Kinetic treatment of ions at the tokamak plasma-wall boundary

Alessandro Geraldini$^{1,2}$, Felix I. Parra$^{1,2}$, Fulvio Militello$^2$

1 Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford OX1 3NP, UK
2 CCFE, Culham Science Centre, Abingdon, OX14 3DB, UK
E-mail: ale.gerald@gmail.com

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Motivation

• Boundary conditions needed for fluid codes used to simulate the SOL plasma

• Ones currently used [1] are obtained using fluid equations: aim is to obtain boundary conditions using a kinetic treatment

• Could be used with drift kinetic/gyrokinetic codes of SOL

• Interesting problem from a purely theoretical point of view: generalizing gyrokinetics to strongly distorted orbits in the magnetic presheath geometry

Geometry of divertor region

Scrape Off Layer

main plasma

private plasma

divertor targets

Assume $\alpha \ll 1$
Boundary layers

<table>
<thead>
<tr>
<th></th>
<th>Width</th>
<th>Estimate*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collisional presheath</td>
<td>$\alpha \lambda_{\text{mfp}}$</td>
<td>100 mm</td>
</tr>
<tr>
<td>Magnetic presheath [2]</td>
<td>$\rho_i$</td>
<td>0.7 mm</td>
</tr>
<tr>
<td>Debye sheath</td>
<td>$\lambda_D$</td>
<td>0.02 mm</td>
</tr>
</tbody>
</table>

$\Rightarrow \lambda_D << \rho_i << \alpha \lambda_{\text{mfp}}$

*Estimates using data from [3]

The magnetic presheath

- Focus on ion Larmor radius scale!
- **Quasineutral** $Zn_i(x) = n_e(x)$
- Use Boltzmann electrons
  \[ n_e(x) = n_\infty \exp \left( \frac{e\phi(x)}{T_e} \right) \]
- Ion orbits heavily distorted by the strong presheath electric field
- Ion density obtained with kinetic treatment
Ion trajectories with $\alpha=0$

- Equations of motion:
  \[
  \begin{align*}
  \dot{x} &= -\frac{Ze \, d\phi(x)}{m_i} \frac{dx}{dx} + \Omega v_y \\
  \dot{y} &= -\Omega v_x \\
  \dot{z} &= 0
  \end{align*}
  \]

  
  Orbit position: \( \ddot{x} = x + (1/\Omega)v_y \)

  Perpendicular energy: \( U_\perp = \frac{1}{2}v_x^2 + \frac{1}{2}v_y^2 + \frac{Ze \phi}{m_i} \)

  Total energy: \( U = U_\perp + \frac{1}{2}v_z^2 \)

- Motion periodic if particle is trapped around a minimum of the effective potential

  \( \chi(x, \bar{x}) = \frac{1}{2} \Omega^2 (x - \bar{x})^2 + \frac{Ze \phi(x)}{m_i} \)

  \[
  \begin{align*}
  v_x &= \pm V_x (\bar{x}, U_\perp, x) = \pm \sqrt{2 \left( U_\perp - \chi(x, \bar{x}) \right)} \\
  v_y &= \Omega (\bar{x} - x) \\
  v_z &= V_\parallel (U_\perp, U) = \sqrt{2 \left( U - U_\perp \right)}
  \end{align*}
  \]
Ion trajectories in system with $\alpha << 1$

- Orbit parameters $\bar{x}$ and $U_\perp$ slowly varying: $\dot{\bar{x}} \approx -\alpha v_z$
- Total energy $U$ conserved $\dot{U}_\perp \approx -\alpha \Omega v_y v_z$
- Over small time intervals have $\sim$ closed orbits (rings below)
- $\alpha << 1 \Rightarrow$ adiabatic invariant: $\mu(\bar{x}, U_\perp) = \frac{1}{\pi} \int_{x_b}^{x_t} \sqrt{2(U_\perp - \chi(s, \bar{x}))} ds$
- Ion trajectories conserve $\mu$ and $U$ to lowest order across magnetic presheath [4]

Closed orbit ion density

- Distribution function $F$ constant when written in terms of $\mu$ and $U$
- If **boundary condition** at $x \to \infty$ is $F(\mu, U)$, closed ion orbits have distribution function $F(\mu, U)$ across the whole magnetic presheath!
- Ion density $= \text{integral in velocity of } F$, holding particle position $x$ fixed
- To compute it, change velocity space variables $(v_x, v_y, v_z) \to (U_\perp, \bar{x}, U)$ [5]

\[
\begin{align*}
n_{i,\text{closed}}(x) &= \int_{\bar{x}_m(x)}^{\infty} \Omega d\bar{x} \int_{\chi(x, \bar{x})}^{\chi_M(\bar{x})} \frac{2dU_\perp}{\sqrt{2(U_\perp - \chi(x, \bar{x}))}} \int_{U_\perp}^{\infty} \frac{dU}{\sqrt{2(U - U_\perp)}} F(\mu(\bar{x}, U_\perp), U)
\end{align*}
\]

Open orbit ion density

• Closed orbit density is not enough to solve for electrostatic potential in magnetic presheath

• Largeness of potential drop $\phi_0 \leftrightarrow$ smallness of ion density at $x=0$

• Quantifying $\phi_0$ requires keeping contribution to ion density at $x=0$ which comes from open orbits

\[
\begin{align*}
n_{i,\text{closed}}(0) &= 0 \\
\Rightarrow 0 &= n_\infty \exp \left( \frac{e\phi_0}{T_e} \right) \\
\Rightarrow \phi_0 &= -\infty
\end{align*}
\]

• Calculated $n_{i,\text{open}}(x)$ which has size $\sqrt{\alpha} \leq n_{i,\text{open}}(x)/n_\infty \leq \alpha$

• With the open orbits included, recover known scaling for potential drop $\phi_0$ across magnetic presheath

\[
\frac{e\phi_0}{T_e} = \ln \left( \frac{n_{i,\text{open}}(0)}{n_\infty} \right) \sim \ln \alpha
\]
Conditions on distribution function

- Asymptotic analysis of boundary layers leads to **solvability conditions** at interface between different layers.
- Define **sonic speed** \( c_s = \sqrt{T_e/m_i} \).
- We solve magnetic presheath numerically by using a distribution function that marginally satisfies **Chodura** condition at \( x \to \infty \):

\[
f_\infty(v) \propto v_z^2 \exp \left( -\frac{m_i}{2T_i} \left( v_x^2 + v_y^2 + v_z^2 \right) \right) \quad \text{with} \quad T_i = T_e
\]

\[
F(\mu, U) \propto (U - \Omega \mu) \exp \left( -\frac{m_i U}{T_i} \right)
\]

\[
u_{\|\infty} = 2\sqrt{\frac{2}{\pi}} c_s \simeq 1.60 c_s
\]

- Should find **Bohm** condition satisfied at \( x = 0 \).


Numerical results: electrostatic potential

Increasing $\alpha$
Distribution function at $x=0$

- $\alpha = 0.01$
  - Very thin and symmetric
  - Centred at sonic speed

- $\alpha = 0.02$
  - Marginally satisfy kinetic Bohm

- $\alpha = 0.05$
  - Wider and asymmetric

- $\alpha = 0.1$
  - Average flow $> \text{sonic}$
Conclusion

- Exploited conservation of total energy and adiabatic invariant to solve for ion distribution function in magnetic presheath
- For a given potential profile, found expressions for lowest order density of ions throughout magnetic presheath including open orbits that matter near $x=0$
- Developed a code that computes ion density and iterates over potential until quasineutrality is solved (with Boltzmann electrons)
- Derived kinetic form of Chodura's condition
- Numerical results consistent with kinetic Bohm condition at Debye sheath
- Ions entering the Debye sheath are “colder” at smaller angles

Future work:

- Solve the magnetic presheath numerically for different distribution functions and $T_e/T_i$ ratios
- Study collisional layer to find correct boundary distribution function at magnetic presheath entrance