Dynamo on the OMEGA laser and kinetic problems of proton radiography

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Overview

1. Background/motivation - why achieving dynamo in the laboratory matters

2. Small-scale turbulent dynamo on the OMEGA laser facility
   - Set-up and diagnostics
   - Simulation and plasma characterisation
   - Results – near equipartition of magnetic and kinetic energy

3. Proton radiography of stochastic magnetic fields
   - Kinetic theory of imaging beam
   - Extracting magnetic field statistics from flux images
   - Particle diffusion due to small-scale fields
Background

• Basic question: how did ICM come to be universally magnetised?
  • Typical seed mechanisms inadequate

• Explanation: small-scale turbulent kinematic dynamo

• MHD dynamo (mostly) understood theoretically
  • Stretching of field lines at resistive scale
    \[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \]
  • Process well-demonstrated in simulations – need \( Rm \geq 200 \)

• Understanding of plasma dynamo less developed – but lots of recent progress!

• Turbulent dynamo mechanism never seen in the laboratory due to insufficient \( Rm \)
  • Laboratory experiments allow for confirmation of process in more complicated physical situations

[3] Simulated small-scale dynamo: Top: velocity; Bottom: magnetic field strength
OMEGA laser facility

- 60 beams lines, 40 kJ maximum laser energy (highest energy/pulse until NIF)

Source: http://www.lle.rochester.edu/omega_facility/omega/
Experimental set-up

- Turbulent region created by colliding unstable laser-driven plasma jets

- Two pulse shapes
  - 5 ns drive gives higher flow velocities (so greater $Rm$)

- Diagnostics
  - Thomson scattering
  - X-ray framing camera
  - Proton radiography
  - Polarimetry

Schematic of experimental set-up, featuring pulse shape (top left), foil design (bottom right) and partial diagnostic layout, along with image of actual target (bottom left)
Thomson scattering

- Electron temperature, bulk flow, and ion temperature/turbulent motion measurement in 50 μm³ region
- Typical electron temperature (for 5 ns drive)
  \[ T_e \sim 350 - 450 \text{ eV} \]
- Ion temperature taken to be similar
  - Further broadening attributed to turbulent small-scale motions on scale of TS volume
  \[ u_i \sim 20 - 100 \text{ km/s} \]
- Bulk motions (combined with large scale turbulent motions) found to have
  \[ u_0 \sim 20 - 80 \text{ km/s} \]

*Top: Thomson scattering lineout with instrument function fit, 32 ns; bottom: Electron temperature (red), jet mean velocity (blue) and turbulent velocity (green).*
X-ray emission

- Self-emission measured by X-ray framing camera, with pinhole resolution $l = 50 \mu m$

- Assuming an optically thin plasma, can relate relative emission intensity fluctuations to relative density fluctuations in interaction region (Churazov et. al.), and hence find latter power spectrum

- Results show power spectrum consistent with Kolmogorov scaling
  - In subsonic turbulence, typically expect spectrum of density and velocity fluctuations to have same scaling

- Conclusion: interaction region seems turbulent (or at least manifests stochastic motions)
Proton radiography

- Proton beam (created by capsule implosion – two energies) used to image magnetic fields
- Early time radiographs show fields too weak to create large flux variations
- Later time show strong non-linear features (enhanced in 5 ns shots)
Atomic and Laser Physics

**Polarimetry**

- Polarisation of Thomson scattering beam used to measure Faraday rotation effects
- Initial rotation taken from calibration shot
- Typical rotation found to be $\Delta \theta \sim 5^\circ - 10^\circ$
- For magnetic correlation scale $l_B$, rotation related to magnetic field strength by
  
  $$B_{\parallel} (l_i l_B)^{1/2} \sim 17 (\Delta \theta / 7^\circ)(n_e / 10^{20} \text{ cm}^{-3})^{-1} \text{ kG cm}$$

- Scaling dependence on $l_B$ of proton flux magnitude has different scaling to polarimetry
  
  $$\langle B_{\perp}^2 \rangle^{1/2} \left( \frac{l_i}{l_B} \right)^{1/2} \sim 560 \left( \frac{\delta \Psi}{\Psi_0} \right) \text{ kG}$$

- This allows for self-consistent solution for $\langle B^2 \rangle^{1/2}$ and $l_B$ (assuming isotropy) to give
  
  $$\langle B^2 \rangle^{1/2} \sim 450 - 550 \text{ kG} \quad l_B \sim 0.06 \text{ cm}$$
Flash simulations

- Two-fluid laser-plasma simulation code used to complement experiment
  - Comparative diagnostic outputs generated

- Thomson scattering results for electron temperature and velocities from simulation and experiment consistent
  - Code predicts $T_e \approx T_i$ - justification for previous assumption

- Simulation used to give estimate of absolute density $n_e \sim 10^{20} \text{ cm}^{-3}$ (not measured experimentally)

- Initial (Biermann battery) seed fields found to have $\langle B^2 \rangle^{1/2} \sim 25 \text{ kG}$: shock compression and Biermann battery field generation in interaction region not sufficient to explain field growth
Plasma parameterisation

- Adopting experiment plasma parameters as

<table>
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<tr>
<th>Input quantity</th>
<th>Value</th>
</tr>
</thead>
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<tr>
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<tr>
<td>$Z$</td>
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<tr>
<td>$T = T_e, T_i$ (eV)</td>
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<tr>
<td>$n_e$ (cm$^{-3}$)</td>
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</tr>
<tr>
<td>$B$ (G)</td>
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</table>

we obtain estimates of theoretical plasma parameters shown opposite.

- Plasma is essentially well described as MHD, subsonic
- Prandtl number $Pr \sim 1$, though sensitive to ion temperature assumption
- Ions seem un-magnetised, so resulting plasma closer to typical simulations than ICM
Conclusions and next steps

1. Platform seems to produce MHD-type, 'super-critical' $R_m$ plasma undergoing stochastic motions

2. Significant field amplification (with stochastic structure) generated by interaction of jets
   - Dynamo mechanism likely candidate
   - Magnetic/kinetic field energy ratio approaching unity $-B^2/\mu_0 \rho u^2 \approx 0.1$

   - Second round of experiments due next week
     - Pinhole set-up to be used as alternative (hopefully more quantitative) method for measuring particle deflections
     - Increase time-range of shots
     - New grid pattern to create larger, more symmetric interaction region
**Proton radiography – improved analysis?**

- Estimating magnetic fields correctly crucial to success of experiment – yet techniques described above qualitative.
  - Question: *can radiographs be analysed quantitatively?*

- Radiographic analysis often done by post-processing simulated EM fields
  - Good approach for simple configurations – but less effective for stochastic fields

- Example: FLASH magnetic fields generate radiographic image which seems quite different from actual radiograph

- Alternative approach: use asymptotic analysis to derive expression for flux in terms of magnetic field
  - ‘Small/moderate’ magnetic field *gradients* – RMS values and spectra extractable.
  - ‘Large’ gradients – spatial information lost, field strength statistics still accessible.
Atomic and Laser Physics

Diagnostic set-up

- Beam typically generated by TNSA (thermal spectrum of energies) or capsule implosion (two discrete energies)

- Usual set-up is done in paraxial limit
  
\[ \delta \alpha \equiv \frac{l_i}{r_i} \ll 1 \]

- Typical deflections \( \delta \theta \ll 1 \)

- Fast speed of protons greatly simplifies physics governing beam
  - Self-interactions, kinetic instabilities, collisional effects (basically) negligible

- Electric forces smaller than magnetic forces

Kugland et. al. (2012)
Proton beam evolution equation

- Beam obeys governing Vlasov equation
  \[
  \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{e}{m_p c} \mathbf{v} \times \mathbf{B}(\mathbf{x}) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0
  \]

- Exact solution
  \[
  f(\mathbf{x}, \mathbf{v}, t) = \sum_{\mathbf{x}_0, \mathbf{v}_0} f_0(\mathbf{x}_0(\mathbf{x}, \mathbf{v}, t), \mathbf{v}_0(\mathbf{x}, \mathbf{v}, t)) \left| \frac{\partial (\mathbf{x}_0, \mathbf{v}_0)}{\partial (\mathbf{x}, \mathbf{v})} \right|
  \]

where characteristic ray mapping given by inverting particle trajectories

\[
(\mathbf{x}(t), \mathbf{v}(t)) = \left( \mathbf{x}_0 + \int_0^t \mathbf{v}(t') \, dt', \mathbf{v}_0 + \int_0^t \frac{e}{m c} \mathbf{v}(t') \times \mathbf{B}(\mathbf{x}(t')) \, dt' \right)
\]

- Since deflections \( \delta \theta \ll 1 \), try naïve asymptotic solution \( \mathbf{v} = \mathbf{V} + \mathbf{w} \), with \( \| \mathbf{w} \| \ll \| \mathbf{V} \|
\]

\[
\mathbf{w}(t) = \frac{e}{m_p c} \mathbf{V} \times \int_0^t \mathbf{B}(\mathbf{x}(t')) \, dt' \left[ 1 + \mathcal{O}(\delta \theta, \delta \alpha) \right]
\]

- Generally helpful for simple fields (with some complications!); leads to scaling

\[
\delta \theta \sim \frac{e B l_i}{m_p c V} \approx \frac{B(MG)}{2.6 MG} \quad \text{for DD 3.3 MeV protons, 1mm plasma}
\]
Asymptotic approximations to deflection field

- Total deflection experienced by a particle with initial position $\mathbf{x}_{\perp 0}$ becomes
  \[ w(\mathbf{x}_{\perp 0}) \equiv w\left(\frac{l_i}{V}\right) = \frac{e}{m_p c V} \hat{z} \times \int_0^{l_i} \mathbf{B}(\mathbf{x}(z'), z') \, dz' \left[ 1 + \mathcal{O}(\delta \alpha, \delta \theta) \right] \]

- Deflection ‘field’ irrotational in sense that
  \[ \nabla_{\perp 0} \times w(\mathbf{x}_{\perp 0}) = \mathcal{O}(\delta \alpha, \delta \theta) \]
  and so can be written as the gradient of a potential
  \[ w(\mathbf{x}_{\perp 0}) = \nabla \left( \frac{e}{m_p c V} \int_0^{l_i} \hat{z} \cdot \mathbf{A}(\mathbf{x}(z'), z') \, dz' \right) \left[ 1 + \mathcal{O}(\delta \alpha, \delta \theta) \right] \]

- Further asymptotic approximations possible – for example, expand around initial position
  \[ w(\mathbf{x}_{\perp 0}) = \frac{e}{m_p c V} \hat{z} \times \int_0^{l_i} \mathbf{B}(\mathbf{x}_{\perp 0}, z') \, dz' \left[ 1 + \frac{l_i}{l_B} \mathcal{O}\left(\delta \alpha, \delta \theta, \frac{a}{r_i}\right) \right] \]
Propagation from plasma to imaging screen

- Beyond magnetic field configuration, particles undergo free-streaming: only real parts of phase space coordinates altered.

- The magnetic fields induce a coordinate distortion to an initial regular grid carrying the flux distribution. If $r_s \gg l_i$, then in paraxial limit, perpendicular mapping satisfies

  \[
  x_{\perp}^{(s)} = \frac{r_i + r_s}{r_i} x_{\perp} + r_s \frac{w(x_{\perp})}{V} + \mathcal{O}\left(\frac{l_i}{r_s}\right)
  \]

- For a distribution function arising from a point source, the flux becomes

  \[
  \Psi\left(x_{\perp}^{(s)}\right) = \sum_{x_{\perp}^{(s)} = x_{\perp}^{(s)}(x_{\perp0})} \frac{\left(\frac{r_i}{r_s + r_i}\right)^2 \Psi_{\perp0}\left(x_{\perp0}\left(x_{\perp}^{(s)}, \frac{r_s}{V}\right)\right)}{\left[1 + \frac{r_s r_i}{(r_s + r_i)V} \nabla_{\perp0} \cdot w(x_{\perp0}) + \left(\frac{r_s r_i}{(r_s + r_i)V}\right)^2 \det \frac{\partial w(x_{\perp0})}{\partial x_{\perp0}}\right]}
  \]
Numerical example

- Test theory by comparing results of test particles numerically propagated through field configuration, field strength $\langle B^2 \rangle^{1/2} = 5 \text{kG}$

- Good qualitative (and quantitative) agreement
Smearing

- By choosing a different initial distribution function, the effects of smearing can be included in kinetic model.

- Smeared flux related to point-source flux by

\[
\tilde{\Psi}(\mathbf{x}^{(s)}) = \int d^2 \tilde{\mathbf{x}}^{(s)} \Psi(\tilde{\mathbf{x}}^{(s)}) S(\mathbf{x}^{(s)} - \tilde{\mathbf{x}}^{(s)})
\]

where the point spread function

\[
S(\mathbf{x}^{(s)} - \tilde{\mathbf{x}}^{(s)}) = P\left(\frac{\tilde{\mathbf{x}}^{(s)} - \mathbf{x}^{(s)}}{r_s} V\right)
\]

is related to the initial spread in perpendicular velocities.

- Again well matched by numerical experiments.

- If smearing effect small, original image recoverable with deconvolution algorithm – though such analysis prone to instabilities.
Physical interpretation of radiographic images

- Meaning of flux map dependent on contrast parameter

\[
\mu \equiv \frac{\delta \theta}{\delta \alpha} \frac{r_s}{r_s + r_i} \frac{l_i}{l_B} = \frac{r_s \delta \theta}{\tilde{l}_B}
\]

for \( \tilde{l}_B \) the stochastic field correlation scale.

\( \mu \ll 1 \) - ‘linear regime’, where relative flux is proportional to the path-integrated \( z \)-component of current:

\[
\frac{\delta \Psi(x_{\perp}^{(s)})}{\Psi_0^{(s)}(x_{\perp}^{(s)})} = \frac{r_s r_i}{r_s + r_i} \frac{4\pi e}{m_p c^2 V} \int_0^{l_i} j_z \left( x_{\perp 0} \left( 1 + \frac{z'}{r_i} \right), z' \right) \, dz'
\]

\( \mu \leq \mu_{\text{crit}} \) - ‘non-linear injective regime’ – coordinate distortions, but no multi-valuedness

\( \mu > \mu_{\text{crit}} \) - ‘multivalued regime’ – coordinate distortions dominate flux morphology

\( \mu \gg 1 \) - ‘high contrast’: spatial information about structure size lost, image displays PDF of deflection field
Numerical example

- Flux generated from stochastic Gaussian field with Kolmogorov power law for various contrasts

\[
\langle B^2 \rangle^{1/2} = 2 \text{ kG} \\
r_i = 1 \text{ cm} \\
\mu = 0.2
\]

\[
\langle B^2 \rangle^{1/2} = 20 \text{ kG} \\
r_i = 1 \text{ cm} \\
\mu = 2
\]

\[
\langle B^2 \rangle^{1/2} = 125 \text{ kG} \\
r_i = 1 \text{ cm} \\
\mu = 10
\]

\[
\langle B^2 \rangle^{1/2} = 500 \text{ kG} \\
r_i = 100 \text{ cm} \\
\mu = 5,000
\]
Stochastic fields – theoretical background

- Heuristically, expect particle undergoing motion through fields to experience a velocity deflection accumulating as a random walk (provided small angle deflections)

\[ \delta w \sim \frac{e \langle B^2 \rangle^{1/2}}{m_pc} \sqrt{l_i l_B} \quad \delta \theta_{RW} \sim \frac{w}{V} \sim \frac{e \langle B^2 \rangle^{1/2}}{m_pcV} \sqrt{l_i l_B} \sim \delta \theta_C \sqrt{\frac{l_B}{l_i}} \]

- The contrast then scales as

\[ \mu_{RW} \sim \frac{\delta \theta_{RW}}{\delta \alpha} \frac{r_s l_i}{r_s + r_i l_B} \sim \frac{r_s}{r_s + r_i} \frac{e \langle B^2 \rangle^{1/2} r_i}{m_pcV} \sqrt{\frac{l_i}{l_B}} \sim \mu_C \sqrt{\frac{l_i}{l_B}} \]

- Thus for a fixed field strength, as the correlation scale of a field decreases the typical deflection size is reduced, but contrasts increase.
  - At very small scales, local flux behaviour determined by deflection magnitude – well modelled by a diffusive picture (see later).

- Scalings supported by analytical correlation analysis, and numerical simulations.
Spectral reconstruction

- In the linear regime, the spectra of the deflection field and the flux respectively can be linked to that of magnetic fields:

\[
E_B(k) = \frac{m_p c^2}{2\pi l_i e^2} k^2 \hat{C}(k)
\]

\[
= \frac{1}{2\pi} \frac{m_p c^2 V^2}{e^2 r_s l_i} \hat{\eta} \left( \frac{r_i}{r_s + r_i} k \right)
\]

- Numerical tests on field with power law spectrum (index \(-2\)), and field strength \(\langle B^2 \rangle^{1/2} = 0.5 \text{ kG} \), demonstrate feasibility of method.
Spectral reconstruction and non-linear effects

• Problem: for highest contrasts, optical distortion of flux affects spectral shape.

• Deflection field is (asymptotically) linear in the magnetic field, so is not affected by this.

• Q: can the deflection field be recovered from the flux image (‘reconstruction’)
  • A: yes – but only if the coordinate mapping is injective

• Monge-Ampère equation (opposite) has unique convex solution with Neumann BCs.
Non-linear flux spectral distortion
Non-linear reconstruction
Non-linear reconstruction on multi-valued images

- If an image has multi-valued flux regions, the algorithm will behave as if the flux is single-valued, so the solution will be invalid.
- Can be shown analytically (Gangbo et. al.) that solution of Monge-Ampere equation is always a lower bound on the deflection field RMS (as illustrated numerically).
Small-scale fields – diffusive picture

• There has been extensive work done modelling the evolution of a beam of particles through stochastic magnetic fields diffusively.
  • Bykov and Toptigin (1966) derive a governing equation for the ensemble averaged distribution function of non-interacting, unmagnetised test particles using quasi-linear theory
  • The result of such an attempt is typically a diffusion tensor, the form of which being dependent on both properties of the beam and the field.

• For homogenous, isotropic stochastic magnetic fields satisfying \( l_B \ll l_i \) (and using a typical beam), can deduce diffusion equation using QL theory on Vlasov equation (assuming no collective effects) for particle distribution \( \langle f \rangle \) (ensemble averaged)

\[
\frac{\partial \langle f \rangle}{\partial t} + \mathbf{w} \cdot \tilde{\nabla} \langle f \rangle = D_w \frac{\partial}{\partial \mathbf{w}_\perp} \cdot \frac{\partial \langle f \rangle}{\partial \mathbf{w}_\perp}
\]

for diffusion coefficient and typical diffusion velocity

\[
D_w = \frac{Ve^2 \langle B^2 \rangle l_B}{3m^2c^2} \quad \Delta w = \frac{2}{\sqrt{3}} \frac{e \langle B^2 \rangle^{1/2}}{mc} \sqrt{l_i l_B}
\]
Summary and next steps

1. Physical interpretation of proton images arising from magnetic fields depends on contrast parameter $\mu$
   - Small $\mu$ - flux variations describe current fluctuations
   - Moderate $\mu$ - non-linear coordinate distortion.
   - Large $\mu$ - probability density function of field strengths.

2. Decreasing correlation length of stochastic field decreases deflections, increases contrasts
   - Shift from regime in which spatial statistics of fields reflected in flux/deflections into diffusive picture.

3. For small to moderate $\mu$, spectrum and mean field strengths extractable.
   - Use of reconstruction techniques avoids non-linear distortion.

   - Still work in progress: application to actual radiographs
     - Reconstruction method non-local, so sensitive to boundary conditions