Neoclassical calculation of the tangential electric field in stellarators close to omnigeneity (and tokamaks with broken symmetry)

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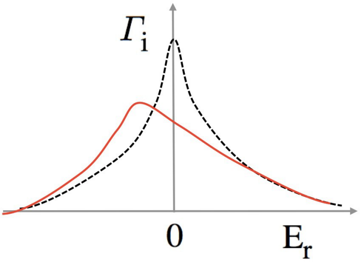
Why the interest in the tangential electric field?

- The **tangential electric field** produces a radial $E \times B$ drift that affects the radial flux of moderate- to high-Z impurities [García-Regaña, PPCF (2013)].
- It has been ignored for a long time in stellarator neoclassical calculations (sometimes this is justified, but not always). Recently, some codes have started to calculate it. Comparisons between EUTERPE and SFINCS in [García-Regaña, NF (2017)].

- The **tangential magnetic drift** has traditionally been ignored as well (again, sometimes this is justified, sometimes not). Global simulations with FORTEC-3D that include it, recently reported [Matsuoka, PoP (2015)], [Huang, PoP (2017)].
- **In situations in which the tangential magnetic drift counts, the behavior of the tangential electric field is particularly interesting.**



Coordinates on phase space

- Spatial coordinates $\{\psi, \alpha, l\}$, where
  - $\psi \in [0, \psi_{\max}]$ determines the magnetic surface and has dimensions of length,
  - $\alpha \in [0, 2\pi)$ is an angle that labels magnetic field lines on the surface,
  - $l \in [0, l_{\max}(\psi, \alpha))$ is the arc length along the field line.

  Then, the magnetic field can be expressed as[1]

$$\mathbf{B} = \Psi_t'(\psi)\nabla\psi \times \nabla\alpha.$$

- Velocity coordinates $\{\mathcal{E}, \mu, \sigma\}$, where
  - $\mathcal{E} = v^2/2 + Ze\varphi/m$ is the total energy per mass unit, $\varphi(\psi(\mathbf{x}), \alpha(\mathbf{x}), l(\mathbf{x}))$ is the electrostatic potential, Ze is the charge of the species, e is the proton charge and m is the mass.
  - μ is the magnetic moment,
  - $\sigma = v_{||}/|v_{||}|$, with $v_{||} = \sigma\sqrt{2(\mathcal{E} - \mu B - Ze\varphi/m)}$.

[1] Primes denote derivatives with respect to ψ.

Drift-kinetic and quasineutrality equations

- The equation for the ion[2] distribution function $F(\psi(\mathbf{x}), \alpha(\mathbf{x}), l(\mathbf{x}), \mathcal{E}, \mu, \sigma)$ is

$$\dot{\mathbf{x}} \cdot \nabla F = C_{ii}[F, F],$$

  with

$$\dot{\mathbf{x}} \cdot \nabla l = v_{||} \sim O(v_t),$$
$$\dot{\mathbf{x}} \cdot \nabla \psi = \mathbf{v}_d \cdot \nabla \psi \sim O(\rho_* v_t),$$
$$\dot{\mathbf{x}} \cdot \nabla \alpha = \mathbf{v}_d \cdot \nabla \alpha \sim O(\rho_* v_t R_0^{-1}).$$

  Here, $\mathbf{v}_d := \mathbf{v}_M + \mathbf{v}_E$ is the sum of the magnetic and $E \times B$ drifts, v_t is the thermal speed, $R_0 \sim |\nabla \ln B|^{-1}$ is a characteristic macroscopic length and $\rho_* = \rho / R_0$ is the normalized Larmor radius.

- Denoting by N_e the electron density, the quasineutrality equation reads

$$Z \int F \mathrm{d}^3 v = N_e.$$

- In a mass ratio expansion $\sqrt{m_e/m} \ll 1$, C_{ie} is negligible and only the adiabatic response of the electrons counts in the quasineutrality equation.

[2] We omit the subscript "i" for almost every ion quantity.

Drift-kinetic equation at low collisionality[3]: lowest-order

- Take a maximal ordering $\nu_* \sim \rho_* \ll 1$, where $\nu_* = \nu_{ii} R_0 / v_t$.
  - This includes the limits $\nu_* \ll \rho_* \ll 1$ and $\rho_* \ll \nu_* \ll 1$.

- Expand the distribution function in ρ_* as

$$F = F_0 + F_1 + \ldots,$$

  where $F_1 \sim \rho_* F_0$ is negligible in this talk.

- To lowest order in ρ_*, the drift-kinetic equation gives

$$v_{||} \partial_l F_0 = 0.$$

- F_0 is determined by averages over l of the drift-kinetic equation to next order in ρ_*.

[3] We do not include aspect ratio factors in the theoretical discussion. If the inverse aspect ratio ϵ is small, low collisionality means $\nu_* \ll \epsilon^{3/2}$.

Drift-kinetic equation at low collisionality: next order

Passing particles

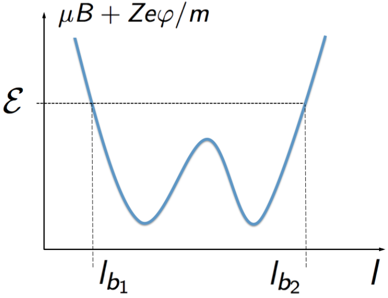
$$\int_0^{2\pi} \mathrm{d}\alpha \int_0^{l_{\max}(\psi,\alpha)} \frac{1}{|v_{||}|} C_{ii}[F_0, F_0]\mathrm{d}l = 0.$$

Trapped particles

$$-\partial_\psi J \partial_\alpha F_0 + \partial_\alpha J \partial_\psi F_0 = \sum_\sigma \frac{Ze\Psi_t'}{m} \int_{l_{b_1}}^{l_{b_2}} \frac{1}{|v_{||}|} C_{ii}[F_0, F_0]\mathrm{d}l,$$

conveniently expressed in terms of the second adiabatic invariant,

$$J(\psi, \alpha, \mathcal{E}, \mu) := 2\int_{l_{b_1}}^{l_{b_2}} |v_{||}|\mathrm{d}l.$$



Second adiabatic invariant and perpendicular drifts

$$2\int_{l_{b_1}}^{l_{b_2}} \frac{1}{|v_{||}|}\mathbf{v}_d \cdot \nabla\psi \, \mathrm{d}l = \frac{m}{Ze\Psi_t'}\partial_\alpha J; \qquad 2\int_{l_{b_1}}^{l_{b_2}} \frac{1}{|v_{||}|}\mathbf{v}_d \cdot \nabla\alpha \, \mathrm{d}l = -\frac{m}{Ze\Psi_t'}\partial_\psi J$$

- Without further assumptions, $\partial_\alpha J|\nabla\alpha| \sim \partial_\psi J|\nabla\psi|$, and the drift-kinetic equation for trapped particles,

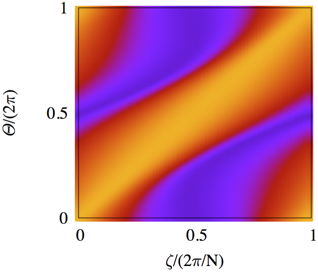
$$-\partial_\psi J\partial_\alpha F_0 + \partial_\alpha J\partial_\psi F_0 = \sum_\sigma \frac{Ze\Psi_t'}{m}\int_{l_{b_1}}^{l_{b_2}} \frac{1}{|v_{||}|} C_{ii}[F_0, F_0]\mathrm{d}l,$$

  is radially non-local.

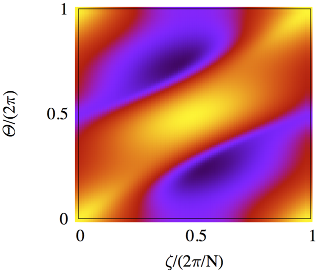
- If the aspect ratio and the radial electric field are sufficiently large, the tangential component of $\mathbf{v}_E$ dominates and the equation becomes radially non-local. But this is not the most general situation.

Omnigeneous stellarators[4]: $\partial_\alpha J = 0$

$\partial_\alpha J = 0$



$\partial_\alpha J \neq 0$



- In these figures we plot $B(\Theta, \zeta)$, where Θ and ζ are Boozer angles.

Expansions around omnigeneity [Calvo, PPCF (2017)]

- $\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}_1$, $0 \leq \delta \ll 1$, where $\mathbf{B}_0$ is omnigeneous.
- If $|\nabla B_1| / |\nabla B_0| \ll \delta^{-1}$, no new wells are created and a linear expansion holds,
  - $J(\psi, \alpha) = J^{(0)}(\psi) + \delta J^{(1)}(\psi, \alpha) + \ldots$.
- We always assume that B is stellarator-symmetric, $B(-\Theta, -\zeta) = B(\Theta, \zeta)$.

[4] [Cary, PoP (1997)], [Landreman, PoP (2012)], [Parra, NF (2015)]

Radially local, linear equations when $\delta \ll 1$

- $\varphi(\psi, \alpha, l) = \varphi_0(\psi) + \delta\varphi^{(1)}(\psi, \alpha, l) + \dots$
- Change velocity coordinates to $v = \sqrt{2(\mathcal{E} - Ze\varphi_0(\psi)/m)}$ and $\lambda = \mu[\mathcal{E} - Ze\varphi_0(\psi)/m]^{-1}$.
- $F_0 = F_M + \delta g^{(1)} + \dots$
  - F_M is a Maxwellian distribution constant on flux surfaces,

$$F_M(\psi, v) = n(\psi)\left(\frac{m}{2\pi T(\psi)}\right)^{3/2} \exp\left(-\frac{mv^2}{2T(\psi)}\right).$$

  - $g^{(1)}$ vanishes for passing trajectories, does not depend on l and can be chosen such that $\int_0^{2\pi} g^{(1)} \mathrm{d}\alpha = 0$.
- Keeping terms linear in δ, **the system of equations consisting of the drift-kinetic equation and the quasineutrality equation**
  - **is linear in $g^{(1)}$ and $\varphi^{(1)}$,**
  - **is radially local,**
  - **rigorously includes the tangential magnetic drift.**

Radially local, linear equations when $\delta \ll 1$

Drift-kinetic equation

$$-\partial_\psi J^{(0)} \partial_\alpha g^{(1)} + \partial_\alpha J^{(1)} \Upsilon F_M = \sum_\sigma \frac{Ze\Psi_t'}{m} \int_{l_{b10}}^{l_{b20}} \frac{\mathrm{d}l}{|v_{||}^{(0)}|} C_{ii}^{\ell(0)}[g^{(1)}],$$

Quasineutrality equation

$$\left(\frac{Z}{T} + \frac{1}{T_e}\right)\varphi^{(1)} = \frac{2\pi}{en}\int_0^\infty \mathrm{d}v \int_{B_{0,\max}^{-1}}^{B^{-1}} \mathrm{d}\lambda \frac{v^3 B_0}{|v_{||}^{(0)}|} g^{(1)},$$

with

$$\partial_\psi J^{(0)} = -\int_{l_{b10}}^{l_{b20}} \frac{\lambda v \partial_\psi B_0 + 2Ze/(mv)\varphi_0'}{\sqrt{1-\lambda B_0}} \mathrm{d}l,$$

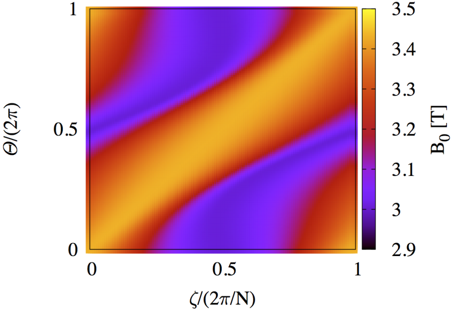
$$J^{(1)} = -\int_{l_{b10}}^{l_{b20}} \frac{\lambda v B_1 + 2Ze/(mv)\varphi^{(1)}}{\sqrt{1-\lambda B_0}} \mathrm{d}l,$$

$$\Upsilon = \frac{n'}{n} + \frac{T'}{T}\left(\frac{mv^2}{2T} - \frac{3}{2}\right) + \frac{Ze\varphi_0'}{T}.$$

- Superindices (0) mean that B is replaced by B_0 in the corresponding quantity.

KNOSOS (KiNetic Orbit-Averaging SOlver for Stellarators) code[5]

- It solves these equations with maximal ordering $\nu_* \sim \rho_*$.
  - For the moment, $C_{ii}^{\ell(0)}$ is a pitch-angle scattering operator.
- In what follows, **we discuss analytically the scalings and spatial structure of $\varphi^{(1)}$** in the $1/\nu$, $\sqrt{\nu}$ and superbanana-plateau collisionality regimes.
- **We complement the discussion with calculations by KNOSOS in an 'academic' stellarator close to omnigeneity.**



- For B_0, we use the field given in [Landreman, PoP (2012)]. Take average magnitude $B_{00} = 3.2$ T, $\iota = 1.05$, inverse aspect ratio $\epsilon = \psi/R_0 = 0.067$, $R_0 = 6$ m and number of periods $N = 4$.
- $B_1 \propto \cos(2\Theta)$.
- H plasma, $\rho_* = 5.6 \cdot 10^{-4}$, $T_i = 10$ keV.

[5] First calculations with KNOSOS presented in [Velasco, EPS Conference (2017)].

$1/\nu$ regime: $\rho_* \ll \nu_* \ll 1$

$$\partial_\psi J^{(0)} \partial_\alpha g^{(1)} \ll \sum_\sigma \frac{Ze\Psi_t'}{m} \int_{l_{b_{10}}}^{l_{b_{20}}} \frac{\mathrm{d}l}{|v_{||}^{(0)}|} C_{ii}^{\ell(0)}[g^{(1)}]$$

$$g^{(1)} \sim \frac{\rho_*}{\nu_*} F_M$$

$$\frac{Ze\varphi^{(1)}}{T} \sim \frac{\rho_*}{\nu_*}$$

$$J^{(1)} \approx J_B^{(1)} = -\int_{l_{b_{10}}}^{l_{b_{20}}} \frac{\lambda v B_1}{\sqrt{1-\lambda B_0}} \mathrm{d}l$$

The drift-kinetic equation does not contain $\varphi^{(1)}$,

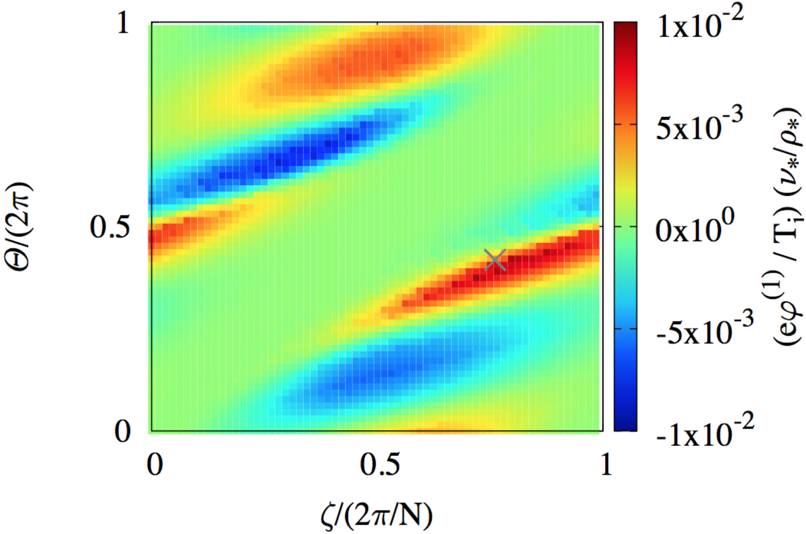
$$\sum_\sigma \frac{Ze\Psi_t'}{m} \int_{l_{b_{10}}}^{l_{b_{20}}} |v_{||}^{(0)}|^{-1} C_{ii}^{\ell(0)}[g^{(1)}]\mathrm{d}l = \partial_\alpha J_B^{(1)} \Upsilon F_M.$$

From the quasineutrality equation,

$$\varphi^{(1)} = \left(\frac{Z}{T} + \frac{1}{T_e}\right)^{-1} \frac{2\pi B_0}{en} \int_0^\infty \mathrm{d}v v^3 \int_{B_{0,\mathrm{max}}^{-1}}^{B^{-1}} \mathrm{d}\lambda |v_{||}^{(0)}|^{-1} g^{(1)}.$$

- $\varphi^{(1)}$ is stellarator antisymmetric.

$1/\nu$ regime: $\rho_* \ll \nu_* \ll 1$



- In this figure, $\nu_* \approx 10^{-2}$ and $\varphi_0' = -10$ kV/m.

$\nu_* \ll \rho_* \ll 1$: It depends on the zeroes of $\partial_\psi J^{(0)}$

- The zeroes of $\partial_\psi J^{(0)}$ correspond to points of phase space where the average tangential $E \times B$ and magnetic drifts cancel each other.
- The condition $\partial_\psi J^{(0)} = 0$ can be conveniently expressed as

$$\lambda \overline{\partial_\psi B_0}(\psi, \lambda) = -\frac{2Ze\varphi_0'(\psi)}{mv^2},$$

where

$$\overline{(\cdot)} = \frac{1}{\tau_b^{(0)}} \sum_\sigma \int_{l_{b_{10}}}^{l_{b_{20}}} (\cdot)|v_{||}^{(0)}|^{-1} \mathrm{d}l, \qquad \tau_b^{(0)}(\psi, v, \lambda) = 2\int_{l_{b_{10}}}^{l_{b_{20}}} |v_{||}^{(0)}|^{-1} \mathrm{d}l.$$

- A necessary condition for this equation to have solutions for $v \lesssim v_t$ is

$$\frac{Ze\varphi_0'}{T} \lesssim \frac{1}{R_0}.$$

- **Regarding transport:**
  - $\nu_* \ll \rho_*$ and $\partial_\psi J^{(0)} \neq 0$ (at least for $v \lesssim v_t$): $\sqrt{\nu}$ regime.
  - $\nu_* \ll \rho_*$ and $\partial_\psi J^{(0)} = 0$ at some point in phase space (with $v \lesssim v_t$): superbanana-plateau regime.

$\nu_* \ll \rho_* \ll 1$ and $\partial_\psi J^{(0)}$ never vanishes

$$\sum_\sigma \frac{Ze\Psi_t'}{m}\int_{l_{b_{10}}}^{l_{b_{20}}} |v_{||}^{(0)}|^{-1} C_{ii}^{\ell(0)}[g^{(1)}]\mathrm{d}l \ll \partial_\psi J^{(0)}\partial_\alpha g^{(1)}$$

Solution of the drift-kinetic equation to lowest order in ν_*/ρ_*, $g^{(1)} = g_0 + \ldots$,

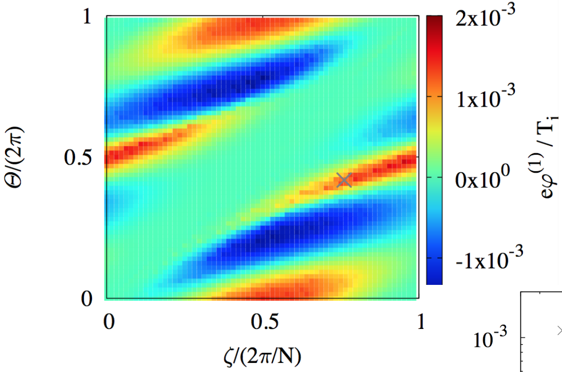
$$g_0 = \frac{1}{\partial_\psi J^{(0)}}\left(J^{(1)} - \frac{1}{2\pi}\int_0^{2\pi} J^{(1)}\mathrm{d}\alpha\right)\Upsilon_i F_{i0}.$$

$\varphi^{(1)}$ is found from the quasineutrality equation, that takes the form

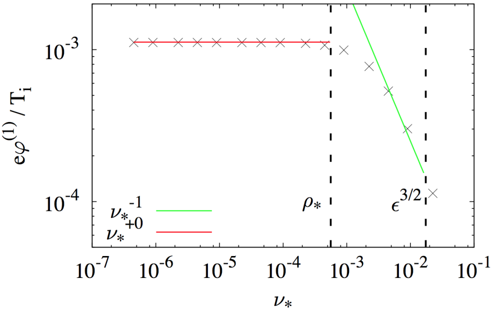
$$\left(\frac{Z}{T} + \frac{1}{T_e}\right)\varphi^{(1)} = \frac{2\pi B_0}{en}\int_0^\infty \mathrm{d}v\, v^3 \int_{B_{0,\max}^{-1}}^{B^{-1}} \mathrm{d}\lambda |v_{||}^{(0)}|^{-1} g_0.$$

- $Ze\varphi^{(1)}/T \sim (\nu_*)^0(\rho_*)^0$.
- $\varphi^{(1)}$ is stellarator symmetric.
- This is the way to calculate $\varphi^{(1)}$ in a plasma in the $\sqrt{\nu}$ regime. g_0 does not give radial transport (this is produced by a layer of size $\Delta_\lambda^{\sqrt{\nu}} \sim (\nu_*/\rho_*)^{1/2} \ll 1$ in the coordinate λ), but however determines $\varphi^{(1)}$.

$\nu_* \ll \rho_* \ll 1$ and $\partial_\psi J^{(0)}$ never vanishes



- $\nu_* \approx 2.2 \cdot 10^{-4}$
- $\varphi_0' = -10$ kV/m



$\nu_* \ll \rho_* \ll 1$ and $\partial_\psi J^{(0)} = 0$ for some value of λ

- Assume that $\partial_\psi J^{(0)}(\psi, v, \lambda_r) = 0$ has one solution, $\lambda_r(\psi, v)$.
- Expand the drift-kinetic equation around the position of the resonance,

$$\partial_\lambda \partial_\psi J_r^{(0)}(\lambda - \lambda_r)\partial_\alpha g_{\mathrm{rl}} + \nu_\lambda k \partial_\lambda^2 g_{\mathrm{rl}} = \left(\partial_\alpha J_{B,r}^{(1)} + \partial_\alpha \widehat{J_\varphi^{(1)}} \right) \Upsilon F_M,$$

where $k(\psi, v) = O(B_0^{-2} R_0 \rho_*^{-1})$, subindices r indicate that the corresponding quantity is evaluated at $\lambda = \lambda_r(\psi, v)$ and $\widehat{J_\varphi^{(1)}}$ is an approximation around λ_r of

$$J_\varphi^{(1)} = -\frac{2Ze}{mv} \int_{l_{b_{10}}}^{l_{b_{20}}} \frac{\varphi^{(1)}}{\sqrt{1 - \lambda B_0}} \mathrm{d}l.$$

- g_{rl} is localized in a layer $B_0 \Delta_\lambda^{\mathrm{sb-p}} \sim (\nu_*/\rho_*)^{1/3} \ll 1$ and $g_{\mathrm{rl}} \sim (B_0 \Delta_\lambda^{\mathrm{sb-p}})^{-1} F_M$.
- This layer is responsible for superbanana-plateau transport.
- g_{rl} does not have definite parity with respect to stellarator symmetry transformations.

Quasineutrality equation when $\nu_* \ll \rho_* \ll 1$ and $\partial_\psi J^{(0)} = 0$ for some value of λ

In principle, the two pieces

$$g_0^{\rm out} = g_0 - \frac{\Upsilon F_M}{(\lambda - \lambda_r)\partial_\lambda \partial_\psi J_r^{(0)}} \left(J_{B,r}^{(1)} - \frac{1}{2\pi}\int_0^{2\pi} J_{B,r}^{(1)} \mathrm{d}\alpha + \widehat{J_\varphi^{(1)}} - \frac{1}{2\pi}\int_0^{2\pi} \widehat{J_\varphi^{(1)}} \mathrm{d}\alpha \right)$$

and $g_{\rm rl}$ contribute to the quasineutrality equation on an equal footing.

$$\left(\frac{Z}{T} + \frac{1}{T_e}\right)\varphi^{(1)} = \frac{2\pi B_0}{en}\left[\int_0^\infty \mathrm{d}v \int_{B_{0,\rm max}^{-1}}^{B^{-1}} \mathrm{d}\lambda \frac{v^3 g_0^{\rm out}}{\left|v_{||}^{(0)}\right|} + \int_{v_{\rm min}}^\infty \mathrm{d}v \int_{\Delta_\lambda^{\rm sb-p}} \mathrm{d}\lambda \frac{v^3 g_{\rm rl}}{\left|v_{||}^{(0)}\right|}\right]$$

Two subcases:

- $2Z_i e[mv^2 \lambda_r \partial_\lambda \overline{\partial_\psi B_0}(\psi, \lambda_r)]^{-1}\varphi_0' \gg \Delta_\lambda^{\rm sb-p}$
- $2Z_i e[mv^2 \lambda_r \partial_\lambda \overline{\partial_\psi B_0}(\psi, \lambda_r)]^{-1}\varphi_0' \ll \Delta_\lambda^{\rm sb-p}$ **More interesting. And new.**

Quasineutrality equation when $\nu_* \ll \rho_* \ll 1$ and $\partial_\psi J^{(0)} = 0$ for some value of λ

$$
\left(\frac{Z}{T} + \frac{1}{T_e}\right)\varphi^{(1)} = \frac{2\pi}{en}\int_0^\infty \mathrm{d}v \int_{B_{0,\mathrm{max}}^{-1}}^{B^{-1}} \mathrm{d}\lambda \frac{v^3 B_0}{|v_{||}^{(0)}|} g_0^{\mathrm{out}}
$$

$$
+\frac{2\pi B_0}{en}\int_{v_{\mathrm{min}}}^\infty \mathrm{d}v\, v^2 \int_{-\infty}^{\lambda_L(l)} \mathrm{d}\lambda \frac{g_{\mathrm{rl}}}{\sqrt{\lambda_r|\partial_l B_0(l_L)|(l - l_L) - (\lambda - \lambda_r)B_0(l_L)}}
$$

$$
+\frac{2\pi B_0}{en}\int_{v_{\mathrm{min}}}^\infty \mathrm{d}v\, v^2 \int_{-\infty}^{\lambda_R(l)} \mathrm{d}\lambda \frac{g_{\mathrm{rl}}}{\sqrt{\lambda_r|\partial_l B_0(l_R)|(l_R - l) - (\lambda - \lambda_r)B_0(l_R)}}
$$

$$
+\frac{2\pi B_0}{en}\int_{v_{\mathrm{min}}}^\infty \mathrm{d}v\, v^2 \left[\frac{1}{\sqrt{1 - \lambda_r B_0(l)}} - \frac{1}{\sqrt{\lambda_r|\partial_l B_0(l_L)|(l - l_L)}}\right.
$$

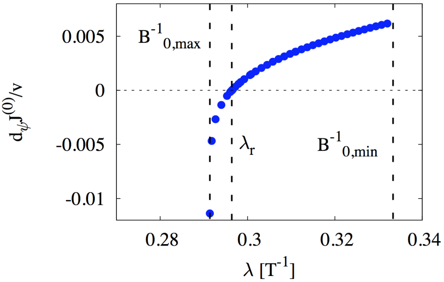
$$
\left. - \frac{1}{\sqrt{\lambda_r|\partial_l B_0(l_R)|(l_R - l)}}\right]\int_{-\infty}^{\infty} \mathrm{d}\lambda\, g_{\mathrm{rl}}.
$$

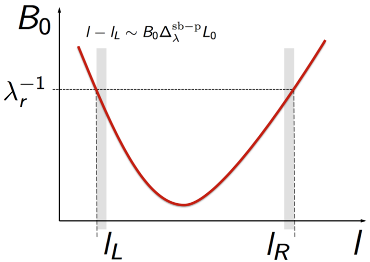
- Let l_L and l_R denote the bounce points of the trajectory $\lambda = \lambda_r$. The red term diverges at $l = l_L$ and $l = l_R$.

Scalings of $\varphi^{(1)}$ when $\nu_* \ll \rho_* \ll 1$ and $\partial_\psi J^{(0)} = 0$ for some value of λ

- If $2Z_i e[m_i v^2 \lambda_r \partial_\lambda \overline{\partial_\psi B_0}(\lambda_r)]^{-1}\varphi_0' \gg \Delta_\lambda^{\mathrm{sb-p}}$, the dependence of λ_r on v is strong enough for the integral over v to smooth out the divergence of $1/\sqrt{1-\lambda_r B_0(l)}$ at $l = l_L$ and $l = l_R$. Then,
  - $Ze\varphi^{(1)}/T \sim (\nu_*)^0(\rho_*)^0$.
- If $2Z_i e[m_i v^2 \lambda_r \partial_\lambda \overline{\partial_\psi B_0}(\lambda_r)]^{-1}\varphi_0' \ll \Delta_\lambda^{\mathrm{sb-p}}$, λ_r is approximately independent of v and no such smoothing happens. Then,
  - $Ze\varphi^{(1)}/T \sim (\nu_*/\rho_*)^{-1/6}$,

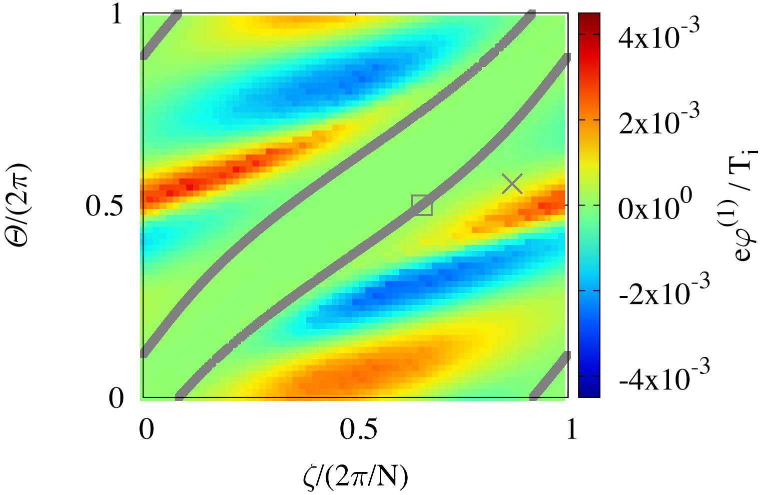
  when $|l - l_j| \sim B_0 \Delta_\lambda^{\mathrm{sb-p}} R_0$, $j = L, R$.





$\nu_* \ll \rho_* \ll 1$, $\partial_\psi J^{(0)} = 0$ for some value of λ and $\varphi_0' = 0$

Collisionality scan from $\nu_* \approx \rho_* = 5.6 \cdot 10^{-4}$ to $\nu_* \approx 5 \cdot 10^{-8}$.



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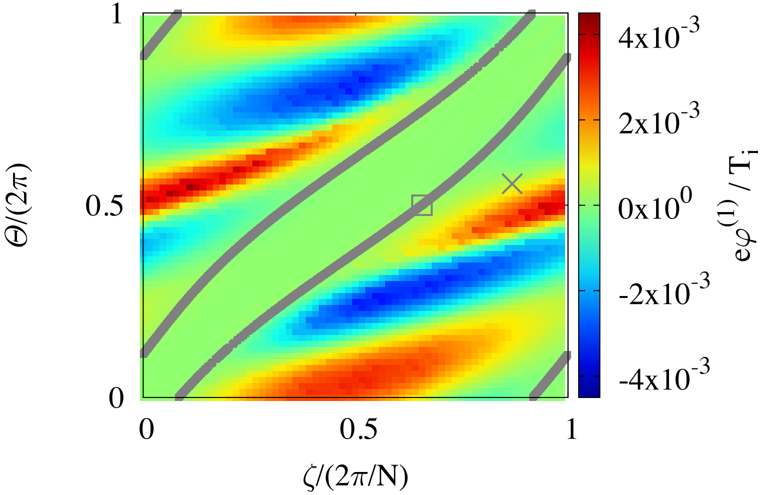

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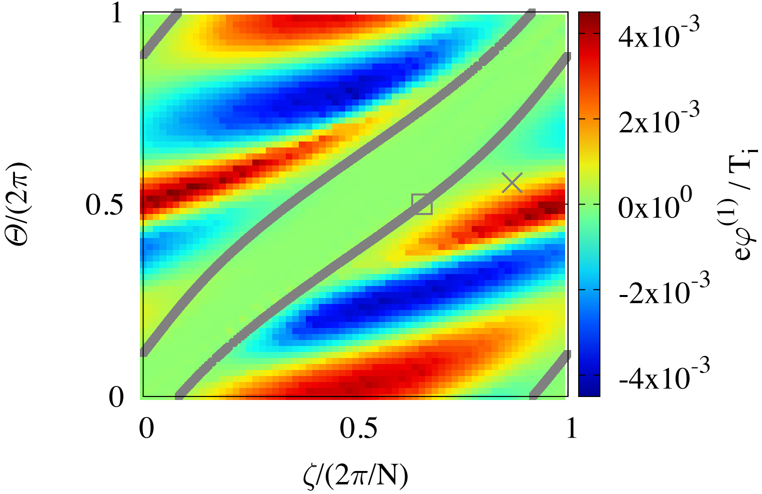

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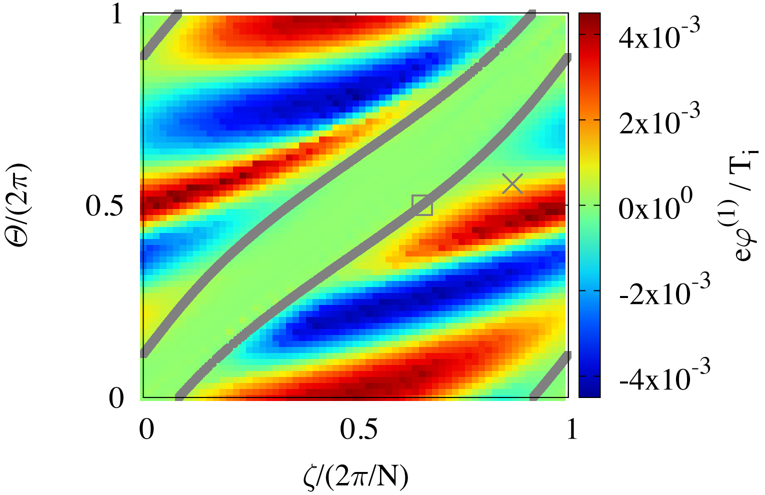
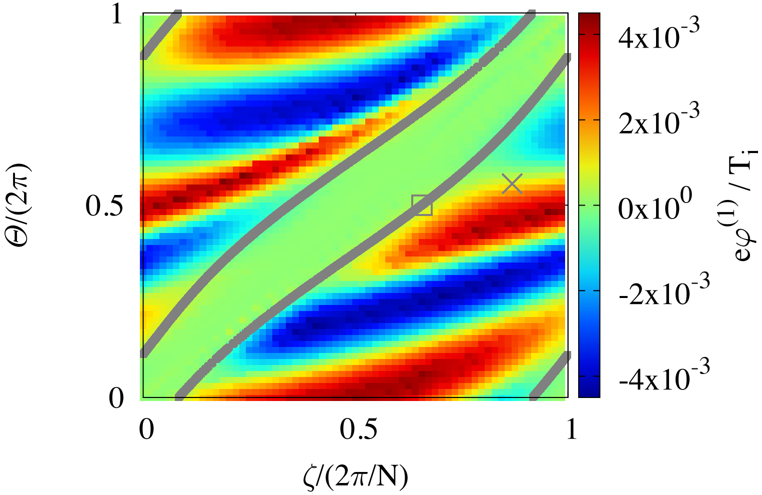


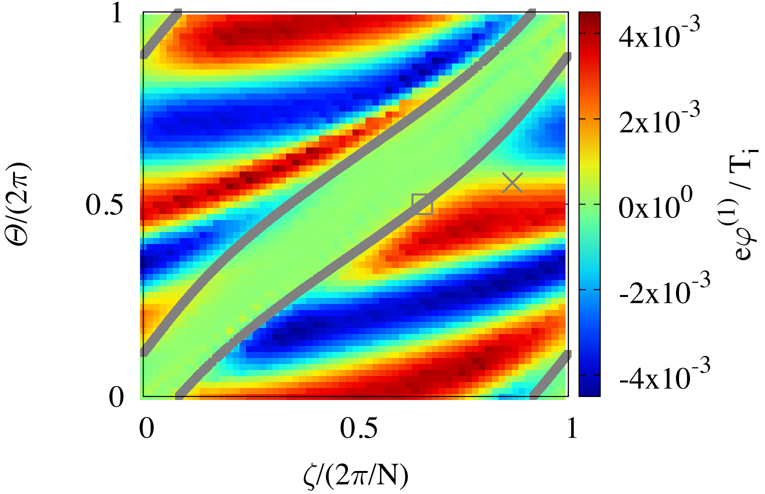
$\nu_* \ll \rho_* \ll 1$, $\partial_\psi J^{(0)} = 0$ for some value of λ and $\varphi_0' = 0$

Collisionality scan from $\nu_* \approx \rho_* = 5.6 \cdot 10^{-4}$ to $\nu_* \approx 5 \cdot 10^{-8}$.



$\nu_* \ll \rho_* \ll 1$, $\partial_\psi J^{(0)} = 0$ for some value of λ and $\varphi_0' = 0$

Collisionality scan from $\nu_* \approx \rho_* = 5.6 \cdot 10^{-4}$ to $\nu_* \approx 5 \cdot 10^{-8}$.



$\nu_* \ll \rho_* \ll 1$, $\partial_\psi J^{(0)} = 0$ for some value of λ and $\varphi_0' = 0$

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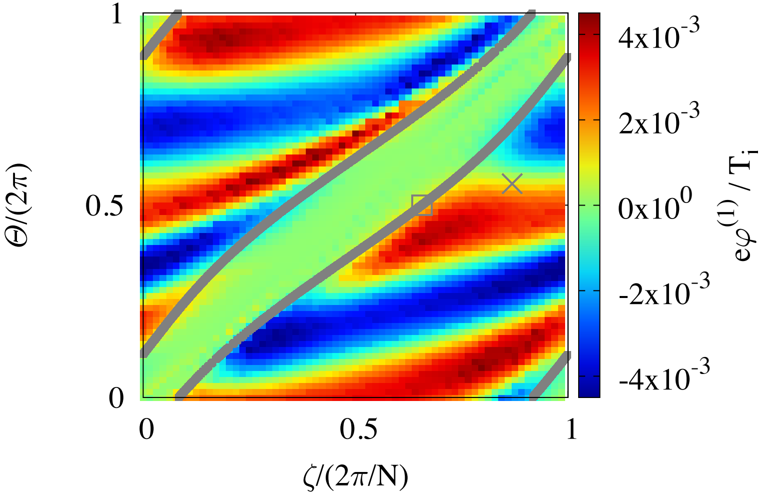

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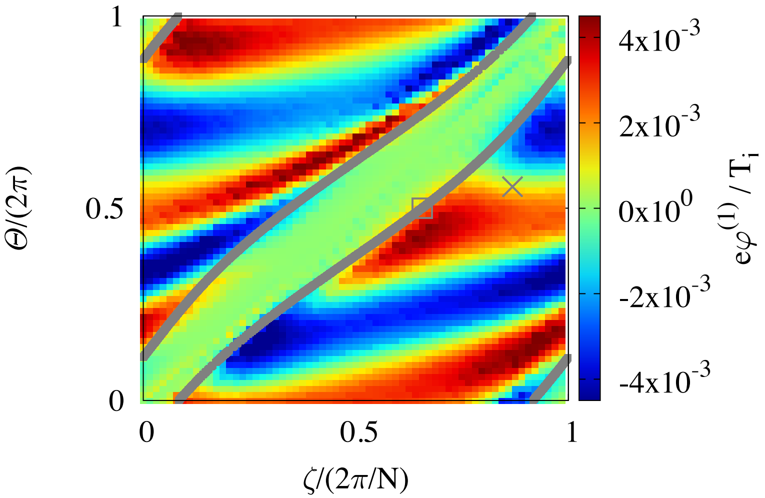

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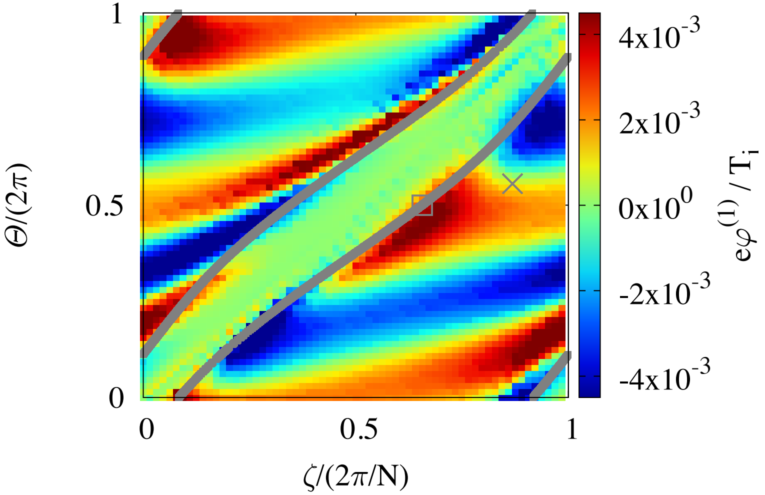

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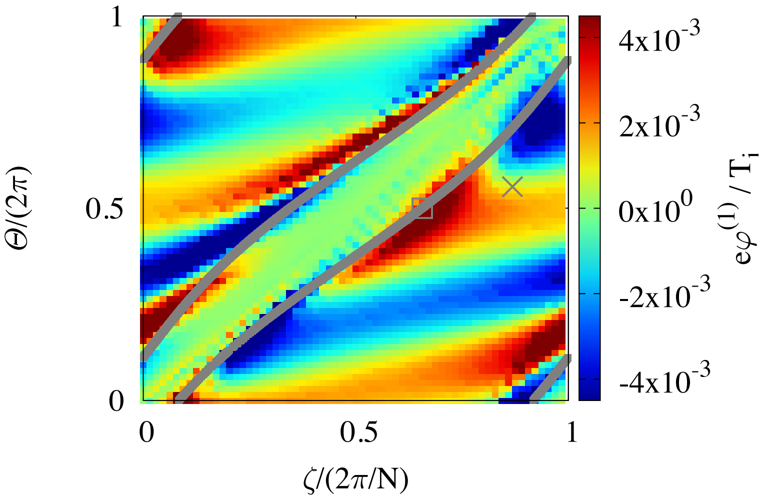

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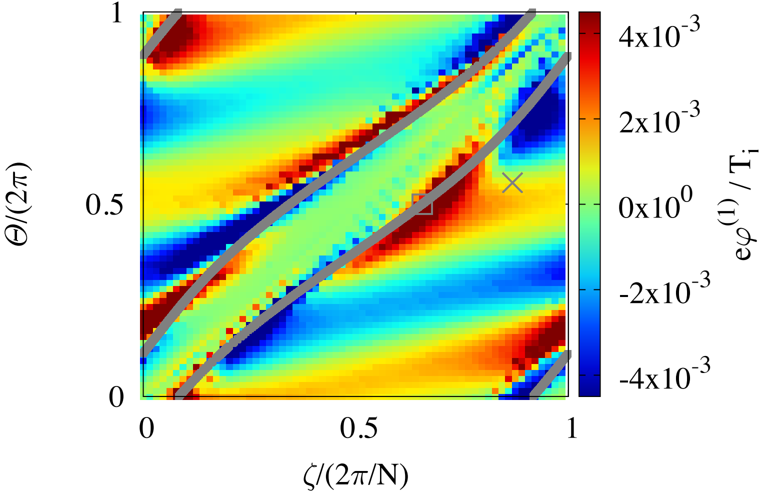

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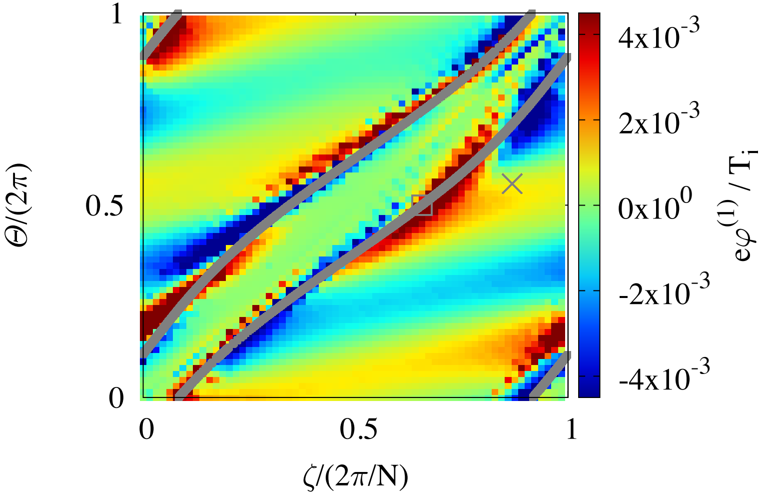

$\nu_* \ll \rho_* \ll 1$, $\partial_\psi J^{(0)} = 0$ for some value of λ and $\varphi_0' = 0$

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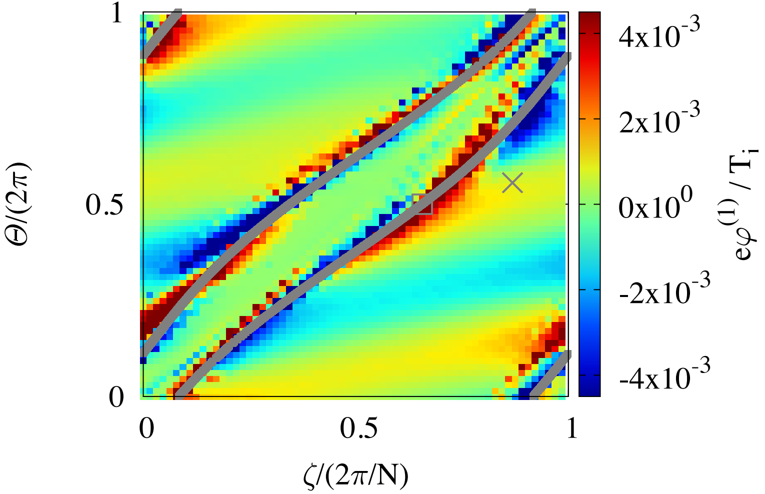

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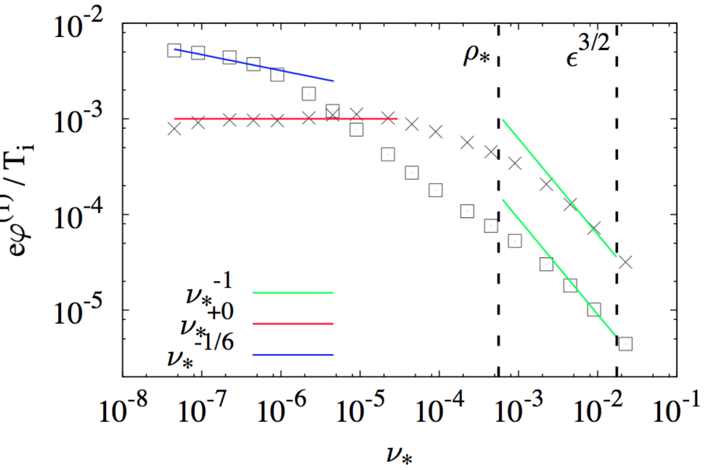

Future work

- Extend the theory and the code to treat more general deviations from omnigeneity.
- Evaluate the impact of $\varphi^{(1)}$ on the radial transport of main ions and impurities in regimes where the tangential magnetic drift is important.
- Analyze how all this depends on the aspect ratio of the stellarator.