Coupled radius-energy turbulent transport of alpha particles

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24 July 2015

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Motivation

- The fate of alpha particles is critical to a burning reactor
- Fast ion destabilize Alfvén eigenmodes
- Damage to wall if energetic particles escape
- Possible sources of alpha particle transport:
  - Neoclassical ripple loss
  - Coherent modes (TAEs, sawteeth, etc.)
  - Anomalous transport from turbulence
What has been done?

- **Estrada-Mila, Candy, Waltz (2006):** Performed transport calculations with an “equivalent Maxwellian”
- **Angioni, Peeters (2008):** Developed a quasilinear model for alpha transport
- **Hauff, Pueschel, Dannert, Jenko (2009):** Scalings of turbulent transport coefficients as a function of energy
- **Albergante, et al. (2009):** Efficient coefficient-based model for transport of Maxwellian alphas.
- **Waltz, Bass, Staebler (2013):** Quasilinear model allowing radial transport to compete with collisional slowing-down.
- **Bass and Waltz (2014):** Stiff transport model for TAEs along with Angioni model for anomalous transport
There is an analytic approximation to the alpha particle slowing-down distribution

- If alpha particle transport is weak compared to collisions:
  \[
  \frac{\partial F_{0\alpha}}{\partial t} = C[F_{0\alpha}(v)] + \frac{\sigma}{4\pi v_\alpha^2} \delta(v - v_\alpha)
  \]

- We can obtain the slowing-down distribution as a steady-state by approximating the collision operator valid for \( v_{ti} \ll v \ll v_{te} \):
  \[
  F_s(v) = A \frac{n_\alpha}{v_c^3 + v^3} \quad \text{for } v < v_\alpha
  \]

- This does not capture the Maxwellianization and buildup of He ash

\[\text{Gaffey, JPP (1976)}\]
\[\text{Helander and Sigmar (2002)}\]
Different physical processes will be dominant at different energy scales:
The radial flux of alpha particles is a strong function of energy.

\[ \phi(\mathbf{r}) \]

\[ \langle \phi \rangle_{R_\alpha} \]

\[ \Gamma(E) \]

\[ \Gamma_p = \int \Gamma(E) \, d^3v \]
Transport can compete with collisional slowing-down at moderate energies

- Define characteristic times such that:
  \[ C[F_0] \sim \frac{F_0}{\tau_c(E)} \]
  \[ \frac{\partial}{\partial r} \Gamma(E) \sim \frac{F_0}{\tau_\Gamma(E)} \]

- When \( \tau_\Gamma < \tau_c \), transport is dominant over collisions and the Gaffey slowing-down distribution is invalid

\[ Wilkie, et al, JPP (2015) \]
Low-collisionality ordering of gyrokinetics

Gyrokinetic equation (electrostatic):

\[
\frac{\partial h_s}{\partial t} + v_\parallel \mathbf{b} \cdot \nabla h_s + \mathbf{v}_{ds} \cdot \nabla h_s + \frac{c}{B} \mathbf{b} \times \langle \phi \rangle \mathbf{R} \cdot \nabla h_s + q \frac{\partial h_s}{\partial E} \frac{\partial \langle \phi \rangle \mathbf{R}}{\partial t} - C[h_s] \\
= -q \frac{\partial F_0}{\partial E} \frac{\partial \langle \phi \rangle \mathbf{R}}{\partial t} - \frac{c}{B} (\mathbf{b} \times \langle \phi \rangle \mathbf{R} \cdot \nabla \psi) \frac{\partial F_0}{\partial \psi}
\]

Caveats:

- Ignoring the parallel nonlinearity out of convenience: would require a factor $O[1/\rho^*]$ more energy grid points
- For the moment, ignoring the collision operator because H-theorem is questionable for non-Maxwellian equilibria. Trace approximation makes this okay?
Transport equation:

\[
\frac{1}{V'} \frac{\partial}{\partial t} (V' F_0) + \frac{1}{V'} \frac{\partial}{\partial \psi} (V' \Gamma_\psi) + \frac{1}{\sqrt{E}} \frac{\partial}{\partial E} \left( \sqrt{E} \Gamma_E \right) = \langle C \left[ F_0 \right] + S \rangle_\psi.
\]

- \( \Gamma_\psi \) and \( \Gamma_E \) are the flux in radius and energy respectively. Both are functions of energy.
- Collision operator appears on the transport scale
- Solve for full \( F_0 \) instead of \( n_0, T_0 \)
Coupled GK-transport workflow

Maxwellian, or other analytic distribution:

\[ \Gamma, \Pi, Q, n(\psi), u(\psi), T(\psi) \]
Coupled GK-transport workflow

Velocity distribution not known \textit{a priori}:

\[ F_0(E, \psi) \]

\[ \Gamma(E) \]

\textbf{Turbulent dynamics (GS2, GENE, GYRO, etc.)}

\textbf{Equilibrium evolution (?)}

Wilkie (Maryland)
Alpha particles are trace and do not affect the linear or nonlinear electrostatic physics at realistic concentrations.

Cyclone base case, electrostatic.

Wilkie (Maryland)

Coupled transport

24 July 2015 15 / 28

Method to calculate flux of an arbitrary equilibrium

The (collisionless) GK equation can be written schematically as:

\[
\mathcal{L}[h] = b_v \frac{\partial F_0}{\partial v} + b_\psi \frac{\partial F_0}{\partial \psi}
\]

where \( b_v, b_\psi, \) and \( \mathcal{L} \) depend on the particular problem, but not on \( F_0 \). So we can write the radial flux as:

\[
\Gamma(E) = -D_v \frac{\partial F_0}{\partial v} - D_\psi \frac{\partial F_0}{\partial \psi}
\]

1. Run a GK simulation with two additional species with the desired charge and mass, but at different temperatures or radial gradients
2. Use \( \Gamma(E) \) for each of these to calculate \( D_v \) and \( D_\psi \) as functions of energy
3. Plug in any desired \( F_0(E, \psi) \), iterate with transport solver to find profile
Two ways to obtain diffusion coefficients

1. Run with multiple species with different gradients
   - Easier to implement, more computationally expensive

2. Obtain directly from $\phi(r, t)$
   - Requires solving the GK equation, one energy at a time, with $\phi$ given

3. Analytic estimate?

$$
\Gamma_{\psi} = \frac{\partial F_0}{\partial \psi} \left< \sum_{\sigma_{\parallel}} \int L^{-1} \left[ \frac{\partial \langle \phi \rangle_R}{\partial y} \right] \frac{c^2}{B^2} \frac{\partial \langle \phi \rangle_R}{\partial y} \frac{\pi B d\lambda}{\sqrt{1 - \lambda B}} \right>_{t, \psi}
$$

$$
\frac{\partial \langle \phi \rangle_R}{\partial y} \equiv b \times \langle \phi \rangle_R \cdot \nabla \psi
$$

$$
L \equiv \frac{\partial}{\partial t} + \left( v_\parallel b + v_d s + \frac{c}{B} b \times \langle \phi \rangle_R \right) \cdot \nabla
$$
Proof of principle: Reconstruct non-Maxwellian flux in ITG turbulence from Maxwellian

**Quasilinear:**

- Slowing-down
- Maxwellian
- Corrected maxw.

**Fully turbulent**

- Reconstructed
- Direct simulation

\[ \Gamma_{\alpha}(E) / |\phi|^2 \] (arb. units)

\[ E / E_{\alpha} \]

\[ |\Gamma(E)| \]

\[ E / E_{\alpha} \]
New tool in development to calculate coupled radius-energy transport of trace energetic particles

\[ \log(F_0) \]

Solves for the steady-state or time-dependent \( F_0(v, r) \)

Utilizes the trace approximation to determine turbulent transport coefficients

Runs on a laptop once the “seed” nonlinear simulations are performed