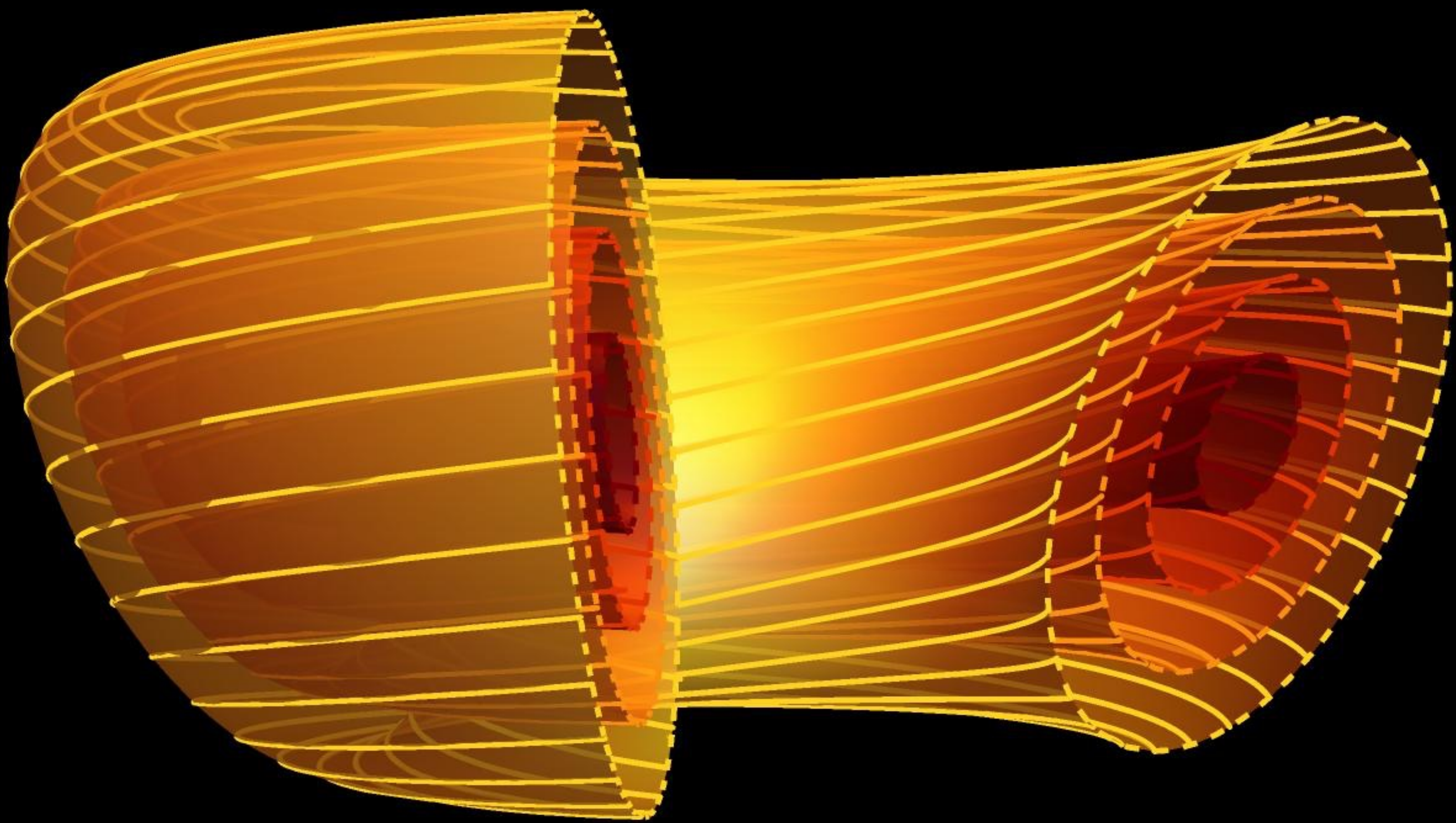
Up-down asymmetric tokamaks



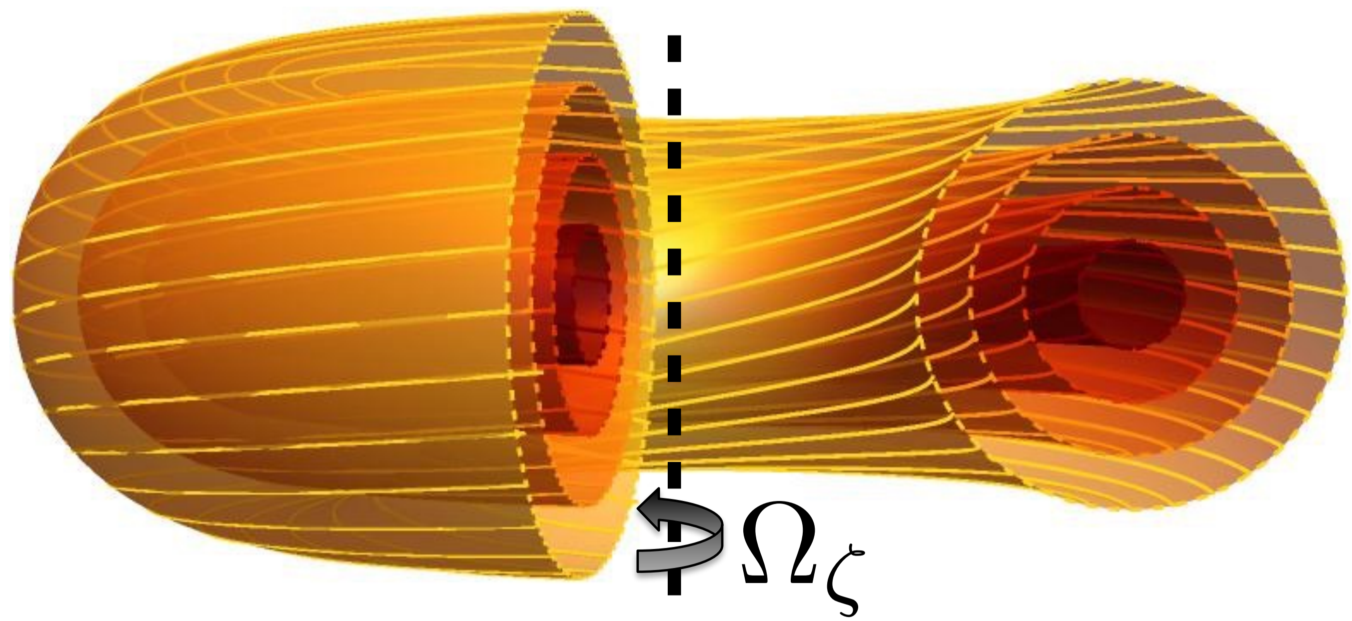
Justin Ball
Prof. Felix Parra and Prof. Michael Barnes
Oxford University and CCFE

9th Gyrokinetic Working Group Meeting
29 July 2016

The problem

Liu et al. *Nucl. Fusion* (2004).

- Toroidal plasma rotation has been used in experiments to significantly increase the plasma pressure



- The usual methods to drive rotation do not appear to scale well to larger devices such as ITER

- Numerical modeling suggests that, to see a benefit, ITER requires rotation with

$$M_A \equiv \frac{R\Omega_\zeta}{v_{\text{Alfvén}}} \approx 0.5 - 5\%^{*}$$

(on-axis)

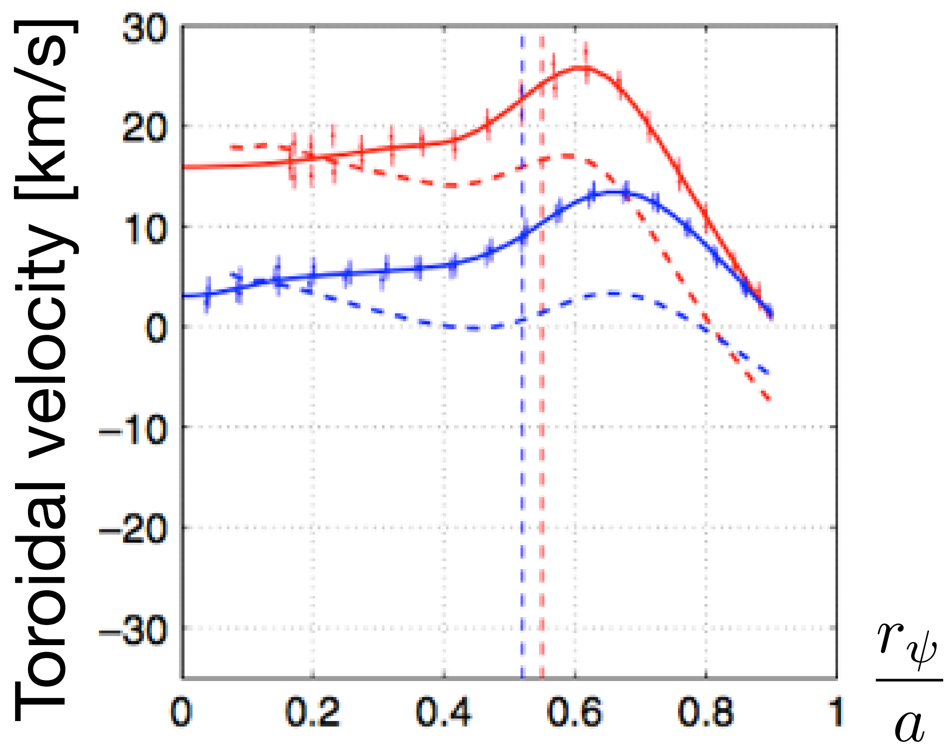
*multiply by ~10 to get

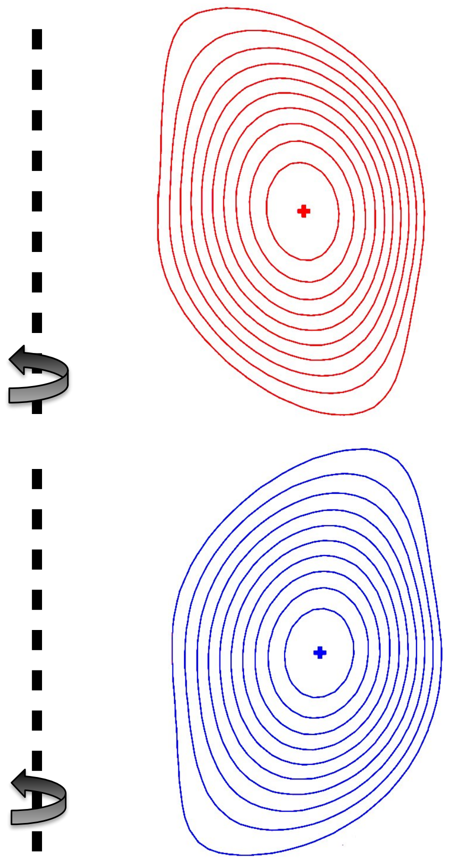
$$M_S \equiv \frac{R\Omega_\zeta}{v_{\text{sound}}}$$

The solution?

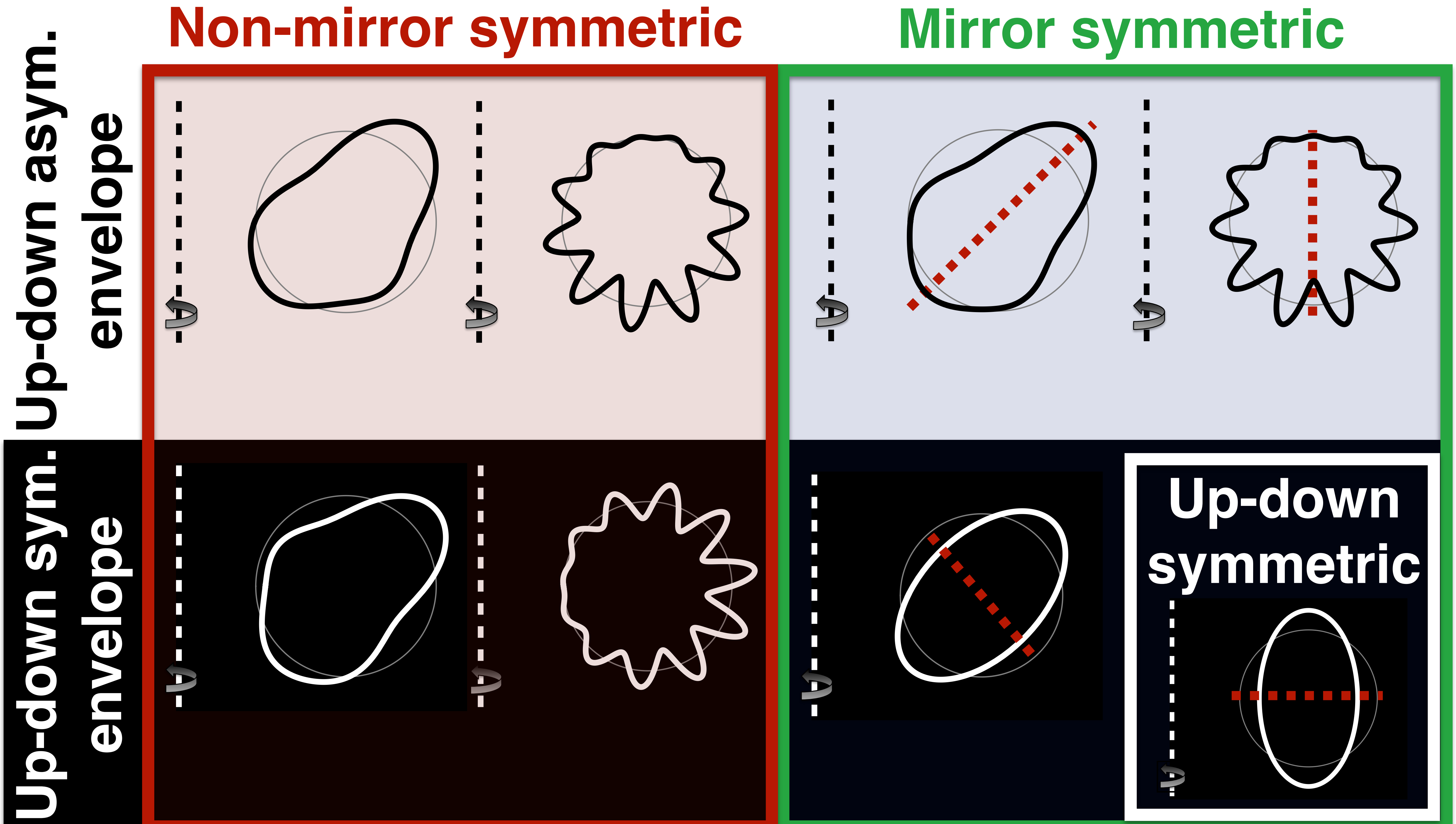
Camenen et al. *PPCF* (2010).

- Use plasma turbulence to transport momentum and spontaneously generate "intrinsic" rotation from a stationary plasma
- For future large devices, we need intrinsic rotation that scales with size
- As we will see, this severely restricts our options: up-down asymmetry in the magnetic equilibrium





Outline


Non-mirror symmetric
Mirror symmetric
Up-down asym. envelope
Up-down sym. envelope
Up-down symmetric

Generalization of Miller local equilibrium

Miller et al. *Phys. Plasmas* (1998).

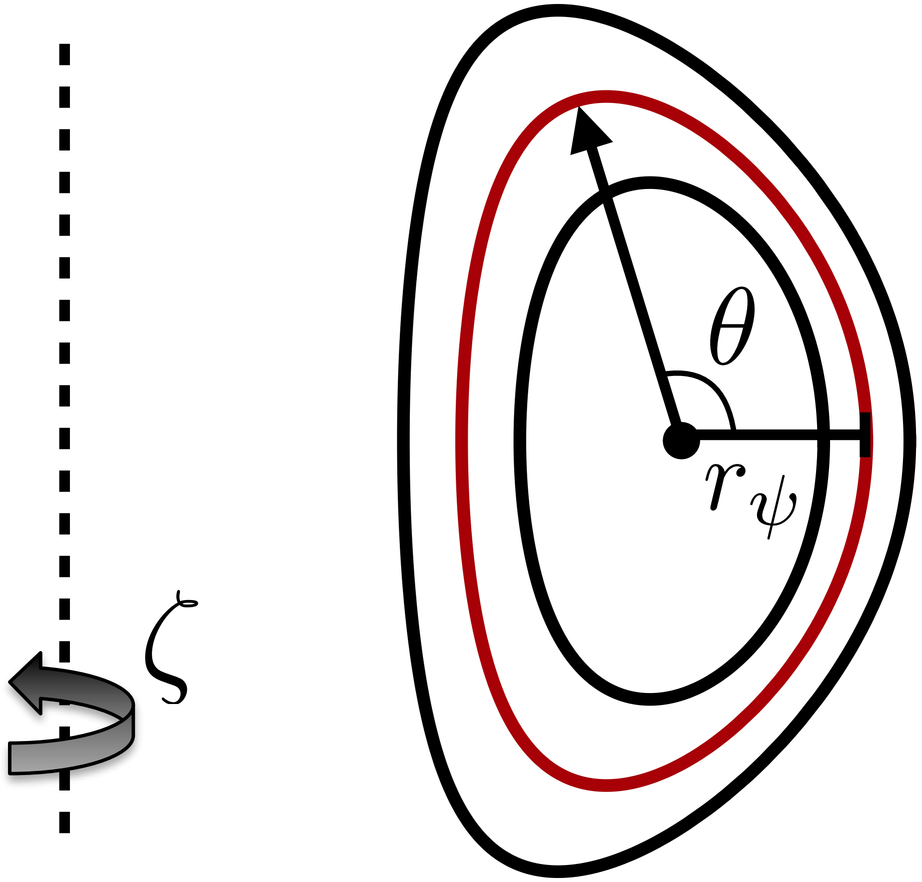
- Works well with GS2, a local δf gyrokinetic code

- Specify the flux surface of interest as a Fourier decomposition:

$$r_0(\theta) = r_{\psi 0}\left(1 - \sum_m C_m \cos\left(m\left(\theta + \theta_{tm}\right)\right)\right)$$

- Specify how it changes with minor radius:

$$\left.\frac{\partial r_0}{\partial r_\psi}\right|_\theta = 1 - \sum_m C'_m \cos\left(m\left(\theta + \theta'_{tm}\right)\right)$$



Gyrokinetics

- Governs turbulence in tokamaks:

$$\frac{\partial h_s}{\partial t} + v_{||}\hat{b}\cdot\vec{\nabla}\theta \left.\frac{\partial h_s}{\partial \theta}\right|_{v_{||}} + i\left(k_\psi v_{ds\psi} + k_\alpha v_{ds\alpha}\right) h_s + a_{||s}\frac{\partial h_s}{\partial v_{||}} - \sum_{s'}\langle C_{ss'}^{(l)}\rangle_\varphi + \left\{J_0\left(k_\perp \rho_s\right)\phi, h_s\right\}$$

$$= \frac{Z_s e F_{Ms}}{T_s}\frac{\partial}{\partial t}\left(J_0\left(k_\perp \rho_s\right)\phi\right) - v_{\phi s\psi} F_{Ms}\left[\frac{1}{n_s}\frac{dn_s}{d\psi} + \left(\frac{m_s v^2}{2T_s} - \frac{3}{2}\right)\frac{1}{T_s}\frac{dT_s}{d\psi}\right]$$

where $k_\perp = \sqrt{k_\psi^2\left|\vec{\nabla}\psi\right|^2 + 2k_\psi k_\alpha \vec{\nabla}\psi\cdot\vec{\nabla}\alpha + k_\alpha^2\left|\vec{\nabla}\alpha\right|^2}$

- Allows us to calculate the turbulent fluxes, such as

$$\Pi = 2\pi i I \sum_{k_\psi, k_\alpha} k_\alpha \left\langle \phi\left(k_\psi, k_\alpha\right)\int dv_{||}d\mu\; v_{||} J_0\left(k_\perp \rho_s\right) h_s\left(-k_\psi, -k_\alpha\right)\right\rangle_\psi$$

- Calculate the eight geometric coefficients from MHD equilibrium

Estimating the Alfvén Mach number

Ball et al. *PPCF* (2014).
Peeters et al. *PRL* (2007).

$$\left\langle \Pi\left(\Omega_\zeta, \frac{d\Omega_\zeta}{dr_\psi}\right)\right\rangle_t = 0$$

$$\langle Q_i \rangle_t = -D_Q \frac{dT_i}{dr_\psi}$$

ignore

$$\langle \Pi\,(0,0)\rangle_t - P_\Pi \Omega_\zeta - D_\Pi \frac{d\Omega_\zeta}{dr_\psi} = 0$$

$$Pr \equiv \frac{D_\Pi}{D_Q} \approx 1 \approx \text{constant}$$

$$M_A \approx \frac{\sqrt{2\beta_T}}{Pr} \frac{\langle \Pi \rangle_t}{\langle Q_i \rangle_t}$$

- Ignoring pinch is conservative, may enhance rotation by a factor of 3

Up-down symmetry argument

Peeters et al. *PoP* (2005). & Parra et al. *PoP* (2011).
Sugama et al. *PPCF* (2011).

- Negating k_ψ, θ, and $v_{||}$ leads to a second solution of the gyrokinetic eq.

$$Q_{\text{geo}}^{\text{ud}} \in \left\{ B, \hat{b}\cdot\vec{\nabla}\theta, v_{ds\psi}, v_{ds\alpha}, a_{||s}, \left|\vec{\nabla}\psi\right|^2, \vec{\nabla}\psi\cdot\vec{\nabla}\alpha, \left|\vec{\nabla}\alpha\right|^2 \right\}$$
$$\rightarrow \left\{ B, \hat{b}\cdot\vec{\nabla}\theta, -v_{ds\psi}, v_{ds\alpha}, -a_{||s}, \left|\vec{\nabla}\psi\right|^2, -\vec{\nabla}\psi\cdot\vec{\nabla}\alpha, \left|\vec{\nabla}\alpha\right|^2 \right\}$$

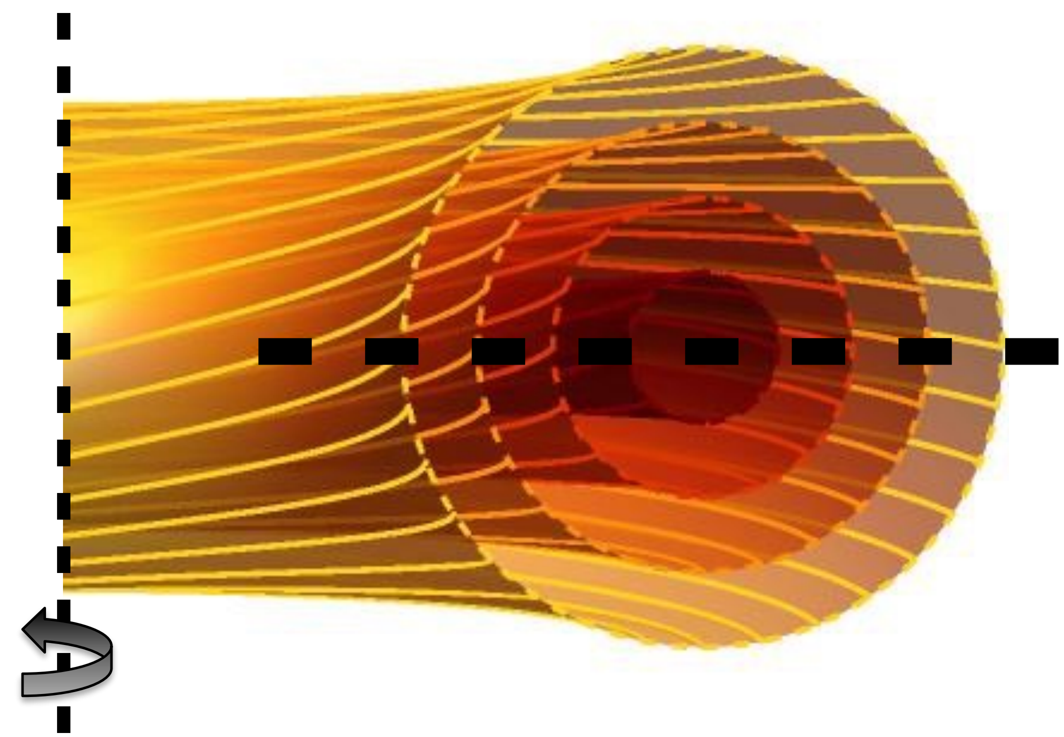
$$h_s\left(k_\psi, k_\alpha, \theta, v_{||}, \mu, t\right) \rightarrow -h_s\left(-k_\psi, k_\alpha, -\theta, -v_{||}, \mu, t\right)$$

- Second solution has a canceling momentum flux:

$$\left\langle \Pi \right\rangle_t \rightarrow -\left\langle \Pi \right\rangle_t$$

$$\Rightarrow \left\langle \Pi \right\rangle_t = 0$$

- Constrains $M_A = 0$ to lowest order in $\rho_* \equiv \rho_i / a \ll 1$



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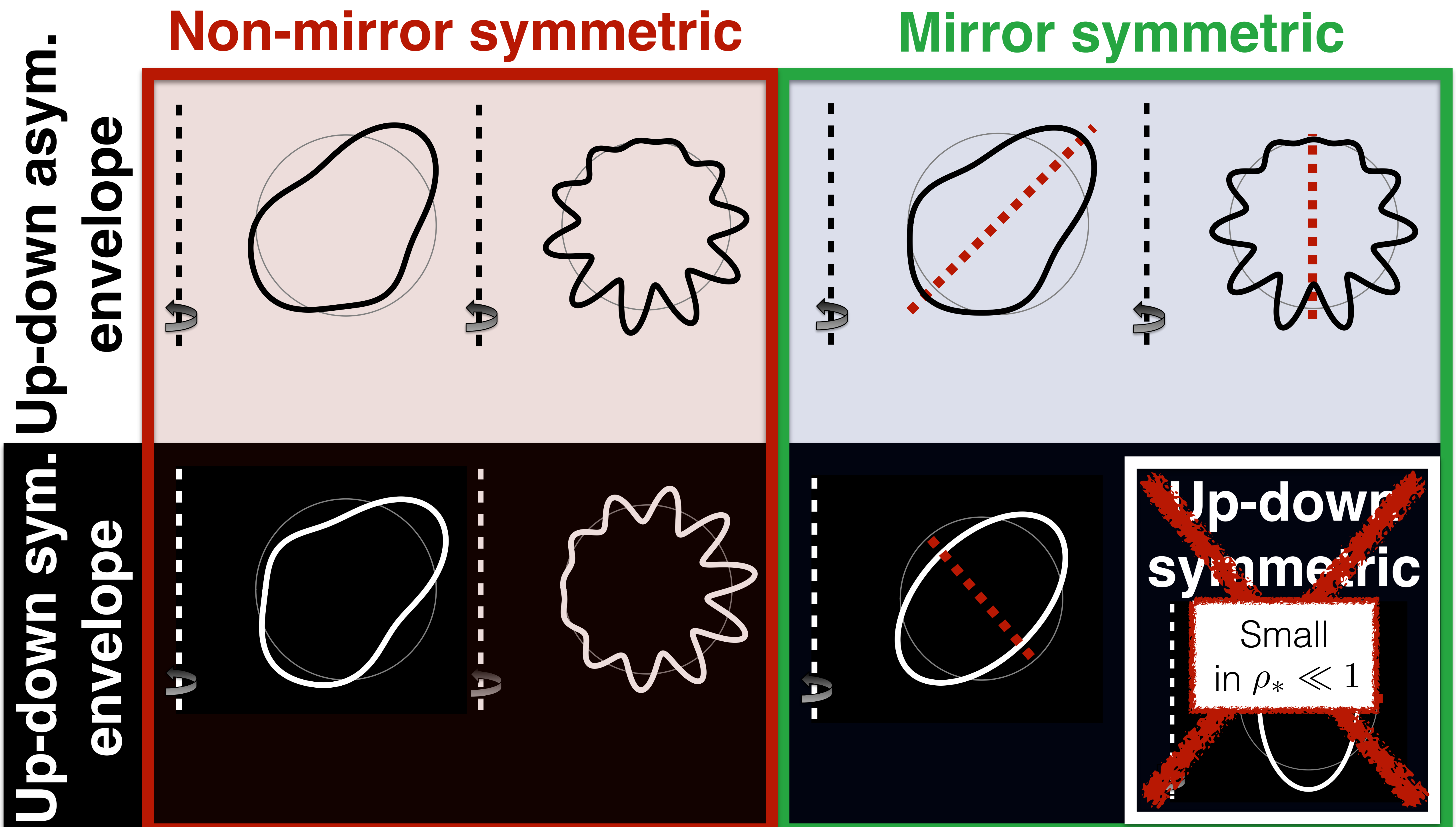


Non-mirror symmetric
Mirror symmetric
Up-down asym. envelope
Up-down sym. envelope
Up-down symmetric
Small in $\rho_* \ll 1$

MHD equilibrium argument

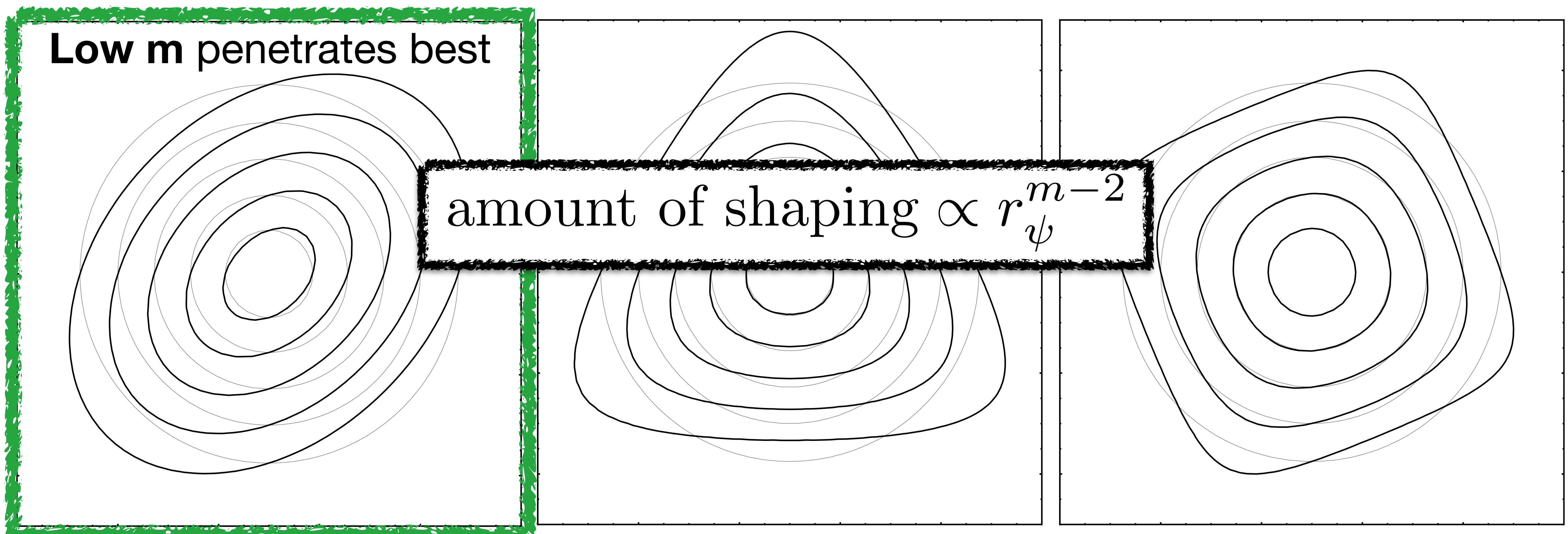
Rodrigues et al. *Nucl. Fusion* (2014).
Ball et al. *PPCF* (2015).

- Grad-Shafranov equation for a constant toroidal current profile:

$$R^2 \vec{\nabla} \cdot \left(\frac{\vec{\nabla}\psi}{R^2} \right) = -\mu_0 R^2 \frac{dp}{d\psi} - I \frac{dI}{d\psi} = \text{const}$$

- To lowest order in aspect ratio, solutions are cylindrical harmonics:

m=2 mode　　m=3 mode　　m=4 mode



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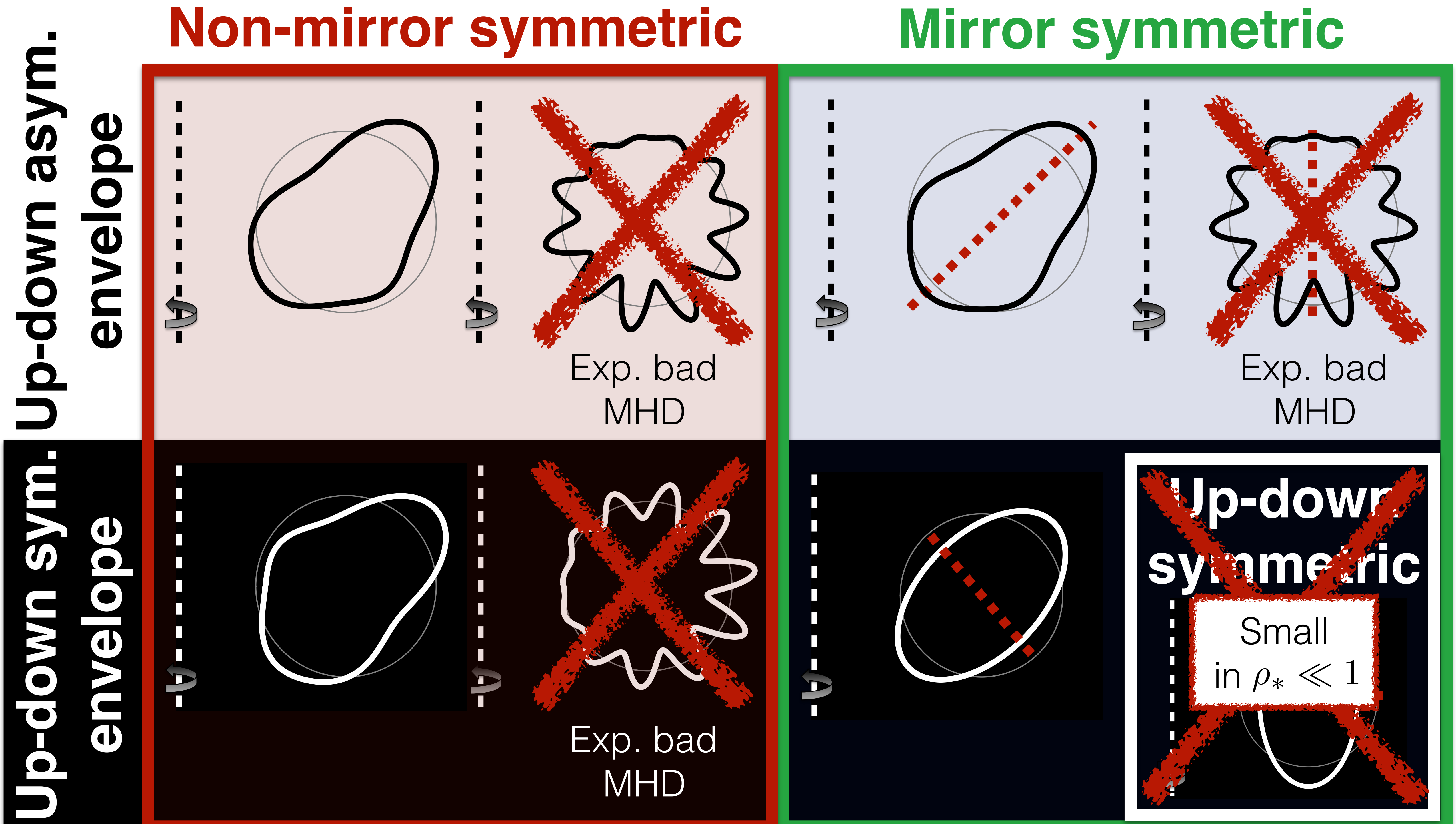
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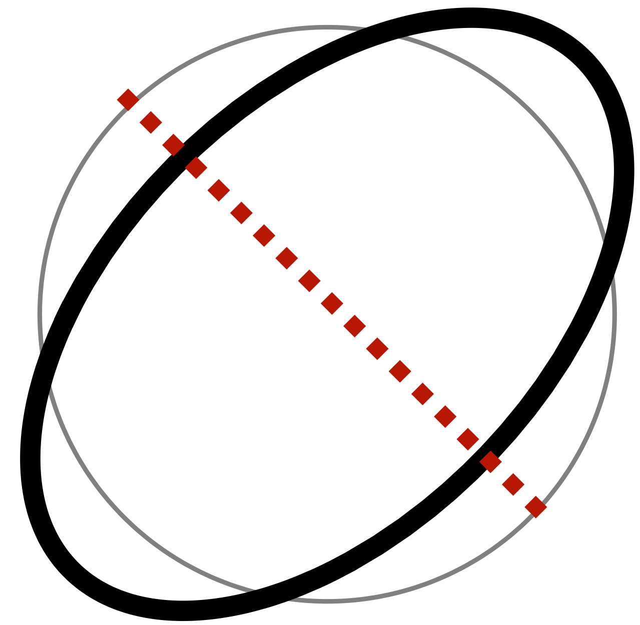
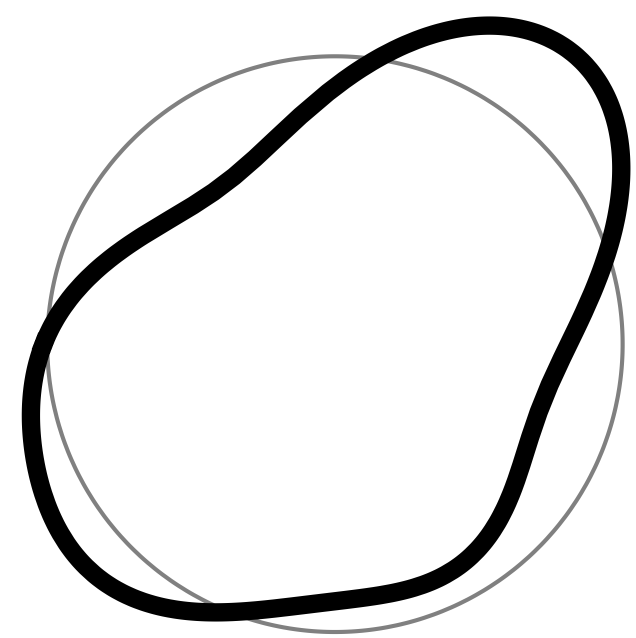
Screw pinch argument

- Screw pinches have no toroidicity, so up-down symmetry has no meaning
- Mirror symmetric flux surfaces generate no rotation
- Rotation can be generated by breaking mirror symmetry (i.e. the direct interaction of two different shaping effects)
- This can occur in tokamaks

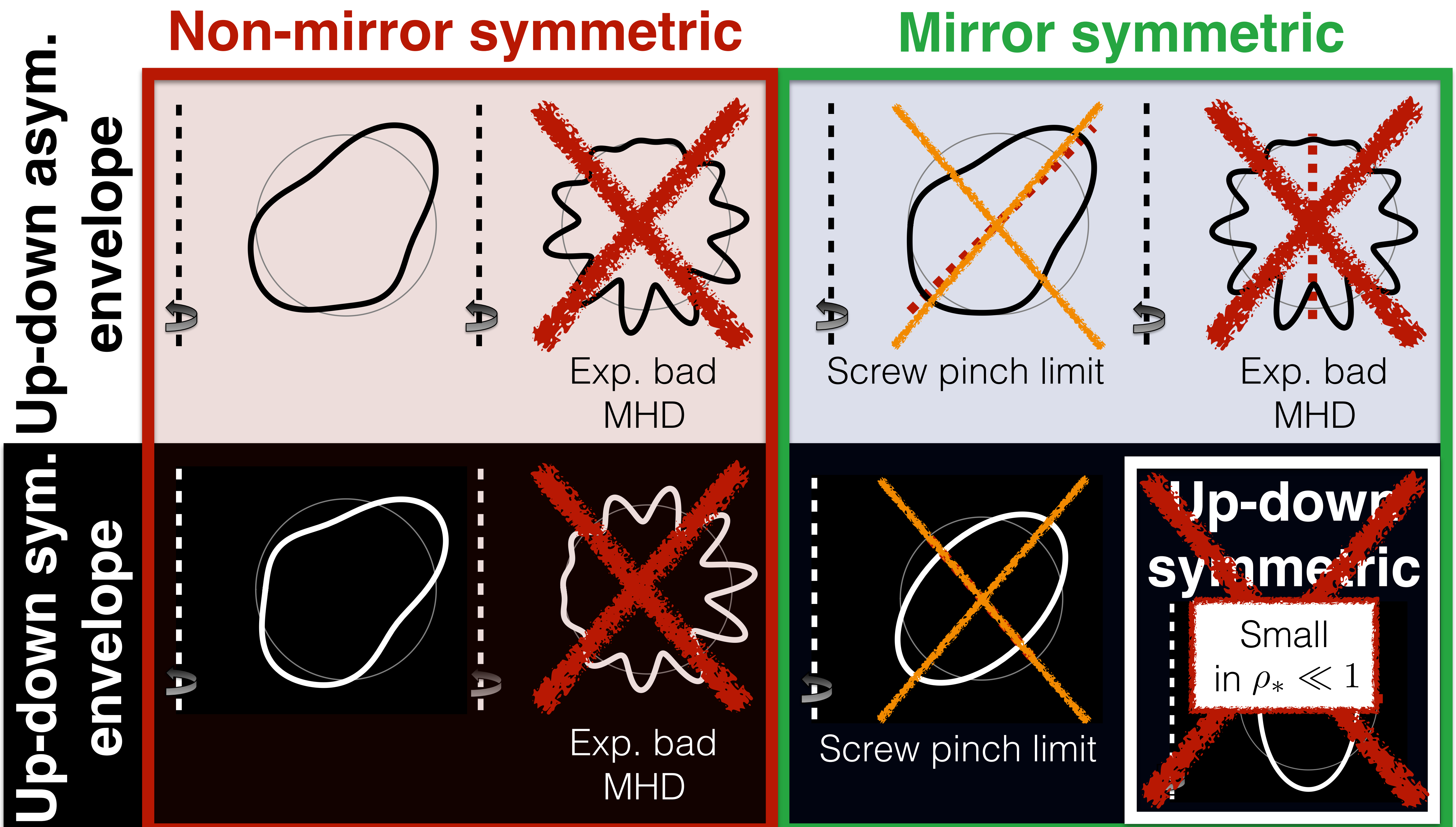


$M_A = 0$



$M_A \neq 0$

Outline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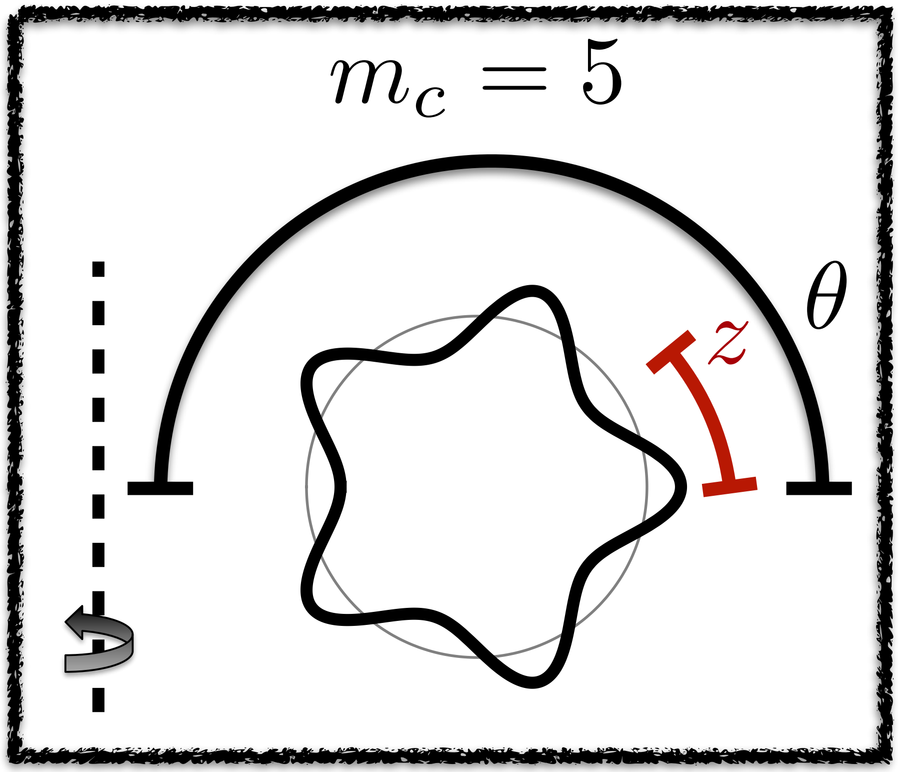
Up-down symmetric
Small in $\rho_* \ll 1$

Poloidal tilting symmetry argument

Ball, et al. *PPCF* **58** 045023 (2016).

- Rewrite geometry specification to distinguish $z \equiv m_c\theta$ (the fast poloidal scale) from θ (the connection length scale):

$$r_0(\theta) = r_{\psi 0}\left(1 - \sum_m C_m \cos\left(m\left(\theta + \theta_{tm}\right)\right)\right)$$



- Convert to the form of a 2-D Fourier series using $k \equiv m - lm_c$

$$r_0(\theta, z) = r_{\psi 0}\Bigg(1 - \sum_{l=0}^{\infty}\sum_{k=0}^{m_c-1} C_{k+lm_c} \times \left[\cos\left(l\left(z + m_c\theta_{tm}\right)\right)\cos\left(k\left(\theta + \theta_{tm}\right)\right) - \sin\left(l\left(z + m_c\theta_{tm}\right)\right)\sin\left(k\left(\theta + \theta_{tm}\right)\right)\right]\Bigg)$$

- Define $l \equiv \lfloor m/m_c \rfloor$ according to the physics of the scale separation (defines any mode $m \geq m_c$ as “fast”)

Poloidal tilting symmetry argument

Ball, et al. *PPCF* **58** 045023 (2016).

- Specify $r_0^{\text{tilt}}(\theta, z) = r_0^{\text{ud}}(\theta, z + z_{\text{tilt}})$

$$Q_{\text{geo}}^{\text{ud}}(\theta, z) \in \left\{ B, \hat{b} \cdot \vec{\nabla}\theta, v_{ds\psi}, v_{ds\alpha}, a_{||s}, \left|\vec{\nabla}\psi\right|^2, \vec{\nabla}\psi \cdot \vec{\nabla}\alpha, \left|\vec{\nabla}\alpha\right|^2 \right\}$$

$$Q_{\text{geo}}^{\text{tilt}} = Q_{\text{geo}}^{\text{ud}}(\theta, z + z_{\text{tilt}})$$

$$\Rightarrow h_s^{\text{tilt}}(\theta, z) = h_s^{\text{ud}}(\theta, z + z_{\text{tilt}})$$

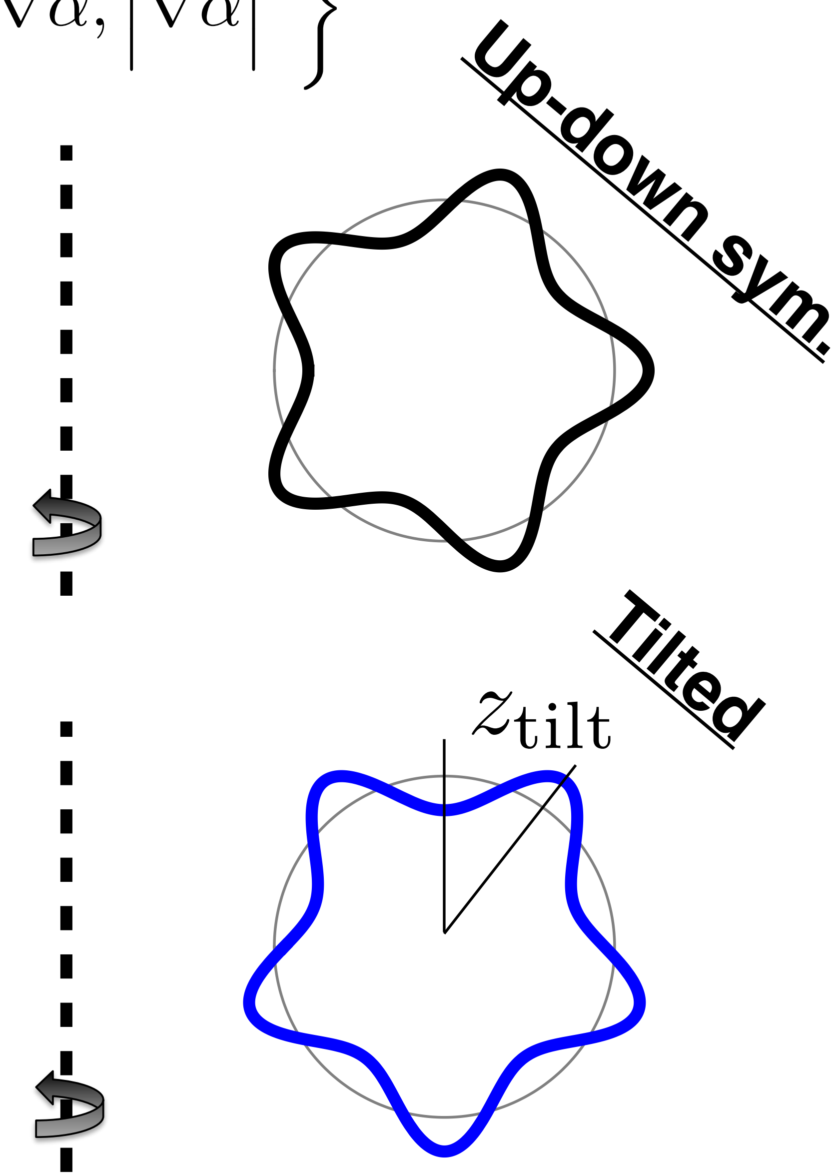
$$\left\langle \left\langle \Pi^{\text{tilt}} \right\rangle_t \right\rangle_z \neq \left\langle \left\langle \Pi^{\text{ud}} \right\rangle_t \right\rangle_z = 0$$

- But remember we expanded in $m_c \gg 1$

$$\Rightarrow \left\langle \left\langle \Pi^{\text{tilt}} \right\rangle_t \right\rangle_z \sim M_A \sim \exp(-m_c)$$

- Verify by looking for

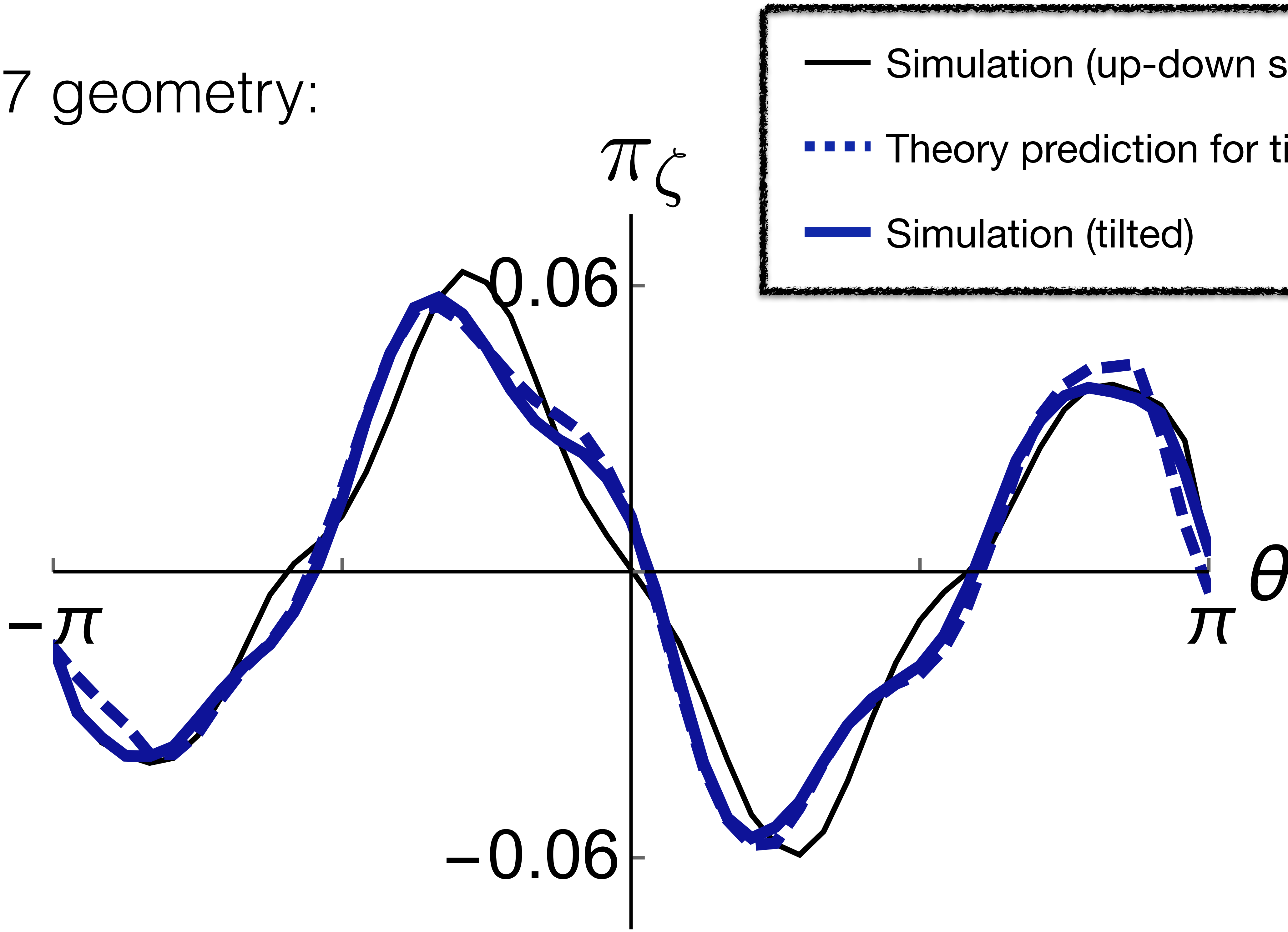
$$\pi_\zeta^{\text{tilt}}(\theta, z) = \pi_\zeta^{\text{ud}}(\theta, z + z_{\text{tilt}})$$



Verify $\pi_\zeta^{\mathrm{tilt}}(\theta, z) = \pi_\zeta^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$

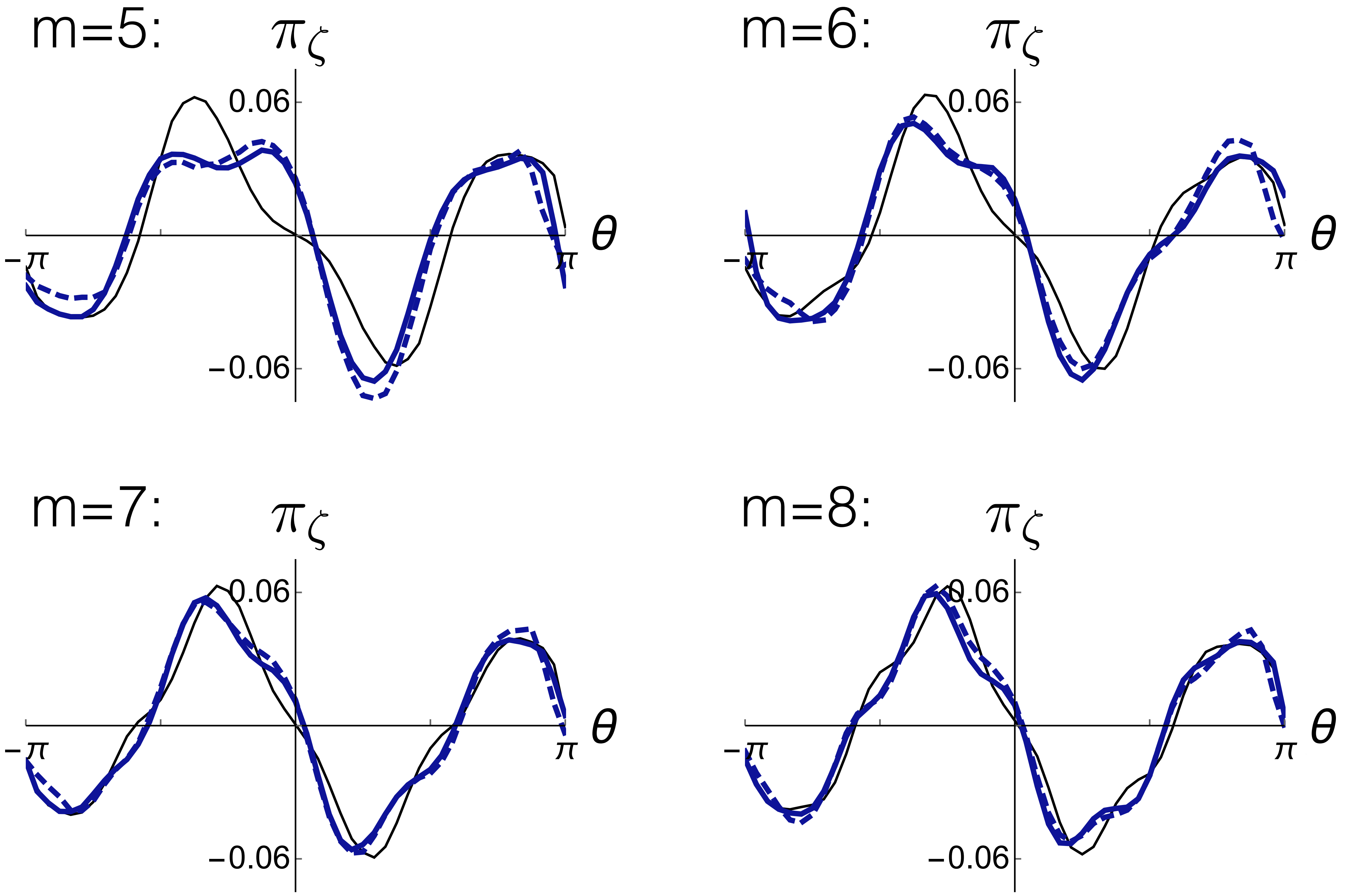
Ball, et al. *PPCF* **58** 045023 (2016).

m=7 geometry:

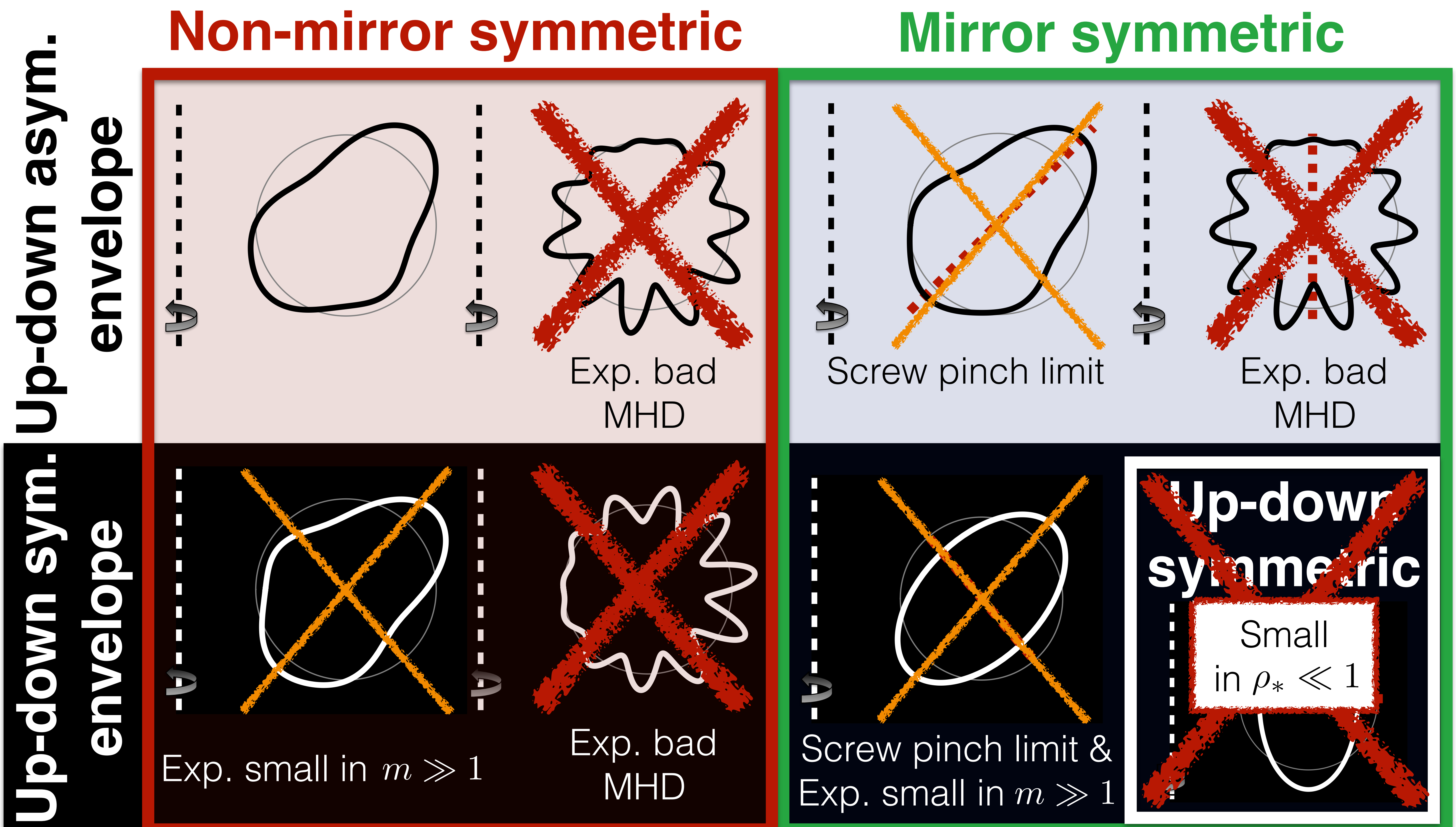


Verify $\pi_\zeta^{\text{tilt}}(\theta, z) = \pi_\zeta^{\text{ud}}(\theta, z + z_{\text{tilt}})$

Ball, et al. *PPCF* **58** 045023 (2016).



Outline



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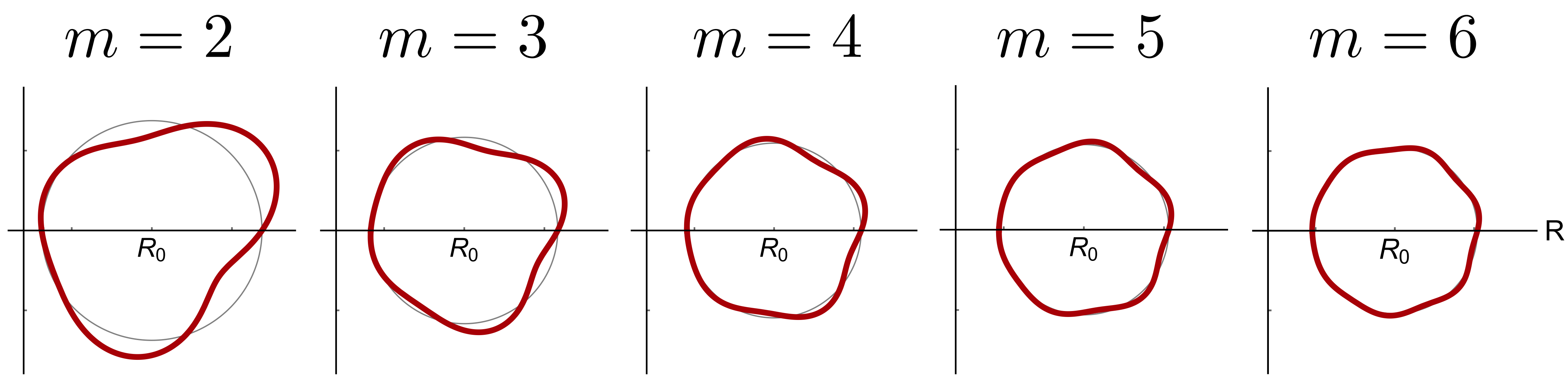
Small in $\rho_* \ll 1$

Envelope argument: Expansion in $m \gg 1$

Ball, et al. *PPCF* **58** *055016* (2016).



- Created using two modes, m and $n = m + 1$, with distinct tilt angles, θ_{tm} and θ_{tn}
- Calculate geometric coefficients order-by-order in $m \gg 1$
- Look for beating between fast shaping effects (creates an envelope on the connection length)

Envelope argument: Expansion in $m \gg 1$

Ball, et al. *PPCF* **58** *055016* (2016).

- Calculate magnetic drift coefficient within flux surface, $v_{ds\alpha}$

$$v_{ds\alpha} = \frac{B_0}{R_0\Omega_s}\frac{dr_\psi}{d\psi}\left(\cos(\theta) + \hat{s}\theta\sin(\theta)\right)$$
$$+ \frac{B_0}{2R_0\Omega_s}\frac{dr_\psi}{d\psi}\left[\left(m^3C_m^2 + n^3C_n^2\right)\theta\sin(\theta)\right.$$
$$- \hat{s}\theta\cos(\theta)\left(mC_m\sin(m(\theta-\theta_{tm})) + nC_n\sin(n(\theta-\theta_{tn}))\right)$$
$$+ \hat{s}\theta\sin(\theta)\left(mC_m\cos(m(\theta-\theta_{tm})) + nC_n\cos(n(\theta-\theta_{tn}))\right)$$
$$+ mn\frac{m+n}{n-m}C_mC_n\sin(\theta)$$
$$\times\left(\sin((n-m)\theta)\cos(n\theta_{tn}-m\theta_{tm}) + \cos((n-m)\theta)\sin(n\theta_{tn}-m\theta_{tm})\right)$$

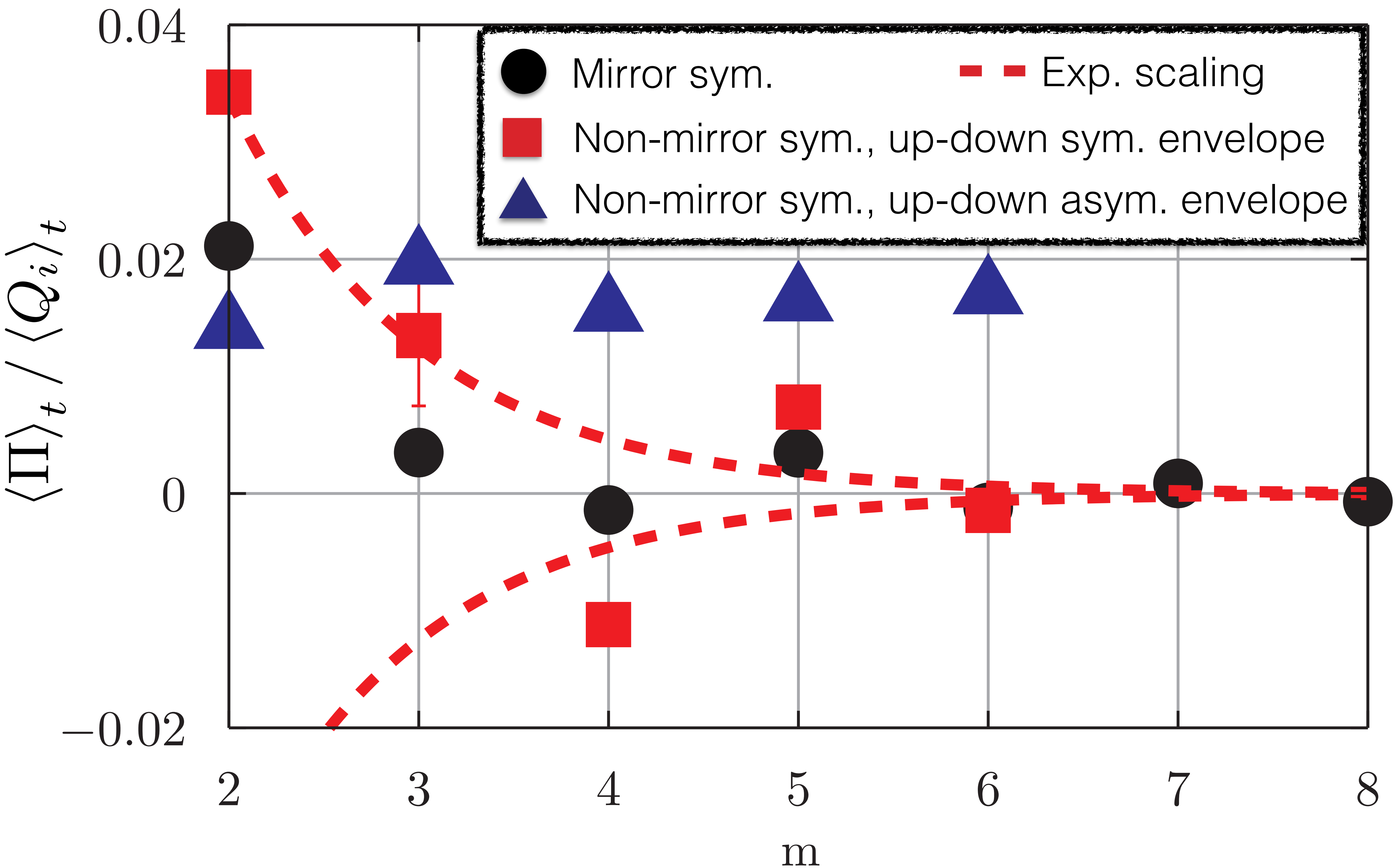
0

breaks symmetry $\sim m^{-1}$

$$\Rightarrow \frac{\langle\Pi\rangle_t}{\langle Q_i\rangle_t} \sim m^{-1} \Rightarrow M_A \propto m^{-1}$$

Envelope argument: Numerical scaling with $m \gg 1$

Ball, et al. *PPCF* **58** *055016* (2016).



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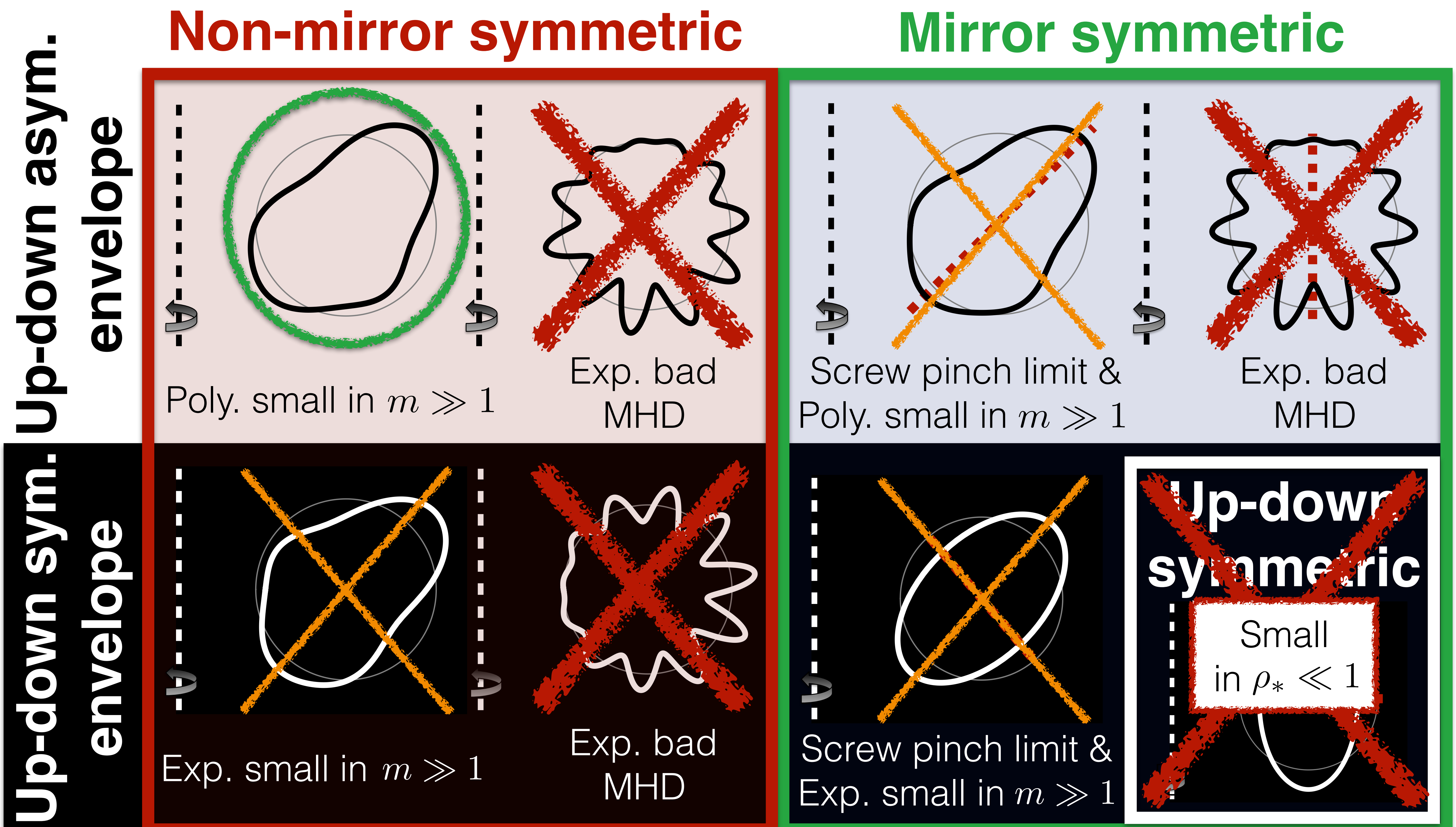

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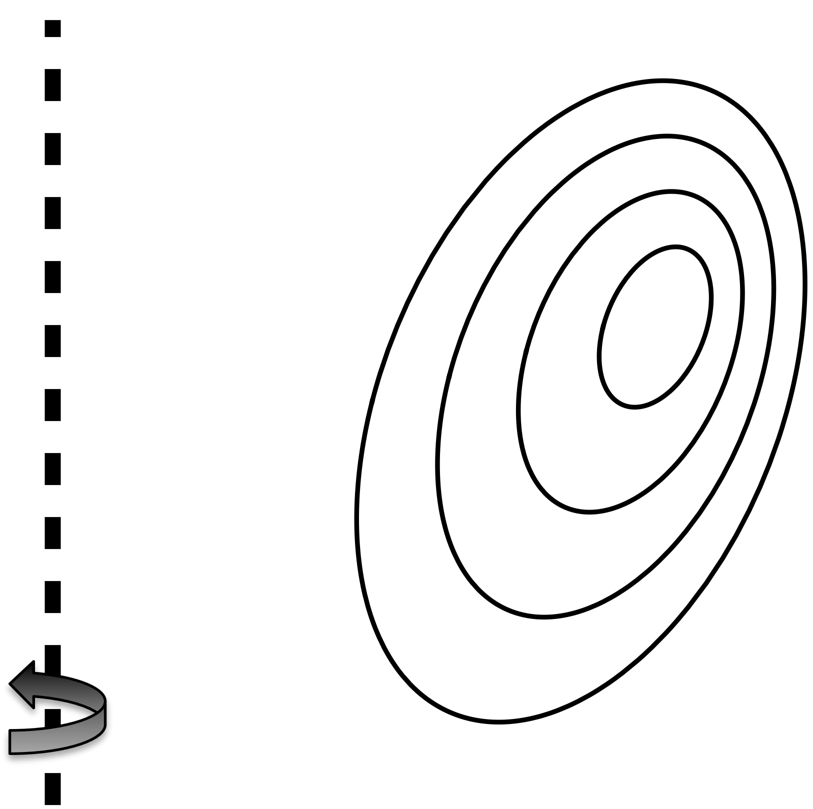
Up-down symmetric
Small in $\rho_* \ll 1$

Two options to maximize rotation

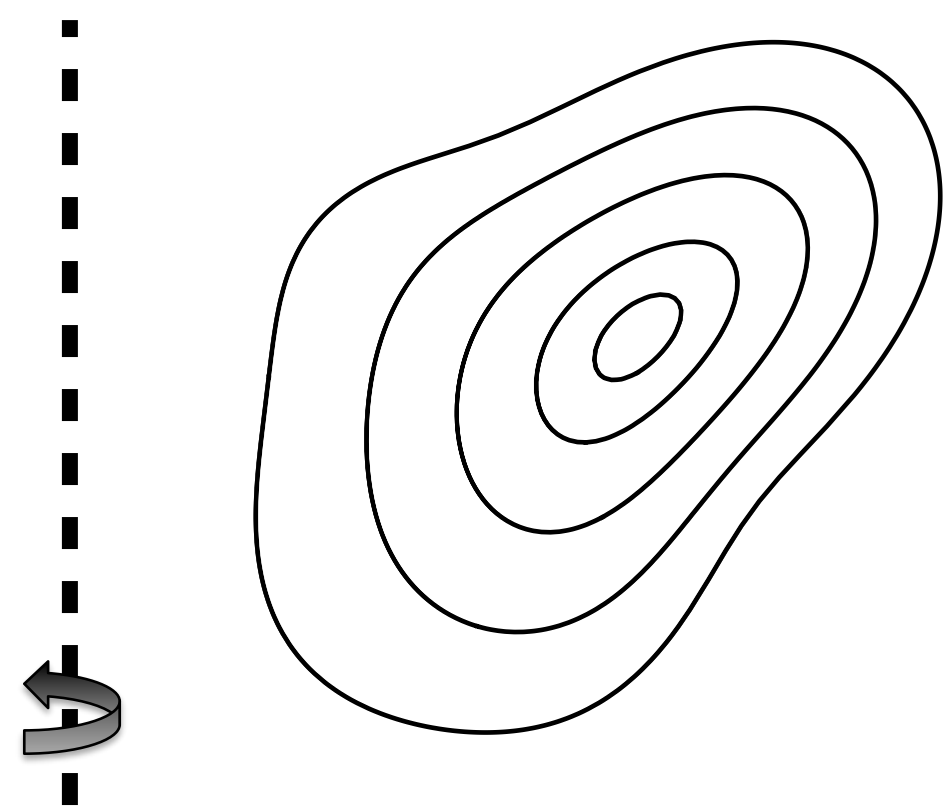
- Use lowest m possible to break mirror symmetry and create up-down asymmetric envelopes

- Prefer modes to be controllable by external shaping magnets

Shafranov shift & elongation
(i.e. $m = 1$ and $m = 2$)



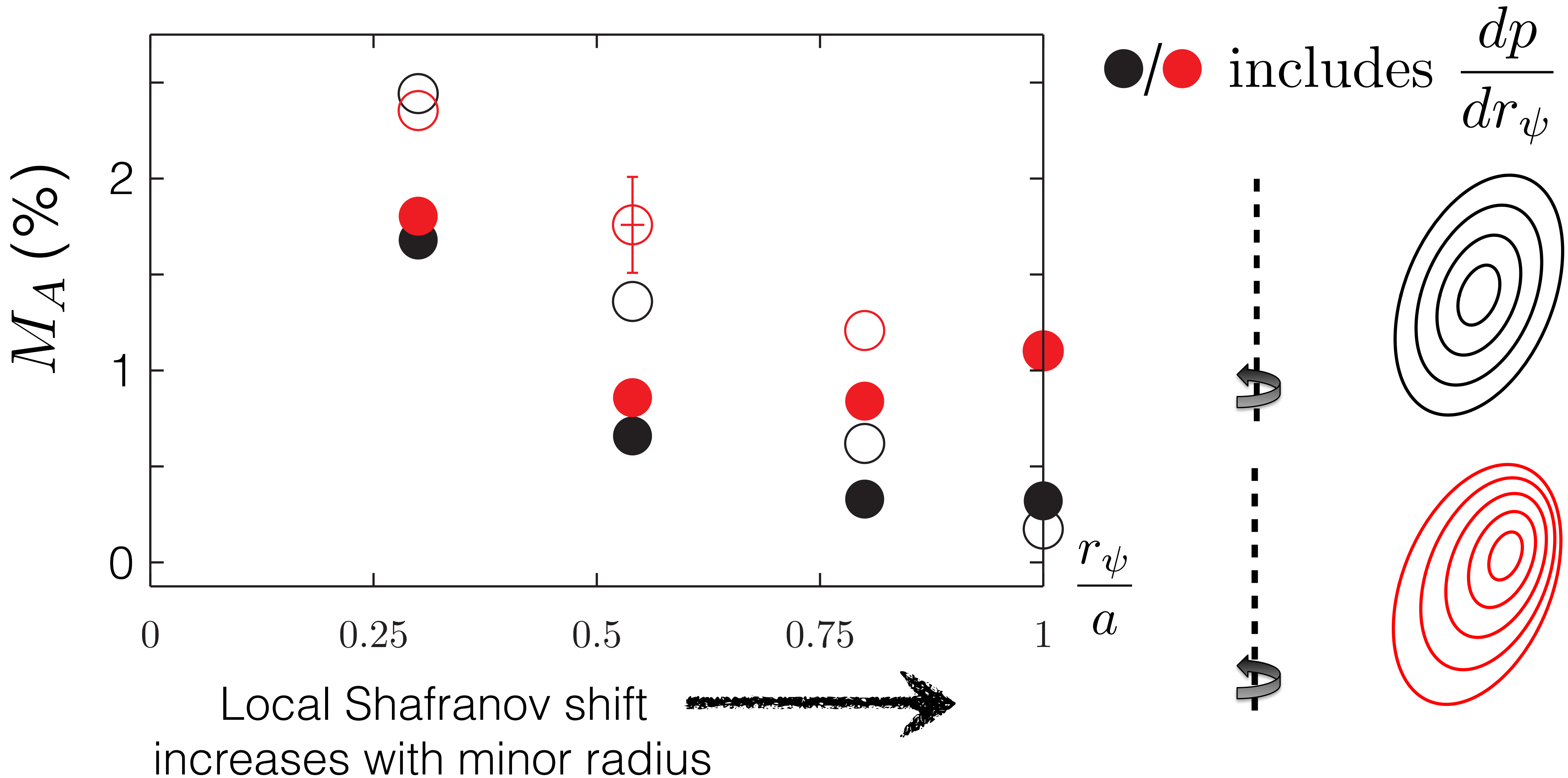
Elongation & triangularity
(i.e. $m = 2$ and $m = 3$)



Momentum flux from Shafranov shift

Ball, et al. *arXiv*:1607.06387 (2016).

- Reduced by including effect of a constant dp/dr_ψ, which affects the magnetic equilibrium through the Grad-Shafranov equation



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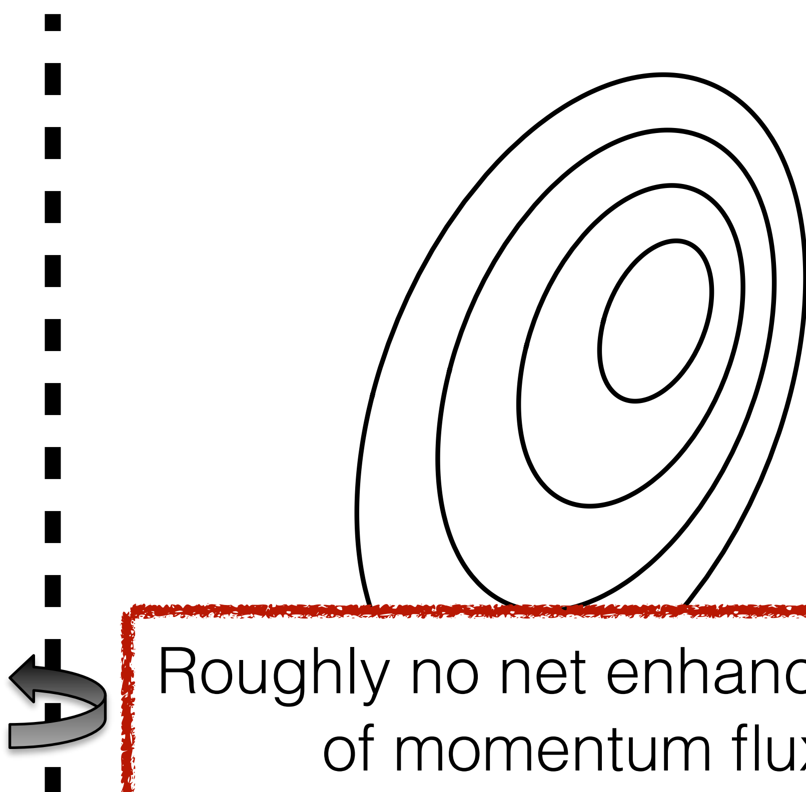
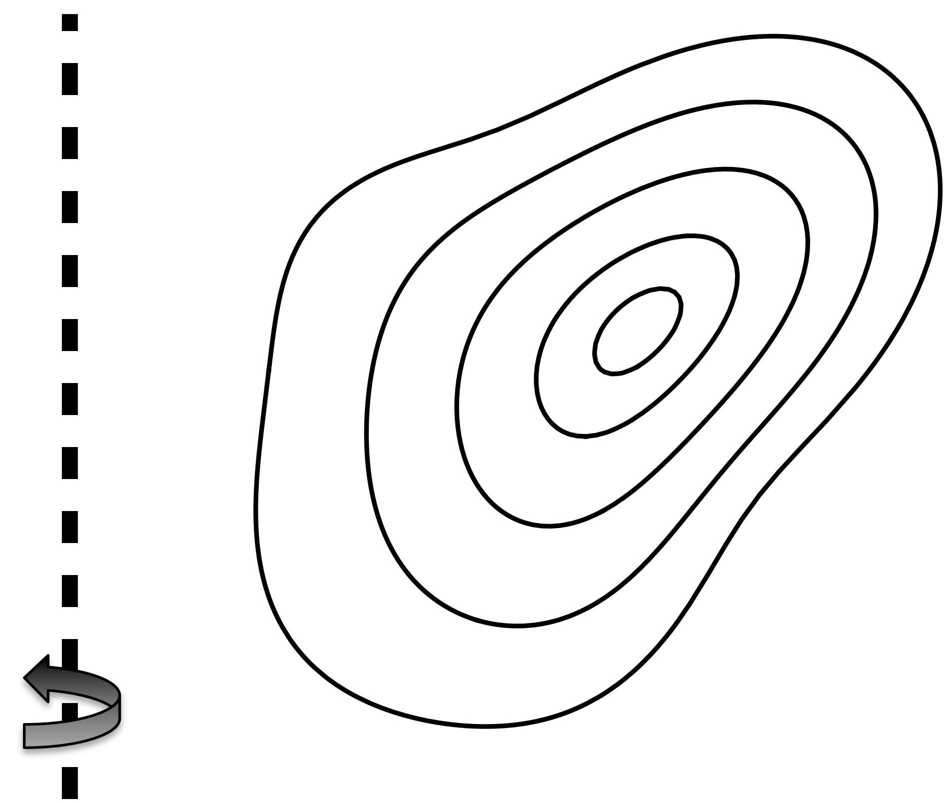


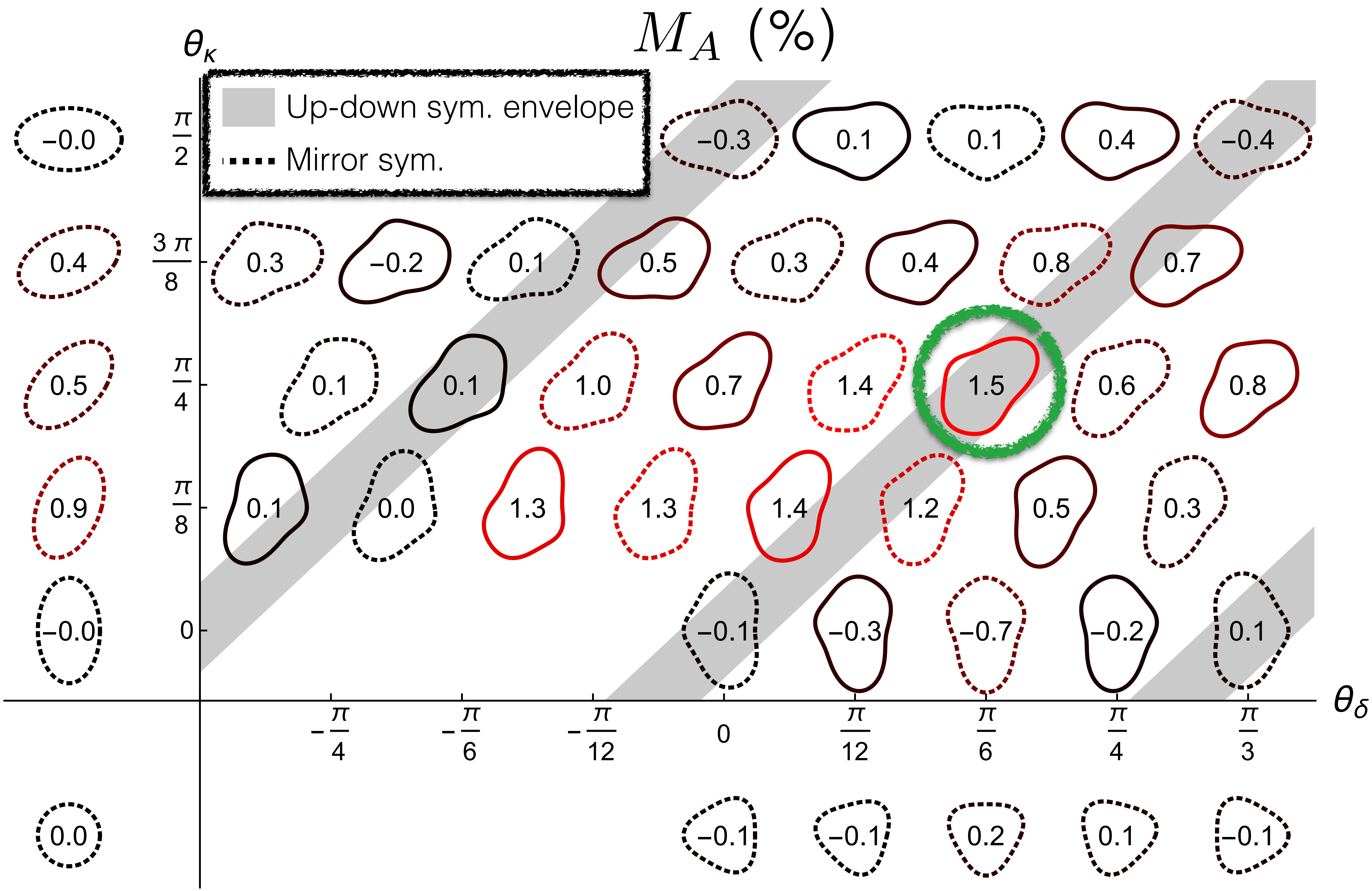
Roughly no net enhancement of momentum flux

Elongation & triangularity

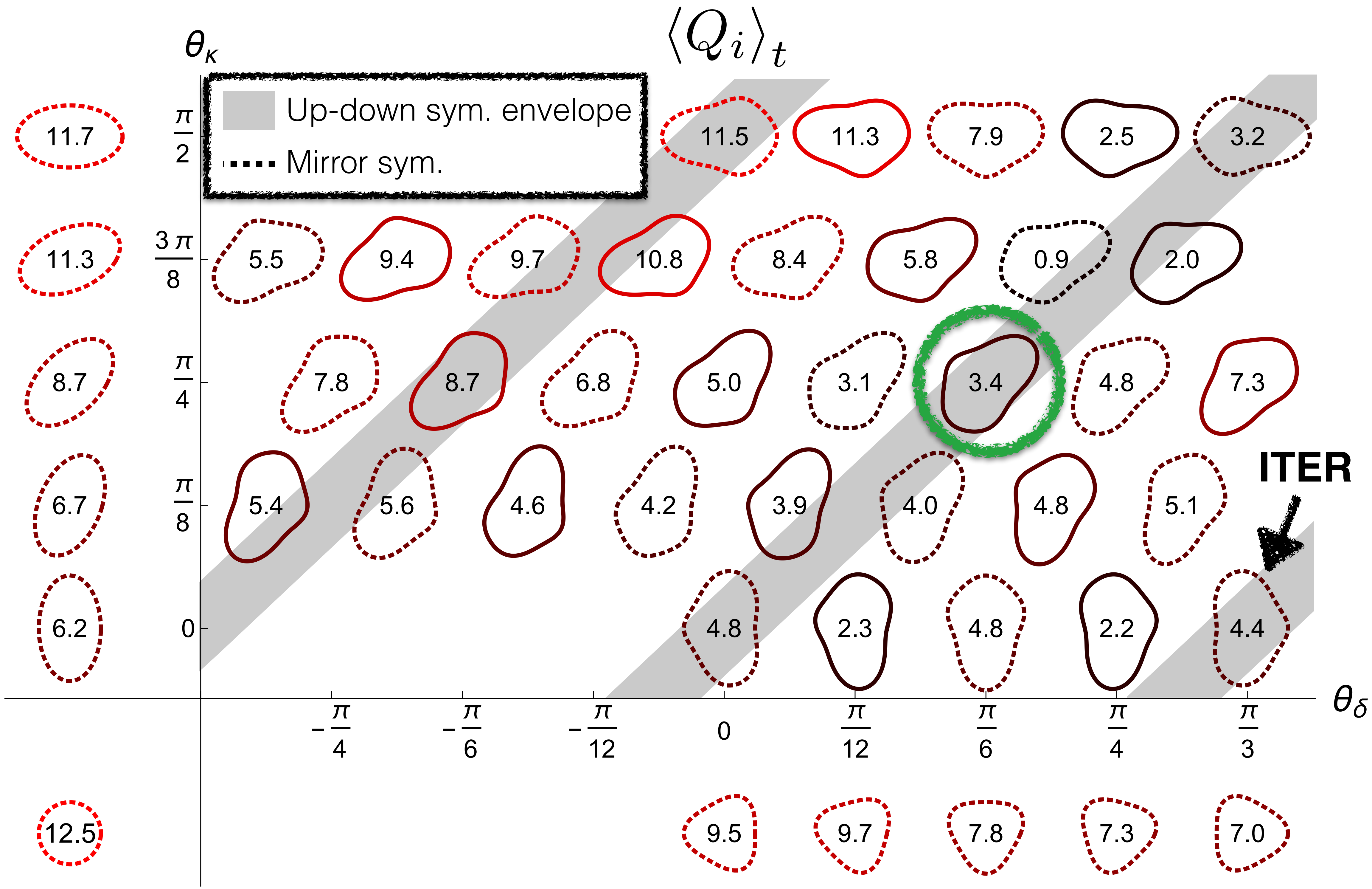
(i.e. $m = 2$ and $m = 3$)



Momentum flux from elongation and triangularity



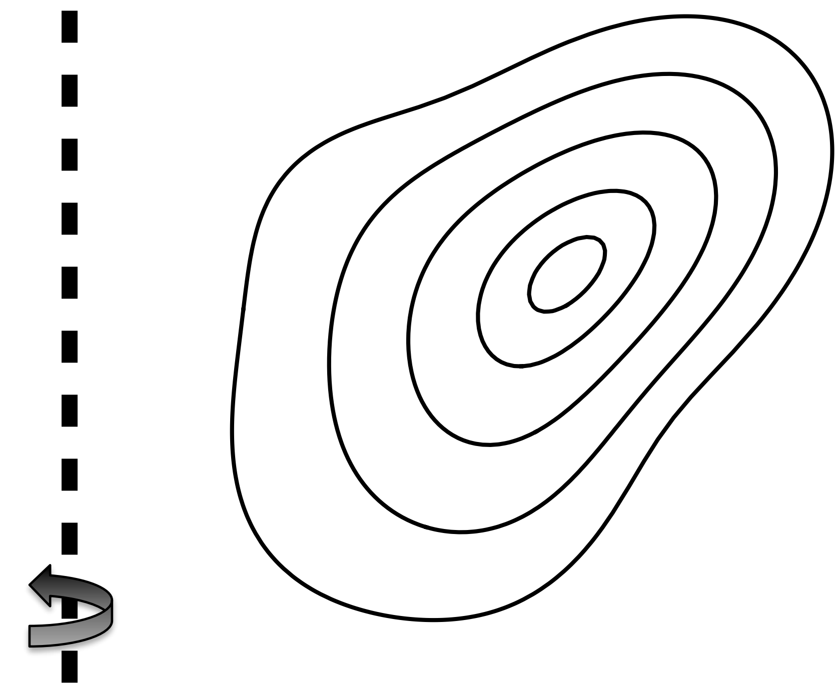
Momentum flux from elongation and triangularity



Conclusions

- Intrinsic rotation generated by up-down asymmetry scales well to larger machines (ITER, DEMO, etc.), unlike other mechanisms
- Tilting the elongation of flux surfaces is a simple way to generate significant rotation
  - The magnitude of rotation is roughly what is needed to permit increasing the plasma pressure in ITER
- Breaking ALL the symmetries, especially with external shaping, can boost the rotation significantly

The "optimal" geometry



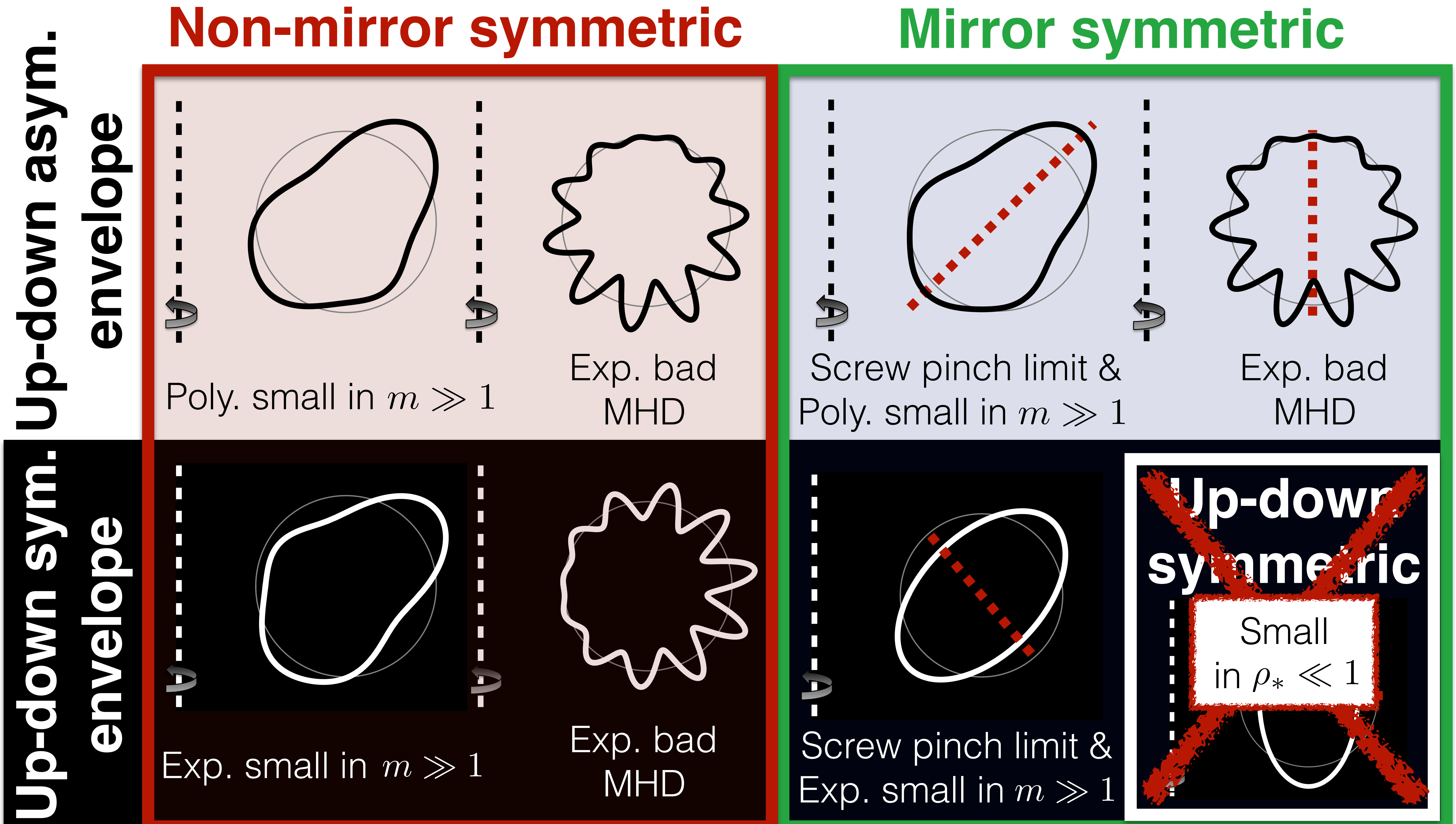
$$\kappa = 1.7 \quad \theta_\kappa = \pi/4$$

$$\underbrace{\delta = 0.35 \quad \theta_\delta = \pi/6}$$

$M_A \approx 1.5\%$ in ITER

Thank you!

Summary



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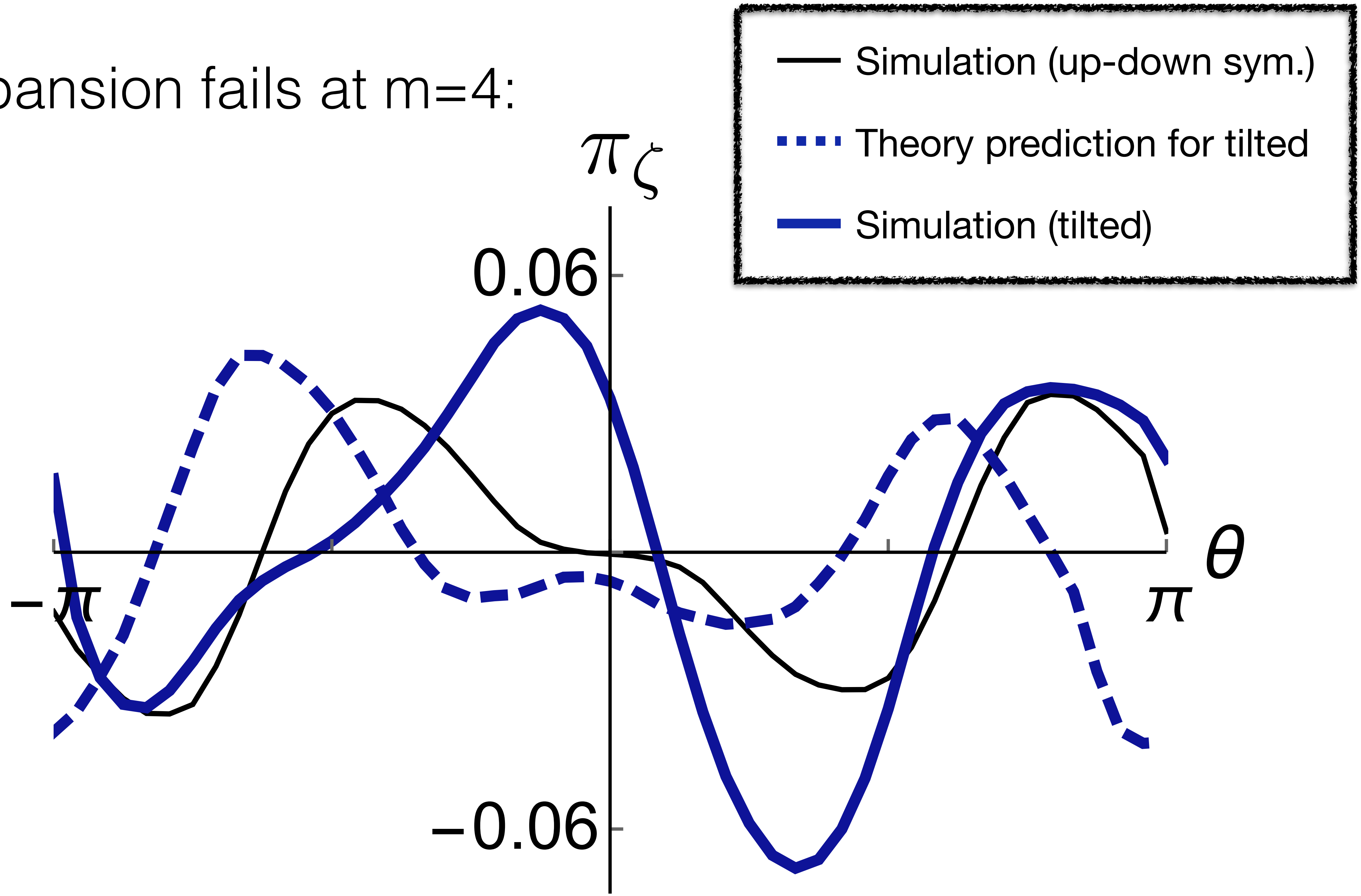
Extra Slides

Verify $\pi_\zeta^{\mathrm{tilt}}(\theta, z) = \pi_\zeta^{\mathrm{ud}}(\theta, z + z_{\mathrm{tilt}})$

Ball, et al. *PPCF* **58** 045023 (2016).

Expansion fails at m=4:



Breakdown in poloidal tilting symmetry

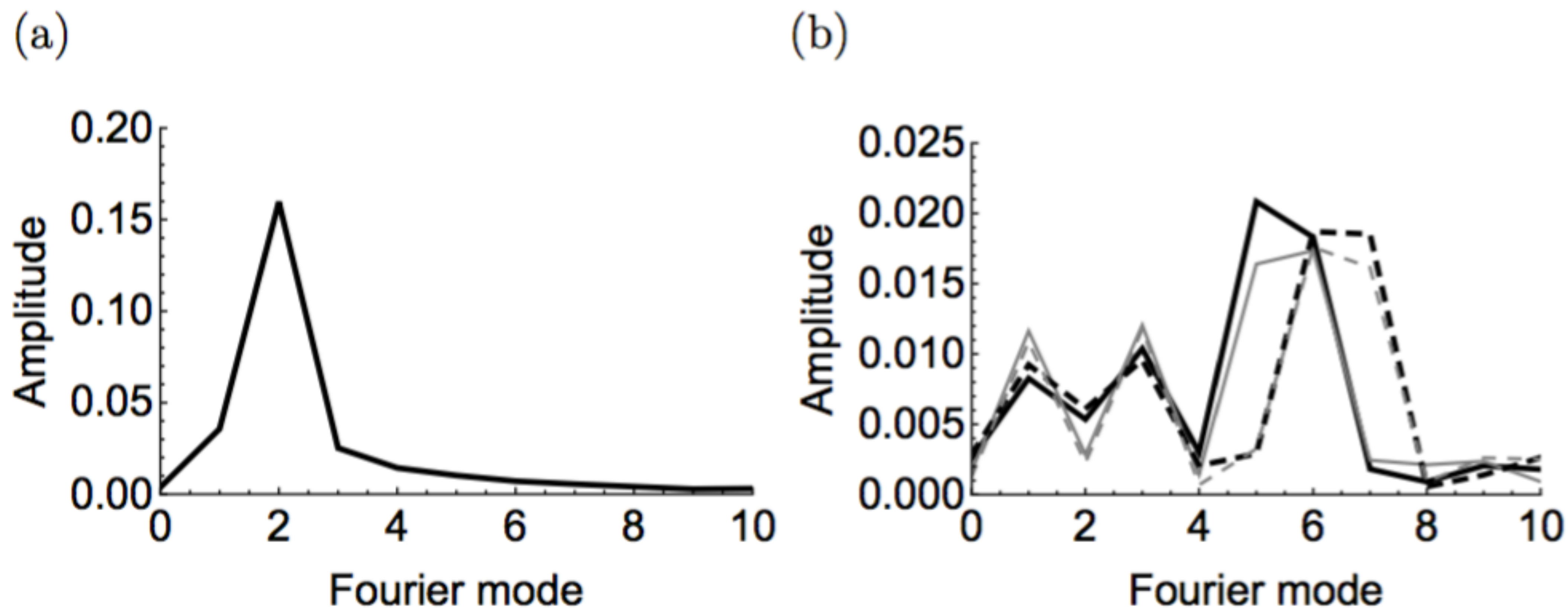
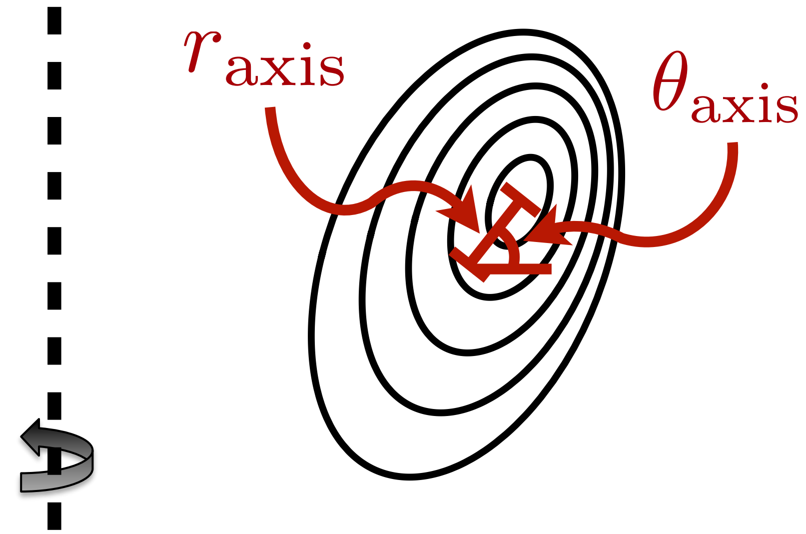


Figure 7.3: (a) The Fourier spectrum of the poloidal distribution of the ion momentum flux generated in circular flux surfaces.
(b) The Fourier spectrum of the poloidal distribution of ion momentum flux after subtracting the flux generated by circular flux surfaces (shown in (a)) for up-down symmetric (grey) and tilted (black) configurations in the $m_c = 7$ (solid) and $m_c = 8$ (dashed) geometries.

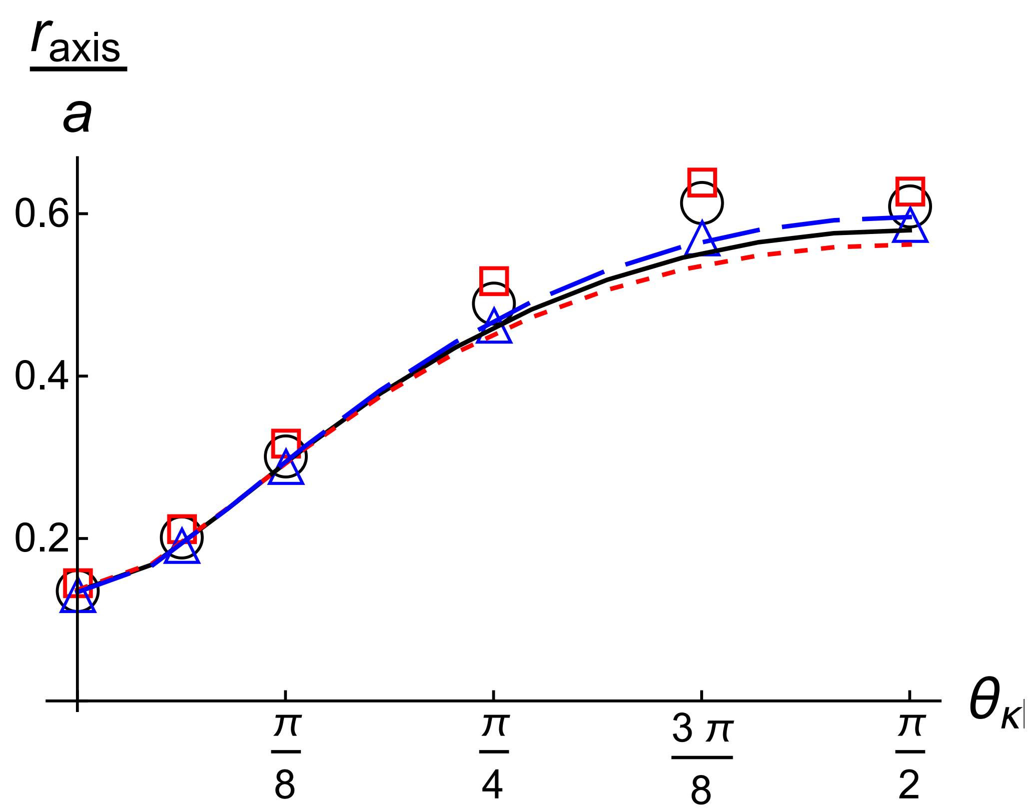
Global Shafranov shift in tilted elliptical geometry

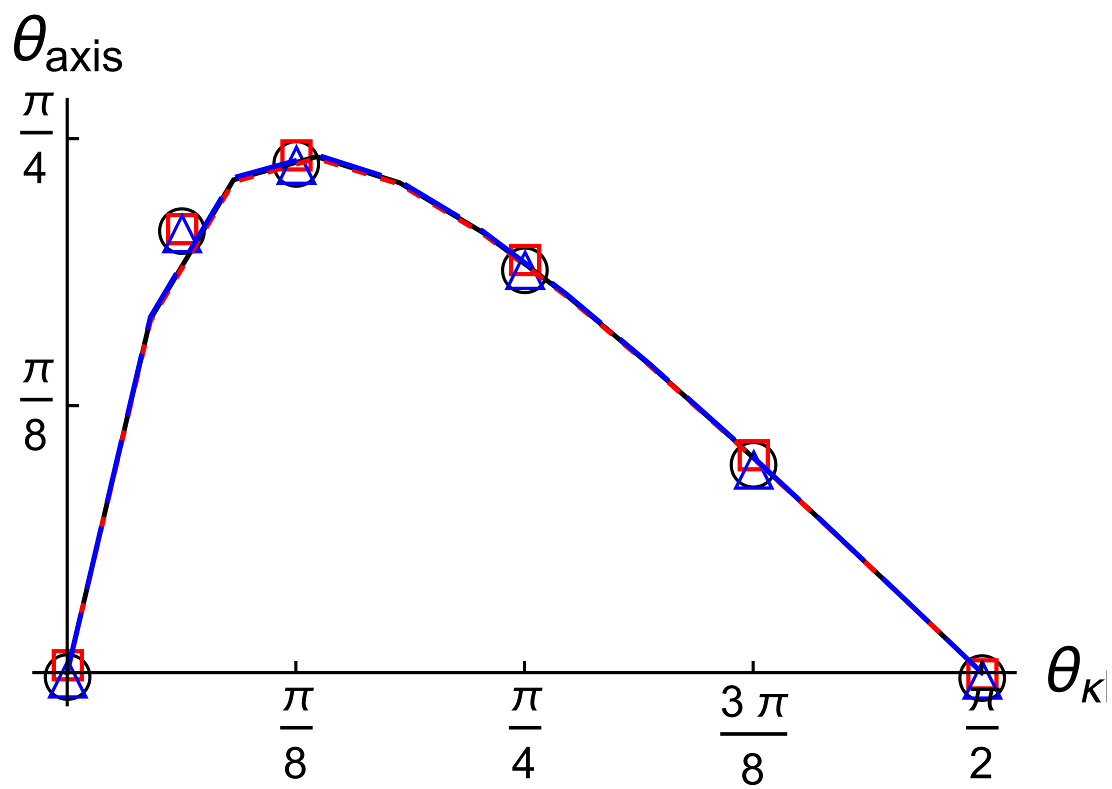
Ball, et al. *arXiv*:1607.06387 (2016).

- Calculate from Grad-Shafranov eq. (to next order in aspect ratio) for ITER-like parameters
- Verify with the equilibrium code ECOM



- Insensitive to shape of **current** profile (holding I_p fixed)



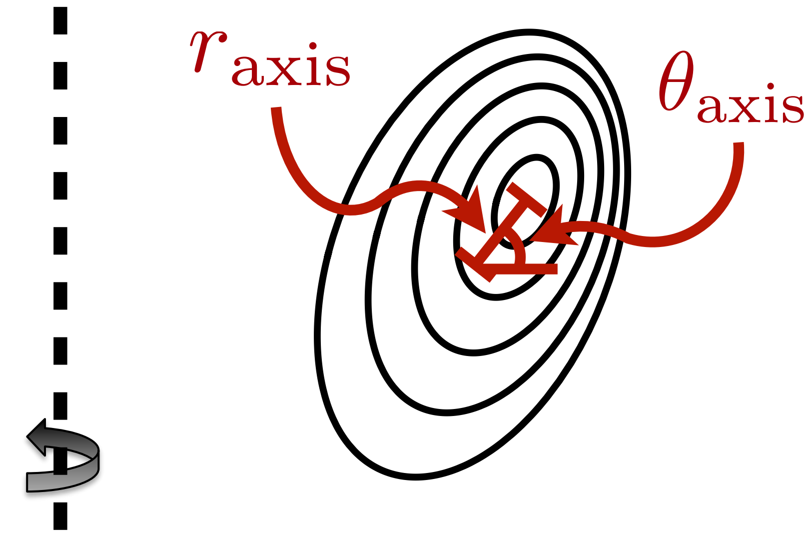


Global Shafranov shift in tilted elliptical geometry

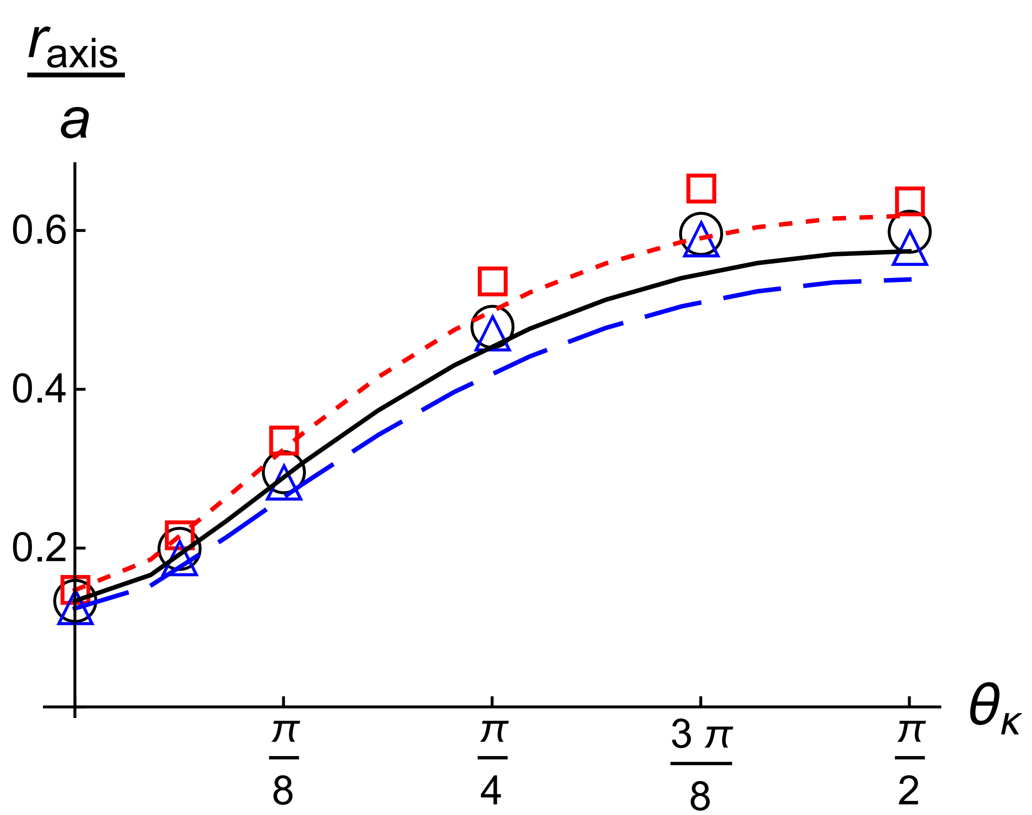
Ball, et al. *arXiv*:1607.06387 (2016).

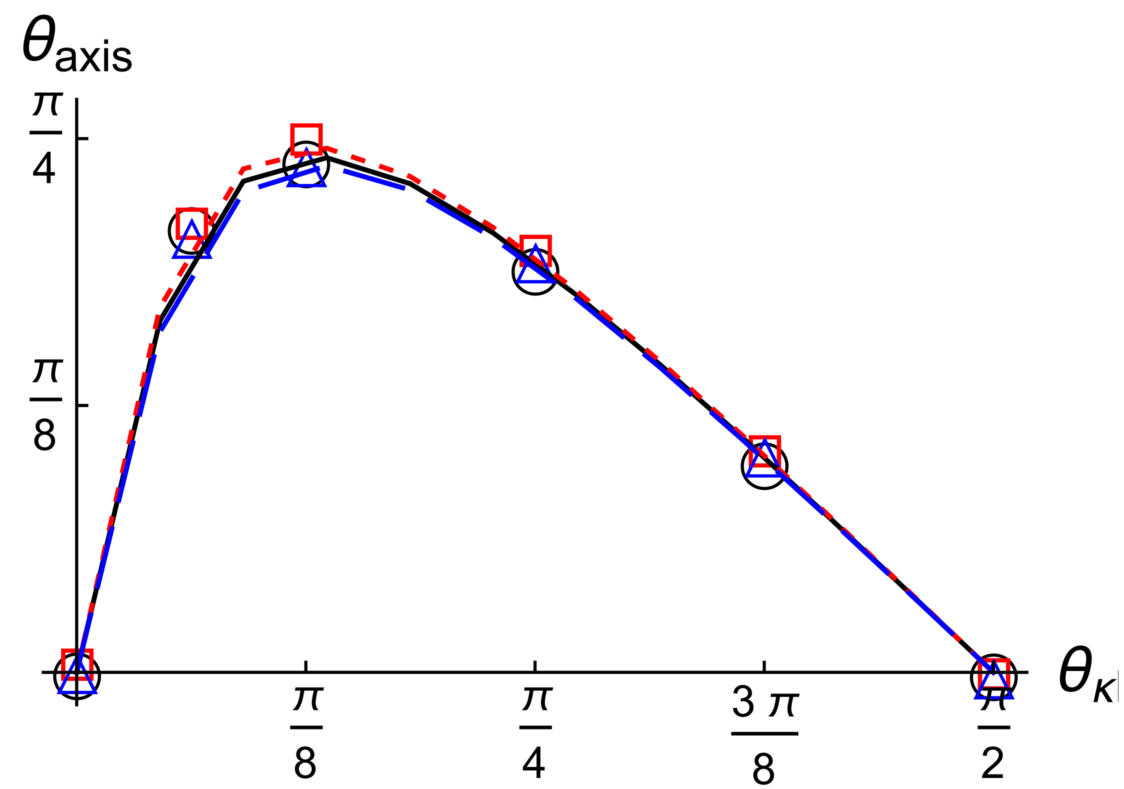
- Calculate from Grad-Shafranov eq. (to next order in aspect ratio) for ITER-like parameters

- Verify with the equilibrium code ECOM



- Insensitive to shape of **current** profile (holding p_{axis}/ψ_b fixed)

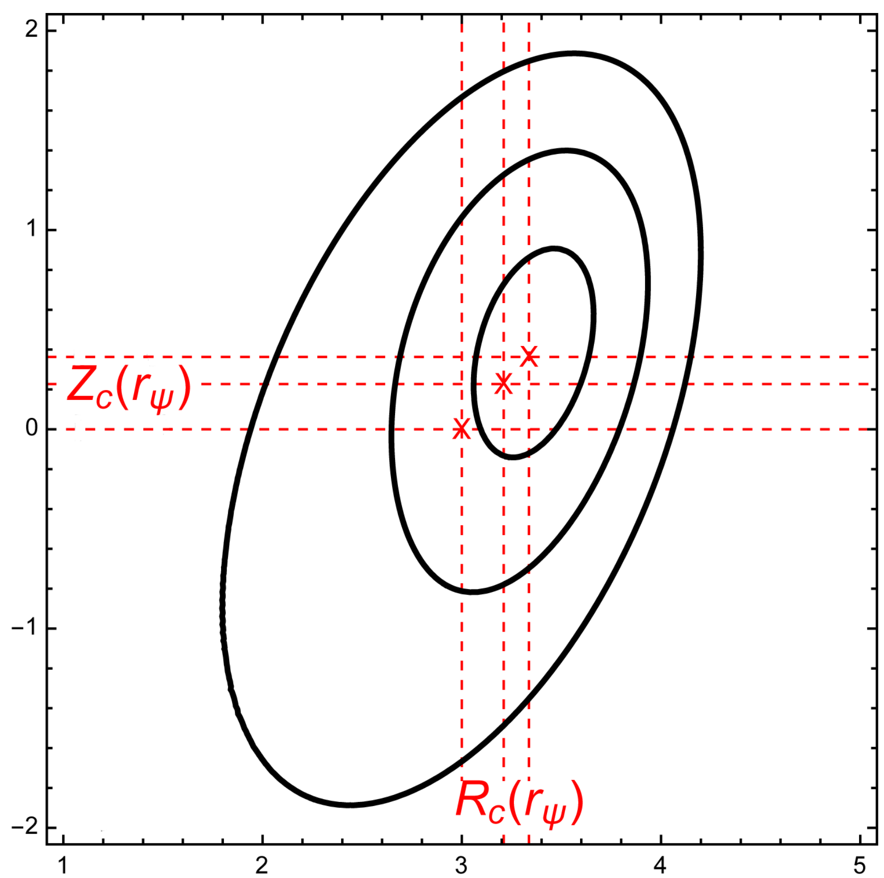


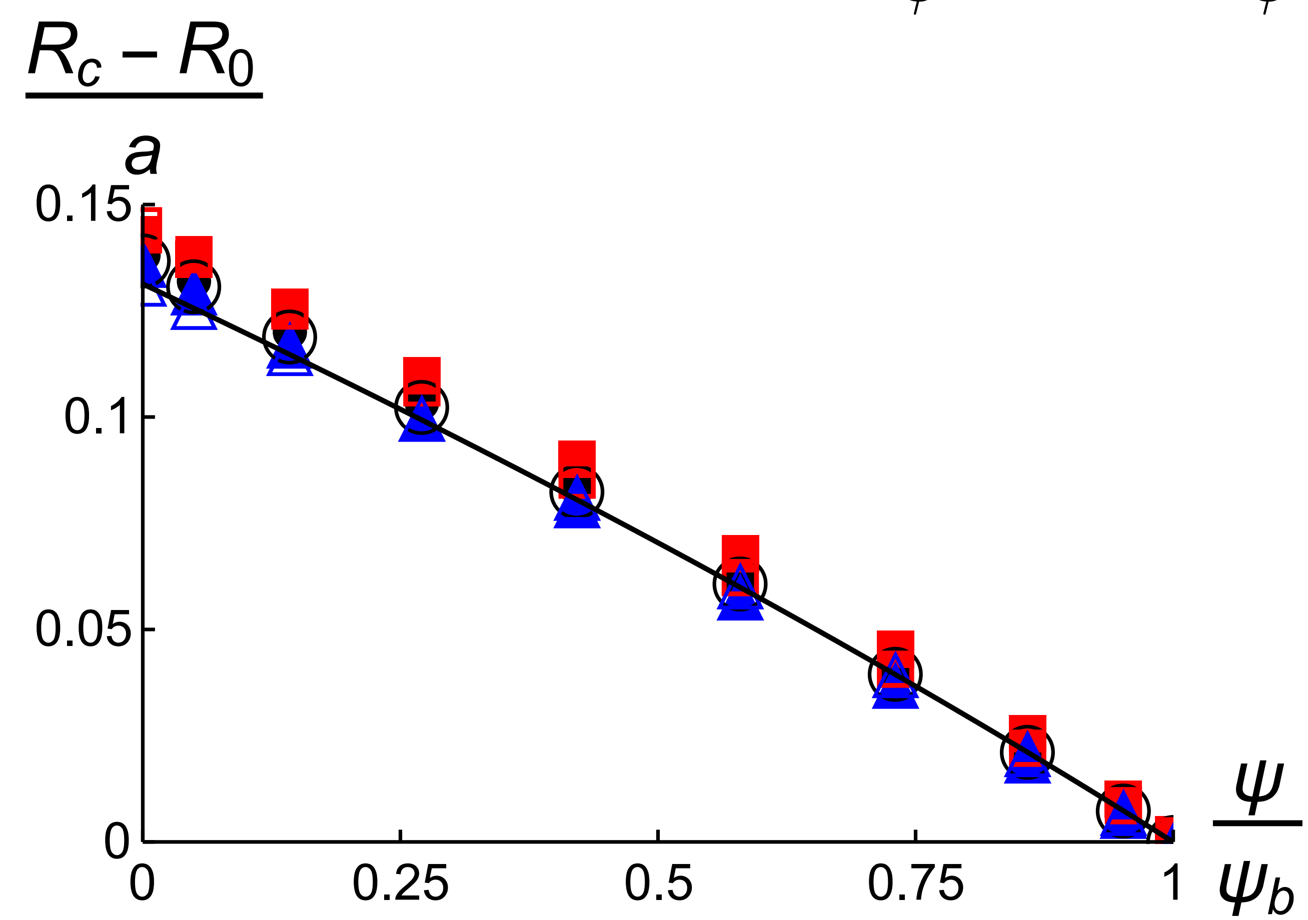


Global to local Shafranov shift

Ball, et al. *arXiv*:1607.06387 (2016).

- GS2 requires the local change in the flux surface center, $\dfrac{dR_c}{dr_\psi}$ and $\dfrac{dZ_c}{dr_\psi}$





- $\dfrac{dR_c}{d\psi} = \text{const}$ ⇒ $\dfrac{dR_c}{dr_\psi} = \dfrac{dR_c}{d\psi}\dfrac{d\psi}{dr_\psi} = -2\dfrac{r_\psi}{a}\dfrac{r_{\text{axis}}}{a}\sin\left(\theta_{\text{axis}}\right)$

Can something like this be done in ITER?

- I think so, but there is a catch
- The shape of the first wall is fixed

  ⇨ Reduced plasma volume

- Each shaping coil has a current limit

  ⇨ Reduced plasma current

- For $\beta_N = 3$, it's worth a shot