AN ALFVÉN WAVE AMPLITUDE LIMIT
IN LOW-COLLISIONALITY PLASMAS

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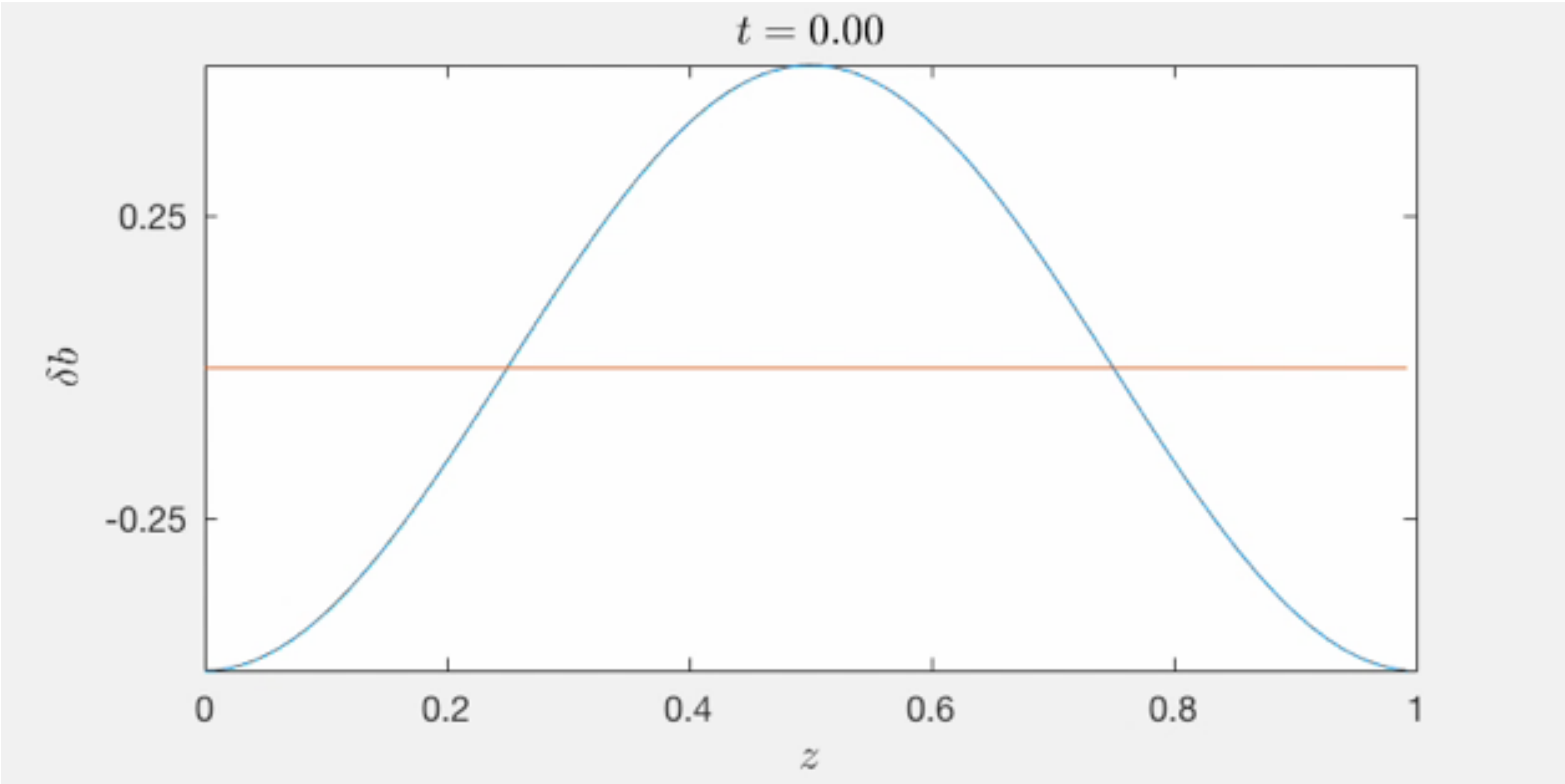
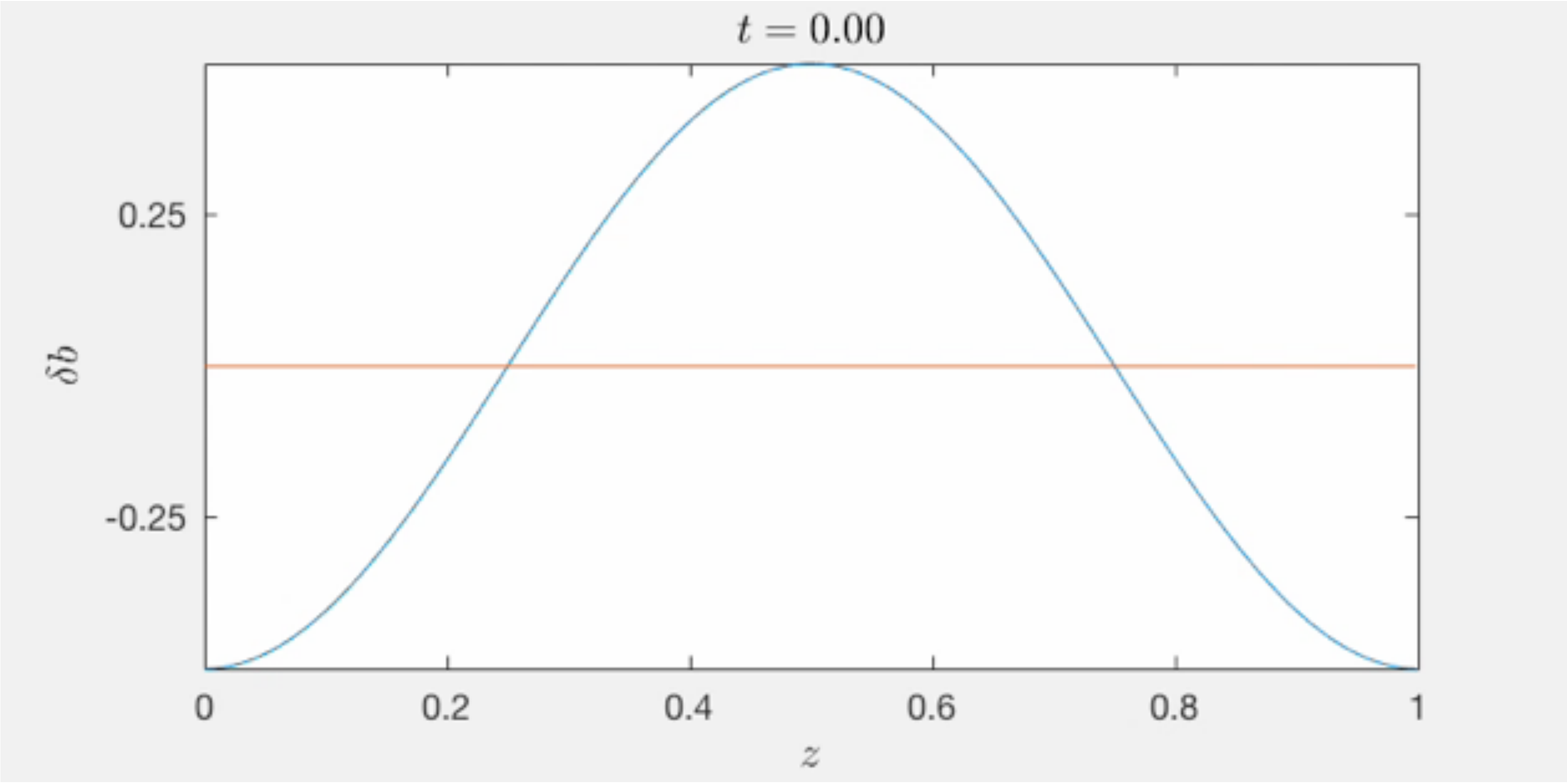
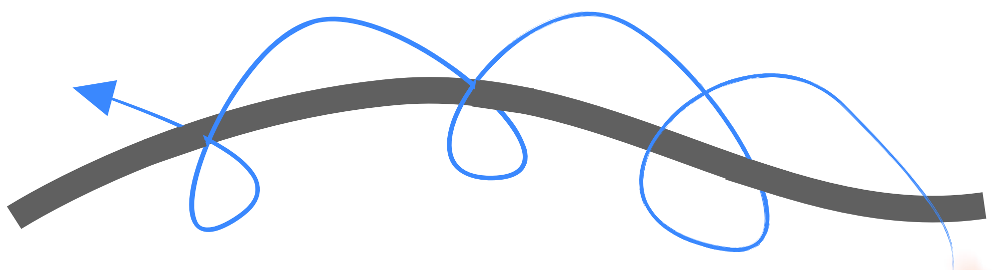
Plasma Kinetics workshop — WPI 2016



$\beta = \frac{8\pi p_0}{B_0^2} = 100$ *Linearly polarized shear-Alfvén fluctuation*





$$\Delta p = p_\perp - p_\parallel$$

If $\nu_c \ll$

B

B

Extra force in momentum equation

$$\nabla \cdot (\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\Delta p) \quad \textit{like} \quad \nabla \cdot (\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}B^2)$$

ASSUMING GYROTROPY, MOMENTS OF VLASOV EQ.

$$\rho(\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}) = -\nabla\left(p_\perp + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\left(\Delta p + \frac{B^2}{4\pi}\right)\right]$$

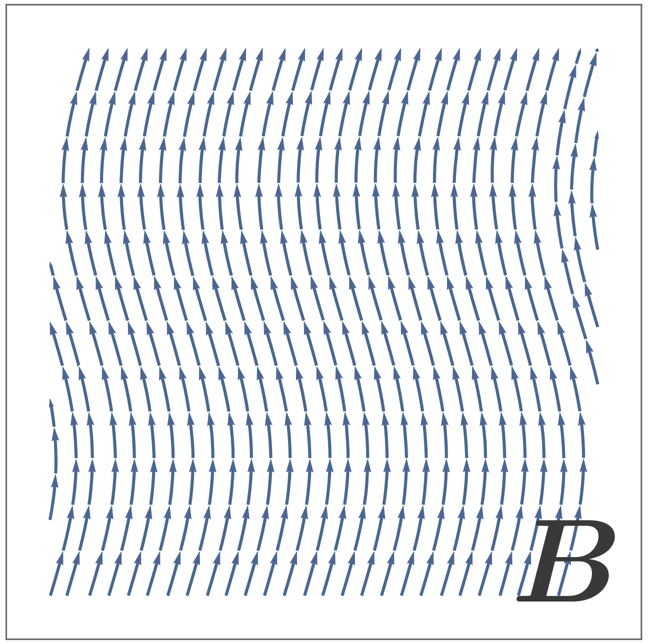
$$\partial_t \boldsymbol{B} = \nabla\times(\boldsymbol{u}\times\boldsymbol{B})$$

$$\frac{d\Delta p}{dt} = 3p_0\frac{1}{B}\frac{dB}{dt} - (3p_\parallel - p_\perp)\frac{1}{\rho}\frac{d\rho}{dt} + \nabla\cdot[\hat{\boldsymbol{b}}(q_\perp - q_\parallel)] - 3q_\perp\nabla\cdot\hat{\boldsymbol{b}} - 3\nu_p\Delta p$$

$$\partial_t E_K = -\frac{1}{8\pi}\partial_t\int d\boldsymbol{x}\, B^2 + \int d\boldsymbol{x}\, p_\parallel \nabla\cdot\boldsymbol{u} - \int d\boldsymbol{x}\,\frac{\Delta p}{B}\frac{dB}{dt}$$

(e.g. Schekochihin+'10)

ALFVÉN WAVES — LINEARLY POLARIZED



- $|B|$ perturbed: $|B| \sim B_0 \left(1 + \frac{1}{2}\frac{\delta B_\perp^2}{B_0^2}\right)$
- $\Delta(\Delta p) \sim p_0 \Delta \ln B \sim -p_0 \frac{\delta B_\perp^2}{B_0^2}$ (collisionless plasma)
- No magnetic tension if $\Delta p \sim -B_0^2$, can occur if $\frac{\delta B_\perp^2}{B_0^2} \sim \frac{1}{\beta}$

WAVE CAN REMOVE ITS OWN RESTORING FORCE IF

$$\frac{\delta B_\perp}{B_0} \gtrsim \beta^{-1/2}$$

- **BRAGINSKII MHD**
  1. Standing wave: *interruption*
  2. Traveling wave: *nonlinear damping*

Limit reduced by $\sqrt{\frac{\nu_c}{\omega_A}}$

- **COLLISIONLESS (LANDAU FLUID MODEL)**
  1. Standing wave: *interruption*
  2. Traveling wave: *nonlinear damping* + *interruption*

Standing vs. Traveling: key difference

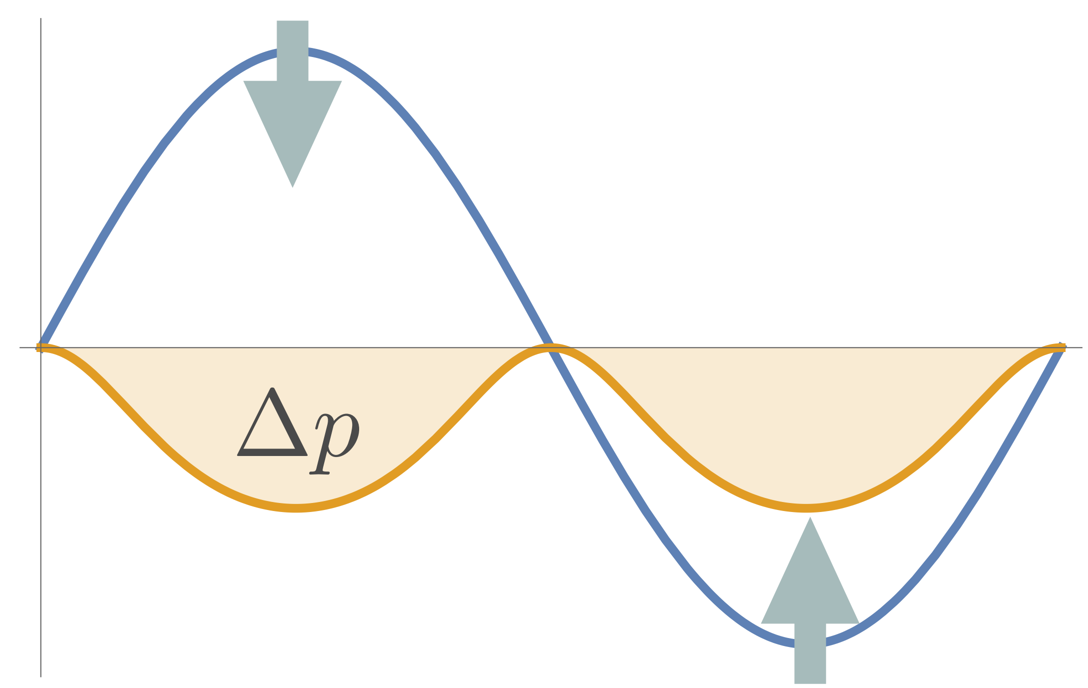
$\langle B \rangle$ *changes during wave evolution*

BRAGINSKII MHD

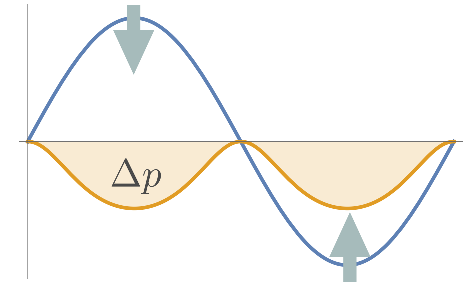
- Collisions balance anisotropy generation $\quad \Omega_c \gg \nu_c \gg |\nabla \boldsymbol{u}|$

$$\frac{d\Delta p}{dt} = 3p_0 \frac{1}{B}\frac{dB}{dt} - (3p_{\|} - p_{\perp})\frac{1}{\rho}\frac{d\rho}{dt} + \nabla\cdot[\hat{\boldsymbol{b}}(q_{\perp} - q_{\|})] - 3q_{\perp}\nabla\cdot\hat{\boldsymbol{b}} - 3\nu_p \Delta p$$

Gives $\Delta \equiv \dfrac{\Delta p}{p_0} \approx \dfrac{1}{\nu_c}\dfrac{1}{B}\dfrac{dB}{dt}$

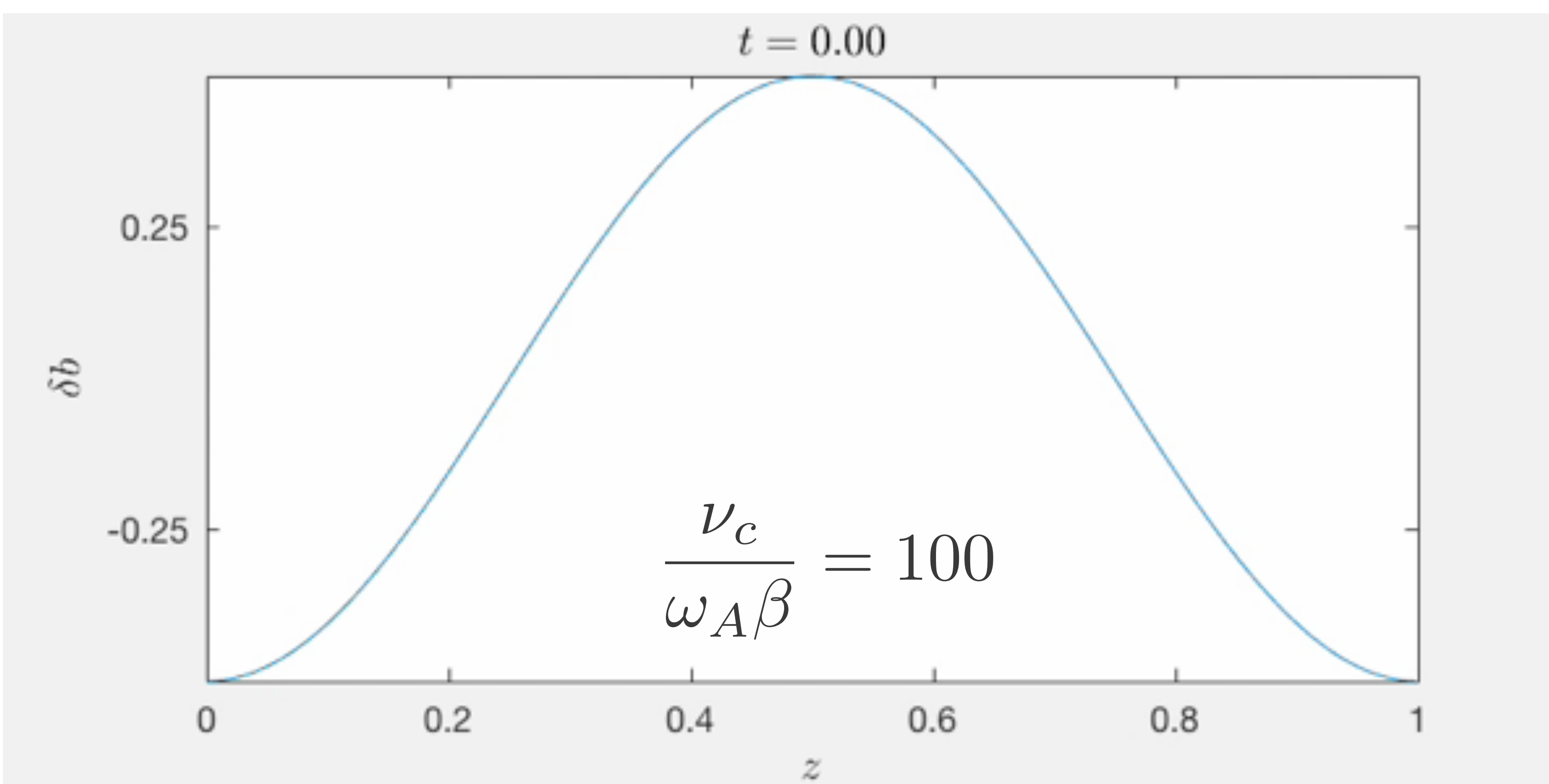


STANDING WAVE: *$\langle B \rangle$ decreases during wave evolution*



(➤ Define $\delta b = \dfrac{\delta B_\perp}{B_0}$)

➤ Use $\partial_t(\delta b) \sim \omega_A \delta b \implies \delta b(0)_{\max} \sim \sqrt{\dfrac{\nu_c}{\omega_A}}\beta^{-1/2}$

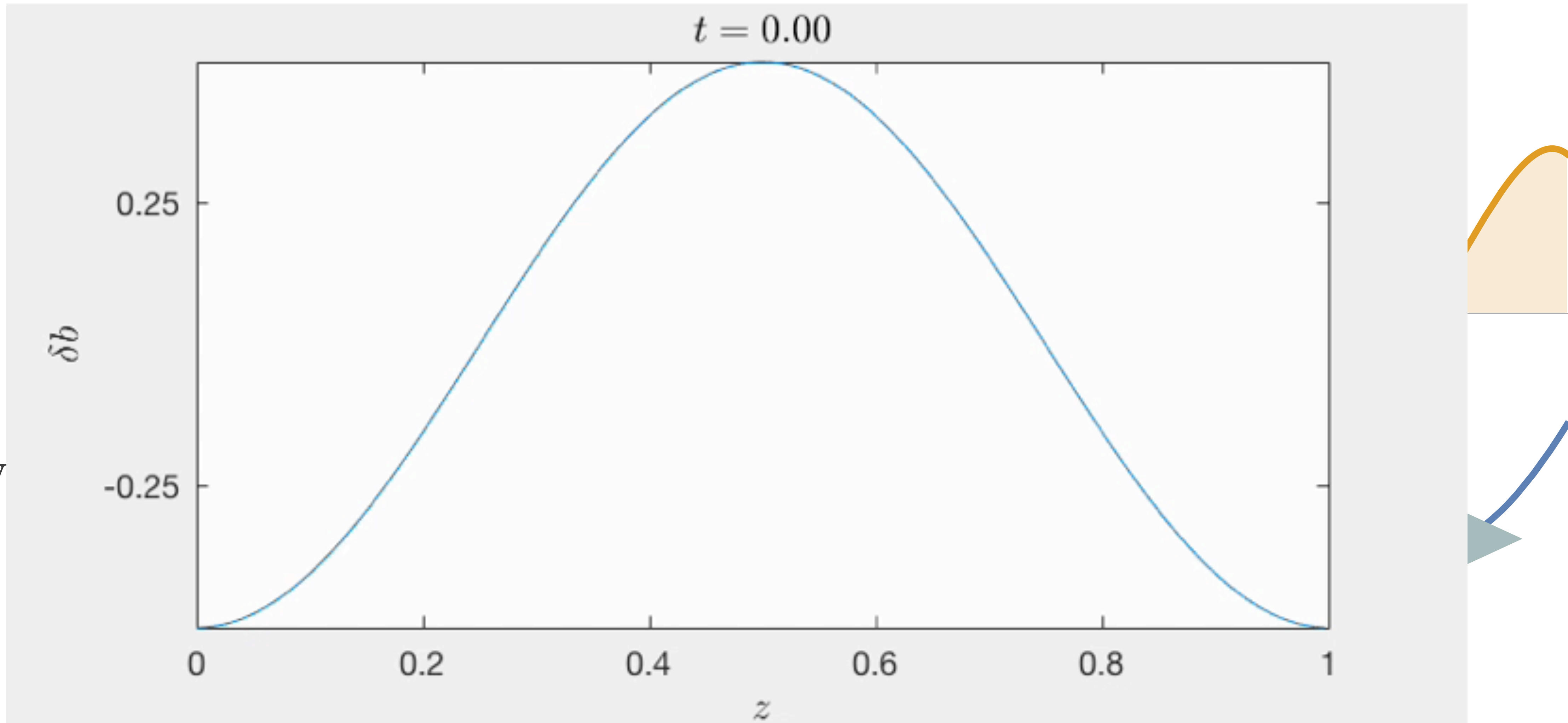


Decay time scale

$\tau_{\text{decay}} \sim \beta\, \delta b(0)^2 / \nu_c$

TRAVELING WAVE: $\langle B \rangle = 0 = const.$

$\partial_t E_\mathrm{w}$



This damping can be very fast!

$$\frac{1}{E_\mathrm{wave}} \frac{dE_\mathrm{wave}}{dt} \sim \frac{\omega_A^2}{\nu_c} \delta b^2 \beta \sim \omega_A \left(\frac{\delta b}{\delta b_\mathrm{max}} \right)^2$$

COLLISIONLESS (LANDAU FLUID MODEL) (e.g., Snyder+'97)

- Heat fluxes important in β>1 plasma

$$\frac{d\Delta p}{dt} = 3p_0 \frac{1}{B}\frac{dB}{dt} - (3p_{\|} - p_{\perp})\frac{1}{\rho}\frac{d\rho}{dt} + \nabla\cdot[\hat{\boldsymbol{b}}(q_{\perp} - q_{\|})] - 3q_{\perp}\nabla\cdot\hat{\boldsymbol{b}} - 3\nu_p\Delta p$$

- After assuming Δ≪1, small *B* perturbation

$$q_{\|} \approx -\sqrt{\frac{8}{\pi}}\rho c_s \frac{\nabla_{\|}}{|k_{\|}|}\left(\frac{p_{\|}}{\rho}\right), \quad q_{\perp} \approx -\sqrt{\frac{2}{\pi}}\rho c_s \frac{\nabla_{\|}}{|k_{\|}|}\left(\frac{p_{\perp}}{\rho}\right)$$

$$\nabla\cdot(\hat{\boldsymbol{b}}q_{\perp}) \sim -\rho c_s|k_{\|}|(p_{\perp}/\rho), \quad \nabla\cdot(\hat{\boldsymbol{b}}q_{\|}) \sim -\rho c_s|k_{\|}|(p_{\|}/\rho).$$

This is just a scale-independent diffusion

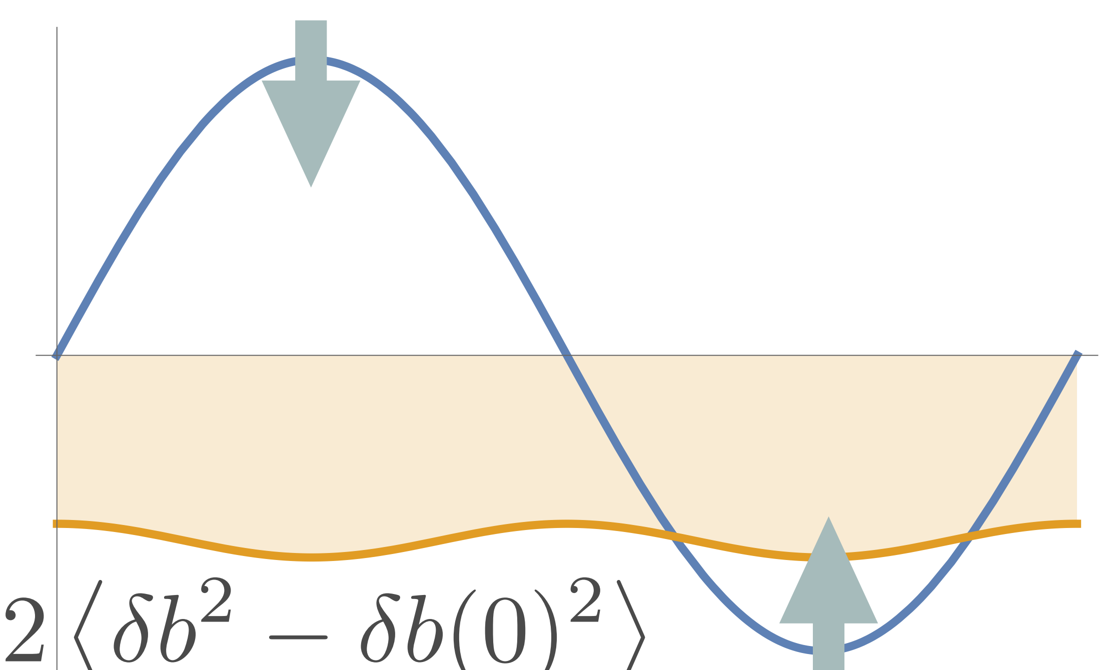
(Medvedev+'97)

COLLISIONLESS (LANDAU FLUID MODEL)

- $k \neq 0$ part of Δ smoothed by $\sim v_A/c_s \sim \beta^{-1/2}$ compared to mean

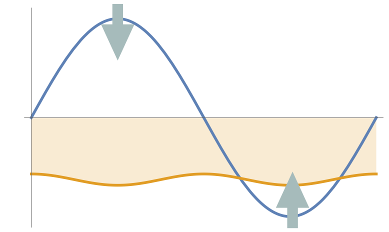
Simple model

$$\Delta = 3\int \left\langle \frac{1}{B}\frac{dB}{dt} \right\rangle dt \left[1 + \mathcal{O}(\beta^{-1/2})(\boldsymbol{x})\right] \approx 3\left\langle \ln \frac{B(t)}{B(0)} \right\rangle$$



$$\Delta \sim 3/2 \left\langle \delta b^2 - \delta b(0)^2 \right\rangle$$

STANDING WAVE $\langle B \rangle$ *decreases during wave evolution*

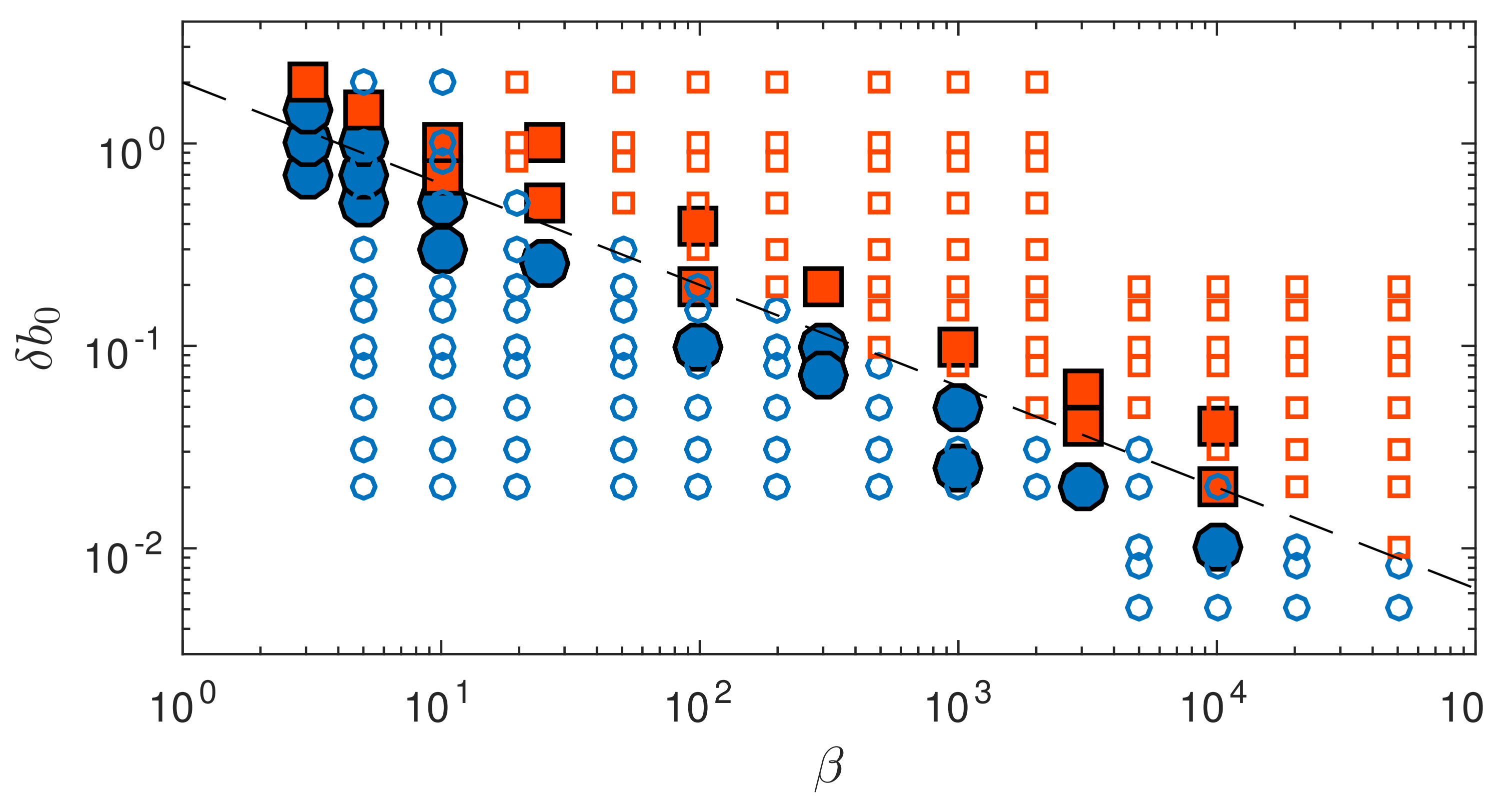


➤ If $3\,\langle \ln[B(t)/B(0)] \rangle = -2/\beta$ wave is “interrupted”

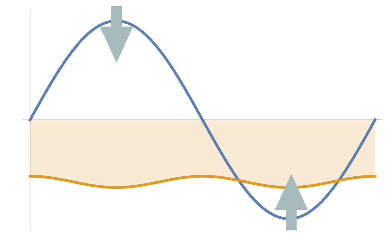
Prediction:

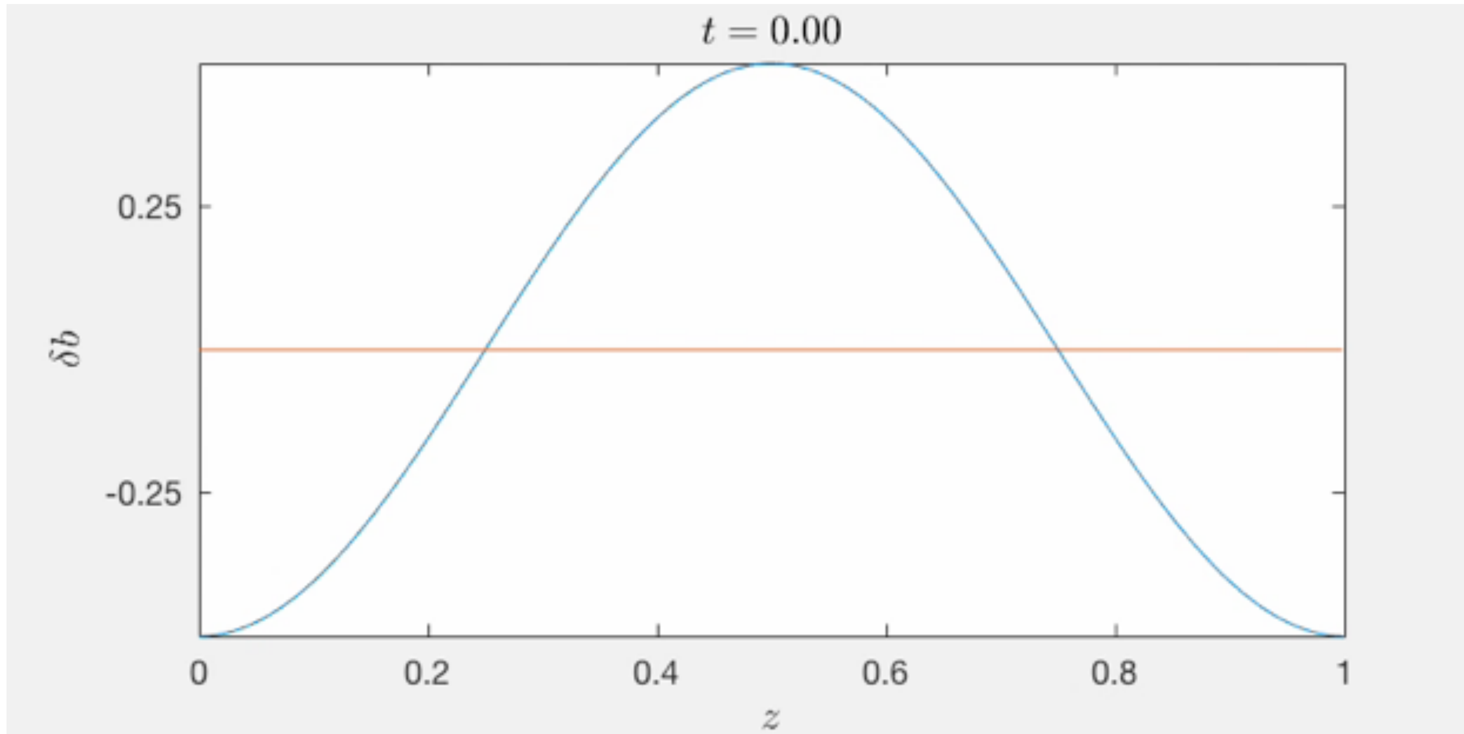
$$\left(\frac{\delta B_\perp}{B_0}\right)_{\max} \approx \sqrt{\frac{8}{3}}\,\beta^{-1/2}$$

(*Actual coeff. is* $\delta b(0)_{\max} \approx 2\beta^{-1/2}$)

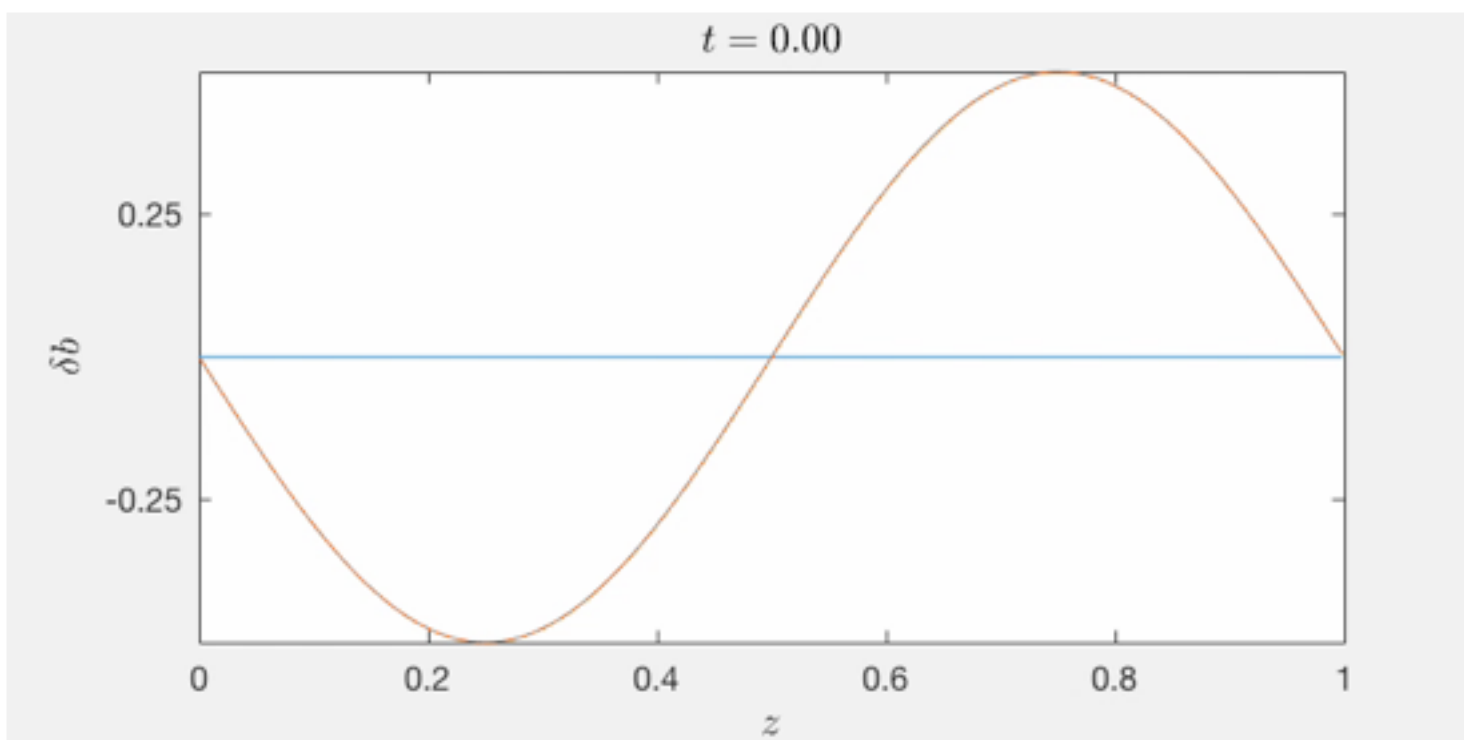


STANDING WAVE $\langle B \rangle$ *decreases during wave evolution*





$$\beta = 100$$



TRAVELING WAVE: $\langle B \rangle = 0 = const.$

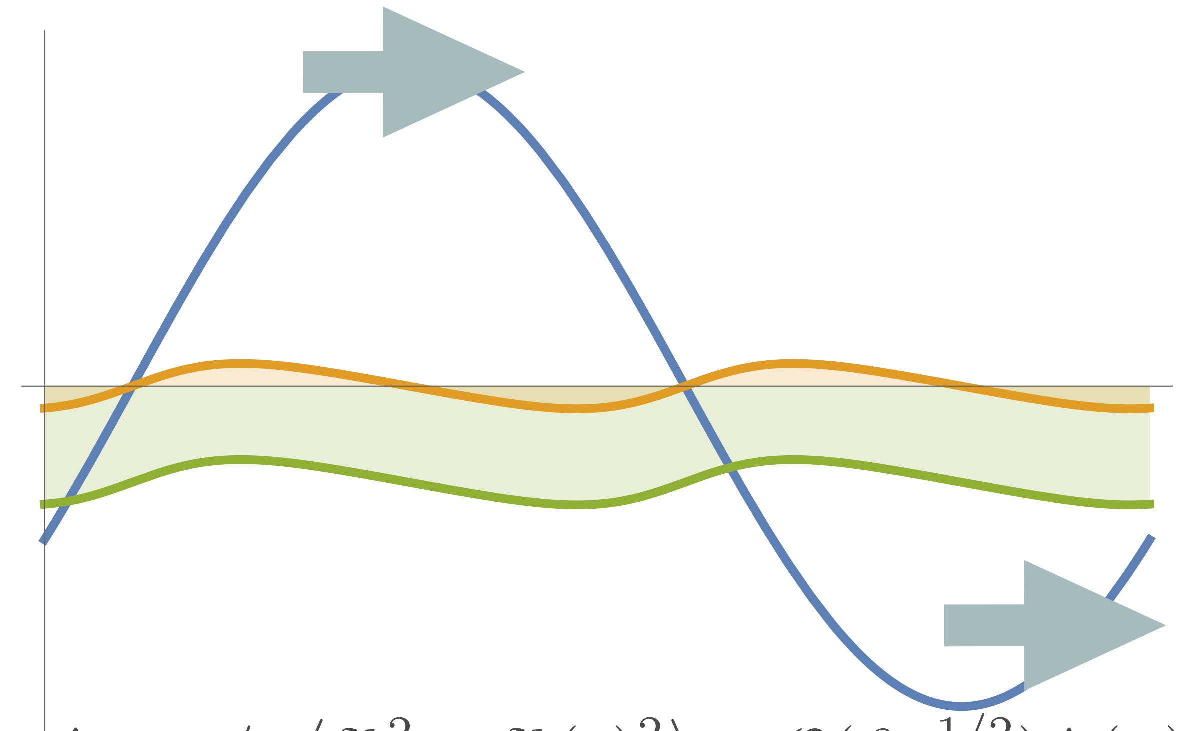
- Again

$$\partial_t E_{\rm wave} \sim -\int d\boldsymbol{x}\, \frac{\Delta p}{B} \frac{dB}{dt}$$

- Heat fluxes reduce Δ

$$d_t \Delta \approx 3 d_t \ln B - \sqrt{\frac{2p_0}{\pi\rho}} |k_\parallel| (2p_\parallel + p_\perp).$$

$$\frac{1}{E_{wave}} \frac{dE_{wave}}{dt} \sim \omega_A\, \delta b^2\, \beta^{1/2}$$

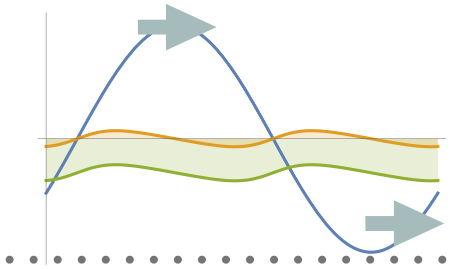


$$\Delta \sim 3/2 \left\langle \delta b^2 - \delta b(0)^2 \right\rangle + \mathcal{O}(\beta^{-1/2}) \Delta(\boldsymbol{x})$$

B decreases

$$\langle \Delta \rangle < 0$$

TRAVELING WAVE: $\langle B \rangle = 0 = const.$

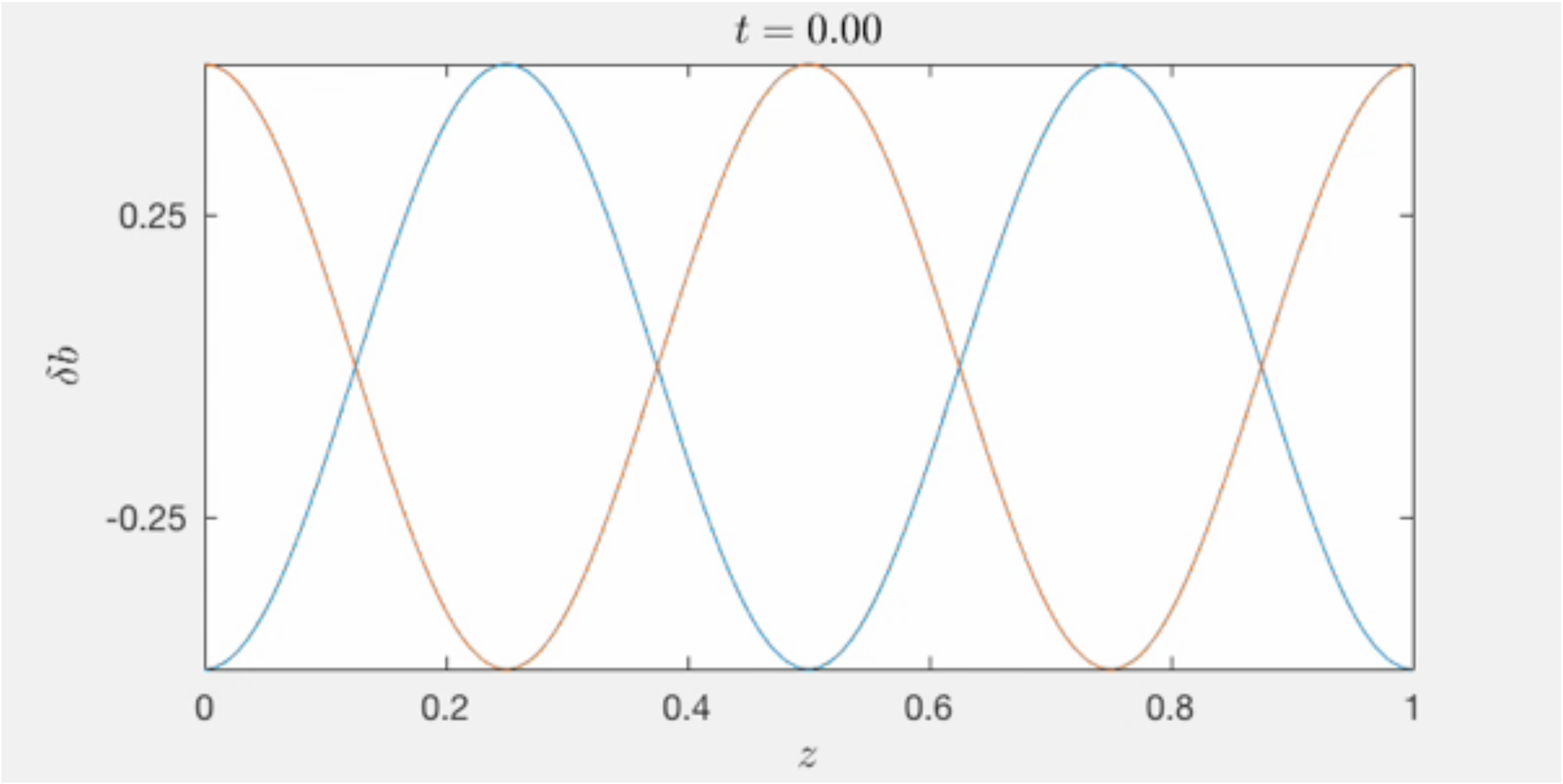


- Accounting for *faster* decrease of $\boldsymbol{u}$ compared to $\boldsymbol{B}$ (since $\Delta < 0$)

$$\gamma_B \sim \omega_A \frac{4\sqrt{\beta}\delta b^2}{8 + 3\beta\delta b^2} \quad \gamma_K \sim \omega_A \frac{\sqrt{\beta}\delta b^2 (4 + 3\beta\delta b^2)}{8 + 3\beta\delta b^2}$$

Faster decay of E_K

- Still "interrupts" with the same limit



$\beta = 100$

➤ BRAGINSKII MHD

1. Standing wave: *interruption* $\delta b(0)_{\max} \sim \sqrt{\frac{\nu_c}{\omega_A}}\beta^{-1/2}$
2. Traveling wave: *nonlinear damping* $\gamma_{\text{wave}} \sim \omega_A \left(\frac{\delta b}{\delta b_{\max}}\right)^2$

➤ COLLISIONLESS

3. Standing wave: *interruption* $\delta b(0)_{\max} \sim \beta^{-1/2}$
4. Traveling wave: *nonlinear damping* + *interruption (wave stops)*

$$\gamma_{\text{wave}} \sim \omega_A \left(\frac{\delta b}{\delta b_{\max}}\right)^2 \beta^{-1/2} \qquad \delta b(0)_{\max} \sim \beta^{-1/2}$$

BUT… (OTHER KINETIC EFFECTS)

- Mirror

  Grow

  These … scatter
  par … e+'16)

- Oblique fi

  Not c … cles.

  Cann … $2/\beta$

- Scattering

  Magnetic "corners" may scatter particles in collisionless case

  Would lead to B field decay

GENERAL CONCLUSION

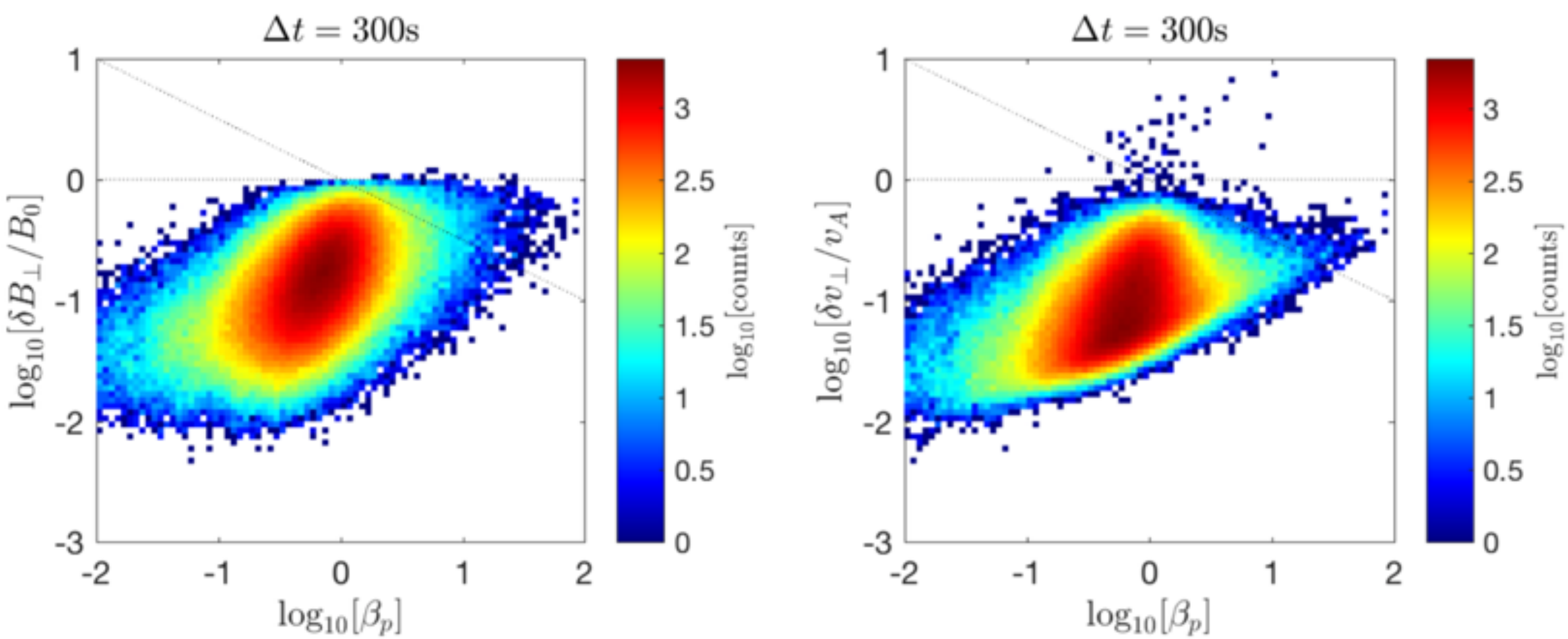
(true also with scattering; e.g., Braginskii)

Magnetic > Kinetic energy

SOME APPLICATIONS

- Solar wind (with Stuart Bale and Chris Chen)

WIND spacecraft data



SOME APPLICATIONS

- Turbulence — cuts off Alfvén wave cascade?

- Energy can go directly to heat through

$$\partial_t E_{u+B} \approx -\int d\boldsymbol{x}\, \frac{\Delta p}{B}\frac{dB}{dt}$$

- No need for cascade to remove energy input? (Kunz+'10)

Braginskii $\beta \approx 8000$

Amp. limit $\delta b_{\max} \sim 0.25$ at large scales