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Sufficient toroidal rotation is critical to ITER’s success
- Magnitude of rotation can stabilize resistive wall modes (RWMs)
- Rotation shear can decrease microinstabilities and promote formation of ITBs

Sources and sinks of rotation in ITER
- Neutral beams: source
- Turbulent intrinsic rotation: source
- Neoclassical toroidal viscosity (NTV): typically sink

I will focus on computing NTV torque caused by symmetry-breaking in ITER
- Finite number (18) of toroidal field (TF) coils
- Test blanket modules (TBMs)
  - Ferritic steel
  - Used to test tritium breeding
  - Installed in 3 equitorial ports (low n perturbation)
  - Experiments on DIII-D using TBM mock-ups found a reduction in rotation by as much as 60%
- Ferritic inserts (FIs)
  - Installed inside TF coils to reduce toroidal ripple
  - FIs implemented on JT-60U and JFT-2M have decreased counter-current rotation

\[^1\text{K. Shinohara et al, Fusion Engineering and Design 84, 24 (2009)}\]
When toroidal symmetry is broken, trapped particles may wander off a flux surface

- Banana diffusion: particles trapped poloidally (bananas) can drift radially, as $J_{\parallel}$ becomes a function of toroidal angle
- Ripple trapping: if local ripple wells exist along a field line and the collisionality is sufficiently small, particles can become helically trapped
  - ITER’s collisionality may be sufficiently low such that this effect is important
- For a general electric field, the ion and electron fluxes are not identical
- Radial current induces a $\mathbf{J} \times \mathbf{B}$ torque, which is typically counter-current

$$
\tau^{\text{NTV}} = -B^\theta \sqrt{g} \sum_a Z_a e \Gamma_{\psi,a}
$$
Classes of Trapped Particles

- The \( n = 18 \) ripple may allow for local trapping in addition to banana diffusion.
- We must compute NTV torque such that all particle trajectories are accounted for.

\[ B \text{ along field line at } r/a = 0.9 \text{ due to TF ripple} \]
Method of Computing NTV

- Model of steady-state scenario profiles using TRANSP\(^2\)
  - Includes NBI model (NUBEAM), EPED1 pedestal model, and current diffusive ballooning mode transport model
- Calculate 3D MHD equilibrium using free boundary VMEC
  - Vacuum FI and TBM magnetic fields (computed with FEMAG\(^1\))
  - Filamentary coil models
  - Pressure and \(q\) profiles (from TRANSP)
- Estimate rotation (\(E_r\)) to find torque
  - Semi-analytic intrinsic rotation model
  - NBI torque (from NUBEAM)
  - Use SFINCS to determine \(E_r\) consistent with rotation
- Solve drift kinetic equation to compute radial particle flux using SFINCS
  - Using profiles, geometry, and \(E_r\) as explained above
  - Makes no assumptions about magnitude of perturbing field and accounts for all collisionality regimes

\(^1\)K. Shinohara et al, *Fusion Engineering and Design* 84, 24 (2009)
Roadmap for Remainder of Talk

- ITER steady state scenario
- VMEC equilibrium with ripple
- Rotation model (without NTV) to estimate $E_r$
- SFINCS calculations of particle and heat fluxes
- Comparison of NTV torque with analytic scaling predictions and other toroidal torques
- Tangential magnetic drift models
ITER Steady State Scenario

Scenario Parameters

- 33 MW NBI, 20 MW EC, 20 MW LH heating
- 9 MA toroidal plasma current
- Fusion gain $Q = 5$

Simulated using Tokamak Simulation Code (TSC) in IPS framework and TRANSP

NBI source modeled using NUBEAM

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2F. Poli et al, Nuclear Fusion 54, 7 (2014)
Free Boundary VMEC Equilibrium

\[ \delta_B = \frac{B_{\text{max}} - B_{\text{min}}}{B_{\text{max}} + B_{\text{min}}} \]

- TBMs increase magnitude of ripple near outboard midplane
- FIs decrease poloidal extent of ripple
- Ripple magnitude decreases with radius
Ferritic inserts decrease magnitude of ripple away from midplane
TBMs locally decrease magnetic flux near midplane
We need to determine a plausible range of toroidal rotation in ITER, as NTV torque is a nonlinear function of $E_r$.

To this end, we consider a model including turbulent momentum transport and an NBI source.

### Time-independent toroidal momentum balance

\[
\nabla \cdot \Pi^\text{turb}_\zeta (\Omega_\zeta) + \nabla \cdot \Pi^\text{NC}_\zeta (\Omega_\zeta) = \tau^\text{NBI}
\]

NTV - will not consider here

\[
\Pi^\text{turb}_\zeta = -m_i n_i \chi_\zeta \langle R^2 \rangle \frac{\partial \Omega_\zeta}{\partial r} + \Pi^\text{int}
\]

### Diffusion

Intrinsic momentum source
Estimating Toroidal Rotation

- We compute $\Omega_\zeta$ due to each source (turbulent intrinsic and NBI) and add results to obtain total $\Omega_\zeta$

\[- \frac{1}{V'} \frac{\partial}{\partial r} \left( V' m_i n_i \chi_i \langle R^2 \rangle \frac{\partial \Omega_\zeta}{\partial r} \right) = \tau^{\text{NBI}} - \nabla \cdot \Pi_{\text{int}}\]

NBI rotation

- NUBEAM computes toroidal rotation by balancing $\tau^{\text{NBI}}$ with turbulent momentum diffusion ($P_r = 1$)

$$\tau^{\text{NBI}} = - \frac{1}{V'} \frac{\partial}{\partial r} \left( V' m_i n_i \chi_i \langle R^2 \rangle \frac{\partial \Omega_\zeta}{\partial r} \right)$$
Estimating Toroidal Rotation

**Turbulent intrinsic rotation model**

- Steady-state: $\Pi_{\text{int}} = m_i n_i \chi_\zeta \langle R^2 \rangle \frac{\partial \Omega_\zeta}{\partial r}$
- Up-down asymmetry due to neoclassical: $\Omega_\zeta \sim \rho_\ast,\theta \left( \frac{v_{ti}}{R} \right)$
- $\Omega_\zeta \Pi_{\text{int}}/Q_i \sim \rho_\ast,\theta$ where $Q_i$ is turbulent heat flux
- Boundary condition: $\Omega_\zeta(a) = 0$
- Consistent with gyrokinetic GS2 simulations and observations on MAST

$$\Omega_\zeta(r) = - \int_r^a \frac{v_{ti} \rho_\ast,\theta}{2L_T^2} dr'$$

- We assume $P_r = 1$ (consistent with NUBEAM calculations)

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Parra\(^4\) scaling:
\[ V_\zeta \sim \frac{T_i}{I_P} \sim 100 \text{ km/s} \]

Rice\(^5\) scaling:
\[ V_\zeta \sim \frac{W_p}{I_p} \sim 400 \text{ km/s} \]

A critical Alfvèn Mach number,
\[ M_A = \frac{\Omega_\zeta(0)}{\omega_A} \gtrsim 5\%, \]
must be achieved for stabilization of the RWM\(^6\)

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Drift kinetic equation solved for $f_{a1}$ on a flux surface

\[
(v_{||b} + v_E + v_{ma}) \cdot (\nabla f_{a1})_{W,\mu} - C(f_{a1}) =
\]

\[
- v_{ma} \cdot \nabla \psi \left( \frac{\partial f_{a0}}{\partial \psi} \right)_{W,\mu} + \frac{Z_a e v_{||} B \langle E_{||} B \rangle}{T_a \langle B^2 \rangle} f_{a0}
\]

- **Tangential magnetic drifts** will not be included for most of the following calculations (stay tuned)
- **Inductive electric field** is small for this steady-state scenario (loop voltage $\sim 10^{-4} V$), so this term is not retained
- $C$ – linearized Fokker-Planck operator
- 3 species ($e, D, T$)
Circle indicates neoclassical offset rotation ($\approx -10 \text{ km/s}$)

Increased $|\tau^{NTV}|$ due to $1/\nu$ transport near $E_r = 0$

Addition of FIs decreases $|\tau^{NTV}|$ by $\sim 75\%$
TBM Ripple Does Not Cause Significant NTV Torque

- $|n| = 18$: TF ripple and Fls (and $|n| = 18$ component of TBM ripple)
- $|n| < 18$: TBMs
- Torque due to $|n| < 18$ component of TBM ripple $\approx 1\%$ of that due to $|n| = 18$ ripple
- More difficult to form local minima with low $n$ ripple

$r/a = 0.9$
At large $E_r$, $\tau^{NTV}$ does not depend strongly on radius

At small $|E_r|$, $|\tau^{NTV}|$ increases with decreasing radius due to $T_i^{7/2}$ scaling in the $1/\nu$ regime
NTV - Rippled Tokamak Collisionality Regimes

Small $E_r$ ($\nu_{\text{eff}} > q\omega_E$)

Superbanana plateau

Superbanana

Collisional boundary layer ($\nu - \sqrt{\nu}$)

Collisionless detrapping ($\nu$)

Large $E_r$ ($\nu_{\text{eff}} < q\omega_E$)

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Scaling of NTV Torque with Ripple Magnitude

**Banana Diffusion Regimes**

- Collisional boundary layer \((\nu - \sqrt{\nu})\)
  - \(\tau_{NTV} \sim (\delta_B)^2\)
- Collisionless detrapping/trapping (CDT)
  - \(\tau_{NTV} \sim (\delta_B)\)

**Ripple-trapping Regimes**

- Tokamak ripple-trapping \(\nu\) regime
  - \(\tau_{NTV} \sim (\delta_B)^0\)
- Results appear consistent with stellarator \(\sqrt{\nu}\) regime
  - \(\tau_{NTV} \sim (\delta_B)^{3/2}\)

\(r/a = 0.9\), TF only geometry
Toroidal symmetry-breaking causes additional radial heat flux over axisymmetric level

Insignificant in comparison to turbulent heat flux $Q_i \approx 0.2 \text{ MW/m}^2$ (estimated from volume integral of heating and fusion power)

$r/a = 0.9$
For calculations presented previously, $\mathbf{v}_m \cdot \nabla f_1$ was not included in the drift kinetic equation solved by SFINCS.

SFINCS does not include radial coupling of $f_1$, so $\nabla \psi$ component of $\mathbf{v}_m$ is not retained when this term is included in the kinetic equation, which introduces a coordinate dependence.
Tangential Magnetic Drifts

We compare two tangential drift models

- Typical definition of $\nabla B$ and curvature drifts
  - Coordinate-dependent form and does not conserve phase space when radially local assumption is made
    $$
    \mathbf{v}_m = \frac{v_{\perp}^2 + 2v_{\parallel}^2}{2\Omega B^2} \mathbf{B} \times \nabla B + \frac{v_{\parallel}^2}{\Omega B} \nabla \times \mathbf{B}
    $$

- For $\mathbf{v}_m$, the $\nabla B$ drift has been projected onto a flux surface
  - This eliminates dependence on choice of toroidal and poloidal angles
  - This form has been shown\(^8\) to eliminate the need for additional particle and heat sources due to the radially local assumption
  - Regularization required for conservation performed by dropping curvature drift (deeply-trapped assumption)
    $$
    \mathbf{v}_{m} = \frac{v_{\perp}^2}{2\Omega B^2} (\mathbf{B} \times \nabla \psi) \frac{\nabla \psi \cdot \nabla B}{|\nabla \psi|^2}
    $$

Comparison of Tangential Magnetic Drift Models

- At this radius, $\rho_* \sim \nu_*$
- Inclusion of $v_m \cdot \nabla f_1$ shifts resonant peak to $E_r$ such that $(v_m + v_E) \approx 0$ (superbanana plateau resonance)
- Differences between tangential drift models are appreciable only for small $|E_r|$
NTV Torque is Comparable to NBI and Turbulent Torques

\( \mathbf{v}_m \cdot \nabla f_1 \) has been included for NTV calculations, as \( E_r \) has a zero-crossing for the NBI rotation model at \( r/a = 0.62 \).

We assume
\[ \Pi_{\text{int}} \sim \rho_* \theta QR / v_{ti} \]
to get scaling for \( \tau^{\text{turb}} \).

\( \tau^{\text{NTV}} \) of similar magnitude to NBI and turbulent torques (without FIs).
Conclusions

- SFINCS can be applied to compute neoclassical ripple transport without the need for simplifying assumptions about transport regimes, the magnitude of ripple, or the collision operator.
- The TBM (low $n$) ripple will not cause significant NTV torque in ITER.
- The TF ripple (without FIs) would cause NTV torque which is similar in magnitude to both the turbulent intrinsic and NBI torques and will significantly damp rotation.
- The inclusion of FIs will decrease the magnitude of NTV torque by $\approx 75\%$.
- Tangential drift models must be considered when $E_r$ is near a zero-crossing.
- Scaling of NTV torque with ripple magnitude indicates local ripple-trapping may be significant for ITER.
Application of Adjoint Methods to Stellarator Coil Optimization (Work in Progress)

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Adjoints & Stellarator Coil Optimization

- Adjoint methods allow rapid calculation of the sensitivity of output quantities with respect to a large number of input parameters in optimization and uncertainty quantification problems.
- These methods have been widely applied in aerospace engineering, aerodynamics, and geophysics.

Figure: CFD car aerodynamics - sensitivity maps of cost functionals

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Statement of Constrained Optimization Problem

\[ \min_p \chi^2(u, p) \text{ subject to } F(u, p) = 0 \]

- Cost functional: \( \chi^2(u, p) \)
- Input parameters: \( p \)
- State variables: \( u(p) \)
- State equation: \( F(u, p) = 0 \)
  - For linear system: \( A(p)u(p) - b(p) = 0 \)
The Hard Way

\[ \frac{\partial \chi^2}{\partial p_i} \bigg|_{F=0} = \frac{\partial \chi^2}{\partial p_i} + \frac{\partial \chi^2}{\partial u} \left( \frac{\partial u}{\partial p_i} \right)_{F=0} \]

- \( \frac{\partial \chi^2}{\partial p_i} \) and \( \frac{\partial \chi^2}{\partial u} \) can be analytically differentiated (easy to compute)
- Computing \( \left( \frac{\partial u}{\partial p_i} \right)_{F=0} \) requires \( n_p = \text{length}(\mathbf{p}) \) solves of linear system
  - Can be obtained by finite differencing in \( p_i \)
  - Or by inverting \( \mathbf{A} \) \( n_p \) times

\[ \left( \frac{\partial u}{\partial p_i} \right)_{F=0} = -\mathbf{A}^{-1} \left( \frac{\partial \mathbf{A}}{\partial p_i} \mathbf{u} - \frac{\partial \mathbf{b}}{\partial p_i} \right) \] (1)
Sensitivity for Linear Constrained Optimization
The Adjoint Way

\[
\left( \frac{\partial u}{\partial p_i} \right)_{F=0} = -A^{-1} \left( \frac{\partial A}{\partial p_i} u - \frac{\partial b}{\partial p_i} \right)
\]

\[
\frac{\partial \chi^2}{\partial p_i} \bigg|_{F=0} = \frac{\partial \chi^2}{\partial p_i} - \frac{\partial \chi^2}{\partial u} A^{-1} \left( \frac{\partial A}{\partial p_i} u - \frac{\partial b}{\partial p_i} \right) q^T
\]

- Solve adjoint equation once: \( A^T q = \left( \frac{\partial \chi^2}{\partial u} \right)^T \)
- Compute \( \frac{\partial \chi^2}{\partial p_i} \bigg|_{F=0} \) for all \( p_i \) with just two linear solves (forward and adjoint)
- Form inner product with \( q \) to compute sensitivity for each \( p_i \)
REGCOIL computes stellarator coil shapes given a target plasma shape and coil winding surface.

- Current density $\mathbf{K} = \mathbf{n} \times \nabla \Phi$

- Current potential $\Phi(\theta, \zeta) = \Phi_{SV}(\theta, \zeta) + \frac{G\zeta}{2\pi} + \frac{I\theta}{2\pi}$ determined by REGCOIL and determined by plasma current.

Figure: Coil and plasma surfaces for W7-X

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REGCOIL finds least squares solution for $\Phi_{SV}$

- Tikhonov regularization (ridge regression) is applied to the ill-posed NESCOIL problem by including $\chi^2_K$ in the objective function.

- Objective function $\chi^2 = \chi^2_B + \lambda \chi^2_K$
  
  - $\chi^2_B = \int_{\text{plasma}} dA B_{\text{normal}}^2$
  
  - $\chi^2_K = \int_{\text{coil}} dA K^2$ (corresponds to inverse distance between coils)

- Least-squares solution for $\Phi_{SV}$ such that $\frac{\partial \chi^2}{\partial \Phi_{SV}} = 0$

- Takes the form $A \Phi_{SV} = b$
Suppose we want to compute sensitivity of some objective function with respect to the coil-winding surface geometry parameters for use in optimization.

Surface is parameterized by Fourier decomposition \( \Omega = r_{mn}^c, z_{mn}^s \)

### Sensitivity of \( \chi^2 \)

- It is straightforward to compute the sensitivity of \( \chi^2 \) (same objective function used to compute \( \Phi_{SV} \))

- As \( \Phi_{SV} \) is solution to the normal equations, \( \frac{\partial \chi^2}{\partial \Phi_{SV}} = 0 \)

- No adjoint solution is required, \( \chi^2 \) can be analytically differentiated

\[
\left. \frac{\partial \chi^2}{\partial \Omega_i} \right|_{F=0} = \frac{\partial \chi^2}{\partial \Omega_i}
\]
Adjoints Are Needed to Compute Sensitivity of Other Objective Functions

- If we would like to find the sensitivity with respect of a different objective function, $\tilde{\chi}^2$, than that used to solve for $\Phi_{SV}$, we must find a solution to the adjoint equation

$$A^T q = \left( \frac{\partial \tilde{\chi}^2}{\partial \Phi_{SV}} \right)^T$$

- $A^T$ must be inverted with a different RHS for each objective function of interest
- For example, we may want to compute sensitivity of $\chi_B^2$ or $\chi_K^2$ with respect to $\Omega$
Sensitivity computation of $\chi^2$ has been implemented. Testing has been performed by finite differencing in W7X and circular torus coil geometry parameters. Next step: complete testing of adjoint solve.
Conclusions

- Adjoint methods allow for fast computation of gradients with respect to input parameters.
- This can be applied to the optimization of a coil winding surface in REGCOIL, requiring only 2 linear solves to compute gradients with respect to all geometry parameters (for W7X, \( \text{length}(\Omega) \approx 50 \)).
- Currently being implemented and tested (stay tuned for results).
- Eventually this extended REGCOIL method could be implemented within an optimization loop.