

Ion vs electron heating in astrophysical gyrokinetic turbulence

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University of Oxford

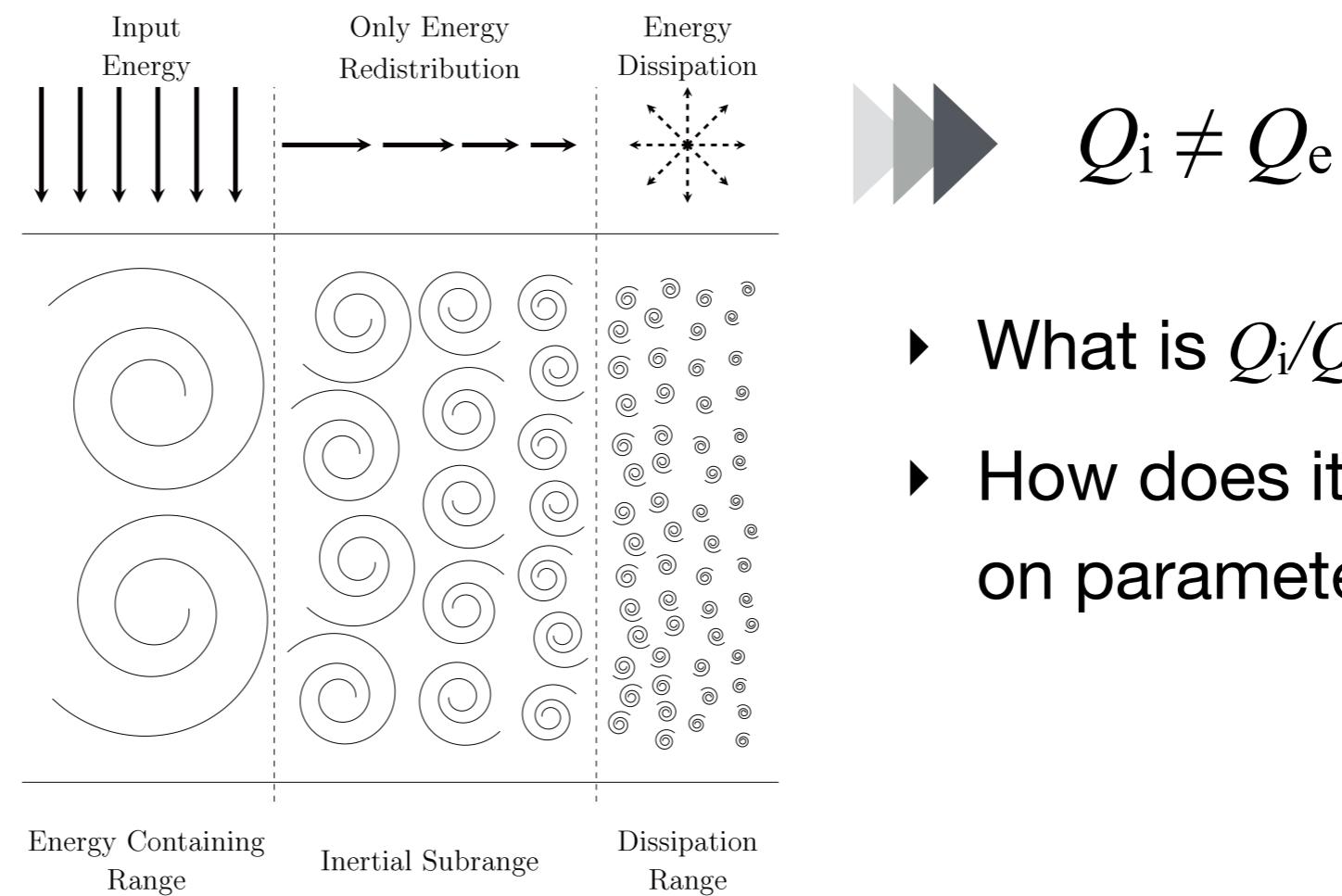
Plasma Kinetics workshop – WPI 2018

Outline

- ✓ Background
- ✓ Ion vs heating ratio
- ✓ k-spectrum ($\beta_i = 0.1$ vs 100)
- ✓ phase space-spectrum ($\beta_i = 0.1$ vs 100)
- ✓ summary
- ✓ Ultrahigh beta ($\beta_i = 10000$) simulation

Turbulent heating in collisionless plasma

- ✓ Large scale free energy source (e.g., Keplerian shear) drives turbulence
- ✓ Turbulence dissipates at small scales
- ✓ Unequal distribution of heating between ions and electrons



- ▶ What is Q_i/Q_e ?
- ▶ How does it depend on parameters?

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- ✓ Turbulence dissipates at small scales
- ✓ Unequal distribution of heating between ions and electrons
- ✓ Does turbulence prefer equilibration between species?

$$T_i > T_e \rightarrow Q_i < Q_e$$

- ✓ Or pushes toward further disequilibration?

$$T_i > T_e \rightarrow Q_i > Q_e$$

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Turbulent heating in collisionless plasma

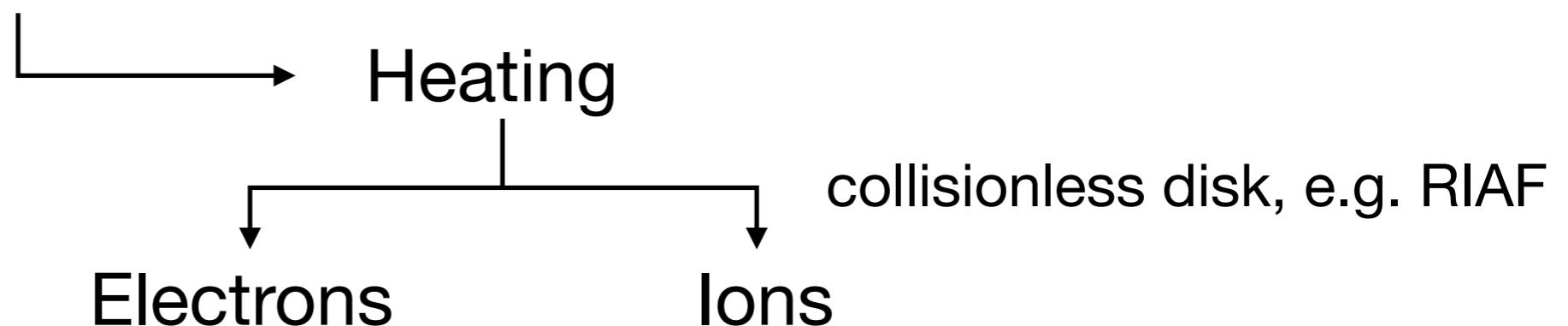
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- ✓ MRI → Turbulence → Momentum transport

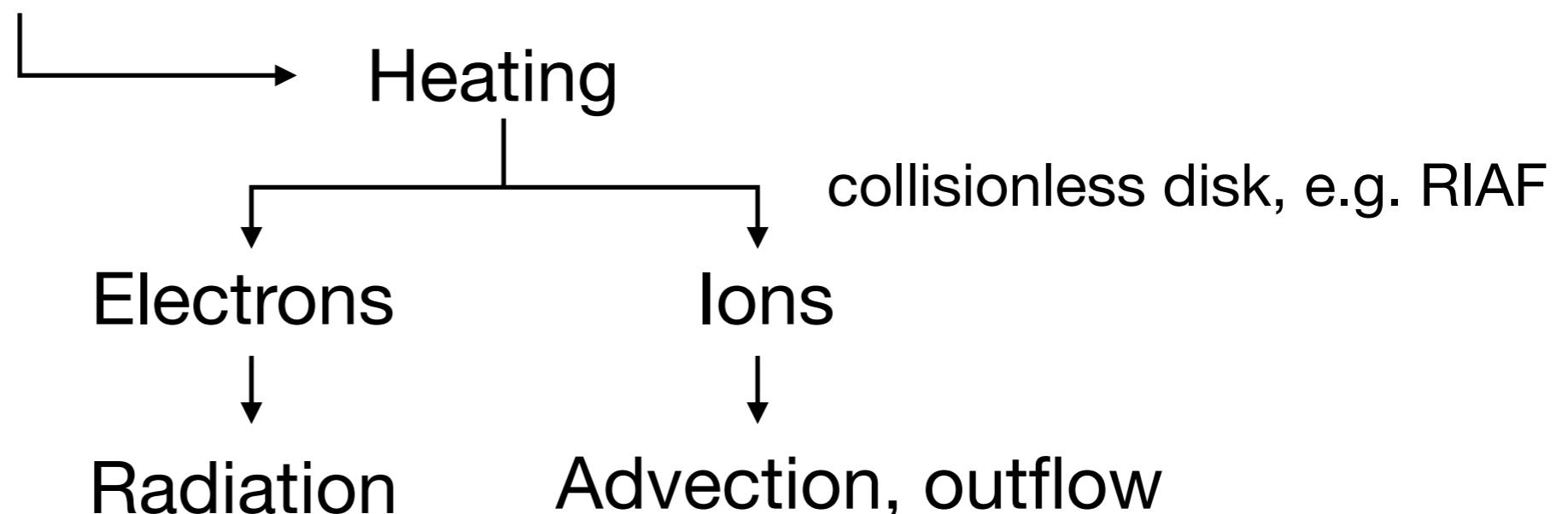
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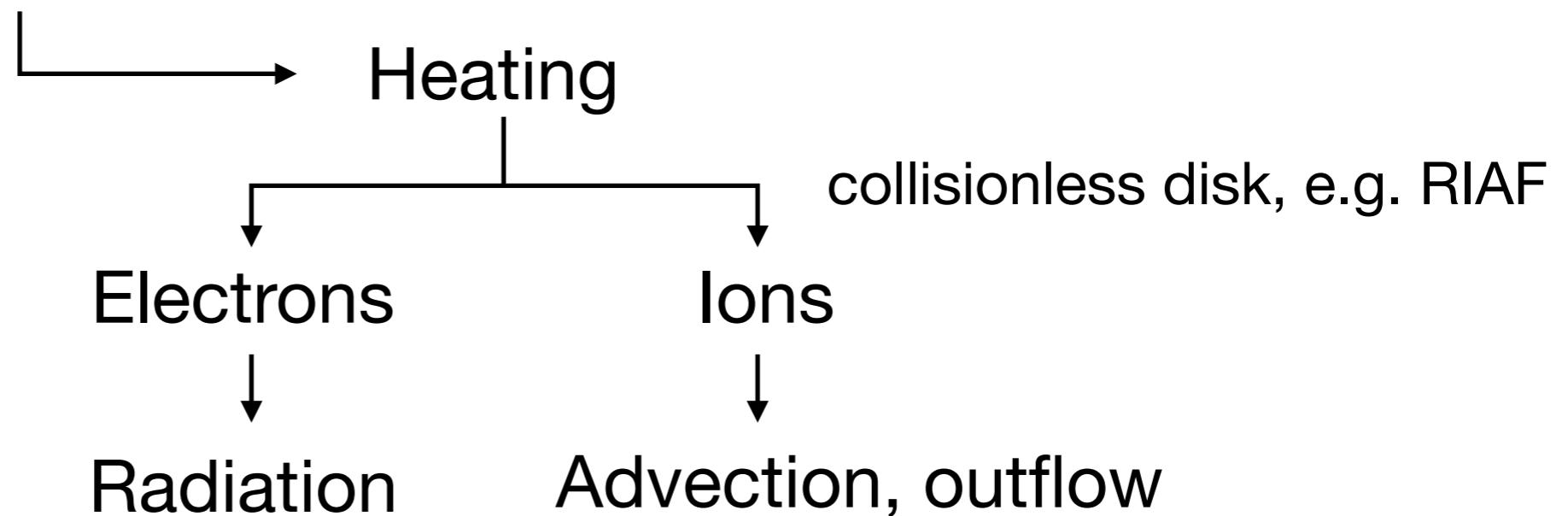
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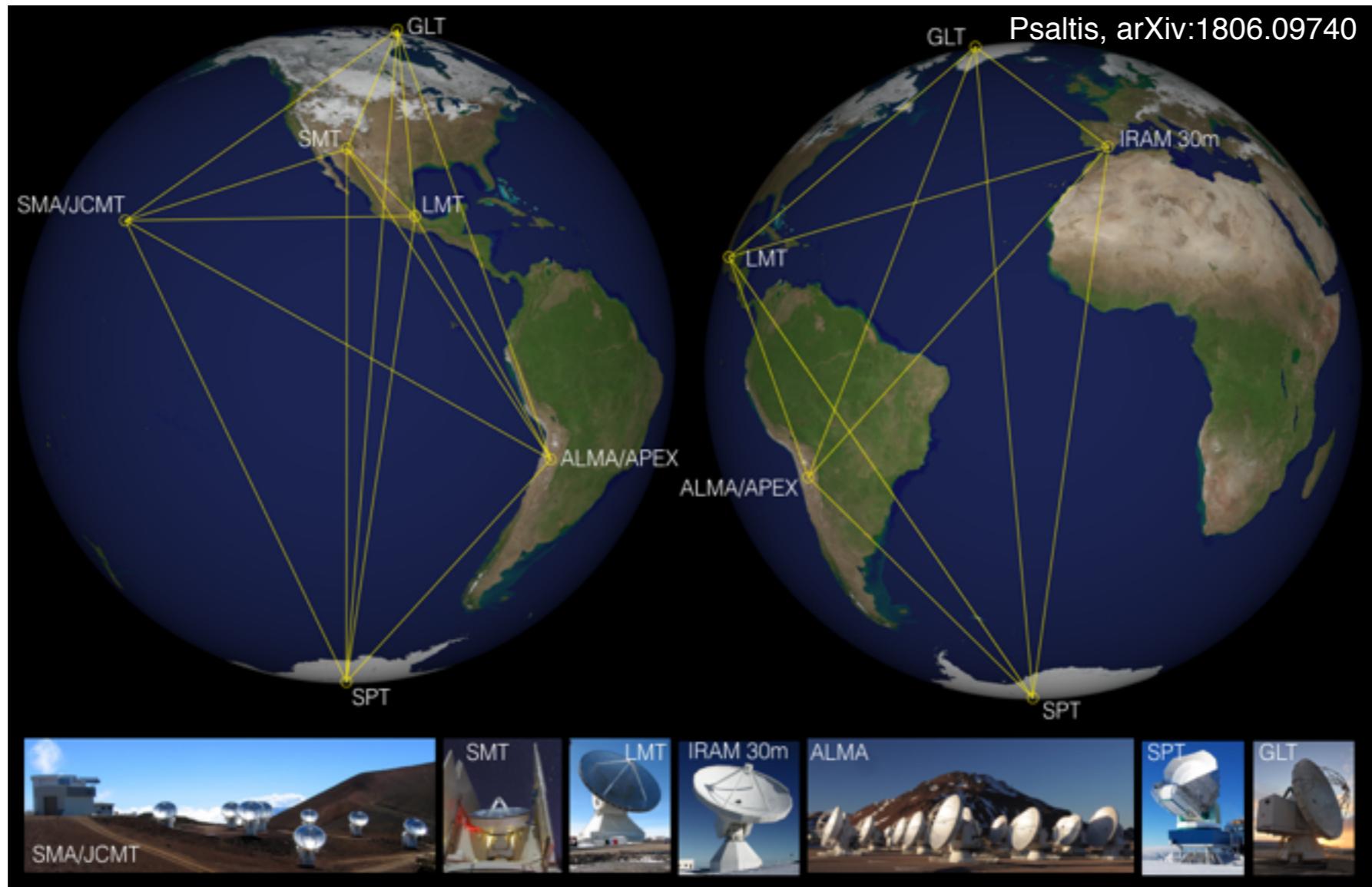


Turbulent heating in collisionless plasma

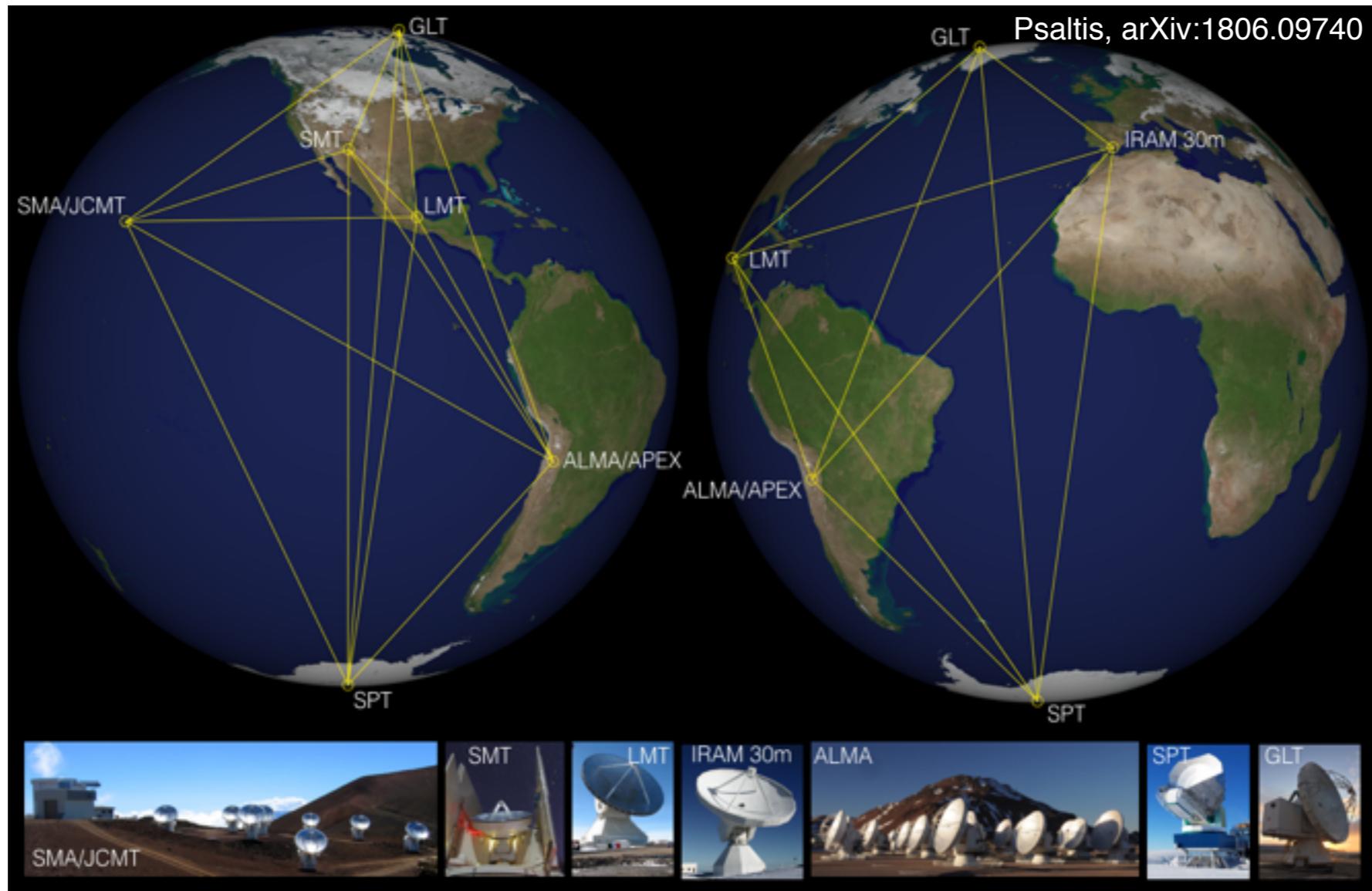
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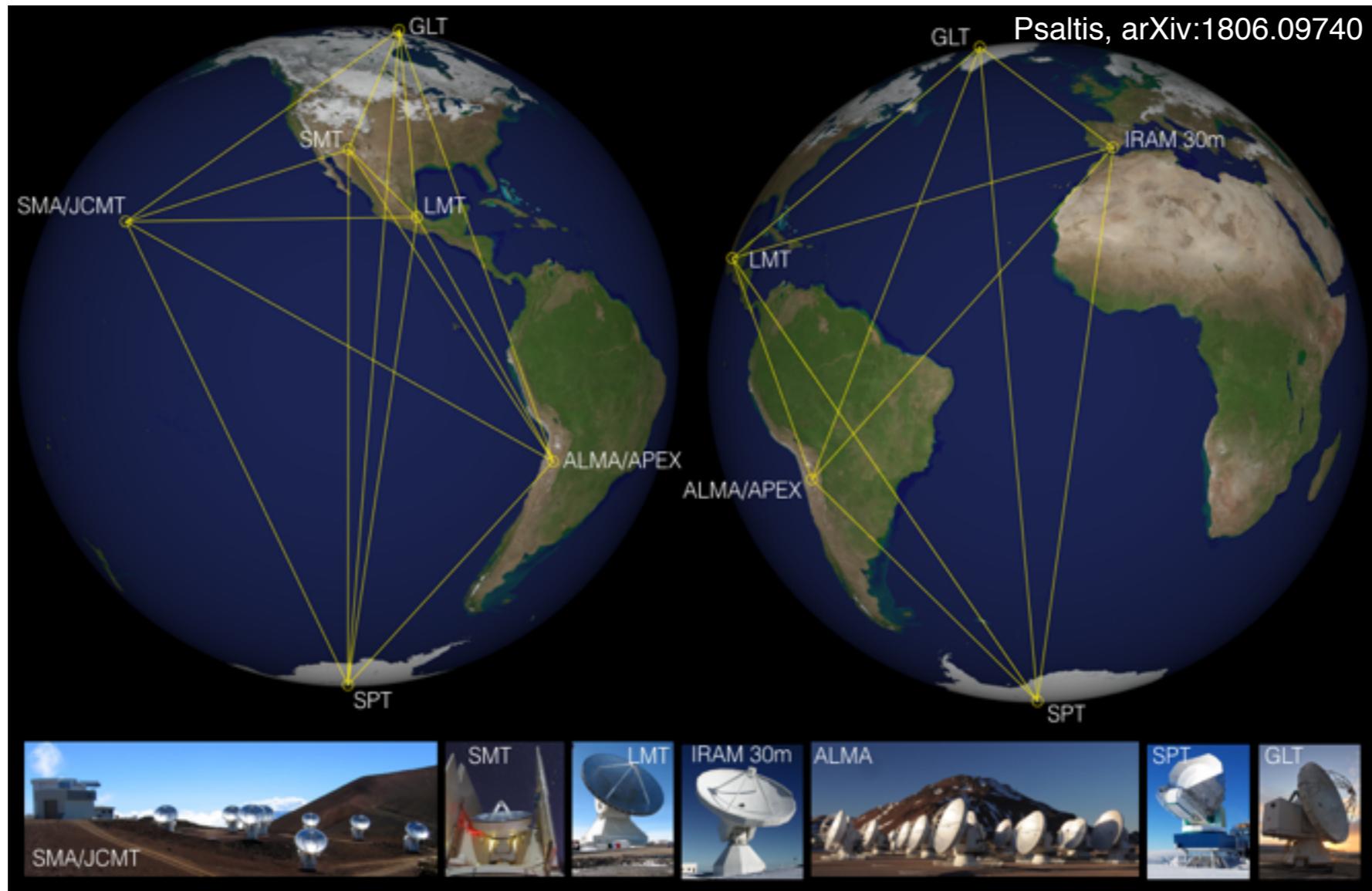
- ✓ To estimate T_i , heating ratio is important



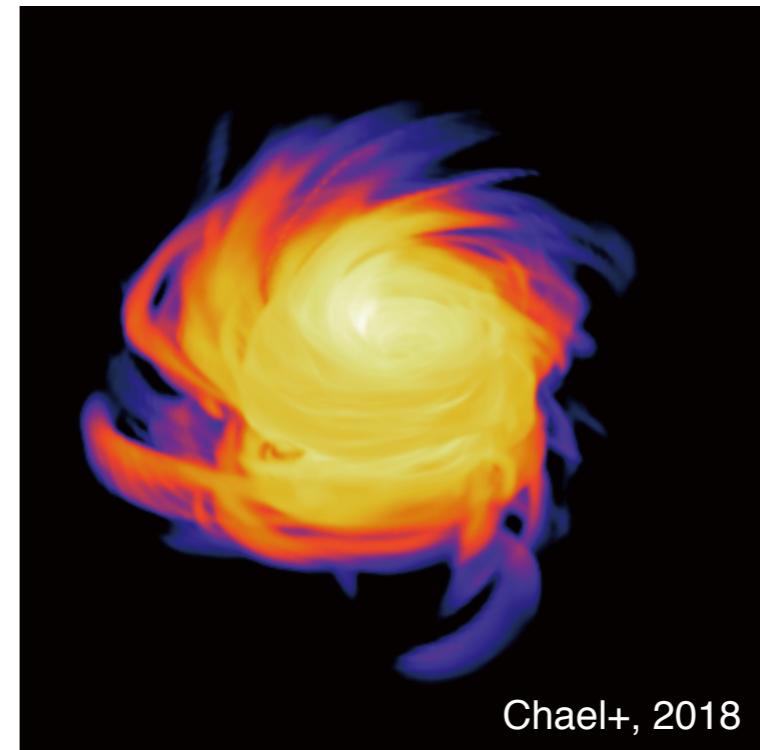
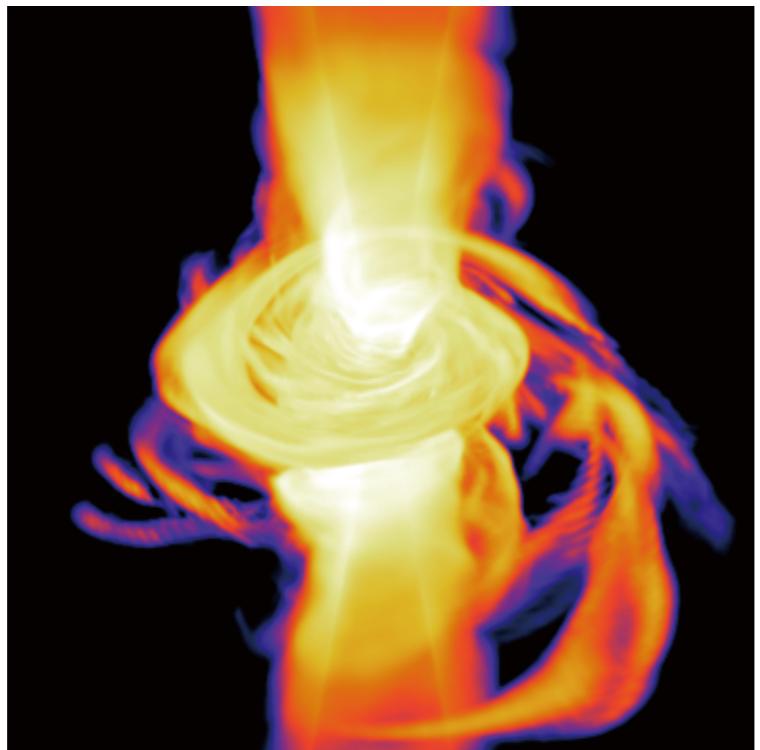
✓ Event Horizon Telescope → Photograph near Sgr A*

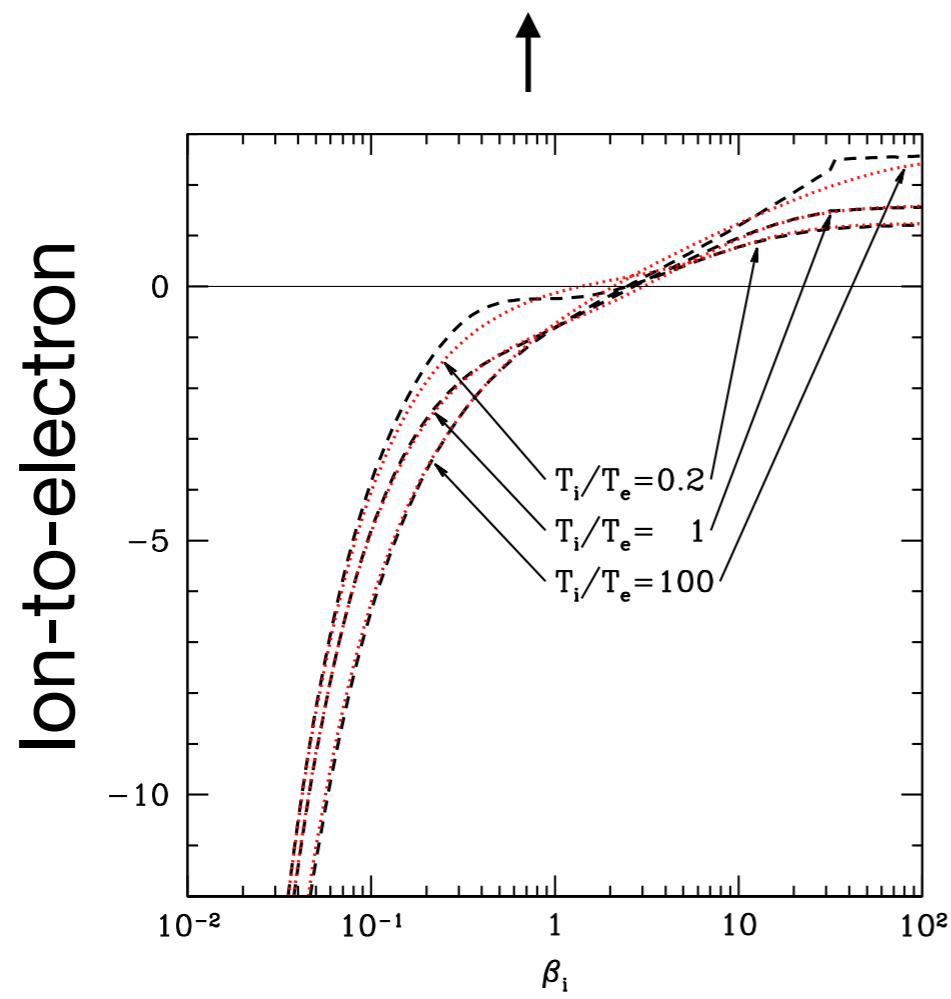
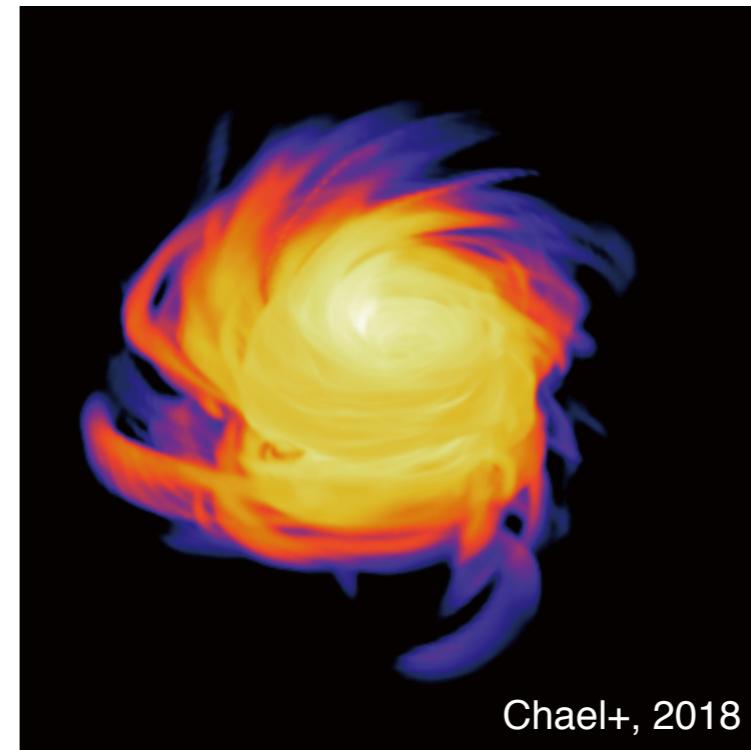
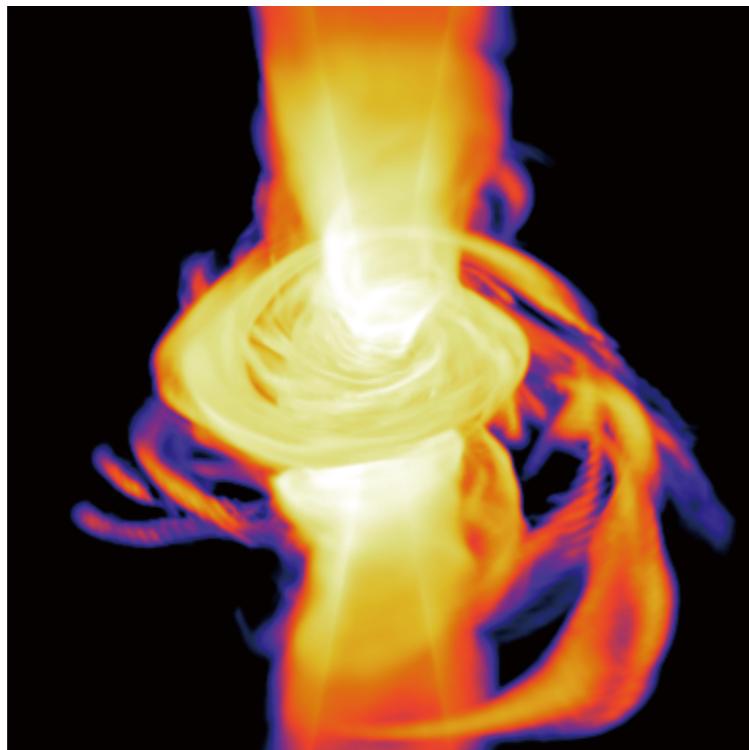


- ✓ Event Horizon Telescope → Photograph near Sgr A*
- ✓ Prediction via GR Radiative MHD simulation

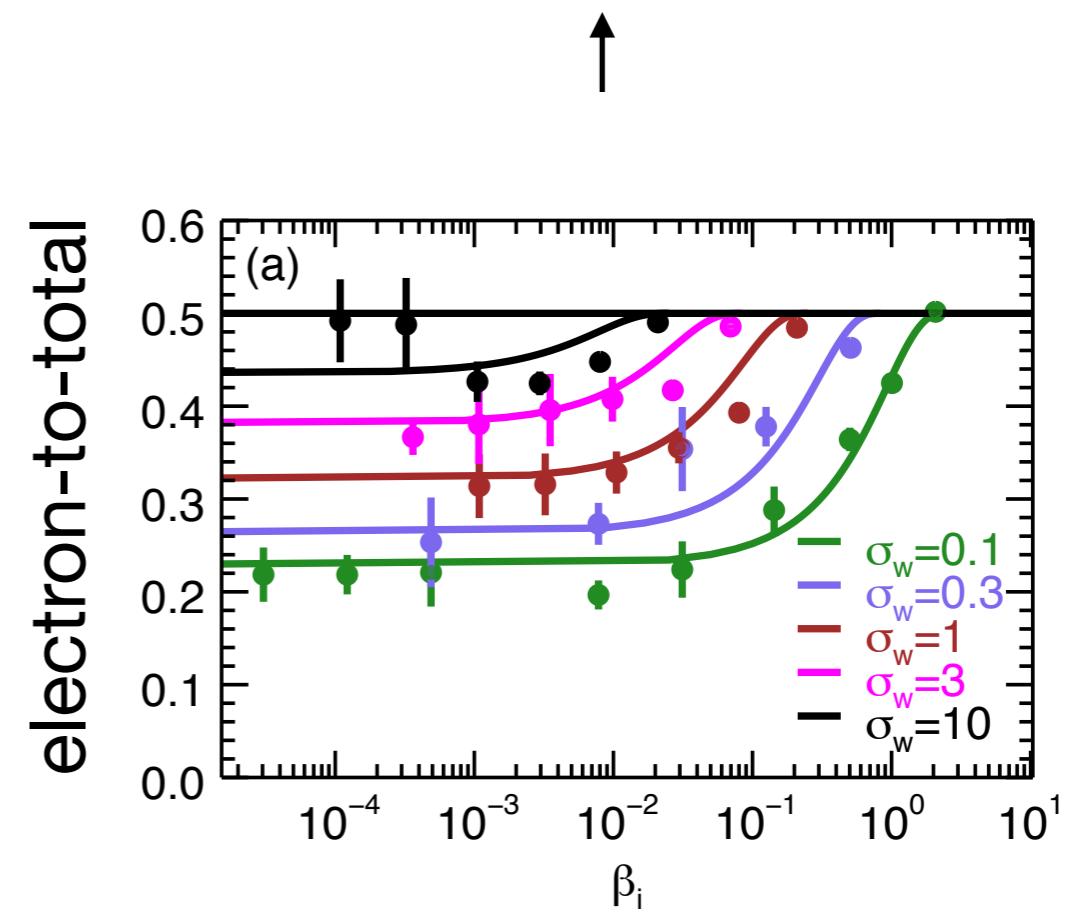


- ✓ Event Horizon Telescope → Photograph near Sgr A*
- ✓ Prediction via GR Radiative MHD simulation
- ✓ Different results depending on the heating prescription

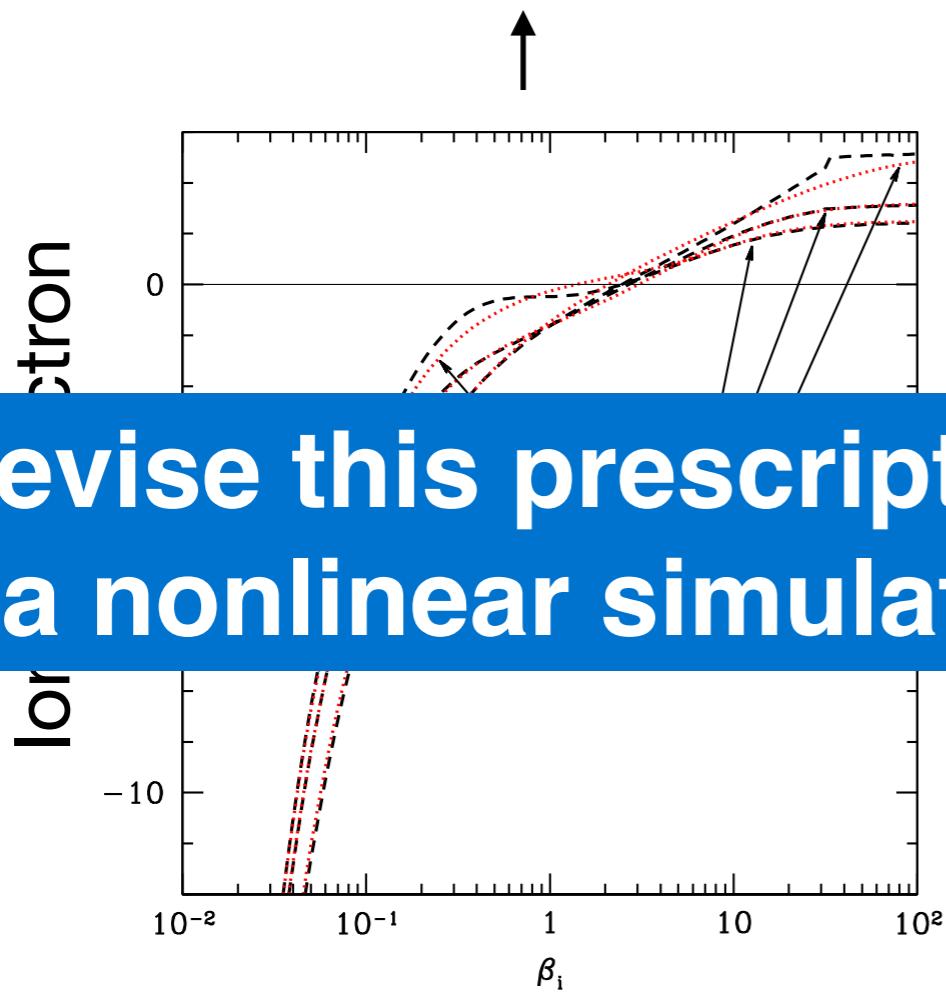
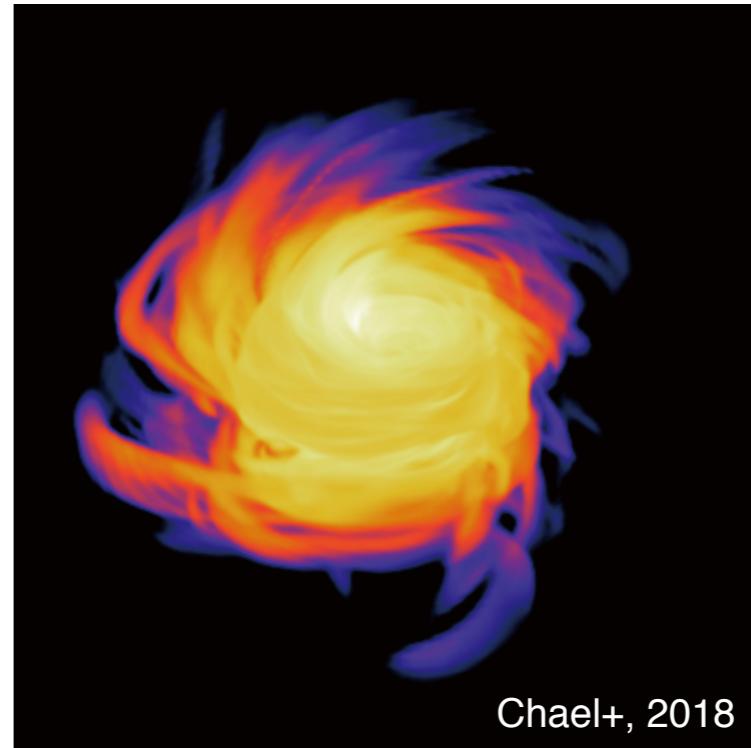
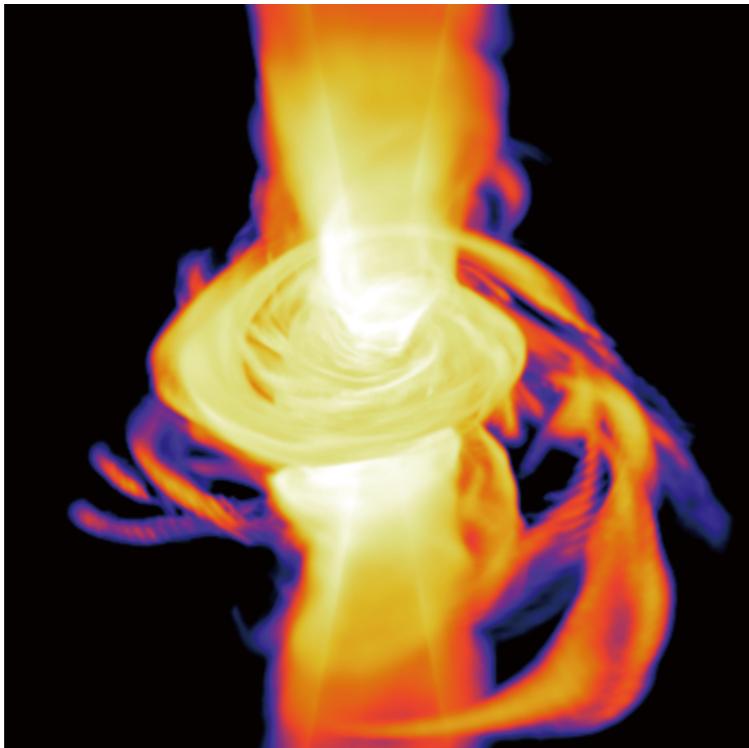




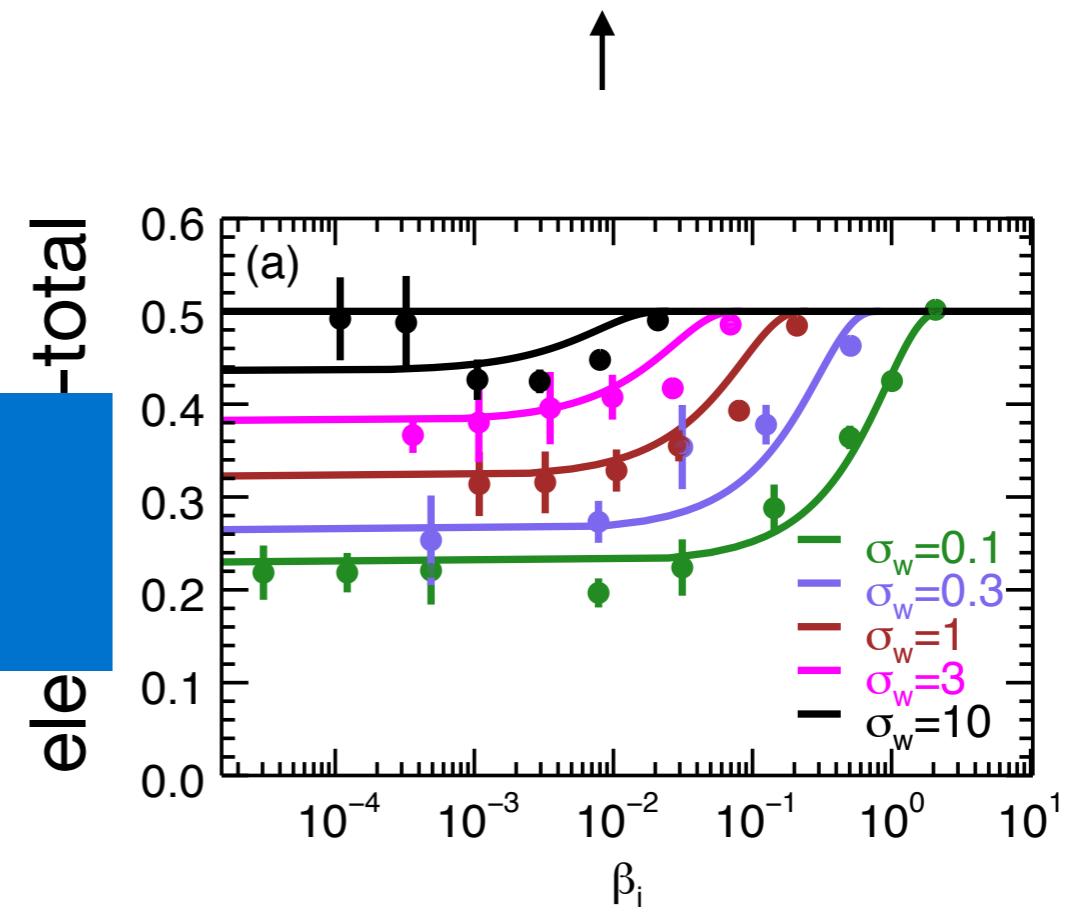
GK turbulence, theory (Howes 2010)



Reconnection, 2D PIC (Rowan+ 2017)

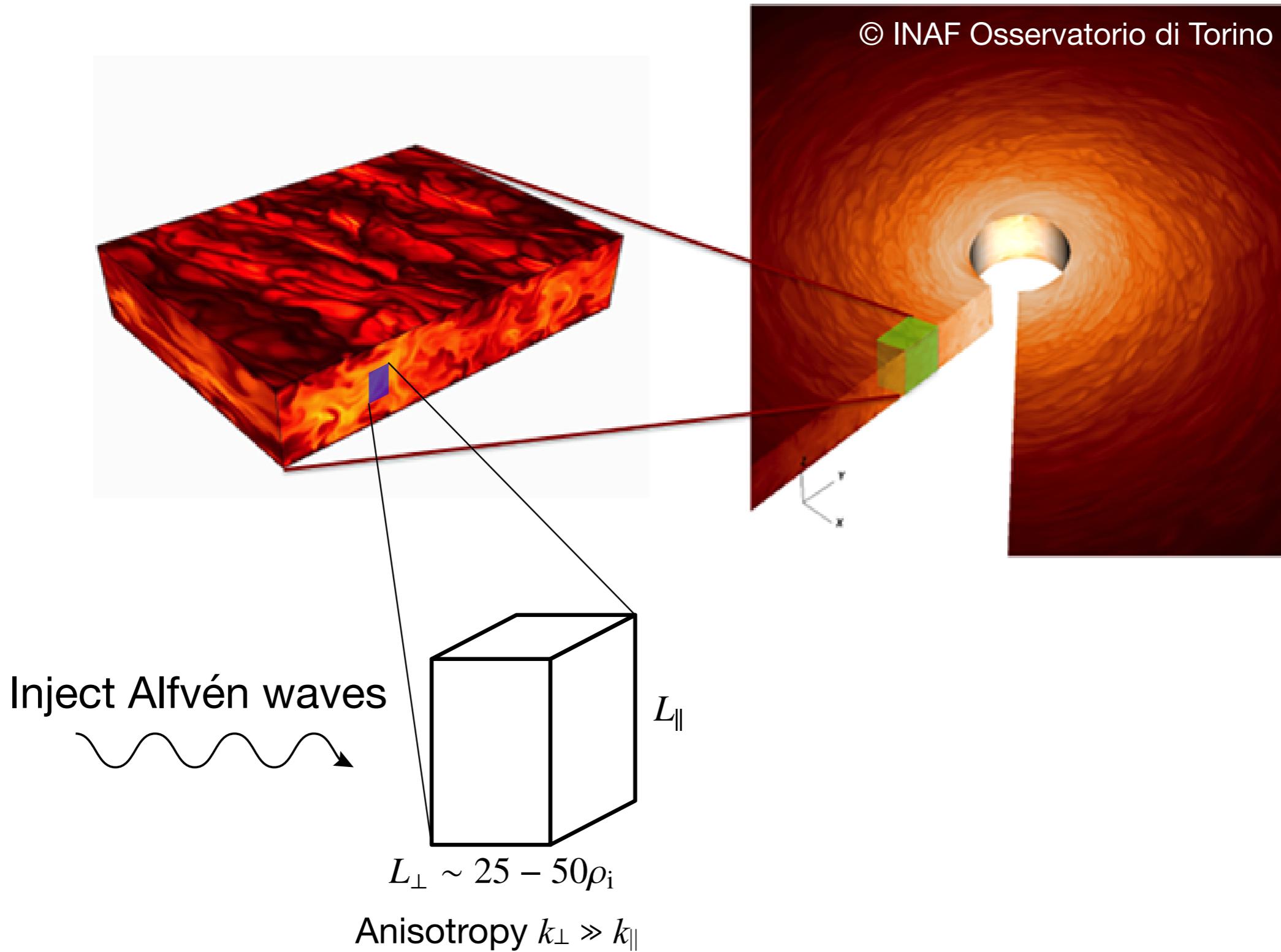


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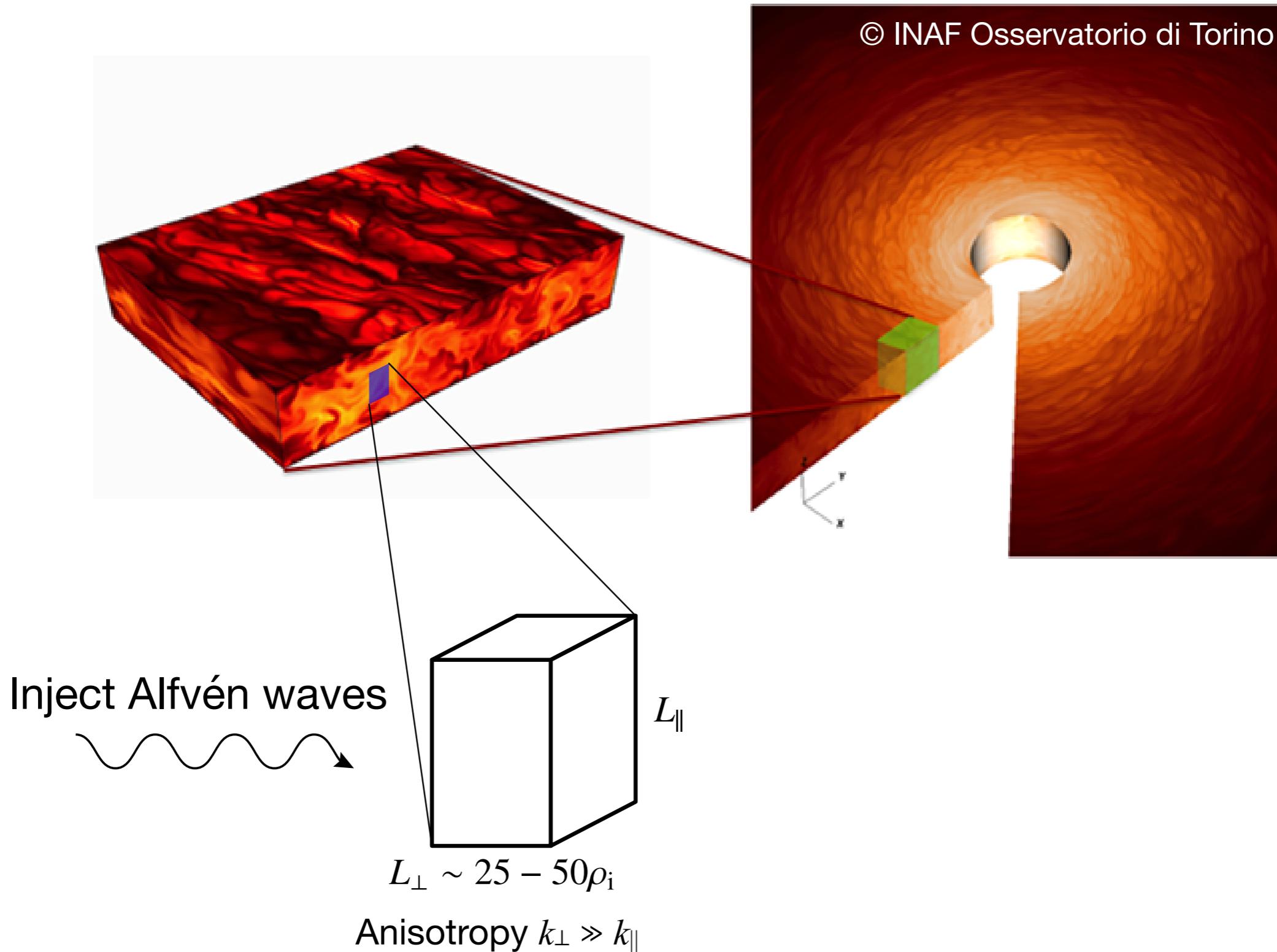


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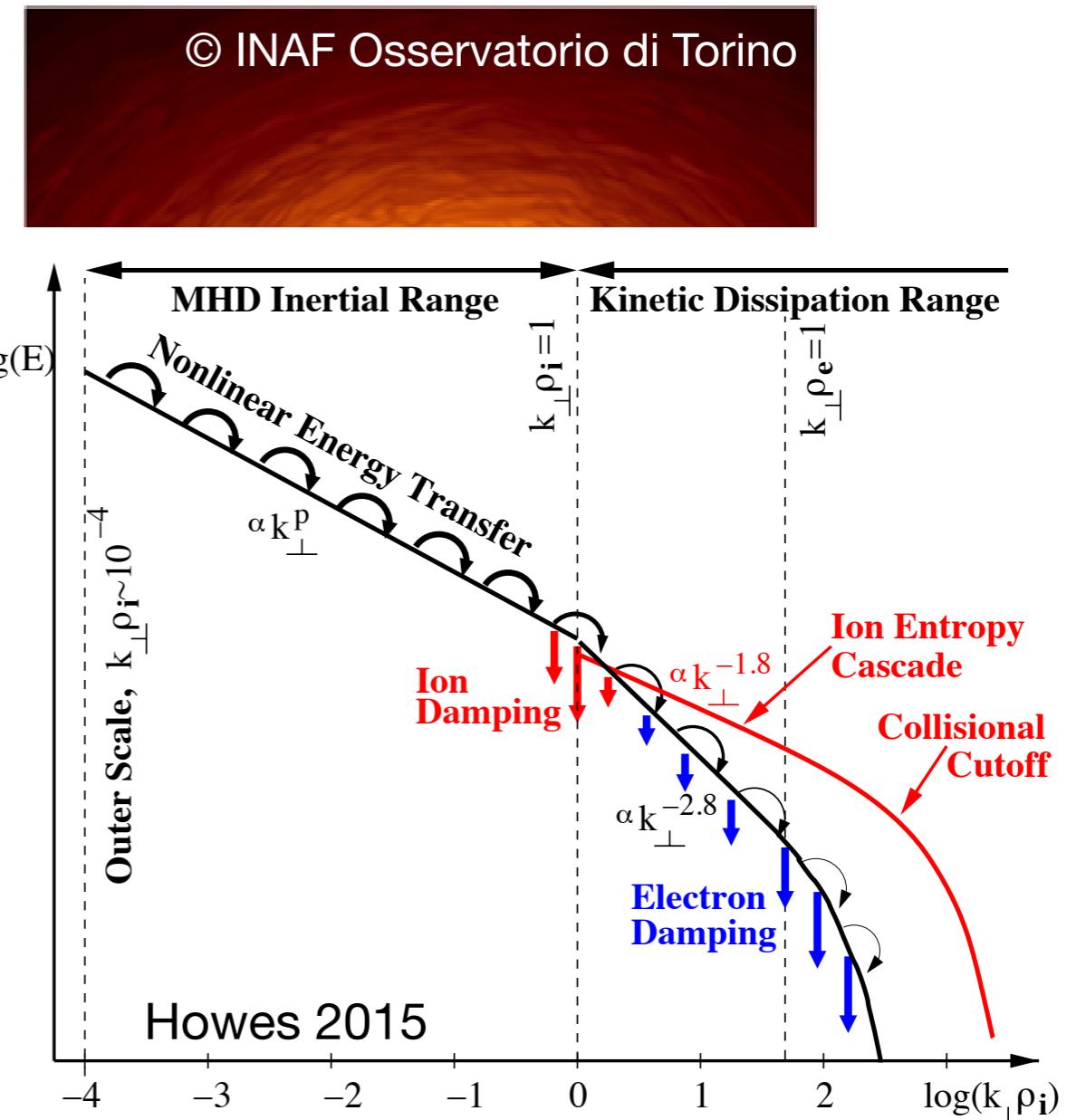
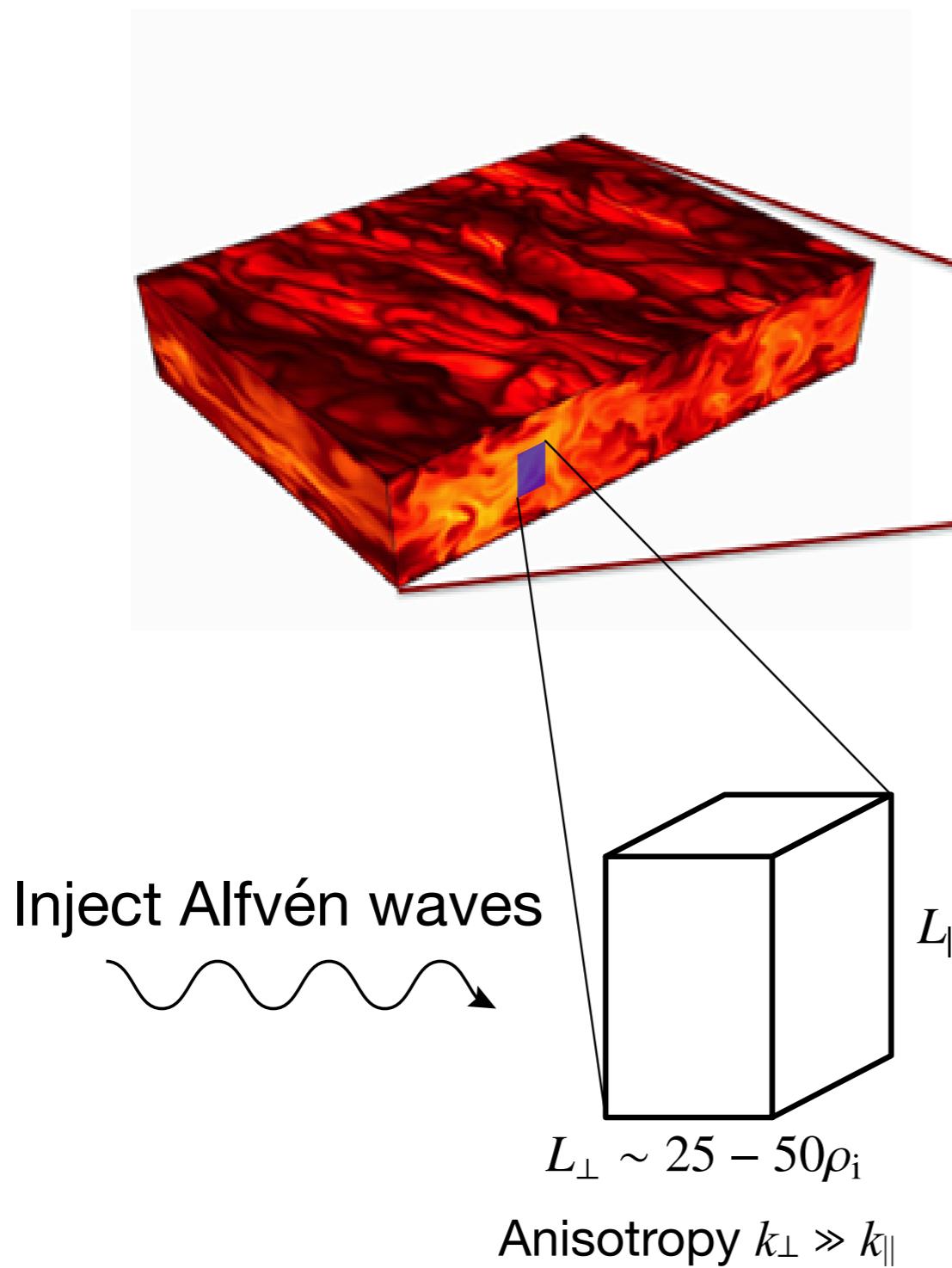
Dissipation scale turbulence in accretion disks



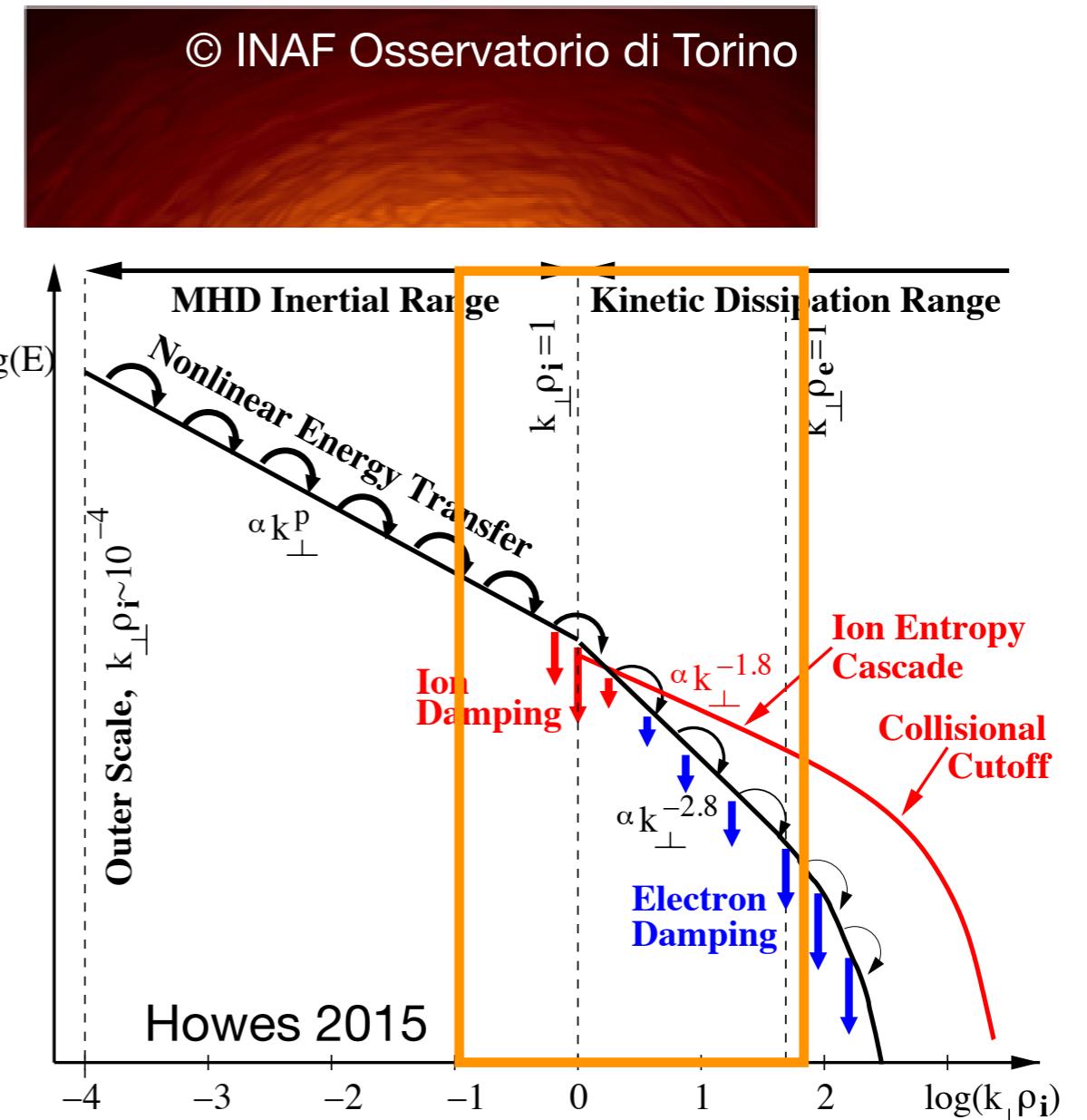
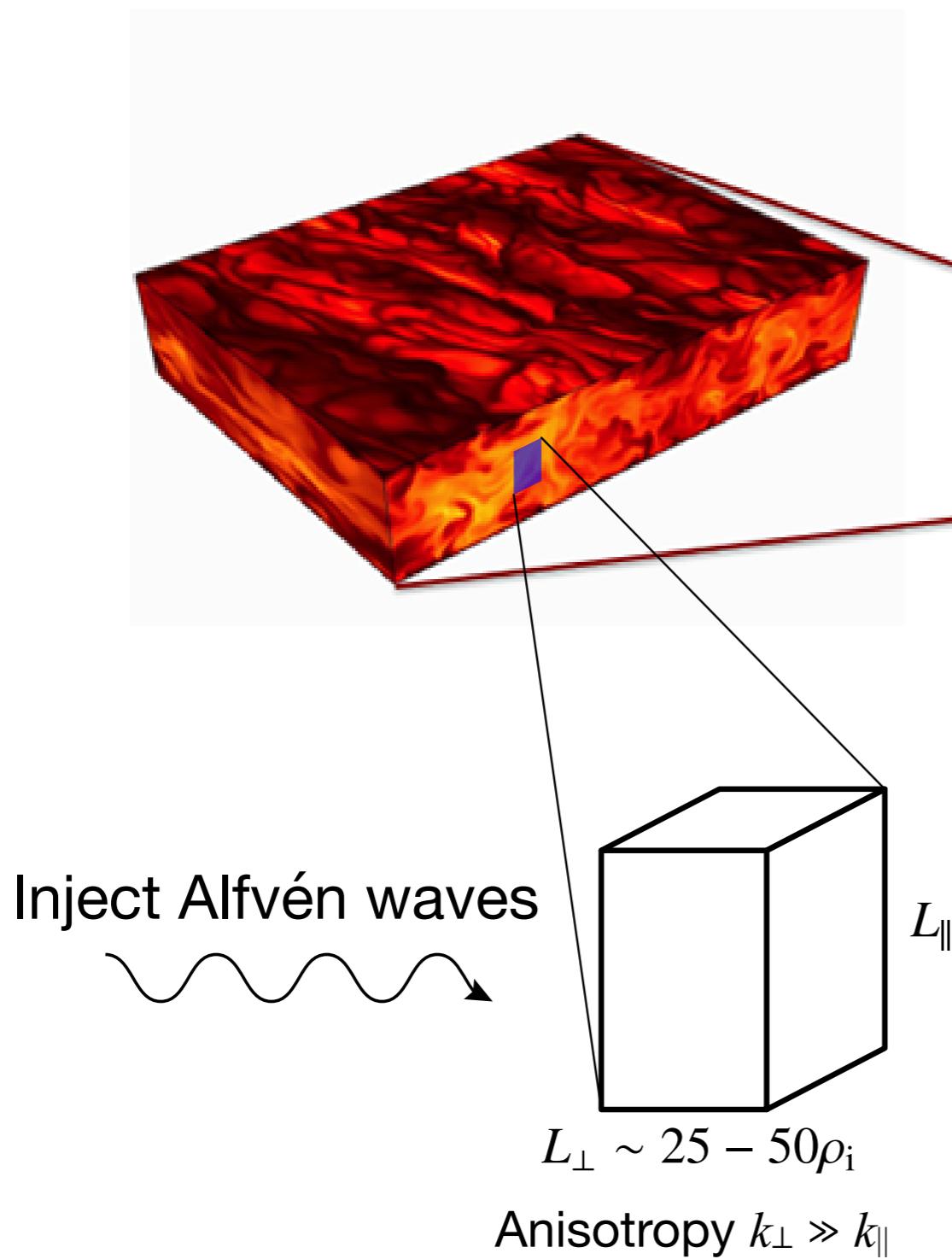
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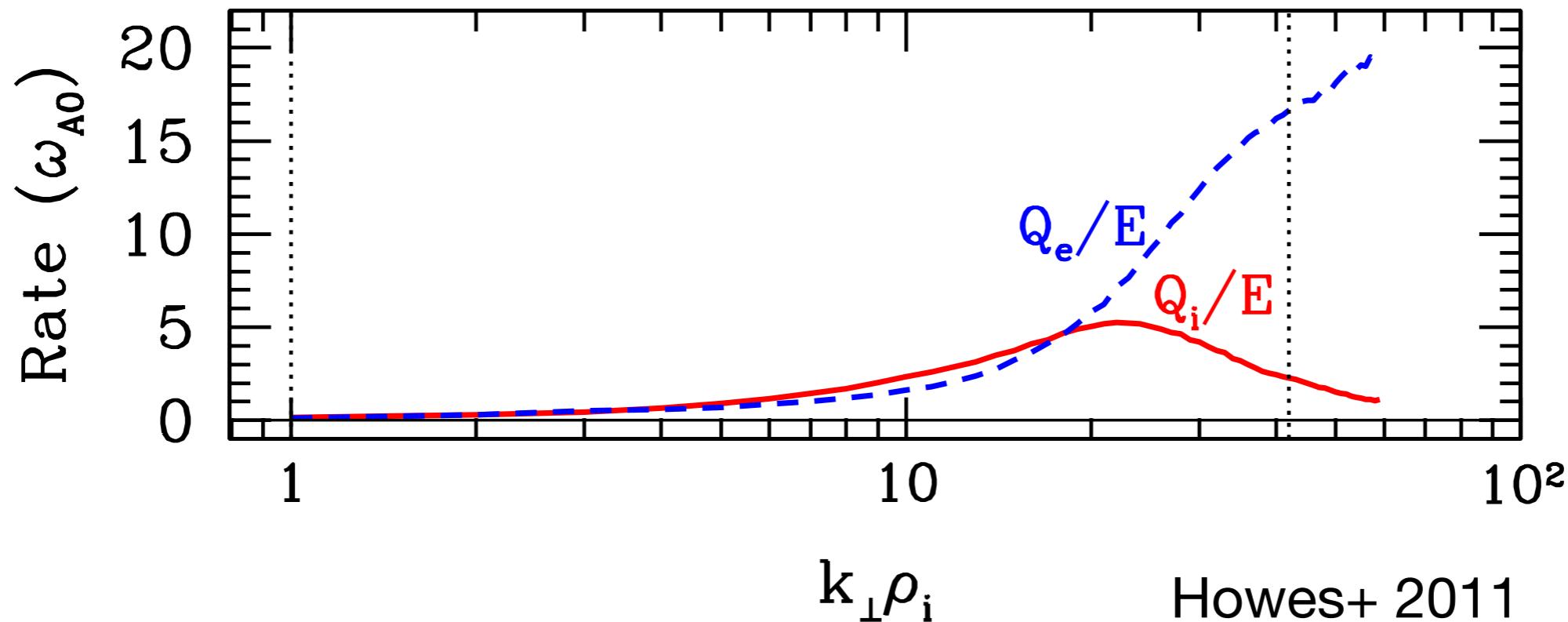


Dissipation scale turbulence in accretion disks



Full GK simulation (ion-electron scales)

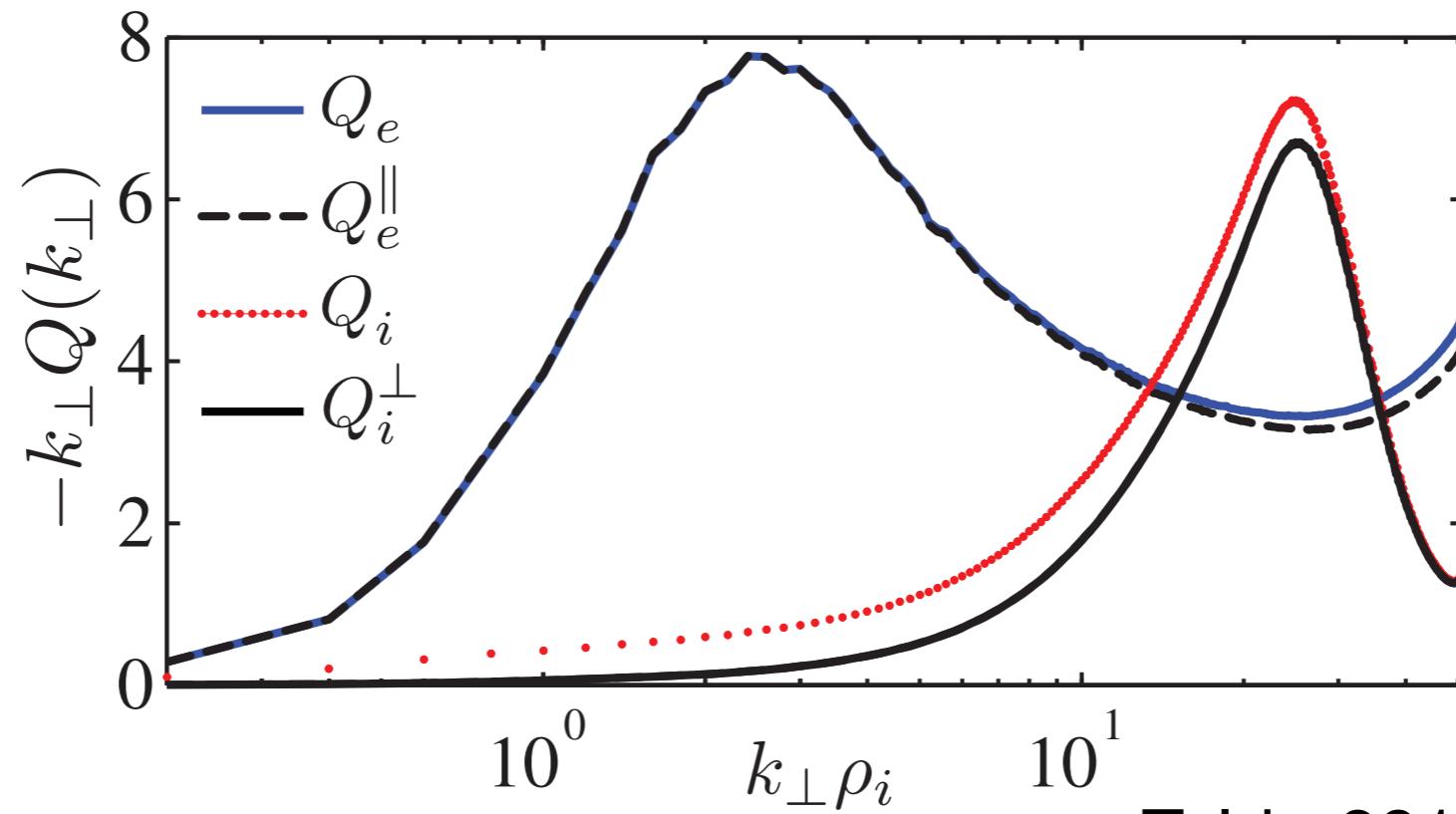
$$\frac{\partial h_s}{\partial t} + v_{\parallel} \frac{\partial h_s}{\partial z} + \frac{c}{B_0} \{ \langle \chi \rangle_{\mathbf{R}_s}, h_s \} = \frac{q_s}{T_s} \frac{\partial \langle \chi \rangle_{\mathbf{R}_s}}{\partial t} F_s + \langle C[h_s] \rangle_{\mathbf{R}_s} \quad \& \text{ Maxwell's eqs}$$



- ✓ For $(\beta_i, T_i/T_e) = (1, 1)$
- ✓ $Q_e > Q_i$
- ✓ Q_i peaks at electron scale ← nonlinear phase mixing
(Tatsuno+ 2009)

Full GK simulation (MHD-ion-electron scales)

$$\frac{\partial h_s}{\partial t} + v_{\parallel} \frac{\partial h_s}{\partial z} + \frac{c}{B_0} \{ \langle \chi \rangle_{\mathbf{R}_s}, h_s \} = \frac{q_s}{T_s} \frac{\partial \langle \chi \rangle_{\mathbf{R}_s}}{\partial t} F_s + \langle C[h_s] \rangle_{\mathbf{R}_s} \quad \& \text{ Maxwell's eqs}$$



Told+ 2015, Navarro+ 2016

✓ For $(\beta_i, T_i/T_e) = (1, 1)$

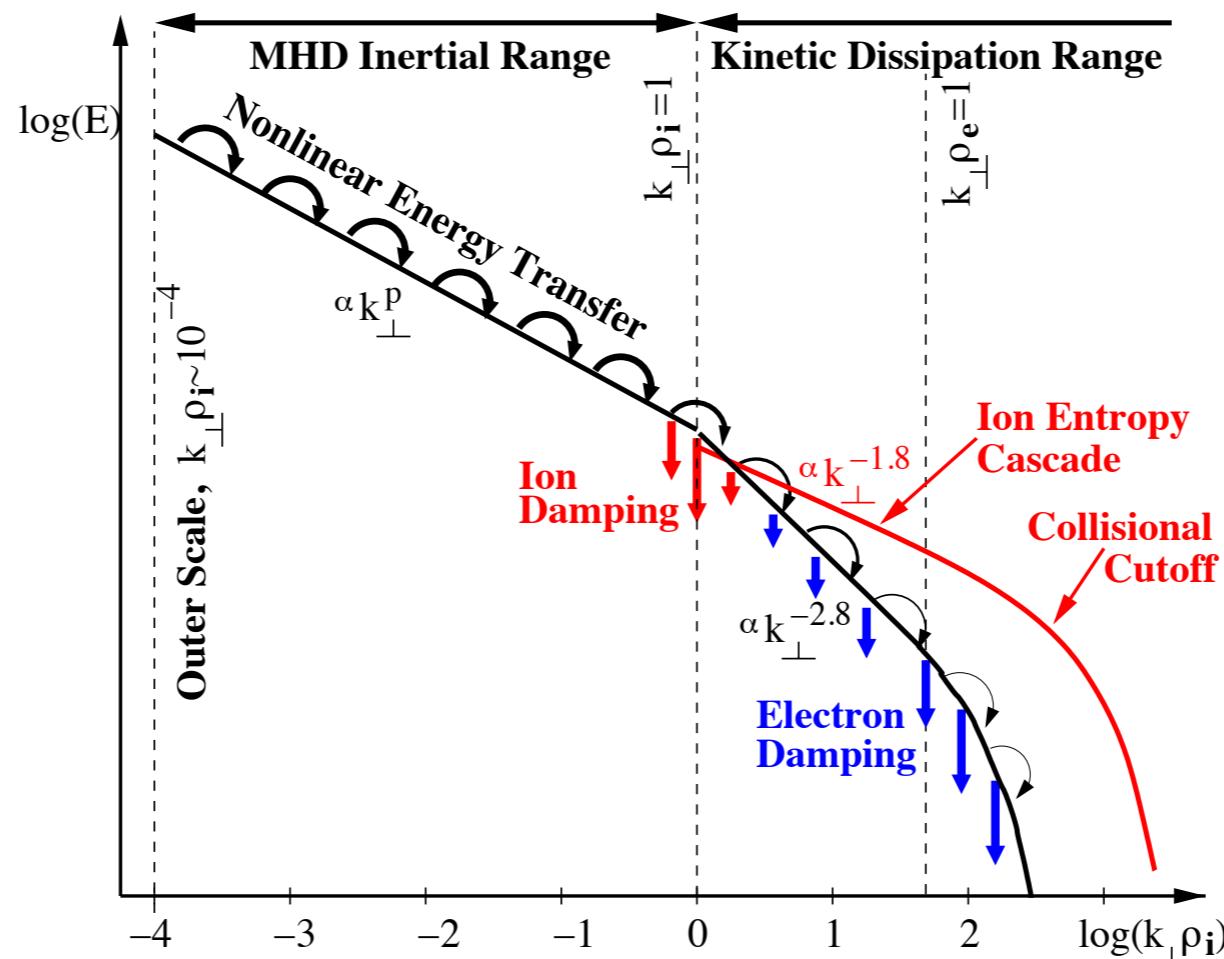
✓ $Q_e/Q_{\text{total}} \sim 70\%$

✓ Q_i via v_{\perp} derivative \leftarrow nonlinear entropy cascade

✓ Q_e via v_{\parallel} derivative \leftarrow linear Landau damping

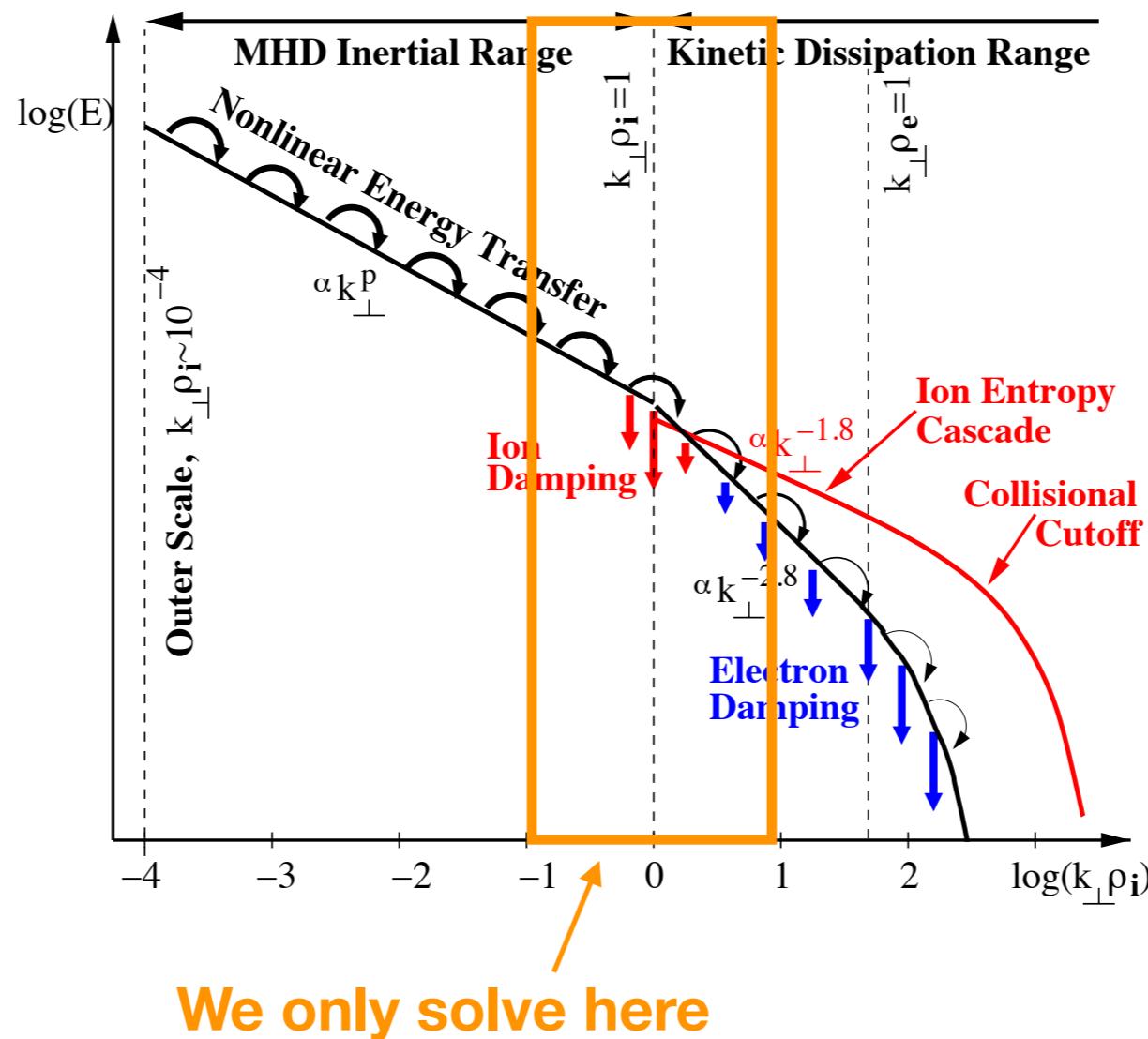
Hybrid GK approach

- ✓ We need wide-ranged scan in the parameter space (β_i , T_i/T_e)
- ✓ Cannot afford full GK approach \rightarrow Hybrid approach
- ✓ Heating partition is decided at $k_{\perp} \rho_i \sim 1$ (Schekochihin+ 2009)



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Hybrid GK approach

✓ Gyrokinetic ions + isothermal electron fluid model

$[(m_e/m_i)^{1/2} \ll 1]$ limit of electron GK eq.]

$$\frac{\partial h_i}{\partial t} + v_{||} \frac{\partial h_i}{\partial z} + \frac{c}{B_0} \{ \langle \chi \rangle_{\mathbf{R}_i}, h_i \} = \frac{Ze}{T_i} \frac{\partial \langle \chi \rangle_{\mathbf{R}_i}}{\partial t} F_i + \langle C[h_i] \rangle_{\mathbf{R}_i}$$

$$\frac{\partial}{\partial t} \left(\frac{\delta n_e}{n_e} - \frac{\delta B_{||}}{B_0} \right) + \frac{c}{B_0} \left\{ \phi - \frac{T_e}{e} \frac{\delta n_e}{n_e}, \frac{\delta n_e}{n_e} - \frac{\delta B_{||}}{B_0} \right\} + \frac{\partial u_{||e}}{\partial z} = \frac{1}{B_0} \{ A_{||}, u_{||e} \}$$

$$\frac{\partial A_{||}}{\partial t} + \frac{c}{B_0} \left\{ \phi - \frac{T_e}{e} \frac{\delta n_e}{n_e}, A_{||} \right\} + c \frac{\partial \phi}{\partial z} - \frac{c T_e}{e} \frac{\partial}{\partial z} \left(\frac{\delta n_e}{n_e} \right) = 0$$

& Maxwell's eqs

✓ Use AstroGK code (Numata+ 2010) with isothermal electron fluid

(Kawazura & Barnes 2018)

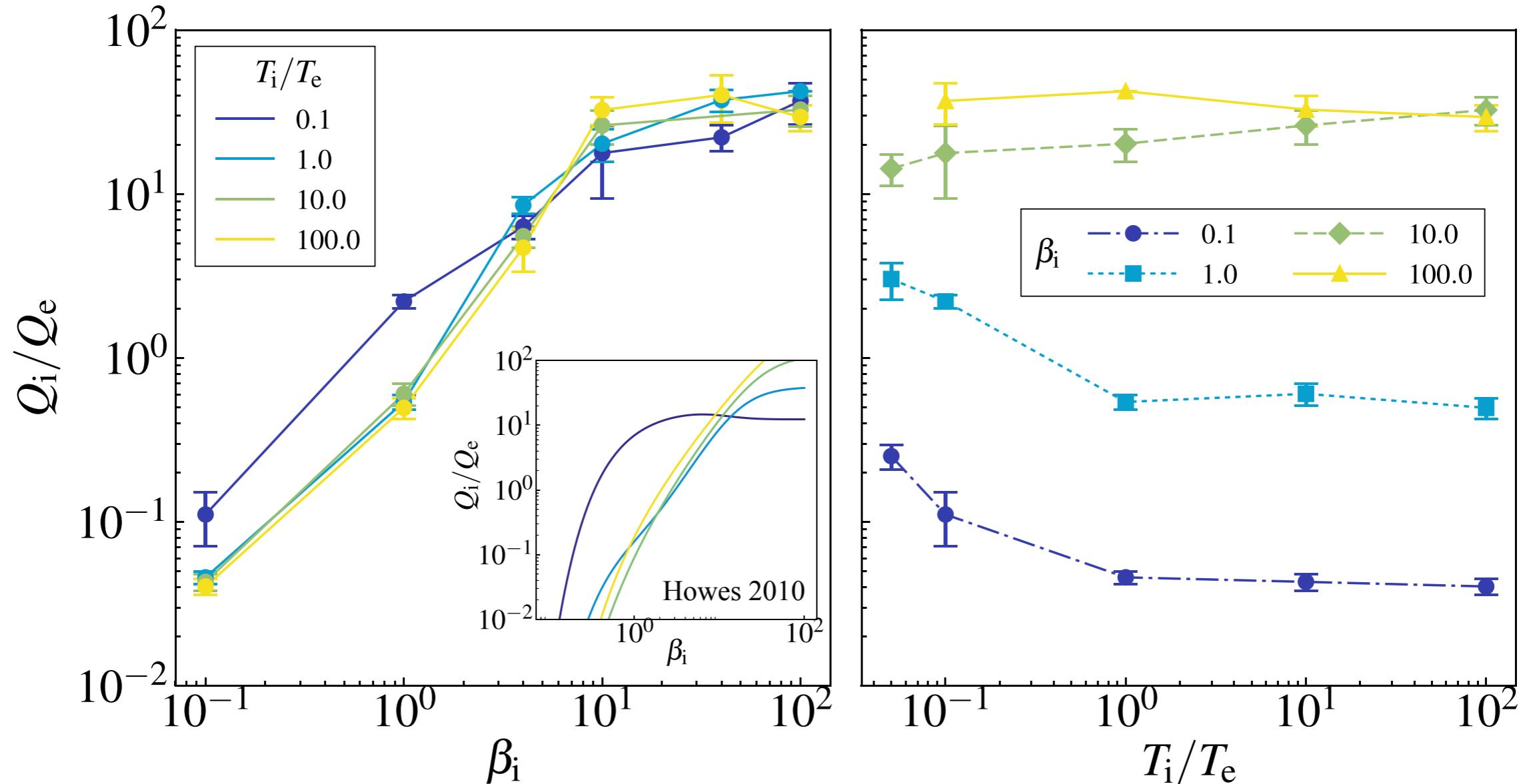
Numerical set up

- ✓ Simulation box : $0.125 \leq k_x \rho_i, k_y \rho_i \leq 5.25$ or $(0.25 \leq k_x \rho_i, k_y \rho_i \leq 5.25)$
- ✓ Inject Alfvén wave at the box scale via oscillating Langevin antennas (TenBarge+ 2014) → P_{ant}
- ✓ Ion dissipation via collision + hypercollision → Q_i
- ✓ Electron dissipation via hyperviscosity + hyperresistivity → Q_e

$$\frac{dW_{\text{tot}}}{dt} = P_{\text{ant}} - Q_i - Q_e$$

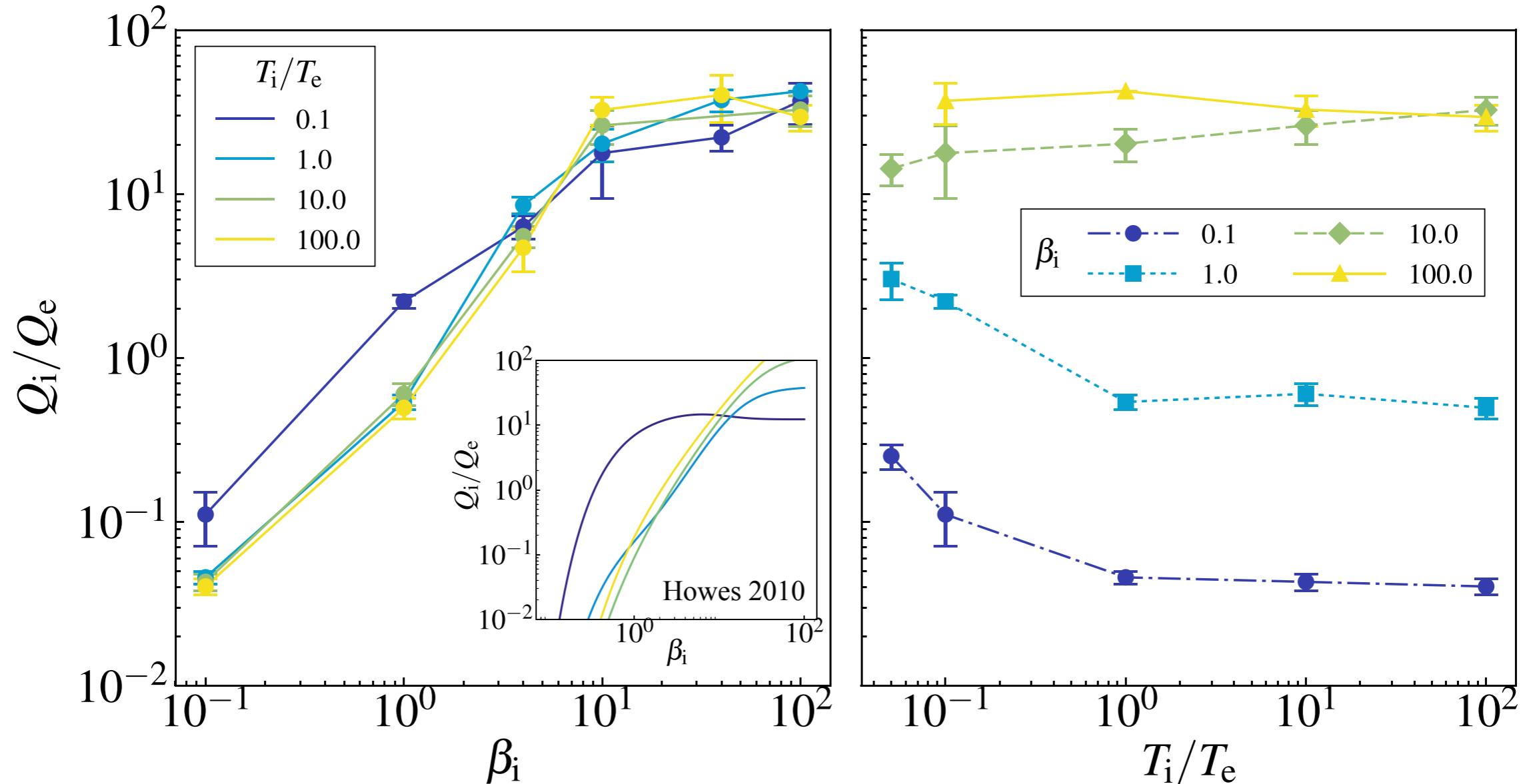
- ✓ In a stationary state, $\langle dW_{\text{tot}}/dt \rangle \simeq 0$

Result : heating ratio



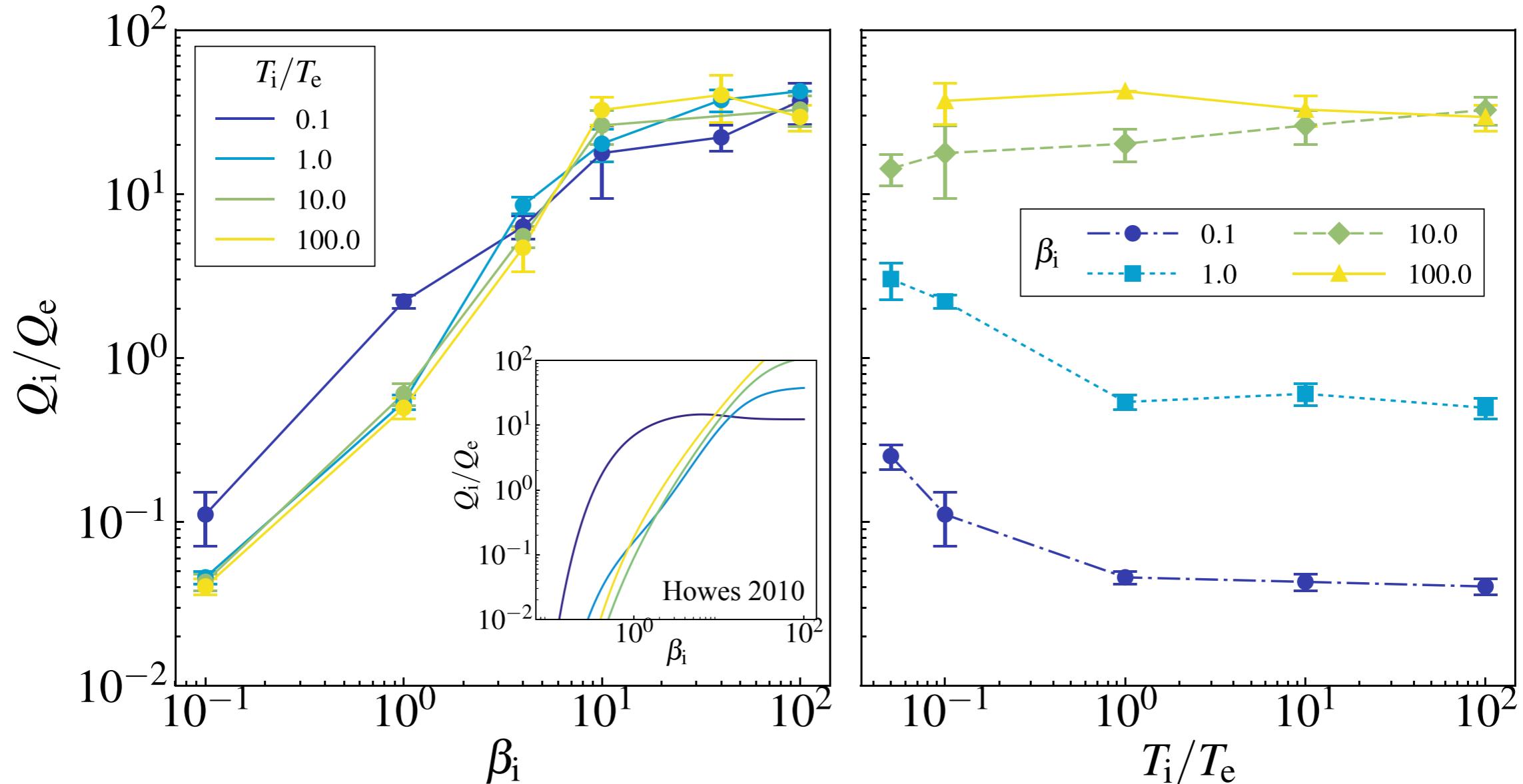
- ✓ $Q_i/Q_e \sim 0.6$ at $(\beta_i, T_i/T_e) = (1, 1)$ ← Consistent with full GK
(Navarro+ 2016)
- ✓ Increasing function of β_i ← The same trend as Howes'
- ✓ Insensitive to T_i/T_e
- ✓ $Q_i/Q_e \rightarrow 0$ when $\beta_i \rightarrow 0$ (as predicted in Zocco & Schekochihin, 2011)

Result : heating ratio



- ✓ Significantly different from Howes' at high β_i and low β_i
- ✓ At high β_i , Q_i/Q_e has ceiling ($\lesssim 30$)
- ✓ At low β_i , Q_i/Q_e is several orders of magnitude larger than Howes'
- ✓ In Sgr A*, mid plane : $\beta_i \sim 10$, Jet region : $\beta_i \sim 0.1$

Result : heating ratio

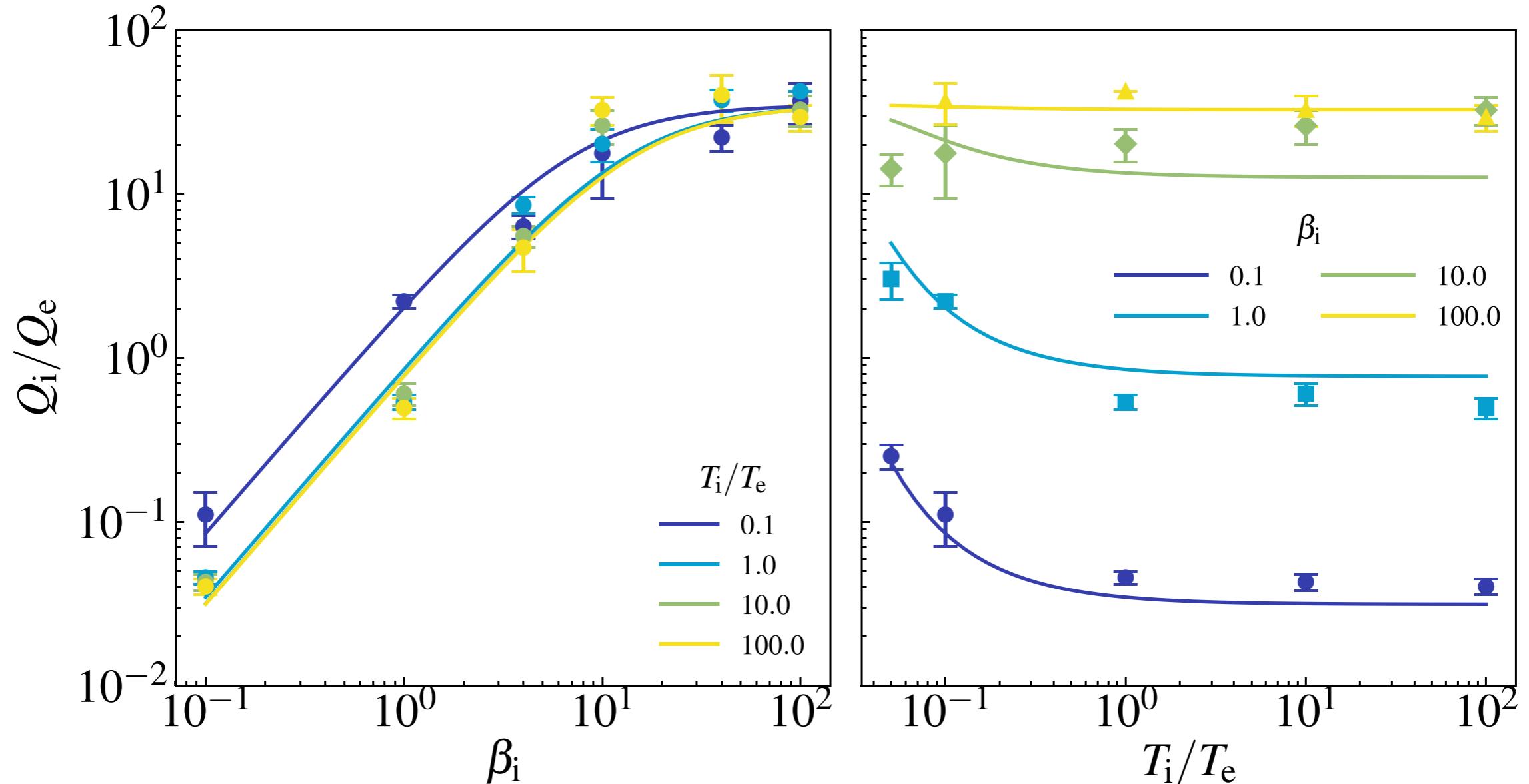


Why is there a ceiling at high β_i ?

Why is it greater than Howes' at low β_i ?

→ Future work

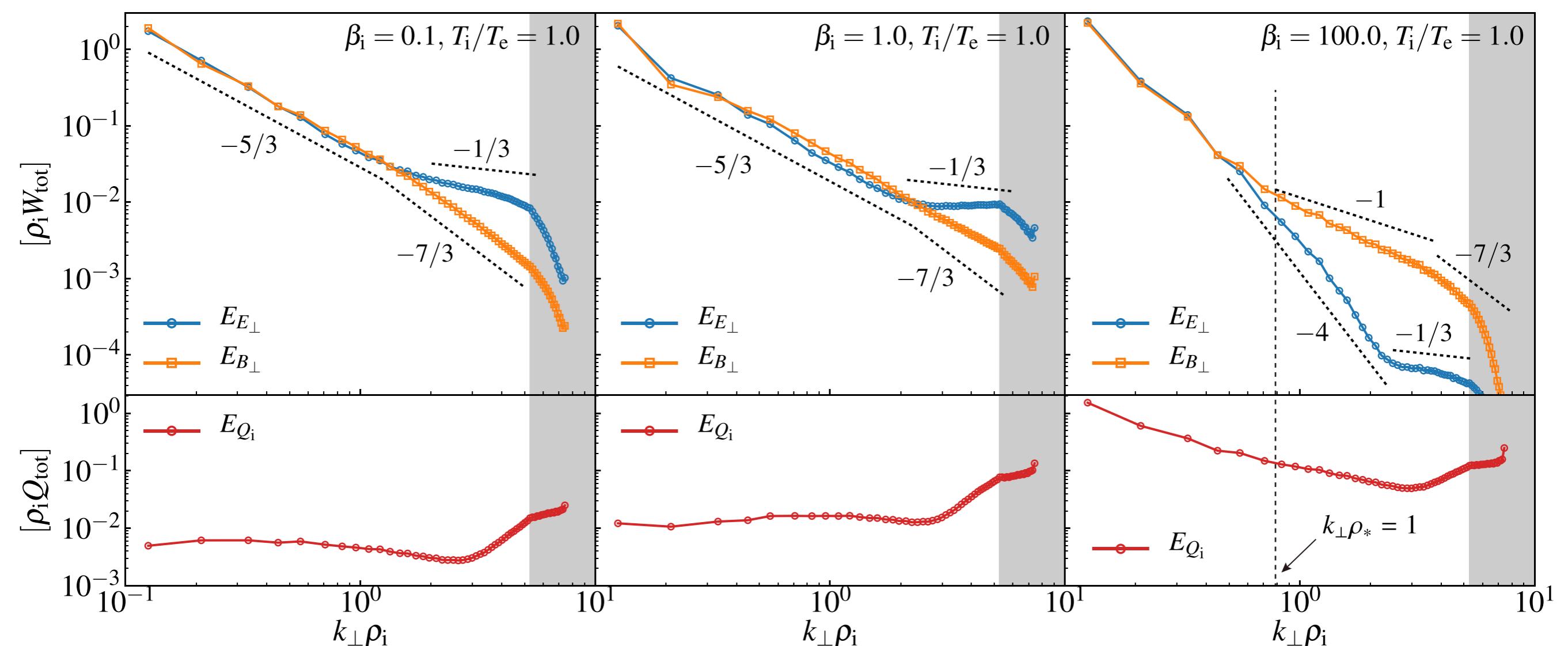
Result : heating ratio



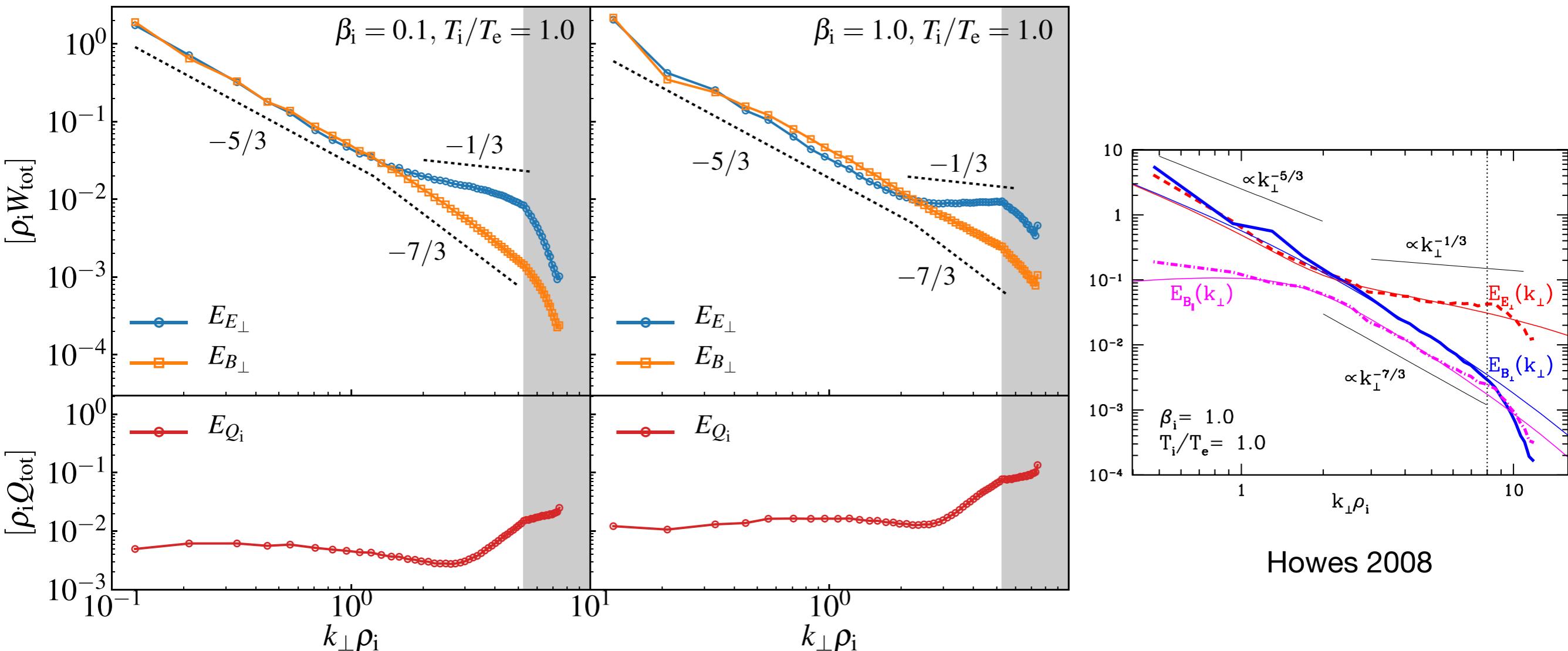
✓ New heating ratio prescription

$$\frac{Q_i}{Q_e}(\beta_i, T_i/T_e) = \frac{35.0}{1 + (\beta_i/15.0)^{-1.4} \exp(-0.1T_e/T_i)}$$

Result : k-spectrum

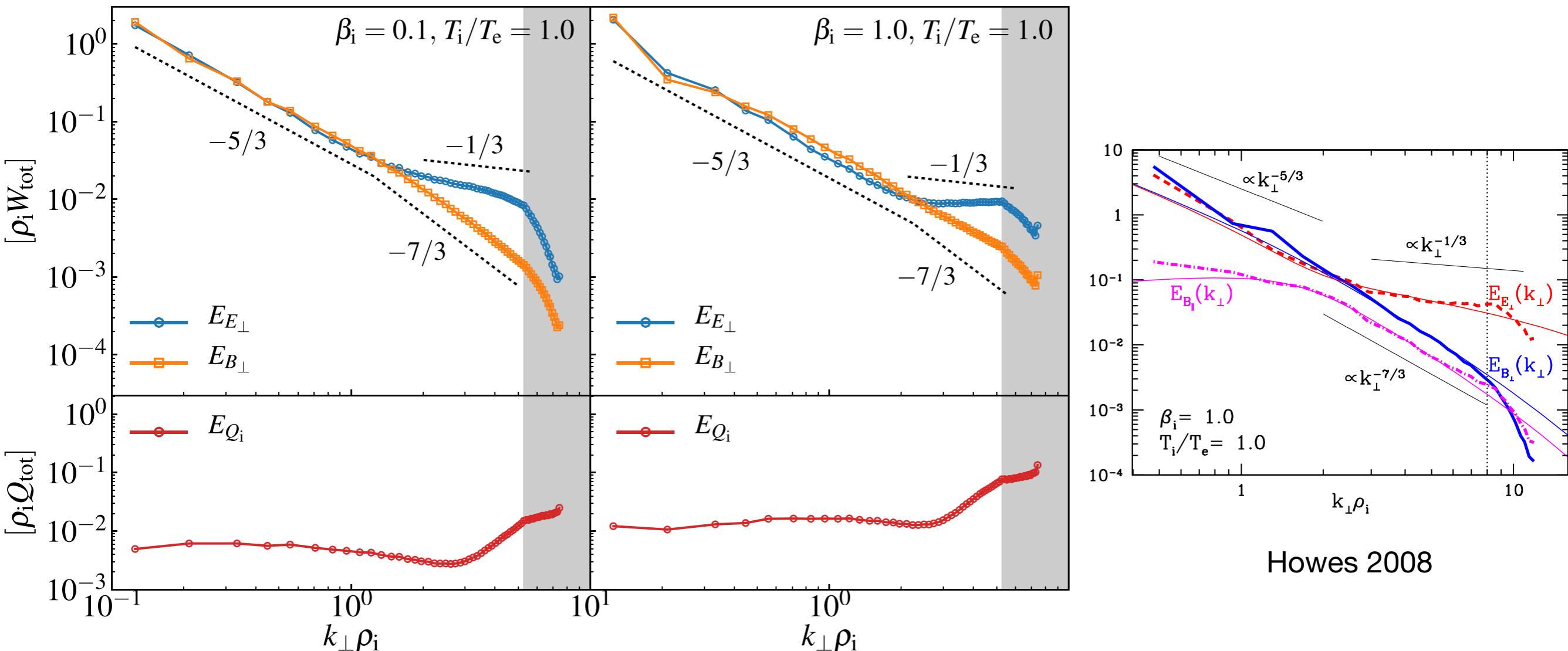


Result : k-spectrum (low-mid β_i)



- ✓ For $\beta_i = 0.1$ and 1
- ▶ A familiar AW/KAW transition at $k_{\perp} \rho_i = 1$

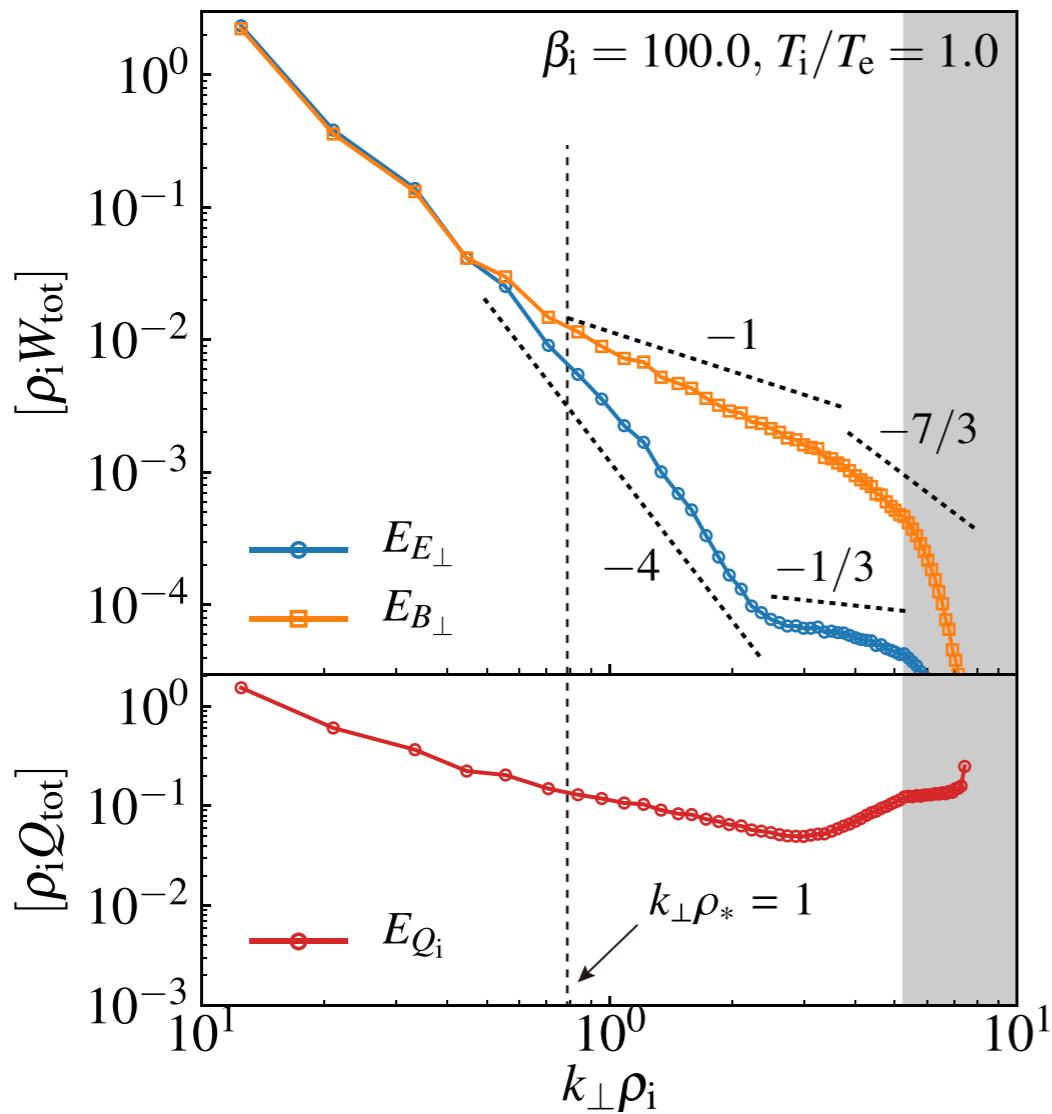
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- ✓ For $\beta_i = 0.1$ and 1
 - ▶ A familiar AW/KAW transition at $k_{\perp} \rho_i = 1$
 - ▶ Heating mainly at a grid scale (electron scale)

↑ consistent with Howes, Told, ...

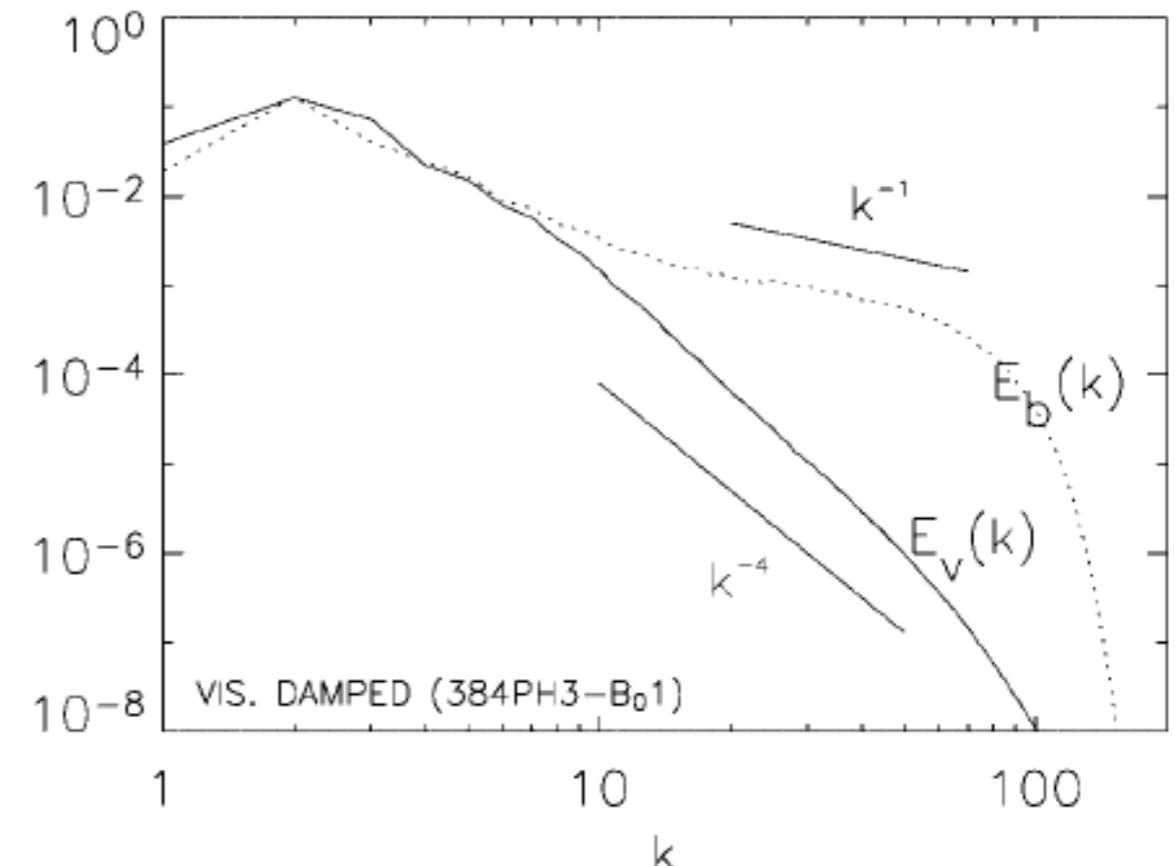
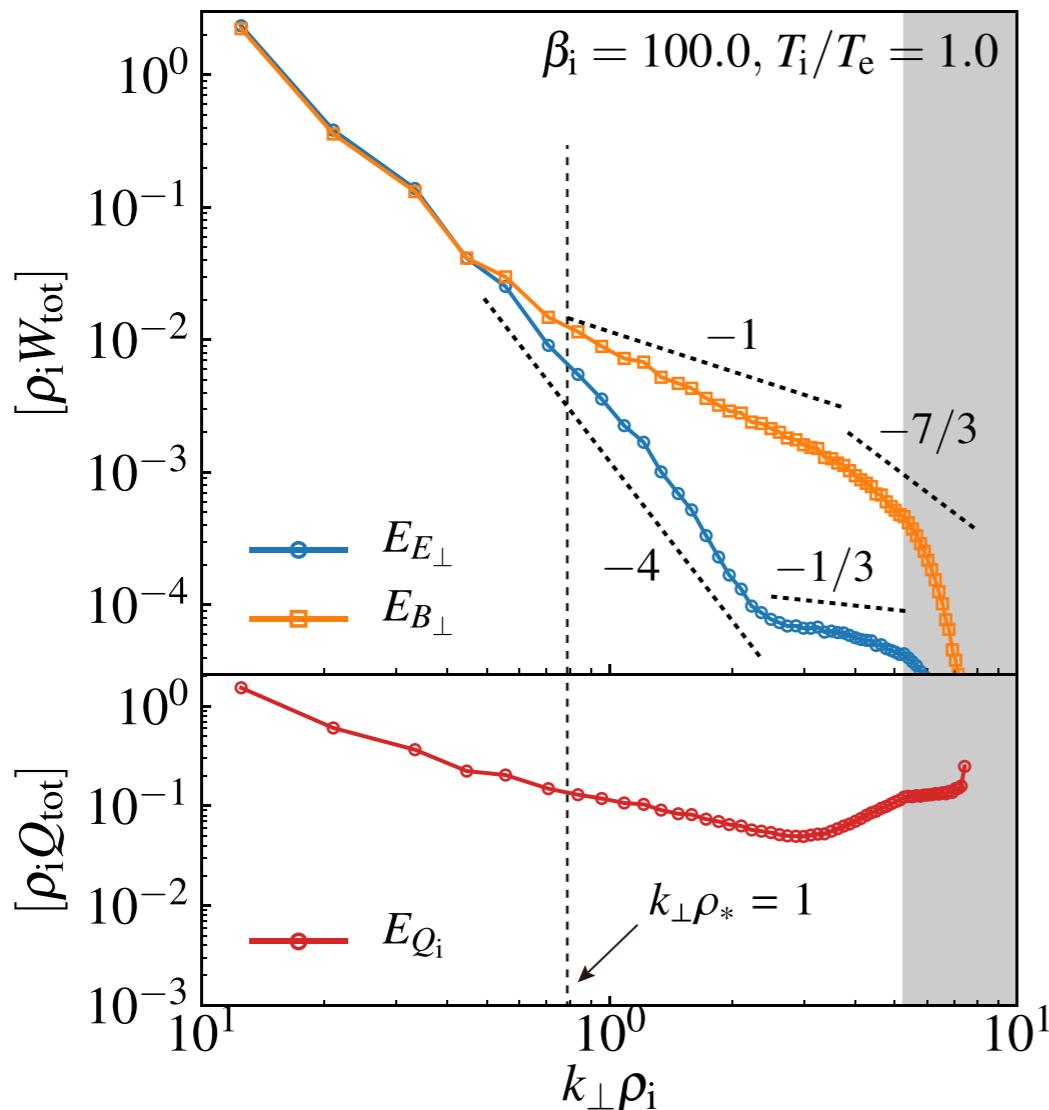
Result : k-spectrum (high β_i)



- ✓ For $\beta_i = 100$
- ▶ Huge drop of electric field $\leftarrow \Phi_{\mathbf{k}} = \pm \frac{k_\perp \rho_i}{\sqrt{(1 + ZT_e/T_i)[2 + \beta_i(1 + ZT_e/T_i)]}} \Psi_{\mathbf{k}}$

(Schekochihin+ 2009)

Result : k-spectrum (high β_i)



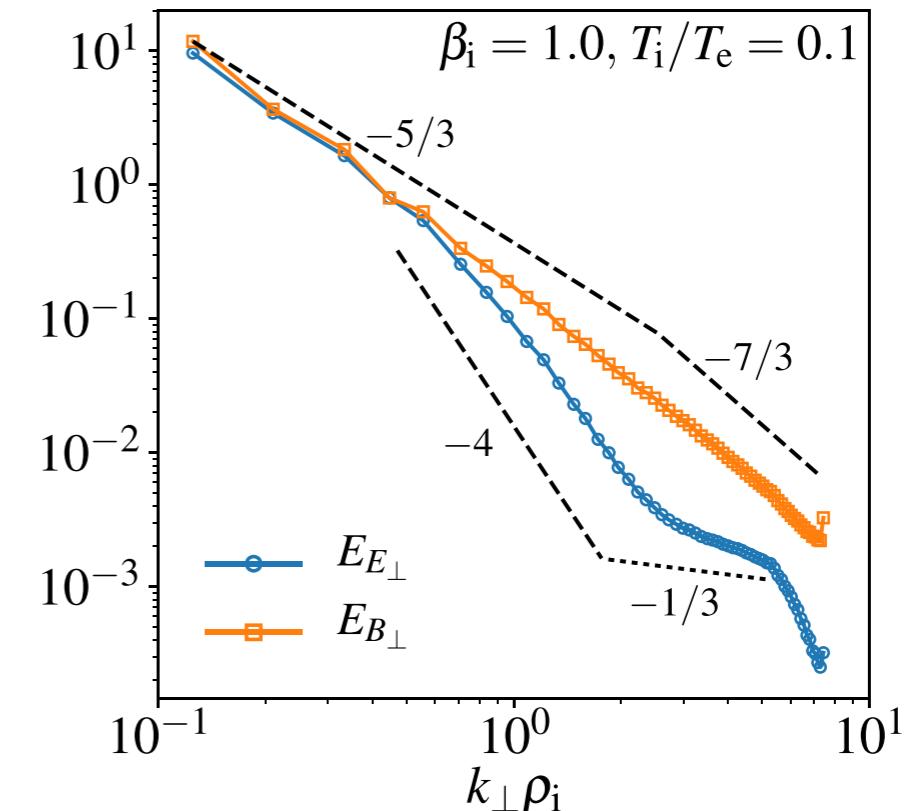
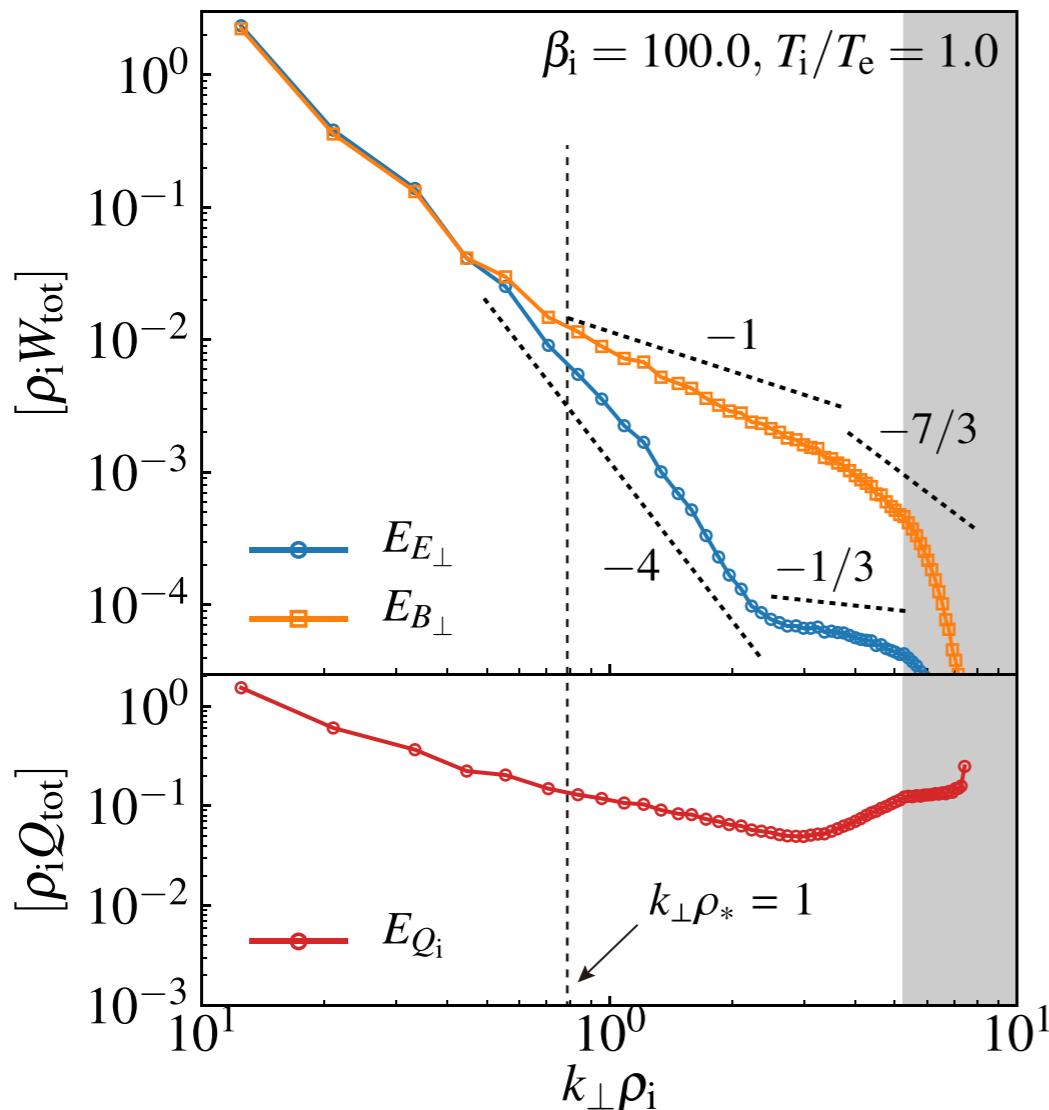
High Pm MHD turbulence, Cho+ 2002

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Result : k-spectrum (high β_i)



Similar drop for low T_i/T_e
but $E_{B\perp}$ is as usual

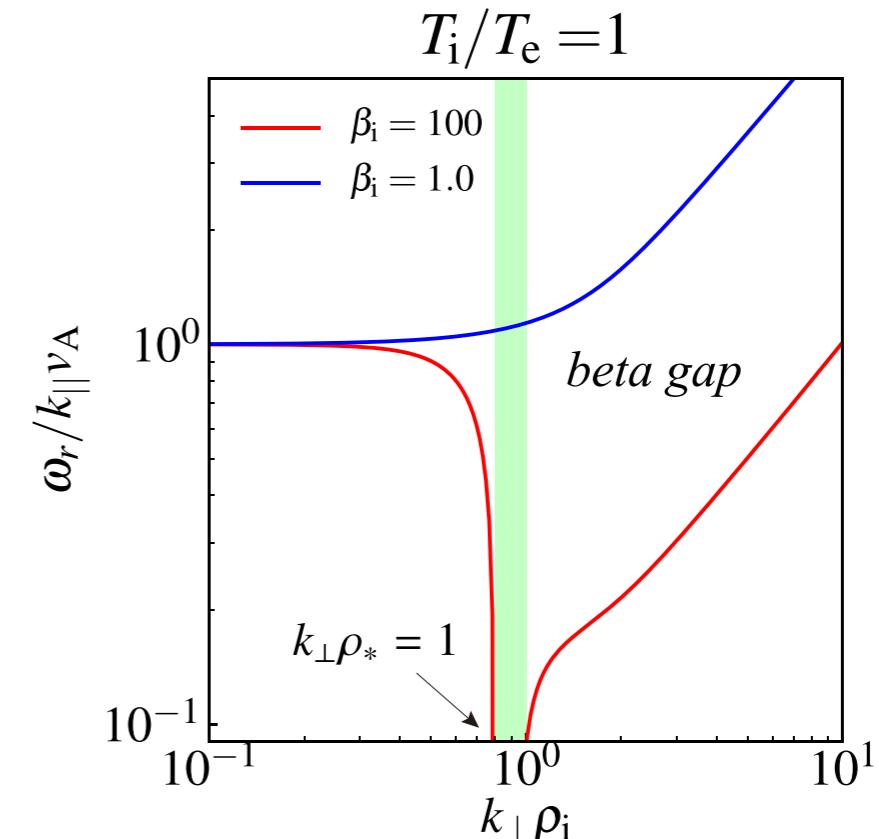
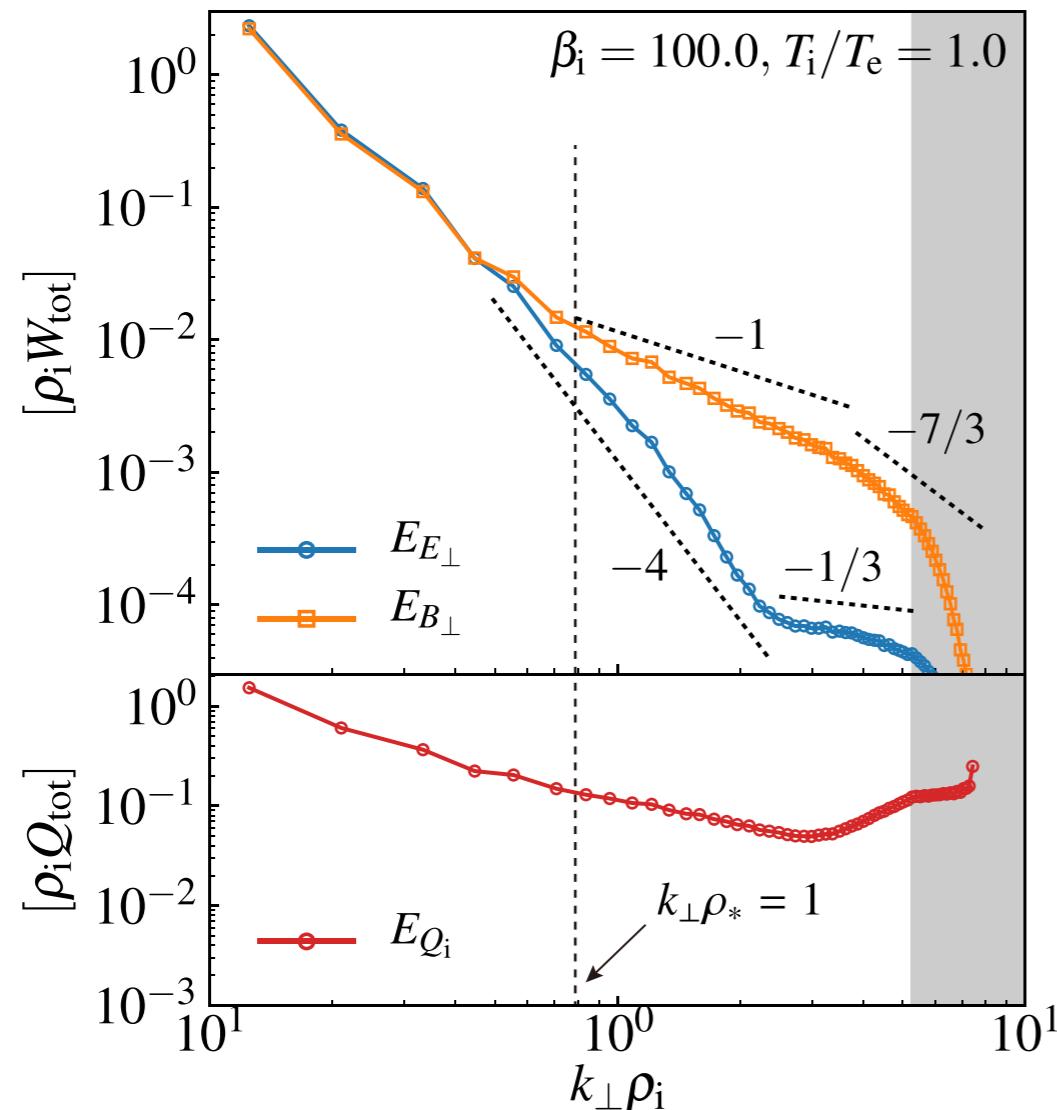
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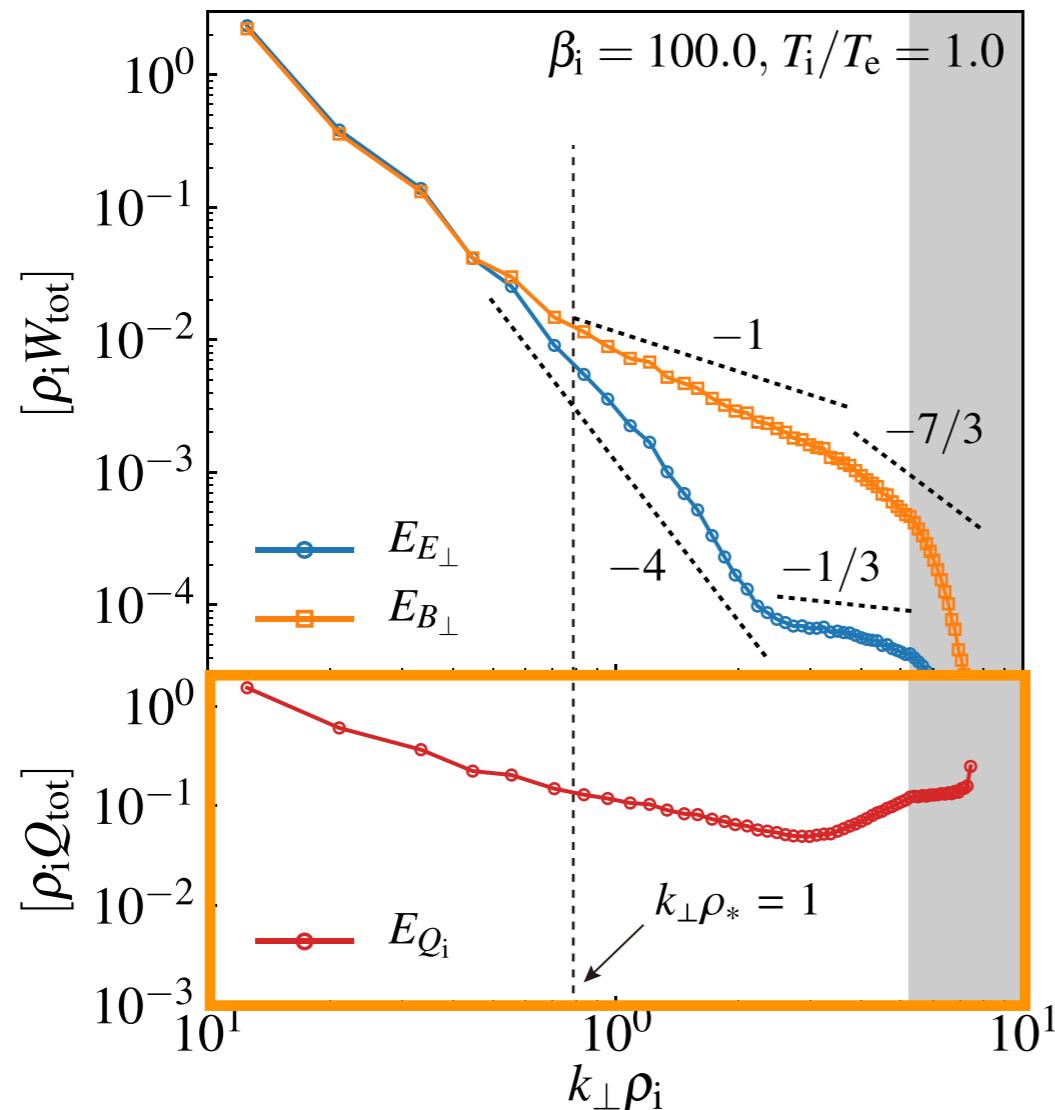
Zero Alfvén frequency region = beta gap
currently working on $\beta_i = 10000$ (in backup slides)

- ✓ For $\beta_i = 100$
- ▶ Huge drop of electric field

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(Schekochihin+ 2009)

Result : k-spectrum (low-mid β_i)



- ✓ For $\beta_i = 100$
- ▶ Huge drop of electric field
- ▶ Heating occurs at $k_{\perp}\rho_i < 1$

$$\Phi_{\mathbf{k}} = \pm \frac{k_{\perp}\rho_i}{\sqrt{(1 + ZT_e/T_i)[2 + \beta_i(1 + ZT_e/T_i)]}} \Psi_{\mathbf{k}}$$

(Schekochihin+ 2009)

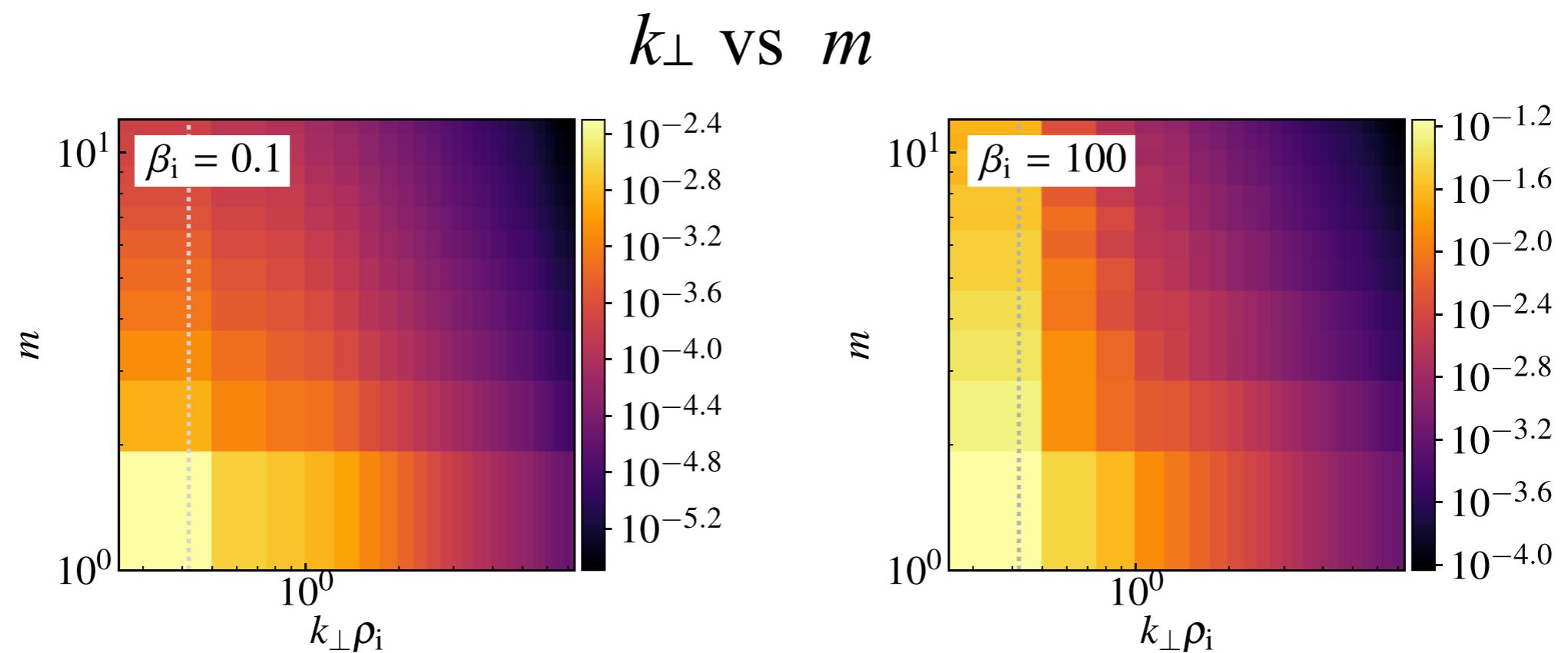
Result : velocity spectrum

- ✓ Hermite-Laguerre spectrum $\hat{g}_{m,\ell} = \int_{-\infty}^{\infty} dv_{\parallel} \frac{H_m(v_{\parallel}/v_{\text{thi}})}{\sqrt{2^m m!}} \int_0^{\infty} d(v_{\perp}^2) L_{\ell}(v_{\perp}^2) g(v_{\parallel}, v_{\perp}^2)$
- ✓ Large m : small scale in v_{\parallel} , Large ℓ : small scale in v_{\perp}

$$g = h - \frac{Ze}{T_i} \langle \phi \rangle_{\mathbf{R}_i} F_i$$

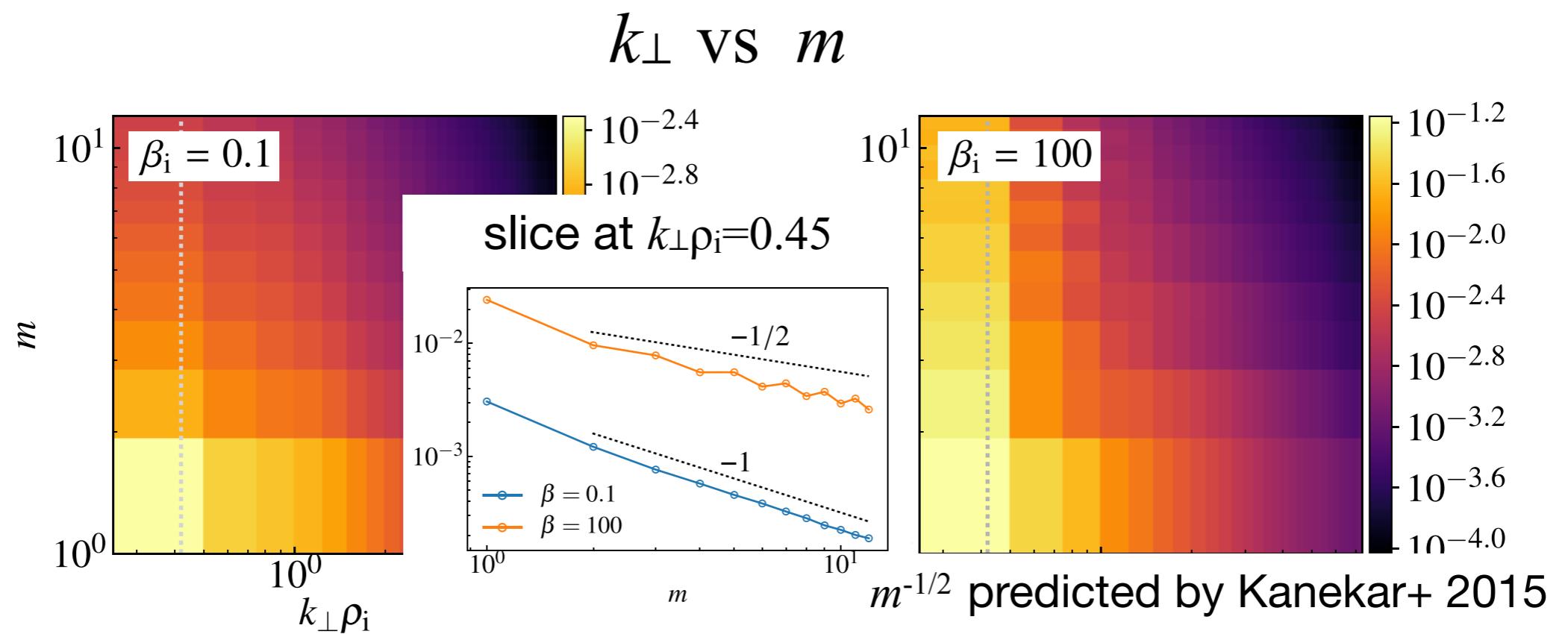
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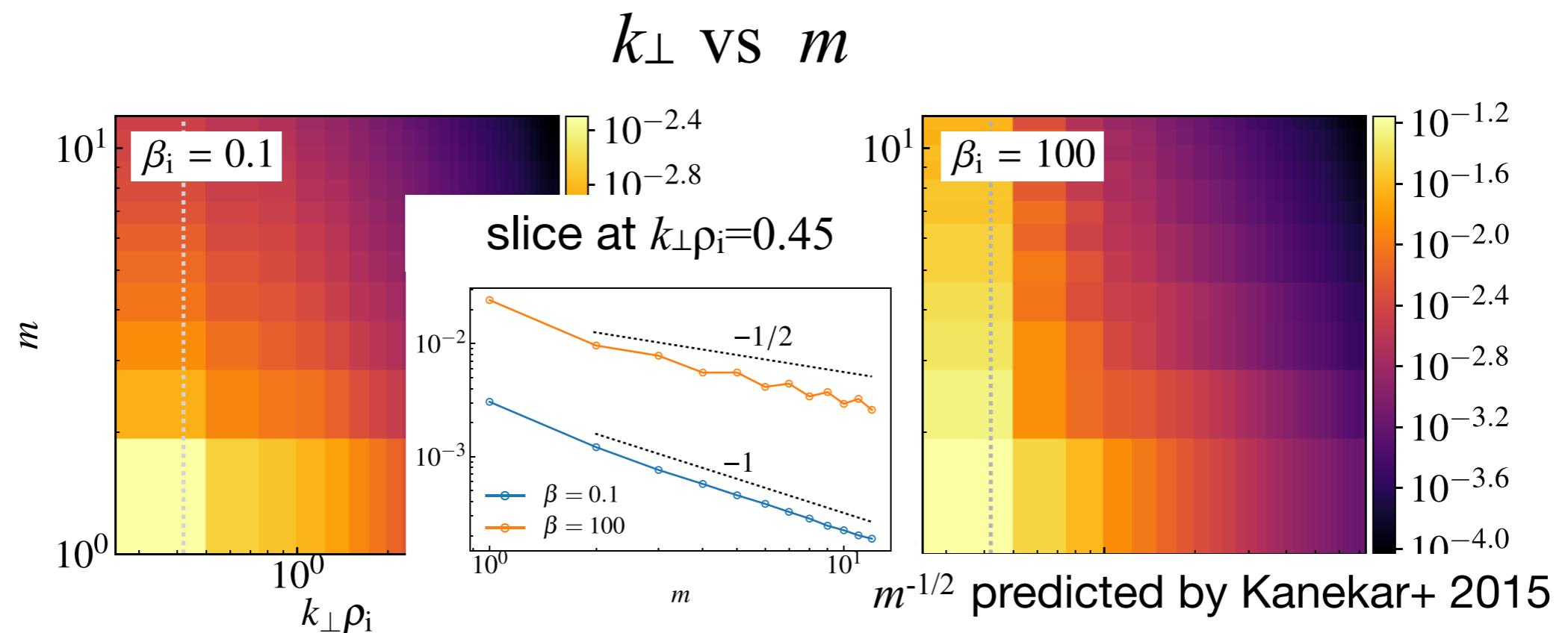
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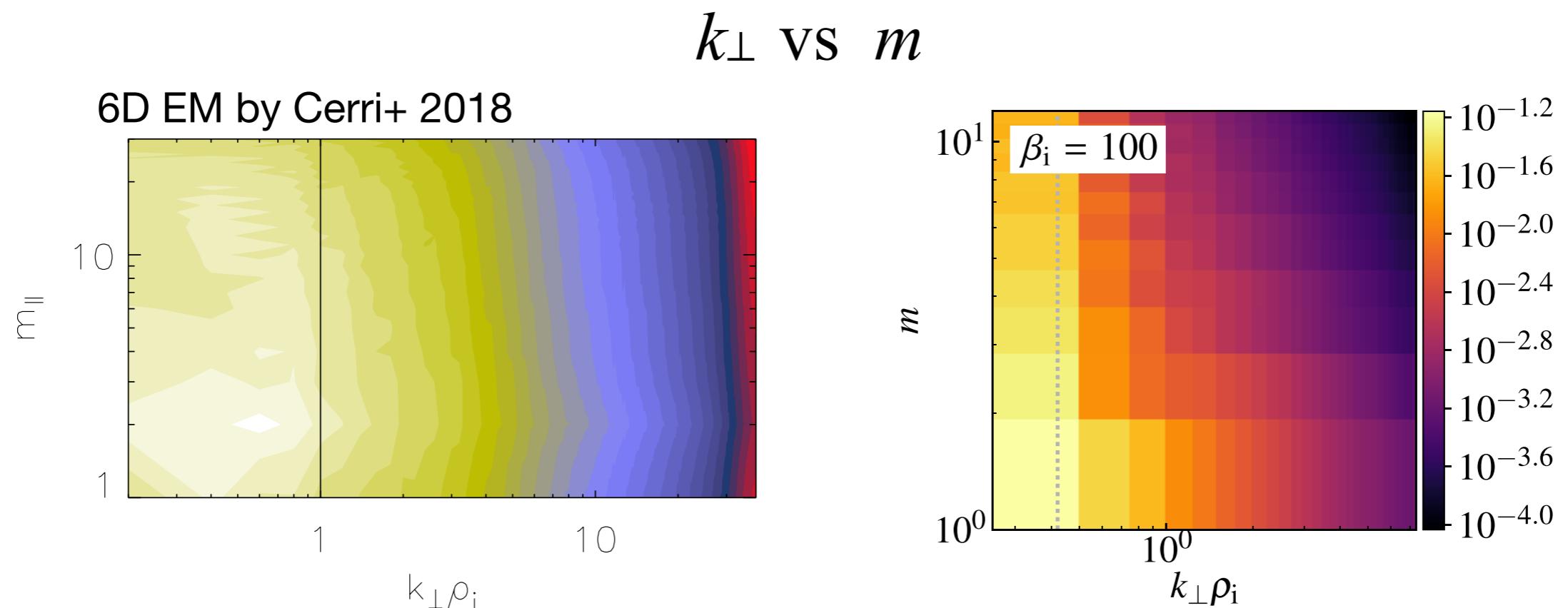
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- ✓ Ion heating at high β_i \leftarrow linear Landau damping



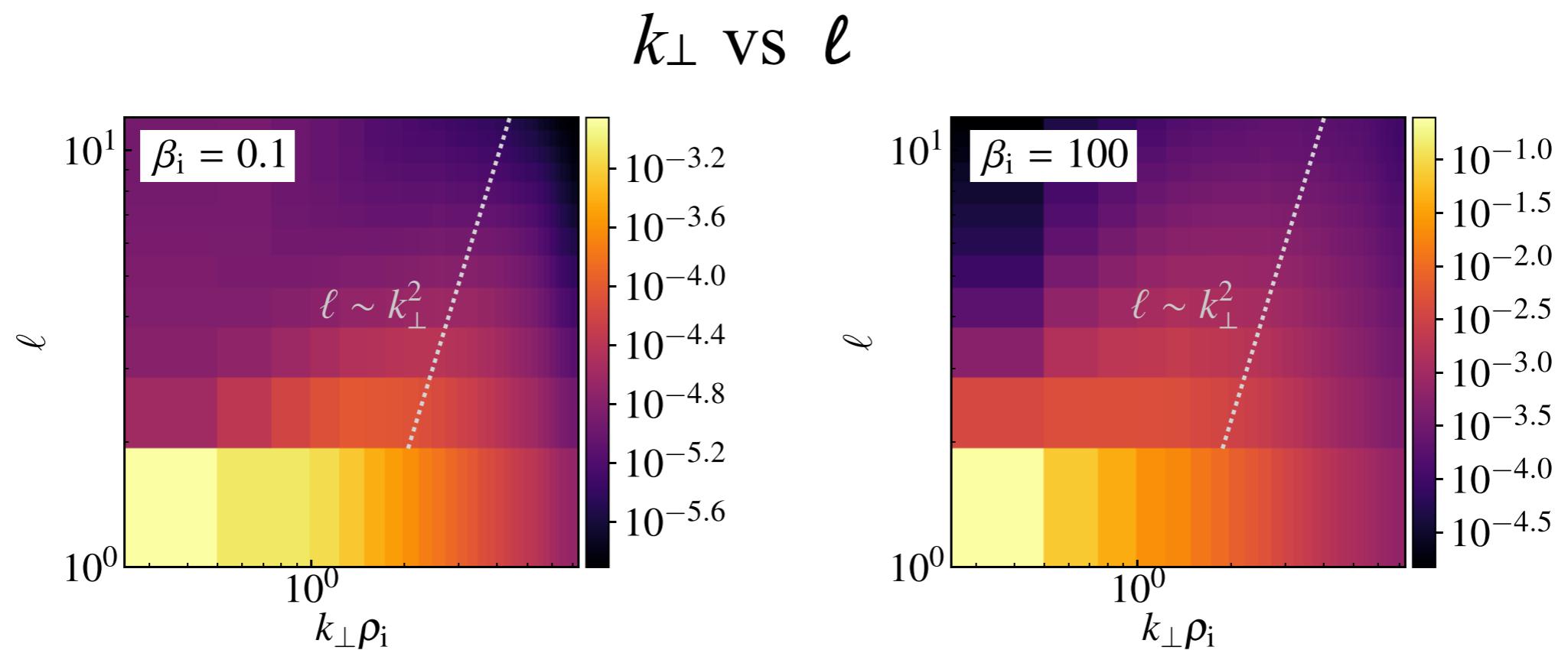
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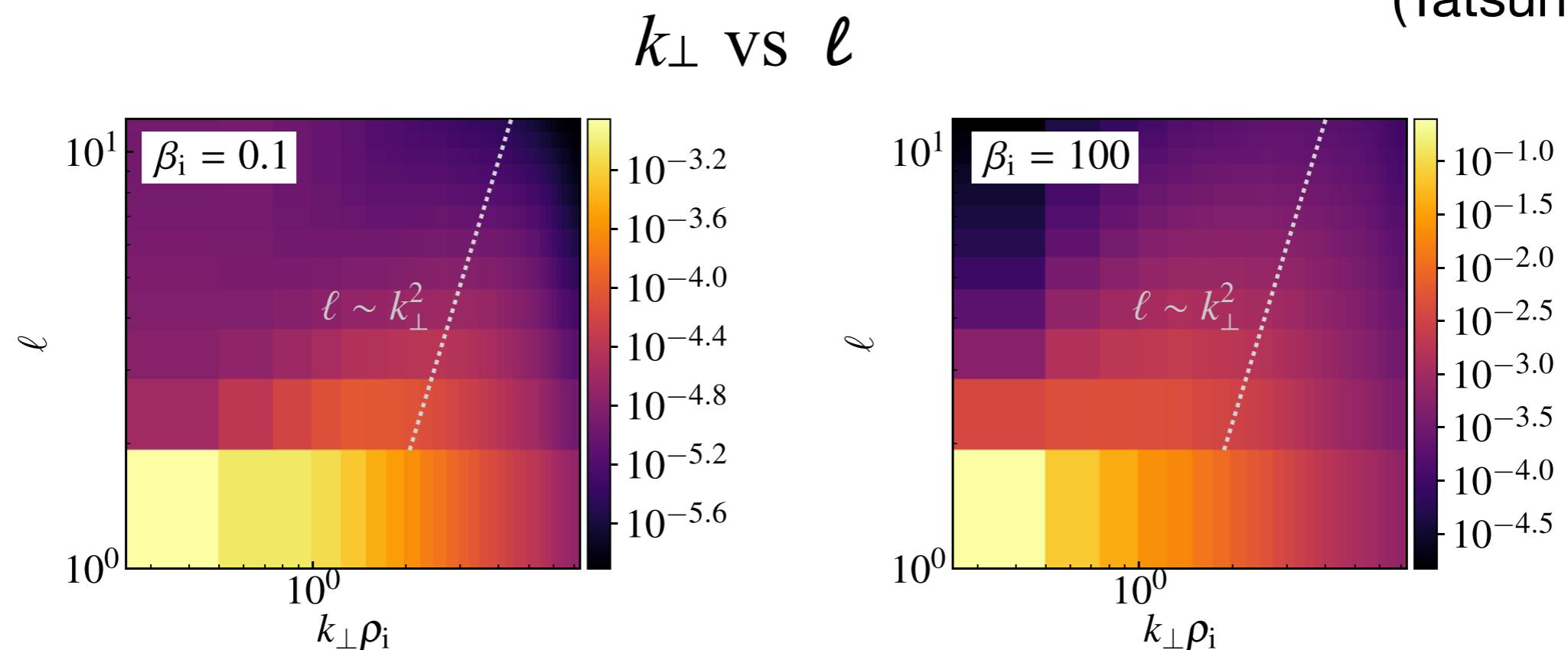
Result : velocity spectrum

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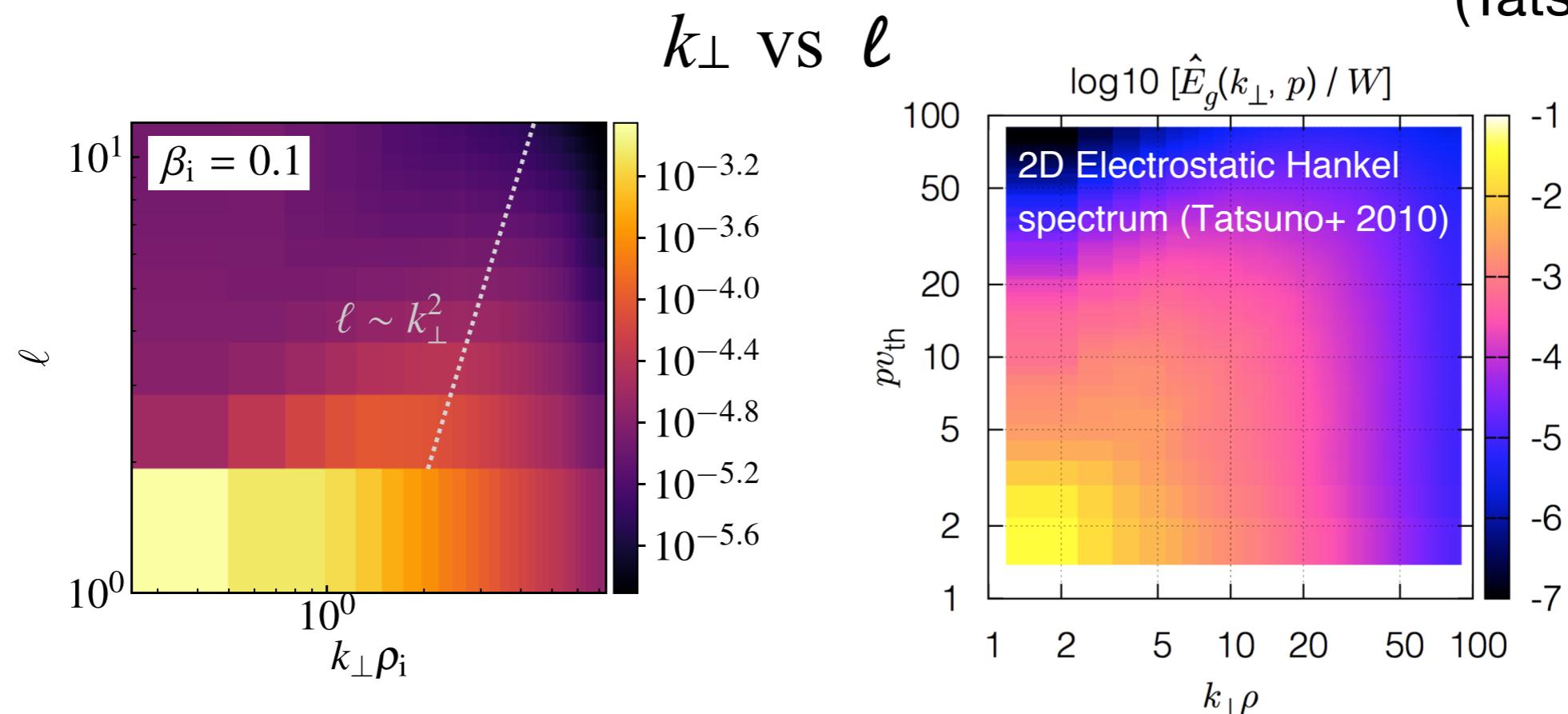
(Tatsuno+ 2009)



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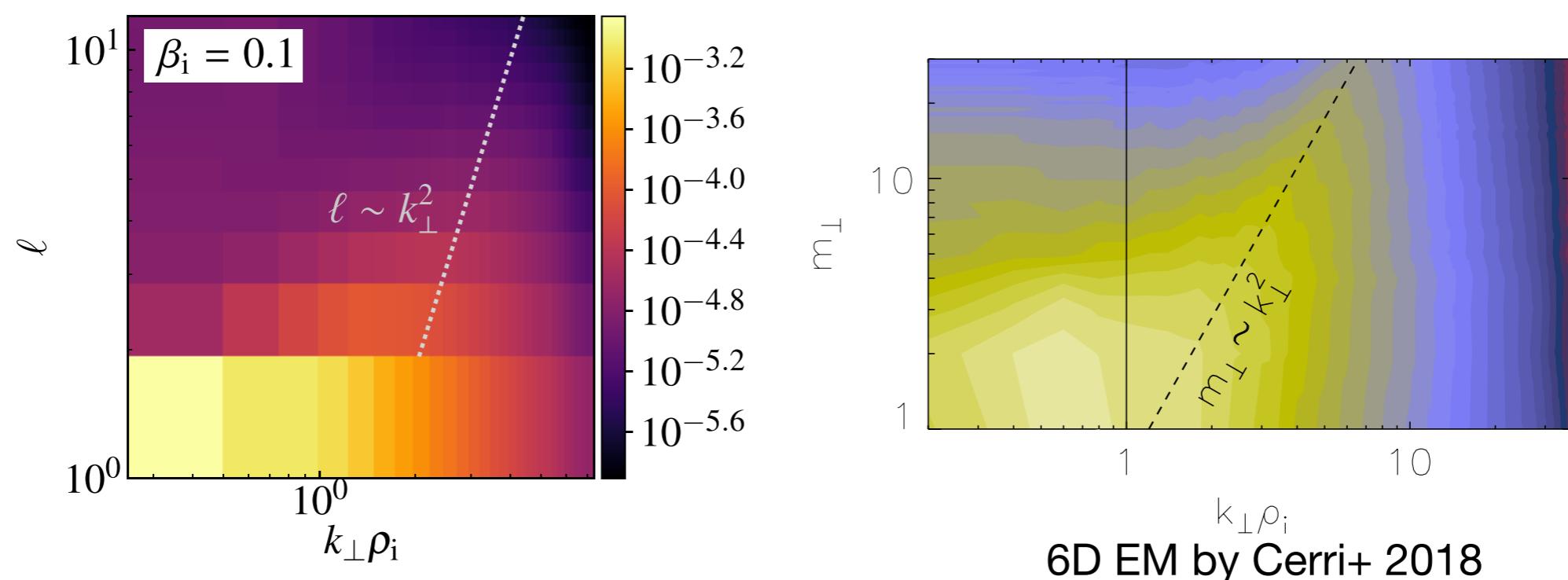


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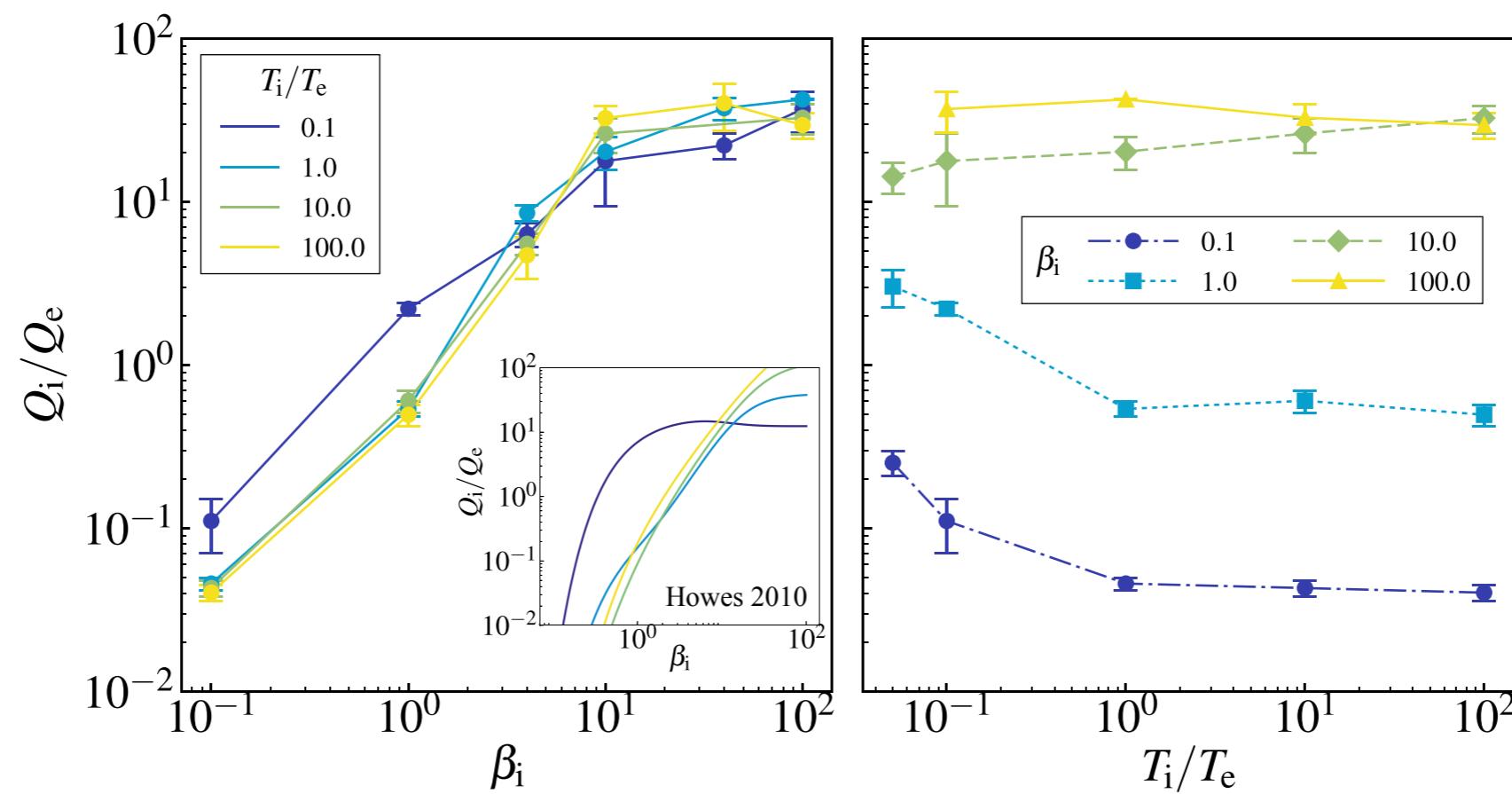
k_{\perp} vs ℓ

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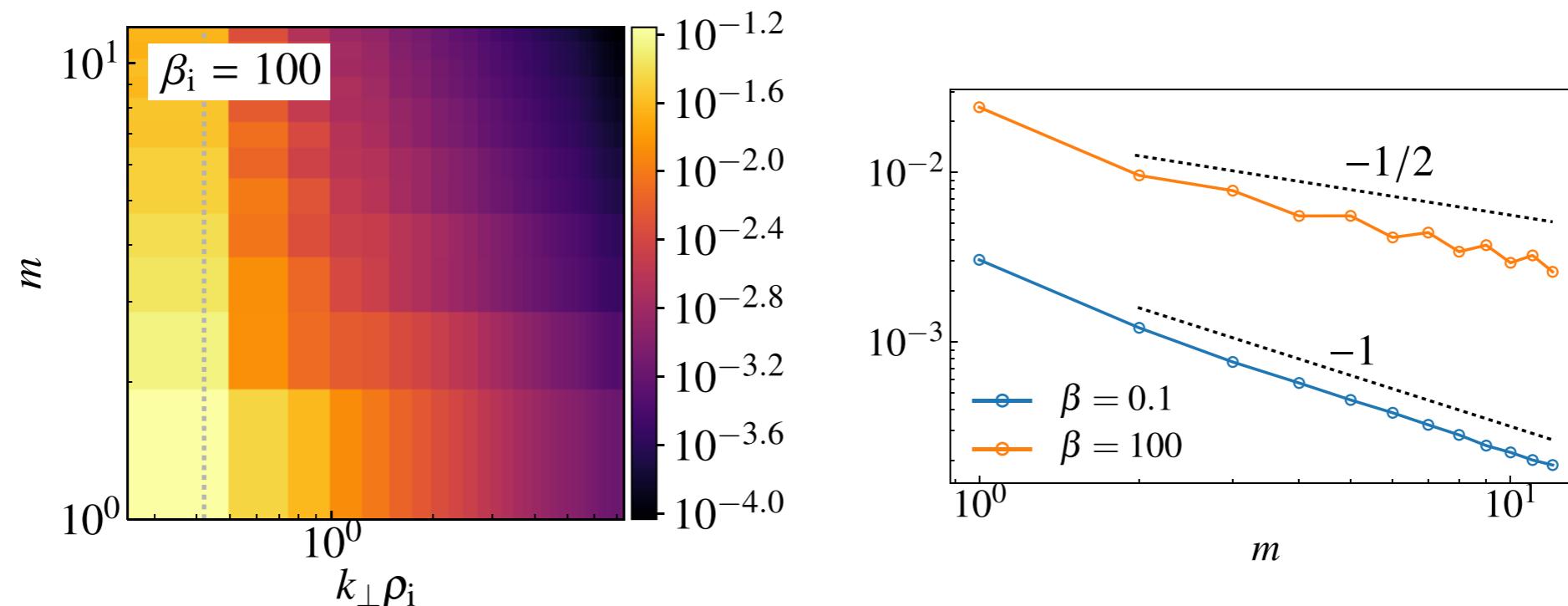
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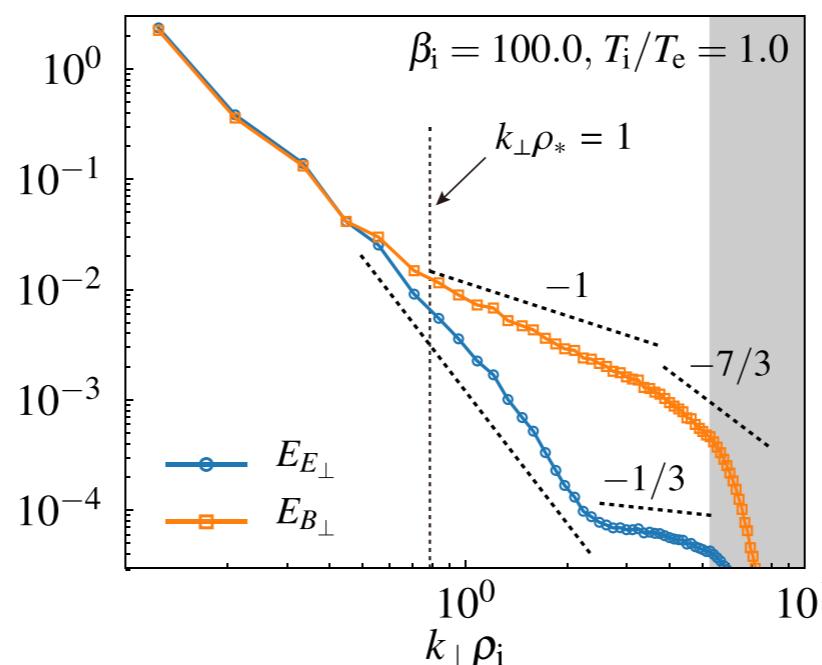
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Y. Kawazura, M. Barnes, and A. A. Schekochihin, arXiv: 1807.07702

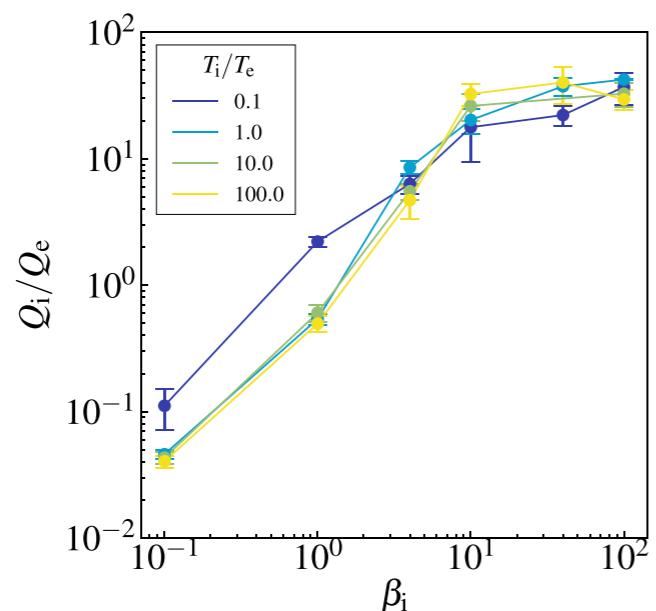
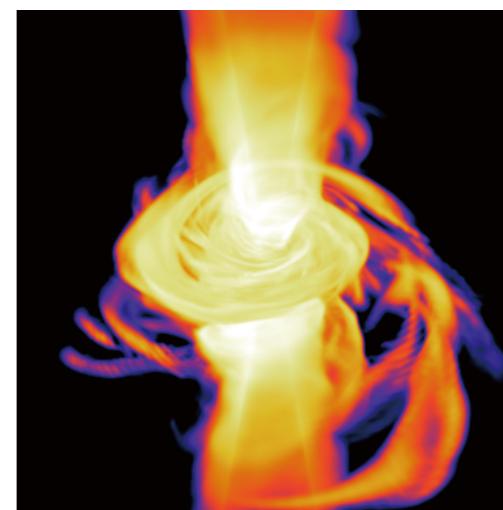
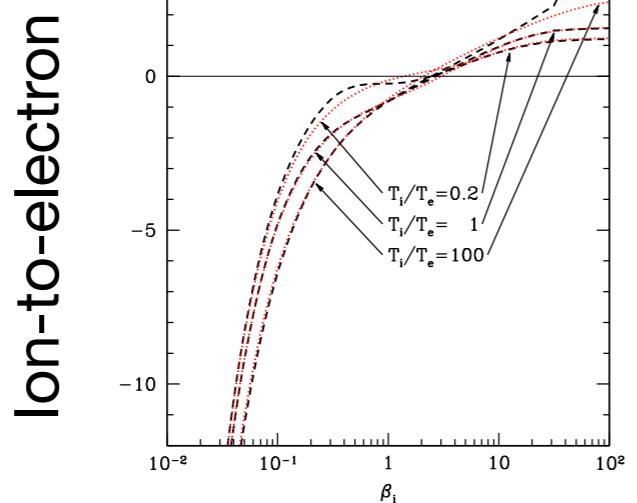
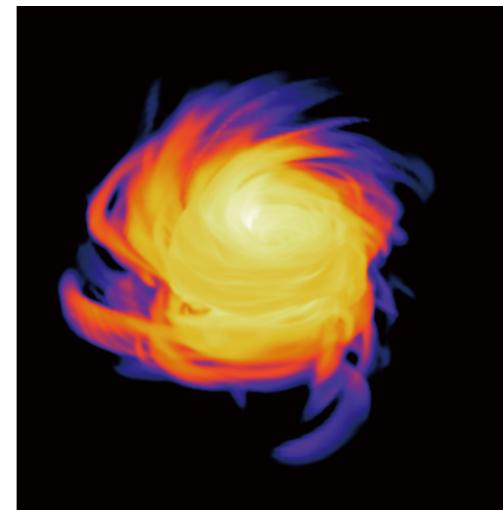
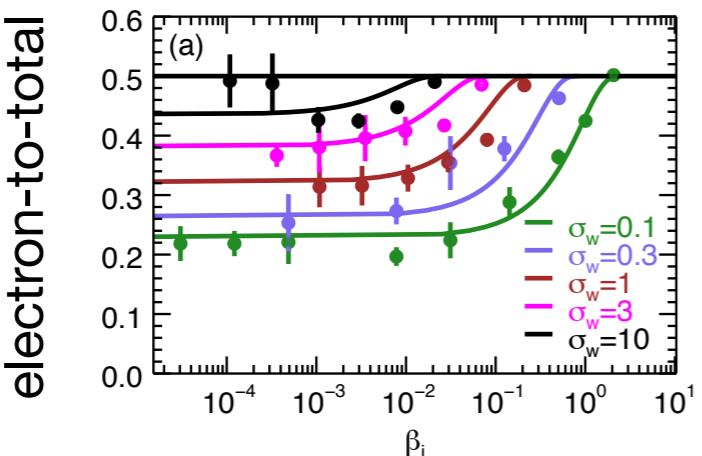
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Future work

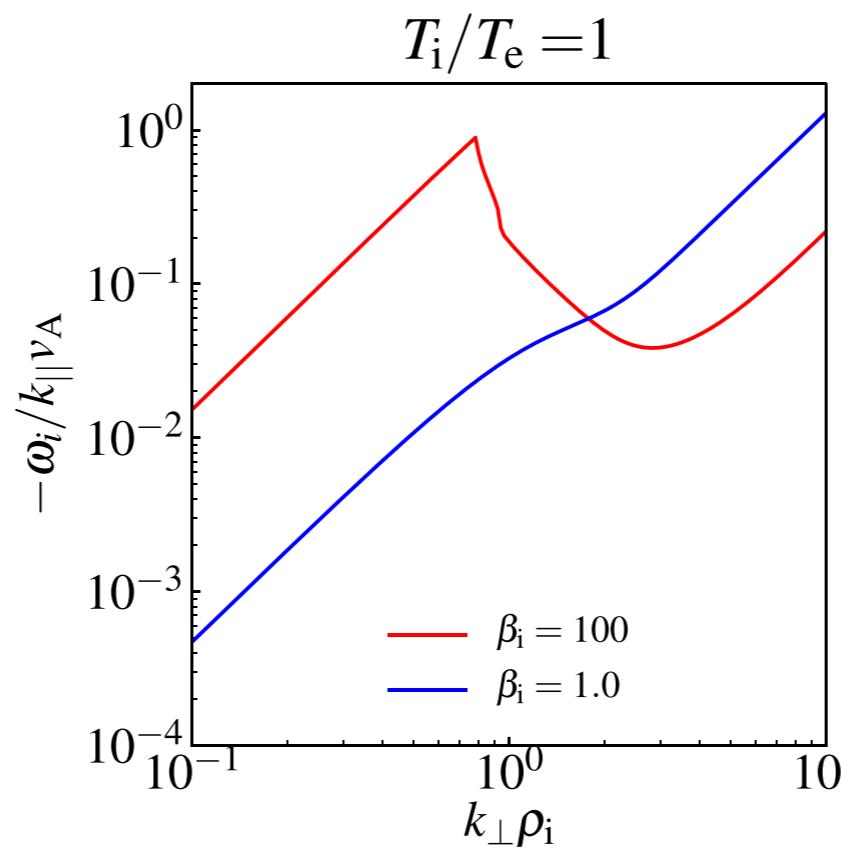
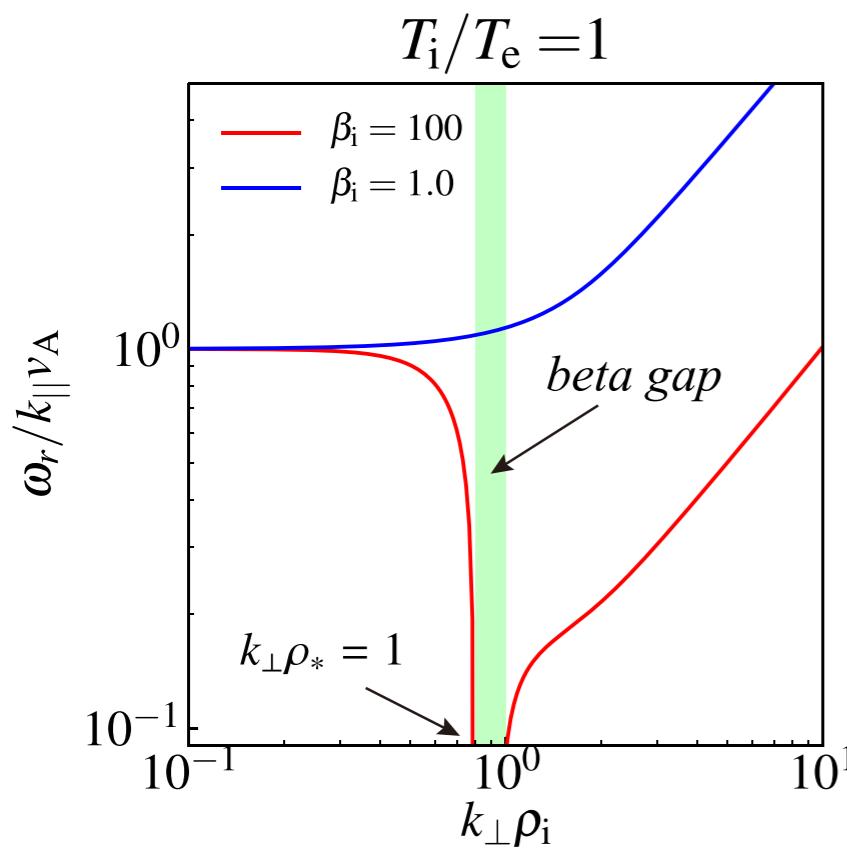
- ✓ Compressive mode driving
- ✓ Beta gap investigation

Application to accretion disks



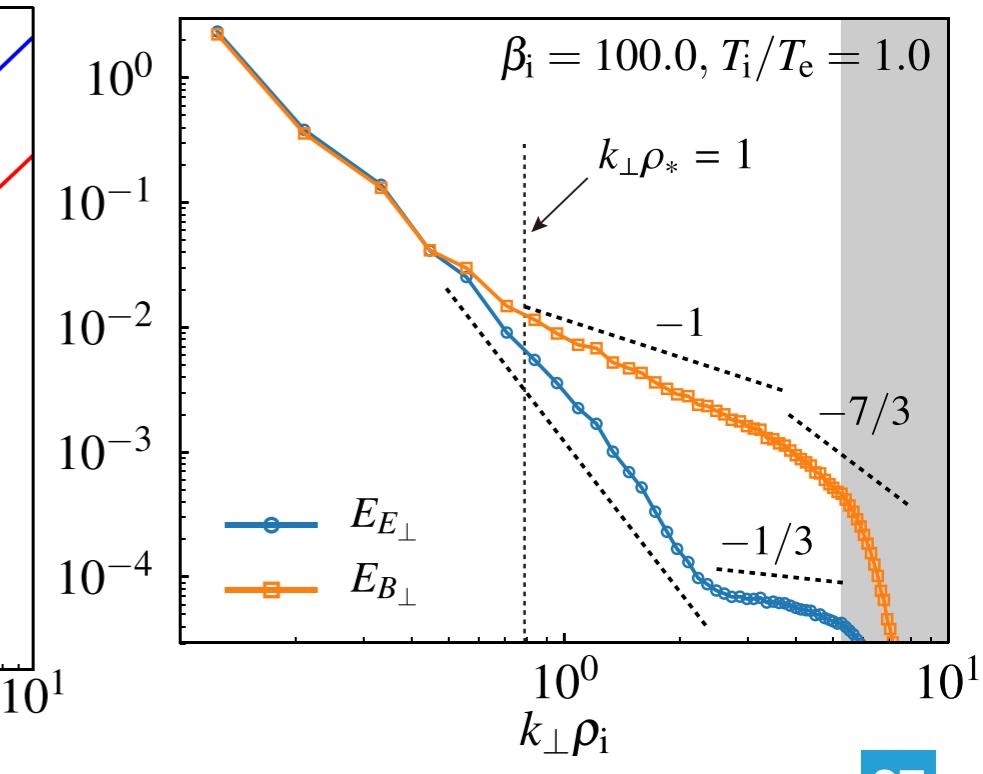
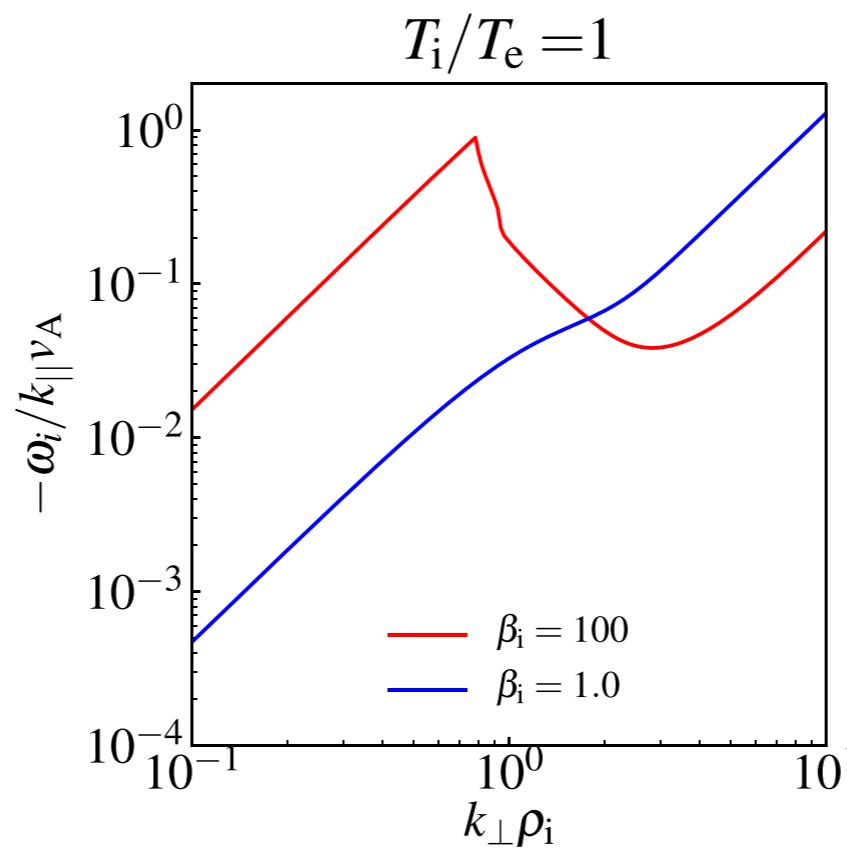
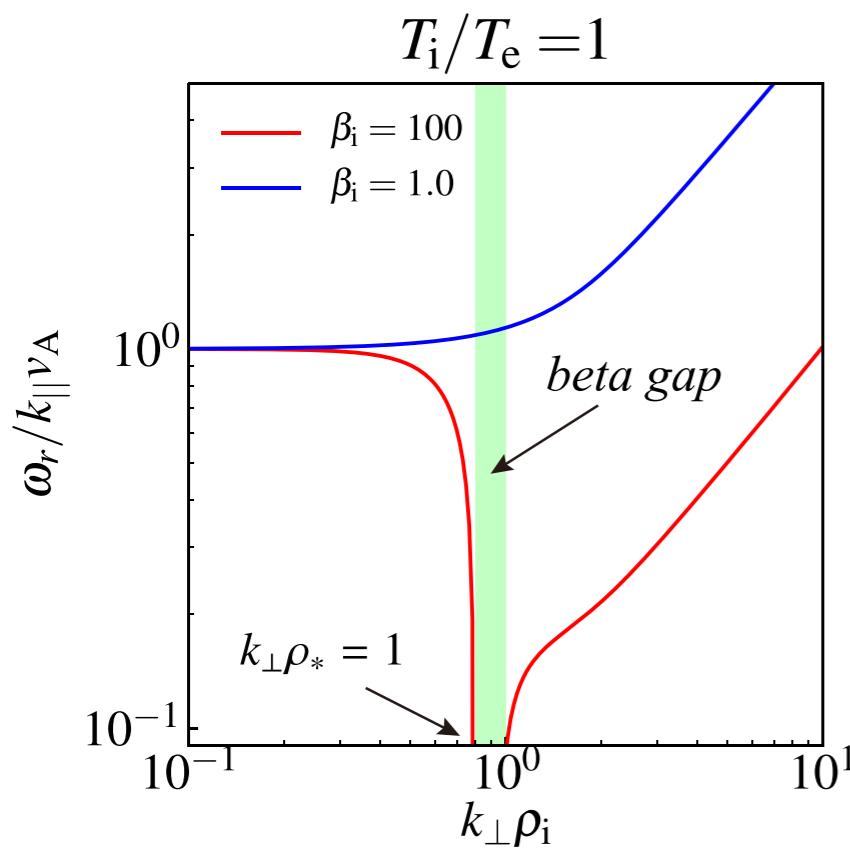
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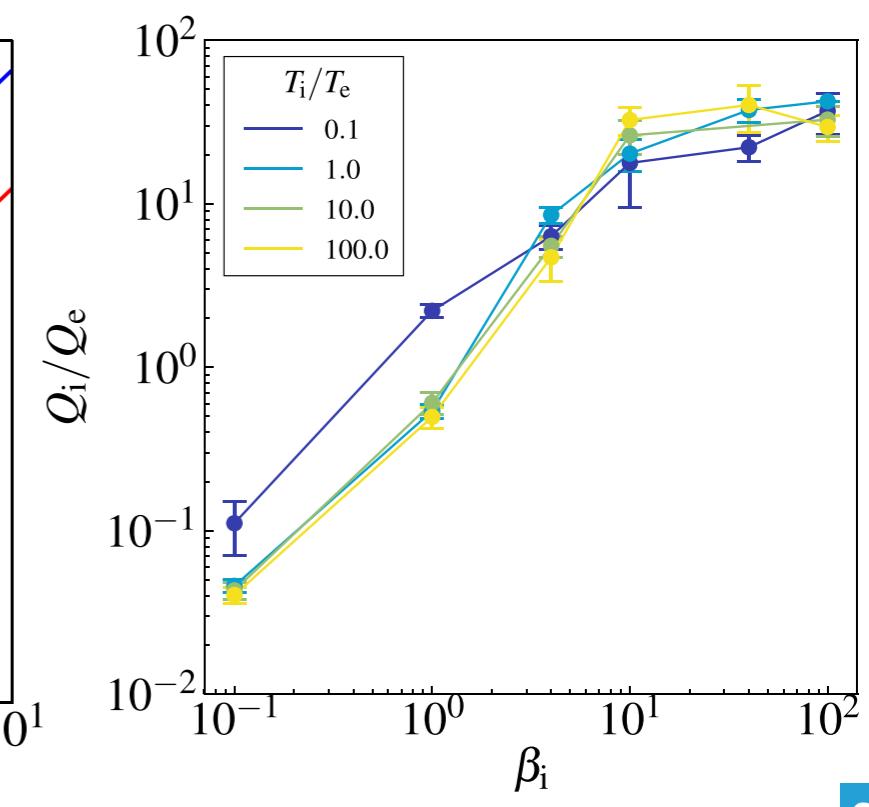
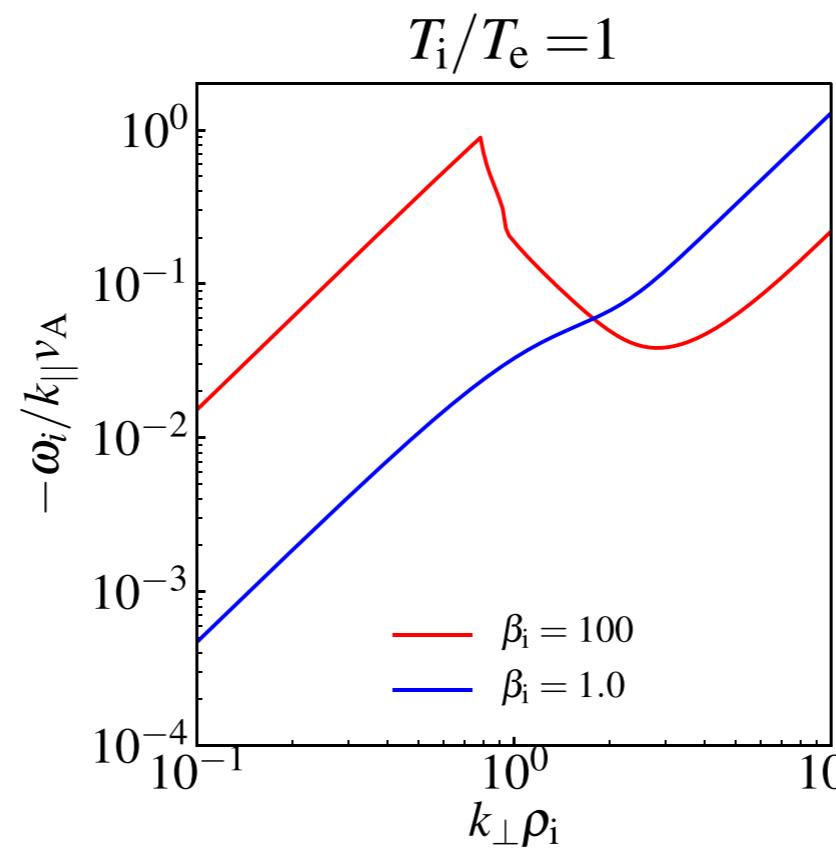
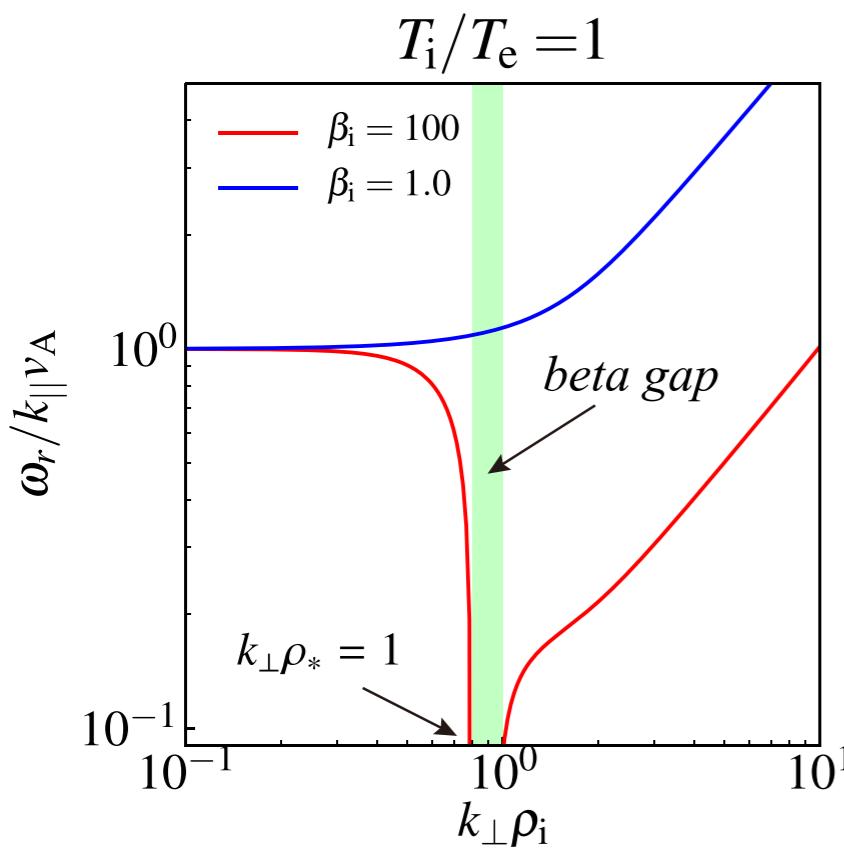
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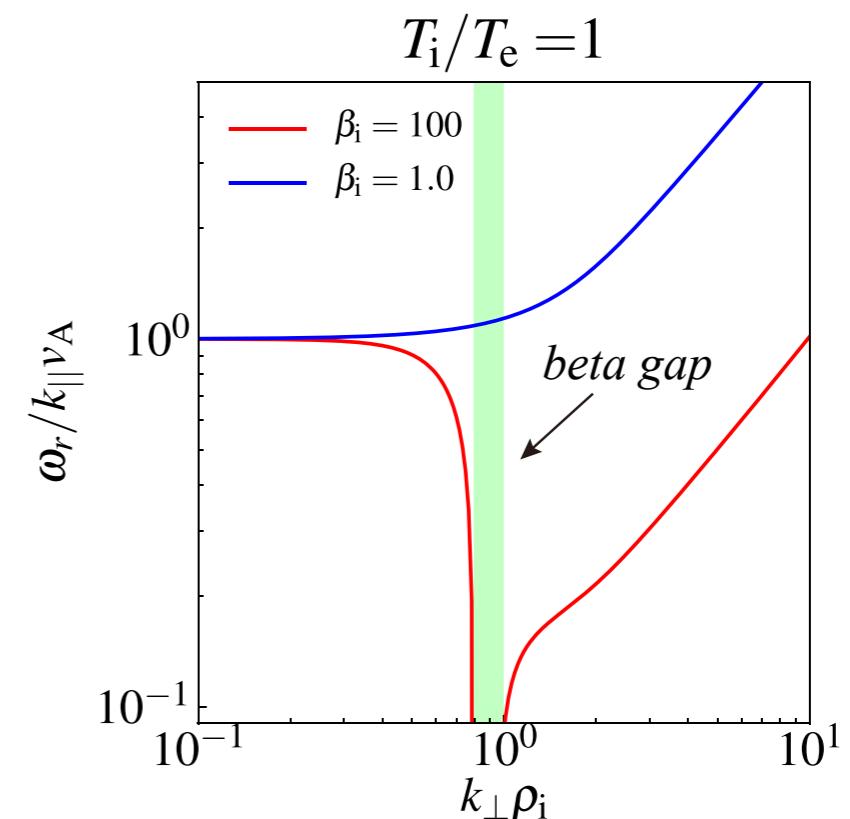
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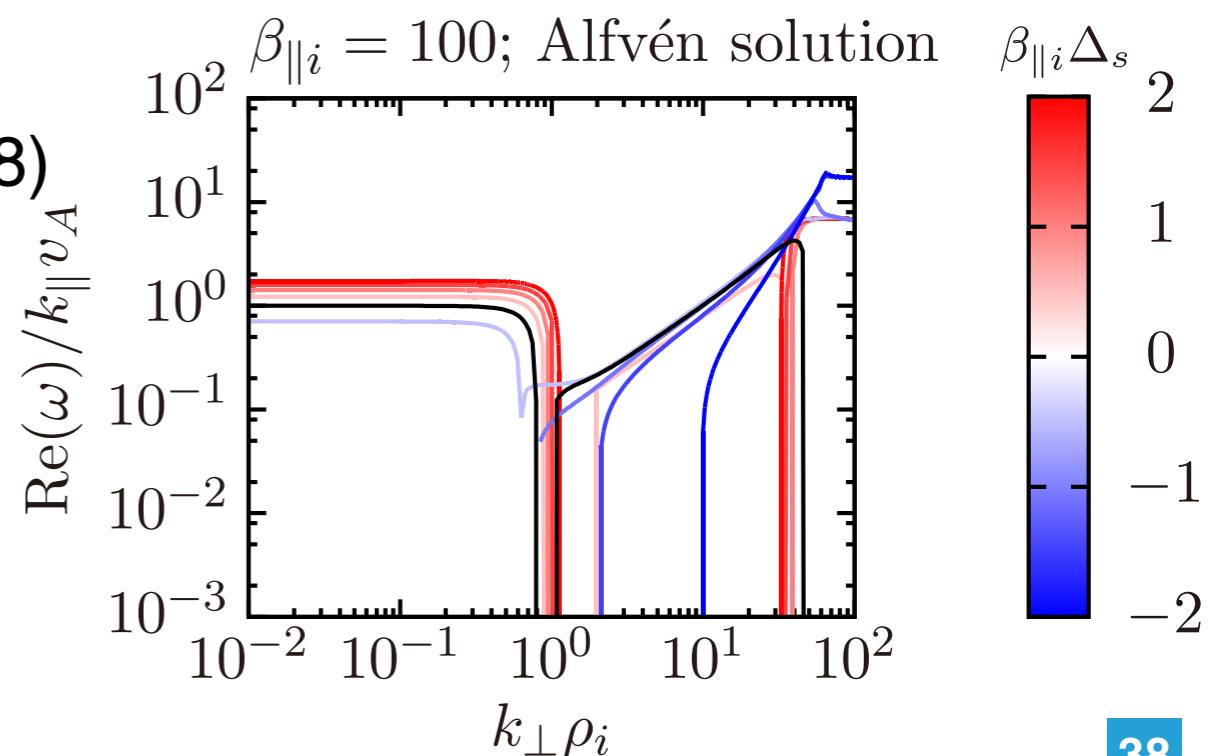
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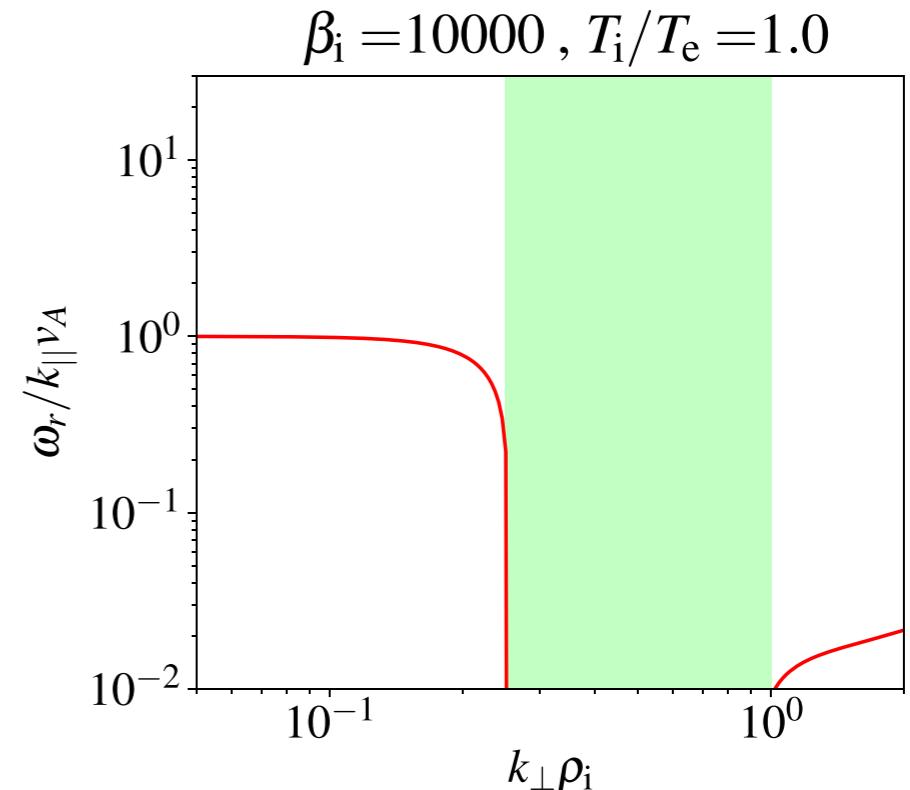
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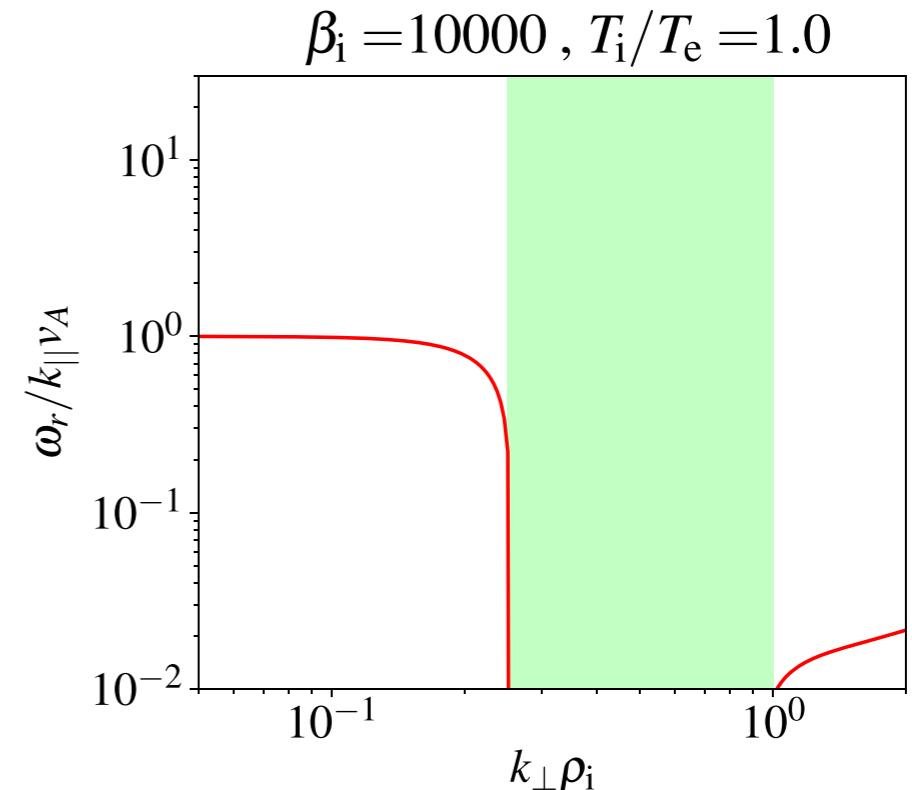
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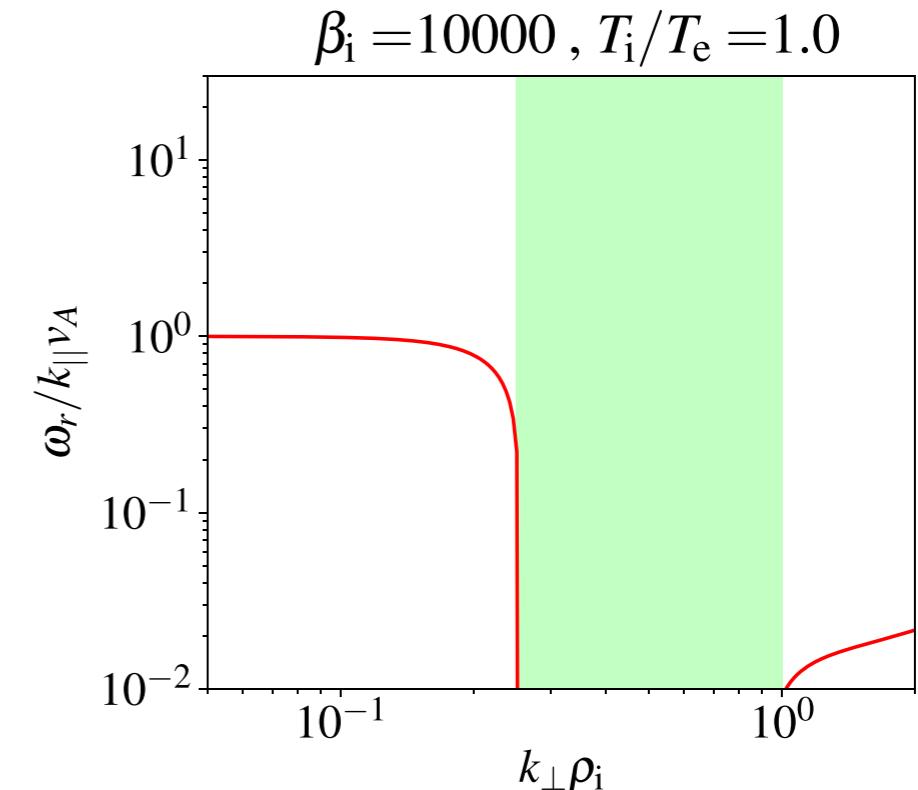
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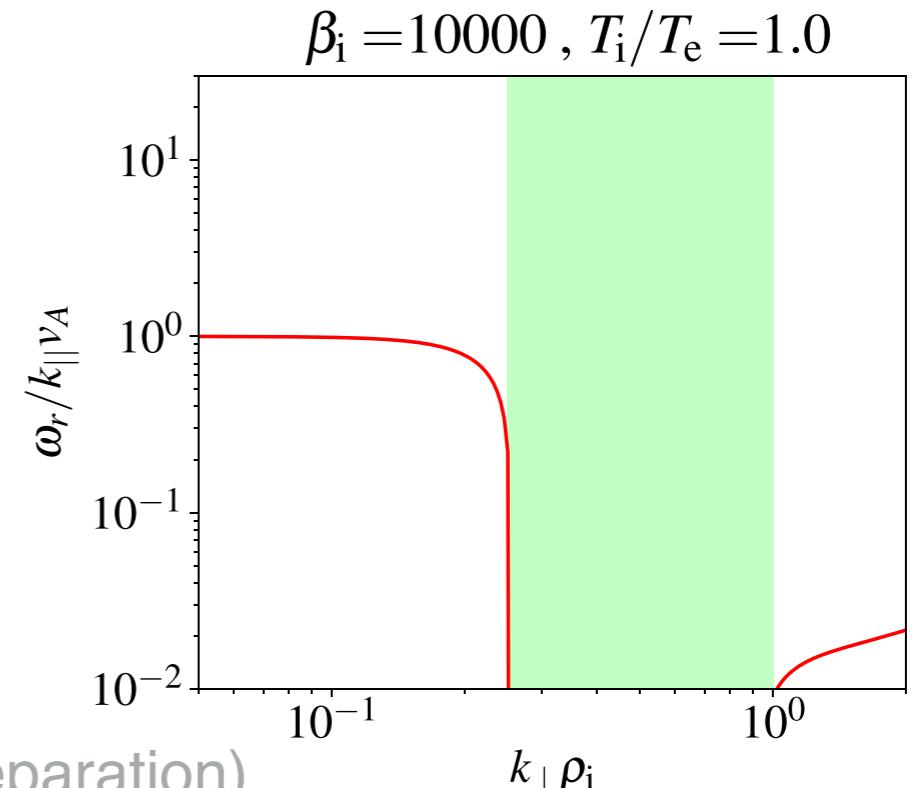
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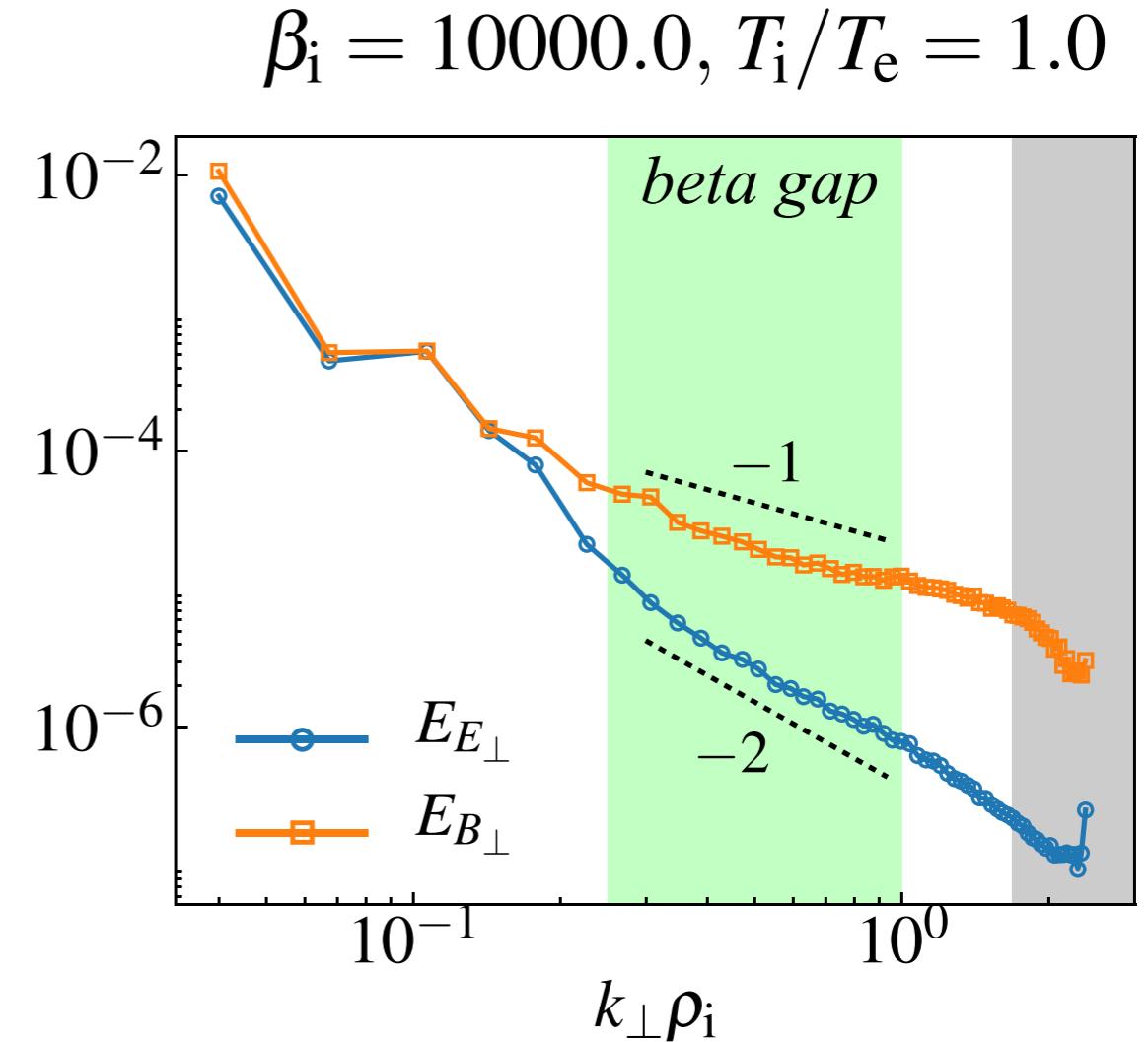
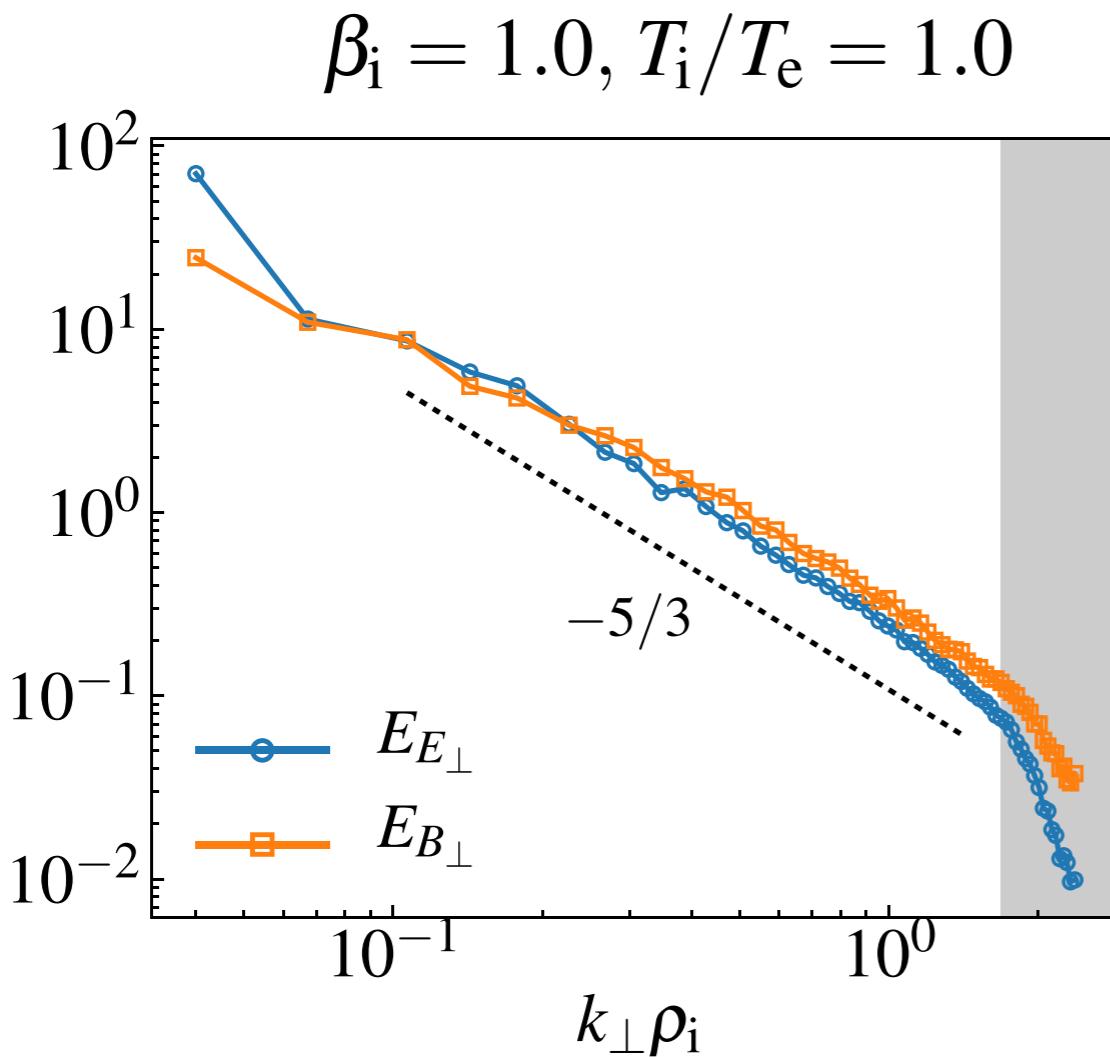
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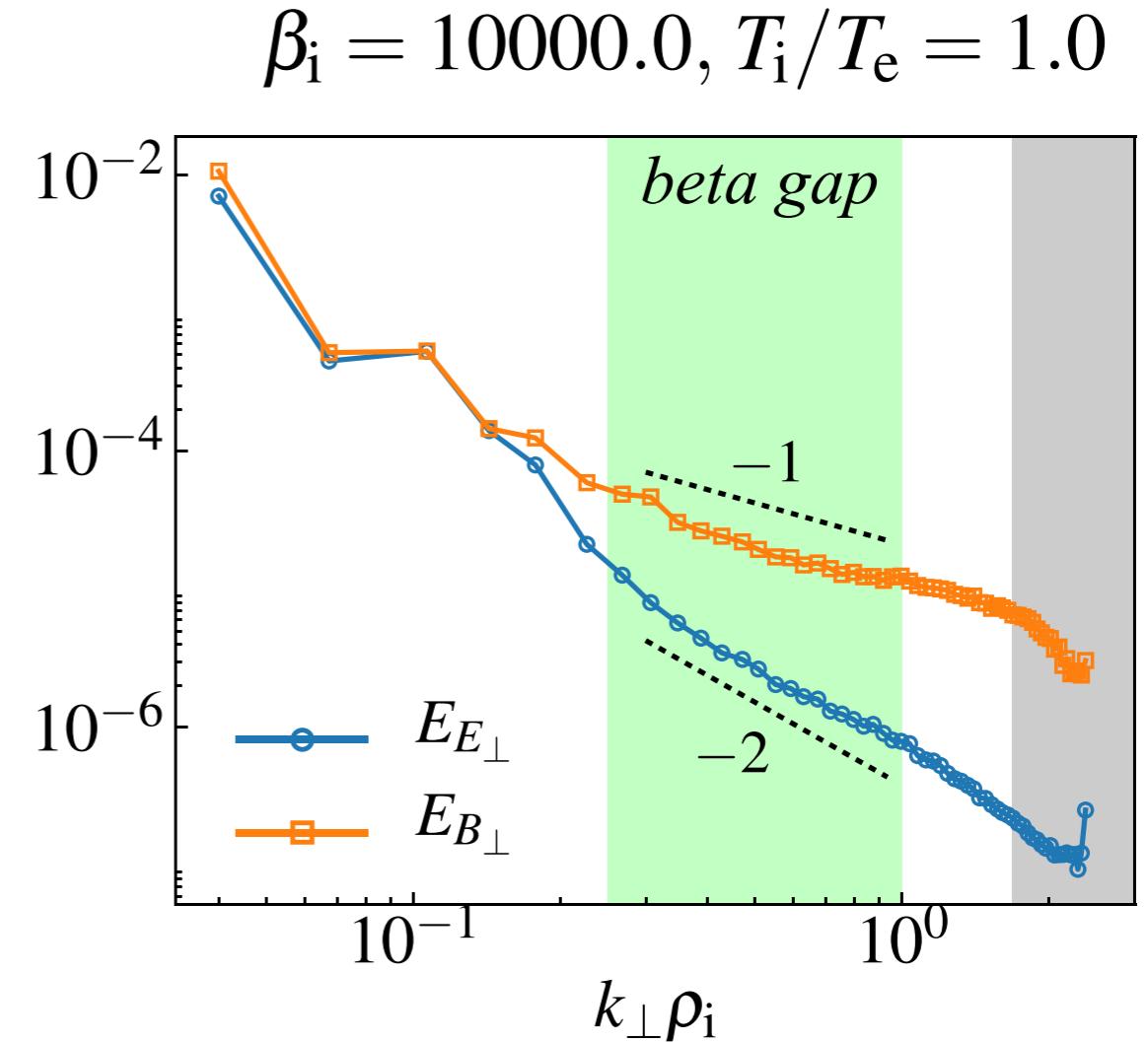
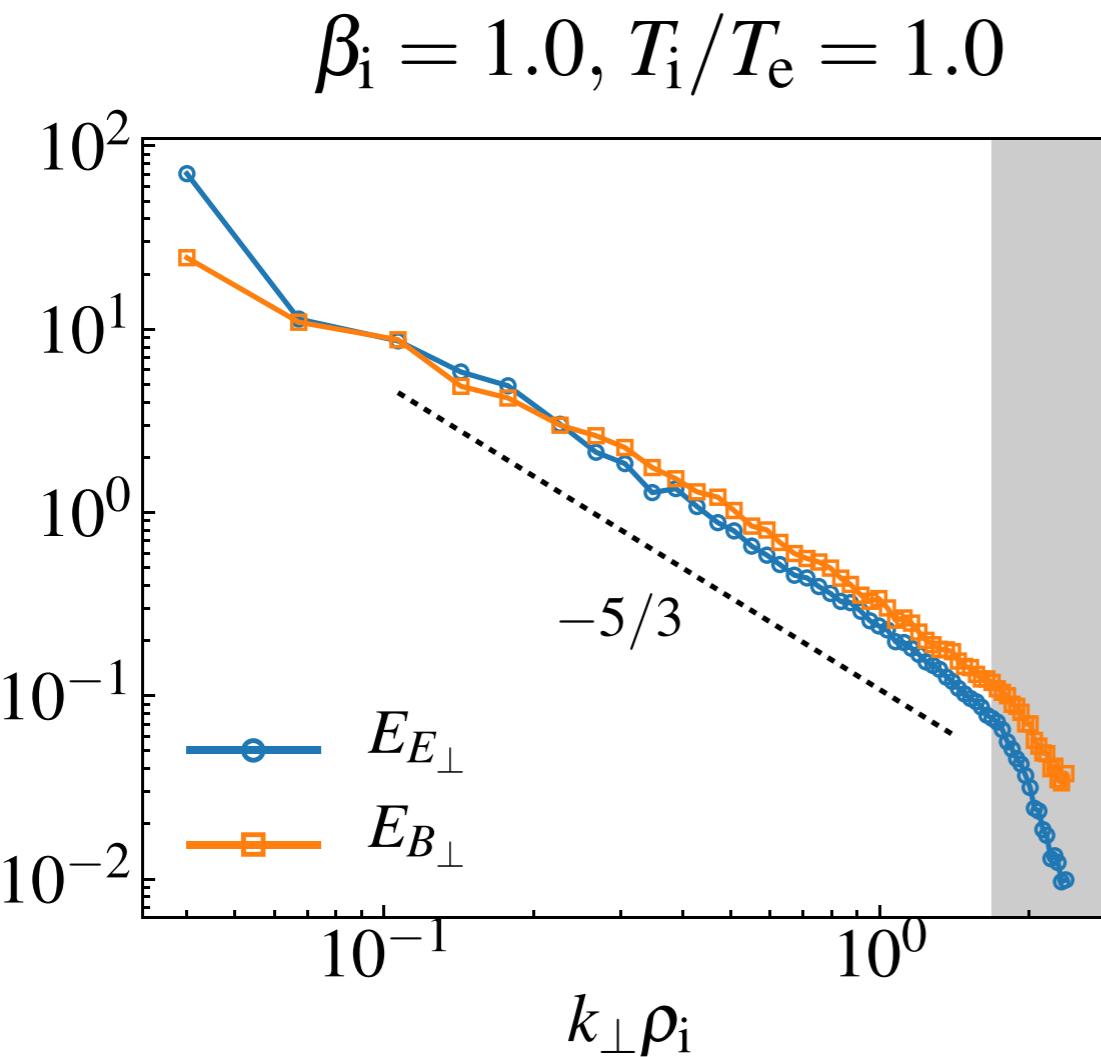
- ✓ Ultra-high β_i is numerically difficult
- ✓ $\beta_i^{1/2}$ times more computational time
- ✓ 4D model for the gap (Schekochihin+ in preparation)



Result : k-spectrum

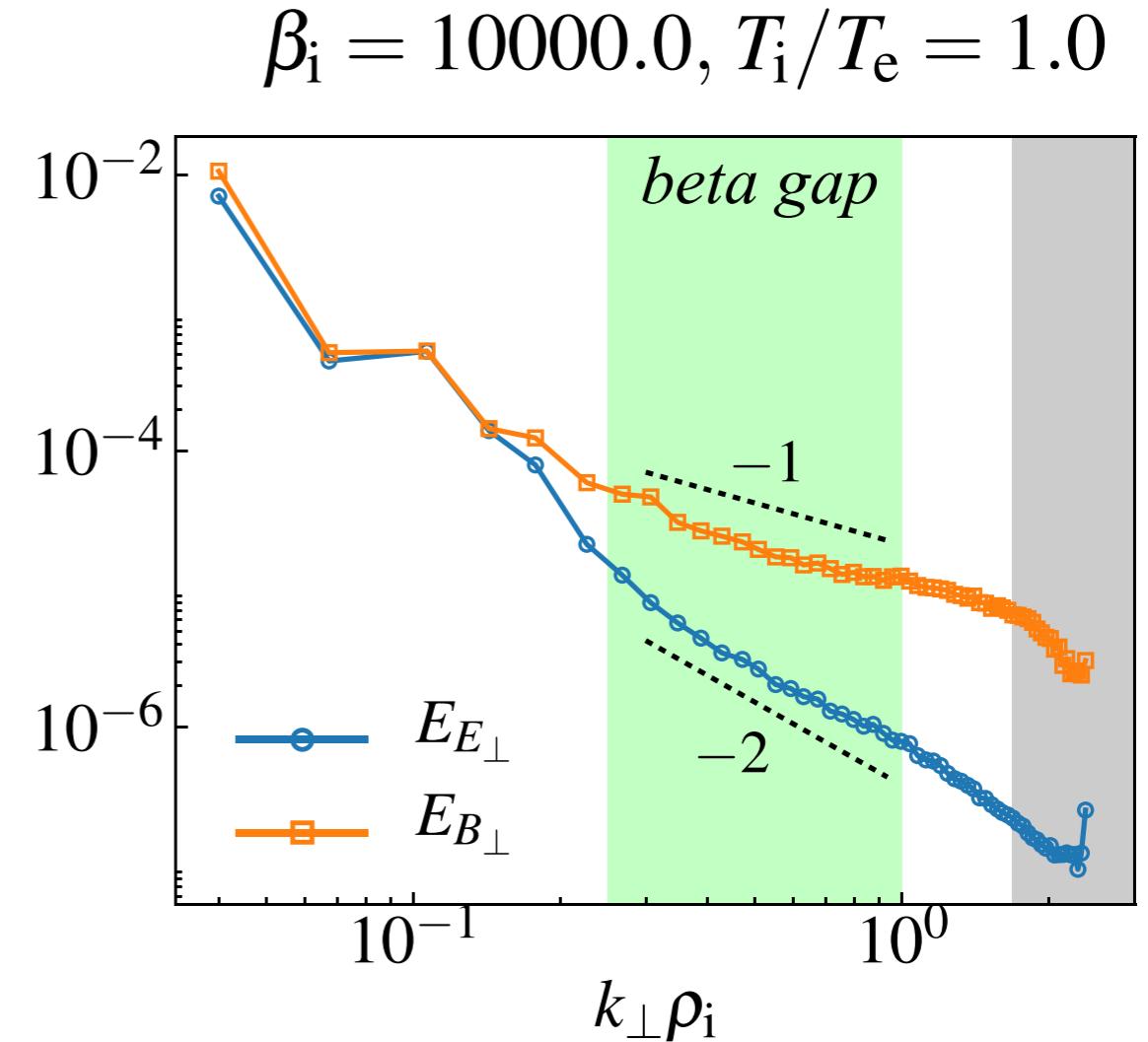
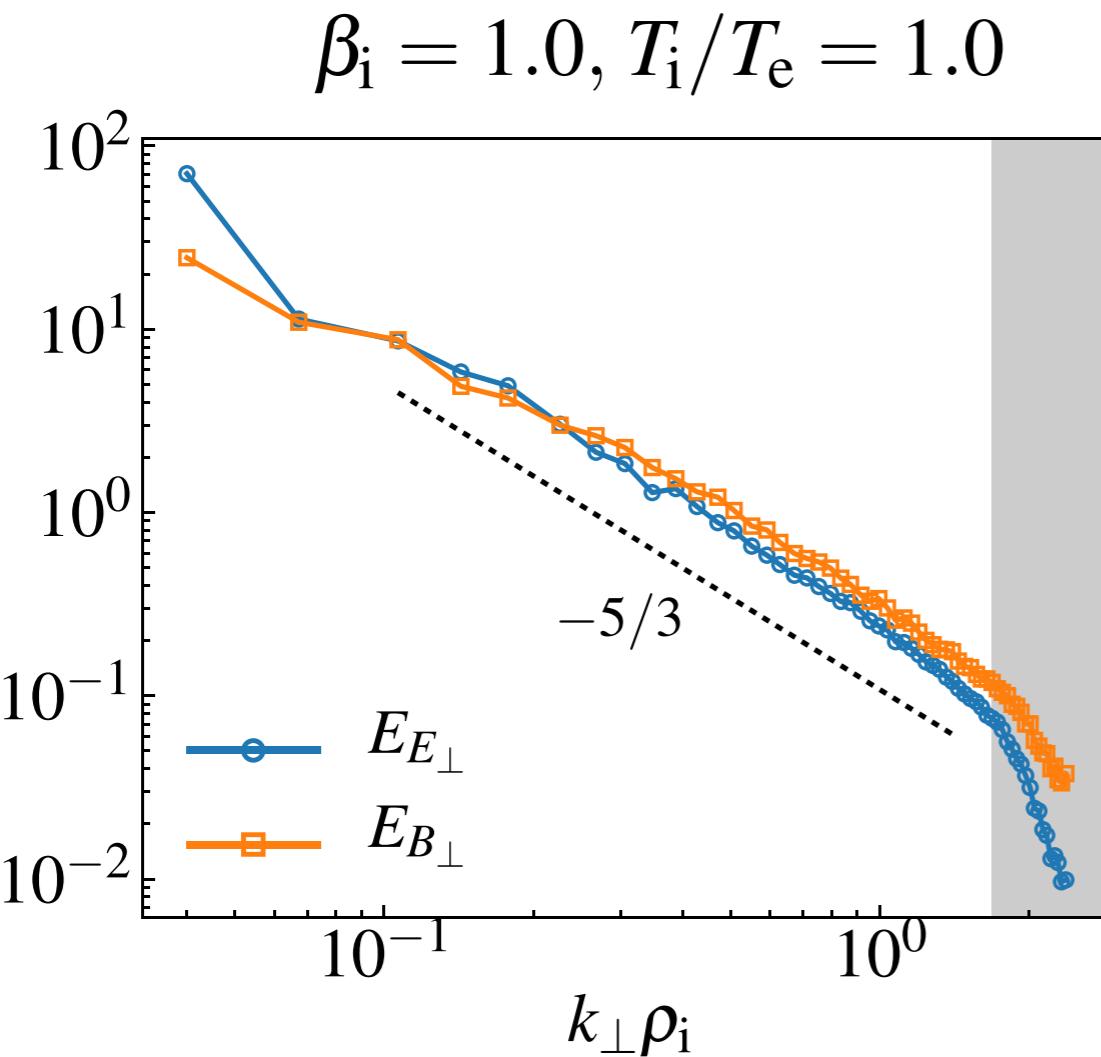


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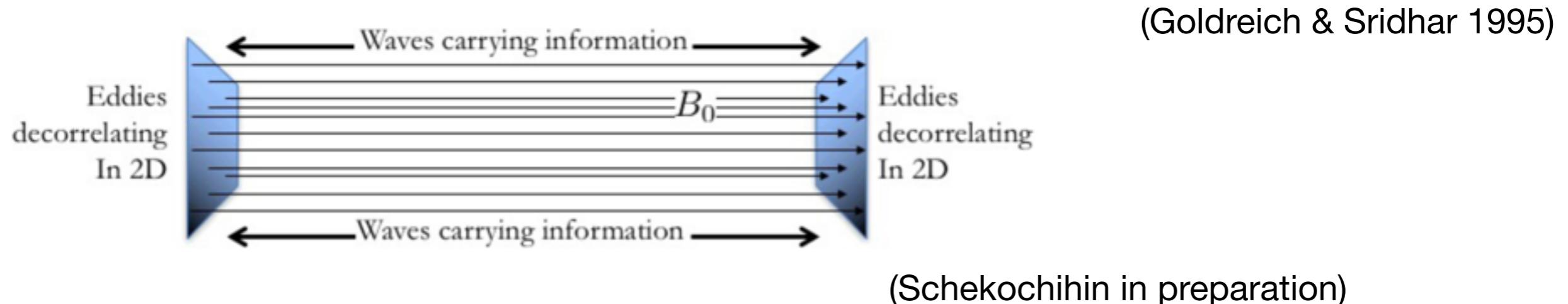
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- ✓ The anomaly of spectrum emerges around $k_\perp \rho^* = 1$
- ✓ Magnetically dominated turbulence in the gap (and maybe further)

Critical balance

- ✓ Nonlinear decorrelation time \sim linear propagation time; $\omega_{\text{nl}} \neq \omega_A$

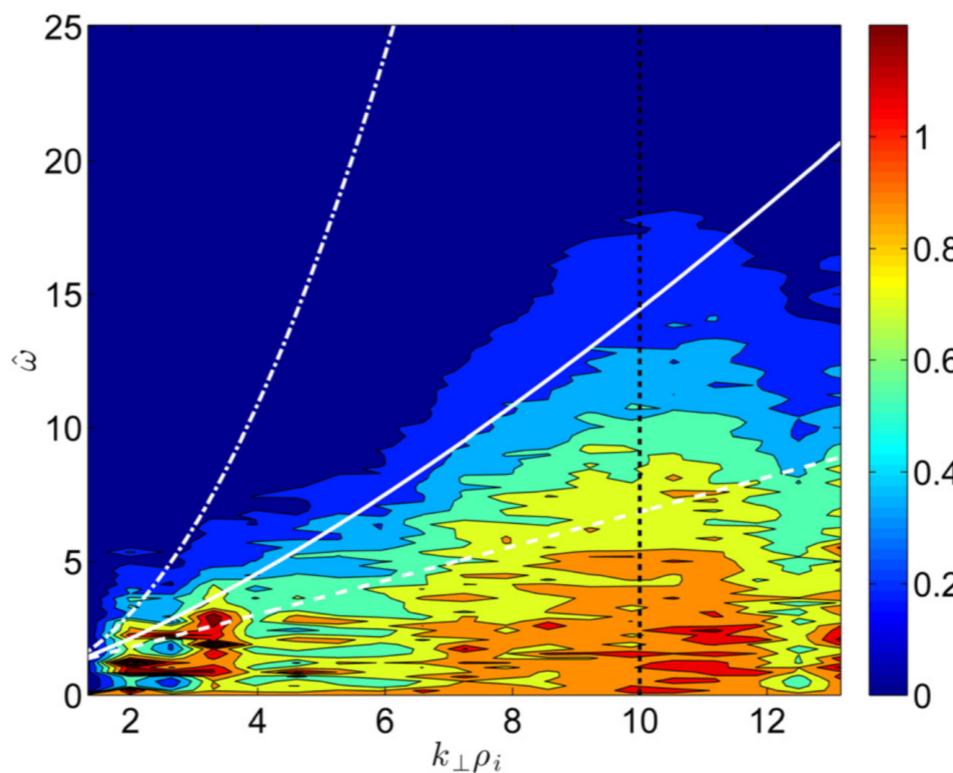


- ✓ Confirmed numerically and experimentally
- ✓ What would happen to CB in the gap, $\omega_A = 0$?
- ✓ CB is an essential assumption in Howes' heating prescription

Result : time-spectrum

✓ Spatio-temporal spectrum

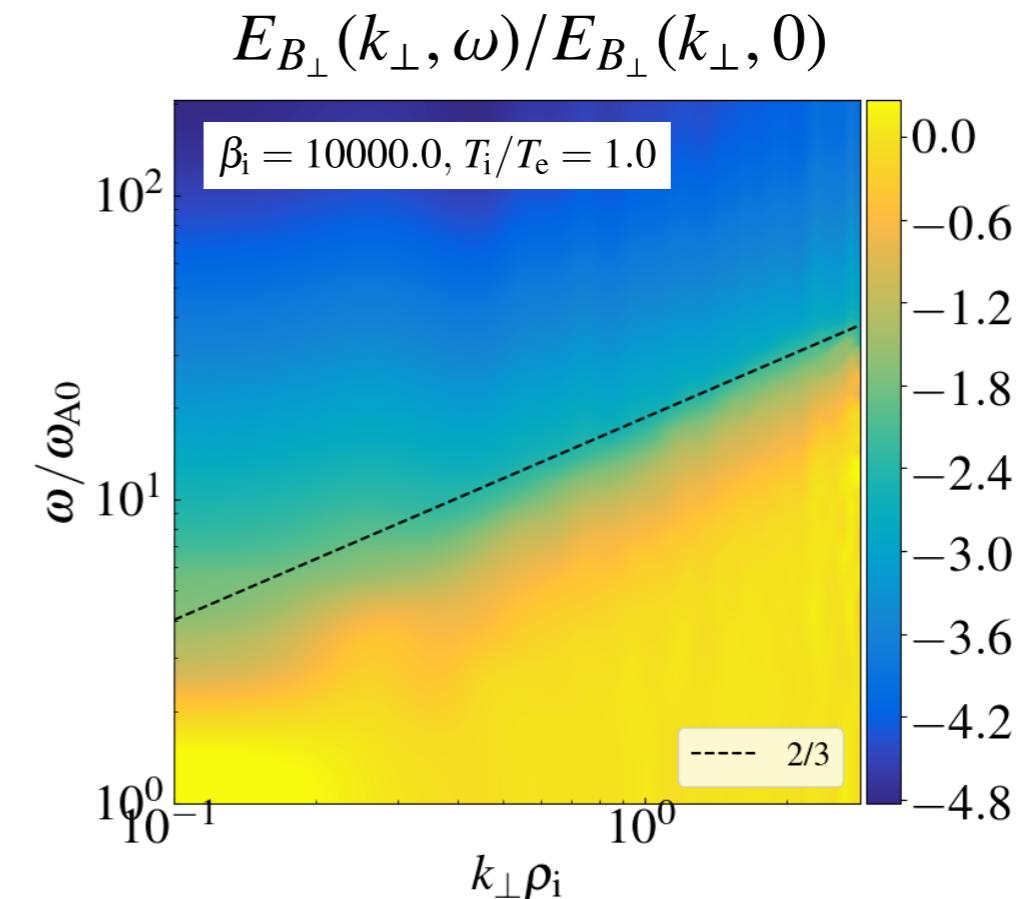
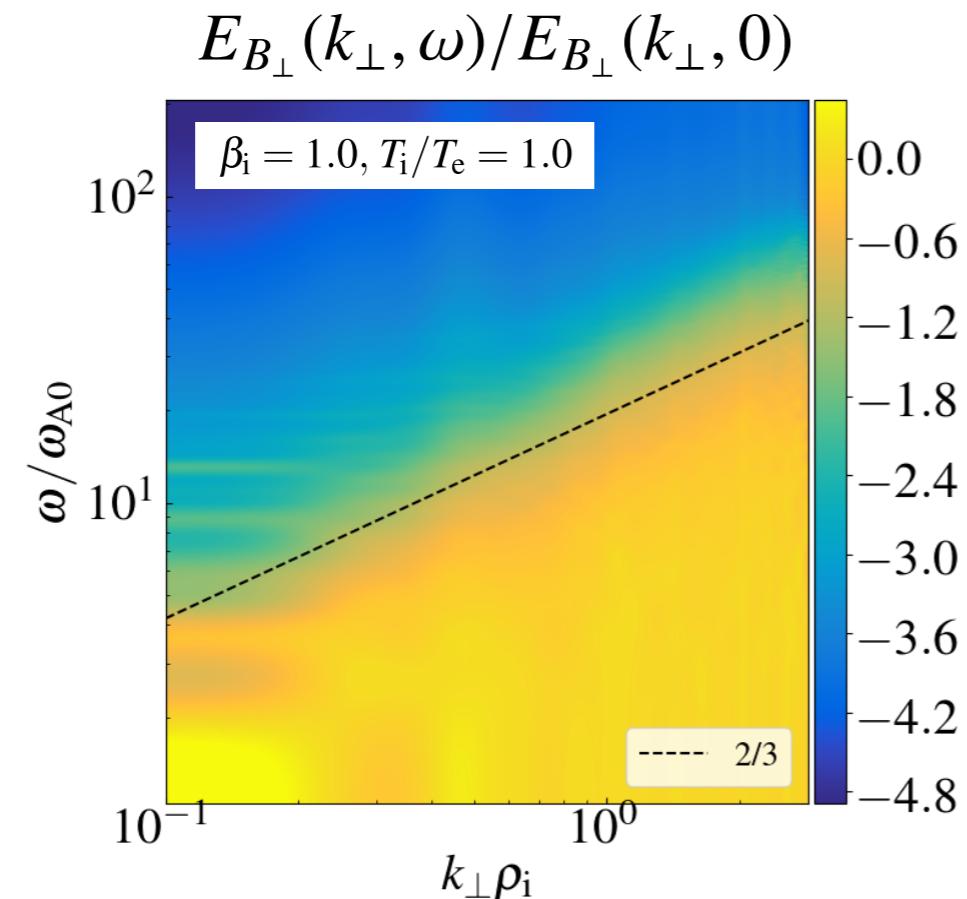
$$E_{B_\perp}(k_\perp, \omega)/E_{B_\perp}(k_\perp, 0)$$



Sub Larmor spectrum for $\beta_i = 1$ (TenBarge & Howes 2012)

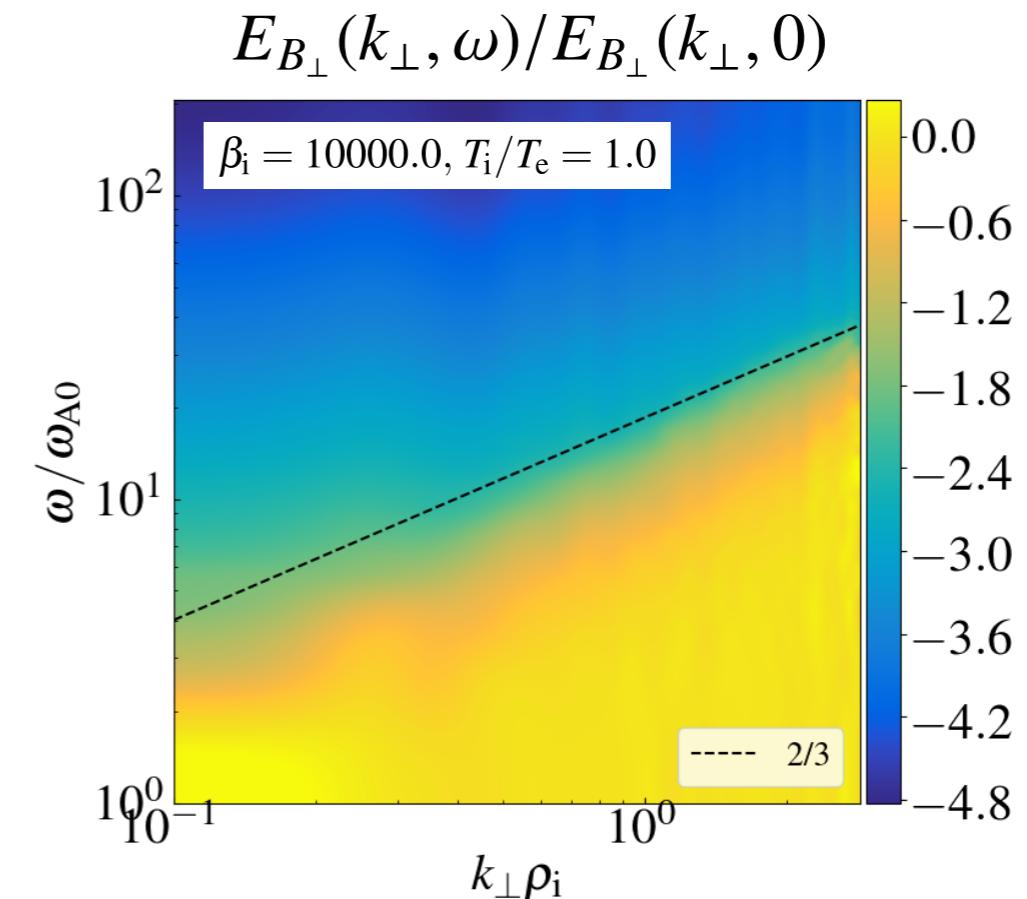
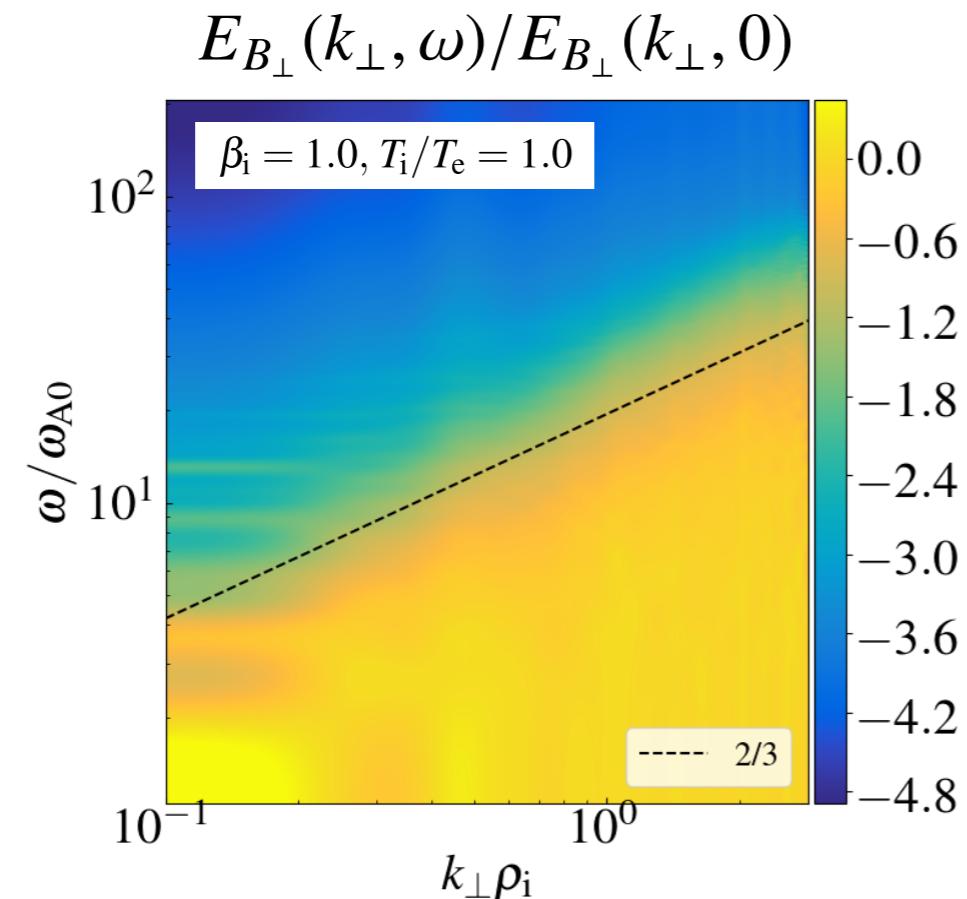
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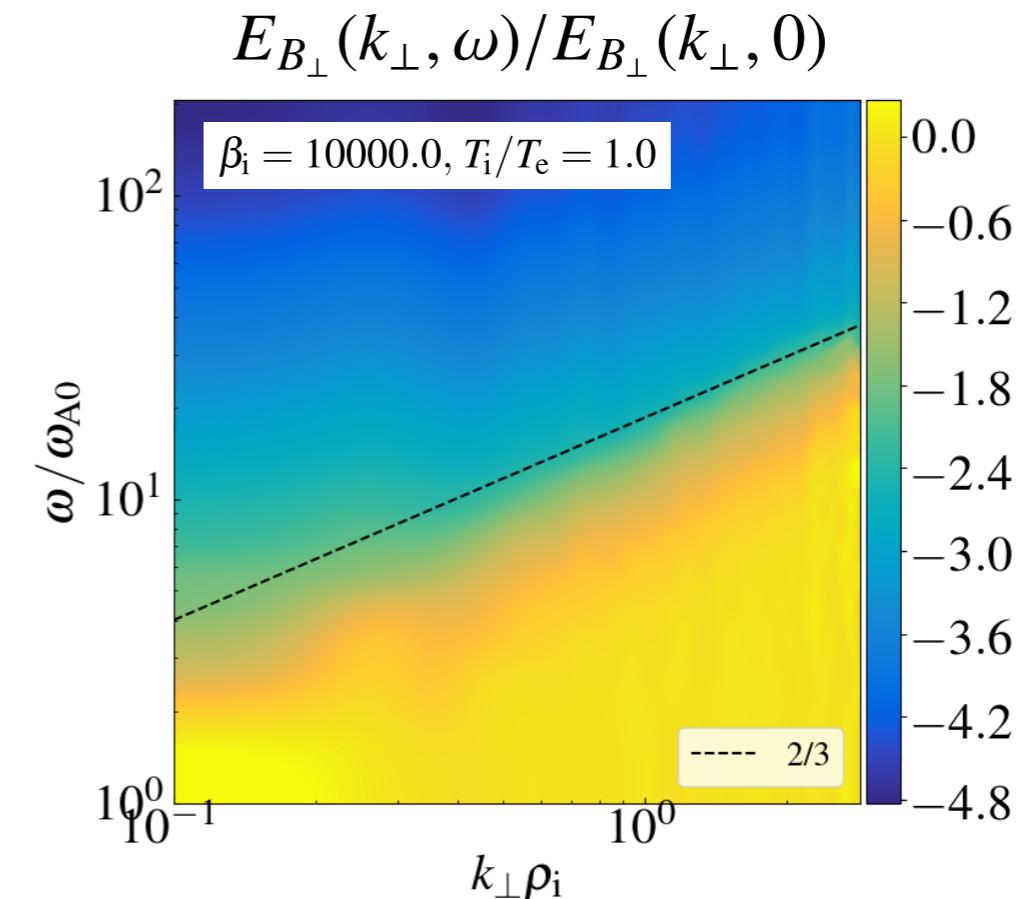
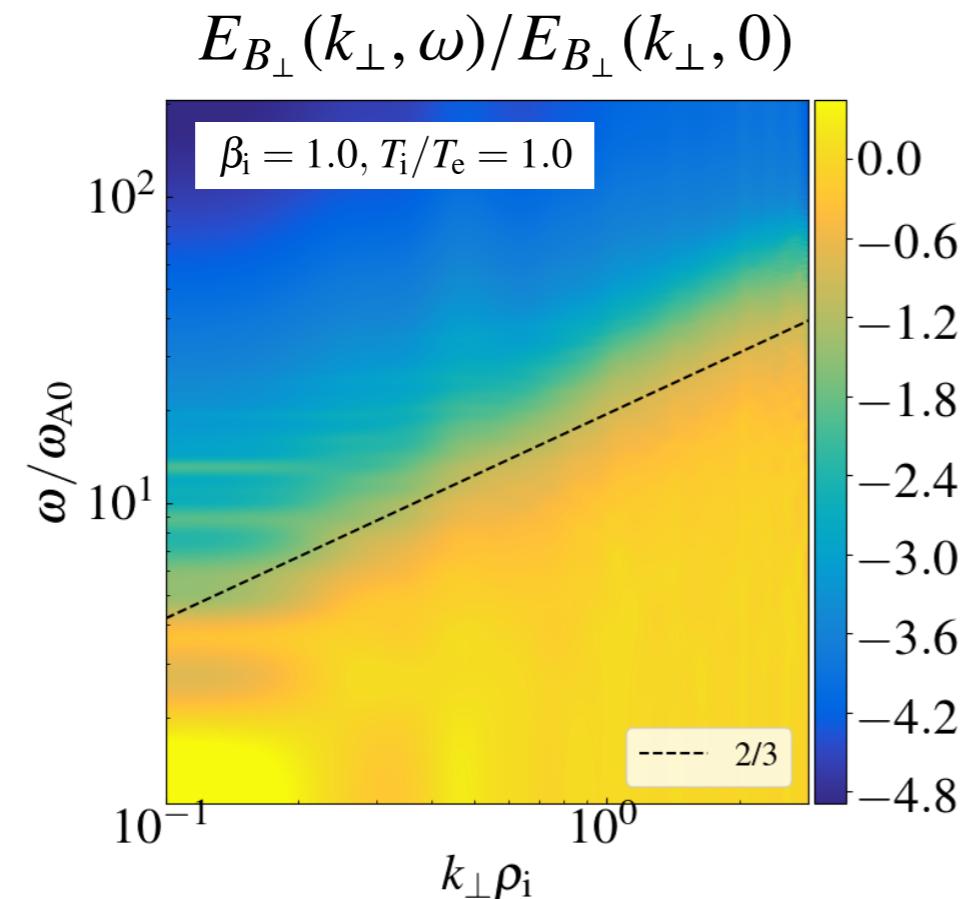
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✓ For $\beta_i = 1$ (the same as MHD), $\omega_{nl} = \omega_{AW} = k_{\parallel} v_A \sim k_\perp^{2/3}$

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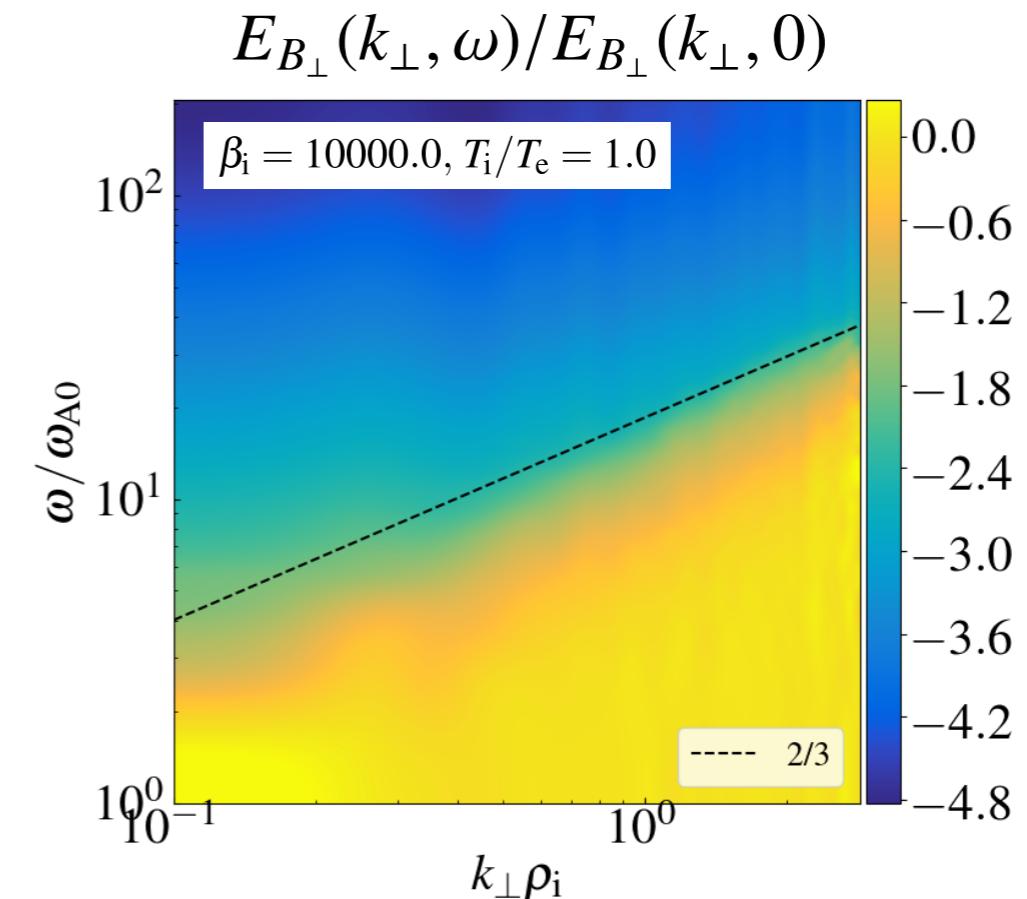
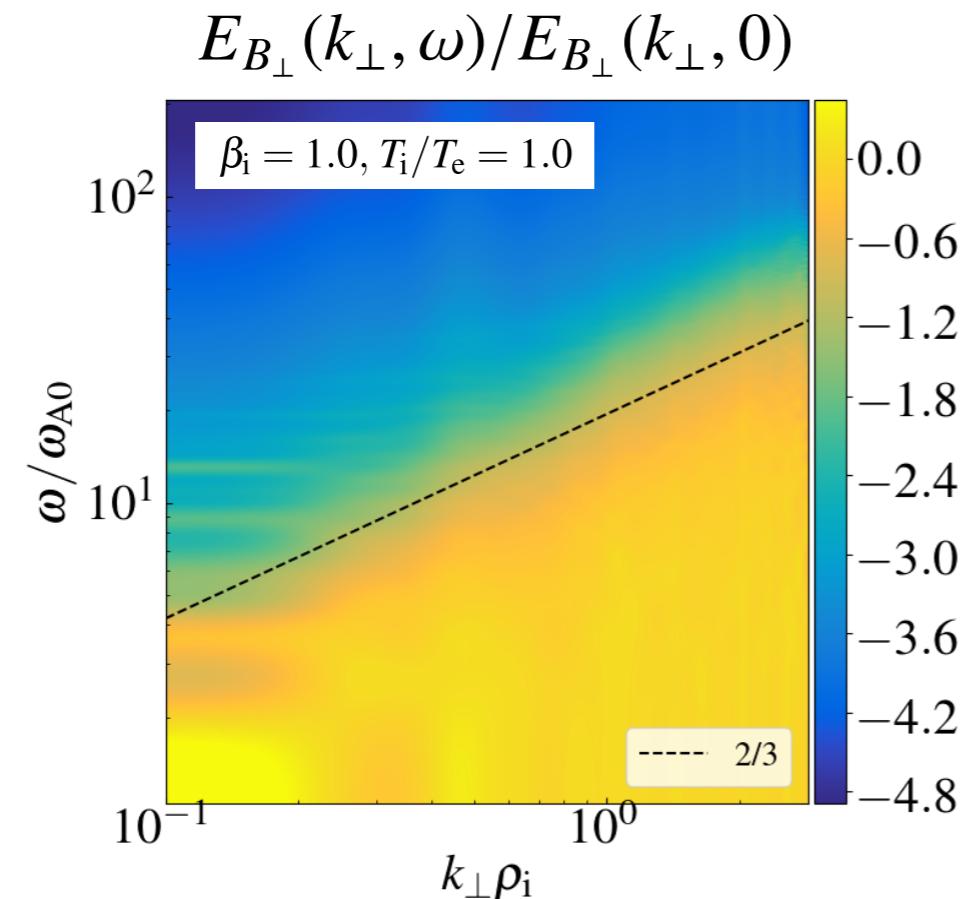
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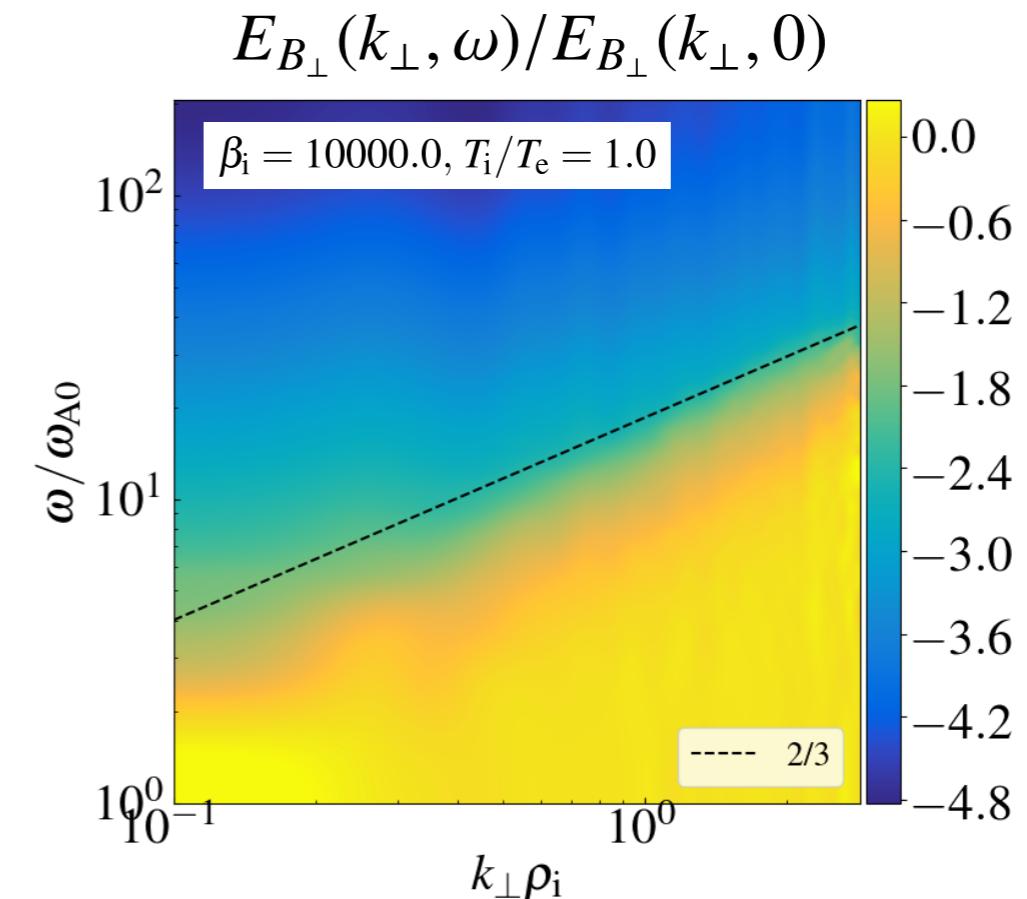
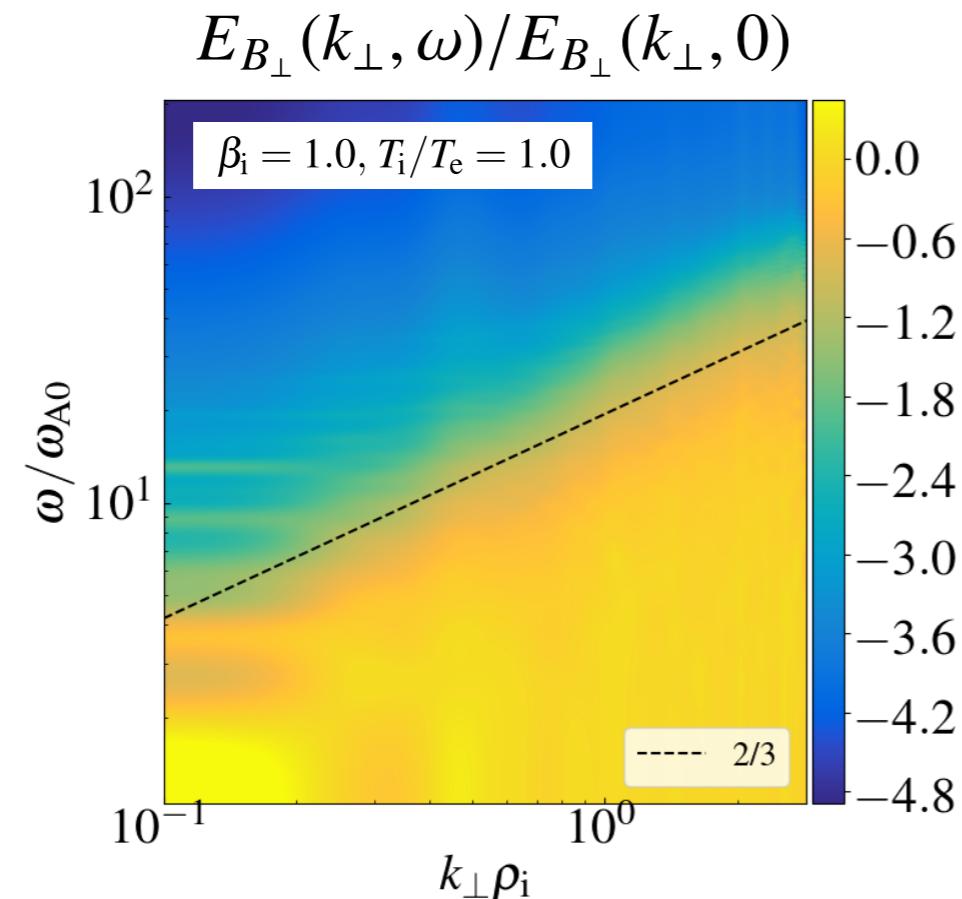
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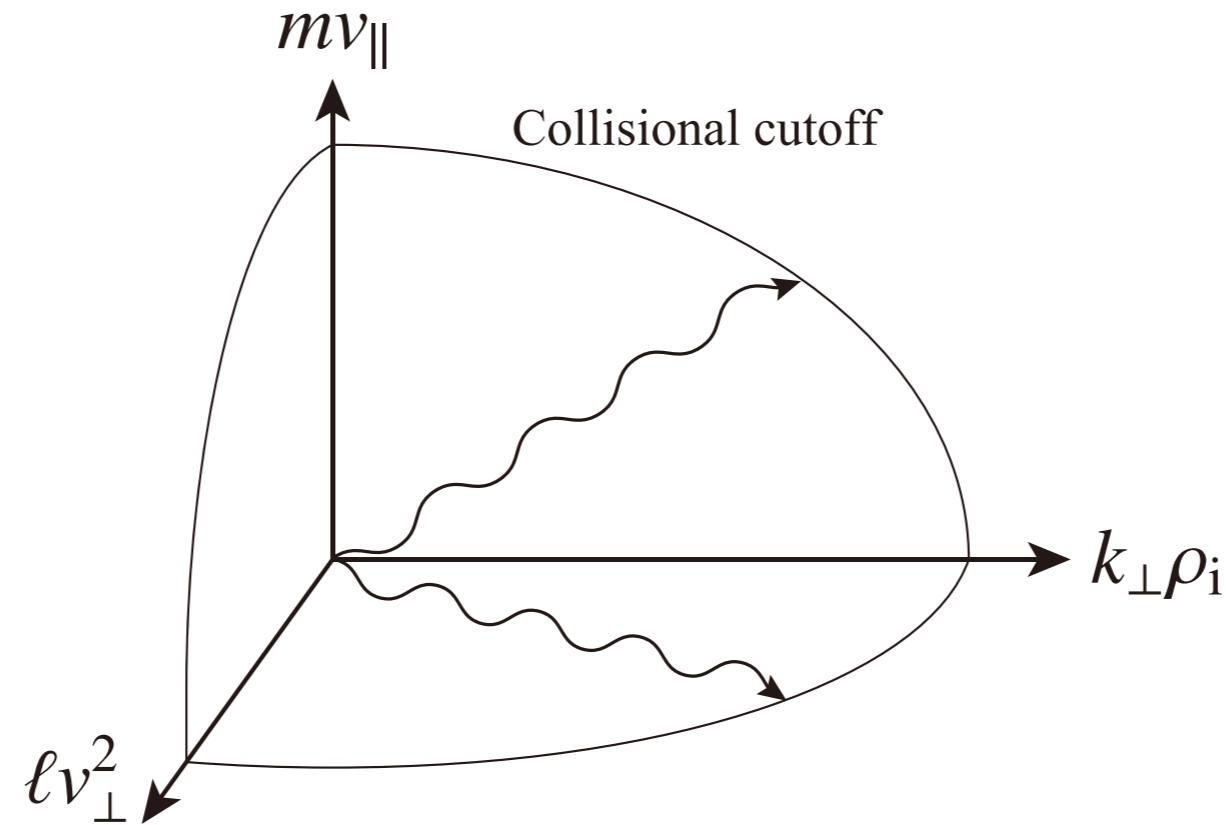
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- ✓ How is this related to the ceiling? → future work

Phase space cascade



- ✓ The energy related to the ion distribution function, $\int d^3x \int d^3v \delta f^2/2$, cascades in 5D phase space
- ✓ Emergence of the small scale structure in the velocity space
- ✓ Eventually hits the collisional cutoff \rightarrow heating
- ✓ Where this dissipation occurs is important
 - ➡ spectral analysis in the phase space