp^2 \tilde{\phi} = -\frac{1}{\varepsilon_0} \sum_s e_s (\tilde{n}_s + \tilde{n}_{s,\mathrm{pol}})$$

$$\nabla_\perp^2 \tilde{A}_\parallel = -\mu_0 \sum_s e_s \tilde{u}_{\parallel s}$$

Example of flux-tube domain



Analysis of the nonlinear mode coupling

Entropy balance is derived from GK eqs.

$$\frac{d}{dt}\left(\sum_{s=i,e} S_{s\boldsymbol{k}} + W_{\boldsymbol{k}}\right) = \sum_{s=i,e}\left(\frac{T_s \Gamma_{s\boldsymbol{k}}}{L_{p_s}} + \frac{\Theta_{s\boldsymbol{k}}}{L_{T_s}} + D_{s\boldsymbol{k}} + E_{s\boldsymbol{k}} + I_{s\boldsymbol{k}}\right)$$

where the nonlinear entropy transfer

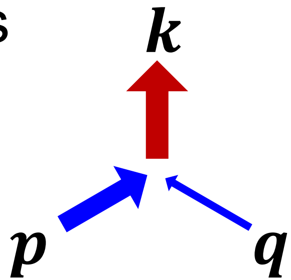
$$I_{s\boldsymbol{k}} = \sum_{\boldsymbol{p}}\sum_{\boldsymbol{q}} J_{s\boldsymbol{k}}^{\boldsymbol{p},\boldsymbol{q}}$$

and triad transfer [Navarro’11PRL; Nakata’12PoP; Hatch’13PRL]

$$J_{s\boldsymbol{k}}^{\boldsymbol{p},\boldsymbol{q}} = \delta_{\boldsymbol{k}+\boldsymbol{p}+\boldsymbol{q},0}\frac{\boldsymbol{b}\cdot\boldsymbol{p}\times\boldsymbol{q}}{2B}\mathrm{Re}\left[\left\langle\int dv^3\left(\chi_{s\boldsymbol{p}} g_{s\boldsymbol{q}} - \chi_{s\boldsymbol{q}} g_{s\boldsymbol{p}}\right)\frac{T_s g_{s\boldsymbol{k}}}{F_{sM}}\right\rangle\right]$$

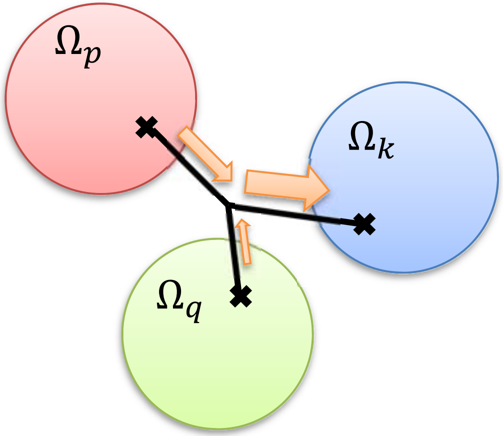
(with $\chi_{s\boldsymbol{k}} = J_{0s\boldsymbol{k}}(\tilde{\phi}_{\boldsymbol{k}} - v_\parallel \tilde{A}_{\parallel\boldsymbol{k}})$, $g_{s\boldsymbol{k}} = f_{s\boldsymbol{k}} + \frac{e_s F_{sM}}{T_s} J_{0s\boldsymbol{k}}\tilde{\phi}_{\boldsymbol{k}}$) satisfies

- Symmetry $J_{\boldsymbol{k}}^{\boldsymbol{p},\boldsymbol{q}} = J_{\boldsymbol{k}}^{\boldsymbol{q},\boldsymbol{p}}$
- Detailed balance $J_{\boldsymbol{k}}^{\boldsymbol{p},\boldsymbol{q}} + J_{\boldsymbol{q}}^{\boldsymbol{k},\boldsymbol{p}} + J_{\boldsymbol{p}}^{\boldsymbol{q},\boldsymbol{k}} = 0$
- $J_{\boldsymbol{k}}^{\boldsymbol{p},\boldsymbol{q}} = 0\ (if\ \boldsymbol{k} \parallel \boldsymbol{p} \parallel \boldsymbol{q})$



Sub-space transfer analysis [Maeyama'17NF]

Dividing the wave-number space into some subspaces, we define the sub-space transfer



$$I_{\Omega_k} = \sum_{\Omega_p} \sum_{\Omega_q} J_{\Omega_k}^{\Omega_p,\Omega_q}$$

$$J_{\Omega_k}^{\Omega_p,\Omega_q} = \sum_{\boldsymbol{k}\in\Omega_k} \sum_{\boldsymbol{p}\in\Omega_q} \sum_{\boldsymbol{q}\in\Omega_q} J_{\boldsymbol{k}}^{\boldsymbol{p},\boldsymbol{q}}$$

which satisfies

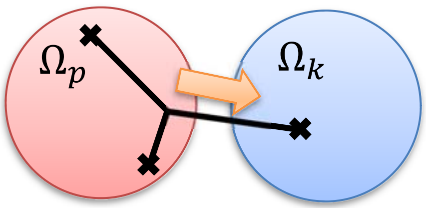
- Symmetry

$$J_{\Omega_k}^{\Omega_p,\Omega_q} = J_{\Omega_k}^{\Omega_q,\Omega_p}$$

- Detailed balance

$$J_{\Omega_k}^{\Omega_p,\Omega_q} + J_{\Omega_q}^{\Omega_k,\Omega_q} + J_{\Omega_p}^{\Omega_q,\Omega_k} = 0$$

- $J_{\Omega_k}^{\Omega_p,\Omega_p} \neq 0 \ (if\ \Omega_p \neq \Omega_k)$



NOTE: Importance of symmetrization for evaluating the interactions among different scales

$$\frac{\partial \tilde{f}}{\partial t} = -\{\tilde{\phi}, \tilde{f}\}, \qquad \text{with } \tilde{f} = \sum_{\Omega_k} \tilde{f}_{\Omega_k} \text{ and } \tilde{\phi} = \sum_{\Omega_k} \tilde{\phi}_{\Omega_k}$$

※Symmetrization is not only the discussion in wavenumber space. Indeed, the above eq. is written in real space. Ω_k is regarded as a filter extracting a certain scale.

Unfortunately, majority of plasma publications use a non-symmetrized transfer function. [Mininni'05PRE; Tatsuno'10PFR; Plunk'12NJP; Navarro'14PoP; Teaca'17NJP]

$$\frac{d}{dt}\frac{\langle \tilde{f}_{\Omega_k}^2 \rangle}{2} = \sum_{\Omega_p}\sum_{\Omega_q} -\left\langle \left\{ \tilde{\phi}_{\Omega_p}, \tilde{f}_{\Omega_q} \right\} \tilde{f}_{\Omega_k} \right\rangle = \sum_{\Omega_q} -\left\langle \left\{ \tilde{\phi}, \tilde{f}_{\Omega_q} \right\} \tilde{f}_{\Omega_k} \right\rangle = \sum_{\Omega_q} T_{\Omega_k}^{\Omega_q}$$

We recommend to use a symmetrized transfer function. [Nakata'12PoP; Maeyama'17NF]

$$\frac{d}{dt}\frac{\langle \tilde{f}_{\Omega_k}^2 \rangle}{2} = \sum_{\Omega_p}\sum_{\Omega_q} -\left\langle \frac{\left\{ \tilde{\phi}_{\Omega_p}, \tilde{f}_{\Omega_q} \right\} + \left\{ \tilde{\phi}_{\Omega_q}, \tilde{f}_{\Omega_p} \right\}}{2} \tilde{f}_{\Omega_k} \right\rangle = \sum_{\Omega_p}\sum_{\Omega_q} J_{\Omega_k}^{\Omega_p,\Omega_q}$$

In neutral fluids, wave-wave interactions are usually analyzed by symmetrized transfer function. [Maltrud'93PFA,Watanabe'07PRE] (※Non-symmetric transfer can only be useful when analyzing wave-mean flow interactions. [Smyth'92PFA])

NOTE: Importance of symmetrization for evaluating the interactions among different scales

An apparent example

Consider interactions among three scales (e.g. low/middle/high-k) $\tilde{f} = \tilde{f}_{\Omega_l} + \tilde{f}_{\Omega_m} + \tilde{f}_{\Omega_h}$.

$$\text{If } \tilde{f} = c\tilde{\phi}, \qquad \frac{\partial \tilde{f}}{\partial t} = -\{\tilde{\phi}, \tilde{f}\} = 0 \qquad \text{No physical interaction}$$

(a) Non-symmetrized transfer

$$\frac{d}{dt}\frac{\langle \tilde{f}_{\Omega_l}^2 \rangle}{2} = T_{\Omega_l}^{\Omega_l} + T_{\Omega_l}^{\Omega_m} + T_{\Omega_l}^{\Omega_h}$$

$T_{\Omega_l}^{\Omega_l} = 0$, but there are fictitious interactions

$$T_{\Omega_l}^{\Omega_m} = -\langle \{\tilde{\phi}, \tilde{f}_{\Omega_m}\} \tilde{f}_{\Omega_l} \rangle = -c^2 \langle \{\tilde{\phi}_{\Omega_h}, \tilde{\phi}_{\Omega_m}\} \tilde{\phi}_{\Omega_l} \rangle \neq 0 \quad \left(\text{also } T_{\Omega_l}^{\Omega_h} \neq 0\right)$$

Find new interactions? No, it's fiction.

(b) Symmetrized transfer

$$\frac{d}{dt}\frac{\langle \tilde{f}_{\Omega_l}^2 \rangle}{2} = T_{\Omega_l}^{\Omega_l,\Omega_l} + T_{\Omega_l}^{\Omega_m,\Omega_m} + T_{\Omega_l}^{\Omega_h,\Omega_h} + 2T_{\Omega_l}^{\Omega_l,\Omega_m} + 2T_{\Omega_l}^{\Omega_h,\Omega_l} + 2T_{\Omega_l}^{\Omega_m,\Omega_h}$$

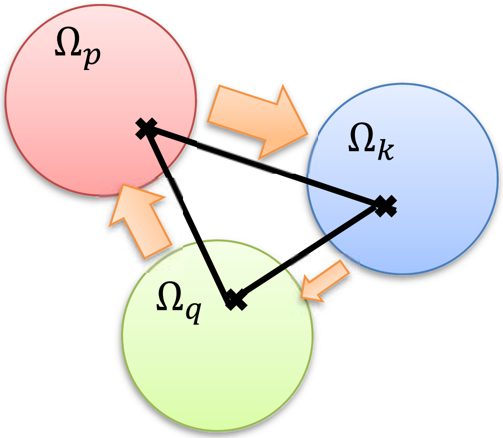
$$= 0 + 0 + 0 + 0 + 0 + 0$$

No fictitious interactions.

NOTE: Importance of symmetrization for evaluating the interactions among different scales

Non-symmetrized transfer

- Fictious interactions arise, because arbitrary circulations among three modes are able to slip into the analysis.

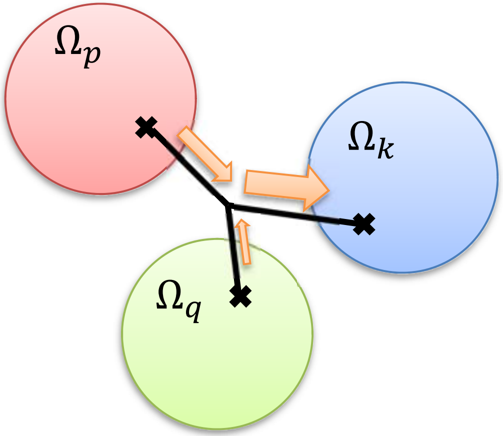


Not recommended.

Symmetrized transfer

- Symmetrization extracts net income/outgo, which rules out the circulations.
- Direction of the transfer should be determined from the detailed balance,

$$J_{\Omega_k}^{\Omega_p,\Omega_q} + J_{\Omega_p}^{\Omega_q,\Omega_k} + J_{\Omega_q}^{\Omega_k,\Omega_p} = 0$$



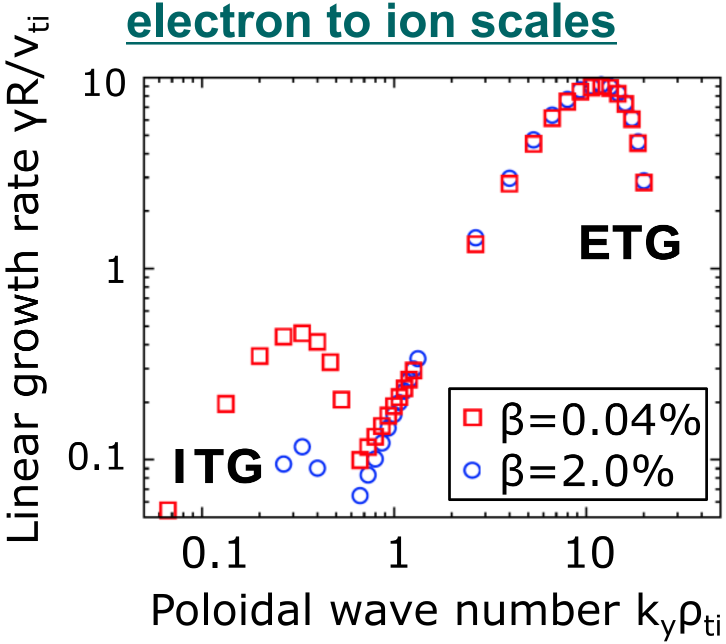
Recommended!

Simulation setup for ITG/ETG case

- High resolution from ion to electron scales
  $(x, y, z, v_{\parallel}, \mu, s) = (1024, 1024, 64, 96, 16, 2) = 2 \times 10^{11}\text{grids}$

- Cyclone-base-case parameters
  [Dimits'00PoP]
  R/L_n=2.2, R/L_{Te}=R/L_{Ti}=6.82,
  β=2.0%, r/R=0.18, q=1.4,
  s=0.786, m_e/m_i=1/1836

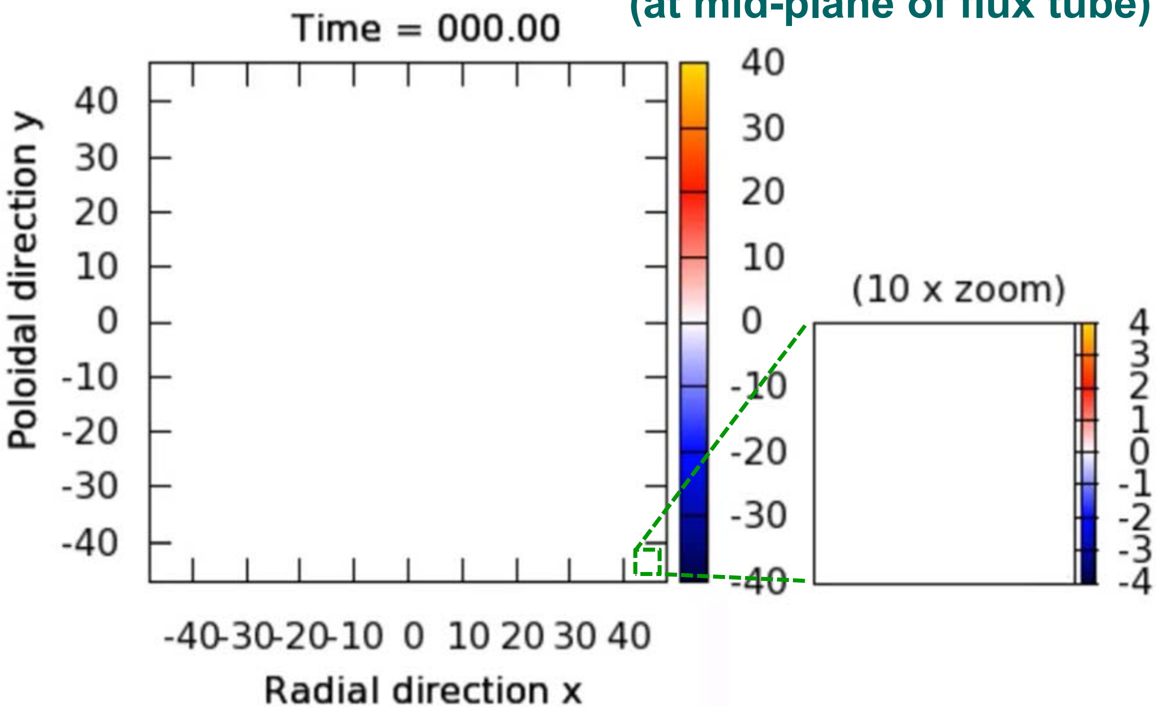
Linear instabilities from electron to ion scales

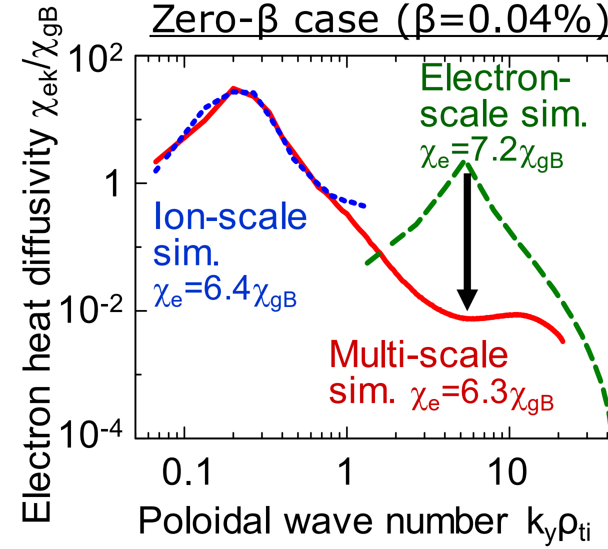


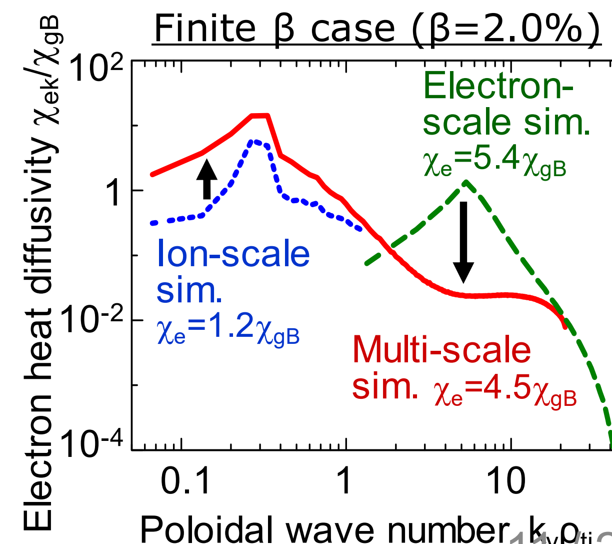
Evolution and spectrum

- ITGs suppress ETGs.
- ETGs enhance near-marginal ITGs.

Time evolution of the electrostatic potential
(at mid-plane of flux tube)







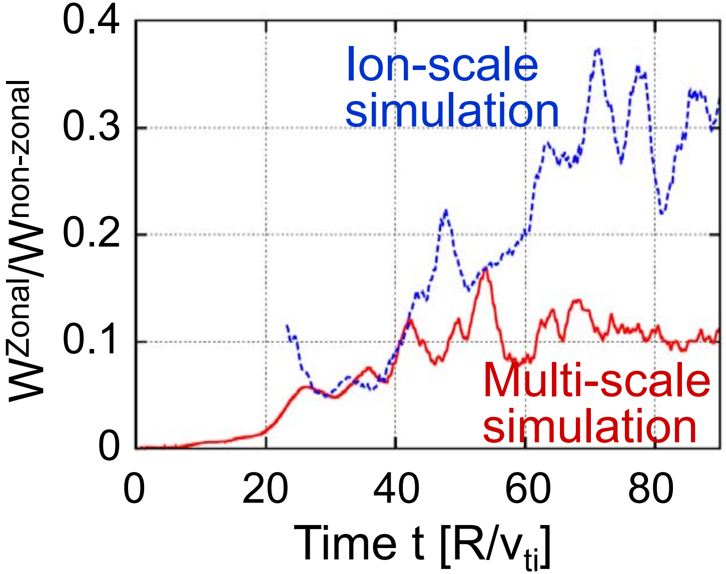
Reduction of ZF by ETG causes enhanced ITG transport.

Sub-space analysis splits ion-scale Ω_i and electron-scale Ω_e effects.

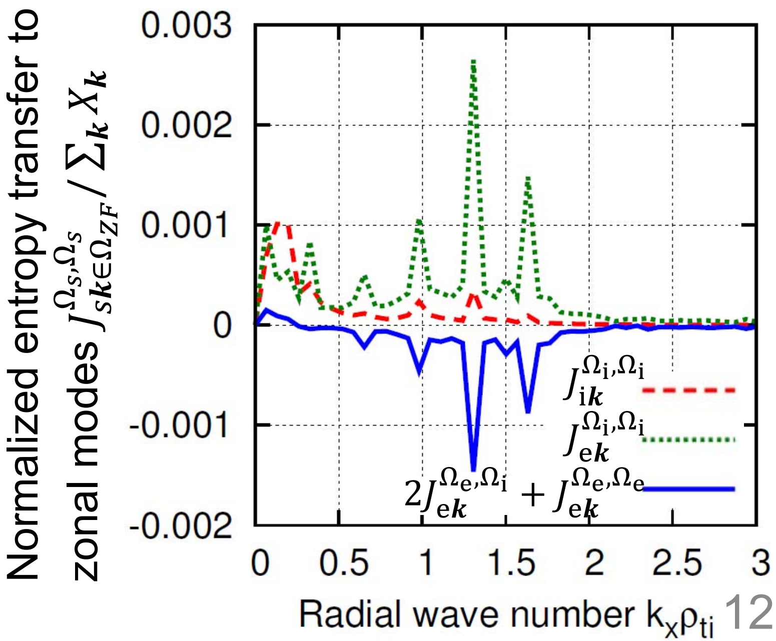
- Kinetic electrons in ITG create high-k_x ZF ($J_{\mathrm{e}\boldsymbol{k}}^{\Omega_\mathrm{i},\Omega_\mathrm{i}} > 0$). [Dominski'15PoP]
- Electron-scale turbulence effectively damps high-k_x zonal flows ($2J_{\mathrm{e}\boldsymbol{k}}^{\Omega_\mathrm{e},\Omega_\mathrm{i}} + J_{\mathrm{e}\boldsymbol{k}}^{\Omega_\mathrm{e},\Omega_\mathrm{e}} < 0$).

Ratio of zonal to non-zonal field energy

→ Weak ZF in multi-scale case.



Entropy transfer to zonal modes



Simulation setup for MTM/ETG case

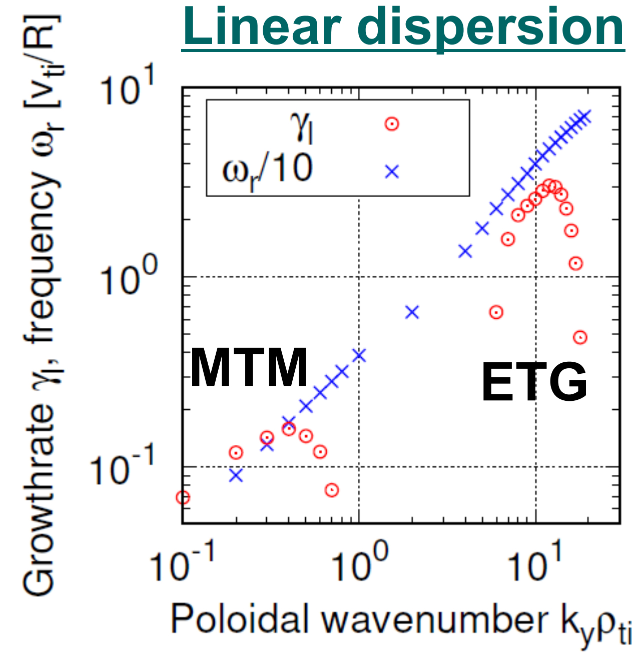
ASDEX-U-like parameters [Doerk'15PoP]

R/L_n=0.26, R/L_{Te}=5.9, R/L_{Ti}=0, β=6.0%,

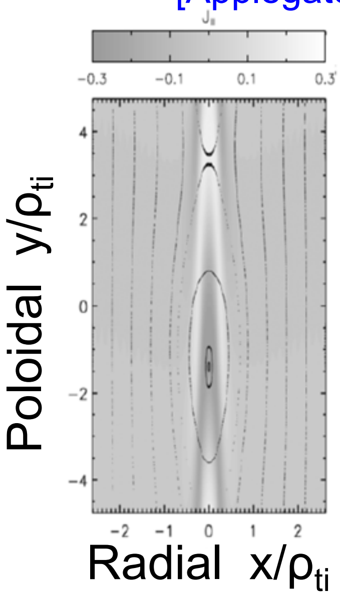
r/R=0.19, q=1.34, s=1.0, m_e/m_i=1/3672

→ MTM and ETG are unstable. (ITG-stable)

Linear dispersion



Poincaré plot of MTM
[Applegate'07PPCF]



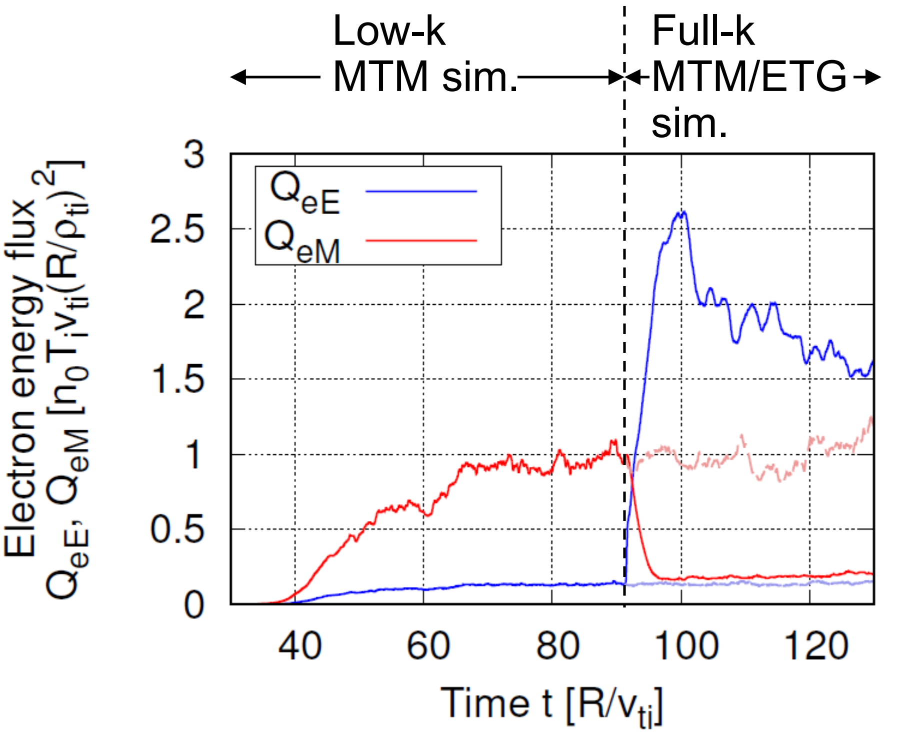
Quick review of micro-tearing modes (**MTM**)

- ✓ Kinetic tearing mode driven by electron temperature gradient [Hazeltine'75PF]
- ✓ Electron heat transport in core/pedestal [Doerk'11PRL,Guttenfelder'11PRL,Hatch'16NF]
- ✓ Radially-localized current sheets

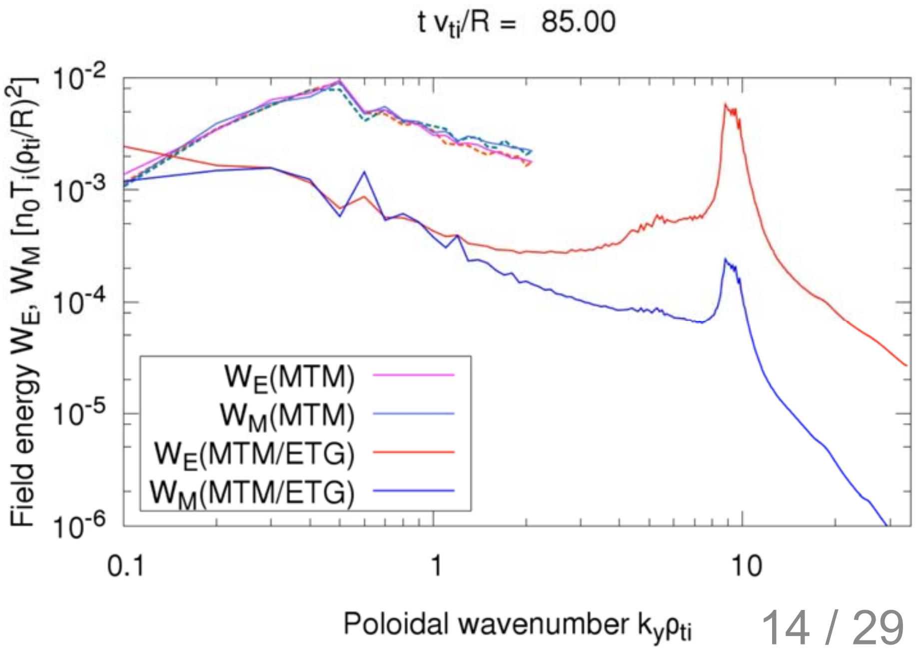
Electron heat flux, and field energy spectra

- ✓ MTM is suppressed as ETG grows up.
- ✓ ETG-driven ExB flows dominate electron heat transport.

ExB flows and magnetic flutter induced electron energy flux Q_{eE}, Q_{eM}



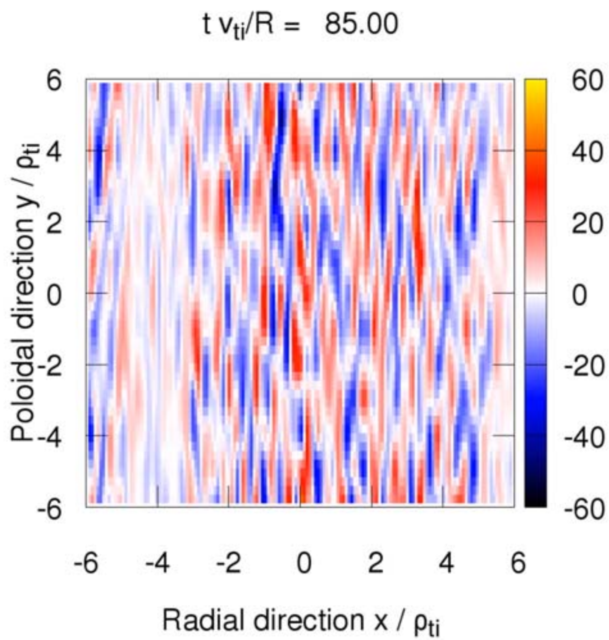
Electrostatic and magnetic field energy spectra W_E, W_M



Electron parallel current profiles

Width of current sheet is consistent with linear theoretical estimate in low-k MTM sim., but is broadened in full-k MTM/ETG sim.
→ ETG destroys current sheets of MTM.

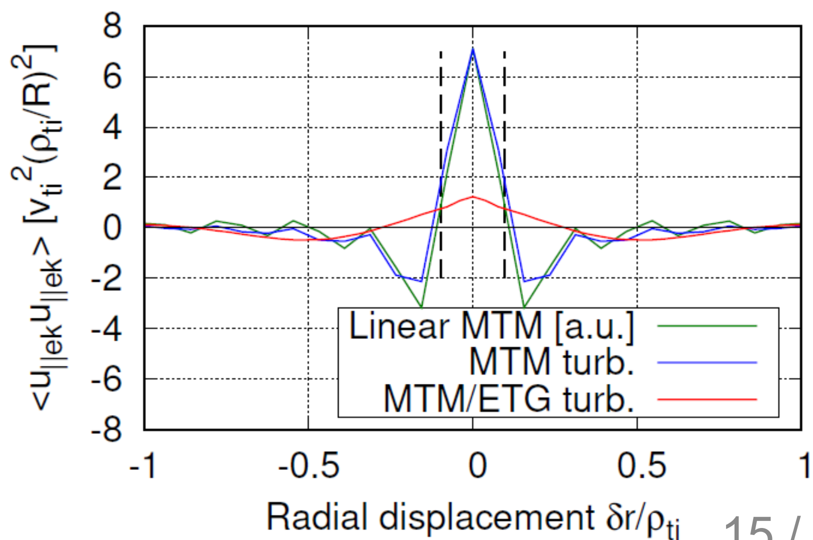
Electron parallel current on poloidal cross-section (at bad curvature region θ=0)



$$\frac{u_{e\parallel}}{v_{ti}\rho_{ti}/R}$$

Auto-covariance of MTM current $(k_y\rho_{ti} = 0.3)$

$$\left\langle u_{\parallel k_y} u_{\parallel k_y} \right\rangle(\delta r) = \iint \mathrm{Re}\left[u_{\parallel k_y}(x,z)\, u^*_{\parallel k_y}(x+\delta r, z)\right]\sqrt{g}\,dx\,dz$$



Cross-scale interactions between MTM/ETG

2D entropy transfer at t=93.4 R/v$_{ti}$

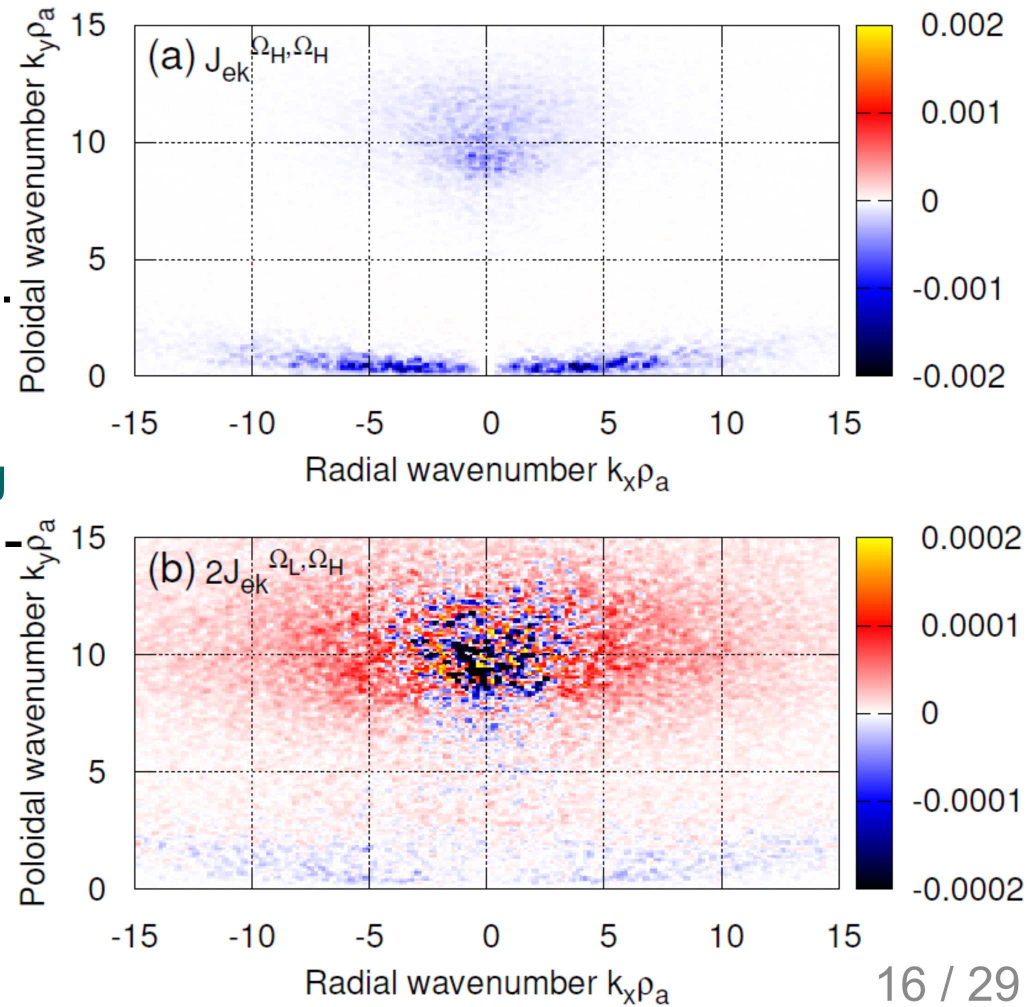
(a) $J_k^{\Omega_H,\Omega_H}$: Contribution by high-k_y and high-k_y coupling

Low-k_y but high-k_x components are effectively damped.

(b) $2J_k^{\Omega_L,\Omega_H}$: Contribution by low-k_y and high-k_y coupling

Entropy is transferred to high-k_y and high-k_x modes.

MTM current sheets (low-k_y but high-k_x) are broken into small-scale eddies via the shearing by ETG streamers.



Summary of 1st part

Multi-scale turbulence is analyzed by new sub-space transfer diagnostics.

- We emphasized the importance of symmetrization for transfer analysis.

ITG/ETG turbulence [Maeyama'15PRL; Maeyama'17NF]

- ➢ **Suppression of ETG by ITG** — Short-wave-length ITG turbulent eddies distort ETG streamers.
- ➢ **Enhancement of ITG by ETG** — Short-wave-length ZF created by ITG with kinetic electrons are damped by ETGs.

MTM/ETG turbulence [Maeyama'17PRL]

- ➢ **Suppression of MTM by ETG** — ETG turbulence destroys radially-localized current sheets of MTM.

Commonality of cross-scale interactions

- Kinetic electron response in low-k scales
- Cross-scale interactions via **sub-ion-scale structures**

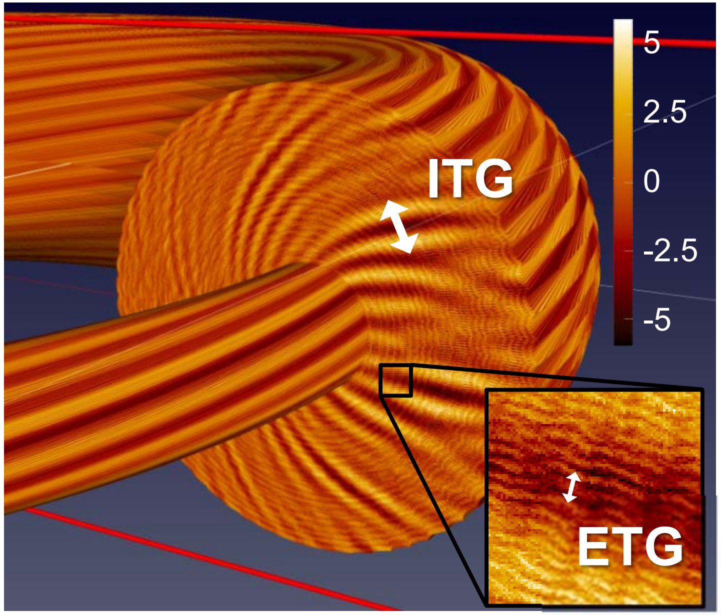
（e.g., ITG eddies, short-wave-length ZF, MTM current sheet, …)

Outline

[20 min.] Cross-scale interactions between electron/ion-scale turbulence

- ✓ ITG/ETG turbulence [Maeyama'15PRL; '17NF]
- ✓ MTM/ETG turbulence [Maeyama'17PRL]

[10 min.] Ongoing work: extracting and modeling cross-scale interactions



Extracting and modeling cross-scale interactions

Q. Can multi-scale simulation be replaced by a couple of single-scale simulations including any cross-scale interaction model?

Denoting low-k ($\boldsymbol{k} \in \Omega_l$) and high-k ($\boldsymbol{k} \in \Omega_h$) components as $\tilde{f} = \tilde{f}_{\Omega_l} + \tilde{f}_{\Omega_h}$,

Multi-scale simulation:

$$\frac{\partial \tilde{f}}{\partial t} = -\{\tilde{\phi}, \tilde{f}\} = -\{\tilde{\phi}_{\Omega_l} + \tilde{\phi}_{\Omega_h}, \tilde{f}_{\Omega_l} + \tilde{f}_{\Omega_h}\}$$

Coupled single-scale simulations:

$$\frac{\partial \tilde{f}_{\Omega_l}}{\partial t} = -\{\tilde{\phi}_{\Omega_l}, \tilde{f}_{\Omega_l}\}_{\Omega_l} + N_{\Omega_l}^{\Omega_h}$$

$$\frac{\partial \tilde{f}_{\Omega_h}}{\partial t} = -\{\tilde{\phi}_{\Omega_h}, \tilde{f}_{\Omega_h}\}_{\Omega_h} + N_{\Omega_h}^{\Omega_l}$$

where $N_{\Omega_l}^{\Omega_h}$ is cross-scale interaction model from high-k to low-k (and v.v. $N_{\Omega_h}^{\Omega_l}$).

NOTE: A LES model will not be useful; high-k modes can be actively excited.

A simplified problem: slab ITG/ETG turbulence

5D multi-scale simulation is too expensive to scan parameters.

In a shearless slab model $\boldsymbol{B}_0 = B_0 \hat{z}$,

$$\frac{\partial \tilde{f}_s}{\partial t} = -v_\parallel \nabla_\parallel \left(\tilde{f}_s + \frac{e_s F_{sM}}{T_s} J_{0s} \tilde{\phi} \right) - \{ J_{0s} \tilde{\phi}, \tilde{f}_s \} + \frac{e_s F_{sM}}{T_s} v_{s*} \cdot \nabla J_{0s} \tilde{\phi} + C_s$$

$$\left[\nabla_\perp^2 - \frac{1}{\varepsilon_0} \sum_s \frac{e_s^2 n_s}{T_s} (1 - \Gamma_{0s}) \right] \tilde{\phi} = -\frac{1}{\varepsilon_0} \sum_{s=i,e} e_s \int J_{0s} \tilde{f}_s \, dv^3$$

we used additional simplifications,

$\nabla_\parallel \to i k_\parallel = \mathrm{const.}$ (mimics parallel compressibility and Landau damping)

$J_{0s} \to \exp\left(-\frac{k_\perp^2 \rho_{ts}^2}{2} \right)$ (mimics FLR by assuming Maxwellian in $v_\perp$)

The reduced 3D problem $\tilde{f}_s(x, y, v_\parallel, t)$ is easy for computation but retains:

- Instability-driven, i.e., the phase between $\tilde{p}_s$ and $\tilde{\phi}$ is determined self-consistently
- FLR for ions and electrons
- Adiabatic-like electrons at low-k, while adiabatic ions at high-k

Examples

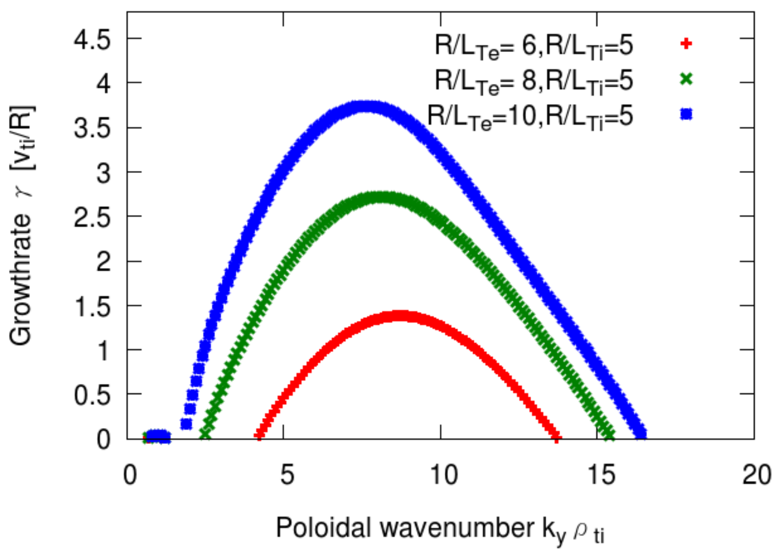
Box sizes & resolution:

$$\left(L_x, L_y, L_{v_\parallel}\right) = (20\pi\rho_{ti}, 20\pi\rho_{ti}, 4.5v_{ts}), \left(N_x, N_y, N_{v_\parallel}\right) = (1024,1024,96)$$

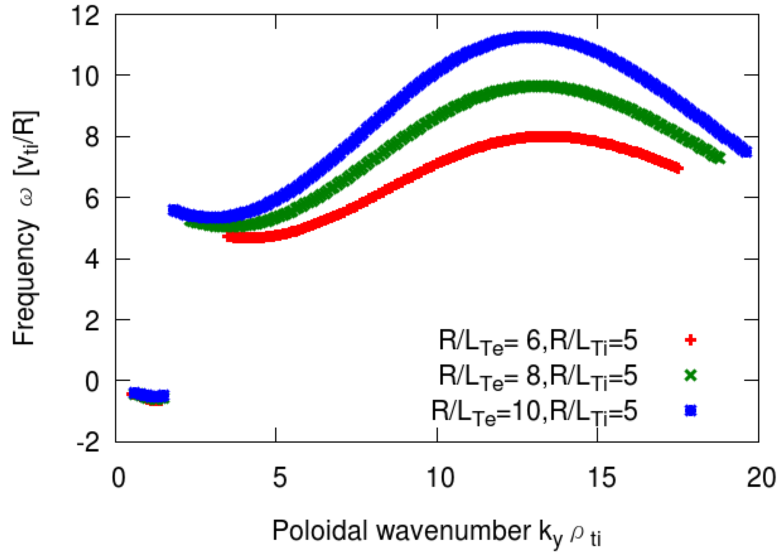
Plasma parameters:

$$\frac{R}{L_n} = 2, \frac{R}{L_{Te}} = 6,8,10, \frac{R}{L_{Ti}} = 5, \frac{m_i}{m_e} = 100, \frac{T_e}{T_i} = 1, Rk_\parallel = 0.5$$

Linear growthrate



Real frequency



Examples

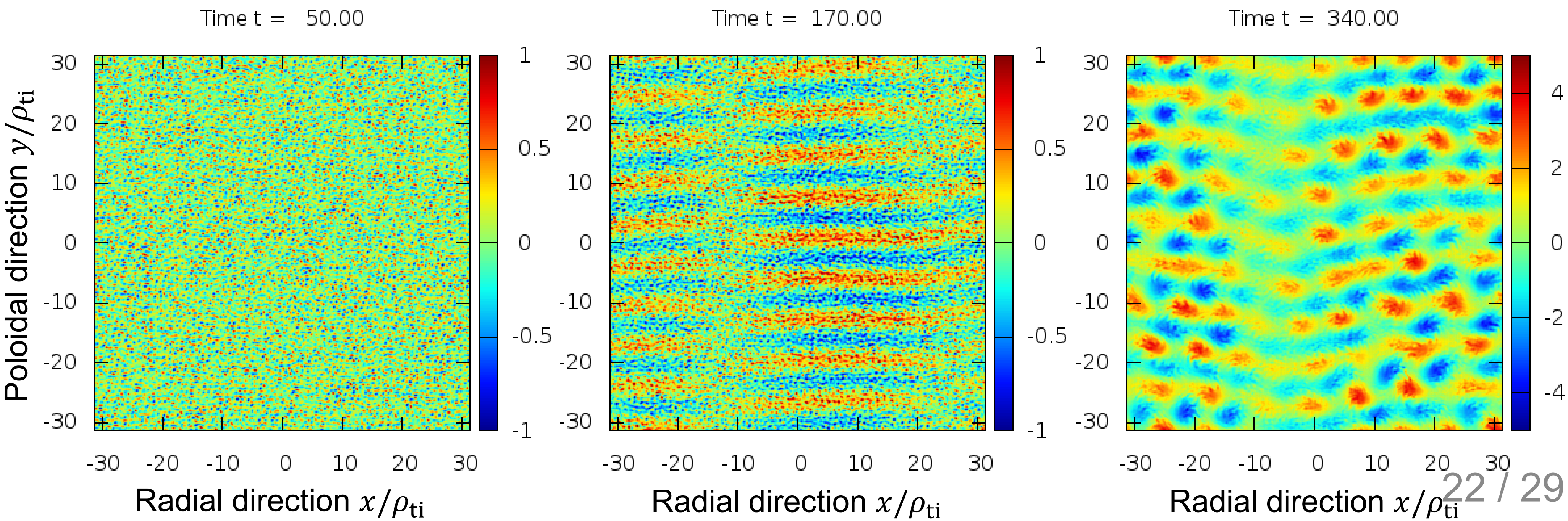
Box sizes & resolution:

$$\left(L_x, L_y, L_{v_\parallel}\right) = (20\pi\rho_{ti}, 20\pi\rho_{ti}, 4.5v_{ts}), \left(N_x, N_y, N_{v_\parallel}\right) = (1024{,}1024{,}96)$$

Plasma parameters:

$$\frac{R}{L_n} = 2, \frac{R}{L_{Te}} = 6{,}8{,}10, \frac{R}{L_{Ti}} = 5, \frac{m_i}{m_e} = 100, \frac{T_e}{T_i} = 1, Rk_\parallel = 0.5$$

Electrostatic potential in slab ITG/ETG turb. (ITG-dominant $R/L_{Te} = 6$ case)

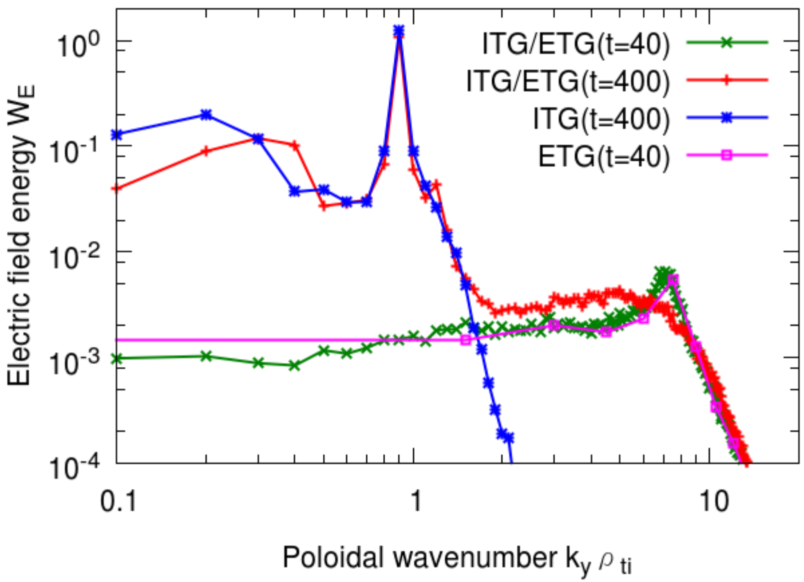


Examples

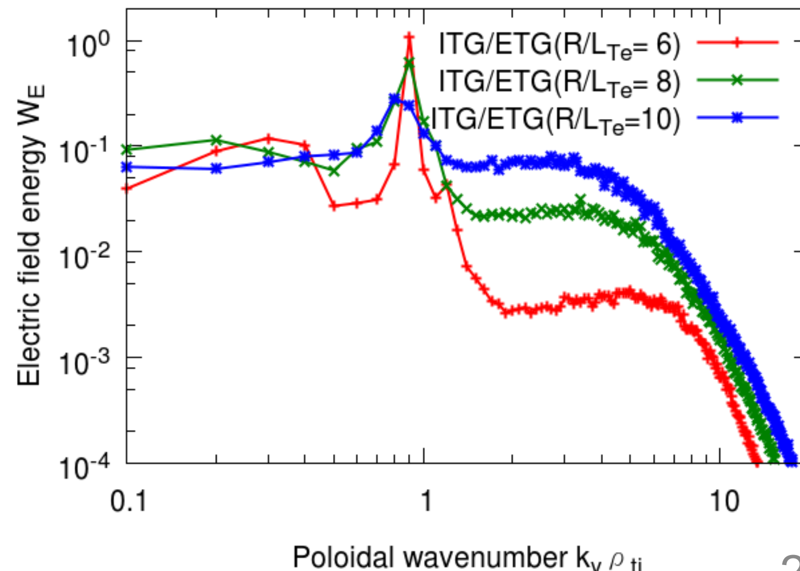
- Suppression of ETG peak when ITG dominates.
- Suppression of ITG as ETG increases.

→ A testbed for extracting and modeling cross-scale interactions.

Field energy spectrum
(ITG-dominant $R/L_{Te} = 6$ case)



Field energy spectrum
($R/L_{Te} = 6, 8, 10$ scan)



A general thinking

Mode coupling appears form ExB nonlinearity,

$$\frac{\partial \tilde{f}_s}{\partial t} = -\{J_{0s}\tilde{\phi}, \tilde{f}_s\}$$

For ions, gyro-phase average $J_{0s} \to \exp\left(-\frac{k_\perp^2 \rho_{ts}^2}{2}\right)$ almost vanishes high-k contributions.

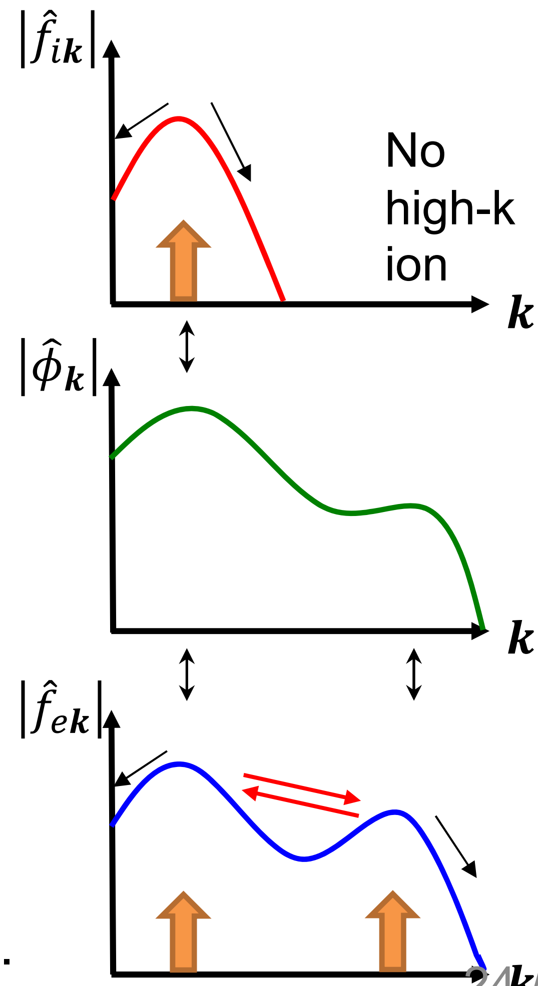
$$\frac{\partial \tilde{f}_i}{\partial t} = -\{J_{0i}\tilde{\phi}_{\Omega_l}, \tilde{f}_{i,\Omega_l}\}$$

$$\frac{\partial \tilde{f}_e}{\partial t} = -\{J_{0e}(\tilde{\phi}_{\Omega_l} + \tilde{\phi}_{\Omega_h}), \tilde{f}_{e,\Omega_l} + \tilde{f}_{e,\Omega_h}\}$$

Trivial consequences:

- Cross-scale interactions directly modify low-k and high-k electrons $\tilde{f}_{e,\Omega_l}, \tilde{f}_{e,\Omega_h}$.
- Through Poisson eq., low-k $\tilde{f}_{e,\Omega_l}$ changes $\tilde{\phi}_{\Omega_l}$ and affects low-k ions $\tilde{f}_{i,\Omega_l}$.

We will here analyze contributions to low-k electrons.



Extracting and modeling cross-scale interactions

$$\frac{\partial \tilde{f}}{\partial t} = -\{\tilde{\phi}, \tilde{f}\}$$

Separating low-k and high-k components, $\tilde{f} = \tilde{f}_{\Omega_l} + \tilde{f}_{\Omega_h}, \tilde{\phi} = \tilde{\phi}_{\Omega_l} + \tilde{\phi}_{\Omega_h}$,

$$\frac{\partial \tilde{f}_{\Omega_l}}{\partial t} = -\{\tilde{\phi}_{\Omega_l}, \tilde{f}_{\Omega_l}\}_{\Omega_l} + N_{\Omega_l}^{\Omega_h} \quad \left(\text{where } N_{\Omega_l}^{\Omega_h} = -\{\tilde{\phi}_{\Omega_h}, \tilde{f}_{\Omega_l}\}_{\Omega_l} - \{\tilde{\phi}_{\Omega_l}, \tilde{f}_{\Omega_h}\}_{\Omega_l} - \{\tilde{\phi}_{\Omega_h}, \tilde{f}_{\Omega_h}\}_{\Omega_l}\right)$$

$$\frac{\partial \tilde{f}_{\Omega_h}}{\partial t} = -\{\tilde{\phi}_{\Omega_h}, \tilde{f}_{\Omega_h}\}_{\Omega_h} + N_{\Omega_h}^{\Omega_l} \quad \left(\text{where } N_{\Omega_h}^{\Omega_l} = -\{\tilde{\phi}_{\Omega_l}, \tilde{f}_{\Omega_l}\}_{\Omega_h} - \{\tilde{\phi}_{\Omega_h}, \tilde{f}_{\Omega_l}\}_{\Omega_h} - \{\tilde{\phi}_{\Omega_l}, \tilde{f}_{\Omega_h}\}_{\Omega_h}\right)$$

Cross-scale effects from high-k to low-k $N_{\Omega_l}^{\Omega_h}$ (v.v. $N_{\Omega_h}^{\Omega_l}$) can be modeled as [Itoh'01PPCF]

$$\frac{\partial \tilde{f}_{\Omega_l}}{\partial t} = -\{\tilde{\phi}_{\Omega_l}, \tilde{f}_{\Omega_l}\}_{\Omega_l} - \gamma_l^h \tilde{f}_{\Omega_l} + \widetilde{w}_l^h$$

$$\frac{\partial \tilde{f}_{\Omega_h}}{\partial t} = -\{\tilde{\phi}_{\Omega_h}, \tilde{f}_{\Omega_h}\}_{\Omega_h} + D_h^l \tilde{f}_{\Omega_h}$$

where

$\gamma_l^h(k_x, k_y, v_\parallel; \tilde{f}_h)$: Coherent turbulent drag (from high-k to low-k)

$\widetilde{w}_l^h(k_x, k_y, v_\parallel, \tilde{f}_h)$: Random noise (from high-k to low-k)

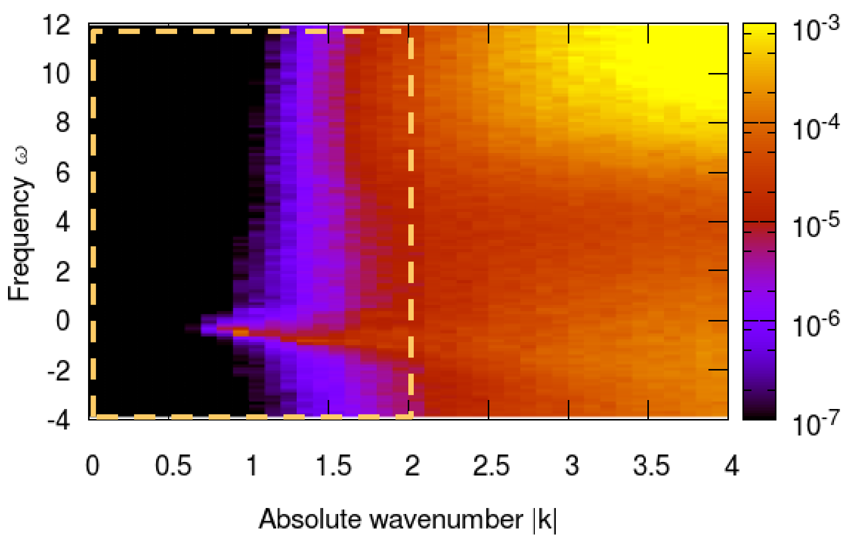
$D_l^h(k_x, k_y, v_\parallel, \tilde{f}_l)$: Mean advection and profile modification (from low-k to high-k)

Spectrum of $N_{\Omega_l}^{\Omega_h}(x,y,t) = \sum_k \sum_\omega \hat{N}_{\Omega_l}^{\Omega_h}(k_x,k_y,\omega) e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}$

- Cross-scale effect from high-k to low-k $N_{\Omega_l}^{\Omega_h}$ seems to consists of a coherent part having $\omega \sim \omega_{*i}$ and a zero-mean noise with finite deviation/correlation.
- Modeled as $N_{\Omega_l}^{\Omega_h} \simeq -\gamma_l^h \tilde{f}_{\Omega_l} + \tilde{w}_l^h$?

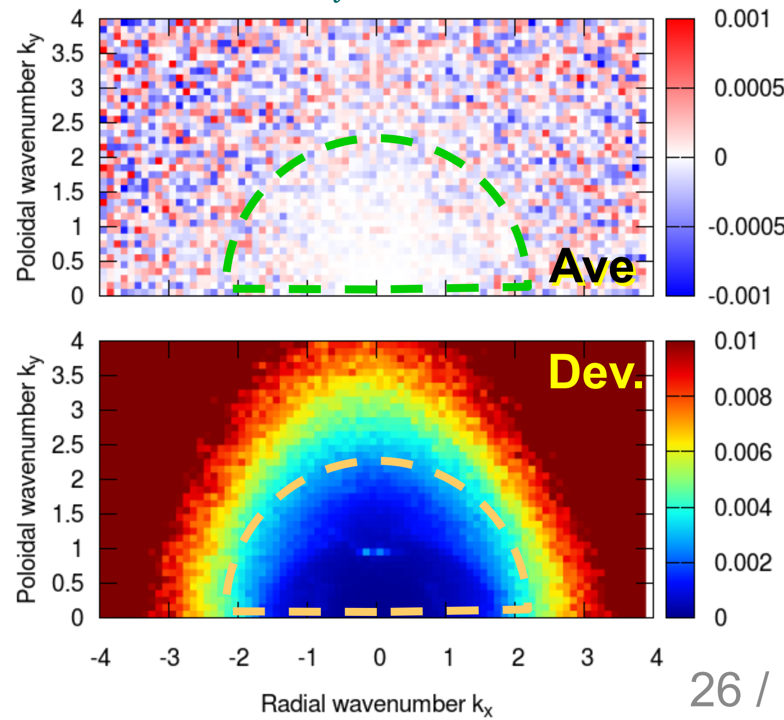
Wavenumber-frequency spectrum

[$\left|\hat{N}_{\Omega_l}^{\Omega_h}(k_x,k_y,\omega)\right|^2$ integrated over angle in k]



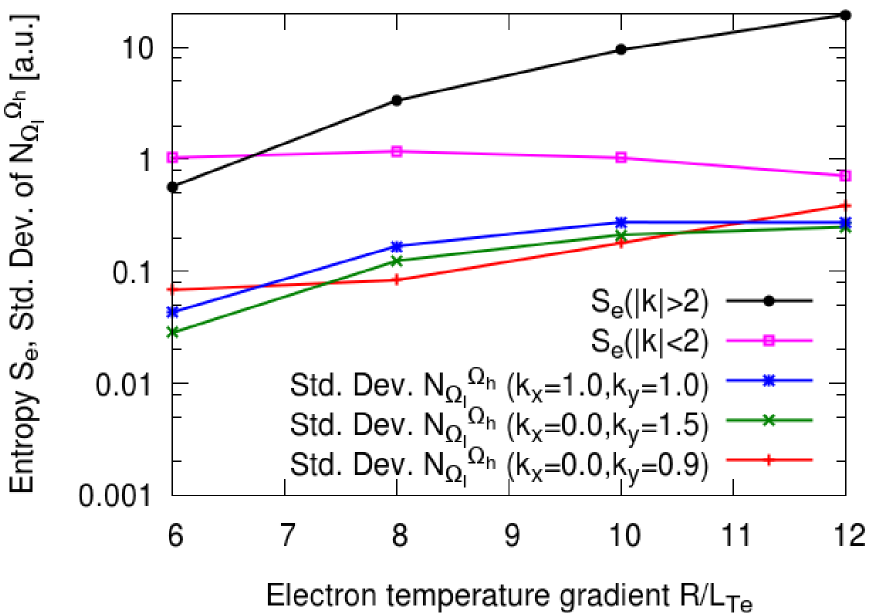
Average and Standard deviation

[Real part of $\hat{N}_{\Omega_l}^{\Omega_h}(k_x,k_y,t)$]

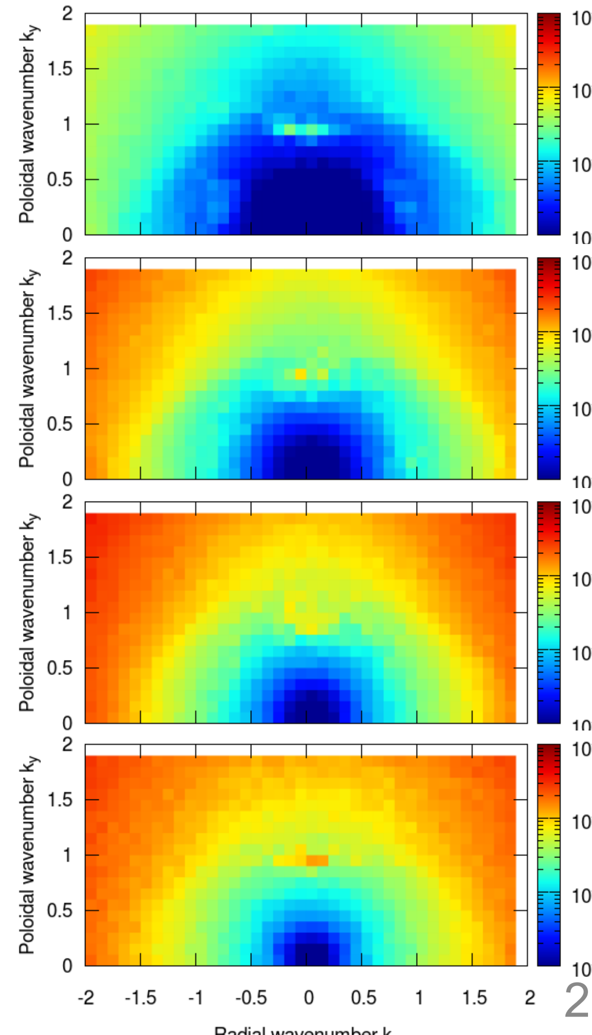


Spectrum of $N_{\Omega_l}^{\Omega_h}$

- Standard deviation of $N_{\Omega_l}^{\Omega_h}$ is roughly proportional to electron-scale entropy S_{e,Ω_h}.
- Modeled as $N_{\Omega_l}^{\Omega_h} \simeq -\gamma_l^h \tilde{f}_{\Omega_l} + \tilde{w}_l^h$, with $\gamma_l^h \propto S_{e,\Omega_h}$ and $\langle \tilde{w}_l^h \tilde{w}_l^h \rangle^{1/2} \propto S_{e,\Omega_h}$?



Standard deviation of $\hat{N}_{\Omega_l}^{\Omega_h}$



$R/L_{Te} = 6$

$R/L_{Te} = 8$

$R/L_{Te} = 10$

$R/L_{Te} = 12$

Discussion

I am trying to extract cross-scale interactions by using a toy model, i.e., slab ITG/ETG turbulence, and testing a possibility of Langevin-type modeling.

✓ From our experiences [Maeyama'15PRL; Maeyama'17PRL], different-scale turbulence tend to be mutually exclusive.

✓ Contribution from high-k to low-k seems to consist of a coherent part having $\omega \simeq \omega_{ITG}$ and a zero-mean noise with finite deviation $\propto S_{e,\Omega_h}$.
  - Modeled by coherent drag and random forcing from high-k to low-k?

✓ There are finite forward/inverse entropy cascades satisfying conservation.
  - Modeled by any other compensation term?

☐ How about the contribution from low-k to high-k?
  - Modeled by mean advection and profile modification?

Thank you for your attention.

I'd appreciate further discussion !

- Symmetrized entropy transfer
- ITG/ETG turbulence (Short-wavelength ITG eddies distort ETG. ETG damps short-wavelength zonal flows.) [Maeyama'15PRL; 17NF]
- MTM/ETG turbulence (ETG destroys radially localized current sheets of MTM.) [Maeyama'17PRL]
- Extracting/modeling cross-scale interactions