Coupled radius-energy turbulent transport of alpha particles

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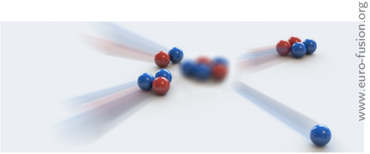
Outline

Motivation



- The fate of alpha particles is critical to a burning reactor
- Fast ion destabilize Alfvén eigenmodes
- Damage to wall if energetic particles escape
- Possible sources of alpha particle transport:
    - Neoclassical ripple loss
    - Coherent modes (TAEs, sawteeth, etc.)
    - **Anomalous transport from turbulence**

What has been done?

- *Estrada-Mila, Candy, Waltz (2006)*: Performed transport calculations with an "equivalent Maxwellian"
- *Angioni, Peeters (2008)*: Developed a quasilinear model for alpha transport
- *Hauff, Pueschel, Dannert, Jenko (2009)*: Scalings of turbulent transport coefficients as a function of energy
- *Albergante, et al. (2009)*: Efficient coefficient-based model for transport of Maxwellian alphas.
- *Waltz, Bass, Staebler (2013)*: Quasilinear model allowing radial transport to compete with collisional slowing-down.
- *Bass and Waltz (2014)*: Stiff transport model for TAEs along with Angioni model for anomalous transport

There is an analytic approximation to the alpha particle slowing-down distribution

- If alpha particle transport is weak compared to collisions:

$$\frac{\partial F_{0\alpha}}{\partial t} = C\left[F_{0\alpha}(v)\right] + \frac{\sigma}{4\pi v_\alpha^2}\delta(v - v_\alpha)$$

- We can obtain the *slowing-down distribution* as a steady-state by approximating the collision operator valid for $v_{ti} \ll v \ll v_{te}$:

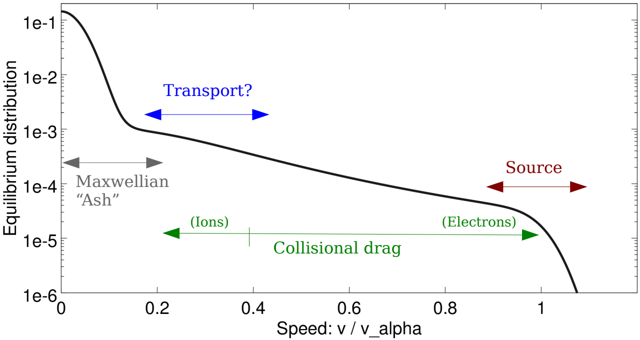
$$F_s(v) = A\frac{n_\alpha}{v_c^3 + v^3} \quad \text{for } v < v_\alpha$$

- This does *not* capture the Maxwellianization and buildup of He ash

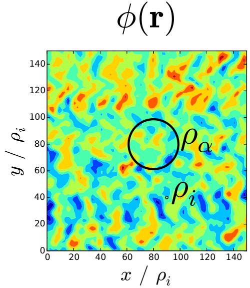
Gaffey, JPP (1976)
Helander and Sigmar (2002)

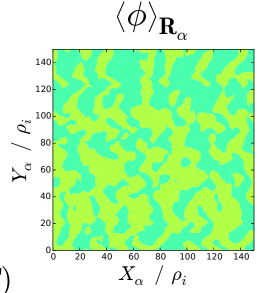
Anatomy of the alpha particle distribution

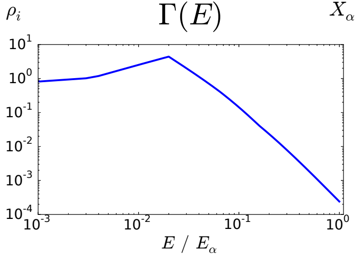
Different physical processes will be dominant at different energy scales:



The radial flux of alpha particles is a strong function of energy

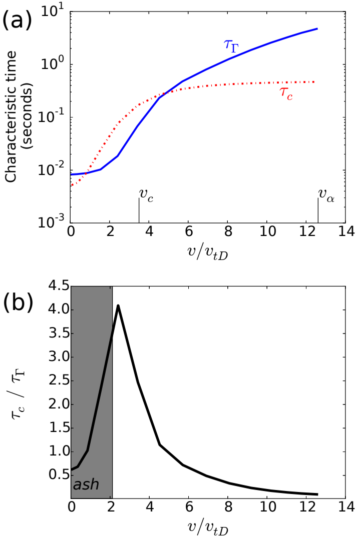






$$\Gamma_p = \int \Gamma(E)\, d^3\mathbf{v}$$

Transport can compete with collisional slowing-down at moderate energies



- Define characteristic times such that:

$$C[F_0] \sim \frac{F_0}{\tau_c(E)}$$

$$\frac{\partial}{\partial r}\Gamma(E) \sim \frac{F_0}{\tau_\Gamma(E)}$$

- When $\tau_\Gamma < \tau_c$, transport is dominant over collisions and the Gaffey slowing-down distribution is invalid

Wilkie, et al, JPP (2015)

Outline

Low-collisionality ordering of gyrokinetics

Gyrokinetic equation (electrostatic):

$$\frac{\partial h_s}{\partial t} + v_\parallel \mathbf{b} \cdot \nabla h_s + \mathbf{v}_{\mathbf{d}s} \cdot \nabla h_s + \frac{c}{B} \mathbf{b} \times \langle \phi \rangle_{\mathbf{R}} \cdot \nabla h_s + \cancel{q \frac{\partial h_s}{\partial E} \frac{\partial \langle \phi \rangle_{\mathbf{R}}}{\partial t}} - \cancel{C[h_s]}$$
$$= -q \frac{\partial F_{0s}}{\partial E} \frac{\partial \langle \phi \rangle_{\mathbf{R}}}{\partial t} - \frac{c}{B} \left(\mathbf{b} \times \langle \phi \rangle_{\mathbf{R}} \cdot \nabla \psi \right) \frac{\partial F_{0s}}{\partial \psi}$$

Caveats:

- Ignoring the parallel nonlinearity out of convenience: would require a factor $\mathcal{O}\left[1/\rho^*\right]$ more energy grid points
- For the moment, ignoring the collision operator because H-theorem is questionable for non-Maxwellian equilibria. Trace approximation makes this okay?

Low-collisionality ordering of gyrokinetics

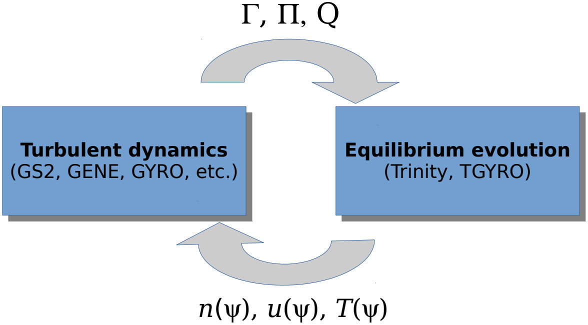
Transport equation:

$$\frac{1}{V'}\frac{\partial}{\partial t}\left(V'F_0\right) + \frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\Gamma_\psi\right) + \frac{1}{\sqrt{E}}\frac{\partial}{\partial E}\left(\sqrt{E}\Gamma_E\right) = \left\langle C\left[F_0\right] + S\right\rangle_\psi .$$

- Γ_ψ and Γ_E are the flux in radius and energy respectively. Both are functions of energy.
- Collision operator appears on the transport scale
- Solve for full F_0 instead of n_0, T_0

Coupled GK-transport workflow

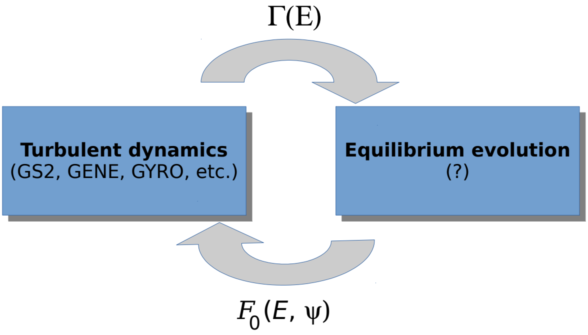
Maxwellian, or other analytic distribution:



Candy, et al. PoP (2009)
Barnes, et al. PoP (2010)
Parra, Barnes. PPCF (2015)

Coupled GK-transport workflow

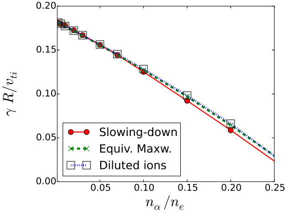
Velocity distribution not known *a priori*:

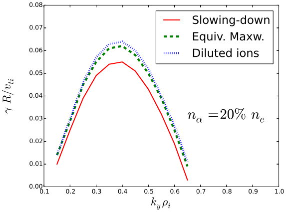


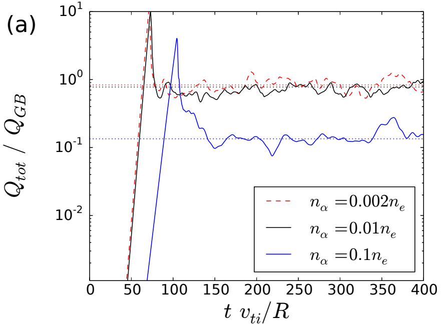
Outline

Alpha particles are trace and do not affect the linear or nonlinear electrostatic physics at realistic concentrations

Cyclone base case, electrostatic.







Tardini, et al. NF (2007)
Holland, et al. PoP (2011)
Wilkie, et al, JPP (2015)

Method to calculate flux of an arbitrary equilibrium

The (collisionless) GK equation can be written schematically as:

$$\mathcal{L}[h] = b_v \frac{\partial F_0}{\partial v} + b_\psi \frac{\partial F_0}{\partial \psi}$$

where b_v, b_ψ, and $\mathcal{L}$ depend on the particular problem, but *not* on F_0.
So we can write the radial flux as:

$$\Gamma(E) = -D_v \frac{\partial F_0}{\partial v} - D_\psi \frac{\partial F_0}{\partial \psi}$$

1. Run a GK simulation with two additional species with the desired charge and mass, but at different temperatures or radial gradients
2. Use $\Gamma(E)$ for each of these to calculate D_v and D_ψ as functions of energy
3. Plug in any desired $F_0(E, \psi)$, iterate with transport solver to find profile

Two ways to obtain diffusion coefficients

1. Run with multiple species with different gradients
   - Easier to implement, more computationally expensive
2. Obtain directly from $\phi(\mathbf{r}, t)$
   - Requires solving the GK equation, one energy at a time, with ϕ given
3. Analytic estimate?

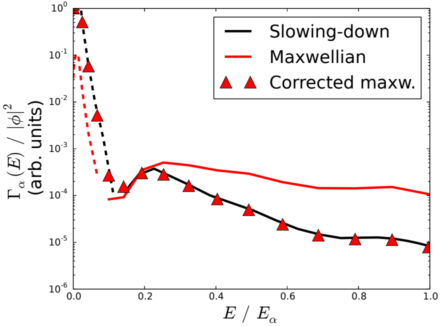
$$\Gamma_\psi = \frac{\partial F_0}{\partial \psi} \left\langle \sum_{\sigma_\parallel} \int \mathcal{L}^{-1} \left[\frac{\partial \langle \phi \rangle_{\mathbf{R}}}{\partial y} \right] \frac{c^2}{B^2} \frac{\partial \langle \phi \rangle_{\mathbf{R}}}{\partial y} \frac{\pi B d\lambda}{\sqrt{1 - \lambda B}} \right\rangle_{t,\psi}$$

$$\frac{\partial \langle \phi \rangle_{\mathbf{R}}}{\partial y} \equiv \mathbf{b} \times \langle \phi \rangle_{\mathbf{R}} \cdot \nabla \psi$$

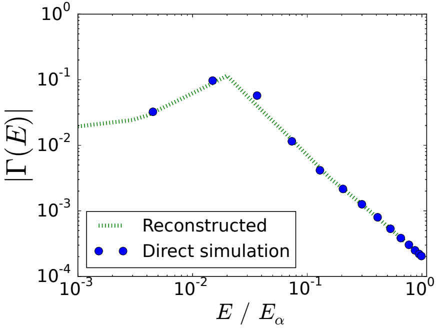
$$\mathcal{L} \equiv \frac{\partial}{\partial t} + \left(v_\parallel \mathbf{b} + \mathbf{v}_{\mathbf{d}_s} + \frac{c}{B} \mathbf{b} \times \langle \phi \rangle_{\mathbf{R}} \right) \cdot \nabla$$

Proof of principle: Reconstruct non-Maxwellian flux in ITG turbulence from Maxwellian

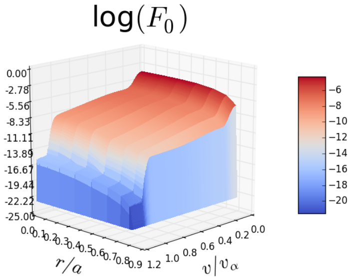
Quasilinear:



Fully turbulent



New tool in development to calculate coupled radius-energy transport of trace energetic particles



T3CORE

Written in julia

Trace **T**urbulent **T**ransport, **CO**upled in **R**adius and **E**nergy

- Solves for the steady-state or time-dependent $F_0(v, r)$
- Utilizes the trace approximation to determine turbulent transport coefficients
- Runs on a laptop once the "seed" nonlinear simulations are performed