A hybrid gyrokinetic ion and isothermal electron fluid code and its application to turbulent heating in astrophysical plasma

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1. Introduction

2. Development of a GKI/ITEF hybrid code

3. Numerical tests

4. Nonlinear simulation of ion/electron heating partitioning

5. Summary

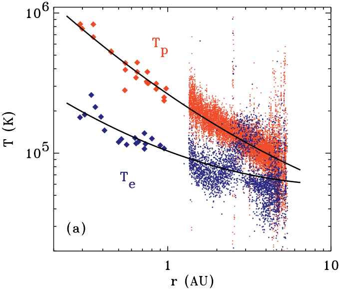
What is the ion–electron temperature ratio in astrophysical systems?

- One of the most important questions in both inner and extra solar systems
- Most of the astrophysical plasmas in a weakly collisional state

  ⇒ Coulomb collisional energy equipartition does not work

  ⇒ In general, $T_\mathrm{i} \neq T_\mathrm{e}$

- Solar wind
- Measurable



[Cranmer et al., ApJ 2009]

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- One of the most important questions in both inner and extra solar systems
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  $\Rightarrow$ In general, $T_\mathrm{i} \neq T_\mathrm{e}$

- Radiatively inefficient accretion flow (RIAF) model
- Very low gas density $\rightarrow$ collisionless
- Prediction of two temperatures with $T_\mathrm{p} \gg T_\mathrm{e}$ [Narayan & Yi 1995]
- Electrons radiate (measurable) but ions are swallowed into the black hole

Two destinations of gravitational potential energy

Mechanisms that heat collisionless plasma

- Dissipation of turbulence [Quataert, ApJ 1998; Quataert & Gruzinov, ApJ 1999; Howes MNRAS, 2010]
- Magnetic reconnection [Quataert & Gruzinov, ApJ 1999]
- Pressure anisotropy driven turbulence [Sironi & Narayan, ApJ 2015; Sironi, ApJ 2015]
- Collisionless shock [Bell, MNRAS 1978; Blandford, ApJ 1978]

Mechanisms of collisionless plasma heating

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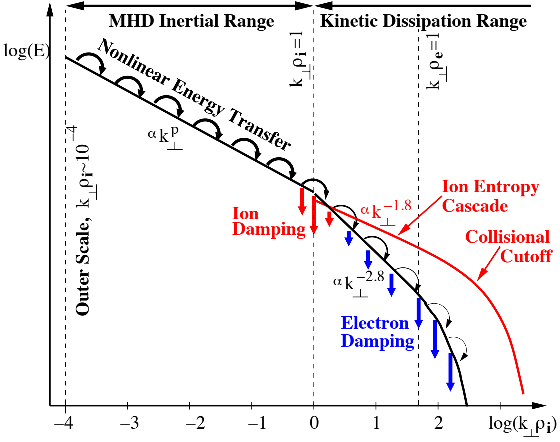
In this study, we focus on dissipation of Alfvénic turbulence

- Especially, we are interested in the dependence of $Q_\mathrm{i}/Q_\mathrm{e}$ on $T_\mathrm{i}/T_\mathrm{e}$
  - If $T_\mathrm{i}/T_\mathrm{e} \nearrow \ \Rightarrow \ Q_\mathrm{i}/Q_\mathrm{e} \nearrow$, there is "positive feedback" to enhance the temperature imbalance
  - If $T_\mathrm{i}/T_\mathrm{e} \nearrow \ \Rightarrow \ Q_\mathrm{i}/Q_\mathrm{e} \searrow$, the system prefers to have a finite temperature ratio

Energy cascade in gyrokinetic turbulence

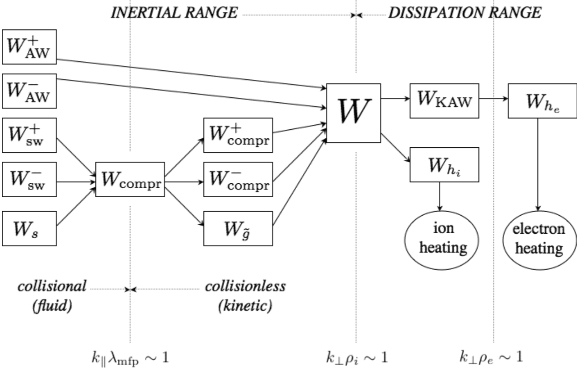
- Energy injected on a larger scale is cascaded to the ion kinetic scale
- Some portion of the energy is damped (ion entropy fluctuation) and the rest (KAW) is cascaded to a smaller scale



[Howes, 2015]

Energy cascade in gyrokinetic turbulence

- Once they are split, they are independently cascaded in the phase space [Schekochihin et al., 2009]
  - Ion entropy fluctuation → ion heating
  - KAW → electron heating
- Therefore, the heating partitioning is decided at $k_{\perp}\rho_{\mathrm{i}} \sim 1$ (damping barrier)



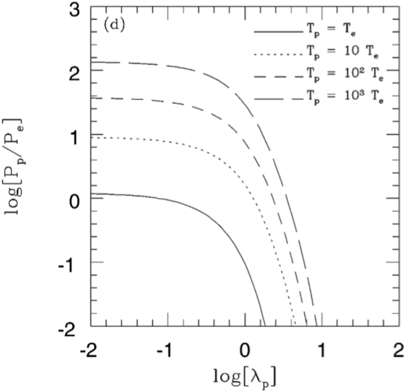
[Schekochihin et al., 2009]

Theoretical estimates of heating ratio

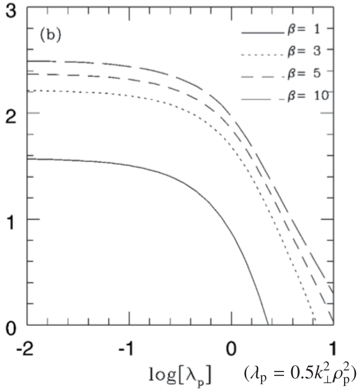
- The rate of energy absorption by Alfvén wave damping [Quataert, ApJ 1998]

$$P_s = \frac{\mathbf{E}^* \cdot \chi_s^a \cdot \mathbf{E}}{4W}$$

$\beta = 1$



$T_\mathrm{p}/T_\mathrm{e} = 100$

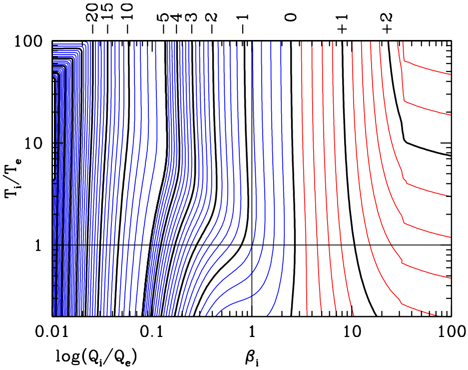


$(\lambda_\mathrm{p} = 0.5 k_\perp^2 \rho_\mathrm{p}^2)$

Theoretical estimates of heating ratio

- An estimate using the gyrokinetic cascade model [Howes, MNRAS 2010]
- All damping is assumed to be linear

$$\frac{\partial b_k^2}{\partial t} = -k_\perp \frac{\partial \epsilon(k_\perp)}{\partial k_\perp} + S(k_\perp) - 2\gamma b_k^2, \quad Q_s(k_\perp) = 2C_1^{3/2} C_2(\overline{\gamma}_s/\overline{\omega})\epsilon(k_\perp)/k_\perp$$



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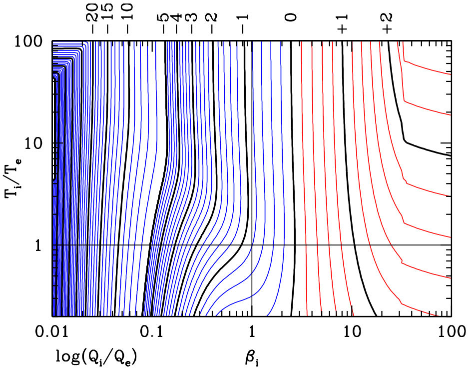
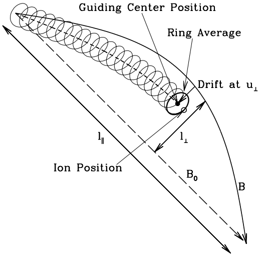


We want to fill this diagram via nonlinear simulation!

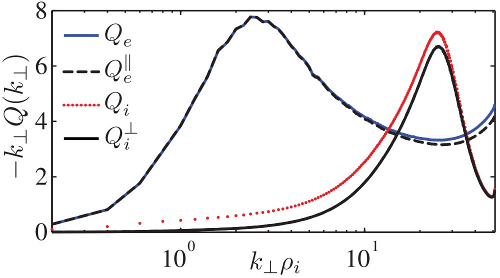
Gyrokinetics

- A reduction of Vlasov–Maxwell system
- In many astrophysical systems, gyrokinetics is an appropriate model
- Scale hierarchy created by the magnetic field: gyrokinetic ordering
  1. fluctuation is much slower than cyclotron motion $\frac{\omega}{\Omega} \ll 1$
  2. fluctuation is anisotropic $\frac{k_\parallel}{k_\perp} \ll 1$
- 5D phase space
- Fast wave and cyclotron resonance are ordered out
- FLR and Landau damping are kept
- Gyrokinetics was originally formulated for fusion studies but has been used in astrophysics in the last decade



[Howes et al., ApJ 2006]

- β_i, $T_\mathrm{i}/T_\mathrm{e} = 1$ case
- Collisional heating $Q_s = \dfrac{2\pi B_0}{m_s} \displaystyle\int \mathrm{d}v_{\|}\mathrm{d}\mu \frac{T_s}{F_{0s}} h_s\, C[f_s]$



[Navarro et al., PRL 2016]

- 70% goes to the electron heating
- The electron heating is caused by parallel Landau damping in the ion scale
- The ion heating is caused by perpendicular phase mixing in the electron scale

Goal

- Direct numerical simulation focusing on the partitioning of heating between ions and electrons
- Scanning β_i and $T_\mathrm{i}/T_\mathrm{e}$ to investigate the dependence

However...

- Parameter scan with gyrokinetics resolving all scales is difficult
- For heating problems, the velocity space resolution must be sufficiently high

On the other hand...

- We do not have to resolve the electron scale because the heating ratio is determined by how much energy bifurcation at $k_\perp \rho_\mathrm{i} \sim 1$
  i.e., how much goes to ion energy fluctuation (to be ion heating); the rest goes to KAW (to be electron heating)
- We utilize the gyrokinetic ions & fluid electron hybrid model [Schekochihin et al., 2009]

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Kinetic ions & fluid electron hybrid model

- Rather long history for FULLY kinetic ions & fluid electron hybrid models [Sgro PoF (1976)]
- Eliminate electron dynamics while keeping all kinetic effects of ions
  $\rightarrow$ improvement of computation time
- PIC type and Eulerian type simulation codes
- Used for both fusion [Sgro PoF (1976)] and astrophysical studies [Kunz JCP (2014); PRL (2016)]
- The hybrid model of gyrokinetic ions & fluid electron [Schekochihin et al., 2009] further improves the computation time (but ignore ion fast kinetic effects)

- In δf gyrokinetics, the distribution function f_s is split into the mean and fluctuating parts: $f_s = F_s + \delta f_s = \left(1 - \frac{q_s \phi(\mathbf{r})}{T_s}\right) F_s(v) + h_s(t, \mathbf{R}_s, v_\parallel, v_\perp)$
- Gyrocenter position $\mathbf{R}_s = \mathbf{r} + \mathbf{v}_\perp \times \hat{\mathbf{z}} / \Omega_s$
- Gyrokinetic equation

$$\frac{\partial h_s}{\partial t} + v_\parallel \frac{\partial h_s}{\partial z} + \frac{c}{B_0} \{\langle \chi \rangle_{\mathbf{R}_s}, h_s\} = \frac{q_s}{T_s} \frac{\partial \langle \chi \rangle_{\mathbf{R}_s}}{\partial t} F_s + \langle C[h_s] \rangle_{\mathbf{R}_s} ,$$

where $\chi = \phi - \mathbf{v} \cdot \mathbf{A}/c$.

- Maxwell's equation

$$\sum_s \frac{q_s^2 n_s}{T_s} \phi = \sum_s q_s \int \mathrm{d}^3\mathbf{v}\ \langle h_s \rangle_{\mathbf{r}} ,$$

$$-\frac{c}{4\pi} \nabla_\perp^2 A_\parallel = \sum_s q_s \int \mathrm{d}^3\mathbf{v}\, v_\parallel \langle h_s \rangle_{\mathbf{r}} ,$$

$$\frac{c}{4\pi} \nabla_\perp \delta B_\parallel = \sum_s T_s \int \mathrm{d}^3\mathbf{v}\ \langle (\hat{\mathbf{z}} \times \mathbf{v}_\perp) h_s \rangle_{\mathbf{r}} ,$$

Isothermal electron fluid (ITEF) [Schekochihin et al., 2009]

- Additional expansion by $\sqrt{m_\mathrm{e}/m_\mathrm{i}} \sim 0.02$ [Snyder & Hammett PoP (2001)]
- For the ion kinetic scale ($k_\perp \rho_\mathrm{i} \sim 1$), $k_\perp \rho_\mathrm{e} \sim k_\perp \rho_\mathrm{i} \sqrt{m_\mathrm{e}/m_\mathrm{i}} \ll 1$
- Ignores all the electron kinetic effects. But improves computational costs ($\sim 2\sqrt{m_\mathrm{i}/m_\mathrm{e}} \sim 100$ times faster)
- From the zeroth order,
  - $h_\mathrm{e}^{(0)}$ is perturbed Maxwellian

$$h_\mathrm{e}^{(0)} = \left[\frac{\delta n_\mathrm{e}}{n_\mathrm{e}} - \frac{e\phi}{T_e} + \left(\frac{v^2}{v_\mathrm{the}^2} - \frac{3}{2}\right)\frac{\delta T_\mathrm{e}}{T_e}\right] F_e$$

  - δT_e is constant along the field line

$$\hat{\mathbf{b}} \cdot \nabla \frac{\delta T_e}{T_e} = 0$$

- Additional assumption of isothermal electron $\delta T_\mathrm{e} = 0$ gives ITEF equations

Isothermal electron fluid (ITEF) [Schekochihin et al., 2009]

- ITEF equations

$$\frac{\partial}{\partial t}\left(\frac{\delta n_{\mathrm{e}\mathbf{k}_\perp}}{n_\mathrm{e}} - \frac{\delta B_{\|\mathbf{k}_\perp}}{B_0}\right) + \frac{c}{B_0}\left\{\phi,\ \frac{\delta n_\mathrm{e}}{n_\mathrm{e}} - \frac{\delta B_\|}{B_0}\right\}_{\mathbf{k}_\perp} + \frac{\partial u_{\|\mathrm{e}\mathbf{k}_\perp}}{\partial z} - \frac{1}{B_0}\{A_\|,\ u_{\|\mathrm{e}}\}_{\mathbf{k}_\perp} + \frac{cT_\mathrm{e}}{eB_0}\left\{\frac{\delta n_\mathrm{e}}{n_\mathrm{e}},\ \frac{\delta B_\|}{B_0}\right\}_{\mathbf{k}_\perp} = 0$$

$$\frac{1}{c}\frac{\partial A_{\|\mathbf{k}_\perp}}{\partial t} + \frac{\partial \phi_{\mathbf{k}_\perp}}{\partial z} - \frac{1}{B_0}\{A_\|,\ \phi\}_{\mathbf{k}_\perp} = \frac{T_\mathrm{e}}{e}\left[\frac{\partial}{\partial z}\left(\frac{\delta n_{\mathrm{e}\mathbf{k}_\perp}}{n_\mathrm{e}}\right) - \frac{1}{B_0}\left\{A_\|,\ \frac{\delta n_\mathrm{e}}{n_\mathrm{e}}\right\}_{\mathbf{k}_\perp}\right]$$

- Maxwell's equations

$$\frac{\delta n_{\mathrm{e}\mathbf{k}_\perp}}{n_\mathrm{e}} = -\frac{Ze\phi_{\mathbf{k}_\perp}}{T_\mathrm{i}} + \frac{1}{n_\mathrm{i}}\int \mathrm{d}^3\mathbf{v}\, J_0(a_\mathrm{i}) h_{\mathrm{i}\mathbf{k}_\perp},$$

$$u_{\|\mathrm{e}\mathbf{k}_\perp} = -\frac{ck_\perp^2}{4\pi e n_\mathrm{e}} A_{\|\mathbf{k}_\perp} + \frac{1}{n_\mathrm{i}}\int \mathrm{d}^3\mathbf{v}\, v_\| J_0(a_\mathrm{i}) h_{\mathrm{i}\mathbf{k}_\perp},$$

$$\frac{\delta B_{\|\mathbf{k}_\perp}}{B_0} = \frac{\beta_\mathrm{i}}{2}\left\{\left(1 + \frac{Z}{\tau}\right)\frac{Ze\phi_{\mathbf{k}_\perp}}{T_\mathrm{i}} - \frac{1}{n_\mathrm{i}}\int \mathrm{d}^3\mathbf{v}\left[\frac{Z}{\tau}J_0(a_\mathrm{i}) + \frac{2v_\perp^2}{v_{\mathrm{thi}}^2}\frac{J_1(a_\mathrm{i})}{a_\mathrm{i}}\right] h_{\mathrm{i}\mathbf{k}_\perp}\right\}$$

where $\tau = T_\mathrm{i}/T_\mathrm{e}$

- Plus ion gyrokinetic equation

Conservation laws for GKI/ITEF

- Generalized energy

$$W = E_{f_\mathrm{i}} + E_{n_\mathrm{e}} + E_B = \int \mathrm{d}^3\mathbf{r} \int \mathrm{d}^3\mathbf{v}\, \frac{T_\mathrm{i}\delta f_\mathrm{i}^2}{2F_\mathrm{i}} + \int \mathrm{d}^3\mathbf{r}\, \frac{n_\mathrm{e} T_\mathrm{e}}{2} \frac{\delta n_\mathrm{e}^2}{n_\mathrm{e}^2} + \int \mathrm{d}^3\mathbf{r}\, \frac{|\delta\mathbf{B}|^2}{8\pi}$$

$$\frac{\mathrm{d}W}{\mathrm{d}t} = P_\mathrm{ext} + \int \mathrm{d}^3\mathbf{R}_\mathrm{i} \int \mathrm{d}^3\mathbf{v}\, \frac{T_\mathrm{i}}{F_\mathrm{i}} \langle h_\mathrm{i} C[h_\mathrm{i}] \rangle_{\mathbf{R}_\mathrm{i}} \,,$$

- 2D invariant

$$I_\mathrm{e} = \int \mathrm{d}^3\mathbf{r}\, \frac{A_\parallel^2}{2}$$

Numerical implementation

- We extend `AstroGK` [Numata et al., JCP 2010] to solve ITEF
- `AstroGK`
  - An Eulerian δf gyrokinetics code specialized to a slab geometry
  - Has been used for solar wind turbulence [Howes et al., PRL 2008; 2011], reconnection [Numata et al., PoP 2011; JPP 2015], and etc. . .
  - Fourier spectral in (x, y) and 2nd order compact finite difference in z
  - Linear terms are solved implicitly
  - Nonlinear terms are solved explicitly (3rd Adams–Bashforth)
  - Linearlized collision operator with pitch angle scattering and energy diffusion satisfying conservation properties [Abel et al., PoP 2008; Barnes et al., PoP 2009]

Numerical implementation

- Maxwell's equations and ITEF equations are combined to a single matrix equation by eliminating $\delta n_e/n_e$ and $u_{\mathrm{e}\|}$

$$\frac{\partial}{\partial t}\left(\frac{\delta n_{\mathrm{e}\mathbf{k}_\perp}}{n_\mathrm{e}} - \frac{\delta B_{\|\mathbf{k}_\perp}}{B_0}\right) + \frac{c}{B_0}\left\{\phi,\ \frac{\delta n_\mathrm{e}}{n_\mathrm{e}} - \frac{\delta B_\|}{B_0}\right\}_{\mathbf{k}_\perp} + \frac{\partial u_{\|\mathrm{e}\mathbf{k}_\perp}}{\partial z} - \frac{1}{B_0}\{A_\|,\ u_{\|\mathrm{e}}\}_{\mathbf{k}_\perp} + \frac{cT_\mathrm{e}}{eB_0}\left\{\frac{\delta n_\mathrm{e}}{n_\mathrm{e}},\ \frac{\delta B_\|}{B_0}\right\}_{\mathbf{k}_\perp} = 0$$

$$\frac{1}{c}\frac{\partial A_{\|\mathbf{k}_\perp}}{\partial t} + \frac{\partial \phi_{\mathbf{k}_\perp}}{\partial z} - \frac{1}{B_0}\{A_\|,\ \phi\}_{\mathbf{k}_\perp} = \frac{T_\mathrm{e}}{e}\left[\frac{\partial}{\partial z}\left(\frac{\delta n_{\mathrm{e}\mathbf{k}_\perp}}{n_\mathrm{e}}\right) - \frac{1}{B_0}\left\{A_\|,\ \frac{\delta n_\mathrm{e}}{n_\mathrm{e}}\right\}_{\mathbf{k}_\perp}\right]$$

$$\frac{\delta n_{\mathrm{e}\mathbf{k}_\perp}}{n_\mathrm{e}} = [\Gamma_0(\alpha_\mathrm{i}) - 1]\frac{Ze\phi_{\mathbf{k}_\perp}}{T_\mathrm{i}} + \frac{1}{n_\mathrm{i}}\int \mathrm{d}^3\mathbf{v}\, J_0(a_\mathrm{i}) h_{\mathrm{i}\mathbf{k}_\perp},$$

$$u_{\|\mathrm{e}\mathbf{k}_\perp} = -\frac{ck_\perp^2}{4\pi e n_\mathrm{e}} A_{\|\mathbf{k}_\perp} + \frac{1}{n_\mathrm{i}}\int \mathrm{d}^3\mathbf{v}\, v_\| J_0(a_\mathrm{i}) h_{\mathrm{i}\mathbf{k}_\perp},$$

$$\frac{\delta B_{\|\mathbf{k}_\perp}}{B_0} = \frac{\beta_\mathrm{i}}{2}\left\{\left(1 + \frac{Z}{\tau}\right)\frac{Ze\phi_{\mathbf{k}_\perp}}{T_\mathrm{i}} - \frac{1}{n_\mathrm{i}}\int \mathrm{d}^3\mathbf{v}\left[\frac{Z}{\tau}J_0(a_\mathrm{i}) + \frac{2v_\perp^2}{v_{\mathrm{thi}}^2}\frac{J_1(a_\mathrm{i})}{a_\mathrm{i}}\right] h_{\mathrm{i}\mathbf{k}_\perp}\right\}$$

Numerical implementation

- Maxwell's equations and ITEF equations are combined to a single matrix equation by eliminating $\delta n_e / n_e$ and $u_{\mathrm{e}\|}$

$$\begin{pmatrix} \mathrm{P}_{11} & \mathrm{P}_{12} & \mathrm{P}_{13} \\ \mathrm{P}_{21} & \mathrm{P}_{22} & \mathrm{P}_{23} \\ \mathrm{P}_{31} & \mathrm{P}_{32} & \mathrm{P}_{33} \end{pmatrix} \begin{pmatrix} \phi_k^* \\ A_{\|k}^* \\ B_{\|k}^* \end{pmatrix} = \begin{pmatrix} \mathrm{Q}_1 \\ \mathrm{Q}_2 \\ \mathrm{Q}_3 \end{pmatrix},$$

  where $\phi^* = \phi^{n+1} - \phi^n$

- P and Q contain finite difference with respect to time and z

- As pointed out by [Schekochihin et al., 2009], the energy of the ion entropy fluctuation and that of KAW indepnedently cascade in $k_\perp \rho_\mathrm{i} \gg 1$ and $k_\perp \rho_\mathrm{e} \ll 1$
- The former is dissipated by ion collision
- The latter is damped by the electron Landau damping or cascaded to the electron kinetic scale
- In GKI/ITEF model, the electron Landau damping does not exist $\Rightarrow$ we need some artifical mechanism to terminate KAW cascade at the smallest scale
- This must not affect the larger scale

Numerical implementation ⟩ hyper dissipation

$$\frac{\partial}{\partial t}\left(\frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}}-\frac{\delta B_{\|}}{B_0}\right)+\frac{c}{B_0}\left\{\phi,\ \frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}}-\frac{\delta B_{\|}}{B_0}\right\}+\frac{\partial u_{\|\mathrm{e}}}{\partial z}-\frac{1}{B_0}\{A_{\|},\ u_{\|\mathrm{e}}\}+\frac{cT_{0\mathrm{e}}}{eB_0}\left\{\frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}},\ \frac{\delta B_{\|}}{B_0}\right\}$$
$$=\nu_h\nabla_{\perp}^{2n}\left(\frac{\delta n_{\mathrm{e}}}{n_{0e}}-\frac{\tau e}{T_{0\mathrm{i}}}\phi\right)$$

- The generalized energy W is split into two pieces

$$W=\int\frac{\mathrm{d}^3\mathbf{r}}{V}\Bigg[\underbrace{\int\mathrm{d}^3\mathbf{v}\,\frac{T_{0\mathrm{i}}}{2F_{0\mathrm{i}}}\langle h_{\mathrm{i}}\rangle_{\mathbf{r}}^2}_{W_{h_{\mathrm{i}}}}\underbrace{-\frac{Z^2e^2n_{0\mathrm{i}}}{2T_{0\mathrm{i}}}\phi^2-Zen_{0\mathrm{i}}\phi\frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}}+\frac{n_{0\mathrm{e}}T_{0\mathrm{e}}}{2}\left(\frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}}\right)^2+\frac{|\delta\mathbf{B}|^2}{4\pi}}_{\widetilde{W}}\Bigg]$$

$$\frac{\mathrm{d}W_{h_{\mathrm{i}}}}{\mathrm{d}t}=\int\frac{\mathrm{d}^3\mathbf{R}_{\mathrm{i}}}{V}\int\mathrm{d}^3\mathbf{v}\,Ze\frac{\partial\langle\chi\rangle_{\mathbf{R}_{\mathrm{i}}}}{\partial t}h_{\mathrm{i}}+\int\frac{\mathrm{d}^3\mathbf{R}_{\mathrm{i}}}{V}\int\mathrm{d}^3\mathbf{v}\,\frac{T_{0\mathrm{i}}}{F_{0\mathrm{i}}}\langle h_{\mathrm{i}}C[h_{\mathrm{i}}]\rangle_{\mathbf{R}_{\mathrm{i}}}$$
$$\frac{\mathrm{d}\widetilde{W}}{\mathrm{d}t}=-\int\frac{\mathrm{d}^3\mathbf{R}_{\mathrm{i}}}{V}\int\mathrm{d}^3\mathbf{v}\,Ze\frac{\partial\langle\chi\rangle_{\mathbf{R}_{\mathrm{i}}}}{\partial t}h_{\mathrm{i}}-\nu_h n_{0\mathrm{e}}T_{0\mathrm{e}}\int\mathrm{d}^3\mathbf{r}\left|\nabla_{\perp}^{n}\left(\frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}}-\tau\frac{e}{T_{0\mathrm{i}}}\phi\right)\right|^2$$

Improvement of the computational time

- For nonlinear runs, `AstroGK` evaluates the nonlinear terms explicitly

  $\Rightarrow$ CFL condition imposes a limitation on the maximum timestep

- In FGK, CFL condition is mainly determined by the electron advection speed $\frac{c}{B_0}\frac{\partial \langle\chi\rangle_{\mathbf{R}_e}}{\partial \mathbf{R}_\mathrm{e}}$

$$\langle\chi\rangle_{\mathbf{R}_e} \simeq \phi - \frac{v_\parallel A_\parallel}{c} - \frac{T_\mathrm{e}}{e}\frac{v_\perp^2}{v_\mathrm{the}}\frac{\delta B_\parallel}{B_0} \quad \text{(for } k_\perp\rho_\mathrm{e} \ll 1)$$

- We may evaluate [Schekochihin et al., 2009] by assuming the critical balance $k_\parallel v_\mathrm{A} \sim k_\perp u_\perp$ where $\mathbf{u}_\perp = -(c/B_0)\nabla\phi\times\hat{\mathbf{z}}$

$$\frac{v_\parallel A_\parallel}{c} \sim \sqrt{\frac{\beta_\mathrm{i}}{\tau}}\sqrt{\frac{m_\mathrm{i}}{m_\mathrm{e}}}\phi, \quad \frac{T_\mathrm{e}}{e}\frac{v_\perp^2}{v_\mathrm{the}^2}\frac{\delta B_\parallel}{B_0} \sim \frac{Z}{\tau}k_\perp\rho_\mathrm{i}\sqrt{\beta_\mathrm{i}}\phi.$$

$$\Rightarrow \quad \frac{c}{B_0}\frac{\partial \langle\chi\rangle_{\mathbf{R}_e}}{\partial \mathbf{R}_\mathrm{e}} \sim \max\left(1,\ \sqrt{\frac{\beta_\mathrm{i}}{\tau}}\sqrt{\frac{m_\mathrm{i}}{m_\mathrm{e}}},\ \frac{Z}{\tau}k_\perp\rho_\mathrm{i}\sqrt{\beta_\mathrm{i}}\right)u_\perp$$

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Improvement of the computational time

- The nonlinear terms in GKI/ITEF are

$$\frac{\partial}{\partial t}\left(\frac{\delta n_{\mathrm{e}\mathbf{k}_\perp}}{n_\mathrm{e}} - \frac{\delta B_{\parallel\mathbf{k}_\perp}}{B_0}\right) + \underbrace{\frac{c}{B_0}\left\{\phi,\ \frac{\delta n_\mathrm{e}}{n_\mathrm{e}} - \frac{\delta B_\parallel}{B_0}\right\}_{\mathbf{k}_\perp}}_{\sim k_\perp u_\perp \epsilon} + \frac{\partial u_{\parallel\mathrm{e}\mathbf{k}_\perp}}{\partial z}$$

$$- \underbrace{\frac{1}{B_0}\{A_\parallel,\ u_{\parallel\mathrm{e}}\}_{\mathbf{k}_\perp}}_{\sim \sqrt{\frac{\beta_\mathrm{i}}{\tau}} k_\perp u_\perp \epsilon} + \underbrace{\frac{cT_\mathrm{e}}{eB_0}\left\{\frac{\delta n_\mathrm{e}}{n_\mathrm{e}},\ \frac{\delta B_\parallel}{B_0}\right\}_{\mathbf{k}_\perp}}_{\sim \frac{Z}{\tau} k_\perp \rho_\mathrm{i} \sqrt{\beta_\mathrm{i}} k_\perp u_\perp \epsilon} = 0$$

$$\frac{1}{c}\frac{\partial A_{\parallel\mathbf{k}_\perp}}{\partial t} + \frac{\partial \phi_{\mathbf{k}_\perp}}{\partial z} - \underbrace{\frac{1}{B_0}\{A_\parallel,\ \phi\}_{\mathbf{k}_\perp}}_{\sim k_\perp u_\perp (A_\parallel/c)} = \frac{T_\mathrm{e}}{e}\frac{\partial}{\partial z}\left(\frac{\delta n_{\mathrm{e}\mathbf{k}_\perp}}{n_\mathrm{e}}\right) - \underbrace{\frac{T_\mathrm{e}}{eB_0}\left\{A_\parallel,\ \frac{\delta n_\mathrm{e}}{n_\mathrm{e}}\right\}_{\mathbf{k}_\perp}}_{\sim \frac{Z k_\perp \rho_\mathrm{i} \sqrt{\beta_\mathrm{i}}}{\tau} k_\perp u_\perp (A_\parallel/c)}$$

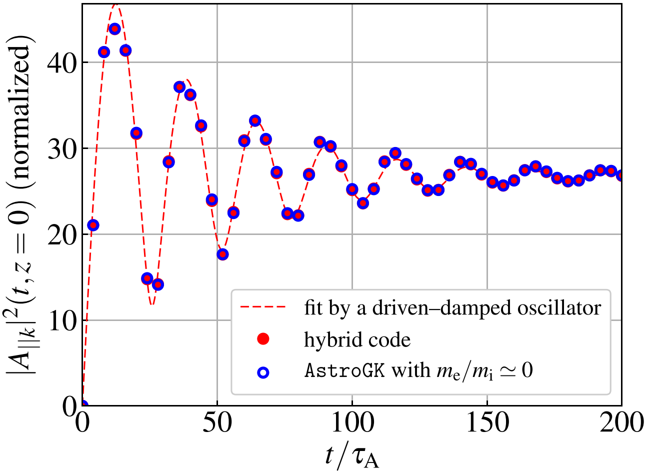
- The maximum timestep can be $\sqrt{m_\mathrm{i}/m_\mathrm{e}}$ times larger

- We do not need to solve the electron GK equation. In total,

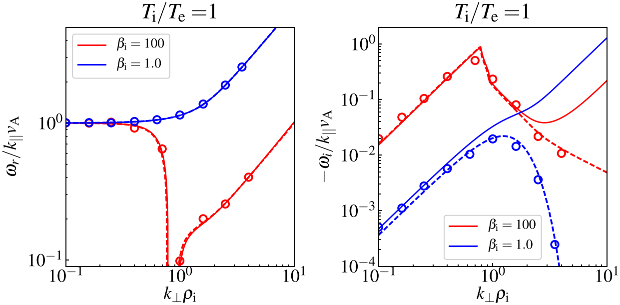
  $2\sqrt{m_\mathrm{i}/m_\mathrm{e}} \sim 100$ times faster

Linear Alfvén wave properties

- Excite Alfvén wave by oscillation antenna
- Set plasma parameter $\beta_\mathrm{i} = 1$, $T_\mathrm{i}/T_\mathrm{e} = 1$, and $k_\perp \rho_\mathrm{i} = 1$
- Compare with the result of `AstroGK` with $m_\mathrm{e}/m_\mathrm{i} = 10^{-10}$



Linear Alfvén wave properties



solid line: FGK
broken line: GKI/ITEF

Discrepancy between FGK and GKI/ITEF is due to the lack of electron damping

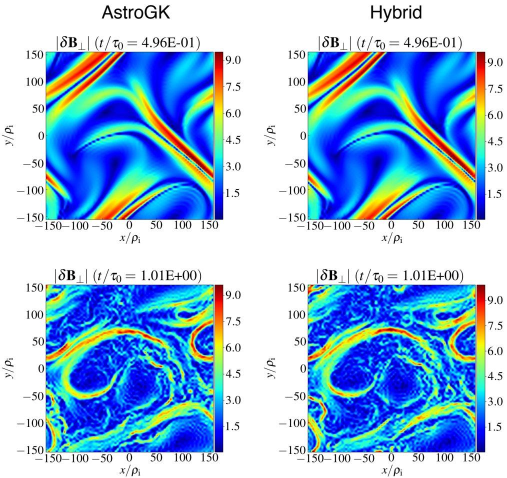
Nonlinear test: Orszag–Tang problem in inertial range

- Standard nonlinear test
- Regularly used to study decaying MHD turbulence
- Asymmetric initial condition similar to [Loureiro2016, CPC 2016]

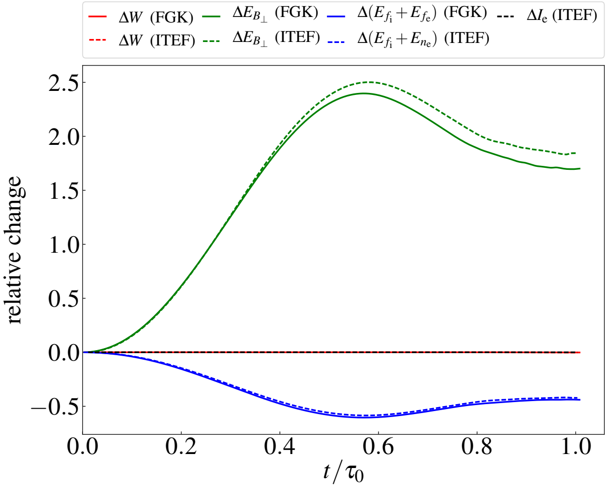
$$
\begin{aligned}
\phi(x,y) &= -\frac{B_0}{c}\delta u_0 \left(\frac{L_\perp}{2\pi}\right)\left[\cos\left(\frac{2\pi x}{L_\perp}+1.4\right)+\cos\left(\frac{2\pi y}{L_\perp}+0.5\right)\right] \\
A_\parallel(x,y) &= \frac{\delta B_{\perp 0}}{2}\left(\frac{L_\perp}{2\pi}\right)\left[\frac{1}{2}\cos\left(\frac{4\pi x}{L_\perp}+2.3\right)+\cos\left(\frac{2\pi y}{L_\perp}+4.1\right)\right],
\end{aligned}
$$

- Set plasma parameter $\beta_\mathrm{i} = 1$, $T_\mathrm{i}/T_\mathrm{e} = 1$, weak ion collision, and no electron collision
- Inertial range $0.02 \le k_\perp \rho_\mathrm{i} \le 0.84$
- Compare with the result of `AstroGK`

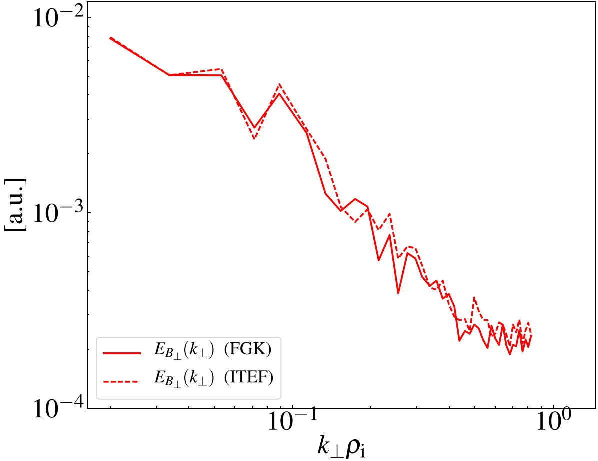
OT in inertial range ⟩ Field profile



- Looks almost identical
- Final $\Delta t/\tau_0 \simeq 3.2 \times 10^{-6}$ for `AstroGK` and 1.6×10^{-4} for the hybrid code $\rightarrow$ ~ 50 times improvement

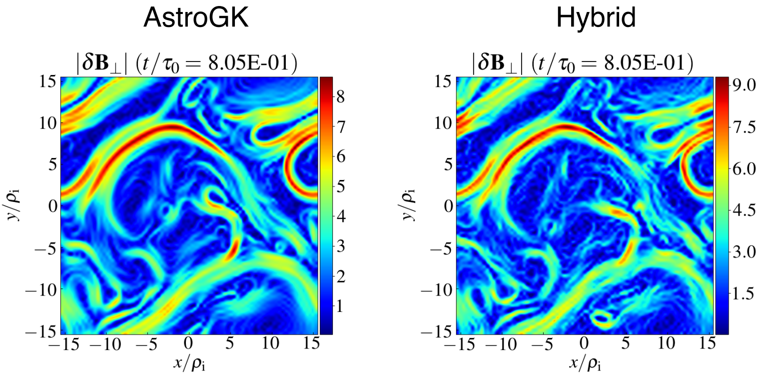


- Agreement in time evolution of energy
- The relative change of $W \sim 10^{-5}$
- The relative change of $I_e \sim 10^{-7} \;\Rightarrow\;$ nice conservation

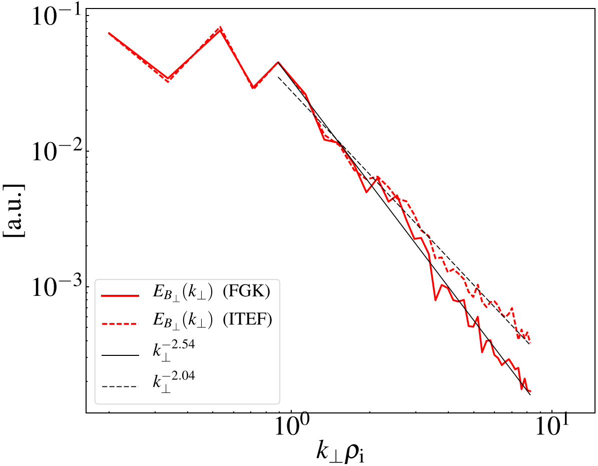


Looks almost identical

- Transition range $0.2 \leq k_\perp \rho_\mathrm{i} \leq 8.4$



Small–scale structures appear in the hybrid model



- For the hybrid model, spectrum gets shallower
- This is consistent with recent comparison of full kinetic and full kinetic ion/ITEF hybrid [Groselj et al., arXiv:1706.02652 2017]

1. Introduction
2. Development of a GKI/ITEF hybrid code
3. Numerical tests
4. Nonlinear simulation of ion/electron heating partitioning
5. Summary

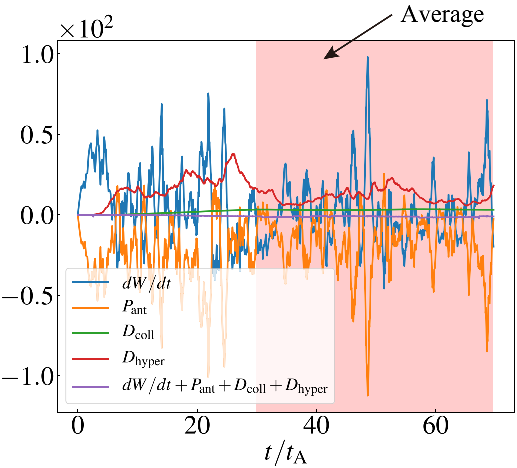
Simulation Setting

- Excite Alfvén wave by oscillation Langevin antenna [TenBarge et al., CPC 2014] at $k_\perp \rho_\mathrm{i} = 0.25$
- Simulation box: $k_\perp \rho_\mathrm{i} = [0.25, 5.25]$
- Simulate time evolution until steady state
- In steady state, energy balance is

$$0 = \overline{\frac{\mathrm{d}W}{\mathrm{d}t}} = \overline{P_\mathrm{ext}} + \underbrace{\overline{\int \mathrm{d}^3\mathbf{r} \int \mathrm{d}^3\mathbf{v}\, \frac{T_\mathrm{i}}{F_\mathrm{i}} \langle h_\mathrm{i} C[h_\mathrm{i}] \rangle_{\mathbf{R}_\mathrm{i}}}}_{\overline{D}_\mathrm{coll}} \underbrace{\overline{-\nu_h n_{0\mathrm{e}} T_{0\mathrm{e}} \int \mathrm{d}^3\mathbf{r} \left| \nabla_\perp^n \left(\frac{\delta n_\mathrm{e}}{n_{0\mathrm{e}}} - \tau \frac{e}{T_{0\mathrm{i}}} \phi \right) \right|^2}}_{\overline{D}_\mathrm{hyper}}$$

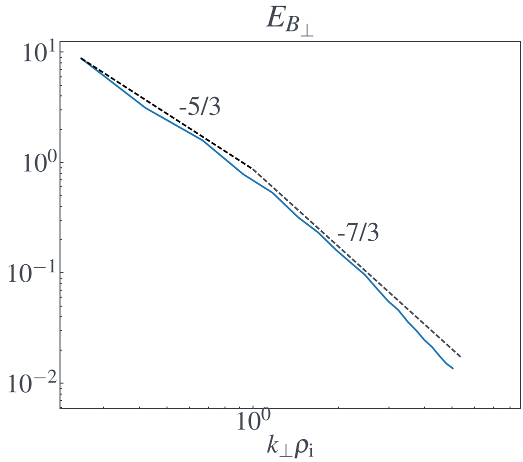
- Electron heating is estimated by $-P_\mathrm{ext} - D_\mathrm{coll}$, which is equivalent to the hyper dissipation D_hyper

$\beta_\mathrm{i} = 1,\ T_\mathrm{i}/T_\mathrm{e} = 1$ case



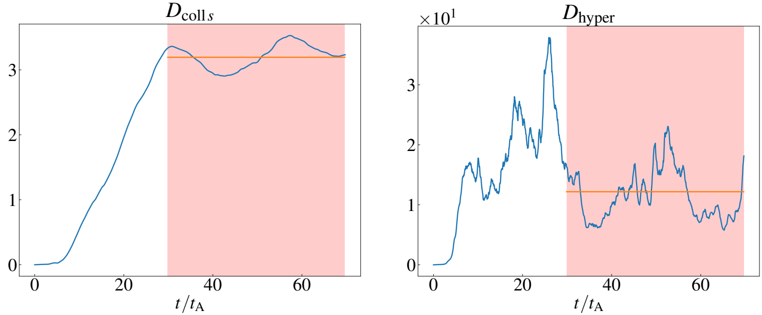
- Good energy conservation (purple)
- Electron heating (red) > ion heating (green)

$\beta_{\rm i} = 1,\ T_{\rm i}/T_{\rm e} = 1$ case



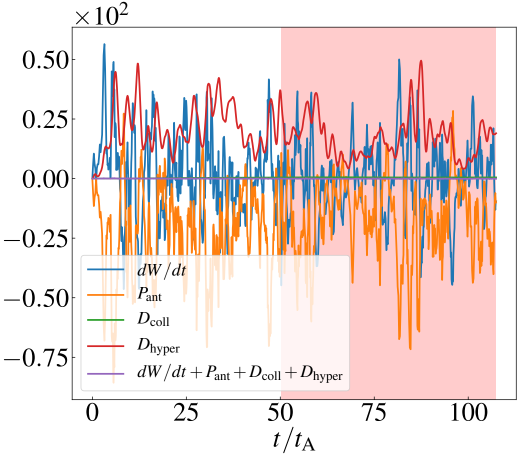
- Good spectral slope [Schekochihin et al., 2009]

$\beta_i = 1,\ T_i/T_e = 1$ case



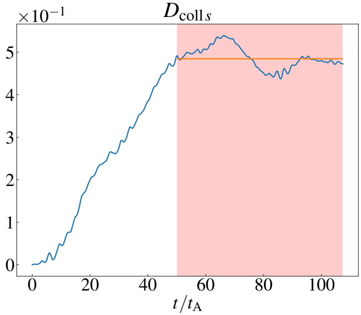
- Electron heating (red) > ion heating (green)
- $Q_i/Q_e \simeq 0.17$
- Howes' estimate [Howes, 2010]: $Q_i/Q_e \sim 0.2$

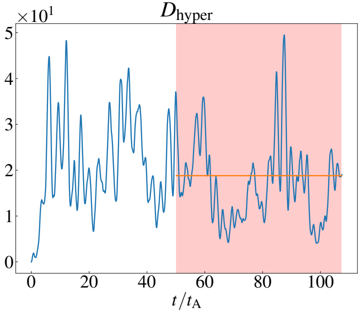
$\beta_{\mathrm{i}} = 1,\ T_{\mathrm{i}}/T_{\mathrm{e}} = 100$ case



- Good energy conservation (purple)
- Electron heating (red) $\gg$ ion heating (green)

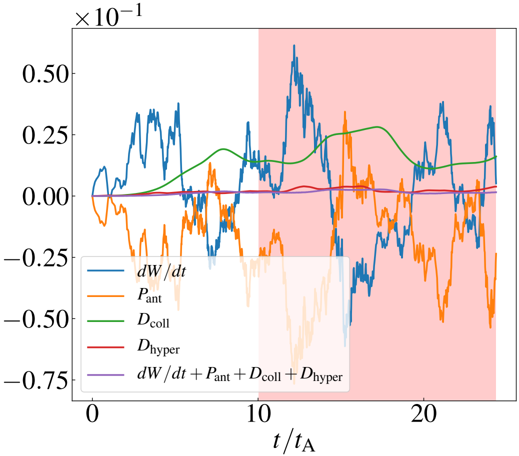
$\beta_{\rm i} = 1,\ T_{\rm i}/T_{\rm e} = 100$ case





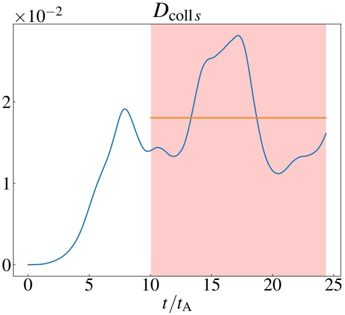
- Electron heating (red) > ion heating (green)
- $Q_{\rm i}/Q_{\rm e} \simeq 0.026$
- Howes' estimate [Howes, 2010]: $Q_{\rm i}/Q_{\rm e} \sim 0.1$

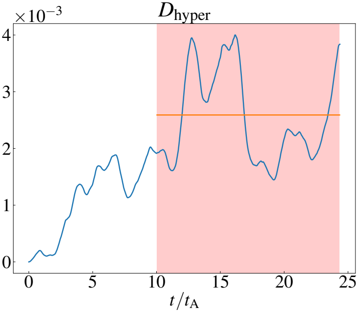
$\beta_i = 100,\ T_i/T_e = 1$ case



- Mildly good energy conservation (insufficient velocity space resolution? necessity of hypercollision?)
- Electron heating (red) $\ll$ ion heating (green)

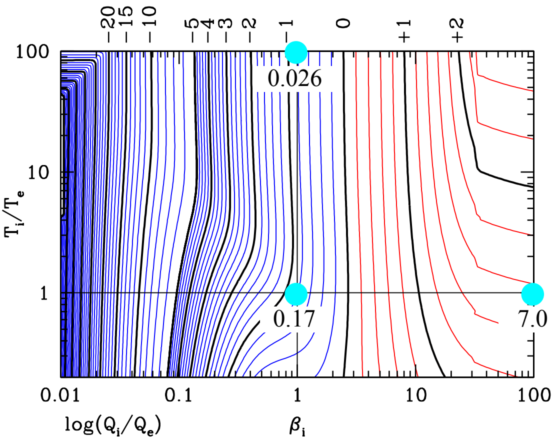
$\beta_i = 100,\ T_i/T_e = 1$ case





- Electron heating (red) $\ll$ ion heating (green)
- $Q_i/Q_e \simeq 7.0$
- Howes' estimate [Howes, 2010]: $Q_i/Q_e \sim 40$

Comparison with Howes' estimate



- Overall tendency is consistent
- However, when $\beta_\mathrm{i} = 1$, $Q_\mathrm{i}/Q_\mathrm{e}$ rapidly decreases as $T_\mathrm{i}/T_\mathrm{e}$ increases

Summary

- We developed the GKI/ITEF hybrid simulation code by extending `AstroGK`
- The new code runs $2\sqrt{m_\mathrm{e}/m_\mathrm{i}}$ times faster than `AstroGK`
- We conducted linear and nonlinear tests
- 2D Orszag–Tang test shows that power spectrum of GKI/ITEF in the ion kinetic region gets shallower than FGK; this is consistent with the recent work of comparison between the full kinetic code and the full kinetic ion/fluid electron code
- We have shown the initial results of 3D driven simulation to investigate the partitioning of turbulent heating
- Overall tendency is consistent with the estimate that uses linear damping whereas the absolute value differs

Future work

- Add more points on the β_{i} vs $T_{\mathrm{i}}/T_{\mathrm{e}}$ diagram
- Careful consideration of (i) the velocity space resolution and (ii) hyper dissipation & hyper collision
- Analysis of the energy transfer route in the phase space
- Power spectrum in the phase space (similar to [Tatsuno et al., 2009] for 2D electrostatic case)

Reference

Y. Kawazura and M. Barnes, in preparation for JCP.