The Onset of Magnetic Reconnection

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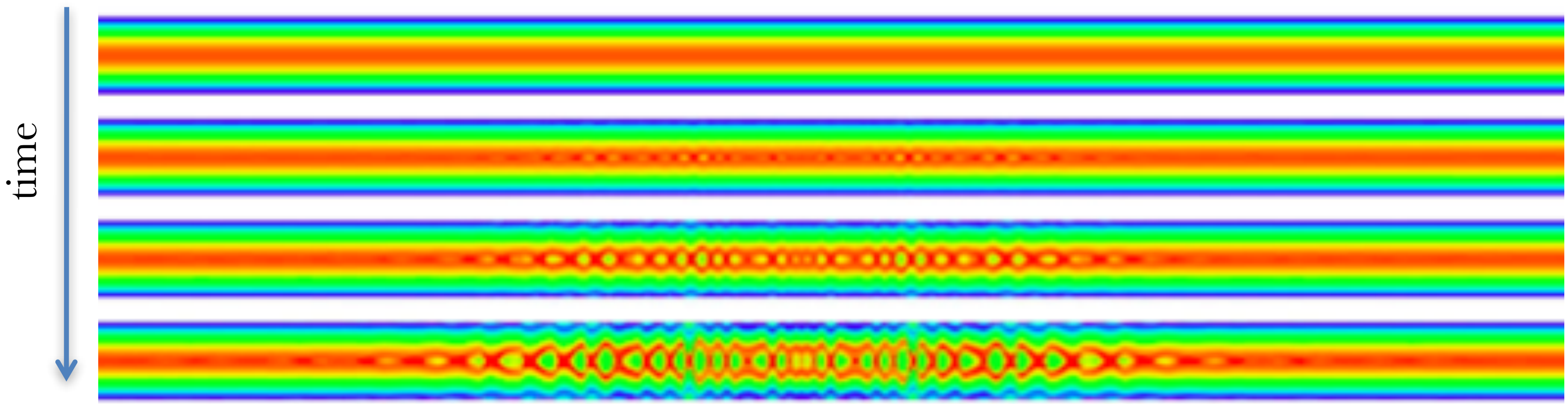
Large aspect-ratio current sheets?

- Large-aspect ratio current sheets are **super-critical** states, i.e., they are violently unstable to the formation of many islands (plasmoids) (see Loureiro & Uzdensky PPCF 2015 for a review)

$$\gamma_{\max}\tau_A \sim S^{1/4}$$

$$k_{\max}L_{CS} \sim S^{3/8}$$



Samtaney *et al.*, PRL'09

Current sheet formation and reconnection onset

- **Implication** is that such current sheets (CSs) cannot form in the first-place; i.e., a forming CS will **disrupt** before reaching those super-critical aspect ratios.
  - *What is the maximum CS aspect ratio?*
  - *How long until disruption of the CS?*
  - *How many islands are generated?*

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  - *What is the maximum CS aspect ratio?*
  - *How long until disruption of the CS?*
  - *How many islands are generated?*
- **Reconnection onset** (the 'trigger', or 'two-time-scale' problem) – perhaps the least understood aspect of reconnection – may be strongly related to this transition.

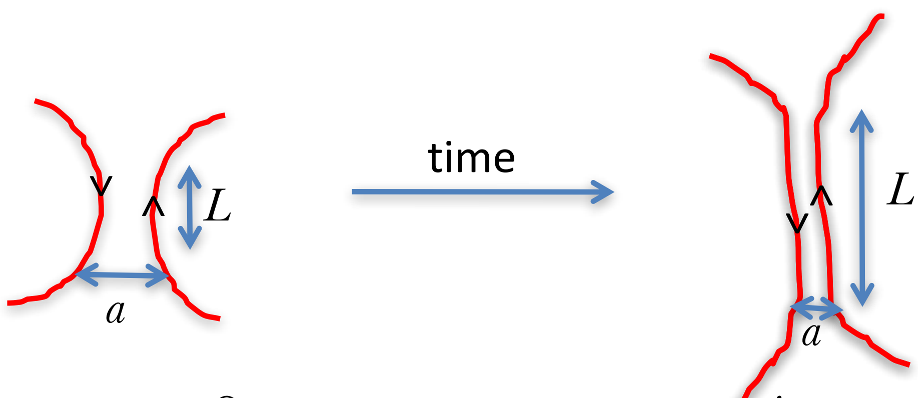
Current Sheet Formation

- **CS formation**: often, ideal-MHD process characterized by:
  - decreasing $a(t)$ -- thinning
  - increasing $L(t)$ -- stretching/lengthening
  - increasing $B_0(t)$ -- strengthening

Current Sheet Formation

- **CS formation**: often, ideal-MHD process characterized by:
  - decreasing *a(t)* -- thinning
  - increasing *L(t)* -- stretching/lengthening
  - increasing $B_0(t)$ -- strengthening
- The particular CS formation mechanism is not of interest here. For our purposes just need the CS formation driving rate:

$$\gamma_{\mathrm{dr}} \equiv \max\left[\dot{a}/a, \dot{L}/L, \dot{B}_0/B_0\right]$$



$t\gamma_{\mathrm{dr}} = 0$ $t\gamma_{\mathrm{dr}} \sim 1$

Aspect ratio L/a increases in time.

Tearing instability of a forming CS

- A current sheet is tearing unstable if the tearing instability parameter $\Delta' > 0$.

Tearing instability of a forming CS

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- For a Harris-type equilibrium $B_y = B_0 \tanh(x/a)$

$$\Delta' a = 2(1/ka - ka) \approx 2/ka \sim L/a$$

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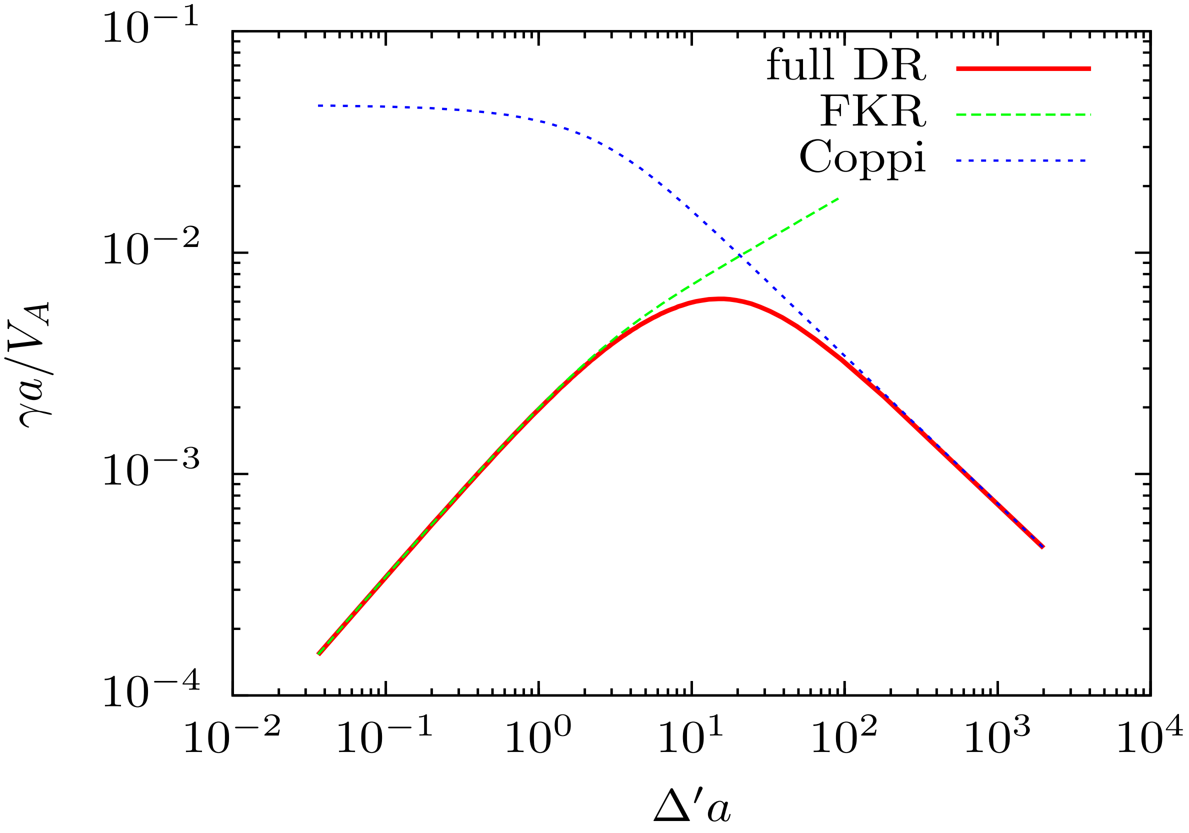
- As soon as Δ' (t)>0, **tearing** instability starts to grow:
  – at first, slow, does not affect CS formation process;
  – then, as layer thickness *a* decreases, $\gamma_{\rm tear}$ (t) increases until

$$\gamma_{\rm tear}(t_c) \sim \gamma_{\rm dr}$$

t_c *is the critical time when the tearing growth rate overcomes the CS formation rate. For the rest of the linear regime can think of CS as frozen*

Linear stage

Consider resistive MHD for simplicity.



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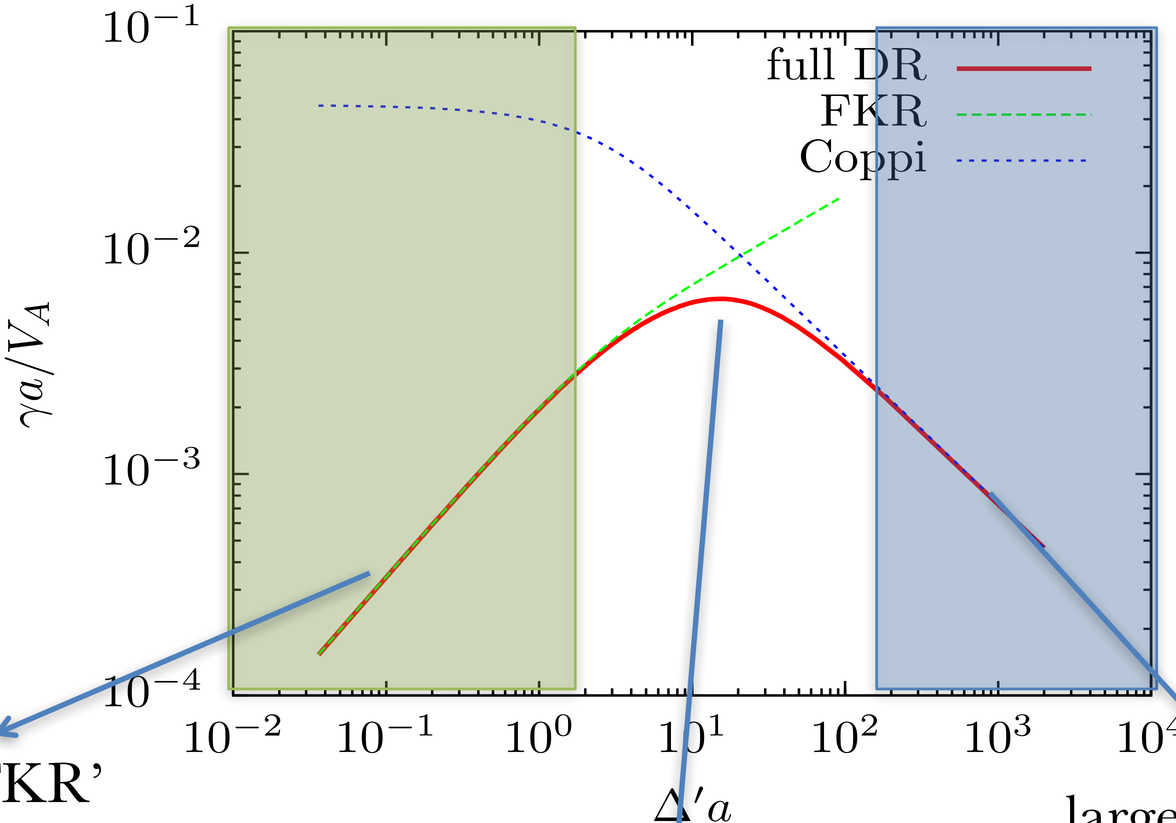


small Δ′, 'FKR' regime

large Δ′, 'Coppi' regime

Fastest growing mode (will call it a Coppi mode)

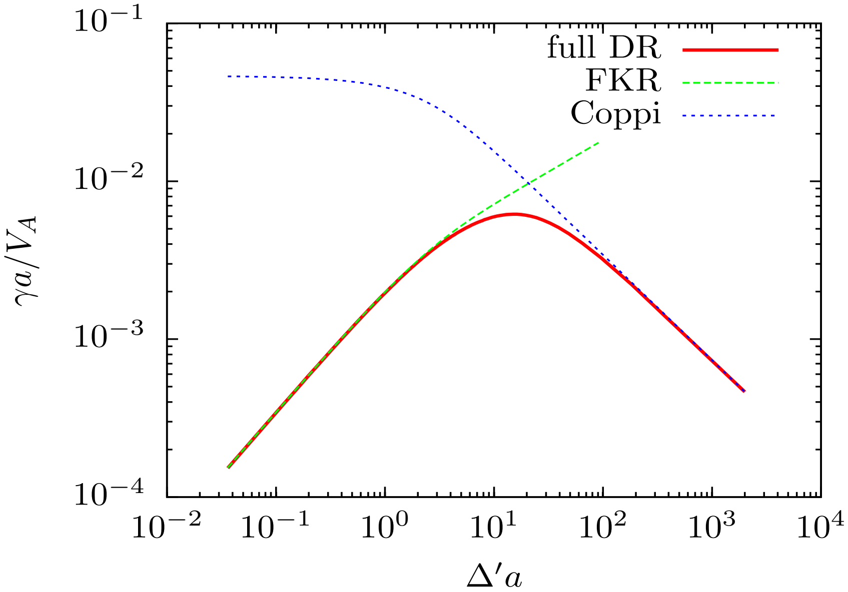
Linear stage

Small Δ': FKR regime (Furth *et al.*, '63)

$$\gamma_{\rm FKR} \sim k^{-2/5} V_A^{2/5} a^{-2} \eta^{3/5}$$

$$k_{\max} \sim 1/L$$

$$N = k_{\max} L \sim 1$$



Linear stage

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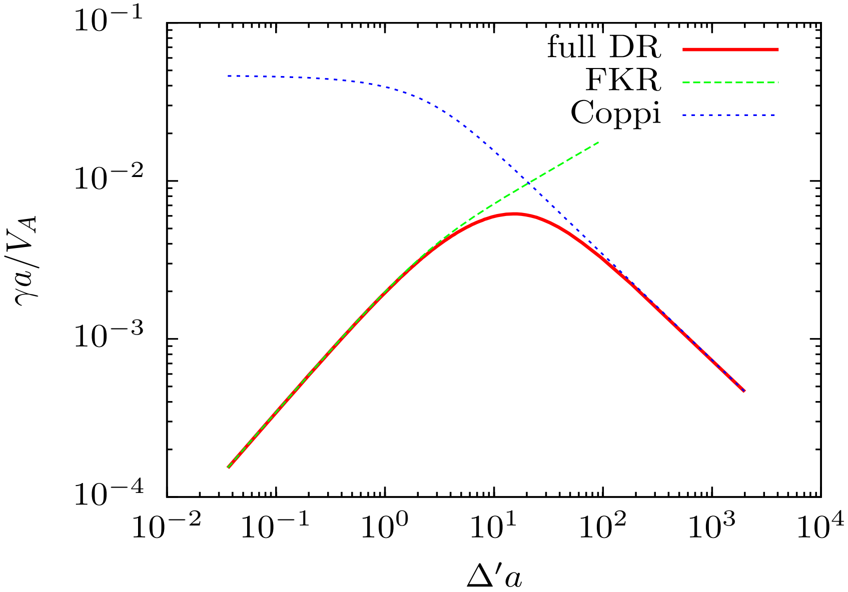
<u>Large Δ'</u>: FKR regime (Coppi *et al.*, '76)

$$\gamma_{\rm Coppi} \sim k^{2/3} V_A^{2/3} a^{2/3} \eta^{-1/3}$$

$$k_{\rm max} \sim a^{-1} S_a^{-1/4}$$

$$N = k_{\rm max} L \sim L/a S_a^{-1/4} \gg 1$$

$$S_a(t) = a(t) V_A / \eta$$



Transition between regimes happens at

$$k_{\rm tr} = a^{-1} S_a^{-1/4}$$

Linear stage

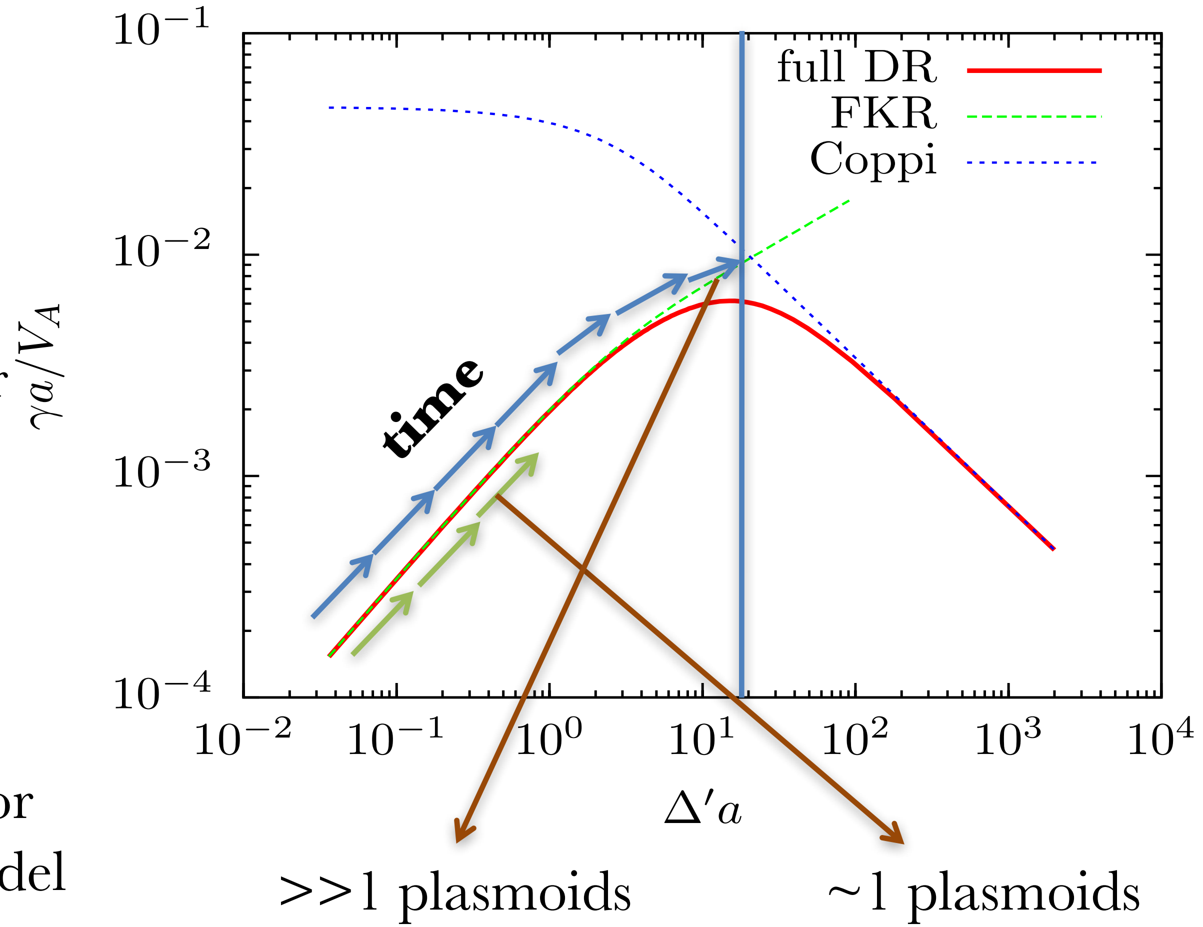
$$\Delta'(t)a(t) \sim 1/k(t)a(t) \sim L(t)/a(t)$$ **increasing in time.**

Two scenarios are possible:

$$\gamma_{\rm FKR}(t=t_c) \sim \gamma_{\rm dr}$$

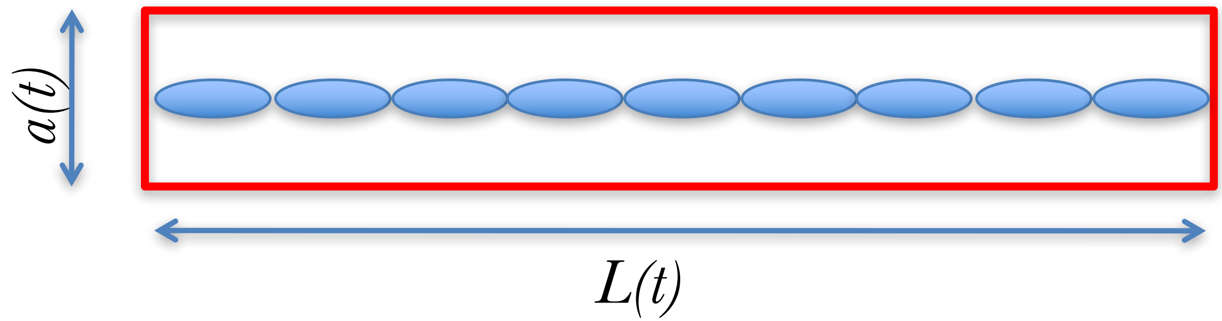
$$\gamma_{\rm Coppi}(t=t_c) \sim \gamma_{\rm dr}$$

– The duration of the linear regime, the number of plasmoids formed and the nonlinear evolution of the system depend on which of these scenarios applies.
– Can be easily computed for any given CS formation model

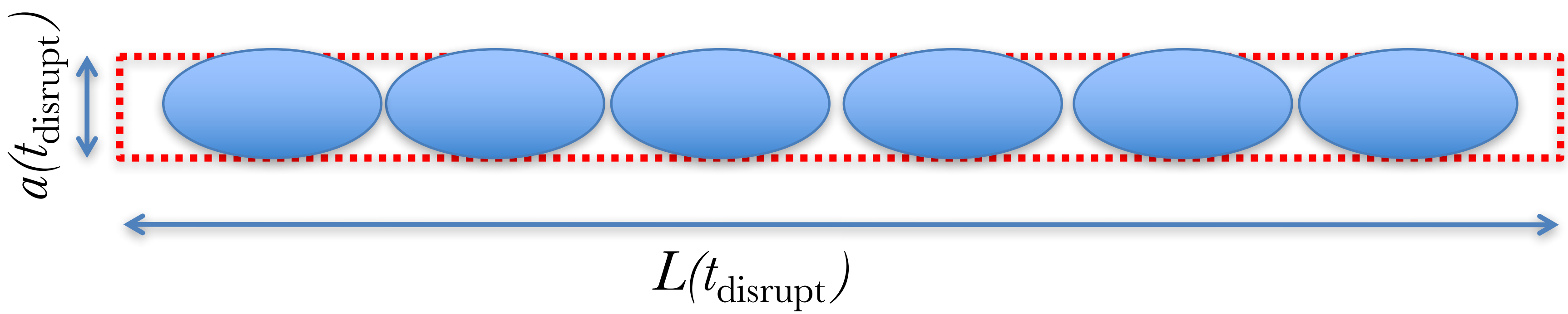


Current sheet disruption

At early stages (i.e., linear and early nonlinear) the tearing instability does not affect the CS formation process.



Current sheet is **disrupted by tearing** when ***w(t)=a(t)***



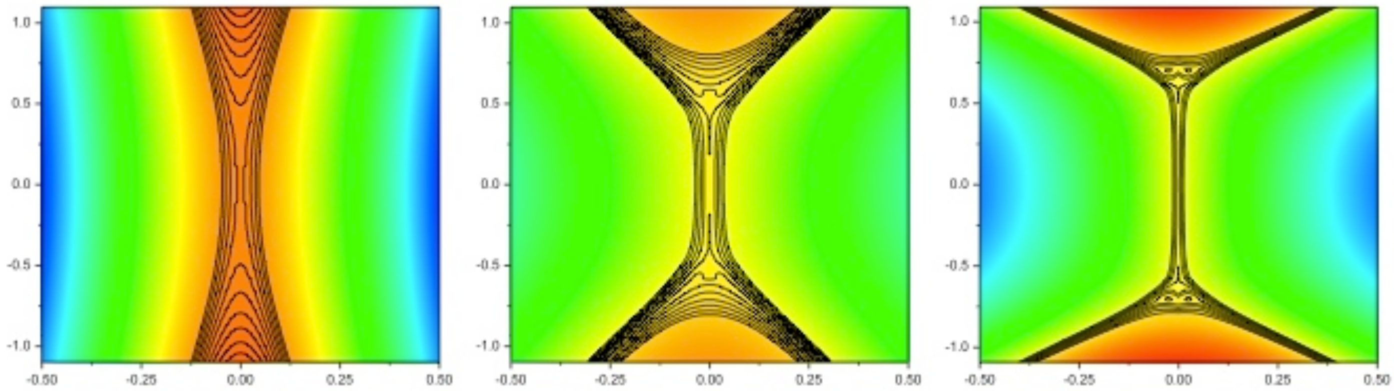
Understanding this process requires analyzing both the **linear** and **nonlinear** evolution of the islands.

Nonlinear Stage

- Linear tearing ends at a very small amplitude:

$$w \sim \delta_{\rm in} \approx a(\gamma a/V_A)^{1/4}(ka)^{-1/2} S_a^{-1/4} \ll a$$

- Nonlinear regime characterized by two stages:
  - *Rutherford '73*: $dw/dt \approx \eta \Delta'(t)$
  - X-point collapse (Waelbroeck '93, Loureiro *et al.* '05): $w(t)\Delta'(t) \sim 1$



X-point collapse leads to very fast island growth
→ **sheet disruption follows immediately.**

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For an FKR mode:

$$\delta_{\text{in,FKR}}\Delta' \ll 1$$

so there is a significant Rutherford stage. Can show that N~1 remains the fastest growing mode

For the Coppi mode:

$$\delta_{\text{in,Coppi}}\Delta' \sim 1$$

so X-point collapse almost immediately follows the linear regime

Example: Chapman-Kendall current sheet model

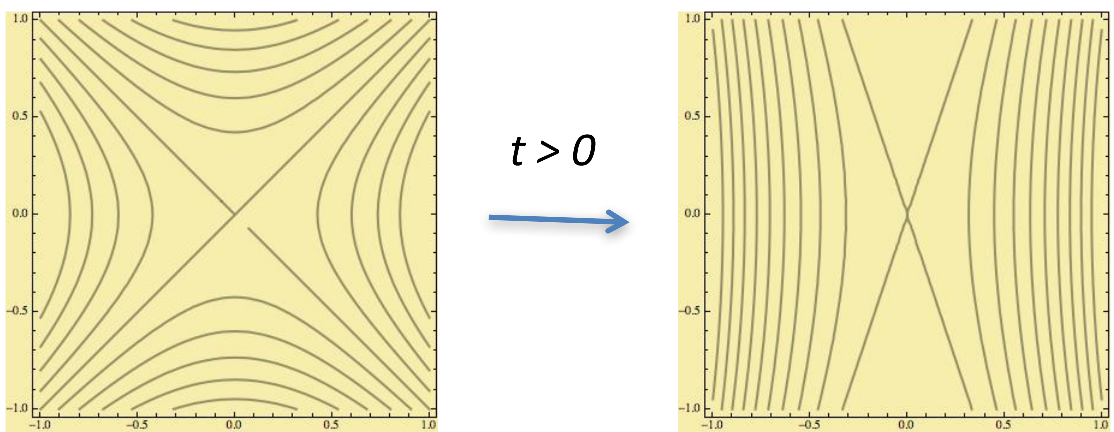
- Crude, but analytically tractable model for current sheet formation (loosely based on Chapman & Kendall, '63). Consider an X-point configuration

$$\phi = v_{\rm dr} xy/L(t), \; \psi = B_0/2[x^2/a(t) - y^2/L(t)]$$

ϕ → stream function ψ → Magnetic flux

- Replace in ideal reduced-MHD equations and solve for *a* and *L*:

$$a(t) = \frac{a_0 L_0}{L_0 + 2v_{\rm dr} t}, \quad L(t) = L_0 + 2v_{\rm dr} t, \quad a_0 \equiv a(t=0) \text{ and } L_0 \equiv L(t=0)$$



Contours of ψ

Tearing instability in CK current sheet

- System is characterized by two dimensionless parameters:

$$M_{\rm dr} \equiv \frac{v_{\rm dr}}{V_A}; \qquad S_0 \equiv \frac{(a_0 L_0)^{1/2} V_A}{\eta}$$

- Fastest growing linear mode in the FKR regime if

$$M_{\rm dr} \ll S_0^{-2/9}$$

- Else, Coppi mode most unstable, with number of plasmoids

$$N = M_{\rm dr}^{9/10} S_0^{1/5} \gg 1$$

FKR: $t_{\rm disrupt}/\tau_{A,0} \sim M_{\rm dr}^{-6/7} S_0^{1/7}$

Coppi: $t_{\rm disrupt}/\tau_{A,0} \sim M_{\rm dr}^{-3/5} S_0^{1/5}$

Application: solar flares

Consider typical solar corona parameters:

$$a_0 = L_0 = 10^4 \text{km}$$

$$n_e = 10^{10} \text{cm}^{-3}$$

$$B_0 = 100G$$

$$V_A = 2000 \text{ km/s}$$

$$S_0 = 3 \times 10^{13}$$

$$\boxed{M_{\text{dr},c} = S_0^{-2/9} \approx 10^{-3} \Rightarrow v_{\text{dr},c} \approx 2 \text{ km/s}}$$

Comparable to typical photospheric velocities

A broad range of drives is likely present in the corona. Consider two cases:

$$M_{\text{dr}} = M_{\text{dr},c} = 0.001 \text{ (FKR)}$$

$$M_{\text{dr}} = 0.05 \text{ (Coppi)} \rightarrow v_{\text{dr}} = 100 \text{ km/s}$$

As may result from ideal MHD instabilities or loss of equilibrium

Application: solar flares (cont'd)

$$M_{\mathrm{dr}} = M_{\mathrm{dr},c} = 0.001 \text{ (FKR)} \Rightarrow \begin{cases} a_{\mathrm{disrupt}} \approx 300 \text{ km} \\ L_{\mathrm{disrupt}} \approx 3 \times 10^5 \text{ km} \\ t_{\mathrm{disrupt}} \approx 40 \text{ h} \\ N = 1 \end{cases}$$

$$M_{\mathrm{dr}} = 0.05 \text{ (Coppi)} \Rightarrow \begin{cases} a_{\mathrm{disrupt}} \approx 70 \text{ km} \\ L_{\mathrm{disrupt}} \approx 1.5 \times 10^6 \text{ km} \\ t_{\mathrm{disrupt}} \approx 4 \text{ h} \\ N \approx 30 \end{cases}$$

- These are very reasonable numbers, considering how crude our CS formation model is.
- In both cases, aspect ratio much smaller than Sweet-Parker would predict

Application: solar flares (cont'd)

In both cases, the smallest scale (the width of the boundary layer of the linear theory at t=$t_{\rm cr}$) remains MHD:

$$\delta_{\rm in}(t_{\rm cr}) \sim 100 - 300\ {\rm m} \gg c/\omega_{pi} \approx 2\ {\rm m}\ ({\rm or}\ \rho_i \approx 0.1\ {\rm m})$$

This validates using MHD to describe reconnection **onset** in the solar corona (in this simple example).

[This does not, of course, imply that the reconnection stage that follows is fully describable by MHD.]

Conclusions

- Current sheet instability implies that very large aspect ratio, super-critical current sheets, cannot form in the first place
  - CS instability must therefore be analyzed in the context of current sheet formation.
  - First analytical model of the reconnection **onset** – we suggest it occurs at the moment of time when plasmoids disrupt the forming CS.
  - Two different regimes – single or multiple plasmoids – are possible, depending on the current sheet formation rate (i.e., the Mach number of the drive).

Uzdensky & Loureiro, Phys. Rev. Lett. **116**, 105003 (2016)