Non-ideal MHD in protoplanetary disks

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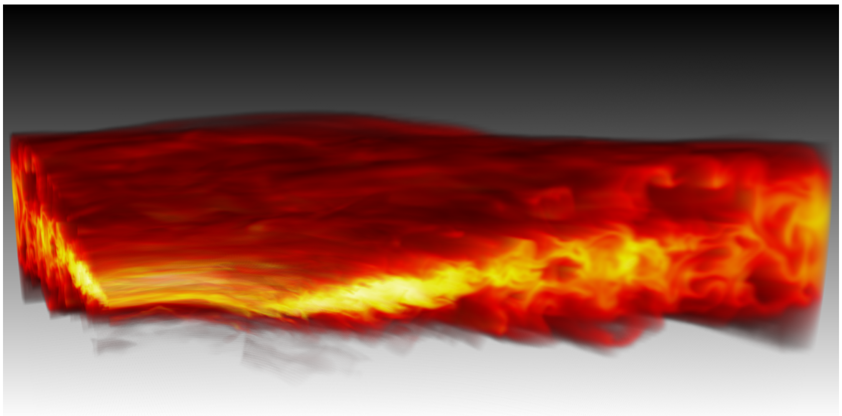
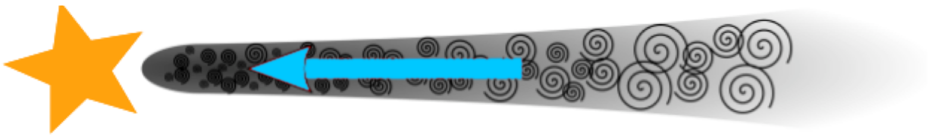
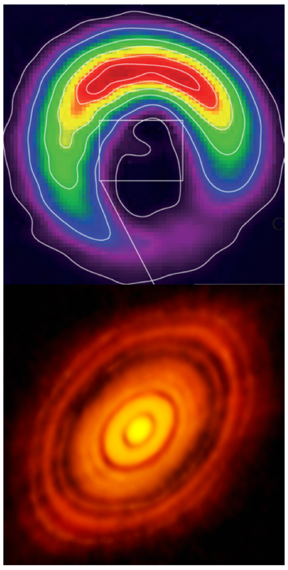


Issues regarding ProtoPlanetary Disks

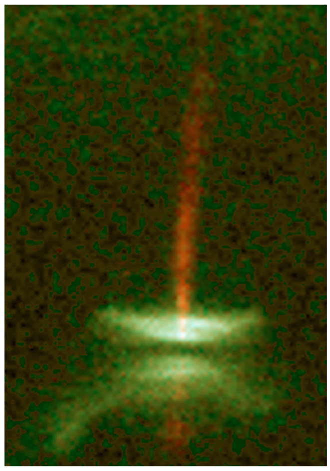


Accretion

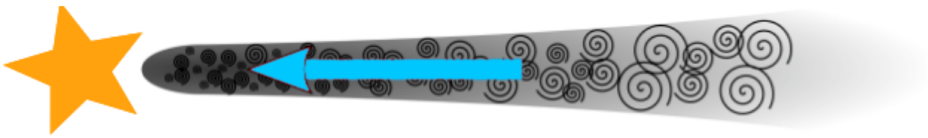


Organization

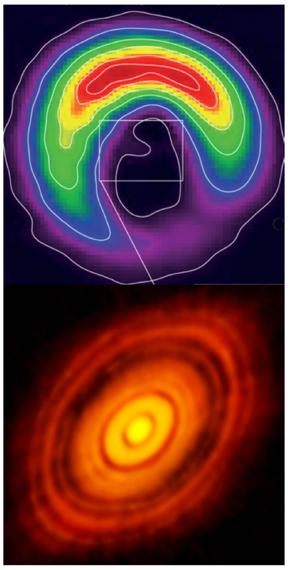
Ejection

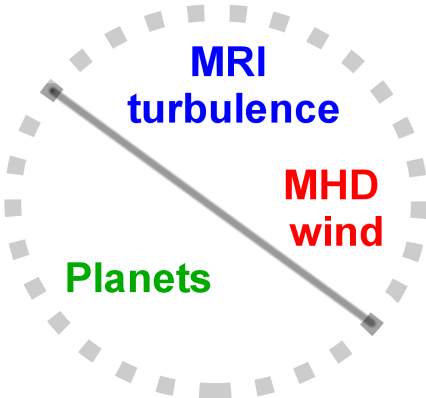


Solutions regarding ProtoPlanetary Disks



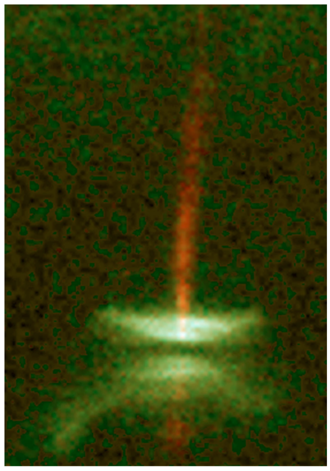
Accretion



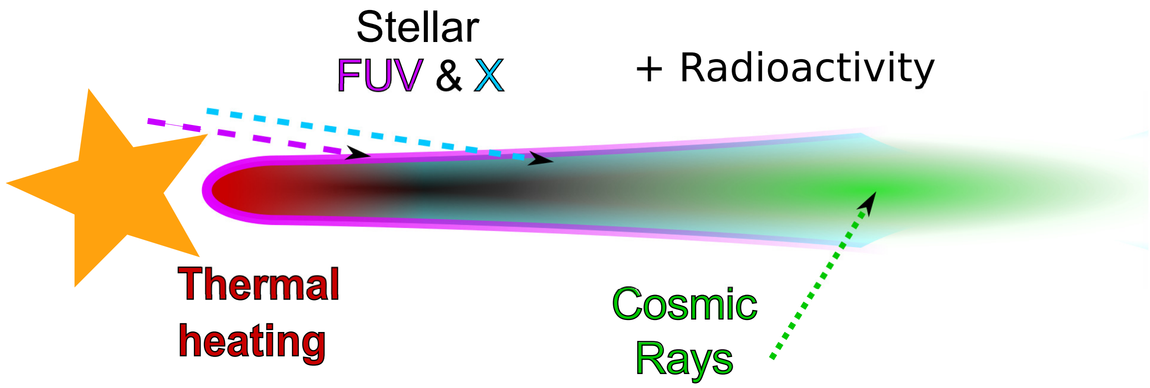


Organization

Ejection



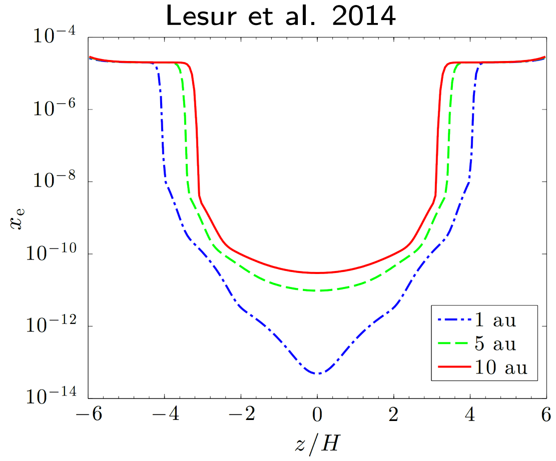
Ionization in PPDs



Ionization fraction $n_e/n < 10^{-10}$

$\longrightarrow$ **ideal MHD does not apply.**

Lesur et al. 2014

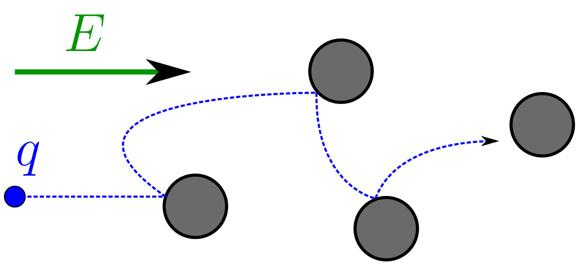


Non-ideal MagnetoHydroDynamics

I. Resistivity

- Electric field and collisions:

$$m\frac{d\boldsymbol{v}}{dt} = q\boldsymbol{E} - \frac{m}{\tau}\boldsymbol{v}$$



- Average over time:

$$\overline{\boldsymbol{v}} = \frac{q\tau}{m}\boldsymbol{E}$$

- Multiply by charge density:

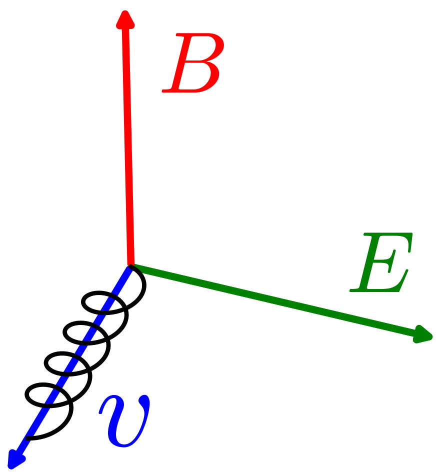
$$\boxed{\boldsymbol{J} = \sigma\boldsymbol{E}} \qquad \text{Ohm's law}$$

Non-ideal MagnetoHydroDynamics

II. Transverse $E \times B$ drift

- No collisions, but a transverse magnetic field:

$$m\frac{d\boldsymbol{v}}{dt} = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$$



- Average over time:

$$0 = \boldsymbol{E} + \overline{\boldsymbol{v}} \times \boldsymbol{B}$$
$$\Longrightarrow \overline{\boldsymbol{v}} = \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2}$$

- Multiply by charge density:

$$\boxed{\boldsymbol{E} = -\frac{1}{qn}\boldsymbol{J} \times \boldsymbol{B}}$$

"Hall effect"

Non-ideal MagnetoHydroDynamics

III. Full induction equation

Assuming $n_e/n \ll 1$,

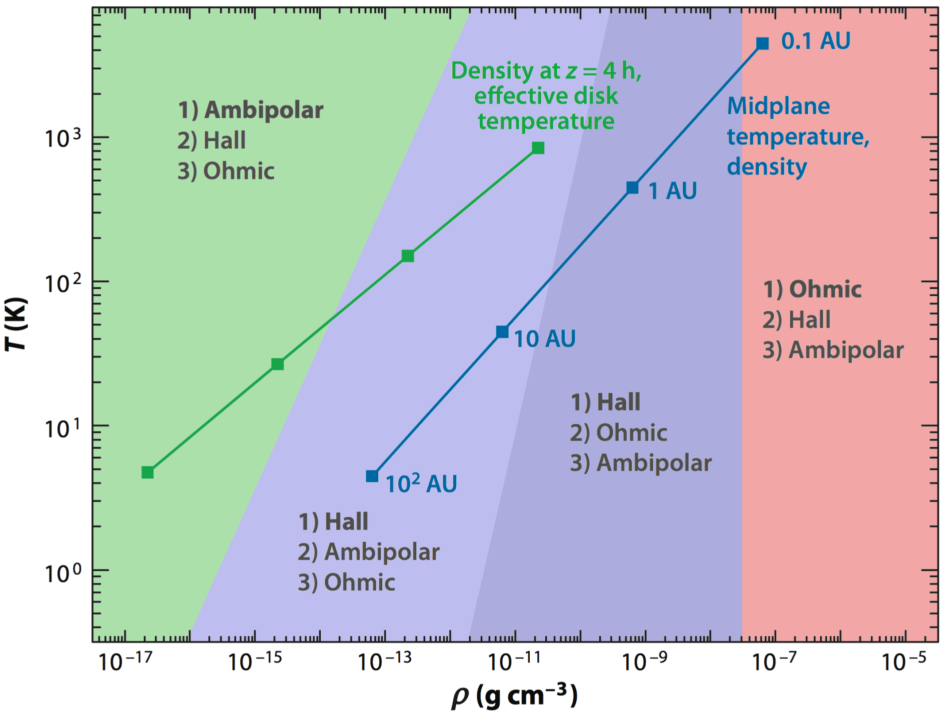
$$\partial_t \boldsymbol{B} = -\nabla \times \boldsymbol{E}$$
$$\simeq \nabla \times \left[\boldsymbol{v} \times \boldsymbol{B} - \eta \boldsymbol{J} - \frac{1}{e n_e} \boldsymbol{J} \times \boldsymbol{B} + \frac{1}{\rho_i \rho_n \gamma_{in}} (\boldsymbol{J} \times \boldsymbol{B}) \times \boldsymbol{B} \right]$$

Ohmic resistivity	Hall effect	Ambipolar diff.
Collisions with electrons	Non-collisional drift	Ion-neutral collisions

Hall length: $\ell_H \equiv \frac{1}{e n_e} \sqrt{\frac{Z_i}{4\pi} \rho_n c^2}$

Relative intensity of non-ideal MHD effects

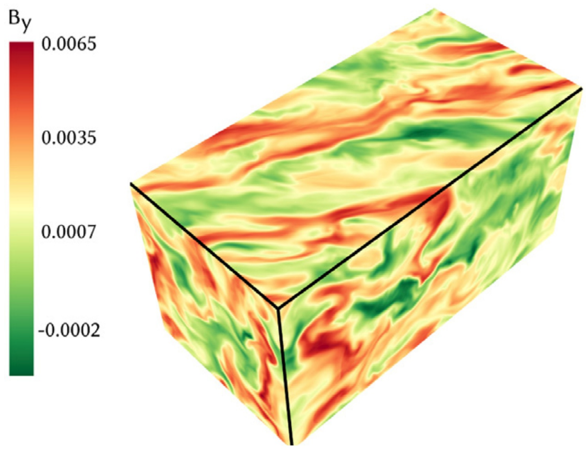
Kunz & Balbus 2004



Ambipolar diff.	Hall effect	Ohmic resistivity
Outer disk & surface	Midplane	Inner disk

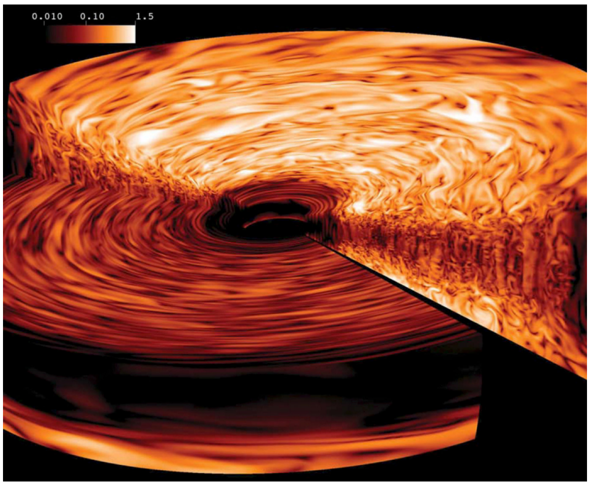
Ideal MRI turbulence

Méheut et al. 2015



'Local', shearing-box simulations

Flock et al. 2011



'Global', stratified simulations

Including non-ideal MHD effects

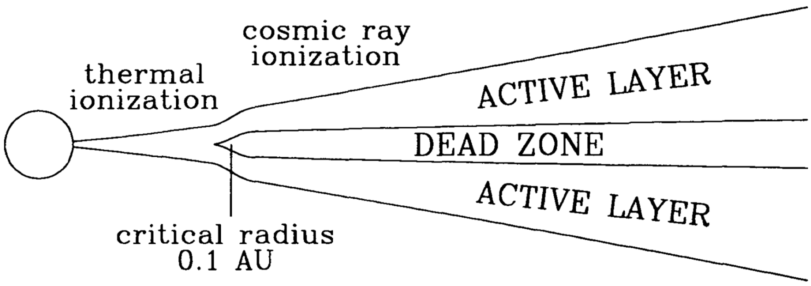
I. Stratified, resistive MRI

Diffusive damping:

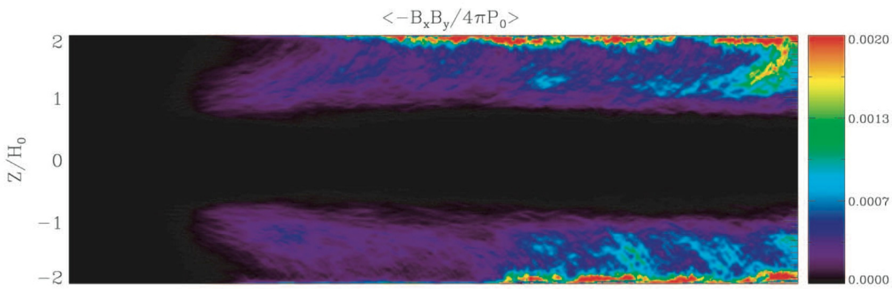
→ **dead-zone**

→ **Layered accretion**

Gammie 1996



Fleming & Stone 2003



Including non-ideal MHD effects

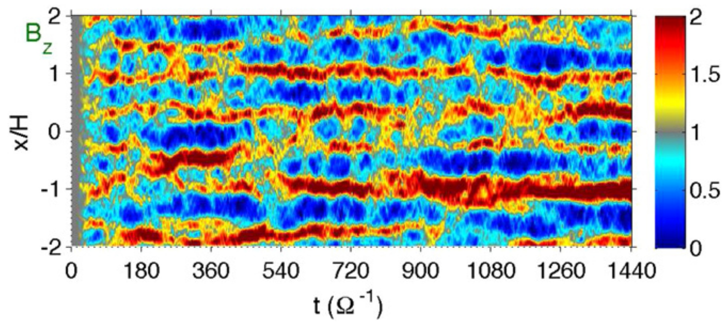
II. Non-stratified, Ambipolar MRI

Diffusivity

$$\eta_{\rm Ambi} \sim \tau_{\rm Ambi}\, v_{\rm Alfven}^2$$

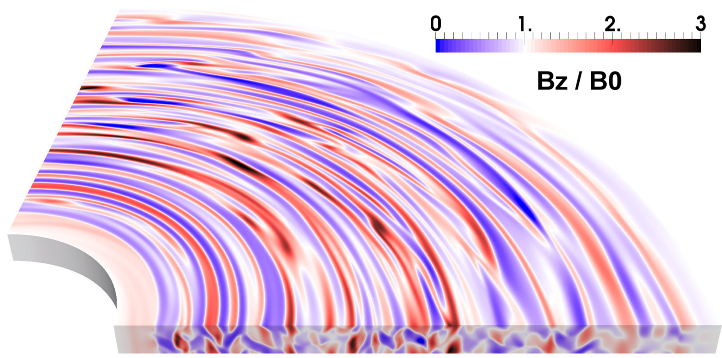
inhomogeneous
&
anisotropic

Bai & Stone 2014



Axisymmetric structures
in non-stratified simulations

Béthune et al. 2016



Including non-ideal MHD effects

III. Stratified Ohmic + ambipolar diffusion

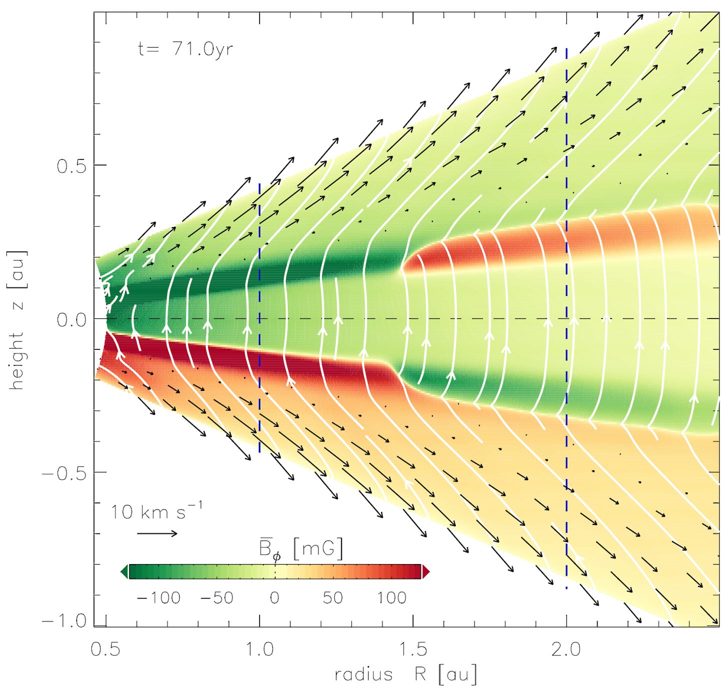
'dead' midplane
+
Ambipolar wind

Layered accretion ✓

No turbulence

No axisymmetric rings ⚠

Gressel et al. 2015



Including non-ideal MHD effects

IV. Non-stratified, Hall MRI

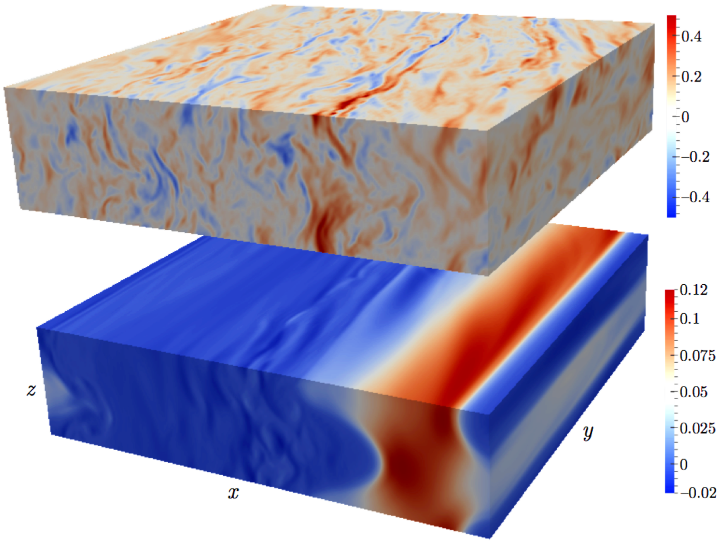
Conserved flux in
incompressible Hall-MHD:

$$\boldsymbol{\varpi} \equiv \nabla \times \boldsymbol{v} + \boldsymbol{B}/\rho\ell_{\mathrm{H}}$$

$\rightarrow$ Self-organization
into **zonal flows**

Do they survive
in global geometry ?

Kunz & Lesur 2013

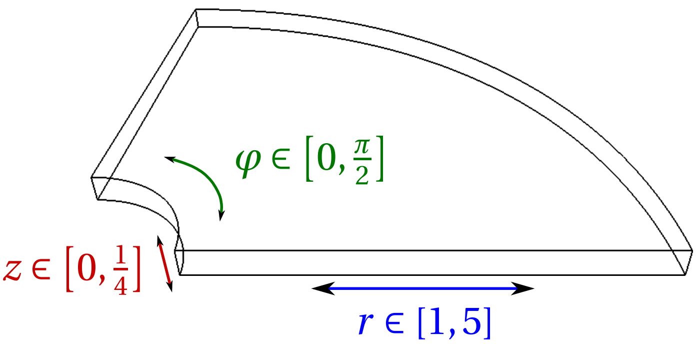


Above : turbulence in ideal MHD
Below : organized flow in Hall-MHD

Global, non-stratified Hall-MHD simulations
I. Numerical setup

Objective: large-scale dynamics, varying Hall and magnetic field intensities.

- Cylindrical domain, vertically periodic
- Keplerian flow, constant initial density, temperature, magnetic field



Global, non-stratified Hall-MHD simulations

II. Main results

Dimensionless measures:

- Torque:

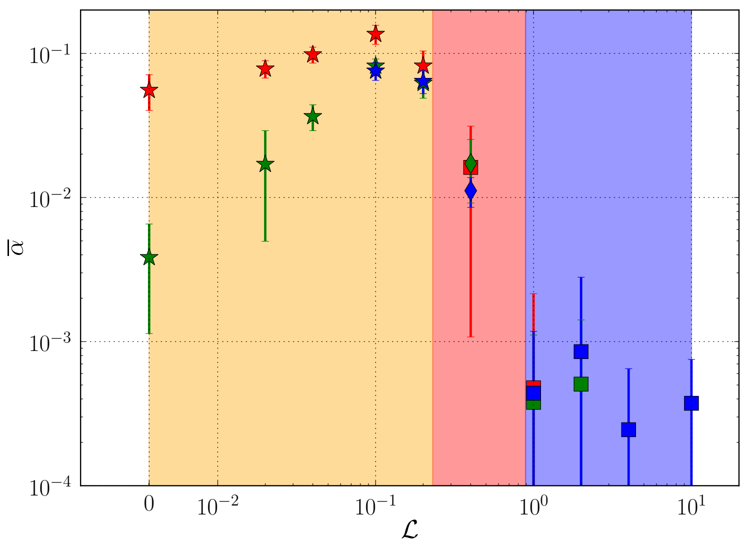
$$\overline{\alpha} \equiv \frac{\langle \rho \tilde{v}_r \tilde{v}_\varphi - B_r B_\varphi \rangle}{\langle \rho \Omega^2 h^2 \rangle}$$

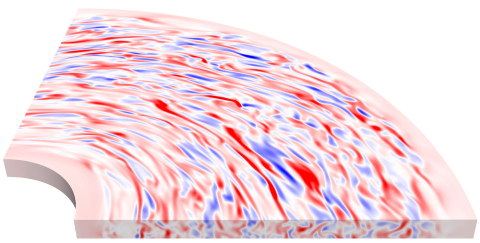
- Hall effect:

$$\mathcal{L} \equiv \ell_H / h$$

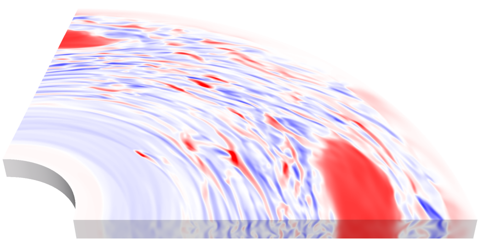
Vertical magnetic field:

Béthune et al. 2016

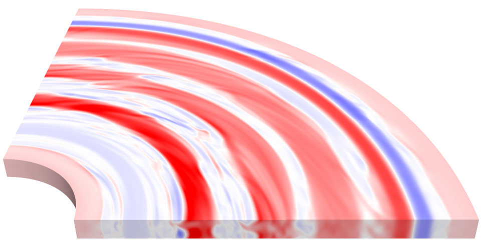




$\mathcal{L} = 0$: turbulence



$\mathcal{L} = 0.4$: vortex



$\mathcal{L} = 1.0$: zonal flows

Blackboard interlude

The instability as a confining mechanism

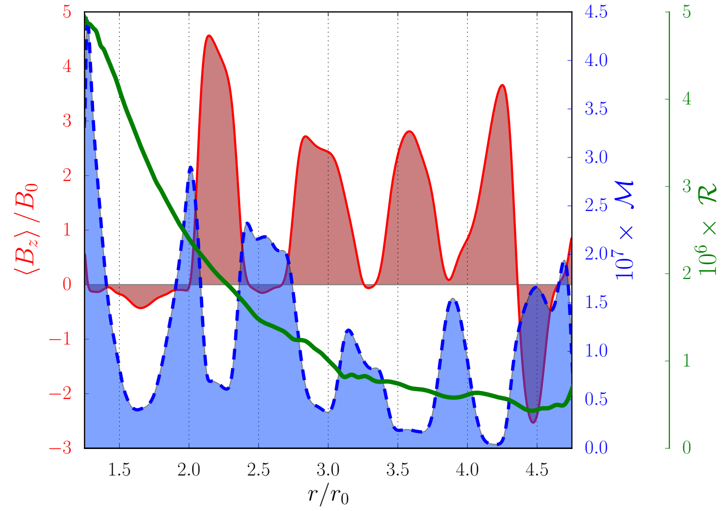
Hall-shear instability:

(Balbus & Terquem 2001, Kunz 2008)

$$0 < \frac{\ell_H B_z k^2}{\Omega\sqrt{\rho}} < 1$$

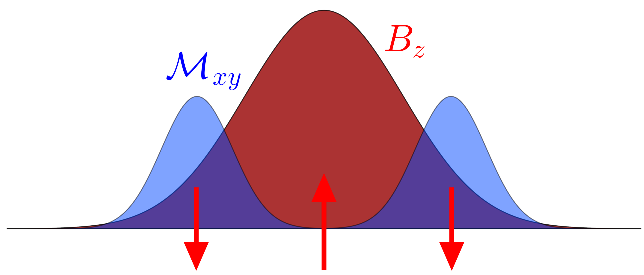
produces a **magnetic torque**

$$\mathcal{M}_{xy} \equiv -B_x B_y > 0$$



Space-averaged induction equation:

$$\partial_t \langle B_z \rangle = \eta \partial_x^2 \langle B_z \rangle + \ell_H \partial_x^2 \langle \mathcal{M}_{xy} \rangle$$



Conclusions from non-stratified Hall-MHD simulations

- Hall can cause magnetized **vortices and rings**
- These structures facilitate **dust accumulation**
- They hold against ohmic and ambipolar diffusion

So what next ?

- **Radial and vertical stratification**
  (self-organization ? wind ? turbulence ?)

- **Full non-ideal MHD**
  (transport ? magnetic polarity & Hall ?)

Global, stratified simulations

Model & setup

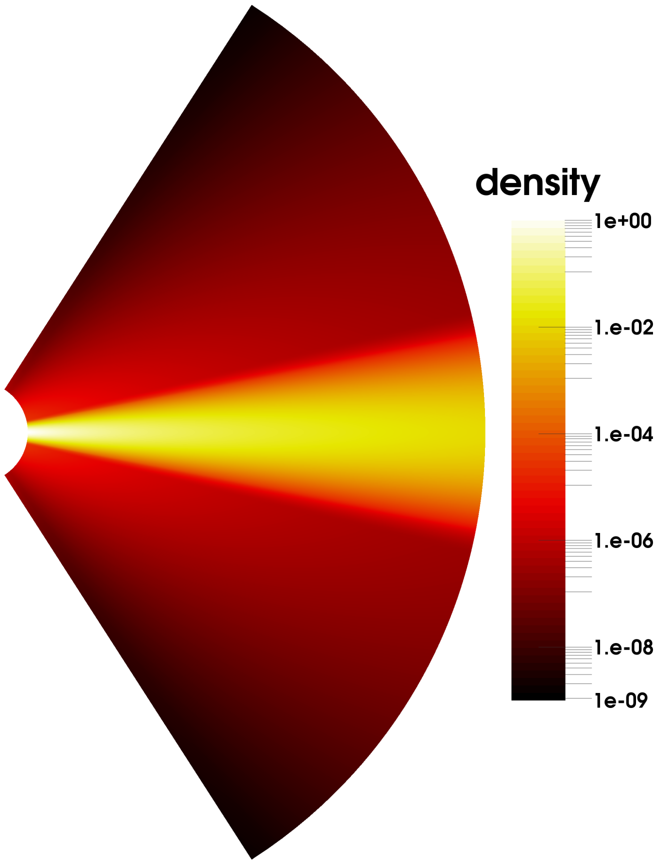
Ingredients

- Radial and vertical $\nabla\rho$
- Large scale, net magnetic field
- A pinch of coronal heating

100% natural ✓

Difficulties

- Large space-time scales
- Orders of magnitude in ρ
- Painful boundary conditions



Preliminary results

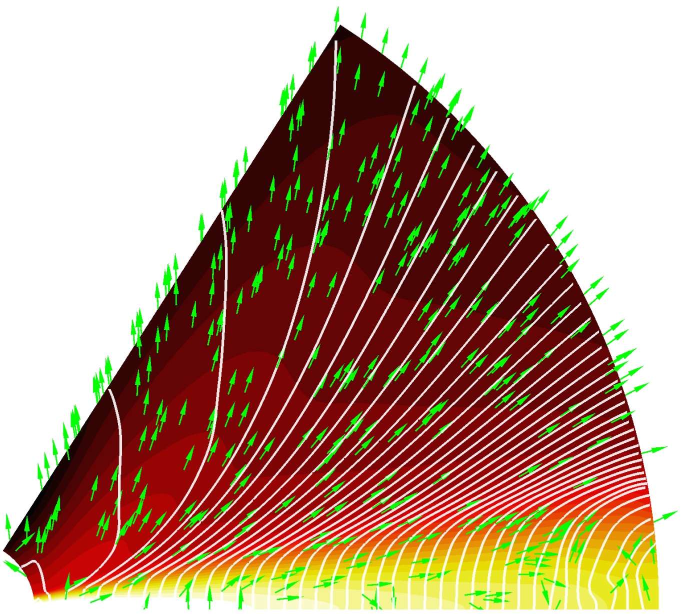
I. Familiar accretion-ejection picture

Disk

- Diffusive disk (but laminar)
- midplane accretion
- No zonal flows

Wind

- Significant disk braking
- Thermo-magnetic acceleration



Magnetized disk wind (in prep)

Preliminary results

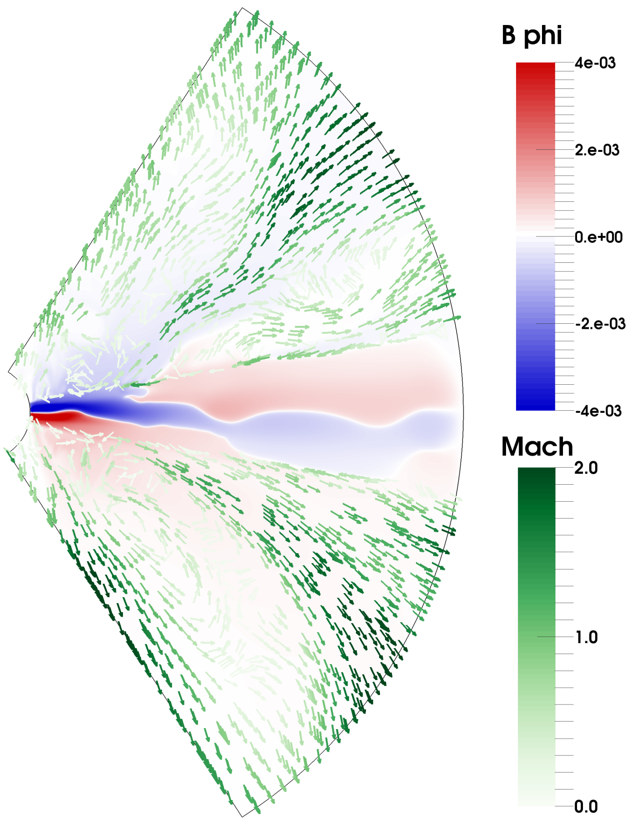
II. The MRI as an ejecting **(or not)** mechanism

Transport of angular momentum
via Maxwell stress

$$\mathcal{M} \equiv -B \otimes B$$

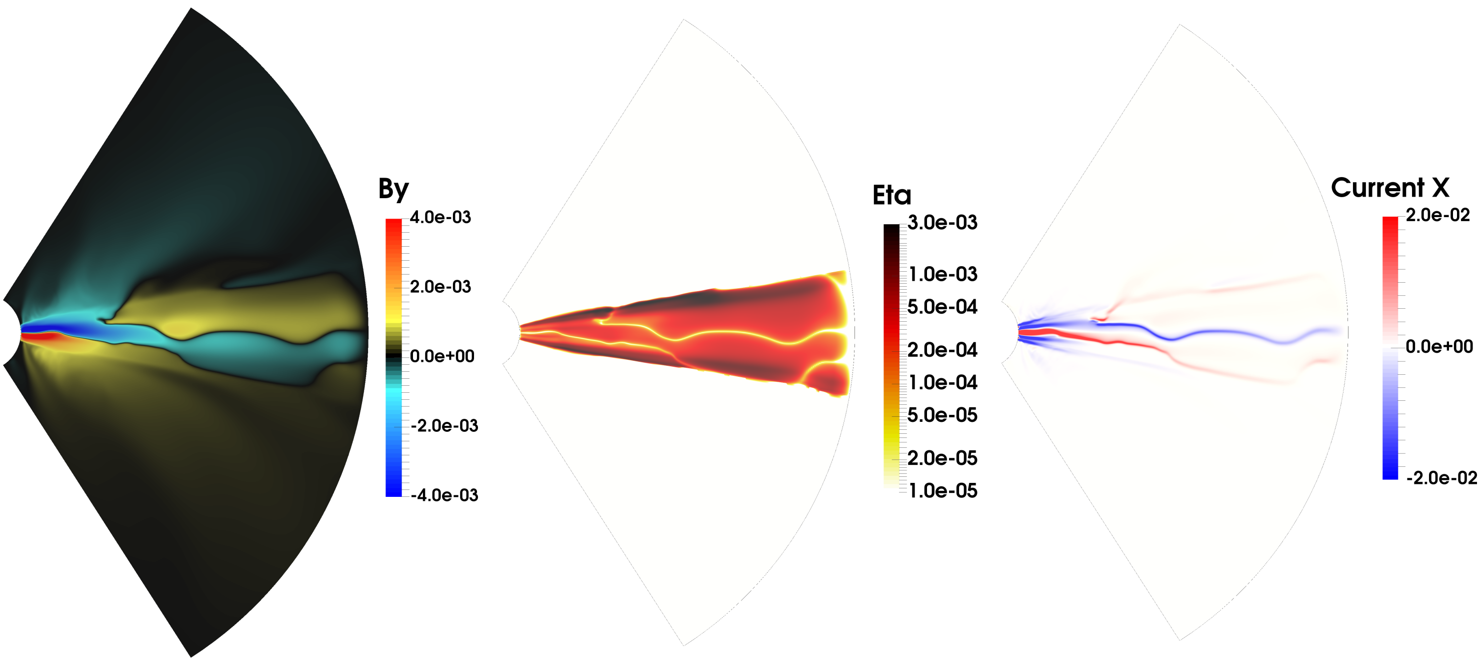
Ejecting **and non-ejecting** disk

depending on MRI phase



Preliminary results

III. The MRI as an insulating mechanism



MRI amplifies B_x and B_y in the same proportions

Ambipolar diffusion in strong field regions

Electric current sheets in low diffusivity tracks

Conclusions

- Always the MRI, but with many modulations...
  (net magnetic flux, stratification, combination of non-ideal effects, etc.)
- ... and a variety of consequences
  (turbulence, vortices, zonal flows, outflows, etc.)

Main caveats:

- Boundary effects
- Thermodynamics (radiative transfert, chemistry)