Kinetic treatment of ions at the tokamak plasma-wall boundary

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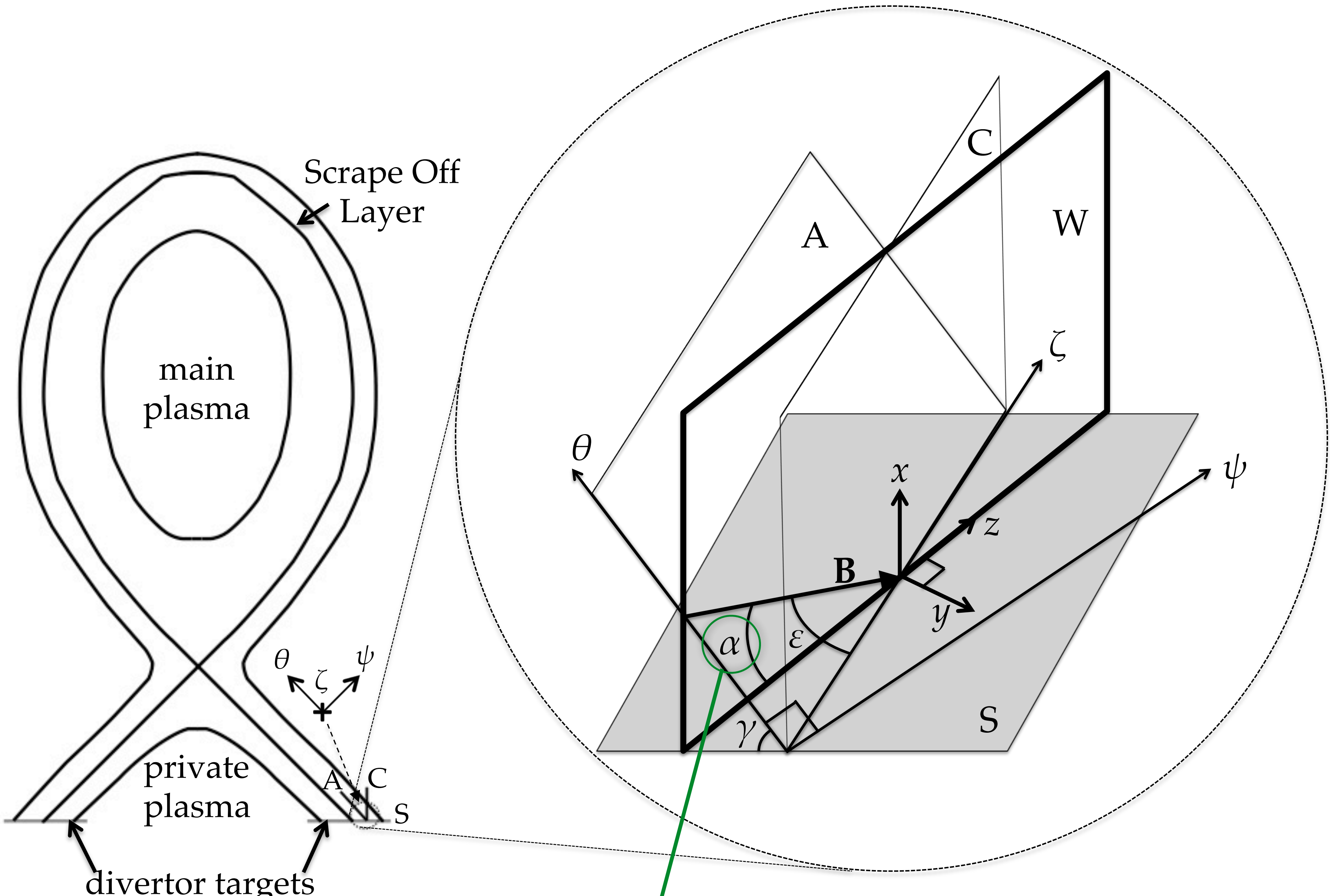
Motivation

- Boundary conditions needed for fluid codes used to simulate the SOL plasma
- Ones currently used [1] are obtained using fluid equations: aim is to obtain boundary conditions using a kinetic treatment
- Could be used with drift kinetic/gyrokinetic codes of SOL
- Interesting problem from a purely theoretical point of view: generalizing gyrokinetics to strongly distorted orbits in the magnetic presheath geometry

[1] J. Loizu, P. Ricci, F.D. Halpern and S. Jolliet, *Phys. Plasmas* **19**, 122307 (2012).

Geometry of divertor region



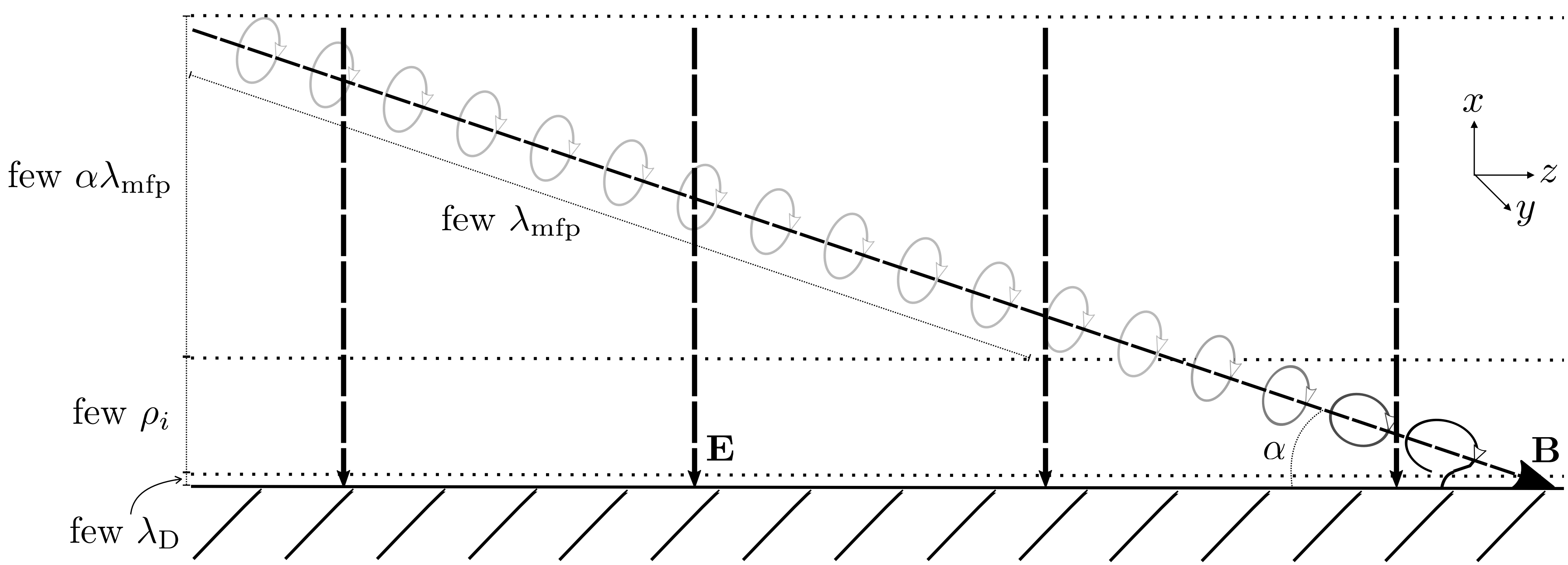
Assume $\alpha << 1$

Boundary layers

	Width	Estimate*
Collisional presheath	$\alpha\lambda_{\mathrm{mfp}}$	100 mm
Magnetic presheath [2]	ρ_{i}	0.7 mm
Debye sheath	λ_{D}	0.02 mm

$\Rightarrow \lambda_{\mathrm{D}} << \rho_{\mathrm{i}} << \alpha\lambda_{\mathrm{mfp}}$

*Estimates using data from [3]



[2] R Chodura, *Phys. Fluids* **25** (1982)

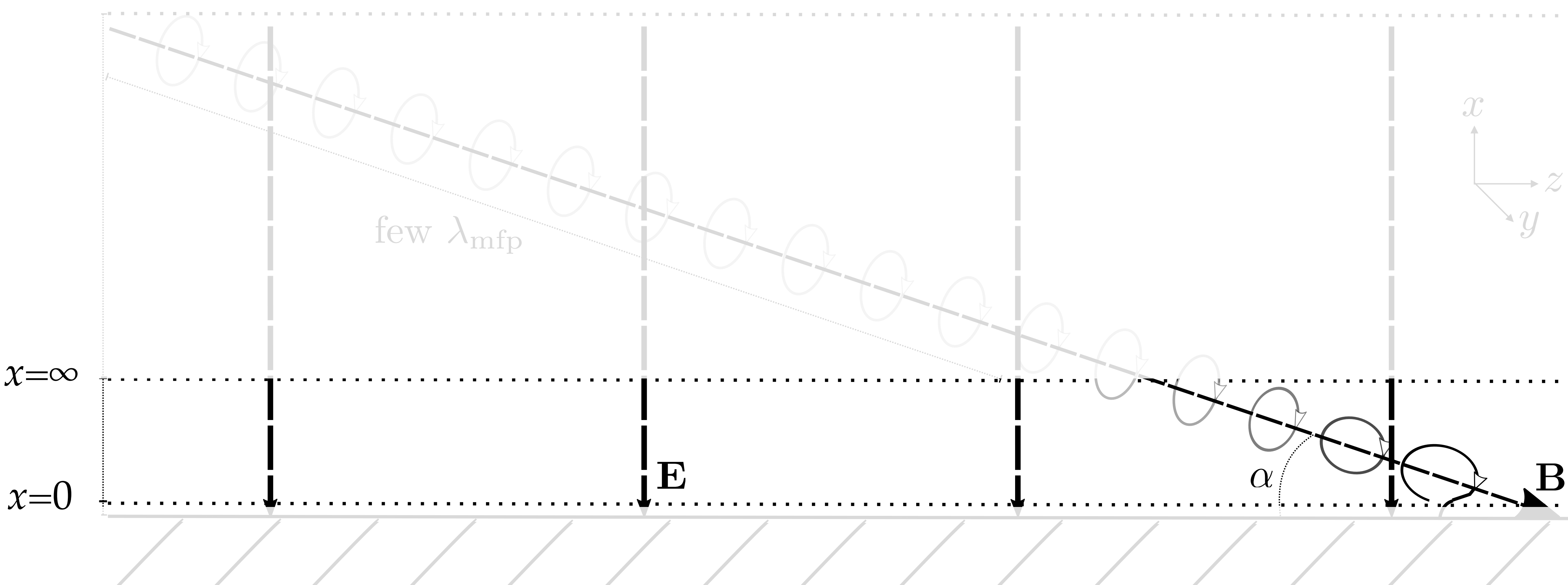
[3] F. Militello and W. Fundamenski, Plasma Phys. Control. Fusion **53**, 095002 (2011)

The magnetic presheath

- Focus on ion Larmor radius scale!
- **Quasineutral** $Zn_i(x) = n_e(x)$
- Use Boltzmann electrons

$$n_e(x) = n_\infty \exp\left(\frac{e\phi(x)}{T_e}\right)$$

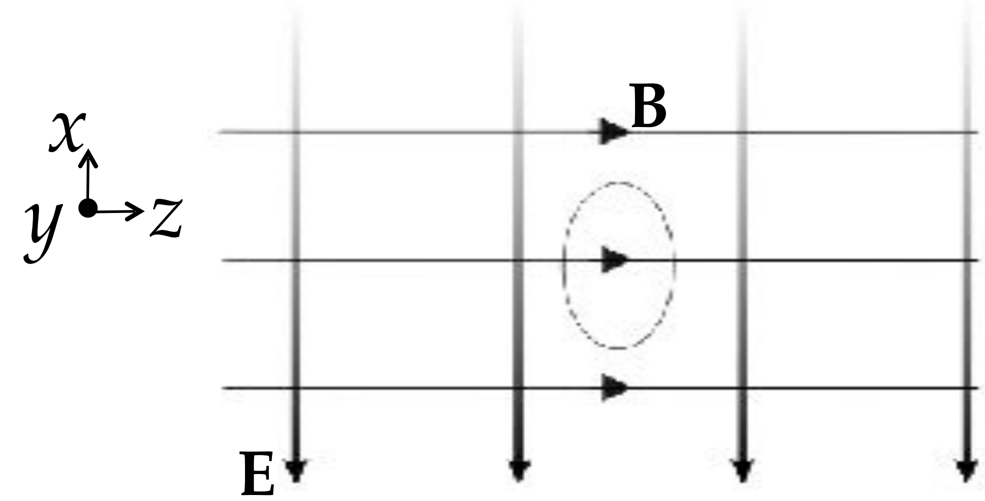
- Ion orbits heavily distorted by the strong presheath electric field
- Ion density obtained with kinetic treatment



Ion trajectories with α=0

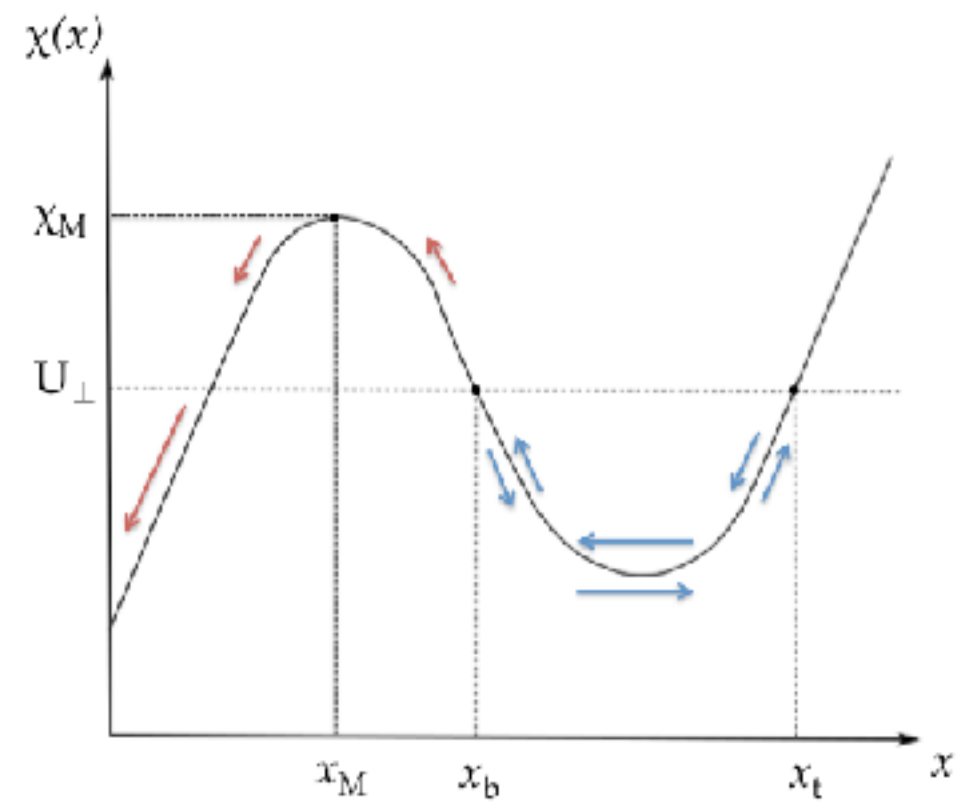
- Equations of motion:

$$\dot{v}_x = -\frac{Ze}{m_\mathrm{i}}\frac{d\phi(x)}{dx} + \Omega v_y$$
$$\dot{v}_y = -\Omega v_x$$
$$\dot{v}_z = 0$$



Orbit position	$\bar{x} = x + (1/\Omega)v_y$
Perpendicular energy	$U_\perp = ½v_x^2 + ½v_y^2 + Ze\phi/m_\mathrm{i}$
Total energy	$U = U_\perp + ½v_z^2$

$$v_x = \pm V_x(\bar{x}, U_\perp, x) = \pm\sqrt{2(U_\perp - \chi(x,\bar{x}))}$$
$$v_y = \Omega(\bar{x} - x)$$
$$v_z = V_\parallel(U_\perp, U) = \sqrt{2(U - U_\perp)}$$

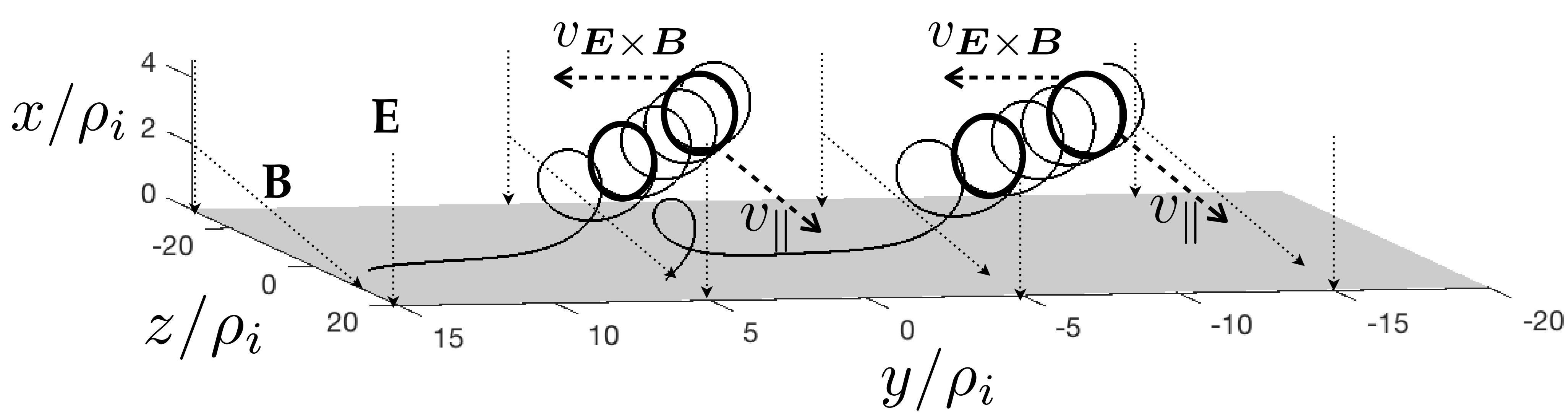


- Motion periodic if particle is trapped around a minimum of the **effective potential**

$$\chi(x,\bar{x}) = \frac{1}{2}\Omega^2(x-\bar{x})^2 + \frac{Ze\phi(x)}{m_i}$$

Ion trajectories in system with α<<1

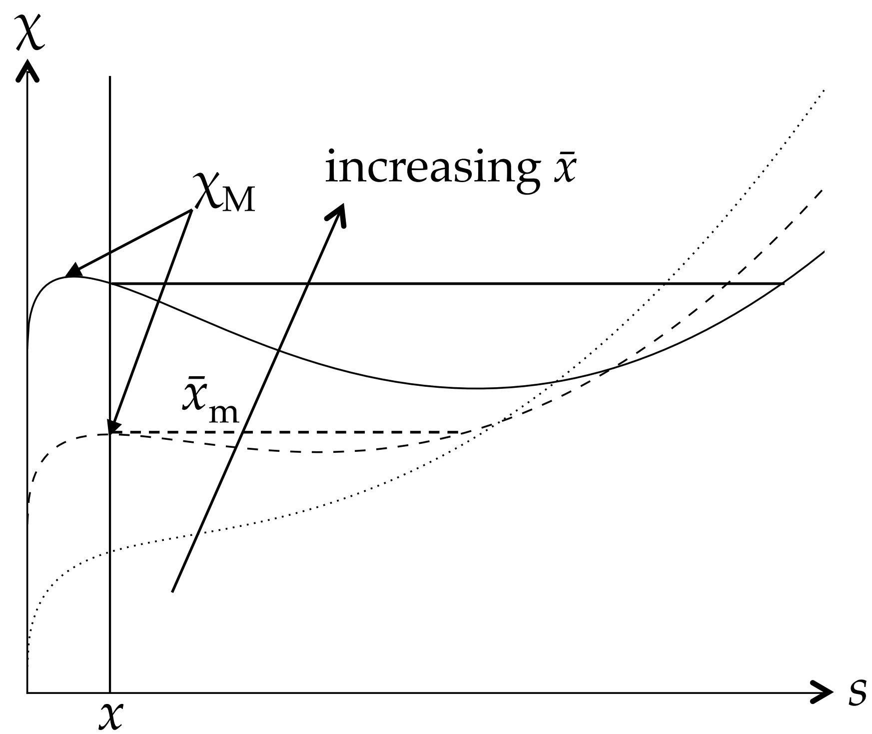
- Orbit parameters $\bar{x}$ and $U_\perp$ slowly varying: $\dot{\bar{x}} \simeq -\alpha v_z$
- Total energy U conserved $\dot{U}_\perp \simeq -\alpha \Omega v_y v_z$
- Over small time intervals have ~ **closed orbits** (rings below)
- α << 1 => adiabatic invariant: $\mu(\bar{x}, U_\perp) = \frac{1}{\pi}\int_{x_b}^{x_t} \sqrt{2(U_\perp - \chi(s,\bar{x}))}ds$
- Ion trajectories **conserve μ and U** to lowest order across magnetic presheath [4]

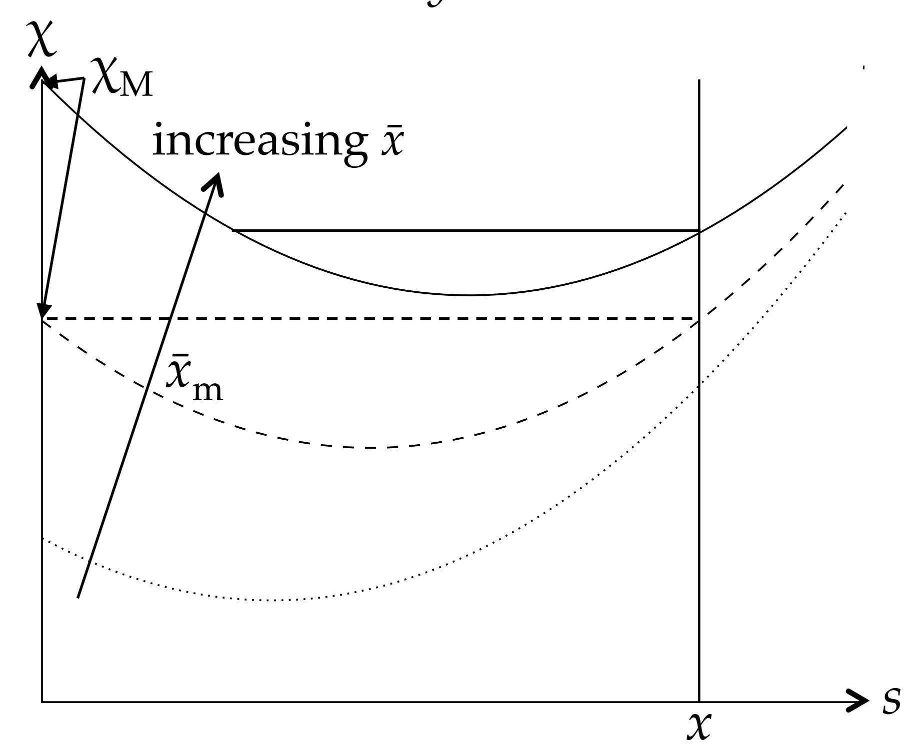


[4] R.H. Cohen and D.D. Ryutov, *Phys. Plasmas* **5**, 808 (1998)

Closed orbit ion density

- Distribution function F constant when written in terms of μ and U
- If **boundary condition** at x→∞ is F(μ, U), closed ion orbits have **distribution function F(μ, U) across the whole magnetic presheath**!
- Ion density = integral in velocity of F, holding particle position x fixed
- To compute it, change velocity space variables (v_x, v_y, v_z) → $(U_\perp, \bar{x}, U)$ [5]





$$n_{i,\mathrm{closed}}(x) = \int_{\bar{x}_m(x)}^{\infty} \underbrace{\Omega d\bar{x}}_{dv_y} \int_{\chi(x,\bar{x})}^{\chi_M(\bar{x})} \underbrace{\frac{2dU_\perp}{\sqrt{2\left(U_\perp - \chi\left(x,\bar{x}\right)\right)}}}_{dv_x} \int_{U_\perp}^{\infty} \underbrace{\frac{dU}{\sqrt{2\left(U - U_\perp\right)}}}_{dv_z} F\left(\mu\left(\bar{x}, U_\perp\right), U\right)$$

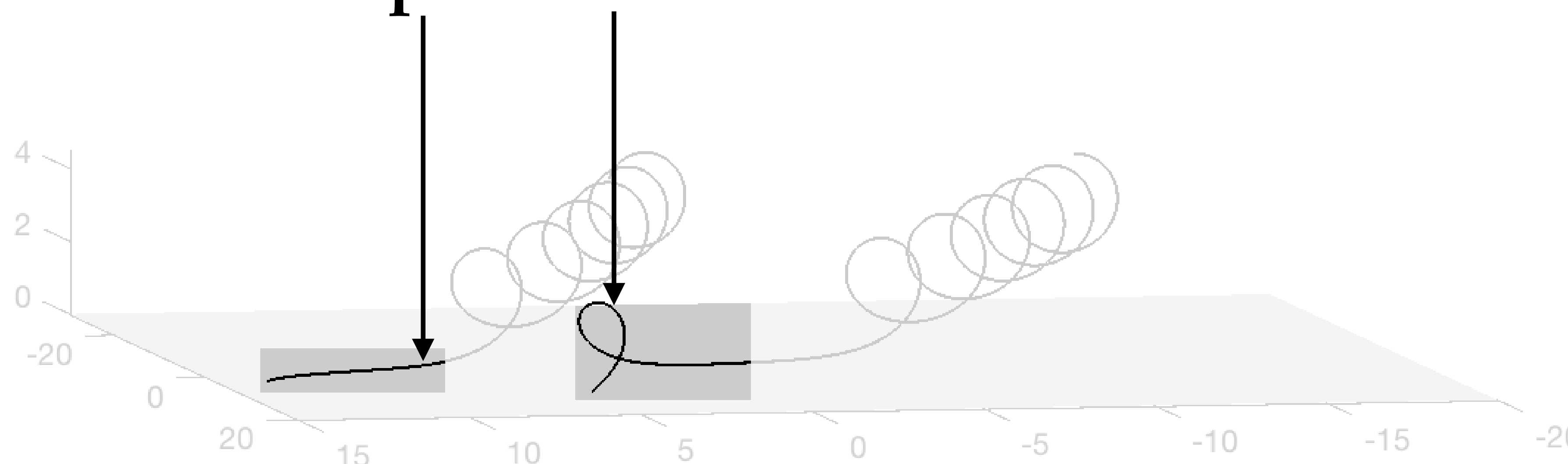
[5] A. Geraldini, F. I. Parra and F. Militello, *Plasma Phys. Control. Fusion* **59**, 025015 (2017)

Open orbit ion density

- **Closed orbit density is not enough** to solve for electrostatic potential in magnetic presheath

$$n_{i,\mathrm{closed}}\left(0\right)=0$$
$$\Longrightarrow 0=n_{\infty}\exp\left(\frac{e\phi_0}{T_e}\right)$$
$$\Longrightarrow \phi_0=-\infty$$

- Largeness of potential drop ϕ_0 <—> smallness of ion density at x=0
- Quantifying ϕ_0 requires keeping contribution to ion density at x=0 which comes from **open orbits**



- Calculated $n_{i,\mathrm{open}}(x)$ which has size $\sqrt{\alpha} \lesssim n_{i,\mathrm{open}}(x)/n_{\infty} \lesssim \alpha$
- With the open orbits included, recover known scaling for potential drop ϕ_0 across magnetic presheath $\dfrac{e\phi_0}{T_e}=\ln\left(\dfrac{n_{i,\mathrm{open}}\left(0\right)}{n_{\infty}}\right)\sim \ln\alpha$

Conditions on distribution function

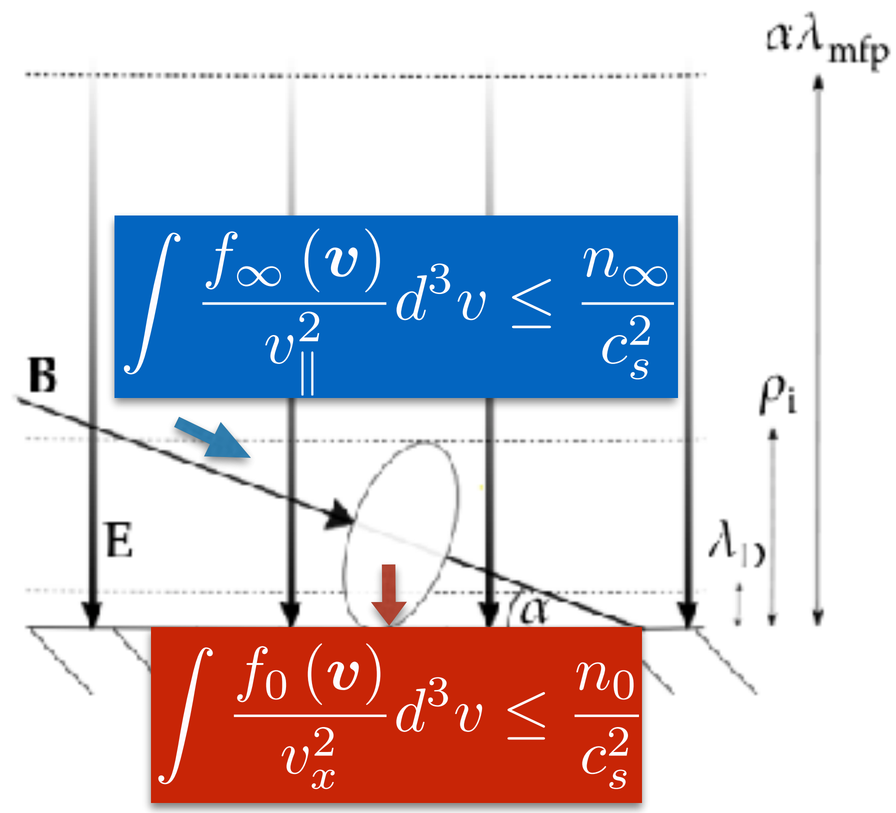
- Asymptotic analysis of boundary layers leads to **solvability conditions** at interface between different layers
- Define **sonic speed** $c_s=\sqrt{(T_e/m_i)}$
- We solve magnetic presheath numerically by using a distribution function that marginally satisfies Chodura condition at $x \rightarrow \infty$

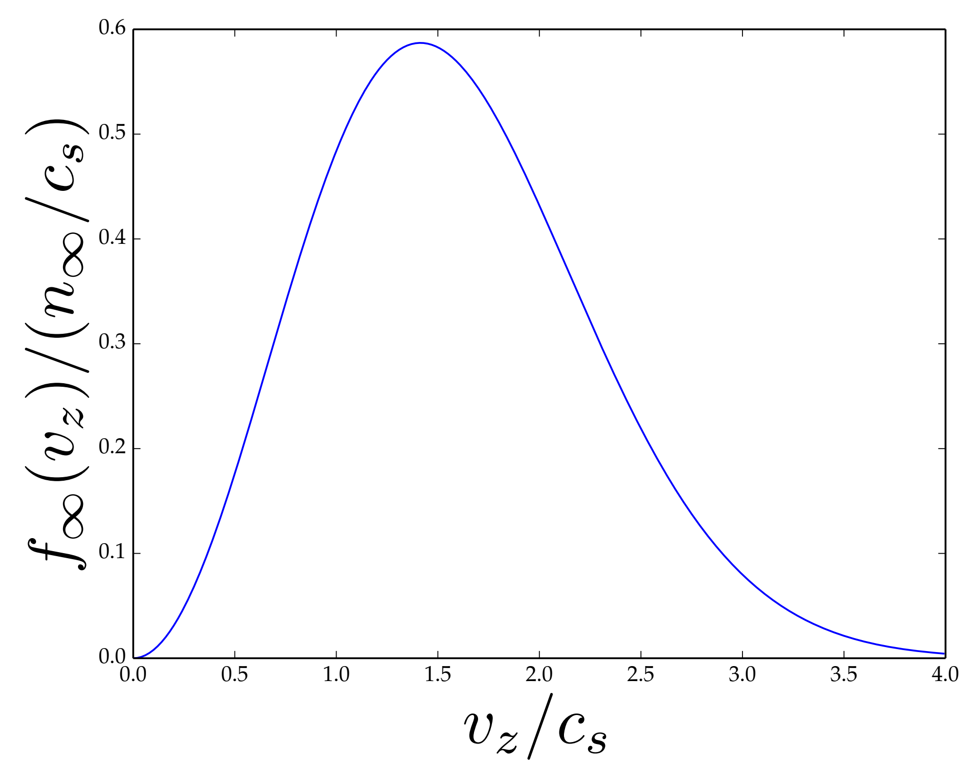
$$f_\infty(\boldsymbol{v}) \propto v_z^2 \exp\left(-\frac{m_i}{2T_i}\left(v_x^2+v_y^2+v_z^2\right)\right) \quad \text{with } T_i = T_e$$

$$F(\mu, U) \propto (U-\Omega\mu)\exp\left(-\frac{m_i U}{T_i}\right)$$

$$u_{\|\infty} = 2\sqrt{\frac{2}{\pi}}c_s \simeq 1.60 c_s$$

- Should find Bohm condition satisfied at x=0

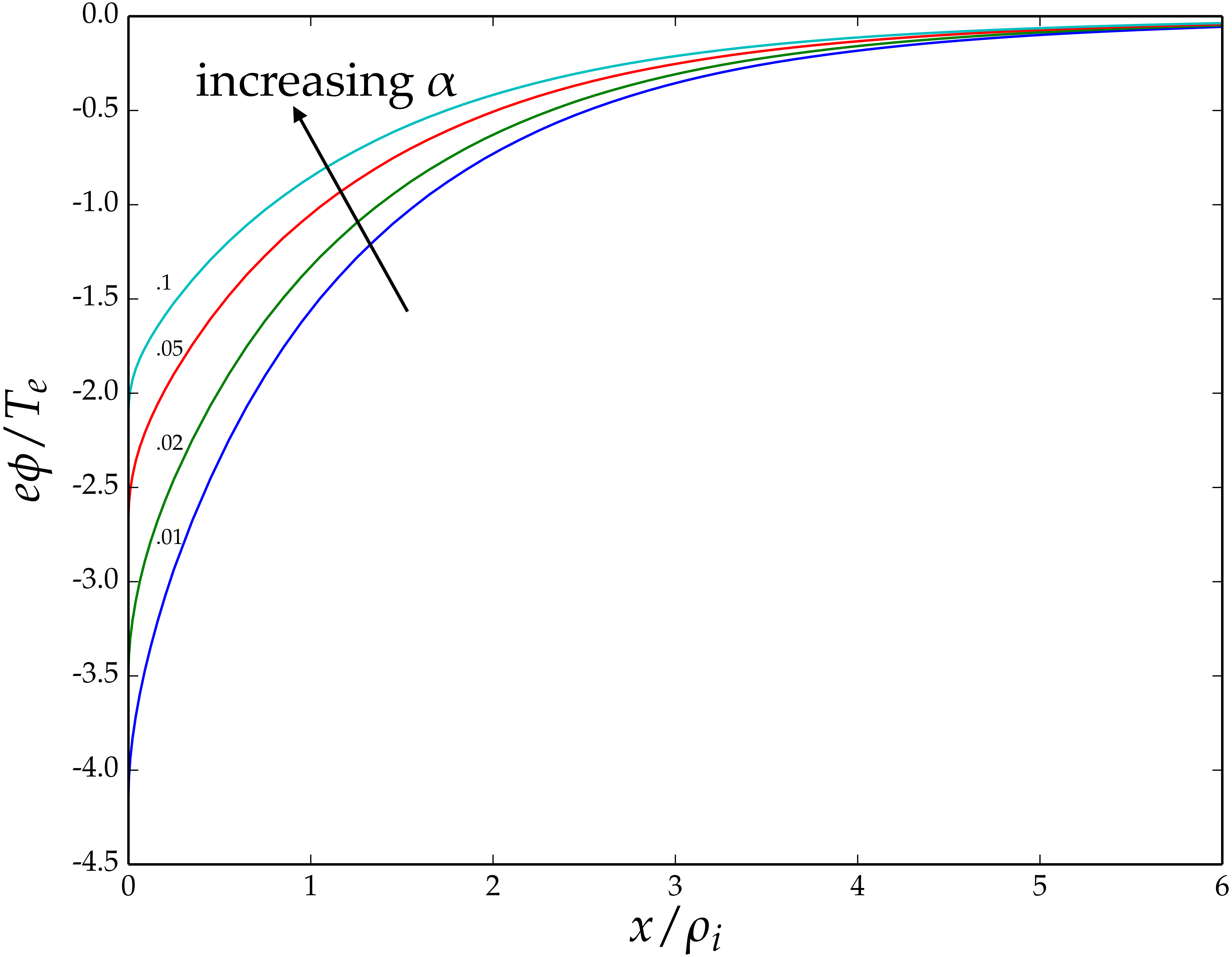




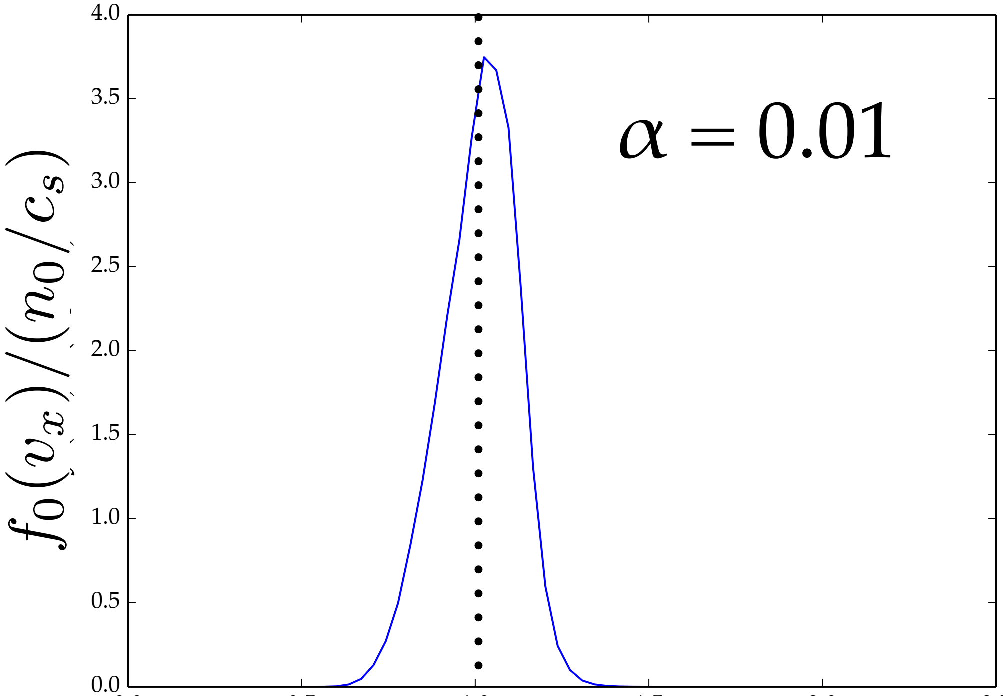
[6] K.-U. Riemann, J. Phys. D: Appl. Phys. **24**, 493-518 (1991)
[7] E. R. Harrison and W. B. Thompson, Proc. Phys. Soc. **74**, 145 (1959)

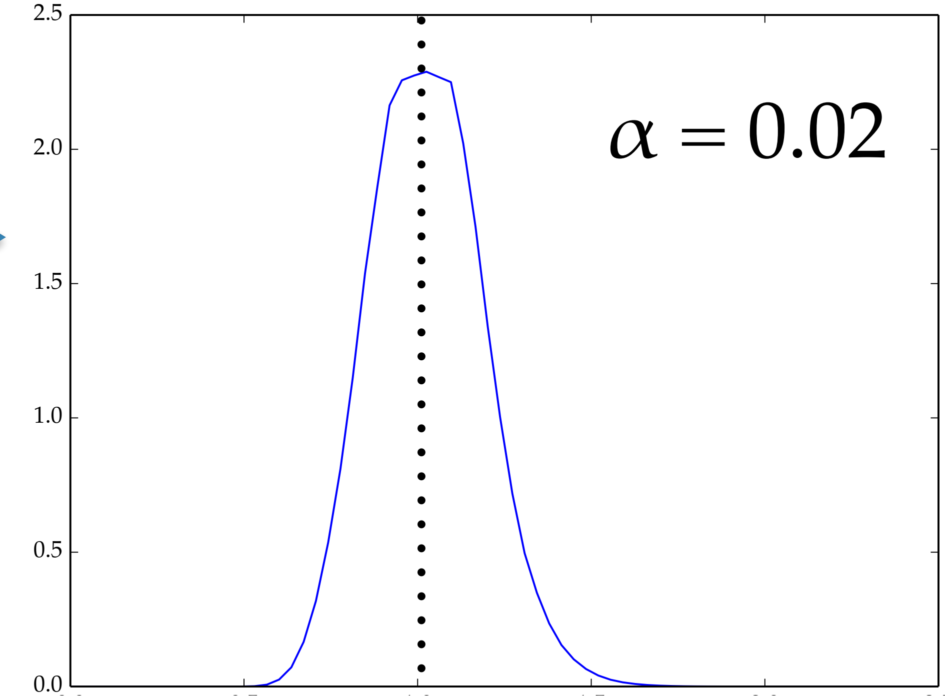
[8] A. Geraldini, F. I. Parra, F. Militello, *in preparation*

Numerical results: electrostatic potential



Distribution function at x=0

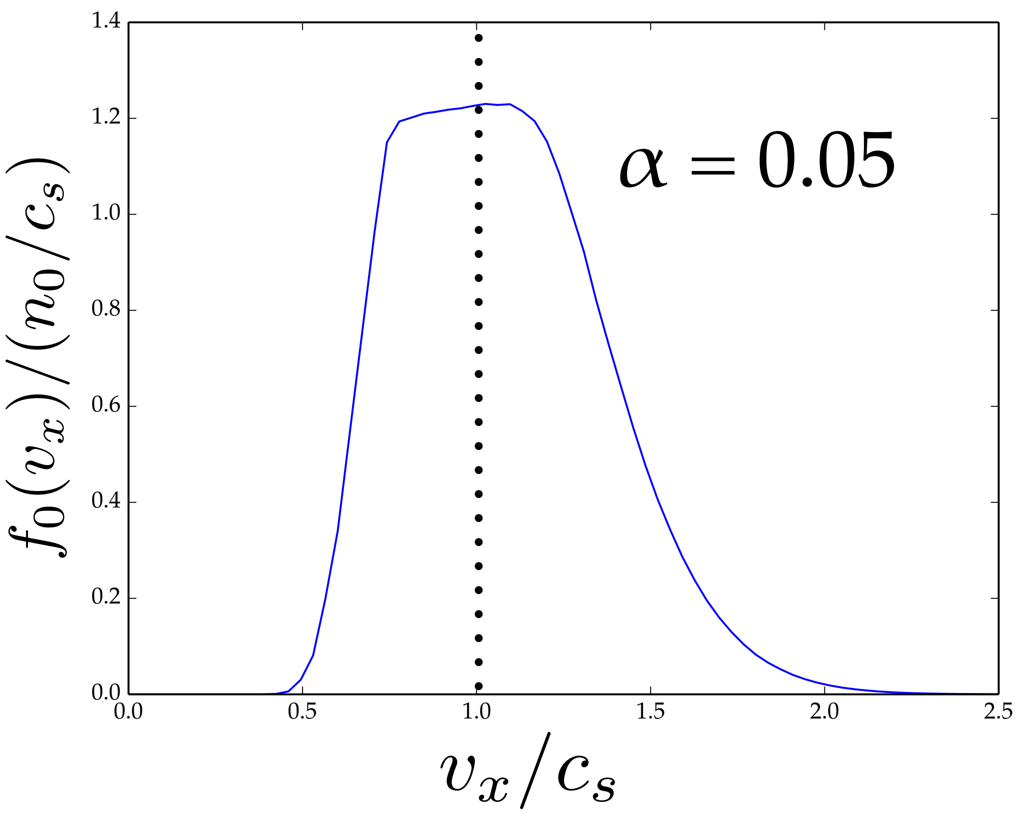


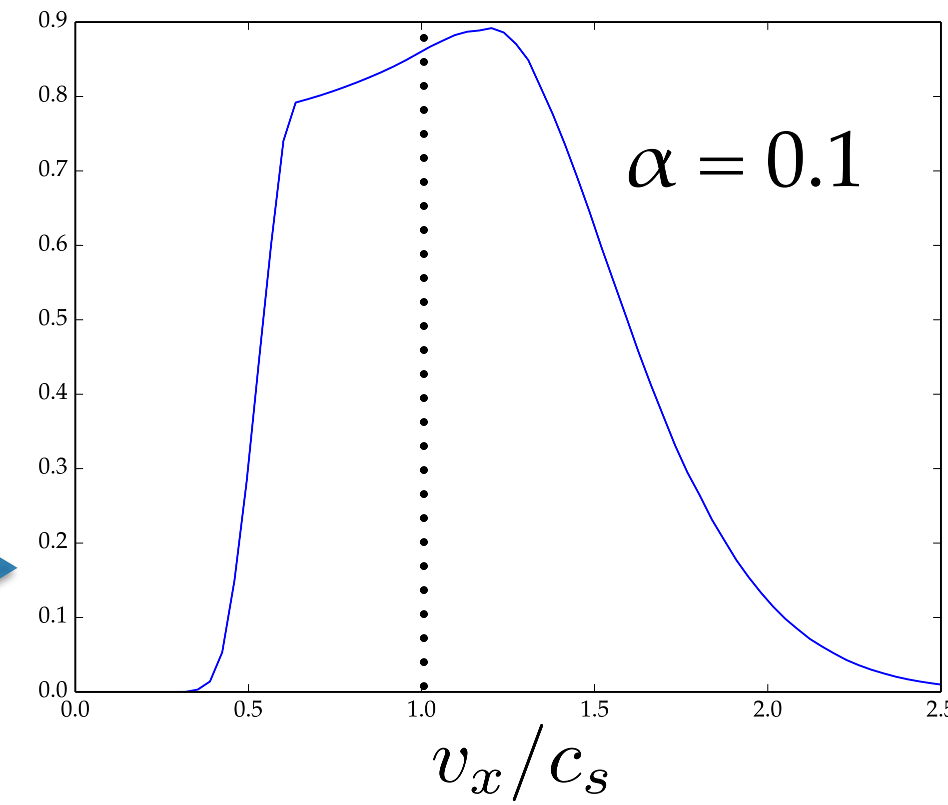


- Very thin and symmetric
- Centred at sonic speed

Marginally satisfy kinetic Bohm

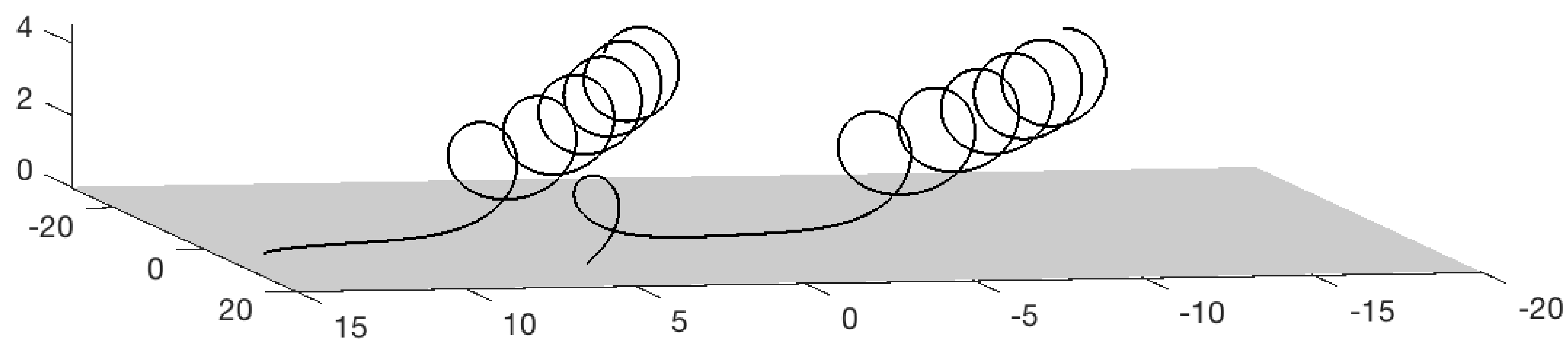
- Wider and asymmetric
- Average flow > sonic





Conclusion

- Exploited conservation of total energy and adiabatic invariant to solve for ion distribution function in magnetic presheath
- For a given potential profile, found expressions for lowest order density of ions throughout magnetic presheath including open orbits that matter near x=0
- Developed a code that computes ion density and iterates over potential until quasineutrality is solved (with Boltzmann electrons)
- Derived kinetic form of Chodura's condition
- Numerical results consistent with kinetic Bohm condition at Debye sheath
- Ions entering the Debye sheath are “colder” at smaller angles



<u>Future work:</u>

- Solve the magnetic presheath numerically for different distribution functions and T_e/T_i ratios
- Study collisional layer to find correct boundary distribution function at magnetic presheath entrance