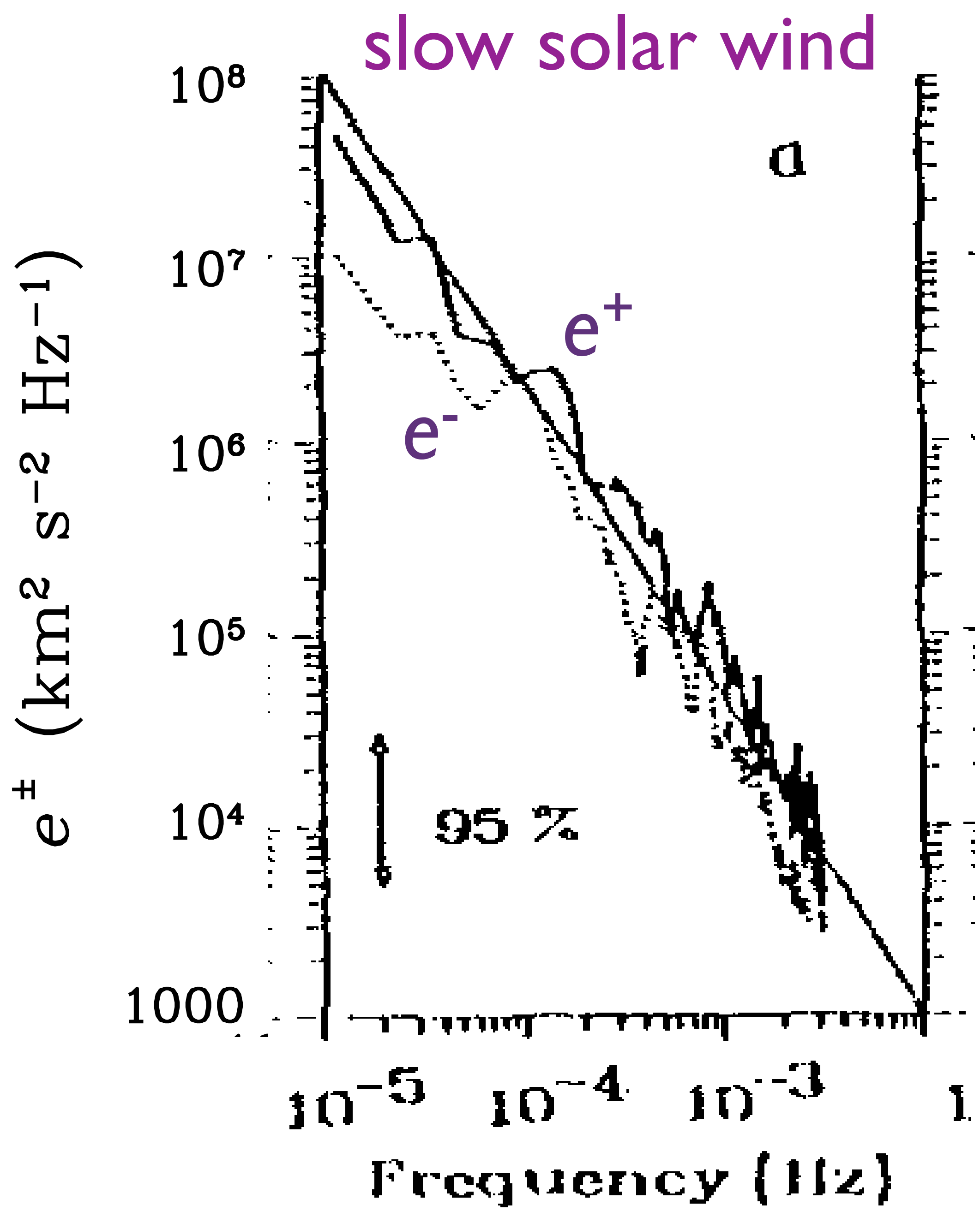
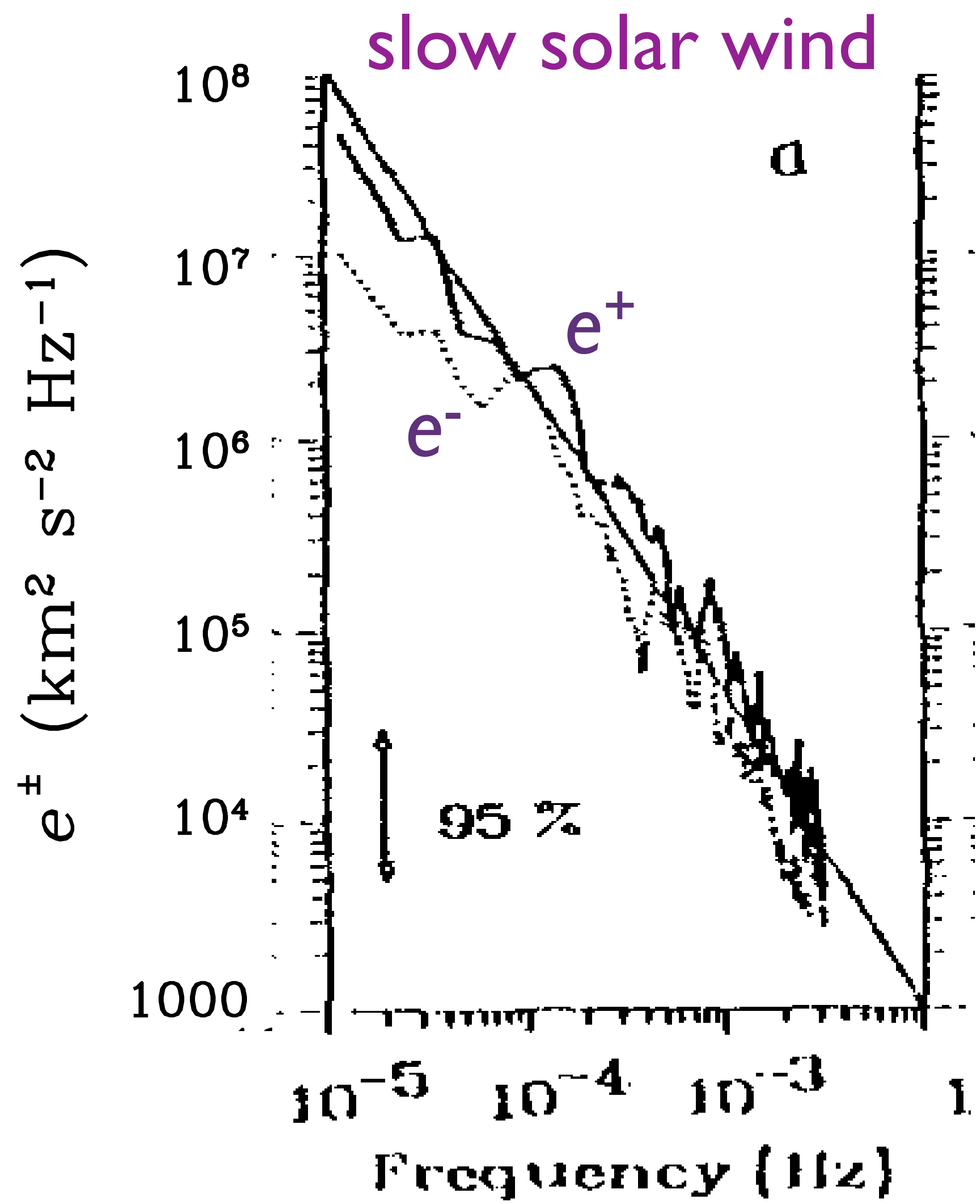
Frequency Spectrum of Outward (e+) and Inward (e-) Propagating Alfven Waves

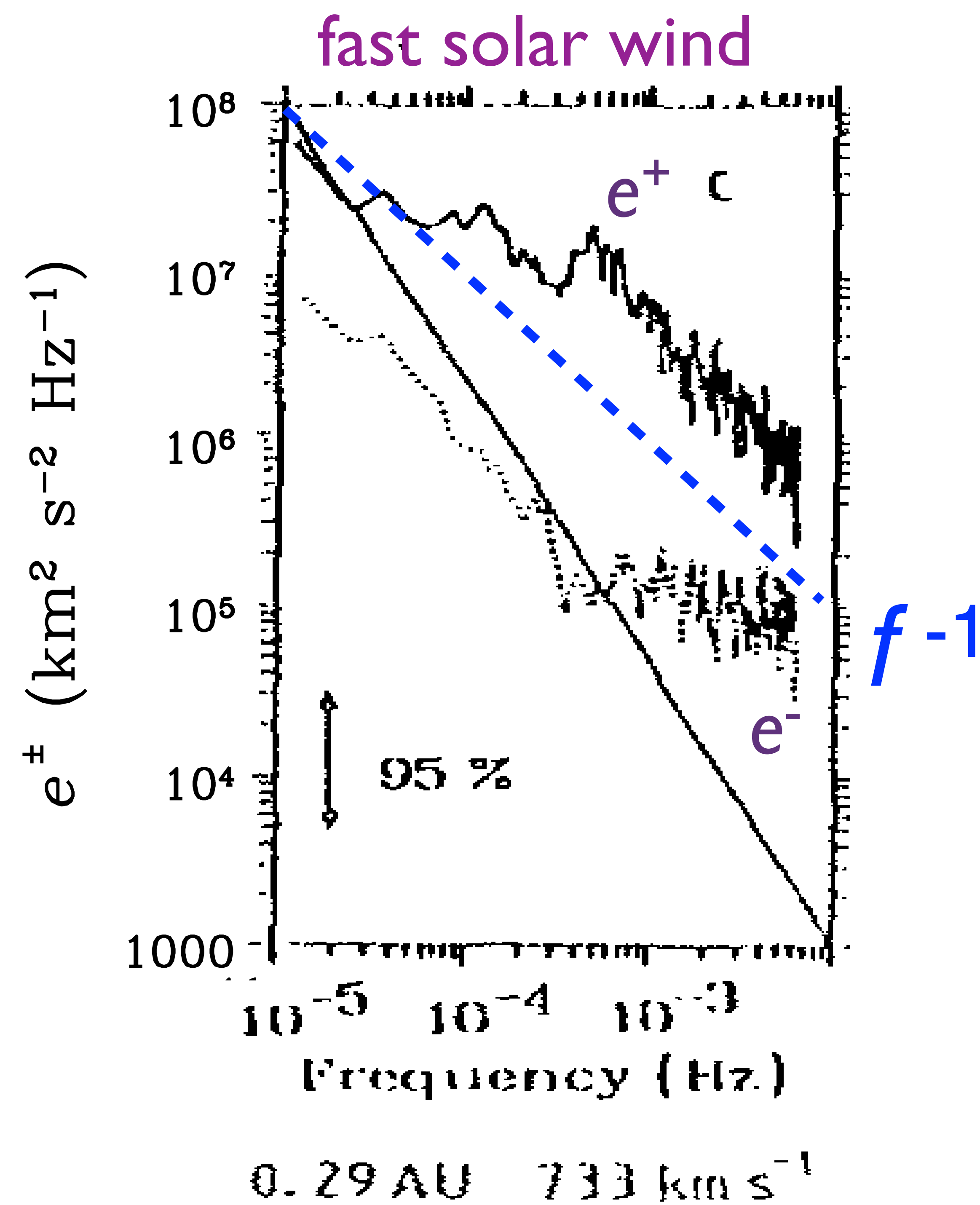


- Solar-wind turbulence is mostly non-compressive ($dn/n_0 << dB/B_0$)
- Incompressible MHD turbulence should have an $f^{-5/3}$ inertial-range power spectrum
- Problem solved.

Helios measurements at 0.3 AU (Tu & Marsch 1995)

Not so *fast* !





Helios measurements at 0.3 AU (Tu & Marsch 1995)

Parametric Instability in the Low-β Solar Wind

- An outward-propagating Alfven wave (AW) decays into an outward-propagating slow magnetosonic wave ("slow wave") and an inward-propagating AW.

- I will focus on fast solar wind at $r < 0.3$ AU.

- I'll take β to be small. ($\beta \sim 0.25$ at r=0.3 AU, and β is smaller at smaller r.)

- I'll use weak turbulence theory: $\omega_{nl}/\omega_{linear} \sim (\delta v_{rms}/v_A)^2 \sim 1/4$ at r=0.4 AU. Even smaller at smaller *r*. (No $k_z = 0$ problem as in incompressible MHD.)

Weak Compressible MHD Turbulence at Low Beta

- Perturbation theory to describe wave-wave interactions. ($\omega_{\text{nonlinear}} << \omega_{\text{linear}}$)
- Add collisionless damping terms post facto. (Strong slow-wave damping.)
- Resonant 3-wave interactions:
  - $\omega_k = \omega_p + \omega_q$
  - $\vec{k} = \vec{p} + \vec{q}$

$$\vec{B}_0 = B_0 \hat{z}$$

- $A_k^{\pm}$ = 3D power spectrum of Alfvén waves propagating in $\pm z$ direction.
- $S_k^{\pm}$ = 3D power spectrum of slow waves propagating in $\pm z$ direction.
- F_k = 3D power spectrum of fast waves propagating in **k** direction.

Wave Kinetic Equations for Weak MHD Turbulence

(Chandran 2008)

$$\frac{\partial S_k^{\pm}}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big[\delta(q_z)4k_\perp^2\overline{m}^2\left(A_q^+ + A_q^-\right)\left(S_p^{\pm}-S_k^{\pm}\right) + \delta(p-q)k_z^2 l^2 F_p F_{-q} + \delta(p_z-q_z)k_z^2 l^2 A_p^+ A_q^-$$
$$+ \delta(p_z+q)k_z^2\bar{l}^2\left(A_p^+F_q + A_p^-F_{-q}\right) + \delta(p_z-q)k_z^2\bar{l}^2\left(A_p^+F_{-q}+A_p^-F_q\right)\Big] - 2\gamma_{s,k}^{\pm}S_k^{\pm},$$

$$\frac{\partial A_k^+}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{\delta(q_z)8(k_\perp n\overline{m})^2A_q^-\left(A_p^+-A_k^+\right) + \delta(k_z+p_z+q)k_z\Lambda_{q-pk}\left(k_zA_p^-F_{-q}+p_zF_{-q}A_k^+ + qA_p^-A_k^+\right)$$
$$+ \delta(k_z+p_z-q)k_z\Lambda_{q-pk}\left(k_zA_p^-F_q + p_zF_qA_k^+ - qA_p^-A_k^+\right) + \delta(k_z-p+q)k_zM_{pk-q}\left(k_zF_pF_{-q} - pF_{-q}A_k^+ + qF_pA_k^+\right)$$
$$+ \delta(q-k_z)p_zA_k^+\left[2(k_z+p_z)F_q + p_zq\frac{\partial F_q}{\partial q}\right] + \delta(q+k_z)p_zA_k^+\left[2(k_z+p_z)F_{-q}+p_zq\frac{\partial F_{-q}}{\partial q}\right]$$
$$+ \varepsilon^{-2}k_z^2\left(S_p^+ + S_p^-\right)\Big[\delta(q-k_z)\overline{m}^2\left(F_q - A_k^+\right) + \delta(q+k_z)\overline{m}^2\left(F_{-q}-A_k^+\right) + \delta(p_z)m^2\left(A_q^+ - A_k^+\right)$$
$$+ \delta(k_z+q_z)m^2\left(A_q^- - A_k^+\right)\Big] + \delta(q_z+k_z)4k_z^2A_k^+\frac{\partial}{\partial q_z}\left(q_zA_q^-\right)\Big\} - 2\gamma_{a,k}^+A_k^+,$$

$$\frac{\partial F_k}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{9\sin^2\theta\Big[\delta(k-p-q)kqF_p\left(F_q-F_k\right) + \delta(k+p-q)k\left(kF_{-p}F_q + pF_qF_k - qF_{-p}F_k\right)\Big]$$
$$+ \delta(k-p_z+q_z)k\Lambda_{kpq}\left(kA_p^+A_q^- - p_zA_q^-F_k + q_zA_p^+F_k\right) + \delta(k-p_z-q)kM_{kpq}\left(kA_p^+F_q - p_zF_qF_k - qA_p^+F_k\right)$$
$$+ \delta(k+p_z-q)kM_{-k-p-q}\left(kA_p^-F_q + p_zF_qF_k - qA_p^-F_k\right) + \delta(k-q)k^{-3}p_zF_k\left[k_z\frac{\partial}{\partial q}\left(q^4F_q\right) - k^2q_z\frac{\partial}{\partial q}\left(q^2F_q\right)\right]$$
$$+ \varepsilon^{-2}k^2\left(S_p^+ + S_p^-\right)\left[\delta(k-q)m^2\left(F_q-F_k\right) + \delta(k-q_z)\overline{m}^2\left(A_q^+ - F_k\right) + \delta(k+q_z)\overline{m}^2\left(A_q^- - F_k\right)\right]$$
$$+ \delta(k-q_z)p_zF_k\left(2k_zA_q^+ + kp_z\frac{\partial A_q^+}{\partial q_z}\right) + \delta(k+q_z)p_zF_k\left(2k_zA_q^- - kp_z\frac{\partial A_q^-}{\partial q_z}\right)\Big\} - 2\gamma_{f,k}F_k,$$

Wave Kinetic Equations for Weak MHD Turbulence

(Chandran 2008)

$$\frac{\partial S_k^{\pm}}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big[\delta(q_z)4k_\perp^2\overline{m}^2\left(A_q^+ + A_q^-\right)\left(S_p^{\pm}-S_k^{\pm}\right) + \delta(p-q)k_z^2 l^2 F_p F_{-q} + \delta(p_z-q_z)k_z^2 l^2 A_p^+ A_q^-$$
$$+ \delta(p_z+q)k_z^2\overline{l}^2\left(A_p^+ F_q + A_p^- F_{-q}\right) + \delta(p_z-q)k_z^2\overline{l}^2\left(A_p^+ F_{-q} + A_p^- F_q\right)\Big] - 2\gamma_{s,k}^{\pm} S_k^{\pm},$$

perpendicular Alfven-wave cascade

$$\frac{\partial A_k^+}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{\boxed{\delta(q_z)8(k_\perp n\overline{m})^2 A_q^-\left(A_p^+ - A_k^+\right)} + \delta(k_z+p_z+q)k_z\Lambda_{q-pk}\left(k_z A_p^- F_{-q} + p_z F_{-q}A_k^+ + qA_p^- A_k^+\right)$$
$$+ \delta(k_z+p_z-q)k_z\Lambda_{q-pk}\left(k_z A_p^- F_q + p_z F_q A_k^+ - qA_p^- A_k^+\right) + \delta(k_z-p+q)k_z M_{pk-q}\left(k_z F_p F_{-q} - pF_{-q}A_k^+ + qF_p A_k^+\right)$$
$$+ \delta(q-k_z)p_z A_k^+\left[2(k_z+p_z)F_q + p_z q\frac{\partial F_q}{\partial q}\right] + \delta(q+k_z)p_z A_k^+\left[2(k_z+p_z)F_{-q} + p_z q\frac{\partial F_{-q}}{\partial q}\right]$$
$$+ \varepsilon^{-2}k_z^2\left(S_p^+ + S_p^-\right)\Big[\delta(q-k_z)\overline{m}^2\left(F_q - A_k^+\right) + \delta(q+k_z)\overline{m}^2\left(F_{-q} - A_k^+\right) + \delta(p_z)m^2\left(A_q^+ - A_k^+\right)$$
$$+ \delta(k_z+q_z)m^2\left(A_q^- - A_k^+\right)\Big] + \delta(q_z+k_z)4k_z^2 A_k^+\frac{\partial}{\partial q_z}\left(q_z A_q^-\right)\Big\} - 2\gamma_{a,k}^+ A_k^+,$$

$$\frac{\partial F_k}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{9\sin^2\theta\Big[\delta(k-p-q)kqF_p\left(F_q - F_k\right) + \delta(k+p-q)k\left(kF_{-p}F_q + pF_q F_k - qF_{-p}F_k\right)\Big]$$
$$+ \delta(k-p_z+q_z)k\Lambda_{kpq}\left(kA_p^+ A_q^- - p_z A_q^- F_k + q_z A_p^+ F_k\right) + \delta(k-p_z-q)kM_{kpq}\left(kA_p^+ F_q - p_z F_q F_k - qA_p^+ F_k\right)$$
$$+ \delta(k+p_z-q)kM_{-k-p-q}\left(kA_p^- F_q + p_z F_q F_k - qA_p^- F_k\right) + \delta(k-q)k^{-3}p_z F_k\left[k_z\frac{\partial}{\partial q}\left(q^4 F_q\right) - k^2 q_z\frac{\partial}{\partial q}\left(q^2 F_q\right)\right]$$
$$+ \varepsilon^{-2}k^2\left(S_p^+ + S_p^-\right)\left[\delta(k-q)m^2\left(F_q - F_k\right) + \delta(k-q_z)\overline{m}^2\left(A_q^+ - F_k\right) + \delta(k+q_z)\overline{m}^2\left(A_q^- - F_k\right)\right]$$
$$+ \delta(k-q_z)p_z F_k\left(2k_z A_q^+ + kp_z\frac{\partial A_q^+}{\partial q_z}\right) + \delta(k+q_z)p_z F_k\left(2k_z A_q^- - kp_z\frac{\partial A_q^-}{\partial q_z}\right)\Big\} - 2\gamma_{f,k}F_k,$$

Wave Kinetic Equations for Weak MHD Turbulence

(Chandran 2008)

passive-scalar mixing

$$\frac{\partial S_k^\pm}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\left[\boxed{\delta(q_z)4k_\perp^2\overline{m}^2\left(A_q^+ + A_q^-\right)\left(S_p^\pm - S_k^\pm\right)} + \delta(p-q)k_z^2 l^2 F_p F_{-q} + \delta(p_z-q_z)k_z^2 l^2 A_p^+ A_q^- \right.$$
$$\left. + \delta(p_z+q)k_z^2\overline{l}^2\left(A_p^+ F_q + A_p^- F_{-q}\right) + \delta(p_z-q)k_z^2\overline{l}^2\left(A_p^+ F_{-q} + A_p^- F_q\right)\right] - 2\gamma_{s,k}^\pm S_k^\pm,$$

$$\frac{\partial A_k^+}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\left\{\delta(q_z)8(k_\perp n\overline{m})^2 A_q^-\left(A_p^+ - A_k^+\right) + \delta(k_z+p_z+q)k_z\Lambda_{q-pk}\left(k_z A_p^- F_{-q} + p_z F_{-q}A_k^+ + qA_p^- A_k^+\right)\right.$$
$$+ \delta(k_z+p_z-q)k_z\Lambda_{q-pk}\left(k_z A_p^- F_q + p_z F_q A_k^+ - qA_p^- A_k^+\right) + \delta(k_z-p+q)k_z M_{pk-q}\left(k_z F_p F_{-q} - pF_{-q}A_k^+ + qF_p A_k^+\right)$$
$$+ \delta(q-k_z)p_z A_k^+\left[2(k_z+p_z)F_q + p_z q\frac{\partial F_q}{\partial q}\right] + \delta(q+k_z)p_z A_k^+\left[2(k_z+p_z)F_{-q} + p_z q\frac{\partial F_{-q}}{\partial q}\right]$$
$$+ \varepsilon^{-2}k_z^2\left(S_p^+ + S_p^-\right)\left[\delta(q-k_z)\overline{m}^2\left(F_q - A_k^+\right) + \delta(q+k_z)\overline{m}^2\left(F_{-q} - A_k^+\right) + \delta(p_z)m^2\left(A_q^+ - A_k^+\right)\right.$$
$$\left.\left. + \delta(k_z+q_z)m^2\left(A_q^- - A_k^+\right)\right] + \delta(q_z+k_z)4k_z^2 A_k^+\frac{\partial}{\partial q_z}\left(q_z A_q^-\right)\right\} - 2\gamma_{a,k}^+ A_k^+,$$

$$\frac{\partial F_k}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\left\{9\sin^2\theta\left[\delta(k-p-q)kqF_p\left(F_q - F_k\right) + \delta(k+p-q)k\left(kF_{-p}F_q + pF_qF_k - qF_{-p}F_k\right)\right]\right.$$
$$+ \delta(k-p_z+q_z)k\Lambda_{kpq}\left(kA_p^+A_q^- - p_zA_q^-F_k + q_zA_p^+F_k\right) + \delta(k-p_z-q)kM_{kpq}\left(kA_p^+F_q - p_zF_qF_k - qA_p^+F_k\right)$$
$$+ \delta(k+p_z-q)kM_{-k-p-q}\left(kA_p^-F_q + p_zF_qF_k - qA_p^-F_k\right) + \delta(k-q)k^{-3}p_zF_k\left[k_z\frac{\partial}{\partial q}\left(q^4F_q\right) - k^2q_z\frac{\partial}{\partial q}\left(q^2F_q\right)\right]$$
$$+ \varepsilon^{-2}k^2\left(S_p^+ + S_p^-\right)\left[\delta(k-q)m^2\left(F_q - F_k\right) + \delta(k-q_z)\overline{m}^2\left(A_q^+ - F_k\right) + \delta(k+q_z)\overline{m}^2\left(A_q^- - F_k\right)\right]$$
$$\left. + \delta(k-q_z)p_zF_k\left(2k_zA_q^+ + kp_z\frac{\partial A_q^+}{\partial q_z}\right) + \delta(k+q_z)p_zF_k\left(2k_zA_q^- - kp_z\frac{\partial A_q^-}{\partial q_z}\right)\right\} - 2\gamma_{f,k}F_k,$$

Wave Kinetic Equations for Weak MHD Turbulence

(Chandran 2008)

$$
\begin{aligned}
\frac{\partial S_k^\pm}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big[&\delta(q_z)4k_\perp^2\overline{m}^2\left(A_q^+ + A_q^-\right)\left(S_p^\pm - S_k^\pm\right) + \delta(p-q)k_z^2 l^2 F_p F_{-q} + \delta(p_z-q_z)k_z^2 l^2 A_p^+ A_q^- \\
&+ \delta(p_z+q)k_z^2\overline{l}^2\left(A_p^+F_q + A_p^-F_{-q}\right) + \delta(p_z-q)k_z^2\overline{l}^2\left(A_p^+F_{-q}+A_p^-F_q\right)\Big] - 2\gamma_{s,k}^\pm S_k^\pm,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial A_k^+}{\partial t} = \frac{\pi}{4v_A}\int d^3p\,d^3q\,\delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{&\delta(q_z)8(k_\perp n\overline{m})^2 A_q^-\left(A_p^+ - A_k^+\right) + \delta(k_z+p_z+q)k_z\Lambda_{q-pk}\left(k_zA_p^-F_{-q} + p_zF_{-q}A_k^+ + qA_p^-A_k^+\right) \\
&+ \delta(k_z+p_z-q)k_z\Lambda_{q-pk}\left(k_zA_p^-F_q + p_zF_qA_k^+ - qA_p^-A_k^+\right) + \delta(k_z-p+q)k_zM_{pk-q}\left(k_zF_pF_{-q} - pF_{-q}A_k^+ + qF_pA_k^+\right) \\
&+ \delta(q-k_z)p_zA_k^+\left[2(k_z+p_z)F_q + p_zq\frac{\partial F_q}{\partial q}\right] + \delta(q+k_z)p_zA_k^+\left[2(k_z+p_z)F_{-q}+p_zq\frac{\partial F_{-q}}{\partial q}\right] \\
&+ \varepsilon^{-2}k_z^2\left(S_p^+ + S_p^-\right)\Big[\delta(q-k_z)\overline{m}^2\left(F_q - A_k^+\right) + \delta(q+k_z)\overline{m}^2\left(F_{-q}-A_k^+\right) + \delta(p_z)m^2\left(A_q^+ - A_k^+\right) \\
&+ \delta(k_z+q_z)m^2\left(A_q^- - A_k^+\right)\Big] + \delta(q_z+k_z)4k_z^2A_k^+\frac{\partial}{\partial q_z}\left(q_zA_q^-\right)\Big\} - 2\gamma_{a,k}^+A_k^+,
\end{aligned}
$$

phase mixing

$$
\begin{aligned}
\frac{\partial F_k}{\partial t} = \frac{\pi}{4v_A}\int d^3p\,d^3q\,\delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{&9\sin^2\theta\Big[\delta(k-p-q)kqF_p\left(F_q-F_k\right) + \delta(k+p-q)k\left(kF_{-p}F_q + pF_qF_k - qF_{-p}F_k\right)\Big] \\
&+ \delta(k-p_z+q_z)k\Lambda_{kpq}\left(kA_p^+A_q^- - p_zA_q^-F_k + q_zA_p^+F_k\right) + \delta(k-p_z-q)kM_{kpq}\left(kA_p^+F_q - p_zF_qF_k - qA_p^+F_k\right) \\
&+ \delta(k+p_z-q)kM_{-k-p-q}\left(kA_p^-F_q + p_zF_qF_k - qA_p^-F_k\right) + \delta(k-q)k^{-3}p_zF_k\left[k_z\frac{\partial}{\partial q}\left(q^4F_q\right) - k^2q_z\frac{\partial}{\partial q}\left(q^2F_q\right)\right] \\
&+ \varepsilon^{-2}k^2\left(S_p^+ + S_p^-\right)\Big[\delta(k-q)m^2\left(F_q-F_k\right) + \delta(k-q_z)\overline{m}^2\left(A_q^+ - F_k\right) + \delta(k+q_z)\overline{m}^2\left(A_q^- - F_k\right)\Big] \\
&+ \delta(k-q_z)p_zF_k\left(2k_zA_q^+ + kp_z\frac{\partial A_q^+}{\partial q_z}\right) + \delta(k+q_z)p_zF_k\left(2k_zA_q^- - kp_z\frac{\partial A_q^-}{\partial q_z}\right)\Big\} - 2\gamma_{f,k}F_k,
\end{aligned}
$$

Wave Kinetic Equations for Weak MHD Turbulence

(Chandran 2008)

$$\frac{\partial S_k^{\pm}}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big[\delta(q_z)4k_\perp^2\overline{m}^2\left(A_q^+ + A_q^-\right)\left(S_p^{\pm}-S_k^{\pm}\right) + \delta(p-q)k_z^2 l^2 F_p F_{-q} + \delta(p_z-q_z)k_z^2 l^2 A_p^+ A_q^-$$
$$+ \delta(p_z+q)k_z^2\overline{l}^2\left(A_p^+ F_q + A_p^- F_{-q}\right) + \delta(p_z-q)k_z^2\overline{l}^2\left(A_p^+ F_{-q} + A_p^- F_q\right)\Big] - 2\gamma_{s,k}^{\pm} S_k^{\pm},$$

$$\frac{\partial A_k^+}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{\delta(q_z)8(k_\perp n\overline{m})^2 A_q^-\left(A_p^+ - A_k^+\right) + \delta(k_z+p_z+q)k_z\Lambda_{q-pk}\left(k_z A_p^- F_{-q} + p_z F_{-q} A_k^+ + q A_p^- A_k^+\right)$$
$$+ \delta(k_z+p_z-q)k_z\Lambda_{q-pk}\left(k_z A_p^- F_q + p_z F_q A_k^+ - q A_p^- A_k^+\right) + \delta(k_z-p+q)k_z M_{pk-q}\left(k_z F_p F_{-q} - pF_{-q}A_k^+ + qF_p A_k^+\right)$$
$$+ \delta(q-k_z)p_z A_k^+\left[2(k_z+p_z)F_q + p_z q\frac{\partial F_q}{\partial q}\right] + \delta(q+k_z)p_z A_k^+\left[2(k_z+p_z)F_{-q} + p_z q\frac{\partial F_{-q}}{\partial q}\right]$$
$$+ \varepsilon^{-2}k_z^2\left(S_p^+ + S_p^-\right)\Big[\delta(q-k_z)\overline{m}^2\left(F_q - A_k^+\right) + \delta(q+k_z)\overline{m}^2\left(F_{-q}-A_k^+\right) + \delta(p_z)m^2\left(A_q^+ - A_k^+\right)$$
$$+ \delta(k_z+q_z)m^2\left(A_q^- - A_k^+\right)\Big] + \delta(q_z+k_z)4k_z^2 A_k^+\frac{\partial}{\partial q_z}\left(q_z A_q^-\right)\Big\} - 2\gamma_{a,k}^+ A_k^+,$$

"radial" fast-wave cascade

$$\frac{\partial F_k}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{9\sin^2\theta\Big[\delta(k-p-q)kqF_p\left(F_q - F_k\right) + \delta(k+p-q)k\left(kF_{-p}F_q + pF_qF_k - qF_{-p}F_k\right)\Big]$$
$$+ \delta(k-p_z+q_z)k\Lambda_{kpq}\left(kA_p^+A_q^- - p_zA_q^-F_k + q_zA_p^+F_k\right) + \delta(k-p_z-q)kM_{kpq}\left(kA_p^+F_q - p_zF_qF_k - qA_p^+F_k\right)$$
$$+ \delta(k+p_z-q)kM_{-k-p-q}\left(kA_p^-F_q + p_zF_qF_k - qA_p^-F_k\right) + \delta(k-q)k^{-3}p_zF_k\left[k_z\frac{\partial}{\partial q}\left(q^4F_q\right) - k^2q_z\frac{\partial}{\partial q}\left(q^2F_q\right)\right]$$
$$+ \varepsilon^{-2}k^2\left(S_p^+ + S_p^-\right)\left[\delta(k-q)m^2\left(F_q - F_k\right) + \delta(k-q_z)\overline{m}^2\left(A_q^+ - F_k\right) + \delta(k+q_z)\overline{m}^2\left(A_q^- - F_k\right)\right]$$
$$+ \delta(k-q_z)p_zF_k\left(2k_zA_q^+ + kp_z\frac{\partial A_q^+}{\partial q_z}\right) + \delta(k+q_z)p_zF_k\left(2k_zA_q^- - kp_z\frac{\partial A_q^-}{\partial q_z}\right)\Big\} - 2\gamma_{f,k}F_k,$$

Wave Kinetic Equations for Weak MHD Turbulence

(Chandran 2008)

parametric instability

$$\frac{\partial S_k^\pm}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big[\delta(q_z)4k_\perp^2\overline{m}^2\left(A_q^+ + A_q^-\right)\left(S_p^\pm - S_k^\pm\right) + \delta(p-q)k_z^2 l^2 F_p F_{-q} + \boxed{\delta(p_z-q_z)k_z^2 l^2 A_p^+ A_q^-}$$
$$+ \delta(p_z+q)k_z^2\overline{l}^2\left(A_p^+F_q + A_p^-F_{-q}\right) + \delta(p_z-q)k_z^2\overline{l}^2\left(A_p^+F_{-q}+A_p^-F_q\right)\Big] - 2\gamma_{s,k}^\pm S_k^\pm,$$

$$\frac{\partial A_k^+}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{\delta(q_z)8(k_\perp n\overline{m})^2A_q^-\left(A_p^+ - A_k^+\right) + \delta(k_z+p_z+q)k_z\Lambda_{q-pk}\left(k_zA_p^-F_{-q} + p_zF_{-q}A_k^+ + qA_p^-A_k^+\right)$$
$$+ \delta(k_z+p_z-q)k_z\Lambda_{q-pk}\left(k_zA_p^-F_q + p_zF_qA_k^+ - qA_p^-A_k^+\right) + \delta(k_z-p+q)k_zM_{pk-q}\left(k_zF_pF_{-q} - pF_{-q}A_k^+ + qF_pA_k^+\right)$$
$$+ \delta(q-k_z)p_zA_k^+\left[2(k_z+p_z)F_q + p_zq\frac{\partial F_q}{\partial q}\right] + \delta(q+k_z)p_zA_k^+\left[2(k_z+p_z)F_{-q} + p_zq\frac{\partial F_{-q}}{\partial q}\right]$$
$$+ \boxed{\varepsilon^{-2}k_z^2\left(S_p^+ + S_p^-\right)}\Big[\delta(q-k_z)\overline{m}^2\left(F_q - A_k^+\right) + \delta(q+k_z)\overline{m}^2\left(F_{-q}-A_k^+\right) + \delta(p_z)m^2\left(A_q^+ - A_k^+\right)$$
$$\times \boxed{+\,\delta(k_z+q_z)m^2\left(A_q^- - A_k^+\right)}\Big] + \boxed{\delta(q_z+k_z)4k_z^2A_k^+\frac{\partial}{\partial q_z}\left(q_zA_q^-\right)}\Big\} - 2\gamma_{a,k}^+A_k^+,$$

parametric instability

$$\frac{\partial F_k}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{9\sin^2\theta\Big[\delta(k-p-q)kqF_p\left(F_q-F_k\right) + \delta(k+p-q)k\left(kF_{-p}F_q + pF_qF_k - qF_{-p}F_k\right)\Big]$$
$$+ \delta(k-p_z+q_z)k\Lambda_{kpq}\left(kA_p^+A_q^- - p_zA_q^-F_k + q_zA_p^+F_k\right) + \delta(k-p_z-q)kM_{kpq}\left(kA_p^+F_q - p_zF_qF_k - qA_p^+F_k\right)$$
$$+ \delta(k+p_z-q)kM_{-k-p-q}\left(kA_p^-F_q + p_zF_qF_k - qA_p^-F_k\right) + \delta(k-q)k^{-3}p_zF_k\left[k_z\frac{\partial}{\partial q}\left(q^4F_q\right) - k^2q_z\frac{\partial}{\partial q}\left(q^2F_q\right)\right]$$
$$+ \varepsilon^{-2}k^2\left(S_p^+ + S_p^-\right)\Big[\delta(k-q)m^2\left(F_q-F_k\right) + \delta(k-q_z)\overline{m}^2\left(A_q^+ - F_k\right) + \delta(k+q_z)\overline{m}^2\left(A_q^- - F_k\right)\Big]$$
$$+ \delta(k-q_z)p_zF_k\left(2k_zA_q^+ + kp_z\frac{\partial A_q^+}{\partial q_z}\right) + \delta(k+q_z)p_zF_k\left(2k_zA_q^- - kp_z\frac{\partial A_q^-}{\partial q_z}\right)\Big\} - 2\gamma_{f,k}F_k,$$

Wave Kinetic Equations for Weak MHD Turbulence

(Chandran 2008)

$$\frac{\partial S_k^{\pm}}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big[\delta(q_z)4k_\perp^2\overline{m}^2\left(A_q^+ + A_q^-\right)\left(S_p^{\pm}-S_k^{\pm}\right) + \delta(p-q)k_z^2 l^2 F_p F_{-q} + \delta(p_z-q_z)k_z^2 l^2 A_p^+ A_q^-$$
$$+\,\delta(p_z+q)k_z^2\bar{l}^2\left(A_p^+F_q + A_p^-F_{-q}\right) + \delta(p_z-q)k_z^2\bar{l}^2\left(A_p^+F_{-q}+A_p^-F_q\right)\Big] - 2\gamma_{s,k}^{\pm}S_k^{\pm},$$

$$\frac{\partial A_k^+}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{\delta(q_z)8(k_\perp n\overline{m})^2A_q^-\left(A_p^+-A_k^+\right) + \delta(k_z+p_z+q)k_z\Lambda_{q-pk}\left(k_zA_p^-F_{-q}+p_zF_{-q}A_k^+ + qA_p^-A_k^+\right)$$
$$+\,\delta(k_z+p_z-q)k_z\Lambda_{q-pk}\left(k_zA_p^-F_q+p_zF_qA_k^+-qA_p^-A_k^+\right) + \delta(k_z-p+q)k_zM_{pk-q}\left(k_zF_pF_{-q}-pF_{-q}A_k^+ + qF_pA_k^+\right)$$
$$+\,\delta(q-k_z)p_zA_k^+\left[2(k_z+p_z)F_q+p_zq\frac{\partial F_q}{\partial q}\right] + \delta(q+k_z)p_zA_k^+\left[2(k_z+p_z)F_{-q}+p_zq\frac{\partial F_{-q}}{\partial q}\right]$$
$$+\,\varepsilon^{-2}k_z^2\left(S_p^+ + S_p^-\right)\Big[\delta(q-k_z)\overline{m}^2\left(F_q-A_k^+\right) + \delta(q+k_z)\overline{m}^2\left(F_{-q}-A_k^+\right) + \delta(p_z)m^2\left(A_q^+-A_k^+\right)$$
$$+\,\delta(k_z+q_z)m^2\left(A_q^- - A_k^+\right)\Big] + \boxed{\delta(q_z+k_z)4k_z^2A_k^+\frac{\partial}{\partial q_z}\left(q_zA_q^-\right)}\Big\} - 2\gamma_{a,k}^+A_k^+,$$

parametric instability when slow waves are strongly damped

$$\frac{\partial F_k}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{9\sin^2\theta\Big[\delta(k-p-q)kqF_p\left(F_q-F_k\right) + \delta(k+p-q)k\left(kF_{-p}F_q+pF_qF_k-qF_{-p}F_k\right)\Big]$$
$$+\,\delta(k-p_z+q_z)k\Lambda_{kpq}\left(kA_p^+A_q^- - p_zA_q^-F_k + q_zA_p^+F_k\right) + \delta(k-p_z-q)kM_{kpq}\left(kA_p^+F_q-p_zF_qF_k-qA_p^+F_k\right)$$
$$+\,\delta(k+p_z-q)kM_{-k-p-q}\left(kA_p^-F_q+p_zF_qF_k-qA_p^-F_k\right) + \delta(k-q)k^{-3}p_zF_k\left[k_z\frac{\partial}{\partial q}\left(q^4F_q\right)-k^2q_z\frac{\partial}{\partial q}\left(q^2F_q\right)\right]$$
$$+\,\varepsilon^{-2}k^2\left(S_p^+ + S_p^-\right)\Big[\delta(k-q)m^2\left(F_q-F_k\right) + \delta(k-q_z)\overline{m}^2\left(A_q^+-F_k\right) + \delta(k+q_z)\overline{m}^2\left(A_q^- - F_k\right)\Big]$$
$$+\,\delta(k-q_z)p_zF_k\left(2k_zA_q^+ + kp_z\frac{\partial A_q^+}{\partial q_z}\right) + \delta(k+q_z)p_zF_k\left(2k_zA_q^- - kp_z\frac{\partial A_q^-}{\partial q_z}\right)\Big\} - 2\gamma_{f,k}F_k,$$

Wave Kinetic Equations for Weak MHD Turbulence

(Chandran 2008)

$$
\begin{aligned}
\frac{\partial S_k^{\pm}}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big[&\delta(q_z)4k_\perp^2\overline{m}^2\left(A_q^+ + A_q^-\right)\left(S_p^{\pm} - S_k^{\pm}\right) + \delta(p-q)k_z^2 l^2 F_p F_{-q} + \delta(p_z-q_z)k_z^2 l^2 A_p^+ A_q^- \\
&+ \delta(p_z+q)k_z^2\bar{l}^2\left(A_p^+ F_q + A_p^- F_{-q}\right) + \delta(p_z-q)k_z^2\bar{l}^2\left(A_p^+ F_{-q} + A_p^- F_q\right)\Big] - 2\gamma_{s,k}^{\pm} S_k^{\pm},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial A_k^+}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{&\delta(q_z)8(k_\perp n\overline{m})^2 A_q^-\left(A_p^+ - A_k^+\right) + \delta(k_z+p_z+q)k_z\Lambda_{q-pk}\left(k_z A_p^- F_{-q} + p_z F_{-q} A_k^+ + q A_p^- A_k^+\right) \\
&+ \delta(k_z+p_z-q)k_z\Lambda_{q-pk}\left(k_z A_p^- F_q + p_z F_q A_k^+ - q A_p^- A_k^+\right) + \delta(k_z-p+q)k_z M_{pk-q}\left(k_z F_p F_{-q} - pF_{-q}A_k^+ + qF_p A_k^+\right) \\
&+ \delta(q-k_z)p_z A_k^+\left[2(k_z+p_z)F_q + p_z q\frac{\partial F_q}{\partial q}\right] + \delta(q+k_z)p_z A_k^+\left[2(k_z+p_z)F_{-q} + p_z q\frac{\partial F_{-q}}{\partial q}\right] \\
&+ \epsilon^{-2}k_z^2\left(S_p^+ + S_p^-\right)\Big[\delta(q-k_z)\overline{m}^2\left(F_q - A_k^+\right) + \delta(q+k_z)\overline{m}^2\left(F_{-q} - A_k^+\right) + \delta(p_z)m^2\left(A_q^+ - A_k^+\right) \\
&+ \delta(k_z+q_z)m^2\left(A_q^- - A_k^+\right)\Big] + \boxed{\delta(q_z+k_z)4k_z^2 A_k^+\frac{\partial}{\partial q_z}\left(q_z A_q^-\right)}\Big\} - 2\gamma_{a,k}^+ A_k^+,
\end{aligned}
$$

parametric instability when slow waves are strongly damped

$$
\begin{aligned}
\frac{\partial F_k}{\partial t} = \frac{\pi}{4v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\Big\{&9\sin^2\theta\Big[\delta(k-p-q)kqF_p\left(F_q - F_k\right) + \delta(k+p-q)k\left(kF_{-p}F_q + pF_qF_k - qF_{-p}F_k\right)\Big] \\
&+ \delta(k-p_z+q_z)k\Lambda_{kpq}\left(kA_p^+A_q^- - p_zA_q^-F_k + q_zA_p^+F_k\right) + \delta(k-p_z-q)kM_{kpq}\left(kA_p^+F_q - p_zF_qF_k - qA_p^+F_k\right) \\
&+ \delta(k+p_z-q)kM_{-k-p-q}\left(kA_p^-F_q + p_zF_qF_k - qA_p^-F_k\right) + \delta(k-q)k^{-3}p_zF_k\left[k\frac{\partial}{\partial q}\left(q^4F_q\right) - k^2q_z\frac{\partial}{\partial q}\left(q^2F_q\right)\right] \\
&+ \epsilon^{-2}k^2\left(S_p^+ + S_p^-\right)\Big[\delta(k-q)m^2\left(F_q - F_k\right) + \delta(k-q_z)\overline{m}^2\left(A_q^+ - F_k\right) + \delta(k+q_z)\overline{m}^2\left(A_q^- - F_k\right)\Big] \\
&+ \delta(k-q_z)p_zF_k\left(2k_zA_q^+ + kp_z\frac{\partial A_q^+}{\partial q_z}\right) + \delta(k+q_z)p_zF_k\left(2k_zA_q^- - kp_z\frac{\partial A_q^-}{\partial q_z}\right)\Big\} - 2\gamma_{f,k}F_k,
\end{aligned}
$$

Integrate the Wave Kinetic Equations over $k_\perp$

$$E^{\pm}(k_z, t) = \int dk_x dk_y A^{\pm}(k_x, k_y, k_z, t)$$

$$\frac{\partial E^+}{\partial t} = \frac{\pi}{v_{\rm A}} k_z^2 E^+ \frac{\partial}{\partial k_z} \left(k_z E^-\right)$$

$$\frac{\partial E^-}{\partial t} = \frac{\pi}{v_{\rm A}} k_z^2 E^- \frac{\partial}{\partial k_z} \left(k_z E^+\right)$$

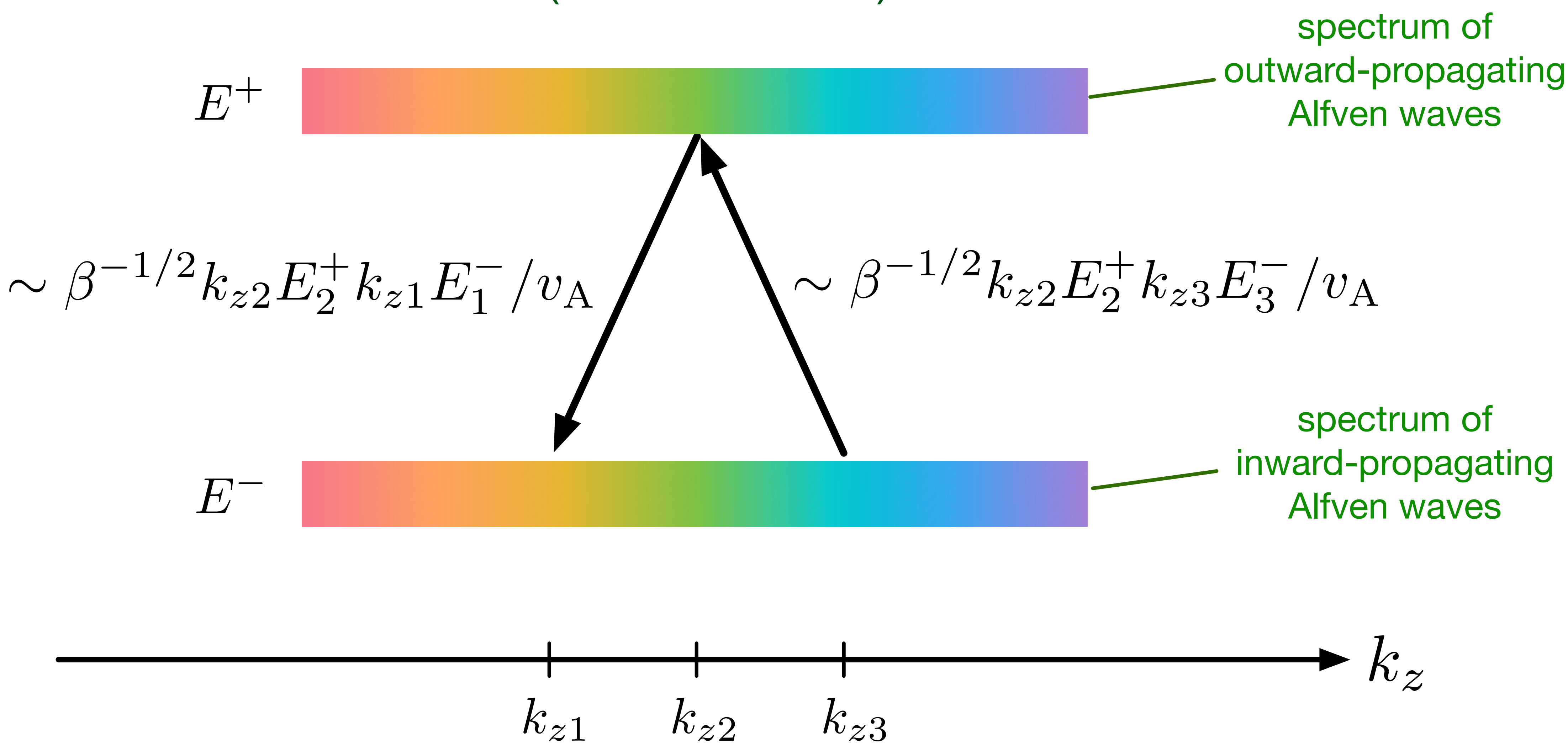
The wave kinetic equations allow for obliquely propagating waves, but these integrated equations depend only on the parallel wavenumber k_z and t.

Alfven Wave Frequency Decreases Slightly During Each Parametric Decay

$$\begin{aligned}
k_z &= p_z + q_z \\
k_z v_{\rm A} &= p_z c_{\rm s} - q_z v_{\rm A} \\
k_z v_{\rm A} &= (k_z - q_z) c_{\rm s} - q_z v_{\rm A} \\
k_z (v_{\rm A} - c_{\rm s}) &= -q_z (v_{\rm A} + c_{\rm s}) \\
k_z \left(\frac{v_{\rm A} - c_{\rm s}}{v_{\rm A} + c_{\rm s}} \right) &= -q_z \qquad (\text{ Note: } c_{\rm s}/v_{\rm A} \sim \beta^{1/2} \ll 1) \\
k_z (1 - 2\beta^{1/2}) &= -q_z
\end{aligned}$$

Why Do the Wave Kinetic Equations Have This Form?

(Chandran 2018)



$$\frac{\partial E_2^+}{\partial t} \sim \frac{\beta^{-1/2} k_{z2} E_2^+}{v_A} \left(k_{z3} E_3^- - k_{z1} E_1^- \right)$$

$$k_{z3} - k_{z1} \sim \beta^{1/2} k_{z2}$$

$$\frac{\partial E_2^+}{\partial t} \sim \frac{\beta^{-1/2} k_{z2} E_2^+}{v_A} \times \beta^{1/2} k_{z2} \frac{\partial}{\partial k_z} \left(k_z E^- \right) \sim \frac{k_{z2}^2 E_2^+}{v_A} \frac{\partial}{\partial k_z} \left(k_z E^- \right)$$

Linear Limit

"Pump-wave" amplitude fixed ($E^+ =$ constant):

$$\frac{\partial E^-}{\partial t} = \frac{\pi}{v_A} k_z^2 E^- \frac{\partial}{\partial k_z}\left(k_z E^+\right)$$

$$\gamma \equiv \frac{\partial \ln E^-}{\partial t} = \frac{\pi}{v_A} k_z^2 \frac{\partial}{\partial k_z}\left(k_z E^+\right)$$

E^- grows exponentially if $E^+ \propto k^{\alpha^+}$ with $\alpha^+ > -1$.

This result was found by Cohen & Dewar (1974) for parallel-propagating waves at low beta, assuming slow waves are strongly damped.

Conservation of Wave Quanta and Inverse Cascade

(Chandran 2018)

$$E^{\pm}(k_z, t) = \int dk_x dk_y A^{\pm}(k_x, k_y, k_z, t)$$

$$\frac{\partial E^+}{\partial t} = \frac{\pi}{v_{\rm A}} k_z^2 E^+ \frac{\partial}{\partial k_z}\left(k_z E^-\right)$$

$$\frac{\partial E^-}{\partial t} = \frac{\pi}{v_{\rm A}} k_z^2 E^- \frac{\partial}{\partial k_z}\left(k_z E^+\right)$$

Divide previous two eqns by $k_z v_{\rm A}$ and add:

$$\frac{\partial}{\partial t}\left(\frac{E^+ + E^-}{k_z v_{\rm A}}\right) = \frac{\partial}{\partial k_z}\left(\frac{\pi k_z^2 E^+ E^-}{v_{\rm A}^2}\right)$$

← inverse cascade of wave quanta

$$\longrightarrow \int_{-\infty}^{\infty} dk_z \left(\frac{E^+ + E^-}{k_z v_{\rm A}}\right) = \text{constant}$$

← conservation of wave quanta (wave action)

Exact Solutions to Wave Kinetic Equation

(Chandran 2018)

$$\frac{\partial E^+}{\partial t} = \frac{\pi}{v_\mathrm{A}} k_z^2 E^+ \frac{\partial}{\partial k_z}\left(k_z E^-\right)$$

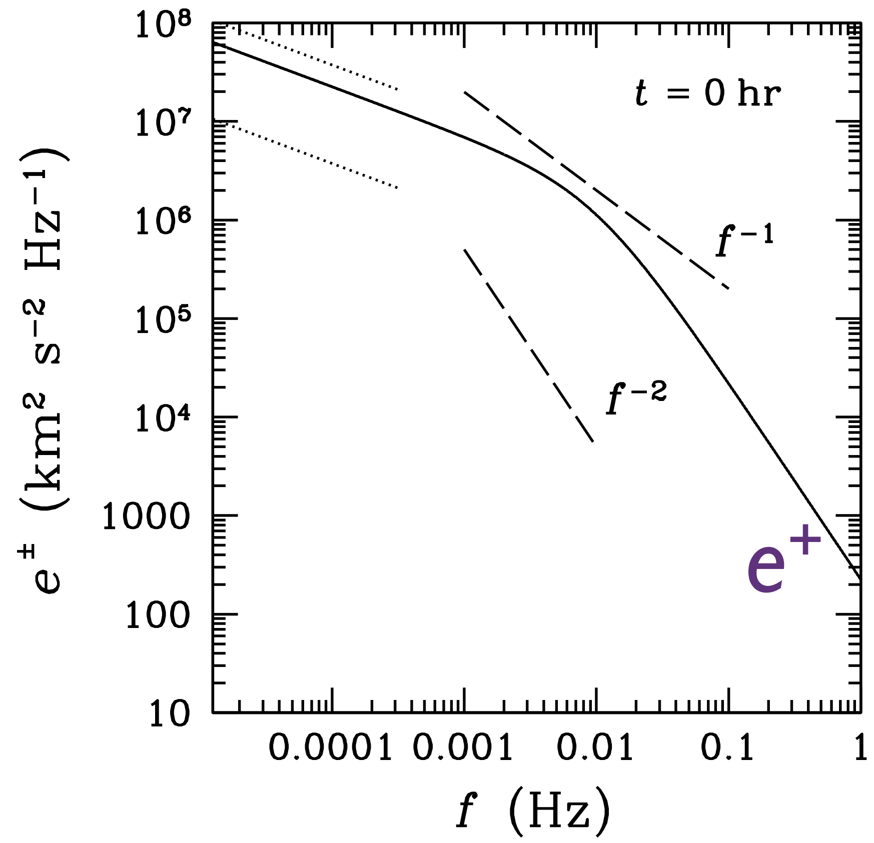
$$\frac{\partial E^-}{\partial t} = \frac{\pi}{v_\mathrm{A}} k_z^2 E^- \frac{\partial}{\partial k_z}\left(k_z E^+\right)$$

$$E^\pm(k_z, t) = \frac{c^\pm}{k_z},$$

$$E^\pm(k_z, t) = \frac{a^\pm(t)}{k_z^2} \qquad a^\pm(t) = \frac{a_0^\pm(a_0^\pm - a_0^\mp)}{a_0^\pm - a_0^\mp e^{-\pi(a_0^\pm - a_0^\mp)t/v_\mathrm{A}}},$$

(can also construct truncated versions of these solutions, and combinations of k_z^{-1} and k_z^{-2} solutions)

Numerical Solution of the Nonlinear Evolution



$$e^{\pm} = \frac{2\pi E^{\pm}}{U} = \text{frequency spectrum}$$

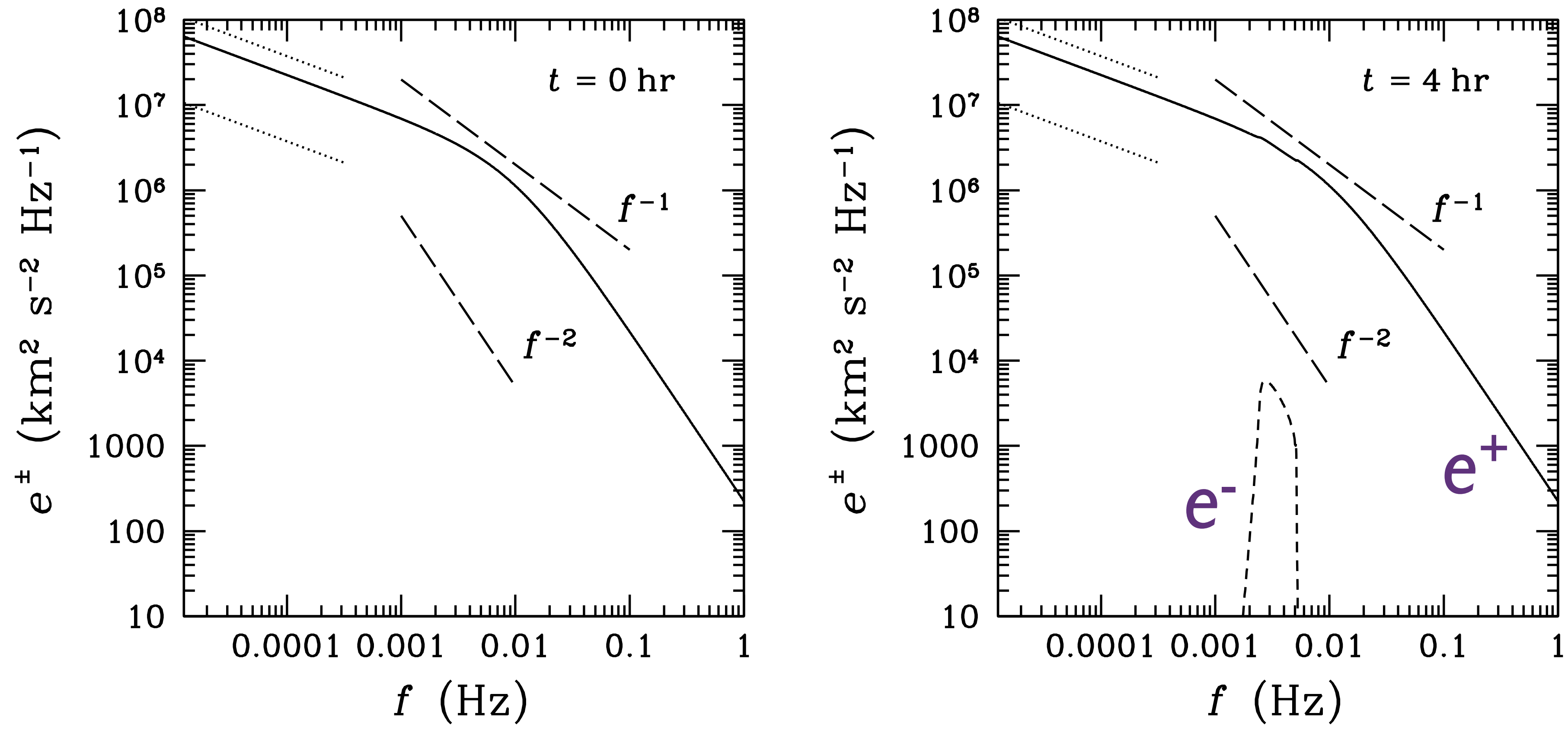
(via Taylor's hypothesis.
U = solar-wind speed = 733 km/s.)

Alfven speed = 150 km/s. Initial dominant frequency (maximum of fe_f) is 0.01 Hz.

$$e^{+}(f, t=0) = \frac{\sigma^{+}(f/f_0)^{-0.5}}{1+(f/f_0)^{1.5}}$$

(Chandran 2018)

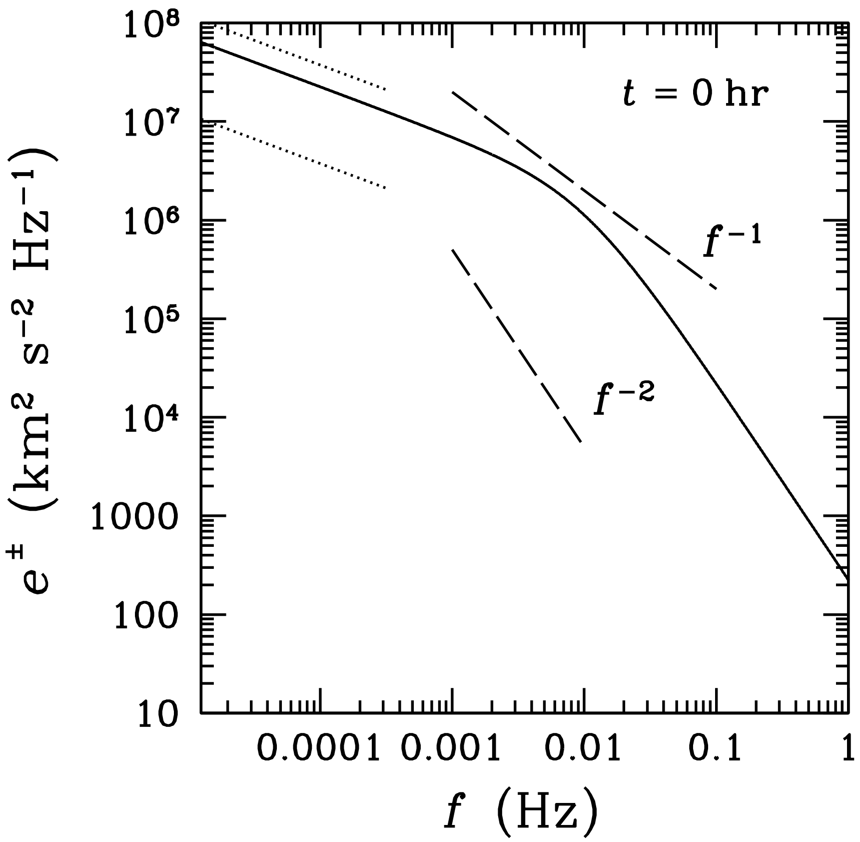
Numerical Solution of the Nonlinear Evolution

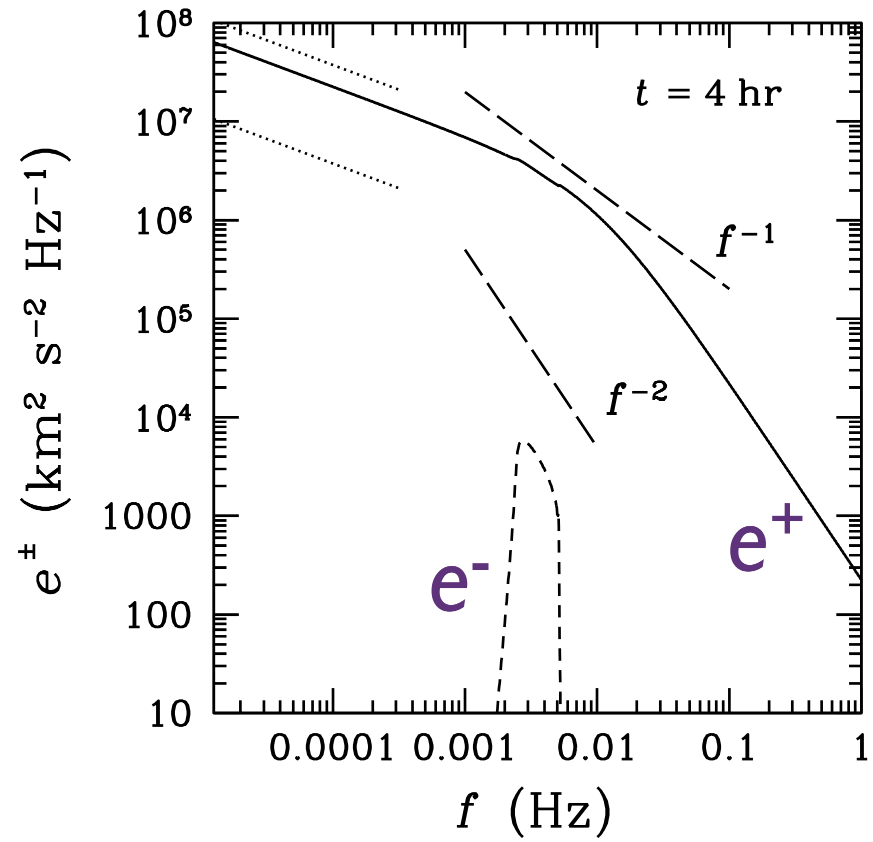


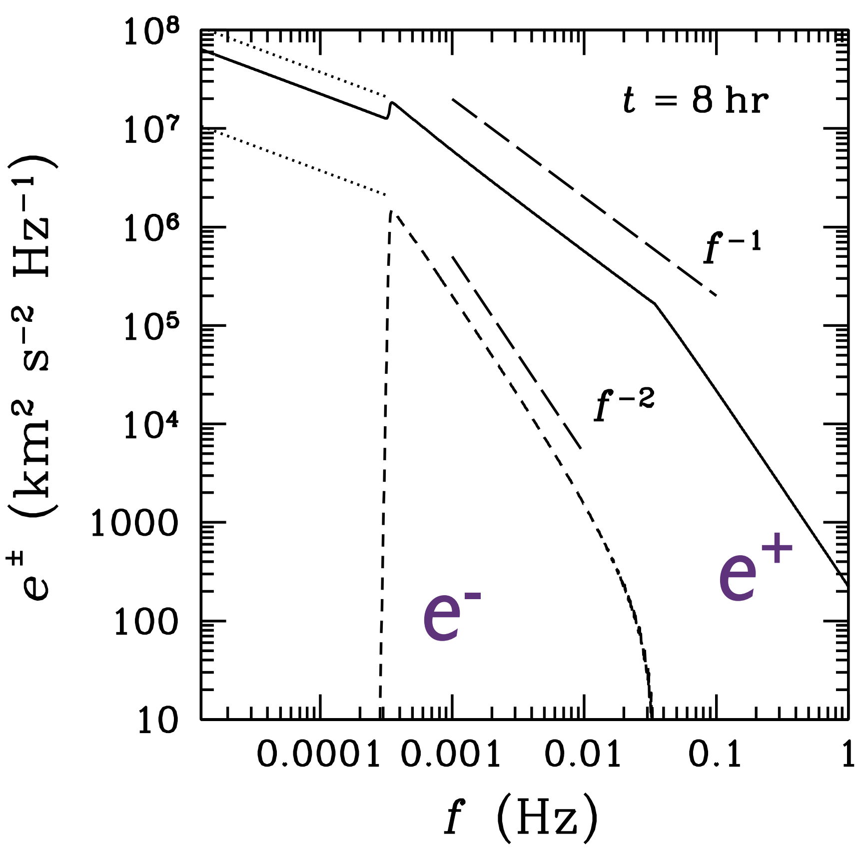
Linear stage: the inward waves grow fastest at the largest wavenumbers where the spectrum is flatter than 1/f.

(Chandran 2018)

Numerical Solution of the Nonlinear Evolution



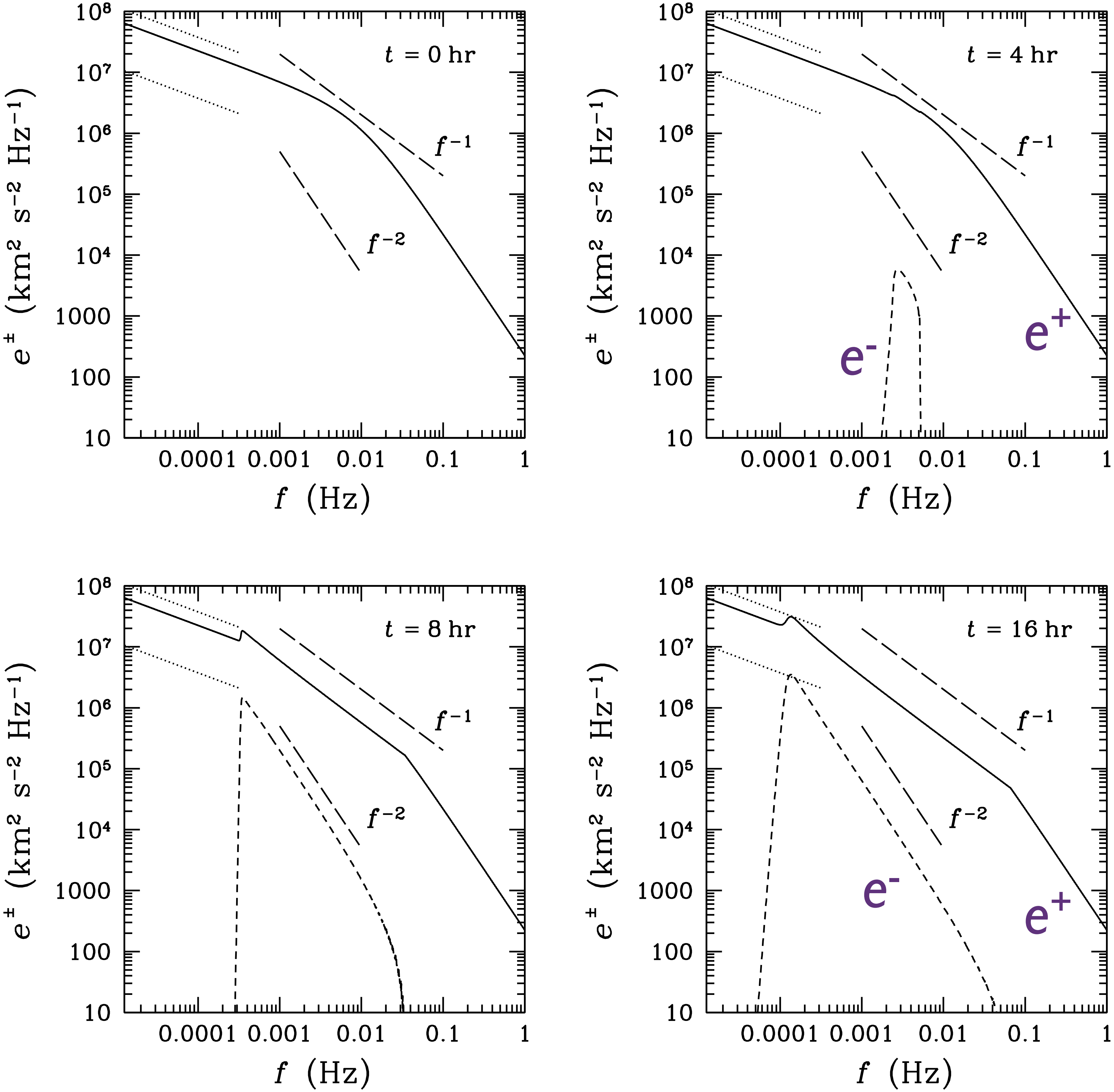




The outward-going "pump waves" acquire a 1/f spectrum in a form of quasilinear "flattening"

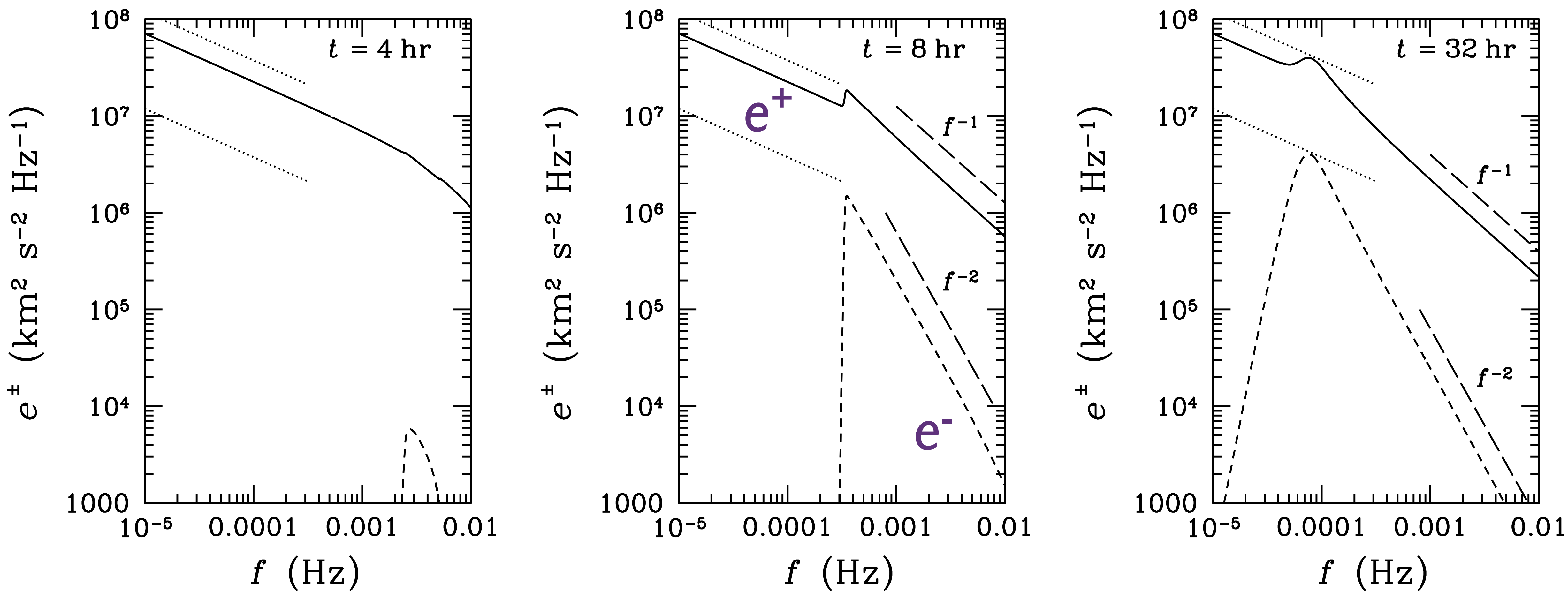
(Chandran 2018)

Numerical Solution of the Nonlinear Evolution



(Chandran 2018)

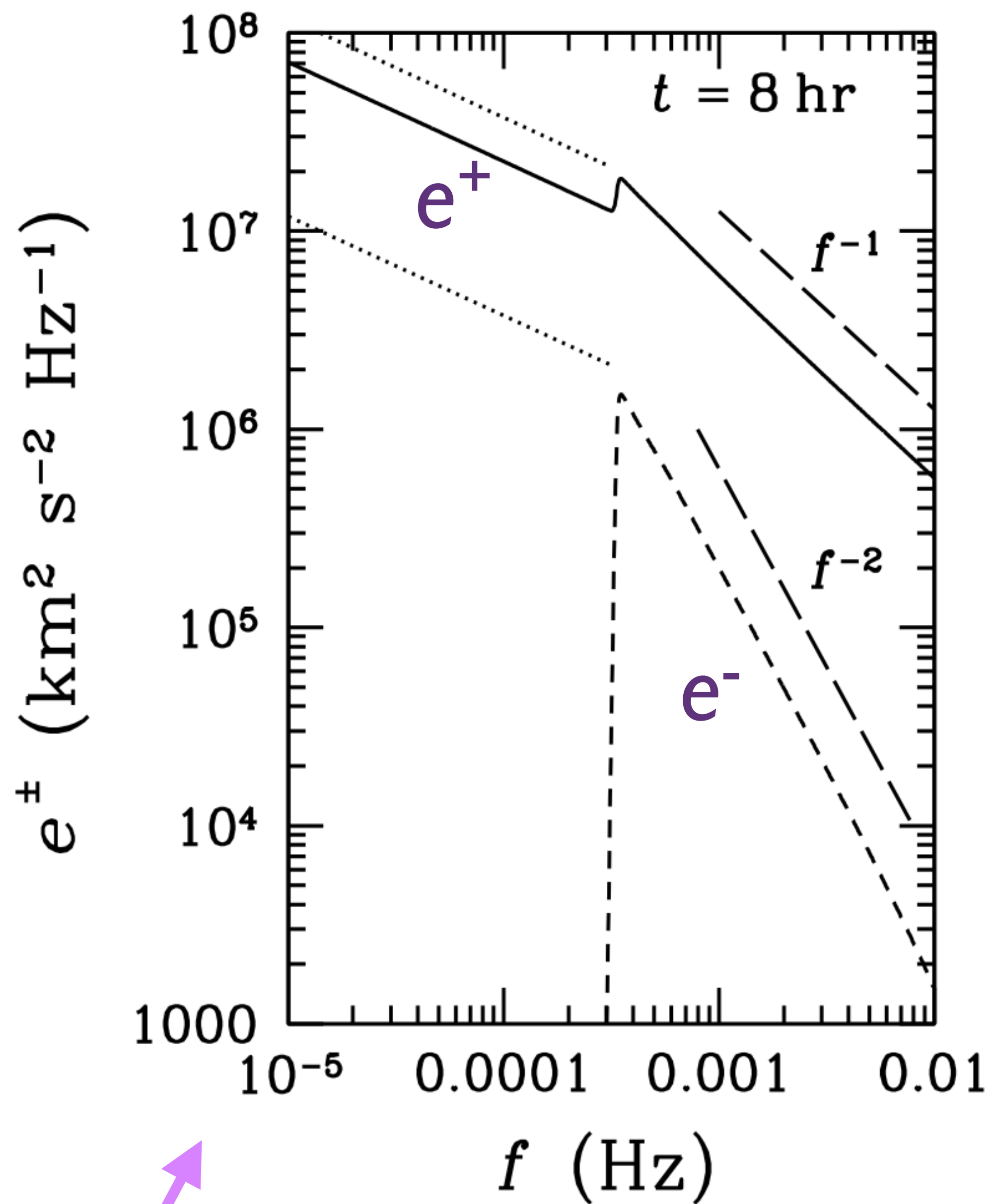
Same Simulation, Plotted over a Smaller Frequency Range, out to 32 Hrs



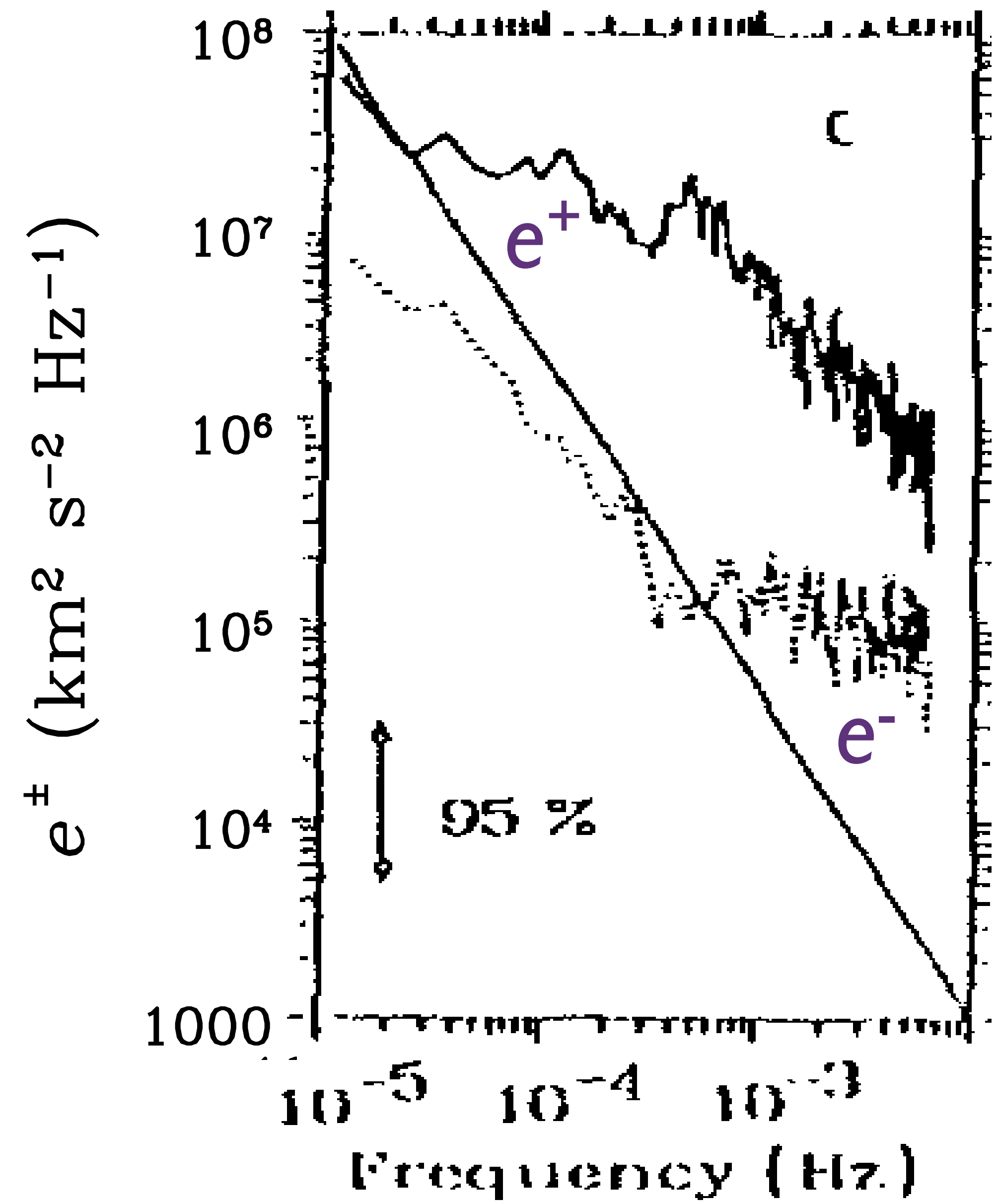
dotted lines in upper left show evolutionary tracks of spectral peaks in an approximate analytic solution

(Chandran 2018)

Comparison Between Numerical Solution and Helios Measurements



Helios Measurements



0.29 AU 733 km s^{-1}

Alfven speed = 150 km/s. Initial dominant frequency (maximum of f x E_f) is 0.01 Hz. Alfven travel time to 0.29 AU is 12 hours.

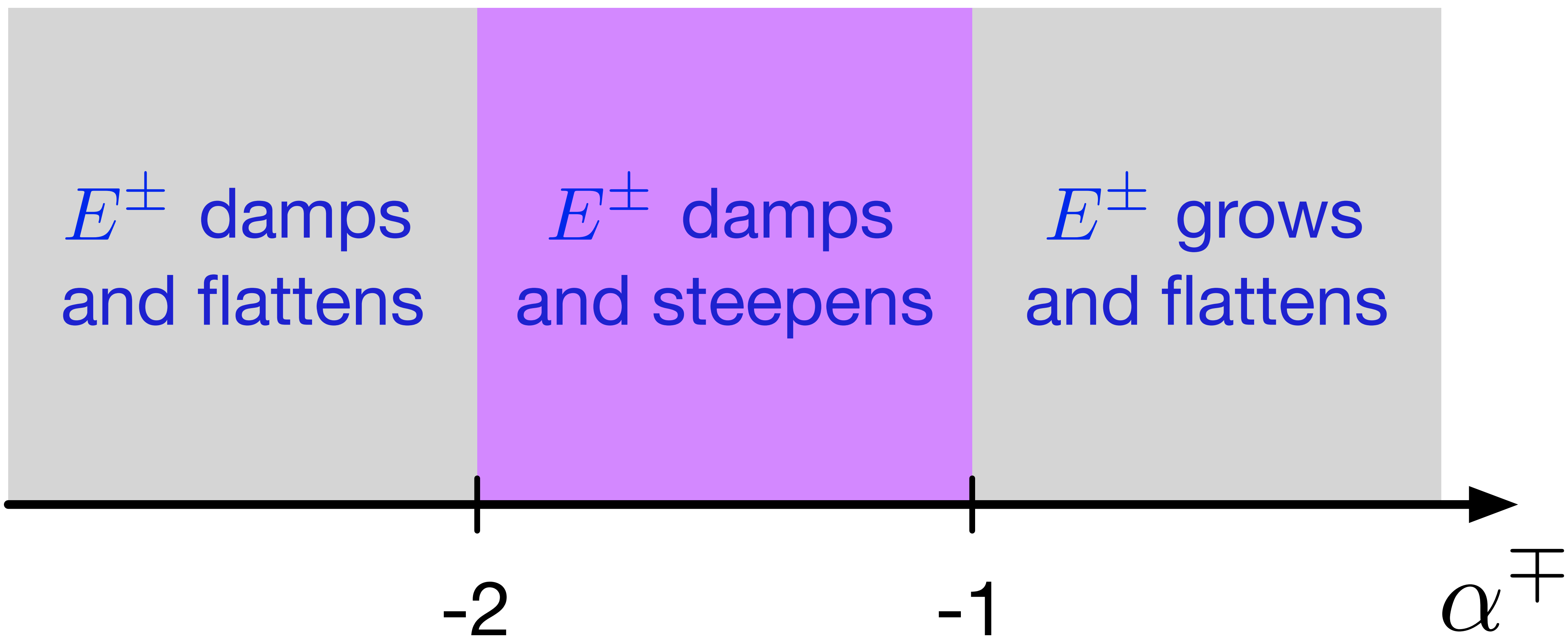
Tu & Marsch (1995)

(Chandran 2018)

Nonlinear Evolution of the Parametric Instability

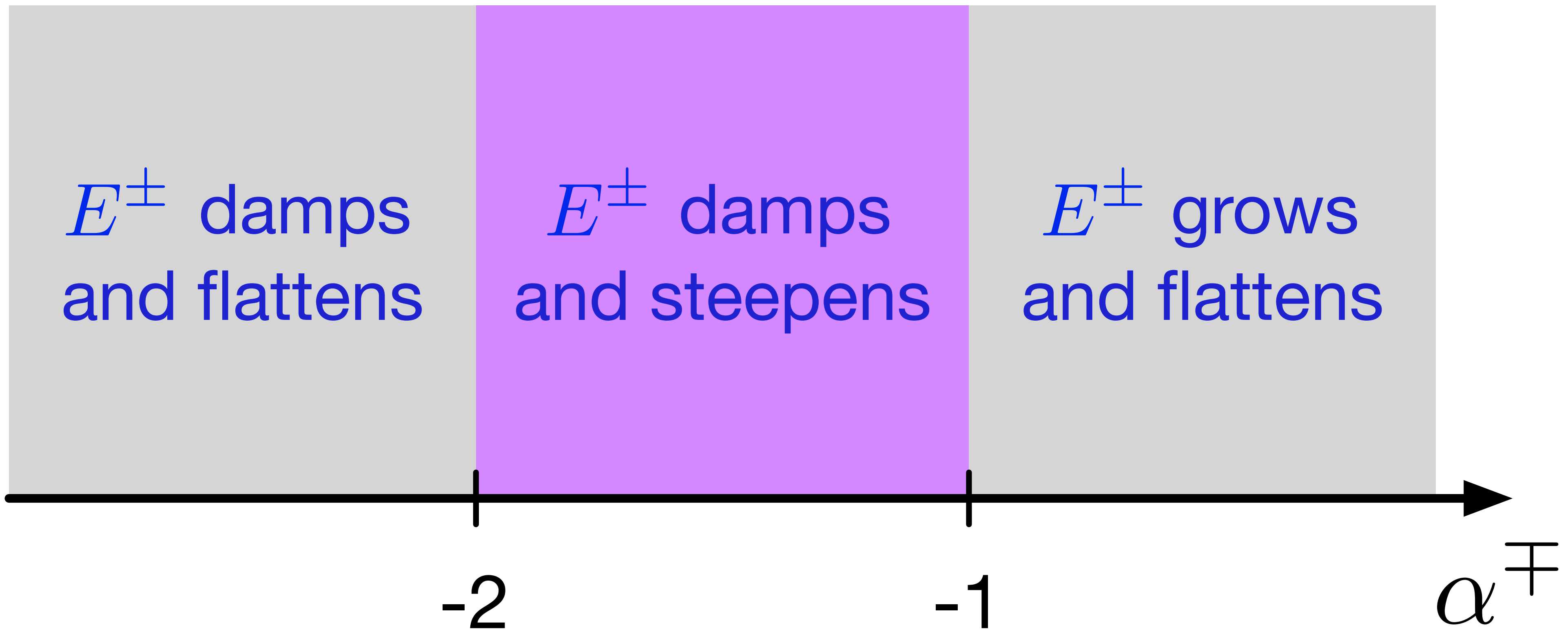
$$\frac{\partial E^{\pm}}{\partial t} = \frac{\pi}{v_{\mathrm{A}}} k_z^2 E^{\pm} \frac{\partial}{\partial k_z} \left(k_z E^{\mp} \right)$$

$$E^{\mp} \propto k_z^{\alpha^{\mp}} \longrightarrow \frac{\partial}{\partial t} \ln E^{\pm} \propto (1 + \alpha^{\mp}) k_z^{2+\alpha^{\mp}}$$



Nonlinear Evolution of the Parametric Instability

$$E^{\mp} \propto k_z^{\alpha^{\mp}} \longrightarrow \frac{\partial}{\partial t} \ln E^{\pm} \propto \left(1 + \alpha^{\mp}\right) k_z^{2+\alpha^{\mp}}$$



Nonlinear Evolution of the Parametric Instability

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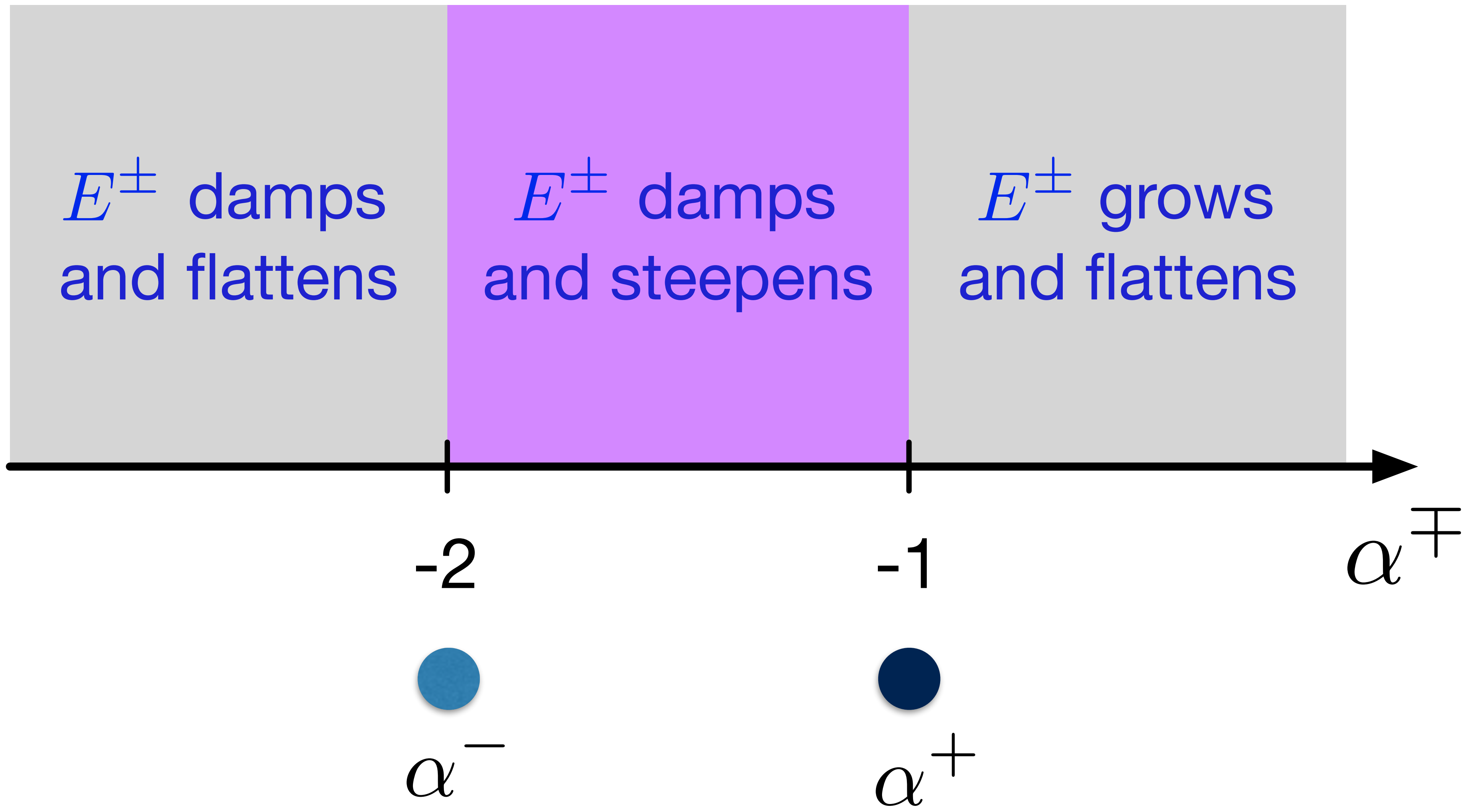

Nonlinear Evolution of the Parametric Instability

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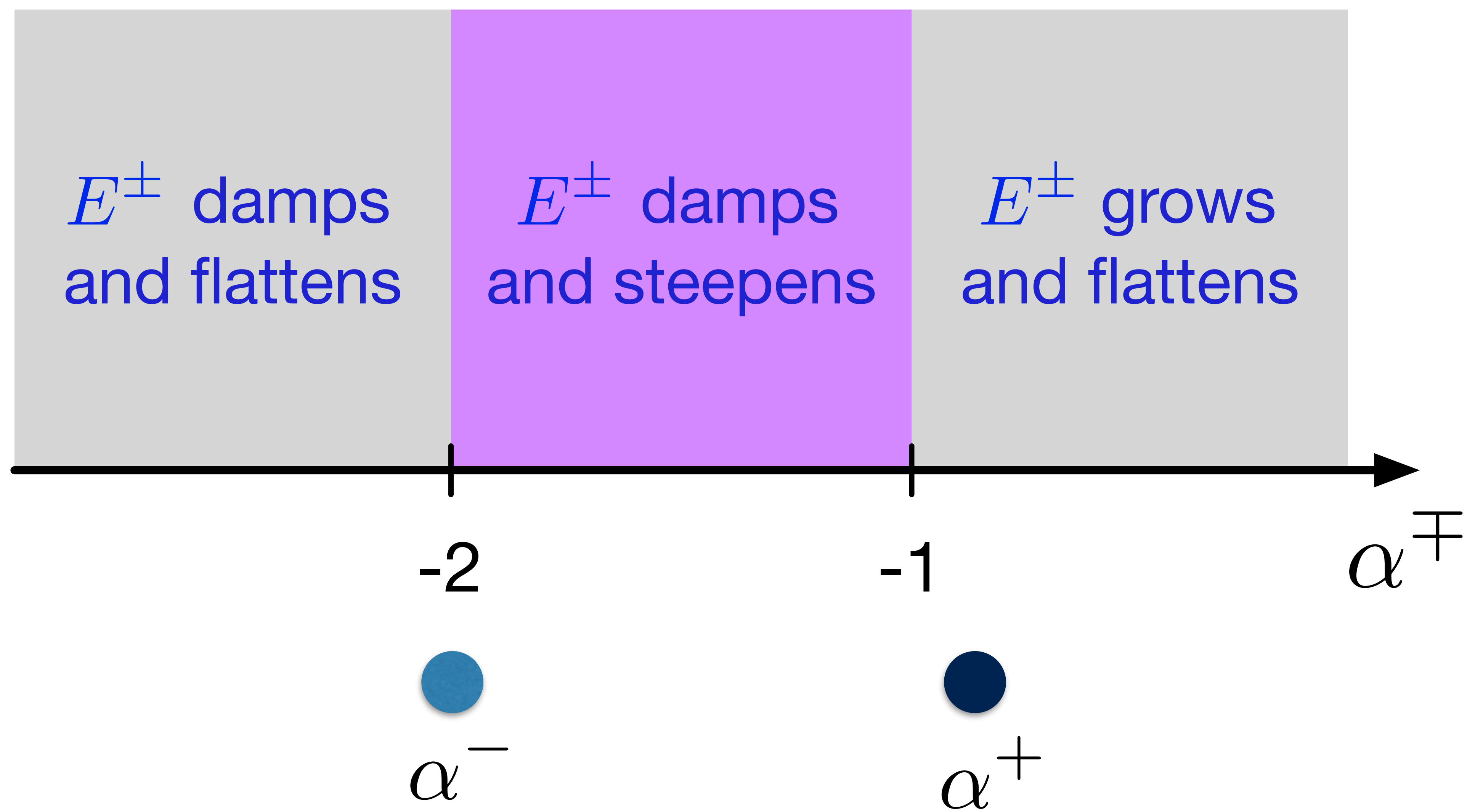
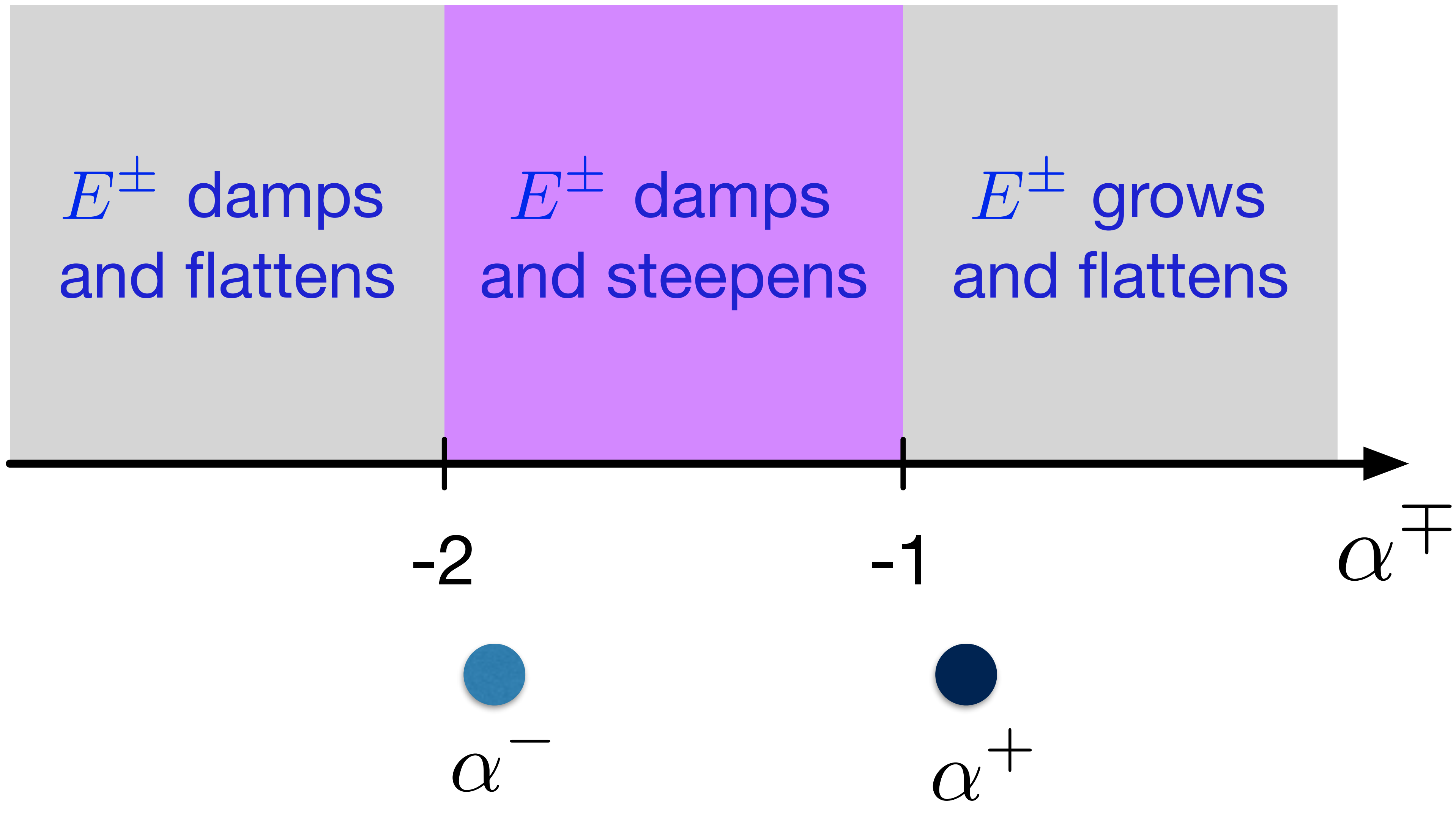


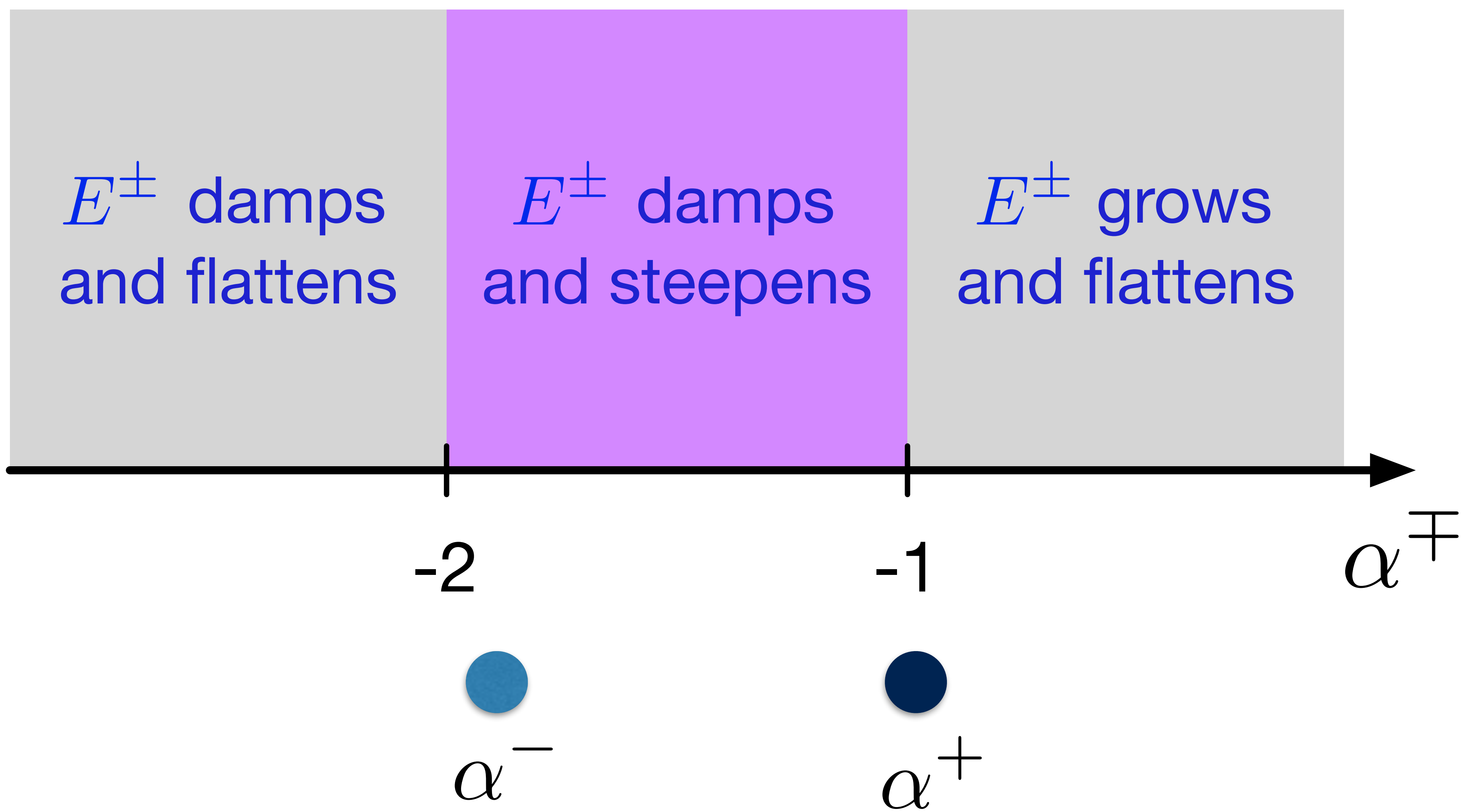
Nonlinear Evolution of the Parametric Instability

$$E^{\mp} \propto k_z^{\alpha^{\mp}} \longrightarrow \frac{\partial}{\partial t} \ln E^{\pm} \propto \left(1 + \alpha^{\mp}\right) k_z^{2+\alpha^{\mp}}$$



Nonlinear Evolution of the Parametric Instability

$$E^{\mp} \propto k_z^{\alpha^{\mp}} \longrightarrow \frac{\partial}{\partial t} \ln E^{\pm} \propto \left(1 + \alpha^{\mp}\right) k_z^{2+\alpha^{\mp}}$$



Future Directions

- assess the errors introduced by weak turbulence theory (via, e.g., numerical simulations)
- determine how results are modified as beta approaches unity.
- better treatment of slow-wave damping
- interplay between parametric instability and other types of nonlinear interactions, as well as linear non-WKB reflection.
- effects of solar-wind expansion and radial evolution

Conclusion and Predictions

- Alfven-wave turbulence is a leading candidate for explaining the heating and acceleration of the solar wind.

- The origin of the 1/f frequency spectrum of outward-propagating Alfven waves is an important unsolved problem.

- Here I have argued that in the fast solar wind, a 1/f magnetic spectrum at sub-hour timescales emerges dynamically between 10 Rs and 60 Rs via parametric instability and inverse cascade.

- Prediction: the 1/f range is much broader at 60 Rs than at 10 Rs.

- Prediction: the 1/f range spreads out in both directions from the initial energy-dominating frequency (at which fe_f is maximized). As PSP gets closer to the Sun, the 1/f range that it sees in fast wind will narrow from both the high and low-frequency ends, eventually disappearing at small enough r.