Modelling Coupled Ion and Electron Scale Turbulence in Magnetic Confinement Fusion Plasmas

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Introduction

Anomalous transport is driven by turbulence,

- at scales where $k\rho_i \lesssim 1$ - ion scale
- at scales where $k\rho_e \sim 1$ - electron scale

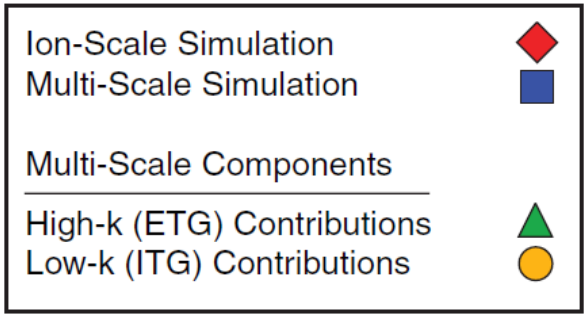
We want to answer the following questions:

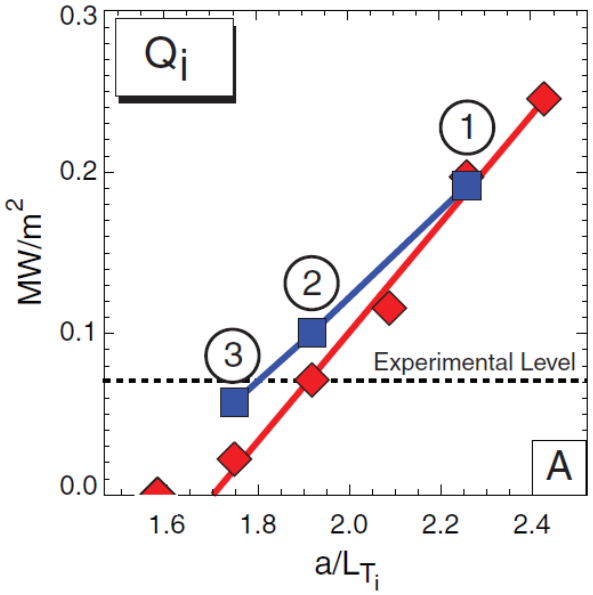
- do all scales matter?
- is cross scale coupling important?

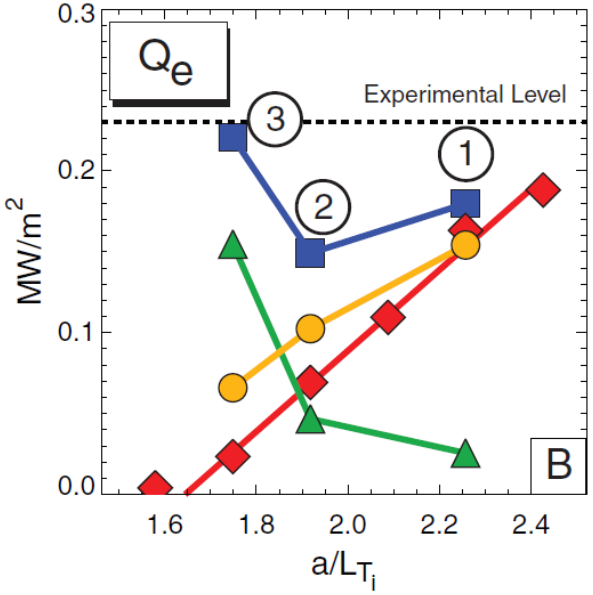
- To answer these questions we take a scale separated approach

Introduction: do all scales matter?

- simulation evidence where $Q_e \sim Q_{i\mathrm{gB}} \sim \sqrt{m_i/m_e}\, Q_{e\mathrm{gB}}$ e.g. Jenko and Dorland (2002)
- recent experimental evidence on NSTX Ren et al. (2017)
- Howard et al. (2016) Fig 3:

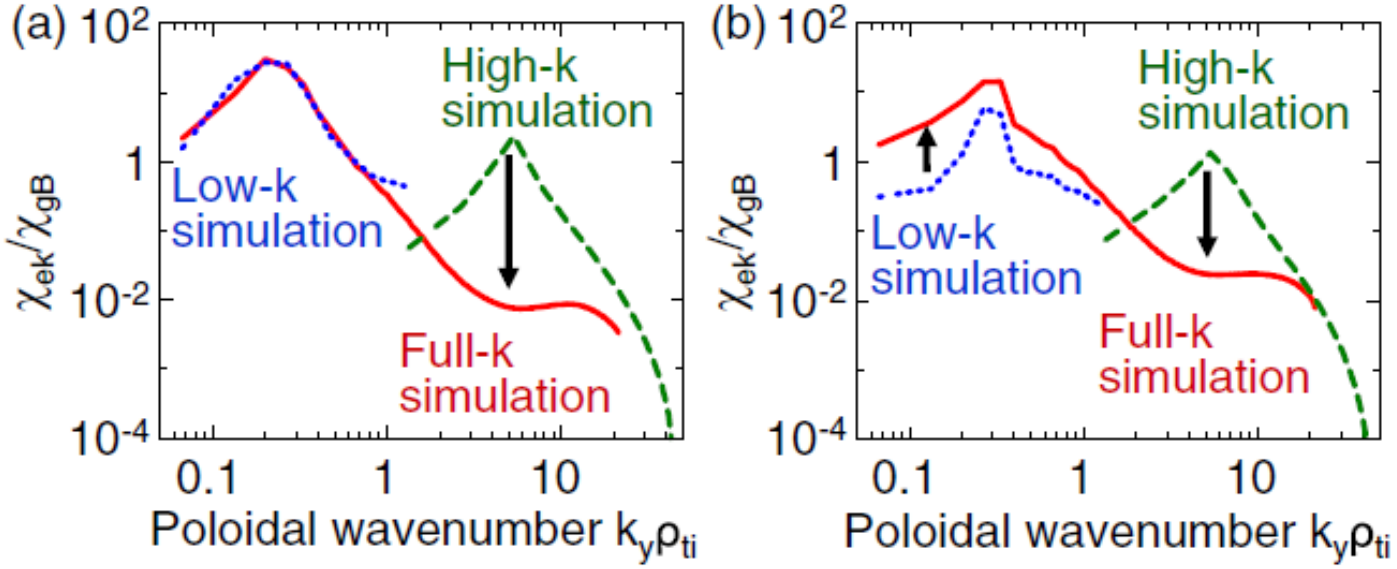






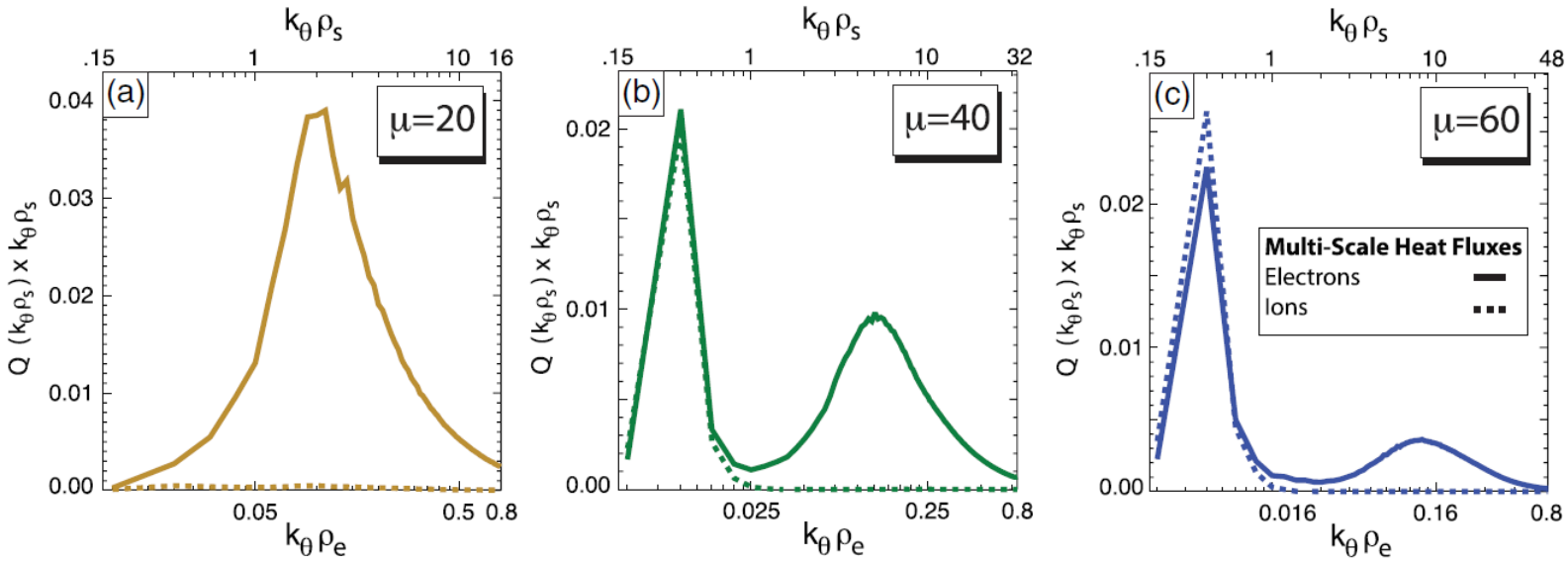
Introduction: is cross scale coupling important?

- Fig 2 from Maeyama et al. (2015):

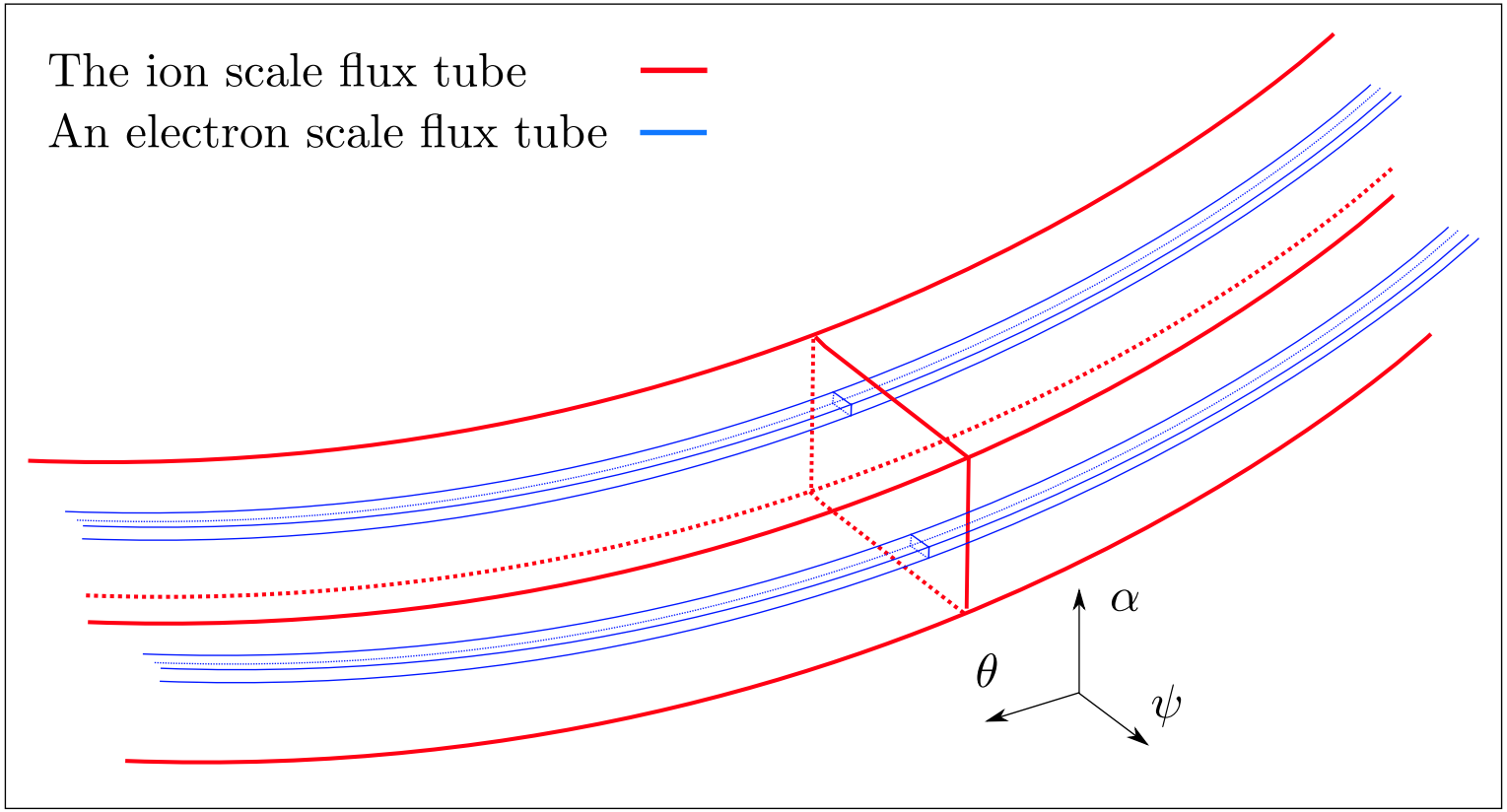


Introduction: can we reduce the mass ratio?

- Fig 5 from Howard et al. (2015):



Introduction: a scale separated approach



A Quick Reminder: The Gyrokinetic Equation

The gyrokinetic equation:

$$\frac{\partial h}{\partial t} + v_{\parallel}\mathbf{b}\cdot\nabla\theta\frac{\partial h}{\partial\theta} + (\mathbf{v}_M + \mathbf{v}_E)\cdot\nabla h + \mathbf{v}_E\cdot\nabla F_0 = \frac{ZeF_0}{T}\frac{\partial\varphi}{\partial t}, \tag{1}$$

where,

$$\mathbf{v}_E = \frac{c}{B}\mathbf{b}\wedge\nabla\varphi. \tag{2}$$

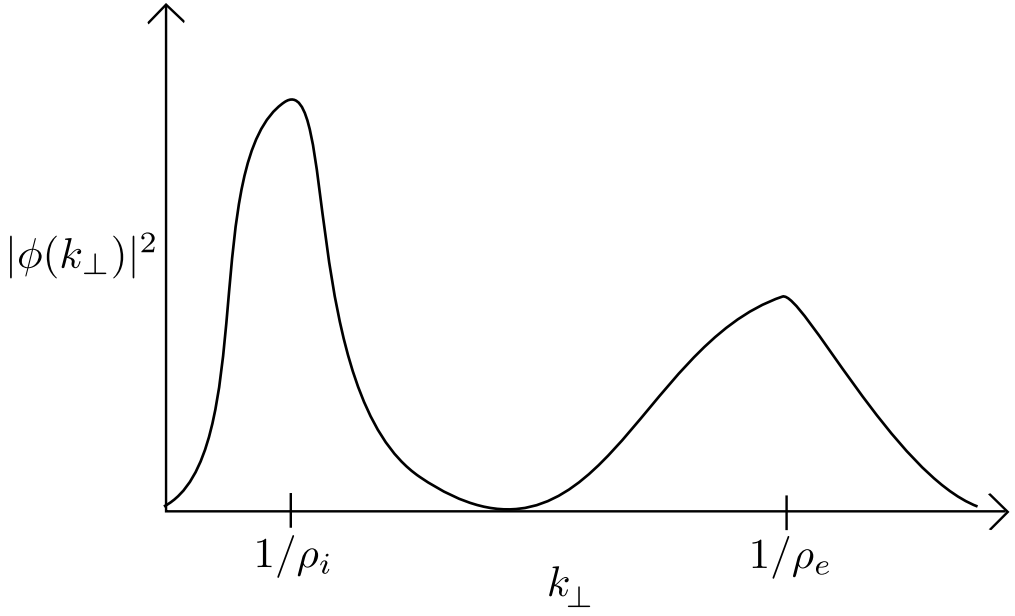
Closed by quasi-neutrality,

$$\sum_\alpha Z_\alpha e\left(\int d^3\mathbf{v}|_{\mathbf{r}} h_\alpha\right) = \sum_\alpha \frac{Z_\alpha^2 e^2 n_\alpha}{T_\alpha}\phi(\mathbf{r}). \tag{3}$$

Ingredients:

- a kinetic equation for f
- scale separation: $\rho_* = \rho/a \to 0, \quad f = F + \delta f$
- statistical periodicity: $\langle\delta f\rangle_{\text{turb}} = 0$
- orderings: $\delta f \sim \rho_* F, \quad \nabla F \sim \nabla_\perp \delta f \sim \rho_*^{-1}\nabla_\parallel \delta f$

Separating Ion and Electron Scale Turbulence



Using the ingredients:

- scale separation: $\sqrt{m_e/m_i} \to 0$, an electron scale average, $\left\langle \cdot \right\rangle$
- scale separation: $\left\langle \delta f \right\rangle = \overline{\delta f}, \quad \delta f = \overline{\delta f} + \tilde{\delta f}$
- electron scale statistical periodicity: $\left\langle \tilde{\delta f} \right\rangle = 0$
- orderings:

$$\nabla_\perp \overline{\delta f} \sim \rho_i^{-1} \overline{\delta f}, \quad \frac{\partial \overline{\delta f}}{\partial t} \sim \frac{v_{ti}}{a} \overline{\delta f}, \quad \nabla_\perp \tilde{\delta f} \sim \rho_e^{-1} \tilde{\delta f}, \quad \frac{\partial \tilde{\delta f}}{\partial t} \sim \frac{v_{te}}{a} \tilde{\delta f}.$$

we can derive the coupled equations!

The Coupled Equations

- ion scale equations, with new back reaction term:

$$\frac{\partial \overline{h}_i}{\partial t} + v_{\|}\mathbf{b}\cdot\nabla\theta\frac{\partial \overline{h}_i}{\partial \theta} + (\mathbf{v}_{Mi} + \overline{\mathbf{v}}_{Ei})\cdot\nabla\overline{h}_i + \overline{\mathbf{v}}_{Ei}\cdot\nabla F_{0i} = \frac{Z_i e F_{0i}}{T_i}\frac{\partial \overline{\varphi}_i}{\partial t}, \tag{4}$$

$$\frac{\partial \overline{h}_e}{\partial t} + v_{\|}\mathbf{b}\cdot\nabla\theta\frac{\partial \overline{h}_e}{\partial \theta} + (\mathbf{v}_{Me} + \overline{\mathbf{v}}_{Ee})\cdot\nabla\overline{h}_e + \overline{\mathbf{v}}_{Ee}\cdot\nabla F_{0e} + \nabla\cdot\left\langle \frac{c}{B}\tilde{h}_e\tilde{\mathbf{v}}_{Ee}\right\rangle = -\frac{eF_{0e}}{T_e}\frac{\partial \overline{\varphi}_e}{\partial t}, \tag{5}$$

$$\int d^3\mathbf{v}|_{\mathbf{r}}(Z_i e\overline{h}_i - e\overline{h}_e) = \left(\frac{e^2 Z_i^2 n_i}{T_i} + \frac{e^2 n_e}{T_e}\right)\overline{\phi}, \tag{6}$$

- electron scale equations, with the new advection and drive terms:

$$\frac{\partial \tilde{h}_e}{\partial t} + v_{\|}\mathbf{b}\cdot\nabla\theta\frac{\partial \tilde{h}_e}{\partial \theta} + (\mathbf{v}_{Me} + \tilde{\mathbf{v}}_{Ee} + \overline{\mathbf{v}}_{Ee})\cdot\nabla\tilde{h}_e + \tilde{\mathbf{v}}_{Ee}\cdot(\nabla\overline{h}_e + \nabla F_{0e}) = -\frac{eF_{0e}}{T_e}\frac{\partial \tilde{\varphi}_e}{\partial t}. \tag{7}$$

$$-\int d^3\mathbf{v}|_{\mathbf{r}} e\tilde{h}_e = \left(\frac{e^2 Z_i^2 n_i}{T_i} + \frac{e^2 n_e}{T_e}\right)\tilde{\phi}, \tag{8}$$

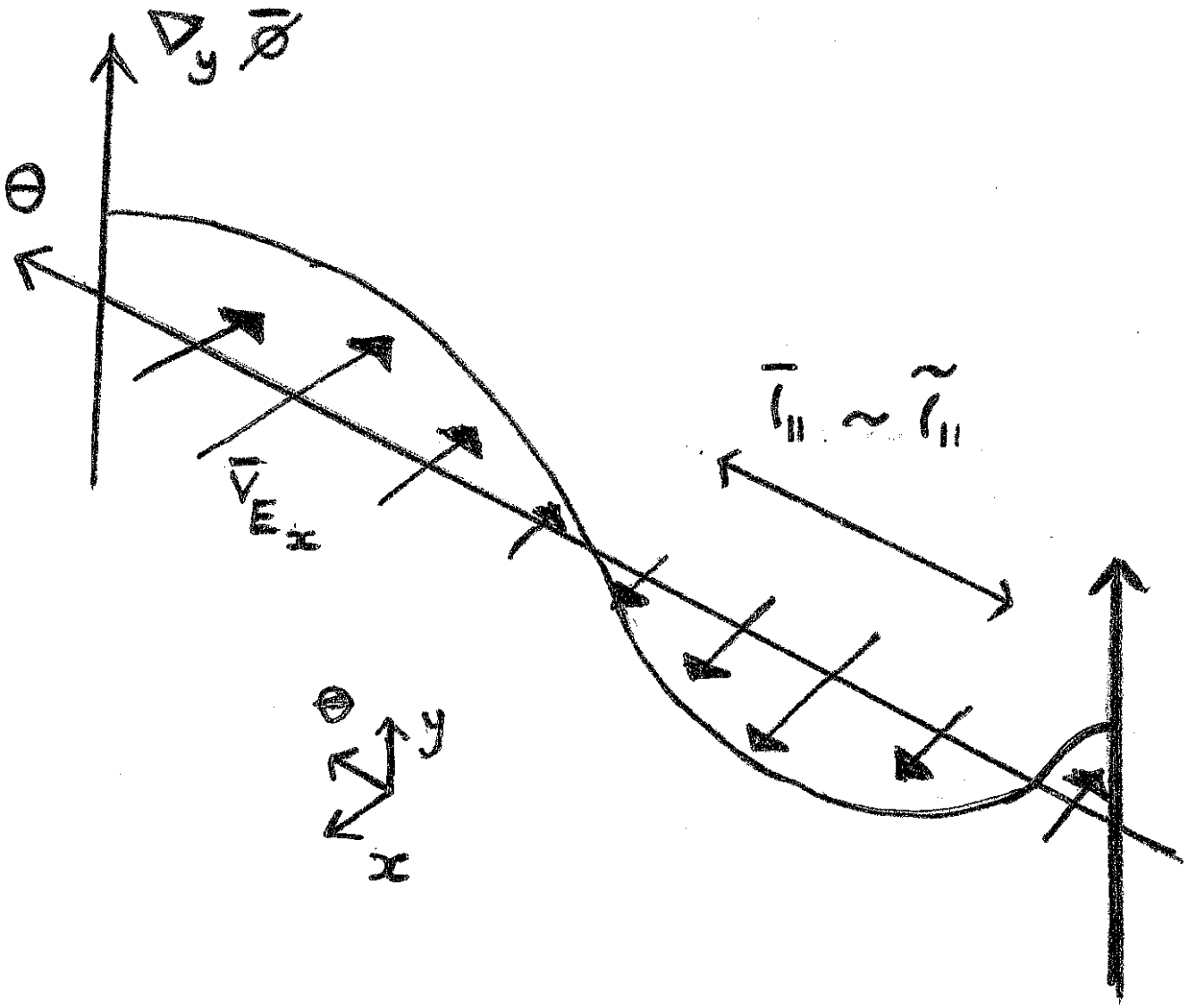
The Coupled Equations: Sticky Points

Deriving parallisable coupled equations requires dealing with:

- non-locality of the gyro average
- the relative size of fluctuations - gyro Boehm scaling
- ions at electron scales
- the parallel boundary condition (*)

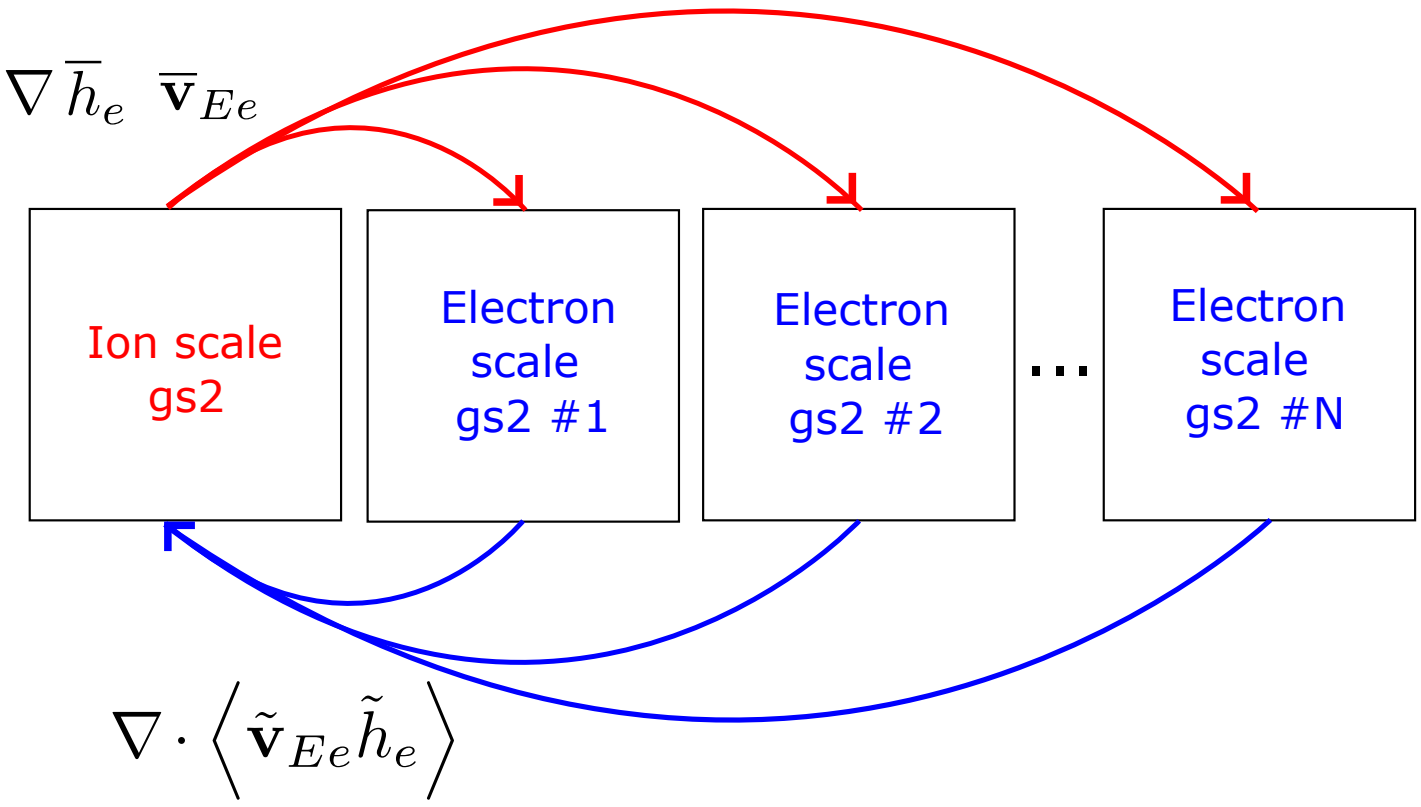
(*) Not yet resolved!

Visualising the Ion Scale E×B Velocity with θ

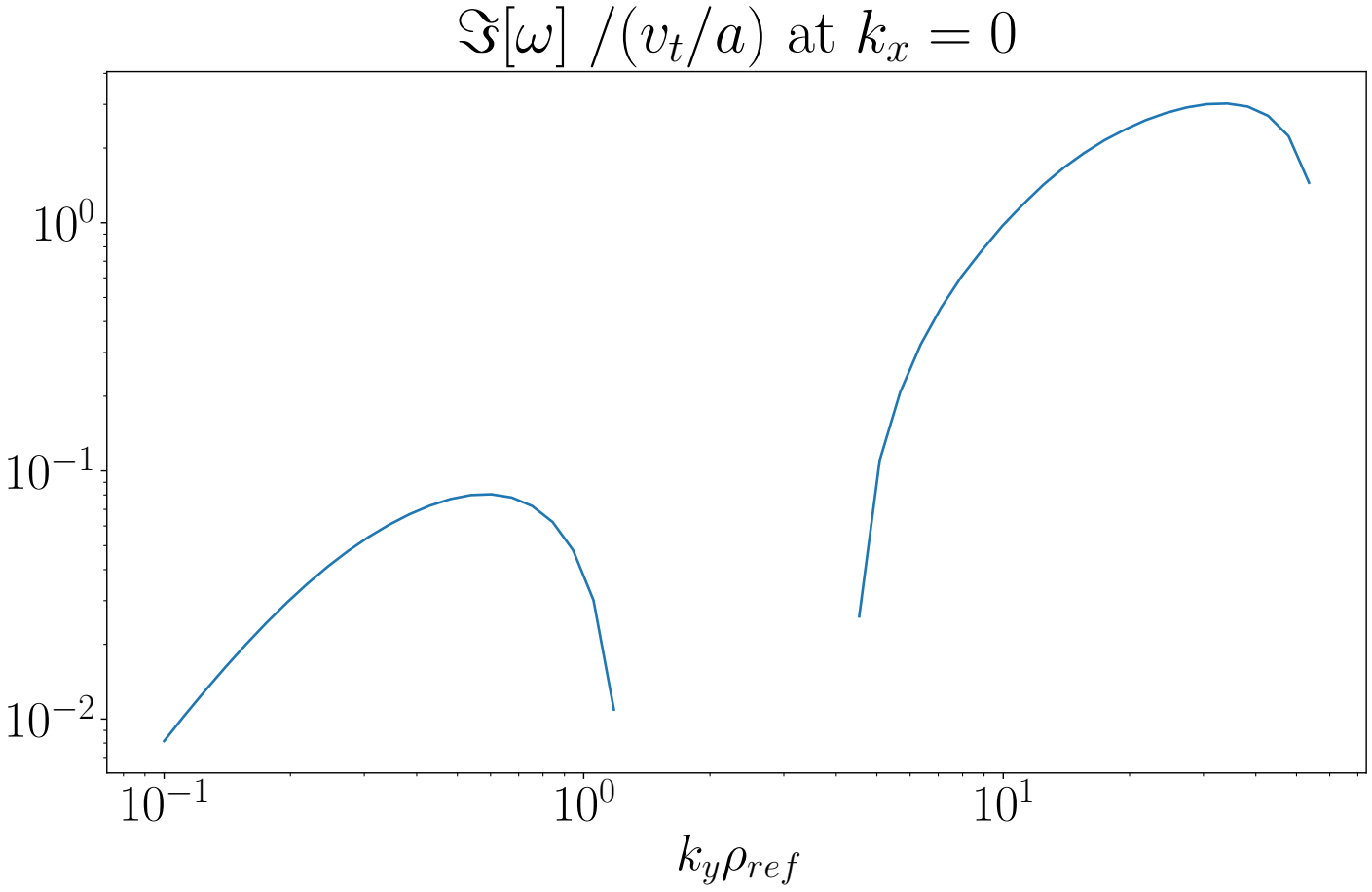


Outline of structure of multigs2: A diagram

multigs2 runs N+1 instances of gs2 and handles communication of gradients and fluxes between them



Electron Scale Simulations: Modification of the Linear Growth Rate



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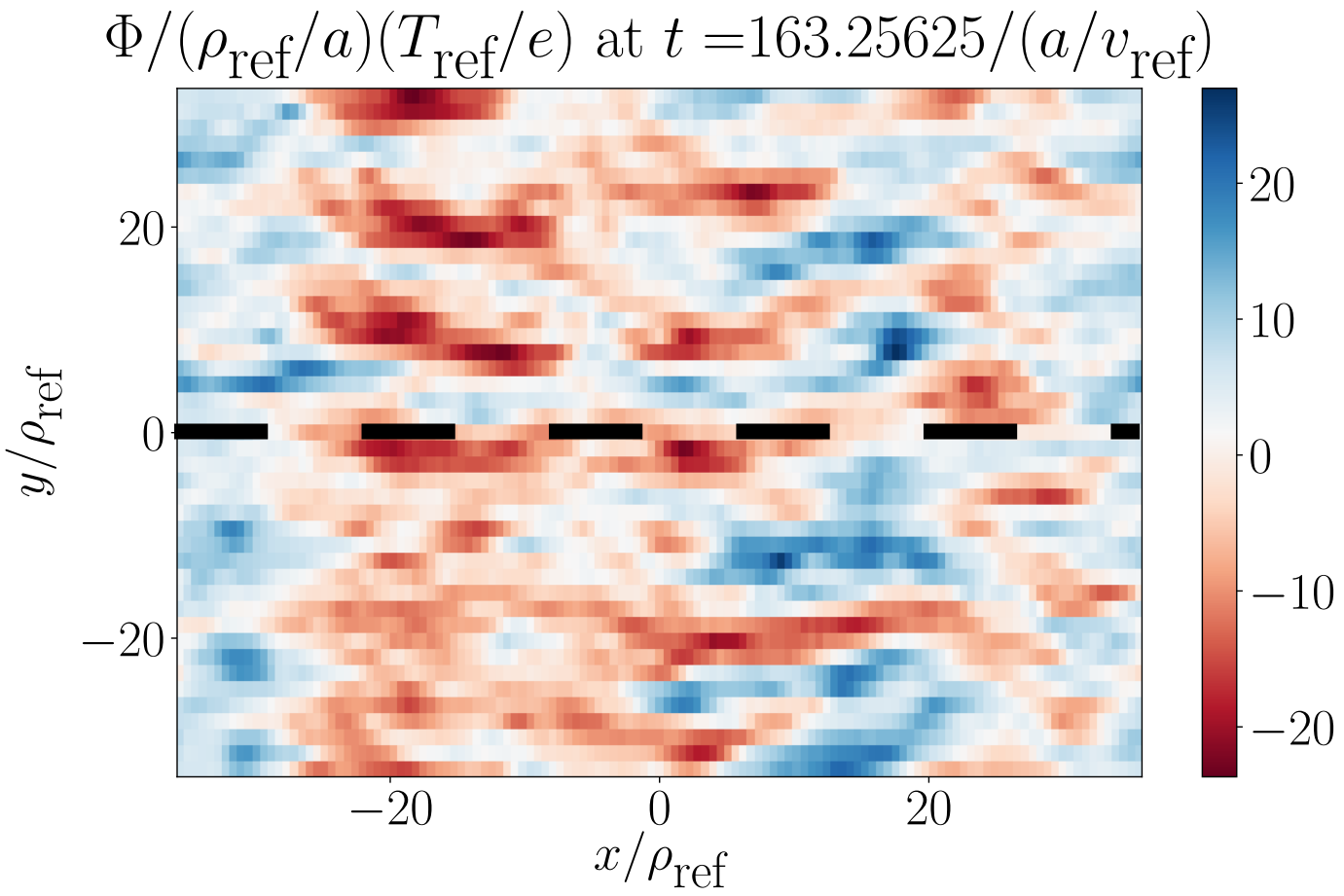

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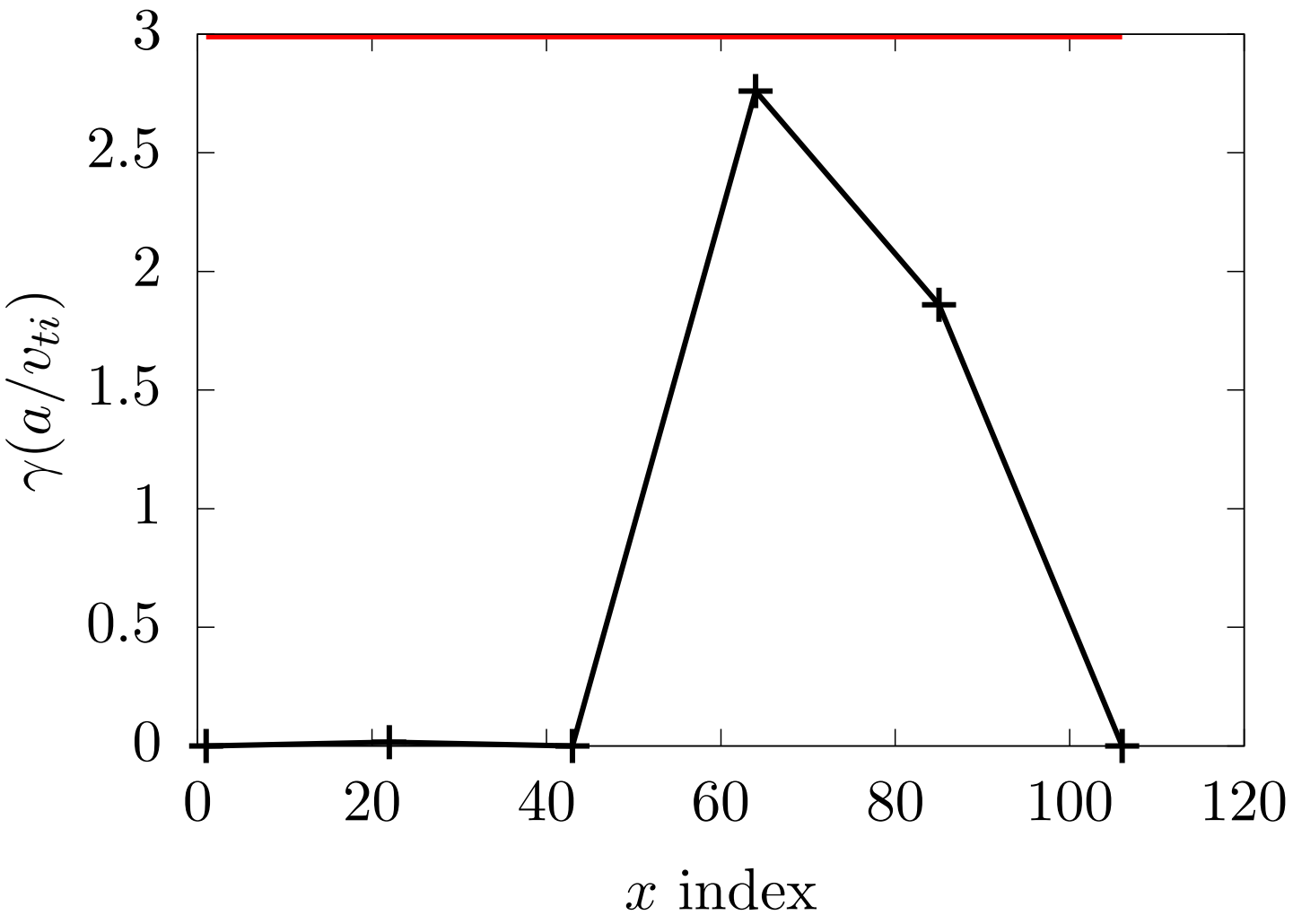
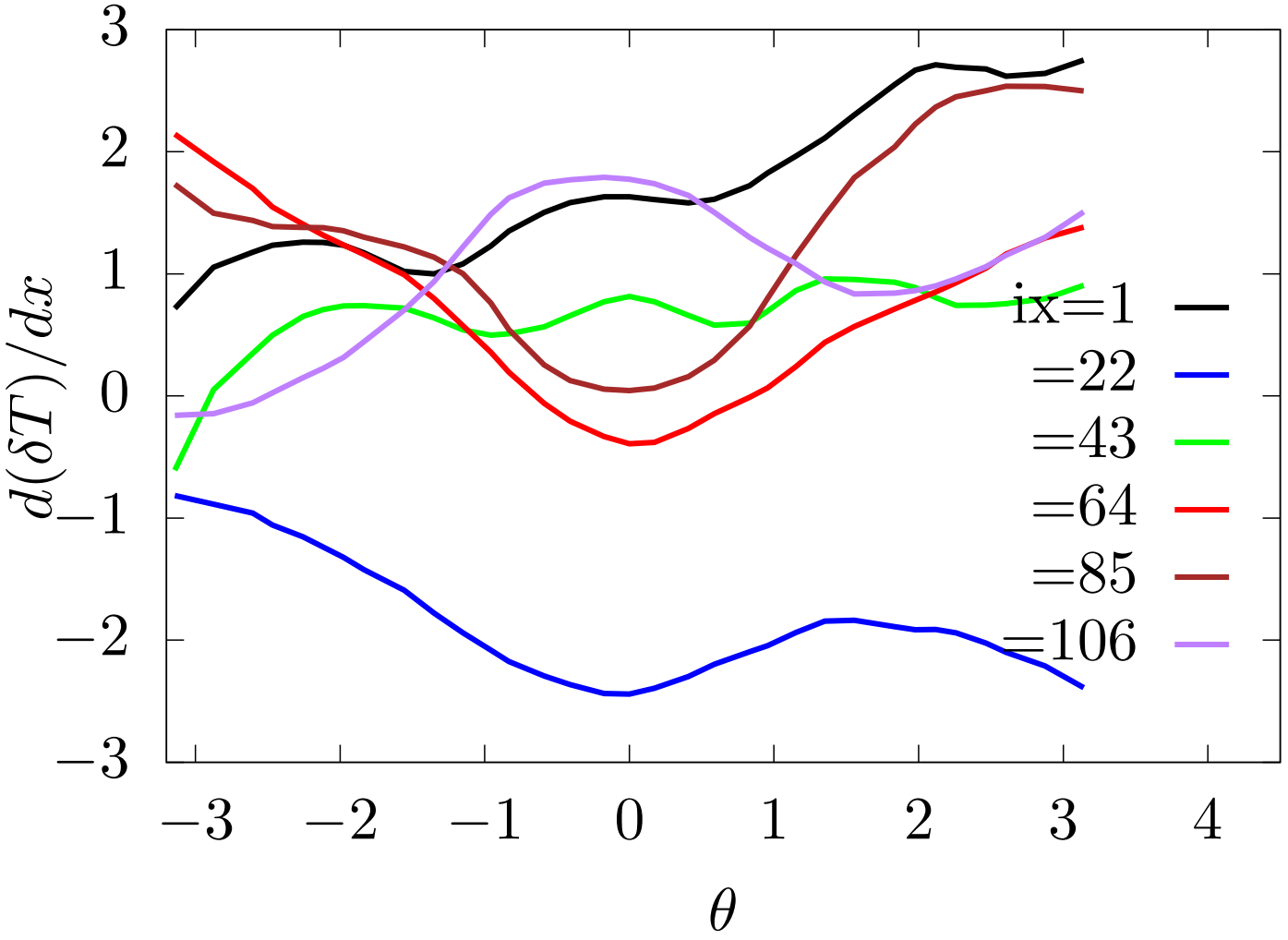
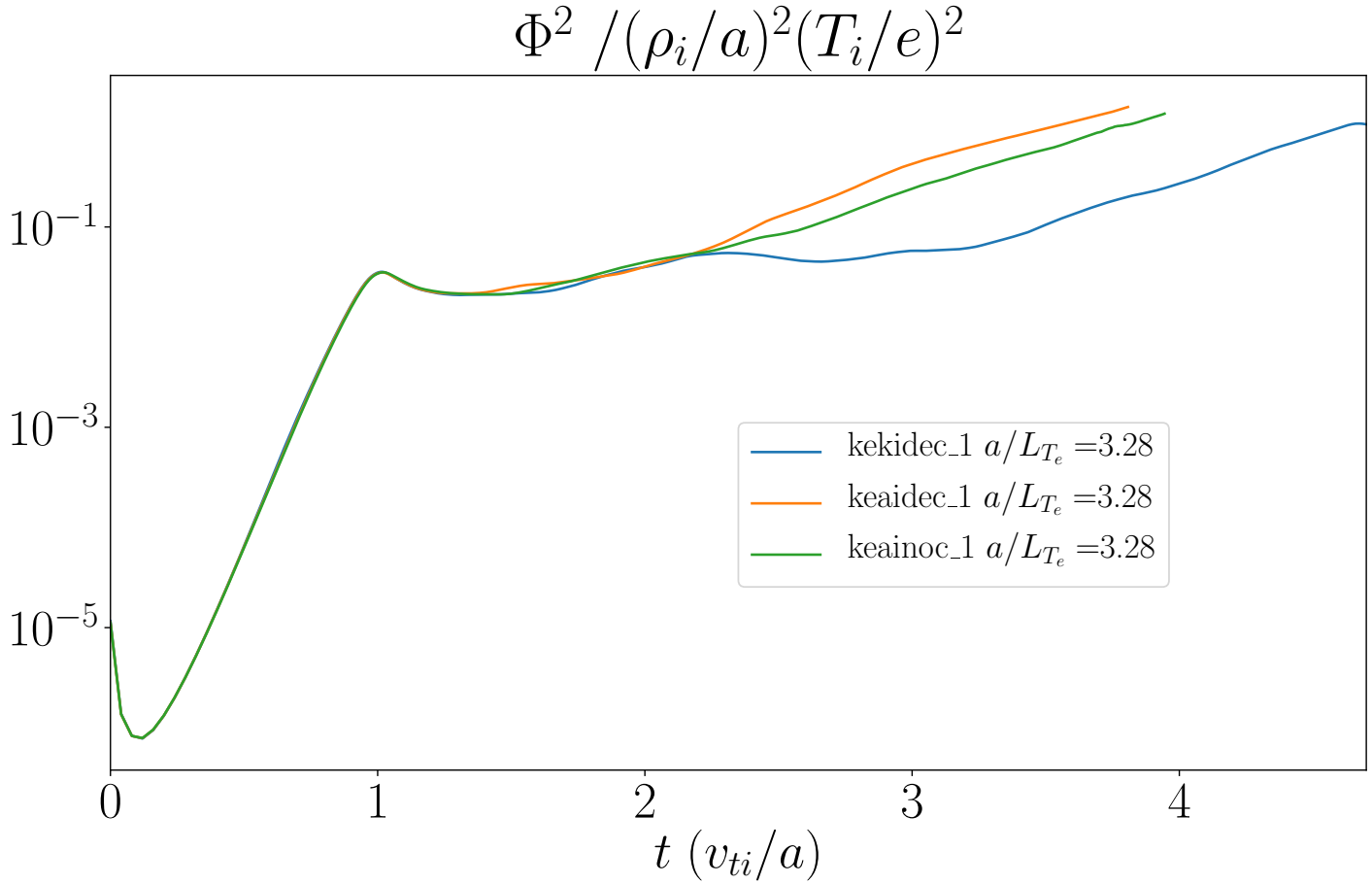


Electron Scale Simulations: Modification of the Linear Growth Rate



Electron Scale Simulations



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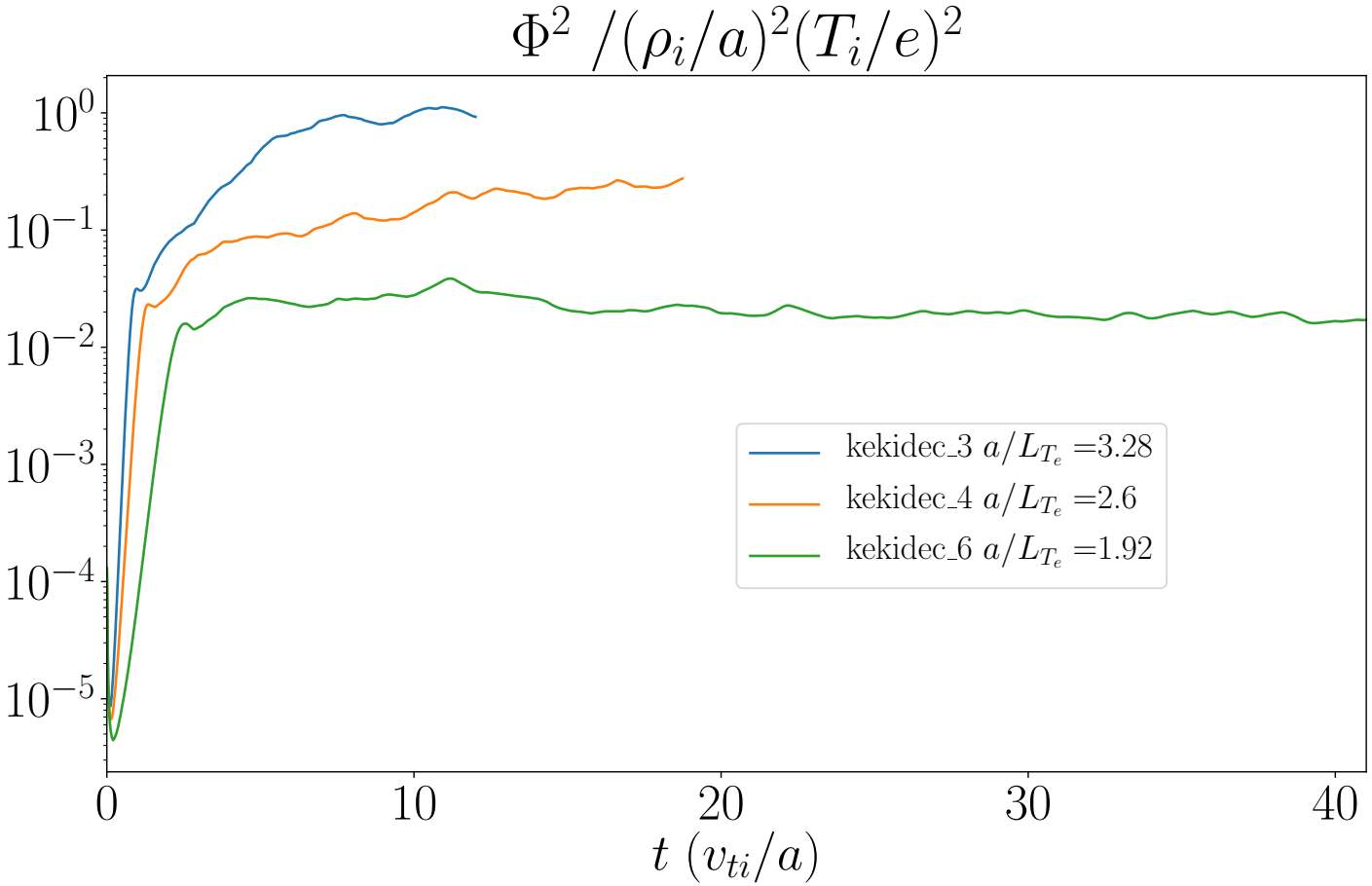
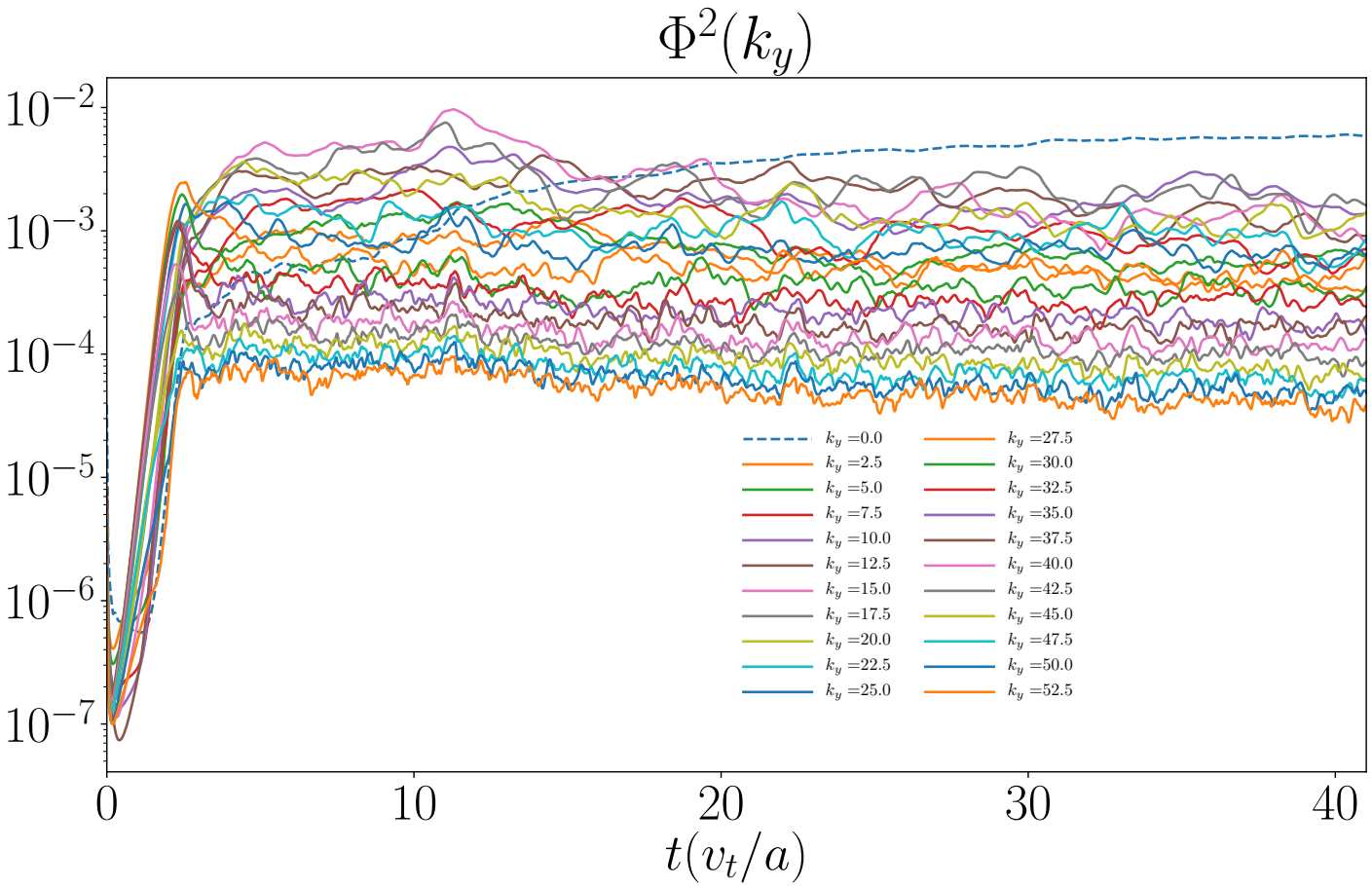


Electron Scale Simulations



Summary

Conclusions:

- we have derived coupled equations for the ion and electron scale turbulence
- the electron scale terms have been implemented in gs2
- we have begun a search for a suitable proof of concept case for simulation

Future Work:

- understanding the electron scale equation through simulation
- including the back reaction in simulations

F. Jenko and W. Dorland. Prediction of significant tokamak turbulence at electron gyroradius scales. *Phys. Rev. Lett.*, 89:225001, Nov 2002. doi: 10.1103/PhysRevLett.89.225001. URL http://link.aps.org/doi/10.1103/PhysRevLett.89.225001.

Y. Ren, E. Belova, N. Gorelenkov, W. Guttenfelder, S.M. Kaye, E. Mazzucato, J.L. Peterson, D.R. Smith, D. Stutman, K. Tritz, W.X. Wang, H. Yuh, R.E. Bell, C.W. Domier, and B.P. LeBlanc. Recent progress in understanding electron thermal transport in nstx. *Nuclear Fusion*, 57(7):072002, 2017. URL http://stacks.iop.org/0029-5515/57/i=7/a=072002.

N.T. Howard, C. Holland, A.E. White, M. Greenwald, and J. Candy. Multi-scale gyrokinetic simulation of tokamak plasmas: enhanced heat loss due to cross-scale coupling of plasma turbulence. *Nuclear Fusion*, 56(1):014004, 2016. URL http://stacks.iop.org/0029-5515/56/i=1/a=014004.

S Maeyama, Y Idomura, T-H Watanabe, M Nakata, M Yagi, N Miyato, A Ishizawa, and M Nunami. Cross-scale interactions between electron and ion scale turbulence in a tokamak plasma. *Physical review letters*, 114(25):255002, 2015.

N T Howard, C Holland, A E White, M Greenwald, and J Candy. Fidelity of reduced and realistic electron mass ratio multi-scale gyrokinetic simulations of tokamak discharges. *Plasma Physics and Controlled Fusion*, 57(6):065009, 2015. URL http://stacks.iop.org/0741-3335/57/i=6/a=065009.

M. A. Beer, S. C. Cowley, and G. W. Hammett. Field aligned coordinates for nonlinear simulations of tokamak turbulence. *Physics of Plasmas*, 2(7):2687–2700, 1995. doi: http://dx.doi.org/10.1063/1.871232. URL http://scitation.aip.org/content/aip/journal/pop/2/7/10.1063/1.871232.

Should We Expect Cross Scale Interaction?

Yes! Because:

- electron scale eddies have $\tilde{l}_\perp \sim \rho_e$
- ion scale eddies have $\bar{l}_\perp \sim \rho_i$
- ambient gradient argument $\Rightarrow \tilde{h}_e \sim \rho_e^* F_{0e}, \quad \bar{h}_e \sim \rho_i^* F_{0e}$
- $\Rightarrow \nabla \tilde{h}_e \sim \nabla \bar{h}_e \sim \nabla F_{0e}$

$\Rightarrow$ gradients of the distribution function are comparable at all scales
$\Rightarrow$ electron scale eddies can be driven by ion scale gradients

- applying the same argument to $\mathbf{E} = -\nabla\phi$
- $\Rightarrow \nabla\tilde{\phi} \sim \nabla\bar{\phi}$

$\Rightarrow$eddy E$\times$B drifts $v_{\mathrm{E}\times\mathrm{B}}$, are comparable at all scales

- applying the critical balance argument
- $v_{te}/\tilde{l}_\parallel \sim \tilde{\tau}_{nl}^{-1} \sim \tilde{v}_{\mathrm{E}\times\mathrm{B}}/\tilde{l}_\perp$
- $v_{ti}/\bar{l}_\parallel \sim \bar{\tau}_{nl}^{-1} \sim \bar{v}_{\mathrm{E}\times\mathrm{B}}/\bar{l}_\perp$
- $\tilde{l}_\parallel \sim \bar{l}_\parallel$

$\Rightarrow$ parallel correlation lengths are the same for ion scale and electron scale eddies
$\Rightarrow$ electron scale eddies are long enough to be differentially advected by $\bar{v}_{\mathrm{E}\times\mathrm{B}}$

Separating Ion and Electron Scale Turbulence: Technicalities

- We introduce a fast spatial variable $\mathbf{r}_f$ and a slow spatial variable $\mathbf{r}_s$ and the fast and slow times t_f, t_s
- In the gyrokinetic equation we send,

$$\delta f(t, \mathbf{r}) \to \delta f(t_s, t_f, \mathbf{r}_s, \mathbf{r}_f), \quad \nabla \to \nabla_s + \nabla_f, \quad \frac{\partial}{\partial t} \to \frac{\partial}{\partial t_s} + \frac{\partial}{\partial t_f}, \tag{9}$$

- then asymptotically expand in the mass ratio $(m_e/m_i)^{1/2}$
- remembering $\nabla_s \sim (m_e/m_i)^{1/2}\nabla_f$, and $\partial/\partial t_s \sim (m_e/m_i)^{1/2}\partial/\partial t_f$
- explicitly define the electron scale average,

$$\overline{\delta f}(t_s, \mathbf{r}_s) = \left\langle \delta f(t_s, t_f, \mathbf{r}_s, \mathbf{r}_f) \right\rangle = \frac{1}{\tau_c A} \int_{t_s - \tau_c/2}^{t_s + \tau_c/2} dt_f \int_{A, \mathbf{r}_s} d^2\mathbf{r}_f \delta f(t_s, t_f, \mathbf{r}_s, \mathbf{r}_f), \tag{10}$$

- We assume that,

$$\delta f(t_s, t_f, \mathbf{r}_s, \mathbf{r}_f) = \delta f(t_s, t_f, \mathbf{r}_s, \mathbf{r}_f + n\Delta_{cx}\hat{\mathbf{x}} + m\Delta_{cy}\hat{\mathbf{y}}), \tag{11}$$

- This enforces $\left\langle \tilde{\delta f} \right\rangle = 0$.

Splitting the Quasi-Neutrality Relation

- We split the guiding centre into a slow $\mathbf{R}_s$ and a fast $\mathbf{R}_f$ part.
- $\mathbf{R} = \mathbf{r} - \rho(\mathbf{r})$, where $\rho(\mathbf{r})$ is the vector gyroradius
- Thus using the periodicity property equation (11) the electron scale average may be taken over guiding centre or real space coordinates.
- This observation allows us to note that the electron scale average commutes with the gyro average,

$$\left\langle \frac{1}{2\pi}\int_0^{2\pi} d\gamma|_{\mathbf{R}}\phi(\mathbf{r}_s,\mathbf{r}_f)\right\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\gamma|_{\mathbf{R}}\left\langle \phi(\mathbf{r}_s,\mathbf{r}_f)\right\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\gamma|_{\mathbf{R}}\overline{\phi}(\mathbf{r}_s), \tag{12}$$

The splitting of the quasi neutrality relation follows directly,

$$\sum_\alpha Z_\alpha e(\int d^3\mathbf{v}|_{\mathbf{r}}\overline{h}_\alpha(\mathbf{R}_s)) = \sum_\alpha \frac{Z_\alpha^2 e^2 n_\alpha}{T_\alpha}\overline{\phi}(\mathbf{r}_s), \tag{13}$$

$$\sum_\alpha Z_\alpha e(\int d^3\mathbf{v}|_{\mathbf{r}}\tilde{h}_\alpha(\mathbf{R}_s,\mathbf{R}_f)) = \sum_\alpha \frac{Z_\alpha^2 e^2 n_\alpha}{T_\alpha}\tilde{\phi}(\mathbf{r}_s,\mathbf{r}_f). \tag{14}$$

Addressing the Non-Locality of the Gyro Average

- Taking the gyro average at fixed guiding centre $\langle\cdot\rangle|_{\mathbf{R}}^{\text{gyro}}$, couples multiple $\mathbf{r}_s$ points.
- but we aim to find parallelisable equations!
- Expanding both the slow and the fast spatial variable in Fourier series we note that,

$$\tilde{\varphi}(t_s, t_f, \mathbf{R}_s, \mathbf{R}_f) = \langle\tilde{\phi}(t_s, t_f, \mathbf{r}_s, \mathbf{r}_f)\rangle|_{\mathbf{R}}^{\text{gyro}} = \frac{1}{2\pi}\int_0^{2\pi} d\gamma|_{\mathbf{R}}\tilde{\phi}(t_s, t_f, \mathbf{r}_s, \mathbf{r}_f)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} d\gamma|_{\mathbf{R}} \sum_{\mathbf{k}_s,\mathbf{k}_f} \tilde{\phi}_{\mathbf{k}_s,\mathbf{k}_f} e^{i\mathbf{k}_s\cdot\mathbf{r}_s} e^{i\mathbf{k}_f\cdot\mathbf{r}_f} = \frac{1}{2\pi}\int_0^{2\pi} d\gamma|_{\mathbf{R}} \sum_{\mathbf{k}_s,\mathbf{k}_f} \tilde{\phi}_{\mathbf{k}_s,\mathbf{k}_f} e^{i\mathbf{k}_s\cdot\mathbf{R}_s} e^{i\mathbf{k}_f\cdot\mathbf{R}_f} e^{-i(\mathbf{k}_s+\mathbf{k}_f)\cdot\rho}$$

$$= \sum_{\mathbf{k}_s,\mathbf{k}_f} \tilde{\phi}_{\mathbf{k}_s,\mathbf{k}_f} e^{i\mathbf{k}_s\cdot\mathbf{R}_s} e^{i\mathbf{k}_f\cdot\mathbf{R}_f} J_0(|(\mathbf{k}_s + \mathbf{k}_f)|\rho), \tag{15}$$

for electrons:

- $|\mathbf{k}_f|\rho_e \sim 1$ and $|\mathbf{k}_s|\rho_e \sim (m_e/m_i)^{1/2}$
- we can expand the Bessel function to return to a local picture in the slow variable with $O(m_e/m_i)^{1/2}$ error.
- We will exploit this in parallelisation.

for ions:

- $|\mathbf{k}_s|\rho_i \sim 1$ and $|\mathbf{k}_f|\rho_i \sim (m_e/m_i)^{-1/2}$.
- we are unable to expand the Bessel function
- we are unable to avoid the coupling of multiple $\mathbf{r}_s$ in the equations for ions at electron scale

Addressing the Non-Locality of the Gyro Average: continued

- assume we can neglect the ion contribution to electronscale quasi neutrality,

$$\tilde{\varphi}_e(t_s, t_f, \mathbf{R}_s, \mathbf{R}_f) = \sum_{\mathbf{k}_s, \mathbf{k}_f} \tilde{\phi}_{\mathbf{k}_s, \mathbf{k}_f} e^{i\mathbf{k}_s \cdot \mathbf{R}_s} e^{i\mathbf{k}_f \cdot \mathbf{R}_f} J_0(|(\mathbf{k}_s + \mathbf{k}_f)|\rho)$$

$$= -\frac{T_e}{n_e e} \sum_{\mathbf{k}_s, \mathbf{k}_f} e^{i\mathbf{k}_s \cdot \mathbf{R}_s} e^{i\mathbf{k}_f \cdot \mathbf{R}_f} \int d^3\mathbf{v}\ \tilde{h}_{e, \mathbf{k}_s, \mathbf{k}_f} J_0^2(|(\mathbf{k}_s + \mathbf{k}_f)|\rho) \tag{16}$$

- now we use that,

$$J_0(|(\mathbf{k}_s + \mathbf{k}_f)|\rho_e) = J_0(|\mathbf{k}_f|\rho_e) + O(\mathbf{k}_s \cdot \mathbf{k}_f \rho_e^2 \frac{dJ_0(z)}{dz}|_{z=|\mathbf{k}_f|\rho_e}), \tag{17}$$

- exploit that $|\mathbf{k}_s|\rho_e \sim (m_e/m_i)^{1/2}$ to bring $\mathbf{R}_s$ under the velocity integral
- regard $\mathbf{R}_s$ as a fixed parameter in the integration, to find,

$$\tilde{\varphi}_e(t_s, t_f, \mathbf{R}_s, \mathbf{R}_f) = -\frac{T_e}{n_e e} \sum_{\mathbf{k}_f} e^{i\mathbf{k}_f \cdot \mathbf{R}_f} \int d^3\mathbf{v}|_{\mathbf{R}_s} \tilde{h}_{e\mathbf{k}_f}(\mathbf{R}_s) J_0^2(|\mathbf{k}_f|\rho_e)(1 + O(m_e/m_i)^{1/2}) \tag{18}$$

- we can evaluate quasi-neutrality purely locally in the slow variable.

Splitting the Gyrokinetic Equation

- we apply the electronscale average to the gyrokinetic equation
- we neglect terms which are small by $(m_e/m_i)^{1/2}$

Ion scale equation:

$$\frac{\partial \overline{h}}{\partial t_s} + v_\parallel \mathbf{b} \cdot \nabla\theta \frac{\partial \overline{h}}{\partial \theta} + (\mathbf{v}_M + \overline{\mathbf{v}}_E) \cdot \nabla_s \overline{h} + \nabla_s \cdot \left\langle \frac{c}{B} \tilde{h} \tilde{\mathbf{v}}_E \right\rangle + \overline{\mathbf{v}}_E \cdot \nabla F_0 = \frac{ZeF_0}{T} \frac{\partial \overline{\varphi}}{\partial t_s}. \quad (19)$$

- we subtract the ion scale equation from the full equation and neglect terms

Electron scale equation:

$$\frac{\partial \tilde{h}}{\partial t_f} + v_\parallel \mathbf{b} \cdot \nabla\theta \frac{\partial \tilde{h}}{\partial \theta} + (\mathbf{v}_M + \tilde{\mathbf{v}}_E + \overline{\mathbf{v}}_E) \cdot \nabla_f \tilde{h} + \tilde{\mathbf{v}}_E \cdot (\nabla_s \overline{h} + \nabla F_0) = \frac{ZeF_0}{T} \frac{\partial \tilde{\varphi}}{\partial t_f}, \quad (20)$$

where

$$\overline{\mathbf{v}}_E = \frac{c}{B} \mathbf{b} \wedge \nabla_s \overline{\varphi}, \quad \tilde{\mathbf{v}}_E = \frac{c}{B} \mathbf{b} \wedge \nabla_f \tilde{\varphi}. \quad (21)$$

Note that,

- there are two additional terms on the electron scale, $\tilde{\mathbf{v}}_E \cdot \nabla_f \tilde{h}$ and $\tilde{\mathbf{v}}_E \cdot \nabla_s \overline{h}$
- there is one new term at the ion scale, $\nabla_s \cdot \left\langle \frac{c}{B} \tilde{h} \tilde{\mathbf{v}}_E \right\rangle$
- $\overline{\mathbf{v}}_E$ cannot be removed with the boost or a solid body rotation because of the θ dependence of $\overline{\varphi}$

Scaling Work: the Relative Size of the Fluctuations

- if we assume the following scalings:

$$\overline{h}_i \sim \frac{e\overline{\phi}}{T} F_{0i}, \quad \tilde{h}_e \sim \frac{e\tilde{\phi}}{T} F_{0e}, \quad \tilde{h}_i \sim (\frac{m_e}{m_i})^{1/4} \frac{e\tilde{\phi}}{T} F_{0i},$$

$$\overline{h}_e \sim (\frac{m_e}{m_i})^{1/2} \frac{e\overline{\phi}}{T} F_{0e} \text{ -parallel gradient term}, \quad \overline{h}_e \sim \frac{e\overline{\phi}}{T} F_{0e} \text{ -}\theta \text{ constant piece.} \tag{22}$$

- Then we can show that:

$$\frac{e\tilde{\phi}}{T} \sim \rho_e^*, \quad \frac{e\overline{\phi}}{T} \sim \rho_i^* \tag{23}$$

Scaling Work: Neglecting Ions at Electron Scales

note that:

- $J_0(\mathbf{k}_f \rho_i) \sim (m_e/m_i)^{1/4}$
- so:

$$\int d^3\mathbf{v}|_{\mathbf{r}} \tilde{h}_i \sim (\frac{m_e}{m_i})^{1/4} (\frac{m_e}{m_i})^{1/4} \frac{en\tilde{\phi}}{T} \tag{24}$$

Ions at electron scales can be neglected to $O((m_e/m_i)^{1/2})$ in the electronscale equations!

note that:

- $\nabla_s \cdot \left\langle \frac{c}{B} \tilde{h}_i \tilde{\mathbf{v}}_{Ei} \right\rangle \sim O((m_e/m_i)^{3/4} \overline{\mathbf{v}}_{Ei} \cdot \overline{h}_i)$

Ions at electron scales can be neglected to $O((m_e/m_i)^{3/4})$ in the ion scale equations!

Scaling Work: which multiscale terms do we keep?

The only remaining multiscale terms are in electron species equations:

note that:

- $\tilde{\mathbf{v}}_{Ee} \cdot \nabla_s \overline{h}_e \sim \overline{\mathbf{v}}_{Ee} \cdot \nabla_f \tilde{h}_e \sim \tilde{\mathbf{v}}_{Ee} \cdot \nabla_f \tilde{h}_e$
- ion scale gradients contribute at $O(1)$ to the electron scale
- ion scale shear can be neglected to $O((m_e/m_i)^{1/2})$ at the electron scale

- $\nabla_s \cdot \left\langle \frac{c}{B} \tilde{h}_e \tilde{\mathbf{v}}_{Ee} \right\rangle \sim O((m_e/m_i)^{1/2} \overline{\mathbf{v}}_{Ee} \cdot \overline{h}_e)$
- back reaction contributes at $O((m_e/m_i)^{1/2})$ to the electron equation at ion scales
- small but can be self consistently included

Scaling work: Heat Flux

Substituting for the gyro Bohm scalings of the potential we find that,

$$\frac{\tilde{Q}_i}{\overline{Q}_i} \sim (\frac{m_e}{m_i}), \quad \frac{\tilde{Q}_e}{\overline{Q}_e} \sim (\frac{m_e}{m_i})^{1/2}, \quad \overline{Q}_i \sim \overline{Q}_e. \tag{25}$$

$(*)$ The Parallel Boundary Condition

using the field line label $\alpha = \xi - q(\psi)\theta \simeq \xi - q_0\theta - q_0'\theta(\psi - \psi_0)$ the fluctuations take the form:

$$A(\theta, \alpha, \psi) = \sum_{n_0, \theta_0} A_{n_0, \theta_0}(\theta) e^{in_0((\alpha - \alpha_0) + q_0\theta_0 + q_0'\theta(\psi - \psi_0))} \tag{26}$$

the parallel boundary condition in these variables is, Beer et al. (1995) ,

$$A_{n_0, \theta_0 + 2\pi N}(\theta + 2\pi N) = A_{n_0, \theta_0}(\theta) \tag{27}$$

If we have parametric ionscale coordinate $(\overline{\alpha}, \overline{\psi})$ dependence then this boundary condition should become:

$$A_{n_0, \theta_0 + 2\pi N}(\theta + 2\pi N, \overline{\alpha}(\theta + 2\pi N), \overline{\psi}) = A_{n_0, \theta_0}(\theta, \overline{\alpha}(\theta), \overline{\psi}), \tag{28}$$

where $\alpha(\theta + 2\pi N) - \alpha(\theta) \simeq -q_0 2\pi N - q_0' 2\pi N(\psi - \psi_0)$

which would couple the electronscale flux tubes together!