Resonant Drag Instabilities

Some specific cases + pretty pictures

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With Phil Hopkins

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Squire+Hopkins arXiv:1706.05020

Hopkins+Squire arXiv:1707.02997

$\mathcal{F}$ = compressible hydro

Ingredients

Resonance condition

$$\omega_{\mathcal{F}} = k c_s$$
$$= \boldsymbol{k} \cdot \boldsymbol{w}_s$$

$$\cos\theta_k = \pm \frac{c_s}{w_s}$$

Jordan-form theory

$$\omega^{(1)} = \pm i \mu^{1/2} \left[\left(\xi^L_{\mathcal{F}} T^{(1)}_{\rho_d} \right) \left(\boldsymbol{k}^T D^{-1}_{\text{drag}} C_{\boldsymbol{v}} \xi^R_{\mathcal{F}} \right) \right]^{1/2}$$

Easy...

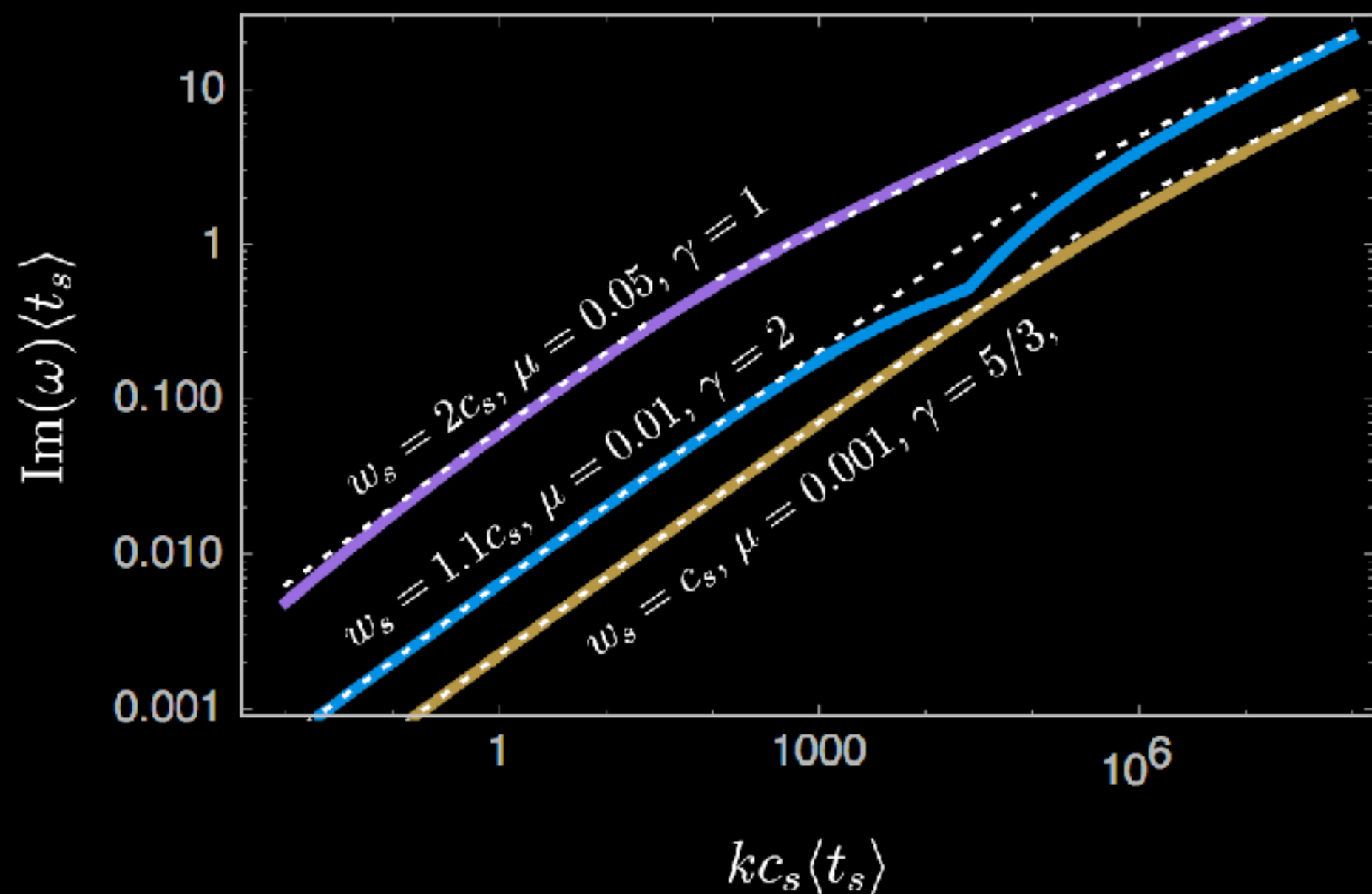
$$\xi^L_{\mathcal{F}} = (\xi^R_{\mathcal{F}})^T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \frac{k_x}{k} & \frac{k_z}{k} \end{pmatrix}$$

$$C = i \begin{pmatrix} 0 & 1 & 0 \\ -w_s \eta & 0 & 1+\zeta \end{pmatrix} \quad T_{1\rho} = \begin{pmatrix} 0 \\ 0 \\ i w_s \end{pmatrix}$$

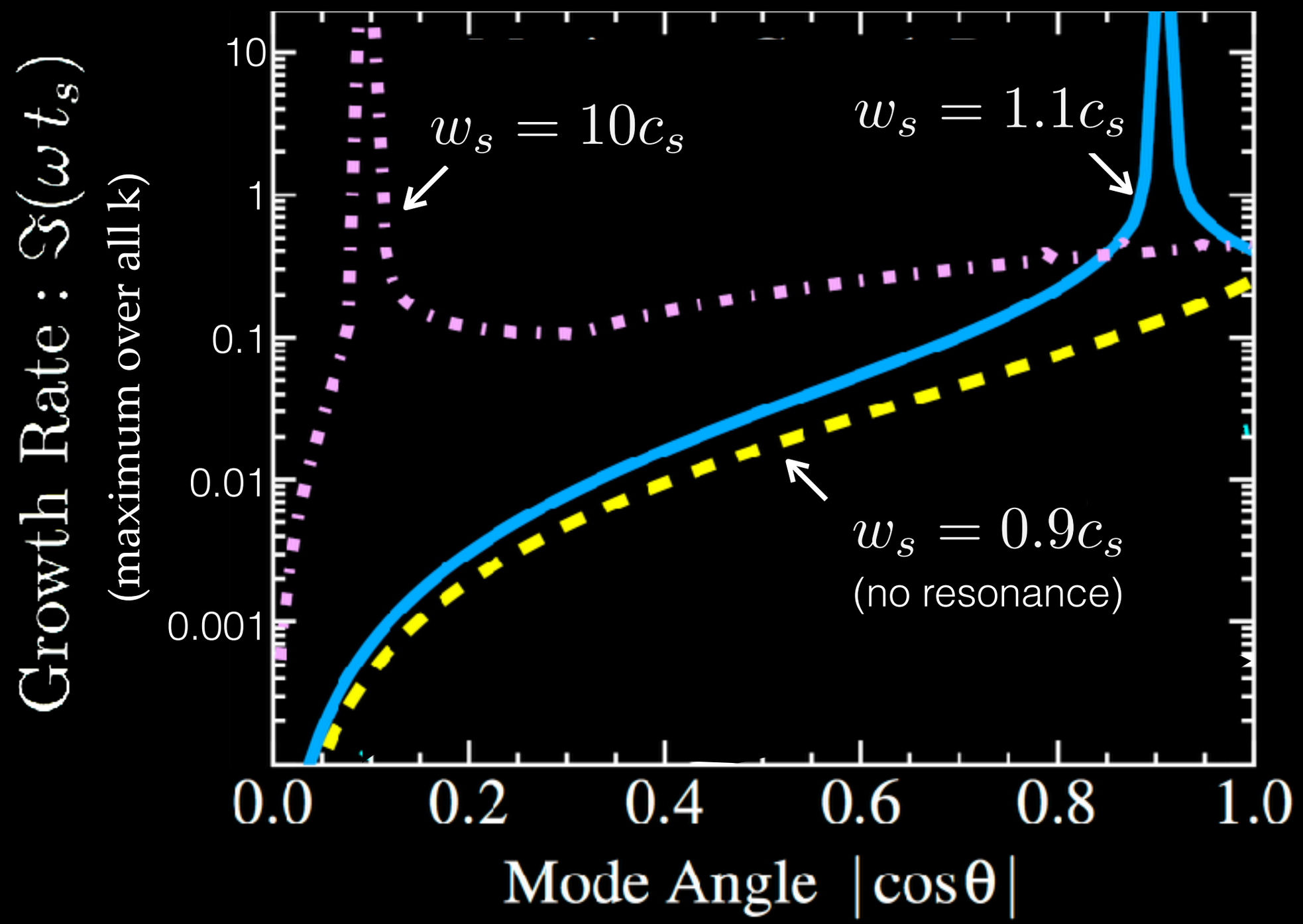
$$D_{\text{drag}} = -i \begin{pmatrix} 1 & 0 \\ 0 & 1+\zeta \end{pmatrix}$$

“Acoustic RDI”

$$\omega \approx kc_s + \mu^{1/2}\frac{1+i}{2}k^{1/2}\sqrt{\frac{c_s}{\langle t_s\rangle}\left|1-\frac{\eta}{1+\zeta}\right|}$$



Always possible+fastest-growing if $w_s > c_s$



$\mathcal{F}$ = MHD

Resonance condition

$$\boldsymbol{k} \cdot \boldsymbol{w}_s = kV_{\mathrm{Slow}}(\hat{\boldsymbol{k}} \cdot \boldsymbol{B})$$

OR

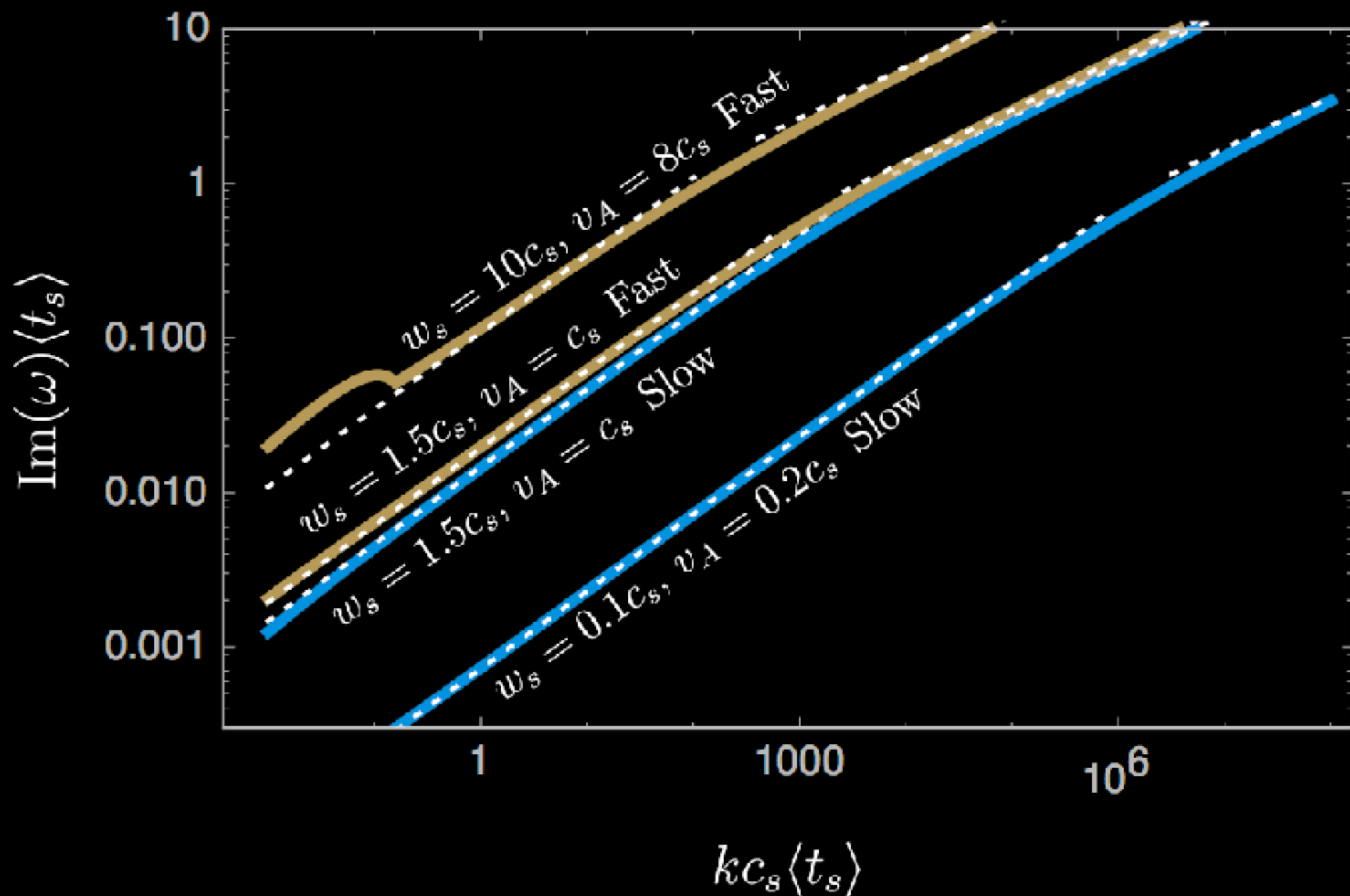
$$\boldsymbol{k} \cdot \boldsymbol{w}_s = kV_{\mathrm{Fast}}(\hat{\boldsymbol{k}} \cdot \boldsymbol{B})$$

Complicated… but

$V_{\mathrm{Slow}}(\hat{\boldsymbol{k}} \cdot \boldsymbol{B}) \to 0$ **as** $\hat{\boldsymbol{k}} \cdot \boldsymbol{B} \to 0$ → **RDI for *any* drift velocity**

$\Im(\omega) \to \infty$ **as** $k \to \infty$

“Magnetosonic RDI”



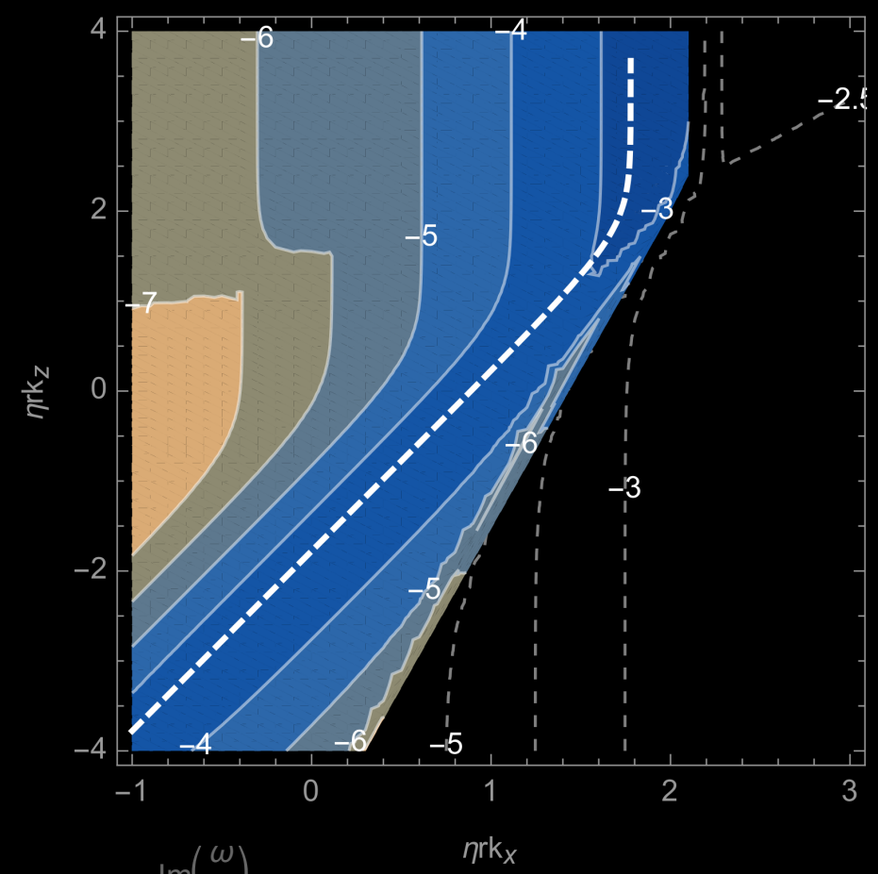
$\mathcal{F}$ = accretion disk

Epicyclic oscillations $\omega_\kappa = \pm \frac{k_z}{k} \Omega$

Resonance occurs only at one k $\omega_\kappa = \boldsymbol{k} \cdot \boldsymbol{w}_s$

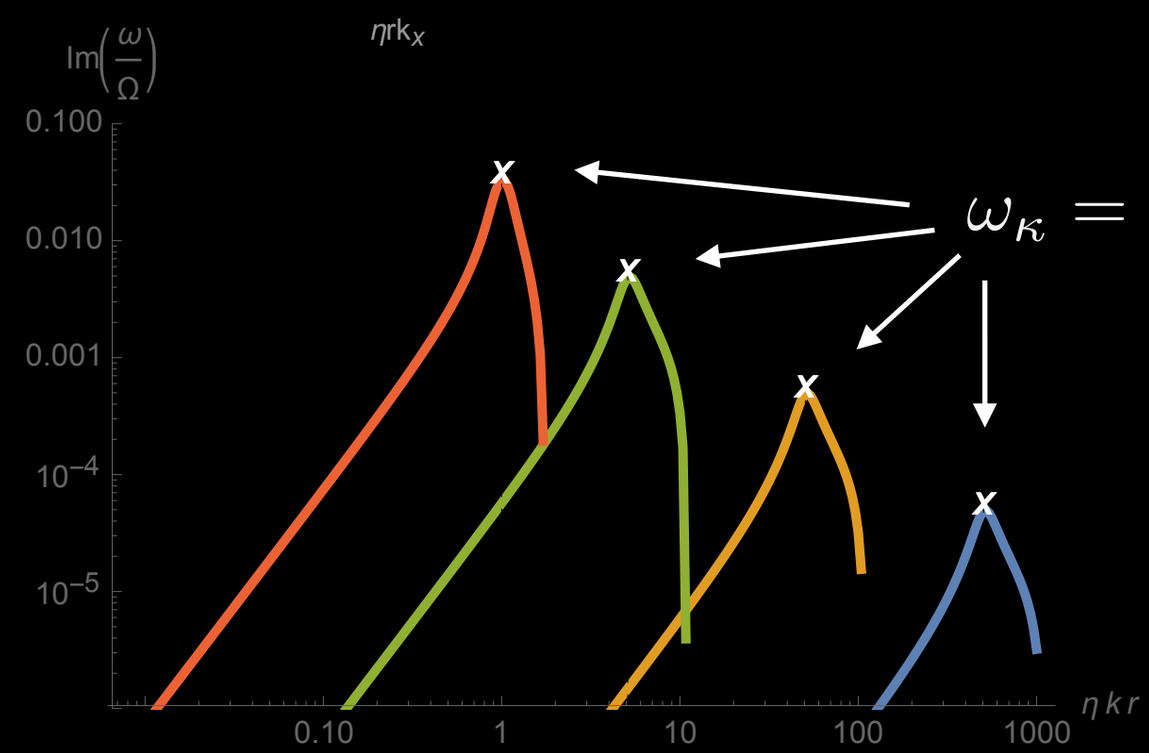
“Epicyclic RDI” = streaming instability

Explains origin



Youdin & Goodman

Simple analytic prediction of growth rate

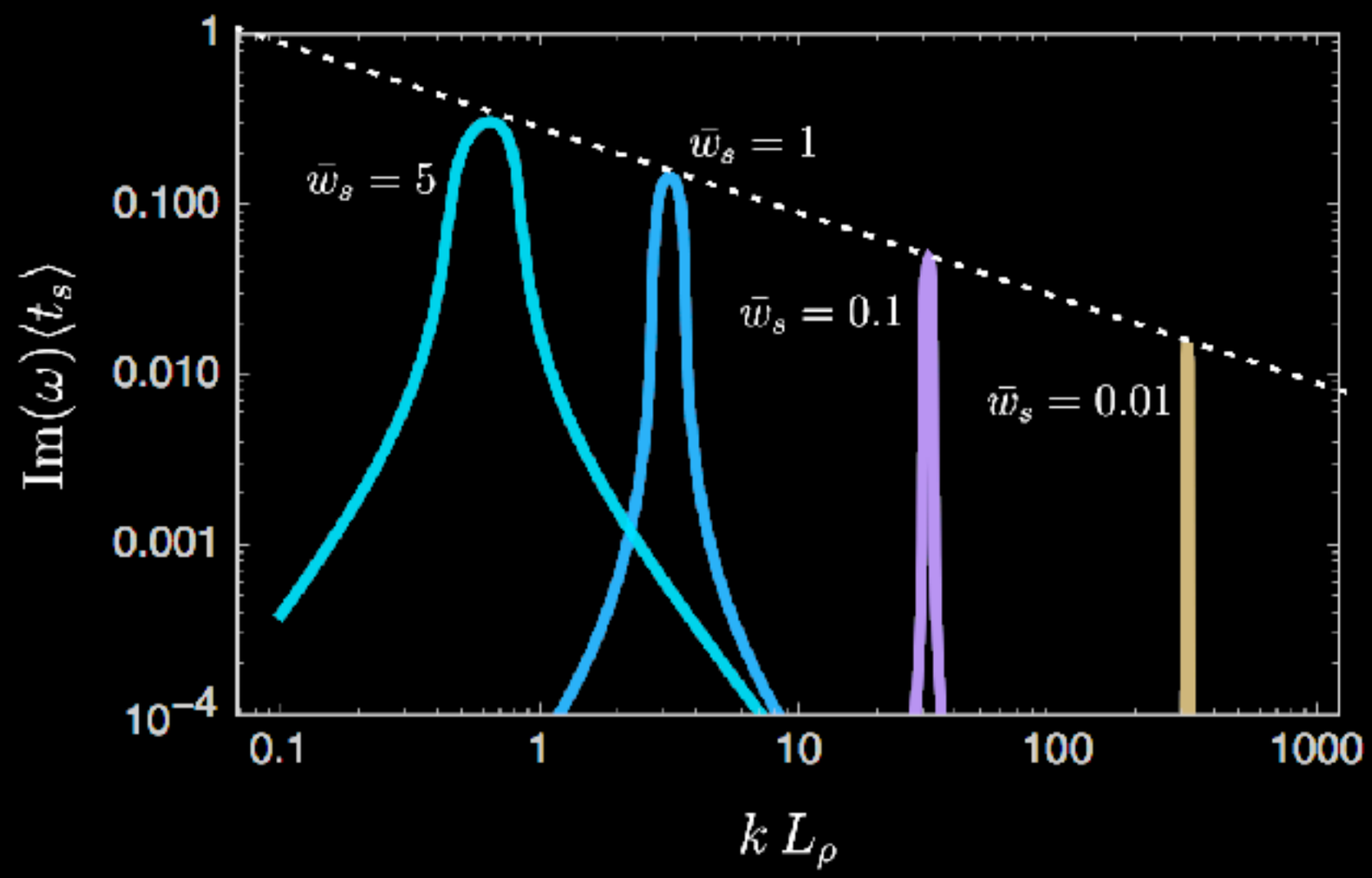


$$\omega_\kappa = \boldsymbol{k} \cdot \boldsymbol{w}_s$$

$\mathcal{F}$ = stratified fluid

Resonance with Brunt-Väisälä oscilations $\pm \frac{k_\perp}{k} N = \boldsymbol{k} \cdot \boldsymbol{w}_s$

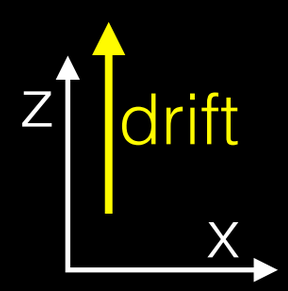
"Brunt-Väisälä" RDI



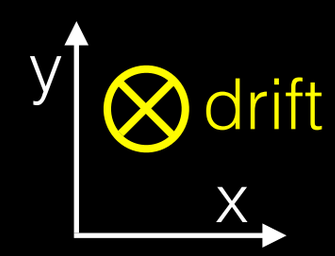
etc…

Nonlinear development $w_s \approx 10 c_s$

$L_{\rm box} \sim 100\, c_s \langle t_s \rangle \qquad \Delta t \sim 500\, \langle t_s \rangle$



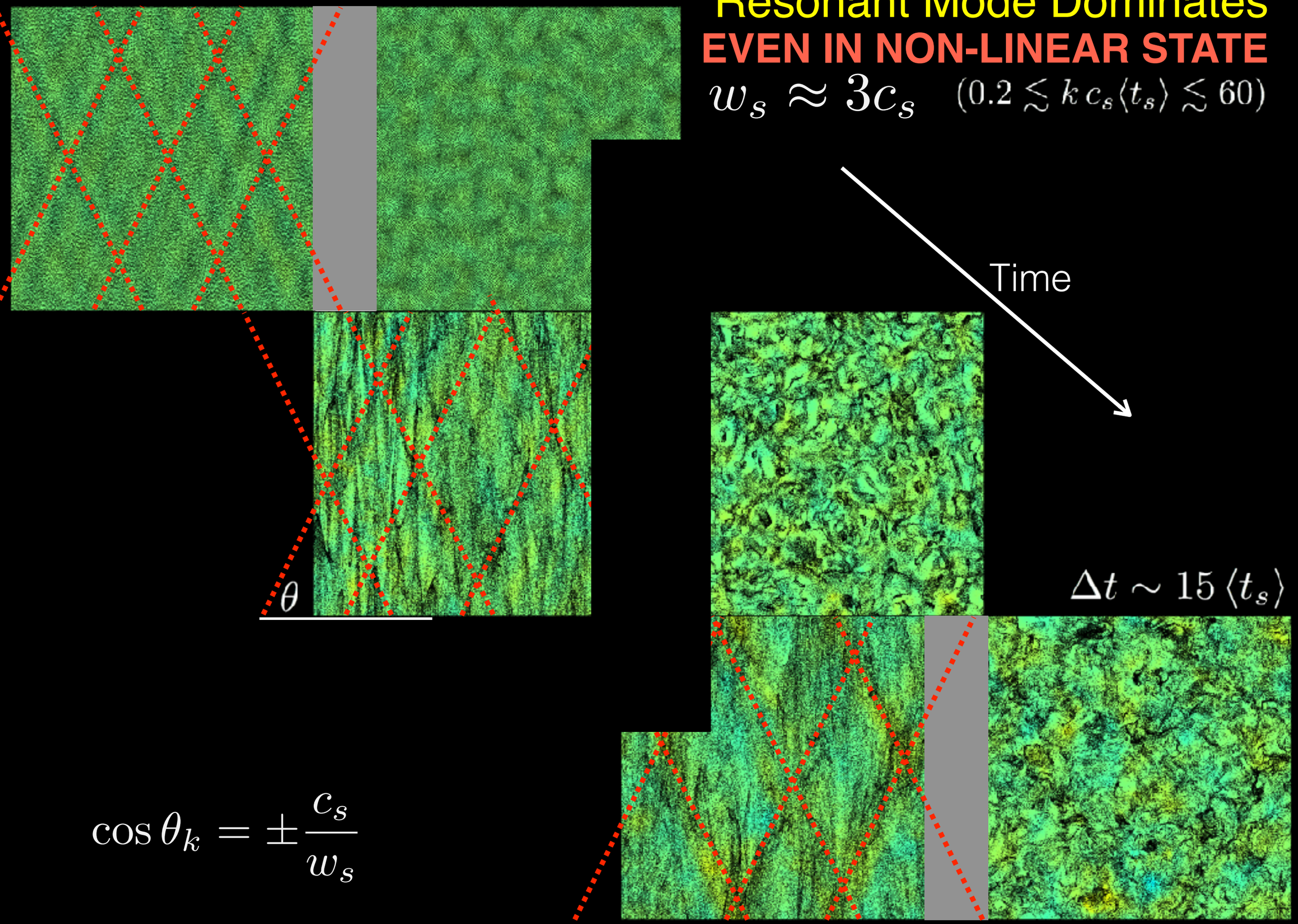
Simulations by Phil Hopkins with GIZMO



Resonant Mode Dominates

EVEN IN NON-LINEAR STATE

$w_s \approx 3c_s \quad (0.2 \lesssim k\, c_s \langle t_s \rangle \lesssim 60)$



$$\cos\theta_k = \pm \frac{c_s}{w_s}$$

The RDI

A “how to”

1. Pick a fluid wave
2. Match the (projected) phase speed to w_s
3. It’s probably unstable with growth rate

$$\omega^{(1)} = \pm i\mu^{1/2}\left[\left(\xi^L_{\mathcal{F}} T^{(1)}_{\rho_d}\right)\left(\boldsymbol{k}^T D^{-1}_{\text{drag}} C_{\boldsymbol{v}} \xi^R_{\mathcal{F}}\right)\right]^{1/2}$$

Application to kinetic instabilities? Cosmic rays?