10th Plasma Kinetics Working Meeting --
WPI, Vienna, Austria -- Jul 28 2017

Quasi-nonlinear approach to the Weibel instability

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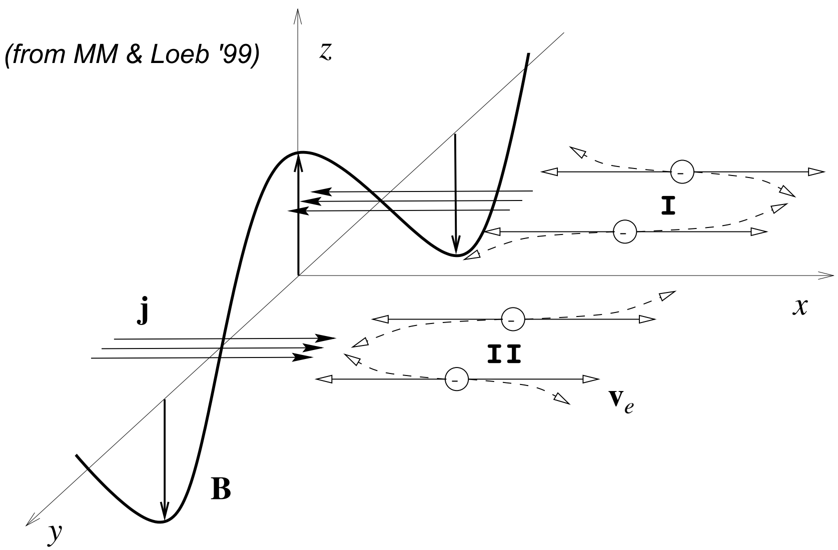
Weibel instability



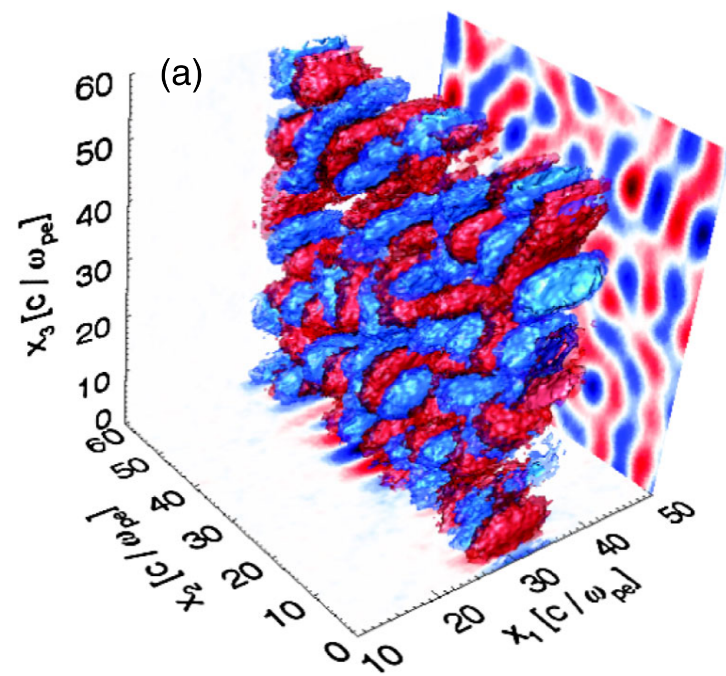
linear instability: *filamentation*

saturation: $k\,\rho \sim 1$ or $\gamma\,\Omega_{bounce} \sim 1$

nonlinear stage: *coalescence*

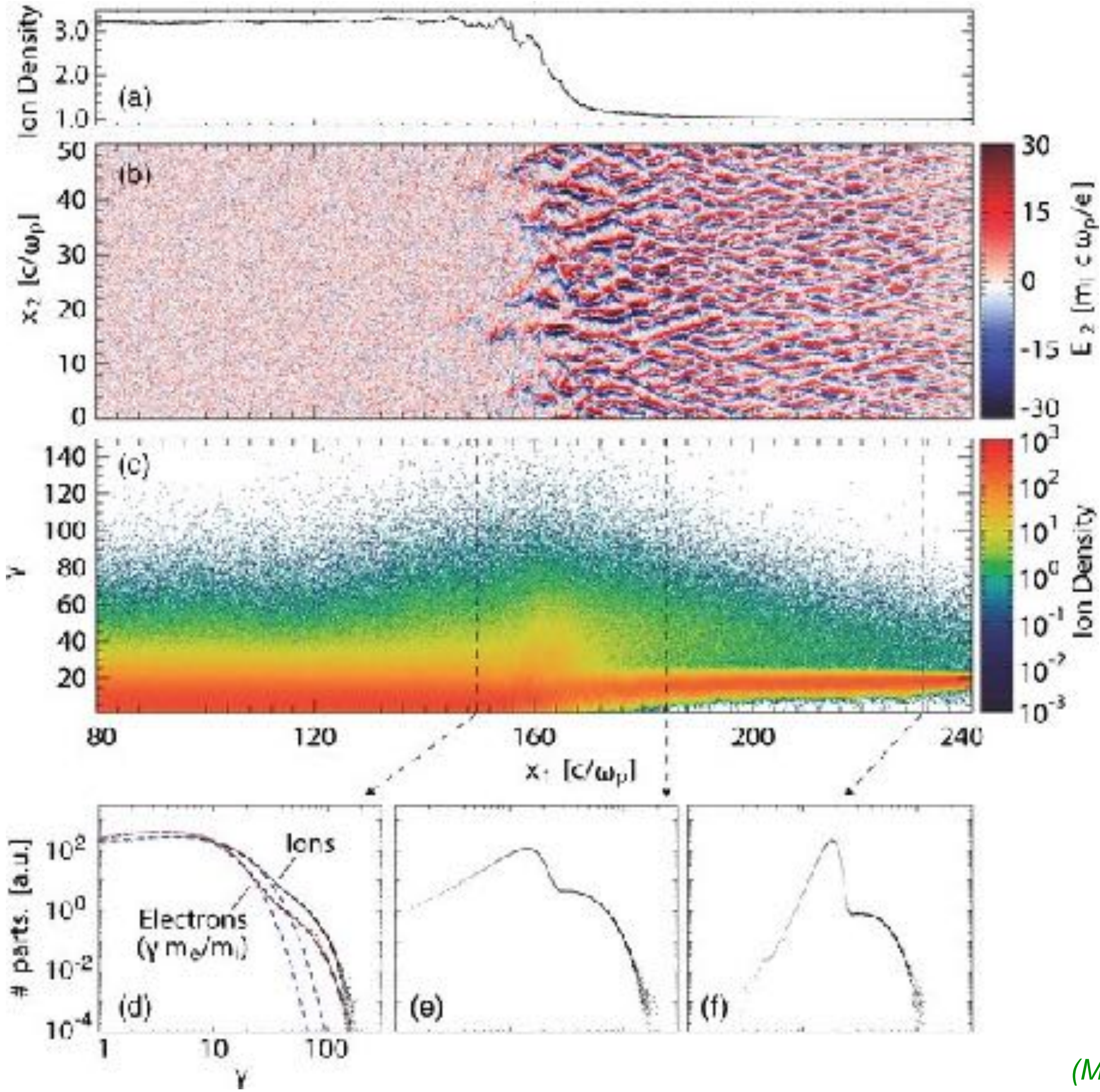
deeply nonlinear regime: -?-

WI with ultra-intense laser beam



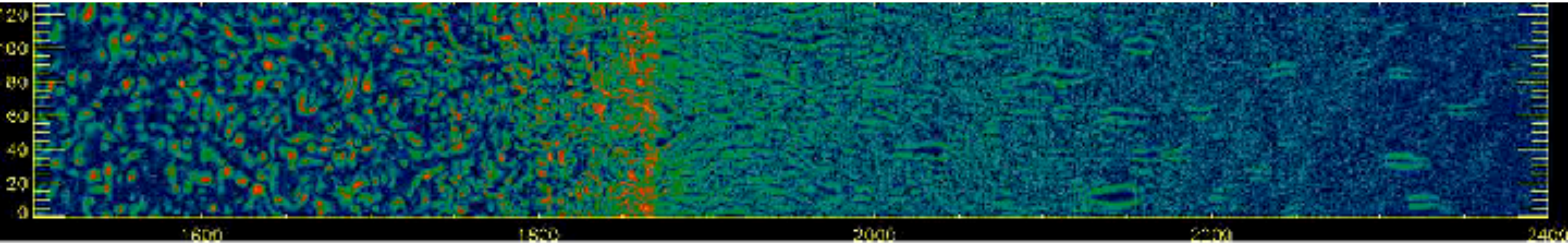
(Stockem et al. '14)

Collisionless shocks with WI



(Martins et al. '09)

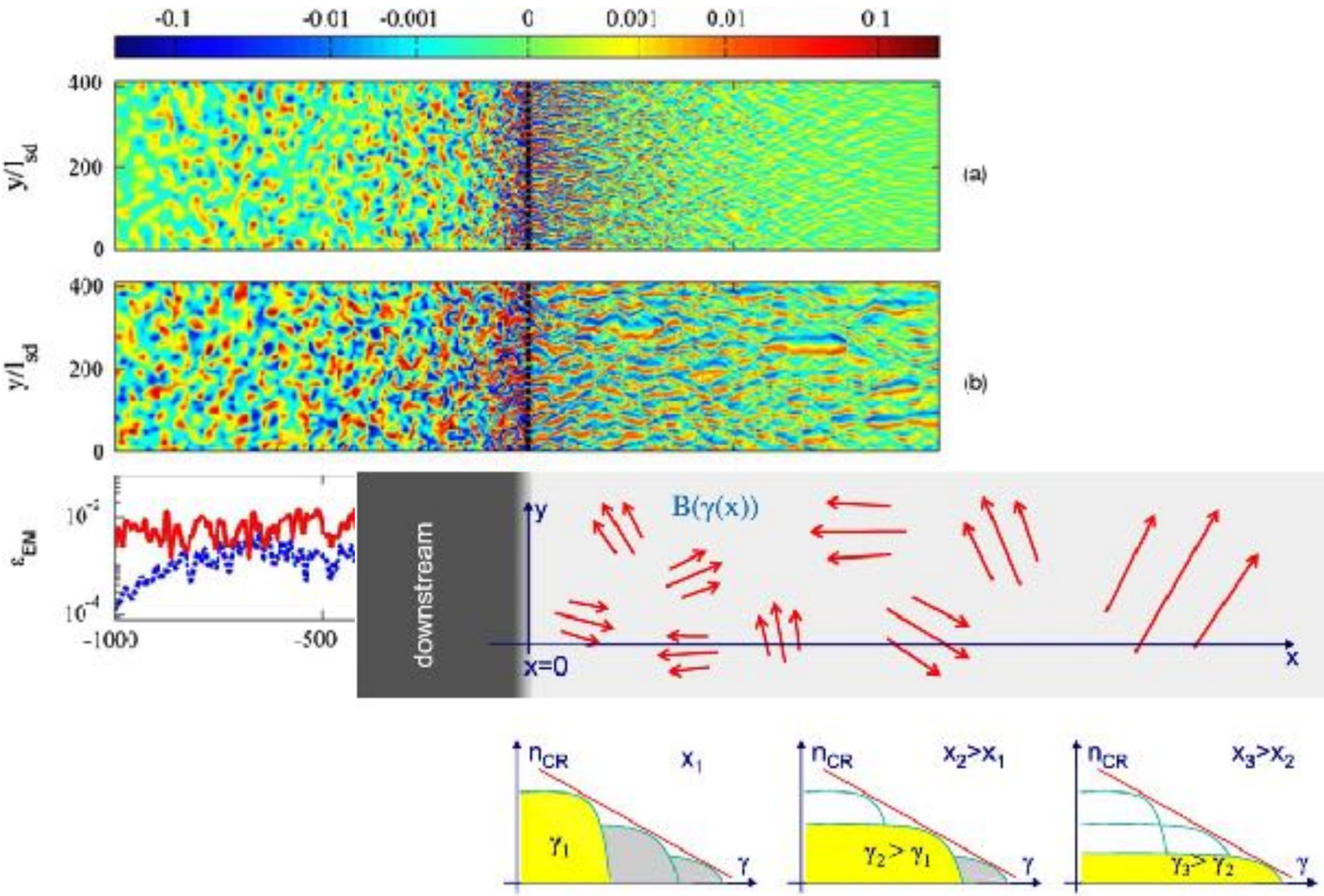
Collisionless shocks with WI



highly viscosity motions => effective collisionality

(Sim. stollen from Anatoly)

Weibel turbulence @ shock & foreshock

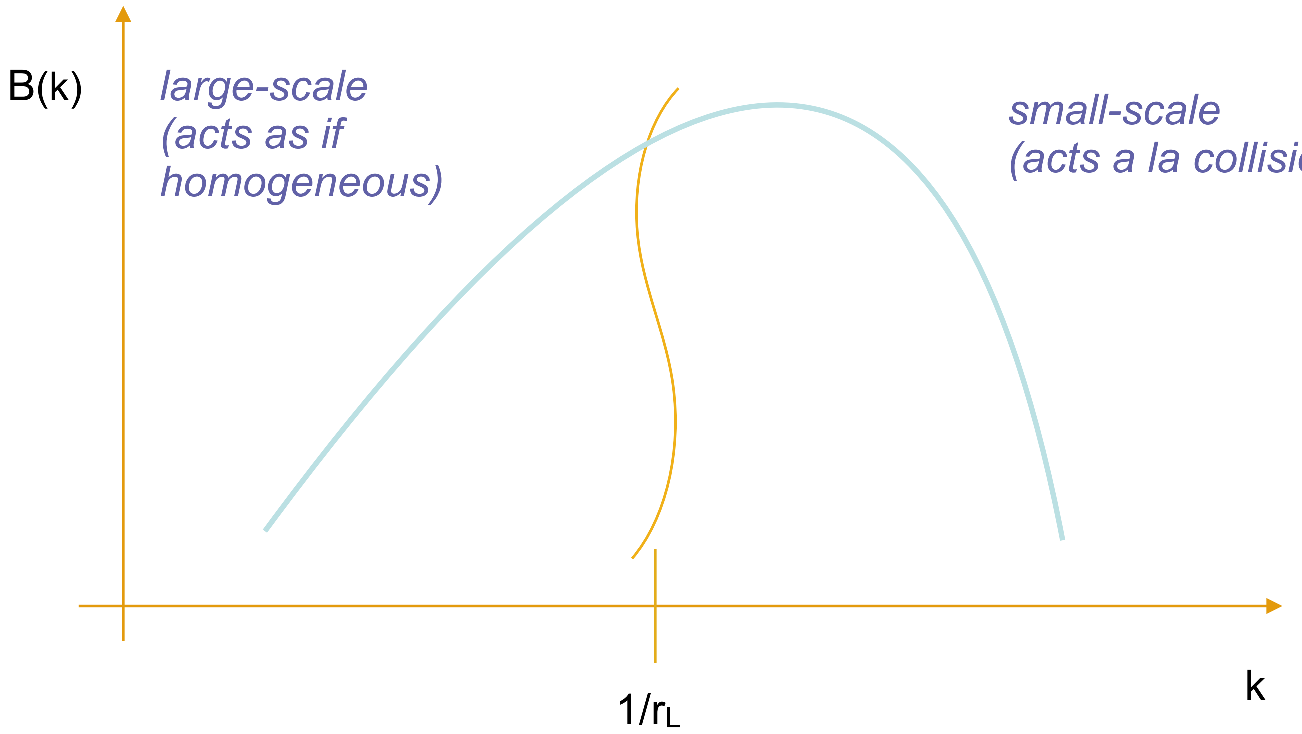


(Keshet et al. '09)

(MM et al. '09)

2 key regimes

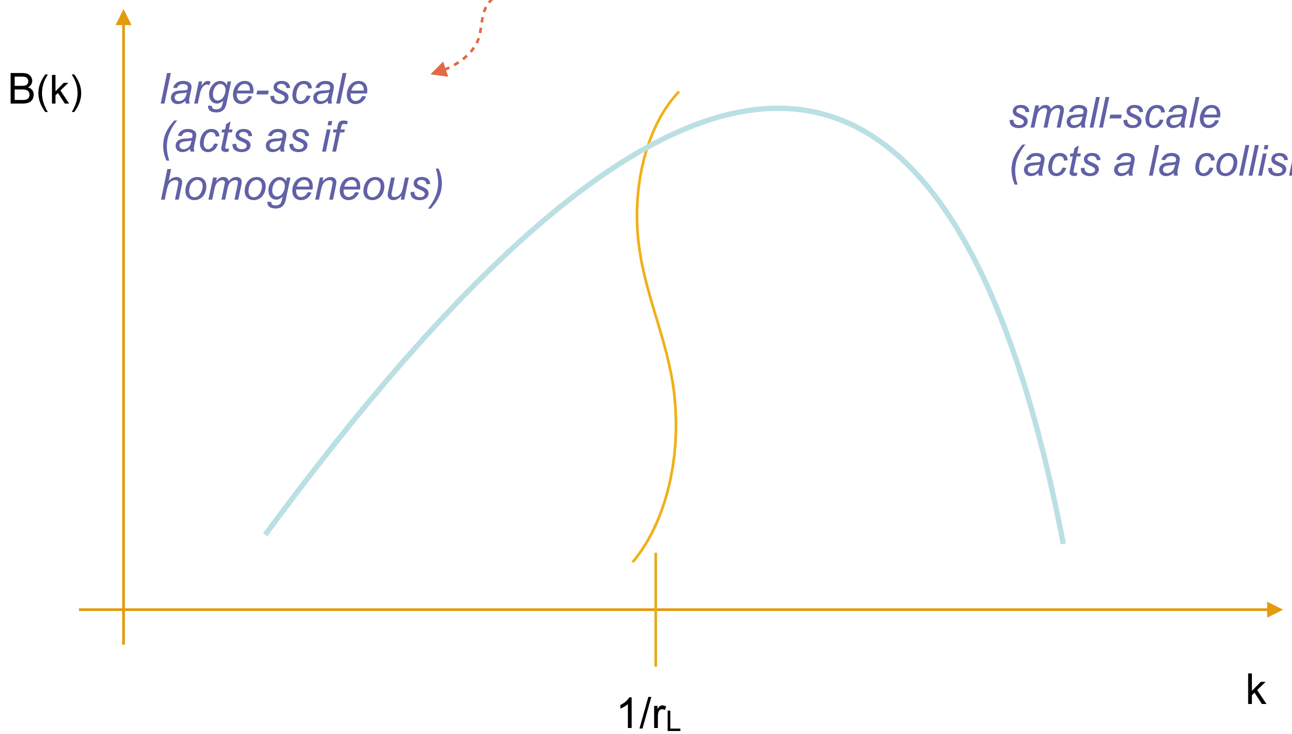
Consider a group of suprathermal particles with some characteristic energy and Larmor scale, propagating in the upstream



2 key regimes

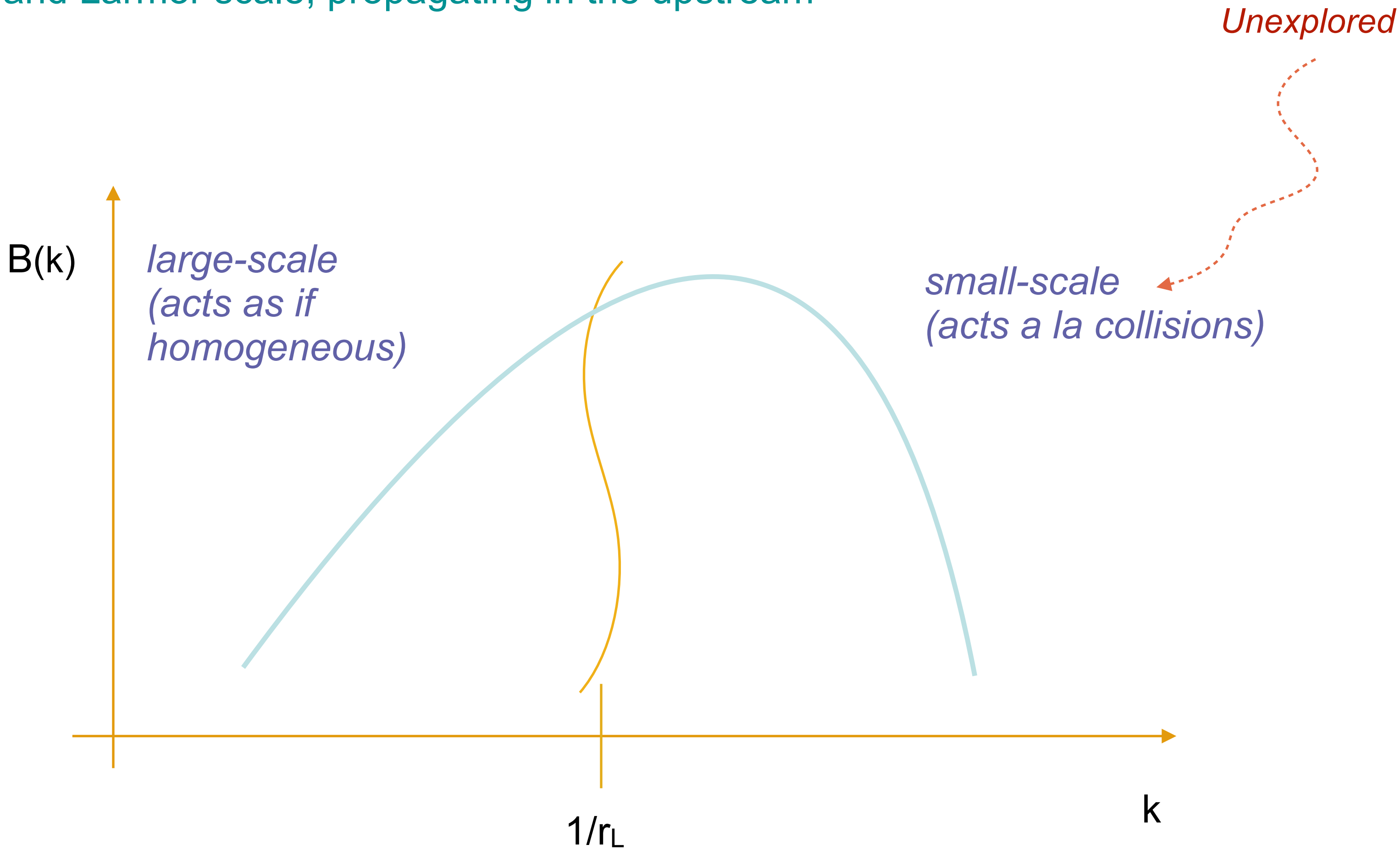
Consider a group of suprathermal particles with some characteristic energy and Larmor scale, propagating in the upstream

B-field does not suppress WI on scales below a Larmor scale (Bret, et al. '16, ...)

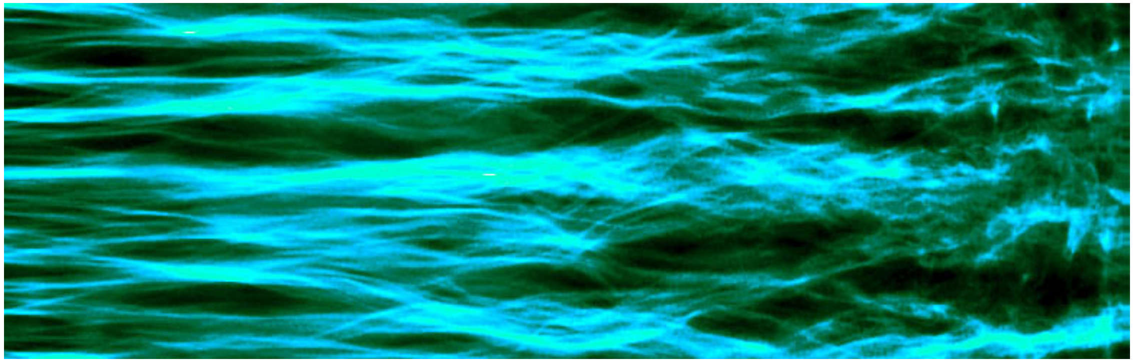


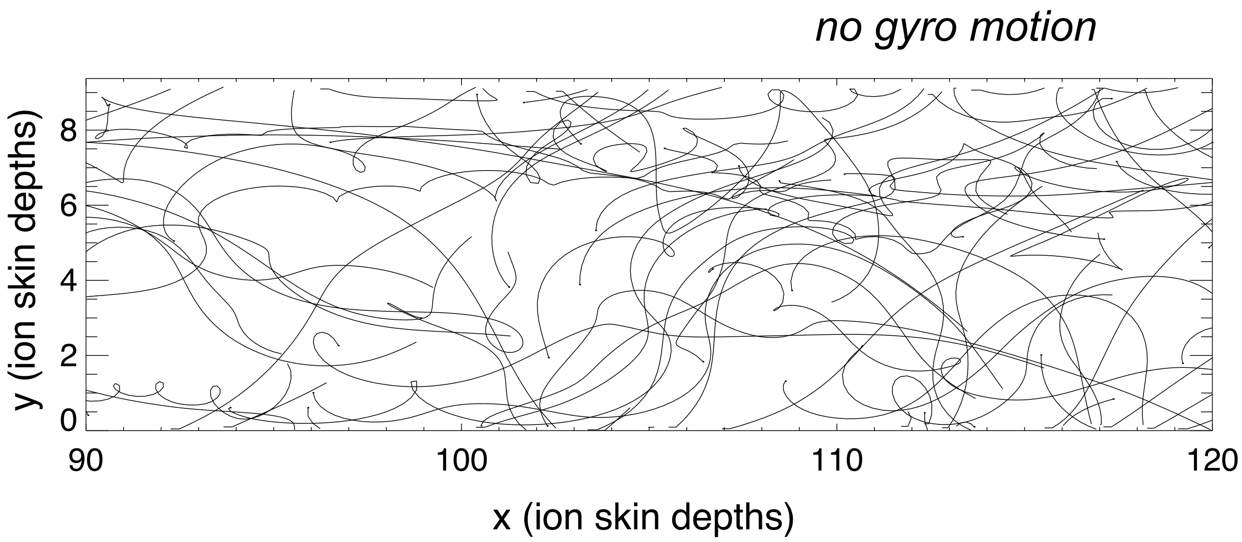
2 key regimes

Consider a group of suprathermal particles with some characteristic energy and Larmor scale, propagating in the upstream



Transport in Weibel turbulence

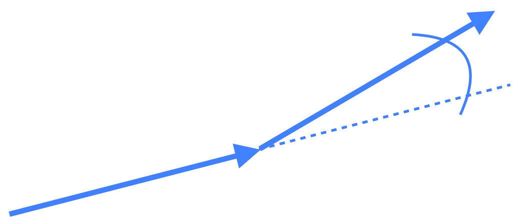




Weibel simulations by Haugboelle, Frederiksen, Nordlund (circa 2004)

Effective collisionality

Pitch angle diffusion



$$\Delta p_\perp \sim F_L \tau_\lambda \simeq (e/c)\,|\mathbf{v}\times\mathbf{B}|\,(\lambda_B/v_\perp)$$

$$\alpha_\lambda \sim \frac{\Delta p_\perp}{p_\perp} \simeq \frac{eB\lambda_B}{\Gamma m c v_\perp}$$

$$\tau_\lambda \simeq \lambda_B/v_\perp$$

$$\boxed{D_{\alpha\alpha} \simeq \frac{\alpha_\lambda^2}{\tau_\lambda} \sim \frac{e^2\langle B^2\rangle\lambda_B}{\Gamma^2 m^2 c^2 \langle v_\perp^2\rangle^{1/2}}}$$

An electron is deflected by one radian, i.e. $\Delta p_\perp/p \sim 1$. Thus:

$$D_{\alpha\alpha}\nu_{\rm eff}^{-1} = \langle\alpha^2\rangle \sim 1$$

$$\boxed{\nu_{\rm eff} \simeq D_{\alpha\alpha} \sim \frac{e^2\langle B^2\rangle\lambda_B}{m^2c^2\Gamma^2 v_{th}}}$$

(Keenan, MM '15, '16; MM '17)

... more accurately:

Field autocorrelation tensor and effective correlation length tensor

$$R^{ij}(\mathbf{r},t) \equiv \langle B^i(\mathbf{x},\tau)B^j(\mathbf{x}+\mathbf{r},\tau+t)\rangle_{\mathbf{x},\tau}$$

$$\lambda_B^{ij}(\hat{\mathbf{r}},t) \equiv \int_0^\infty \frac{R^{ij}(\mathbf{r},t)}{R^{ij}(0,0)}\,\mathrm{d}r$$

Sometimes (homogeneity, isotropy, stationarity)
it can be simplified:

$$\lambda_B \equiv \lambda_B^{zz}(\hat{\mathbf{x}},t) = \int_0^\infty \frac{R^{zz}(x\hat{\mathbf{x}},t)}{R^{zz}(0,0)}\,\mathrm{d}x = \frac{3\pi}{8}\frac{\int_0^\infty k|\mathbf{B}_k|^2\,\mathrm{d}k}{\int_0^\infty k^2|\mathbf{B}_k|^2\,\mathrm{d}k}$$

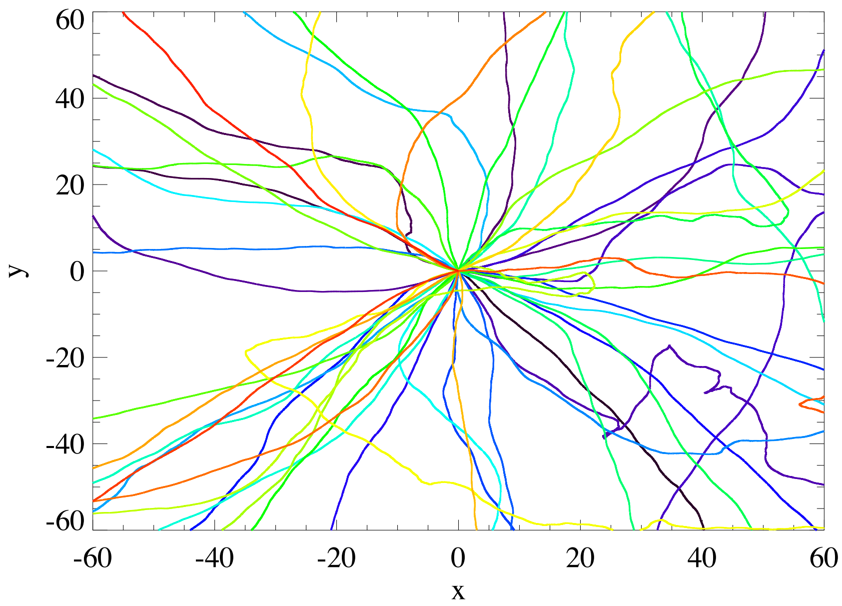
Effective turbulent collisional frequency

$$\boxed{\nu_{\text{eff}} = D_{\alpha\alpha} = \frac{3\pi}{8}\sqrt{\frac{3}{2}}\frac{e^2}{m^2c^2}\left(\frac{\int_0^\infty k|\mathbf{B}_k|^2\,\mathrm{d}k}{\int_0^\infty k^2|\mathbf{B}_k|^2\,\mathrm{d}k}\right)\frac{\langle B^2\rangle}{\Gamma^2 v_{th}}}$$

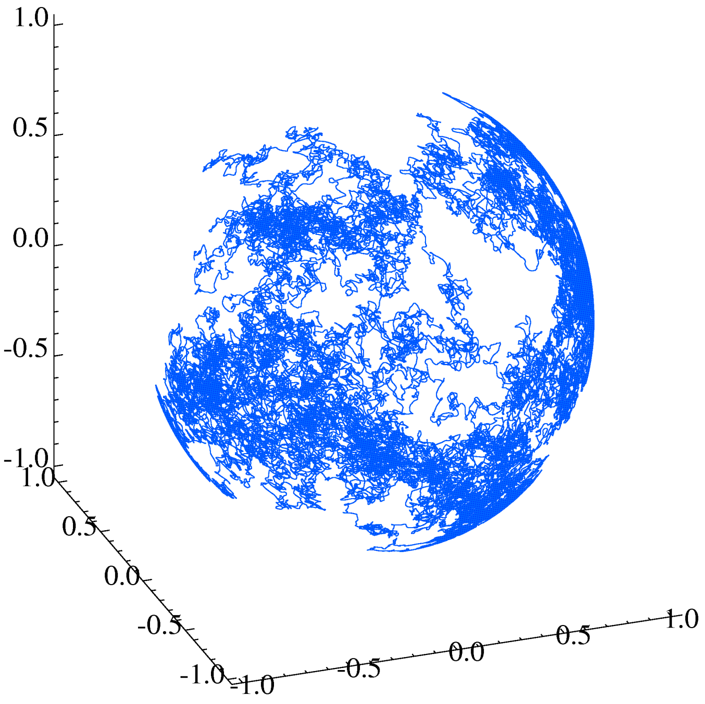
(MM'17; Keenan & MM, PRE'13, PoP'15, PRE'15a, PRE'15b, JPP'16a, JPP'16b)

Numerical modeling

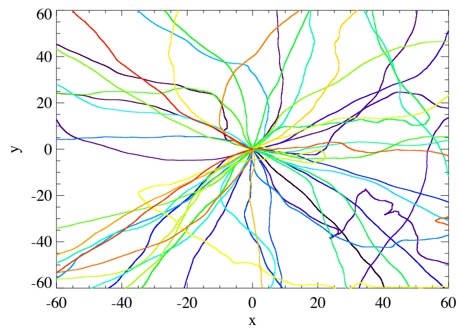
x-space

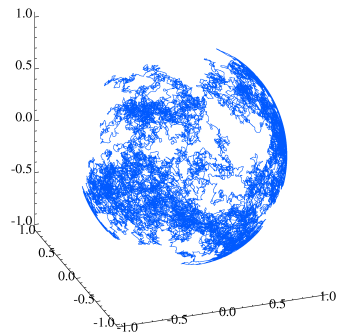


v-space



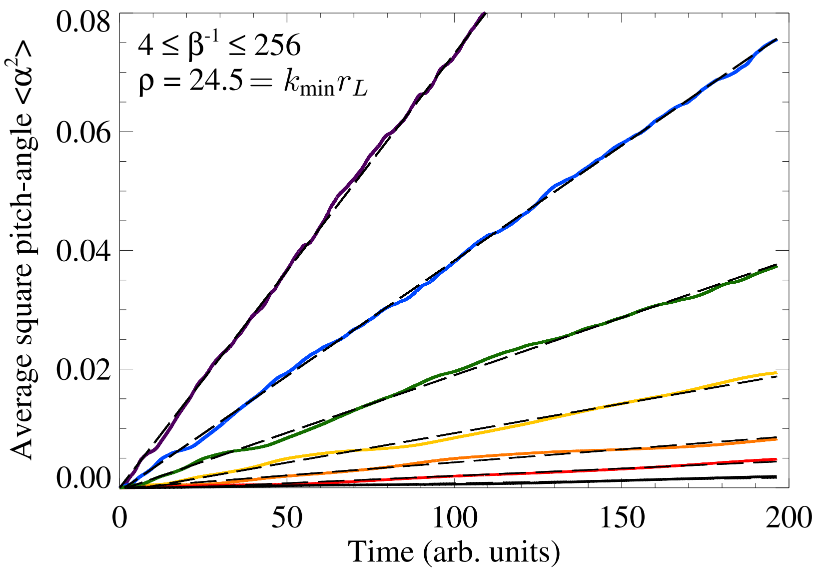
Numerical modeling

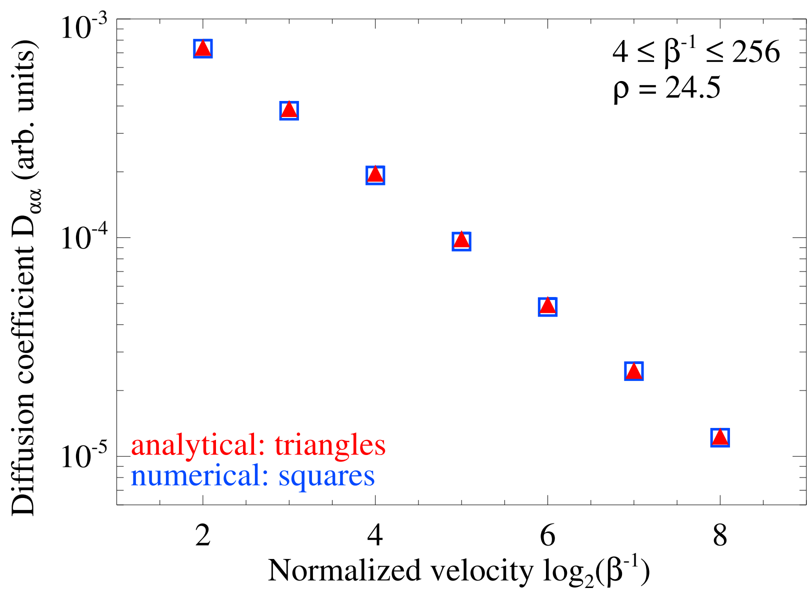




$$D_{\alpha\alpha} = \frac{3\pi}{8}\sqrt{\frac{3}{2}}\frac{e^2}{m^2c^2}\left(\frac{\int_0^\infty k|\mathbf{B}_k|^2\,\mathrm{d}k}{\int_0^\infty k^2|\mathbf{B}_k|^2\,\mathrm{d}k}\right)\frac{\langle B^2\rangle}{\Gamma^2 v_{th}}$$

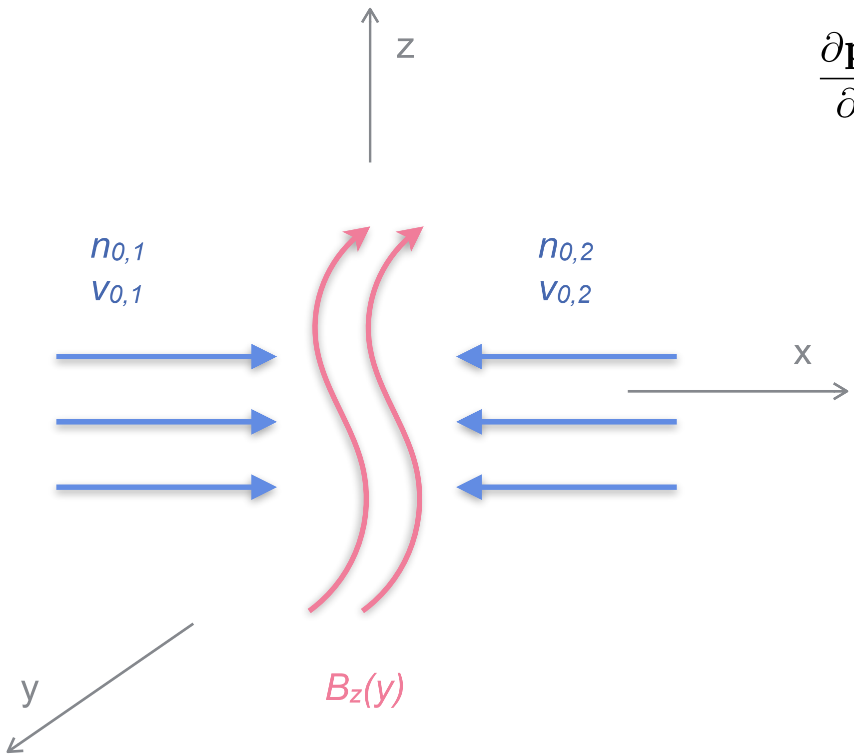
Diffusion





(Keenan & MM, PRE'13, PoP'15, PRE'15a, PRE'15b)

"Quasi"-collisional Weibel (e^-)



$$e^{(ik_y y - i\omega t)}$$

$$\frac{\partial n_a}{\partial t} - \nabla \cdot \mathbf{j}_a = 0,$$

$$\frac{\partial \mathbf{p}_a}{\partial t} + \mathbf{v}_a \cdot \nabla \mathbf{p}_a = -(\mathbf{E} + \mathbf{v}_a \times \mathbf{B}) - \nu_{\text{eff}}(\mathbf{p}_a - \mathbf{p}_{\bar{a}})$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \sum_a \mathbf{j}_a,$$

$$\nabla \cdot \mathbf{E} = 1 - \sum_a n_a,$$

$$\mathbf{j}_a = -n_a \mathbf{v}_a$$

$$\mathbf{v}_a = \mathbf{p}_a / \sqrt{1 + p_a^2}.$$

$$\mathbf{v}_{0,a} = v_{0,a} \hat{\mathbf{x}}$$

$$\sum_a n_{0,a} v_{0,a} = 0$$

(Honda '04 - a la Califano et al '97; MM '17)

Dispersion relation

$$\boxed{\omega^2(1-A_1)(1-A_2)-k^2(1-A_1)(1+A_3)+A_4=0}$$

$$A_1=\sum_a \frac{n_{0,a}}{\Gamma_{0,a}\omega^2},$$

$$A_2=\sum_a \frac{n_{0,a}}{\Gamma_{0,a}^3\omega^2},$$

$$A_3=\sum_a \frac{n_{0,a}v_{0,a}^2}{\Gamma_{0,a}\omega'^2},$$

$$A_4=\left(\sum_a \frac{n_{0,a}v_{0,a}}{\Gamma_{0,a}\omega^2}\right)\left(\sum_a \frac{n_{0,a}v_{0,a}}{\Gamma_{0,a}\omega'^2}\right)$$

$$\omega'^2=\omega^2\,\frac{\omega+2i\nu_{\mathrm{eff}}}{\omega+i(1+v_{0,\bar{a}}/v_{0,a})\nu_{\mathrm{eff}}}$$

$$\Gamma_{0,a}=\left(1-v_{0,a}^2\right)^{-1/2}$$

Dispersion relation: equal streams

$$n_{0,1} = n_{0,2} = 0.5$$

$$v_{0,1} = -v_{0,2}$$

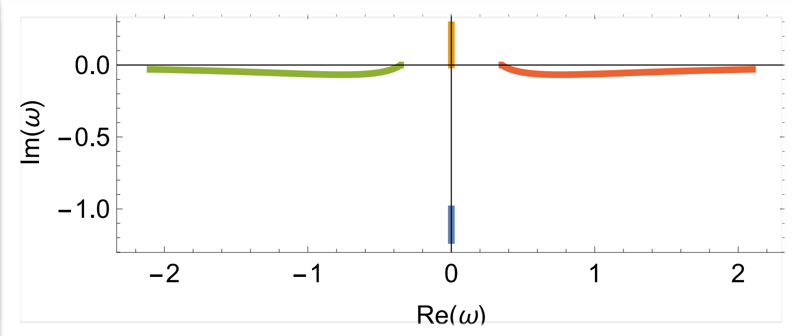
$$\Gamma_0 = (1 - v_0^2)^{-1/2}$$

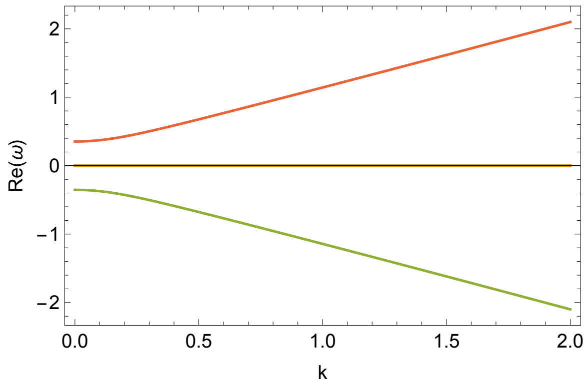
$$\left[\omega'^2\left(\omega^2 - \Gamma_0^{-3}\right) - k^2\left(\omega'^2 + v_0^2\Gamma_0^{-1}\right)\right] = 0$$

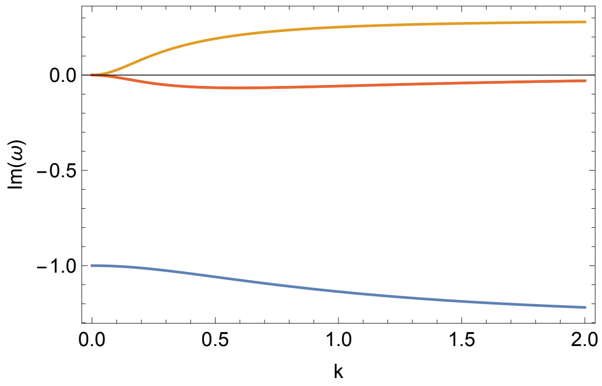
$$\omega'^2 = \omega(\omega + 2i\nu_{\text{eff}})$$

Langmuir factors out

$$\omega = \pm\Gamma_0^{-1/2}$$







(MM '17)

Quasi-collisional Weibel dispersion

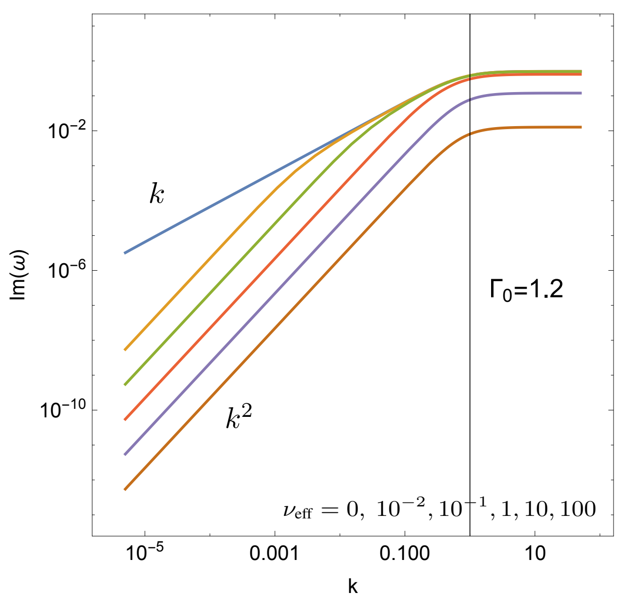
$$\gamma(\gamma+2\nu_{\rm eff})\left(\gamma^2+\Gamma_0^{-3}\right)+k^2\left(\gamma(\gamma+2\nu_{\rm eff})-v_0^2\Gamma_0^{-1}\right)=0$$

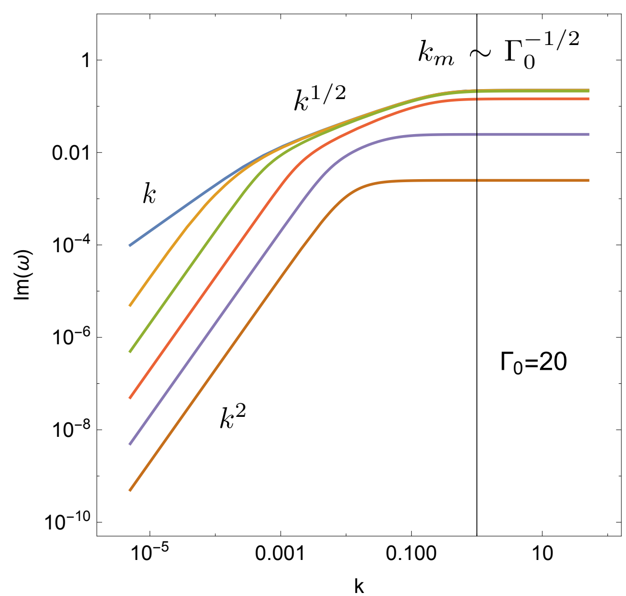
Maximum growth rate

$$\gamma_m=\sqrt{\nu_{\rm eff}^2+v_0^2/\Gamma_0}-\nu_{\rm eff}$$

$$\approx\begin{cases}\gamma_0-\nu_{\rm eff}, & \text{if } \nu_{\rm eff}\ll\gamma_0,\\ \gamma_0^2/(2\nu_{\rm eff}), & \text{if } \nu_{\rm eff}\gg\gamma_0,\end{cases}$$

$$\gamma_0=v_0/\sqrt{\Gamma_0}$$





(MM '17)

Implications: nonlinear Weibel

Restoring units....

Field scale $\lambda_B \sim (c/\omega_p)\Gamma_0^{1/2}$

Quasi-collisional frequency $\nu_{\text{eff}} \sim \dfrac{\omega_p}{\Gamma_0^{1/2}\beta}\dfrac{v_0}{c}$

where we introduced the "*generalized plasma beta*"

$$\beta \equiv \frac{\Gamma_0 n_0 (m v_0^2/2)}{\langle B^2\rangle/8\pi}$$

Growth rate vs self-generated field strength

$$\boxed{\gamma_m(\beta) \sim \frac{\omega_p}{\Gamma_0^{1/2}}\frac{v_0}{c} \times \begin{cases} 1-\beta^{-1}, & \text{if } \beta \gg 1, \\ \beta, & \text{if } \beta \ll 1. \end{cases}}$$

Conclusions

Self-generated Weibel fields act as scatterers --> *quasi-collisions*

Quasi-collisions -- a deeply nonlinear effect affecting further development of the Weibel instability

Dispersion relation with self-induced quasi-collisions has been derived --> obviously, a *quasi-nonlinear* treatment

Weibel instability cannot shut-off itself and the max growth rate is

$$\gamma_m(\beta) \sim \frac{\omega_p}{\Gamma_0^{1/2}} \frac{v_0}{c} \times \begin{cases} 1 - \beta^{-1}, & \text{if } \beta \gg 1, \\ \beta, & \text{if } \beta \ll 1. \end{cases}$$