Magnetic islands and Gyrofluid models

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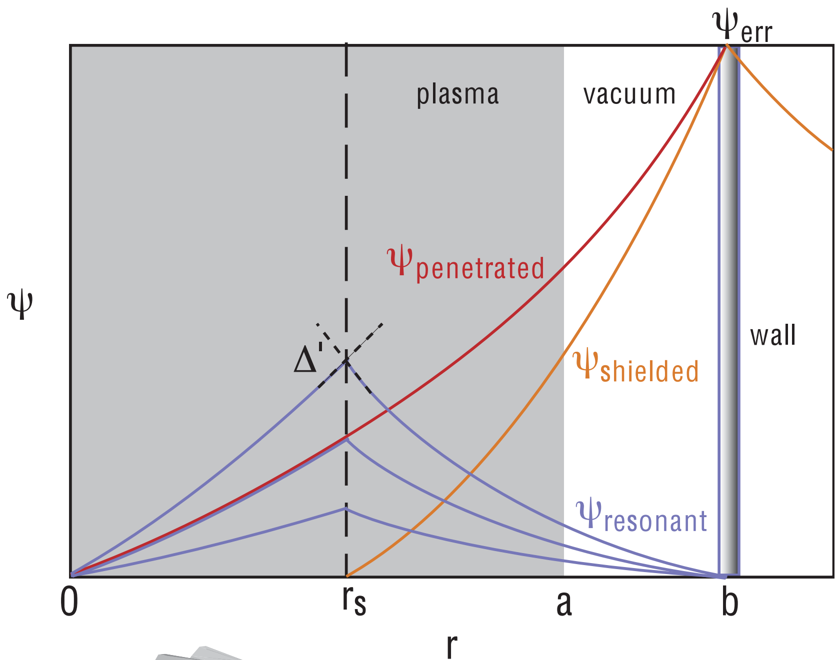
Outline

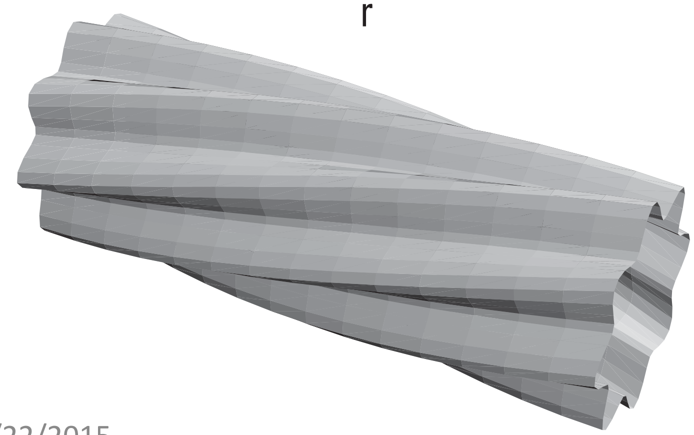
- Introduction
- Gyrokinetic description of islands
- Hamiltonian fluid closures
- Summary

Why study magnetic islands?

- A magnetic island is a flux tube with its own magnetic axis (regardless of size).
- The width of magnetic islands is a measure of the amount of reconnection at resonant surfaces.
- The size of magnetic islands determines the distortions of the flux surfaces *at large distances*, and thus influence mean flows through NTV.
- At finite beta and in 3D systems, magnetic islands *together* with zonal flows, regulate confinement quality.

Islands are the local manifestation of global Alfvén modes





- In a cylinder the exterior perturbation satisfies B·∇J=[ψ,J]=0,
- Note $\psi = B_r/k_y$
- Linearizing yields

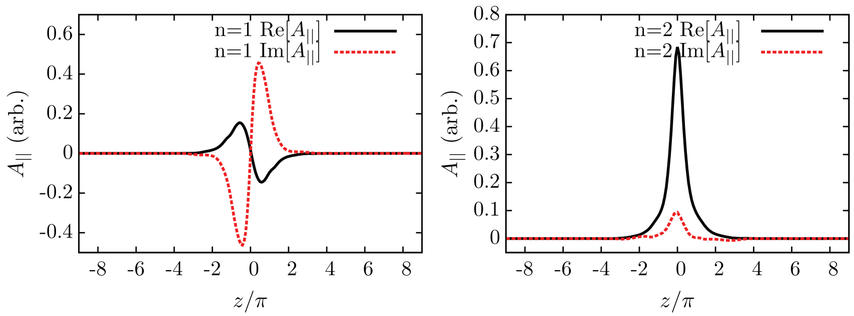
$$B_{y0}'(r)\,\nabla^2\psi - J_0'(r)\psi = 0$$

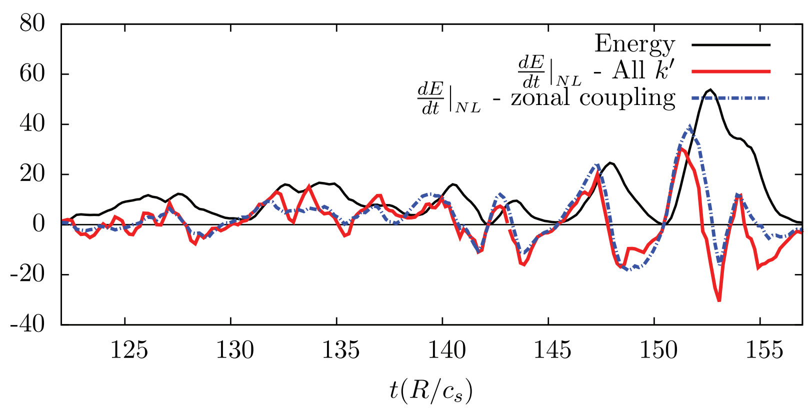
- The matching condition is

$$\Delta'\ \psi + A\,\psi_{err} = \Delta(\omega)\,\psi$$

- Note $\psi \sim r^{-m}$

Micro-Tearing Modes are driven by Zonal Flows

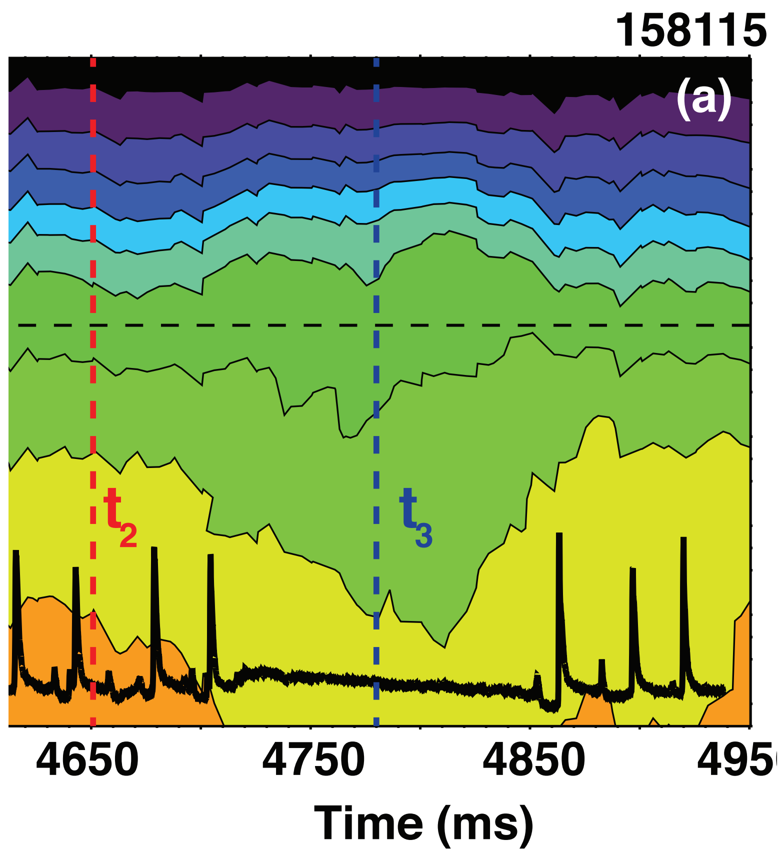


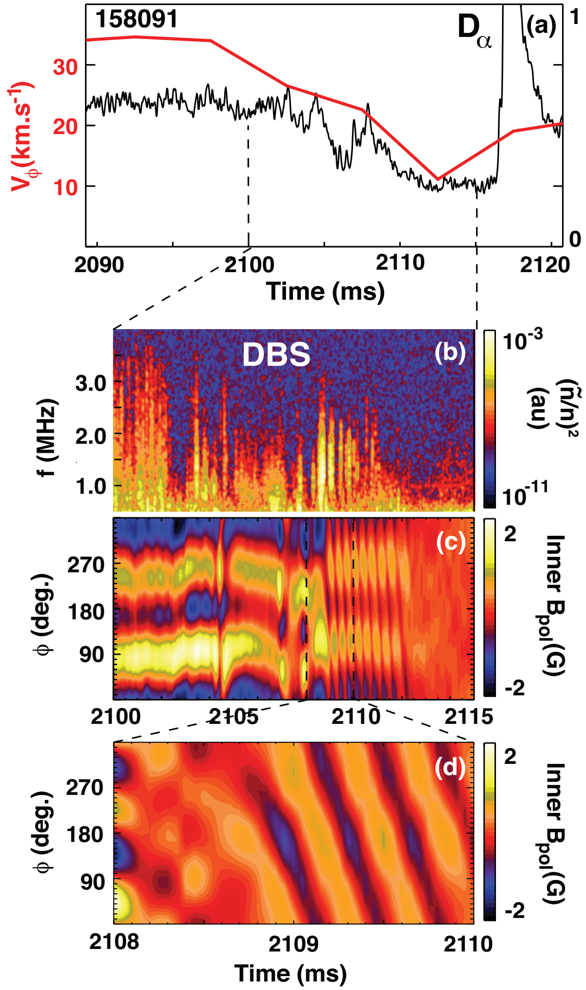


Hatch et al., PRL 2012

- POD analysis of finite-β turbulence shows dominance of an interchange and a tearing parity mode.
- Hatch has shown that the tearing-parity mode is a linearly damped MTM driven by the ZF.

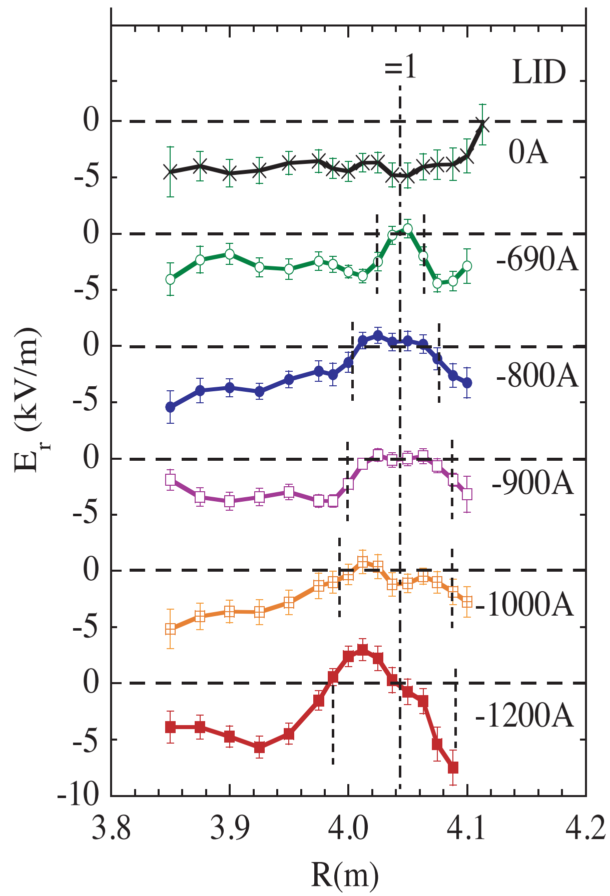
In DIII, the bifurcation to ELM-suppressed regime shows signature of magnetic island



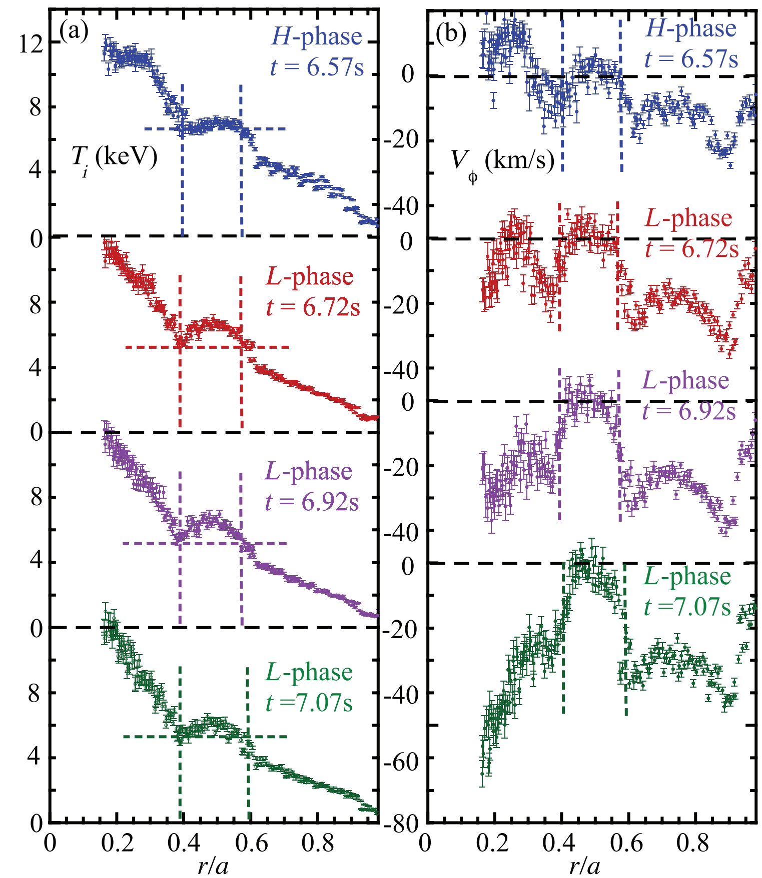


Nazikian et al, PRL 2015

LHD and JT-60 also show profile modification caused by islands.

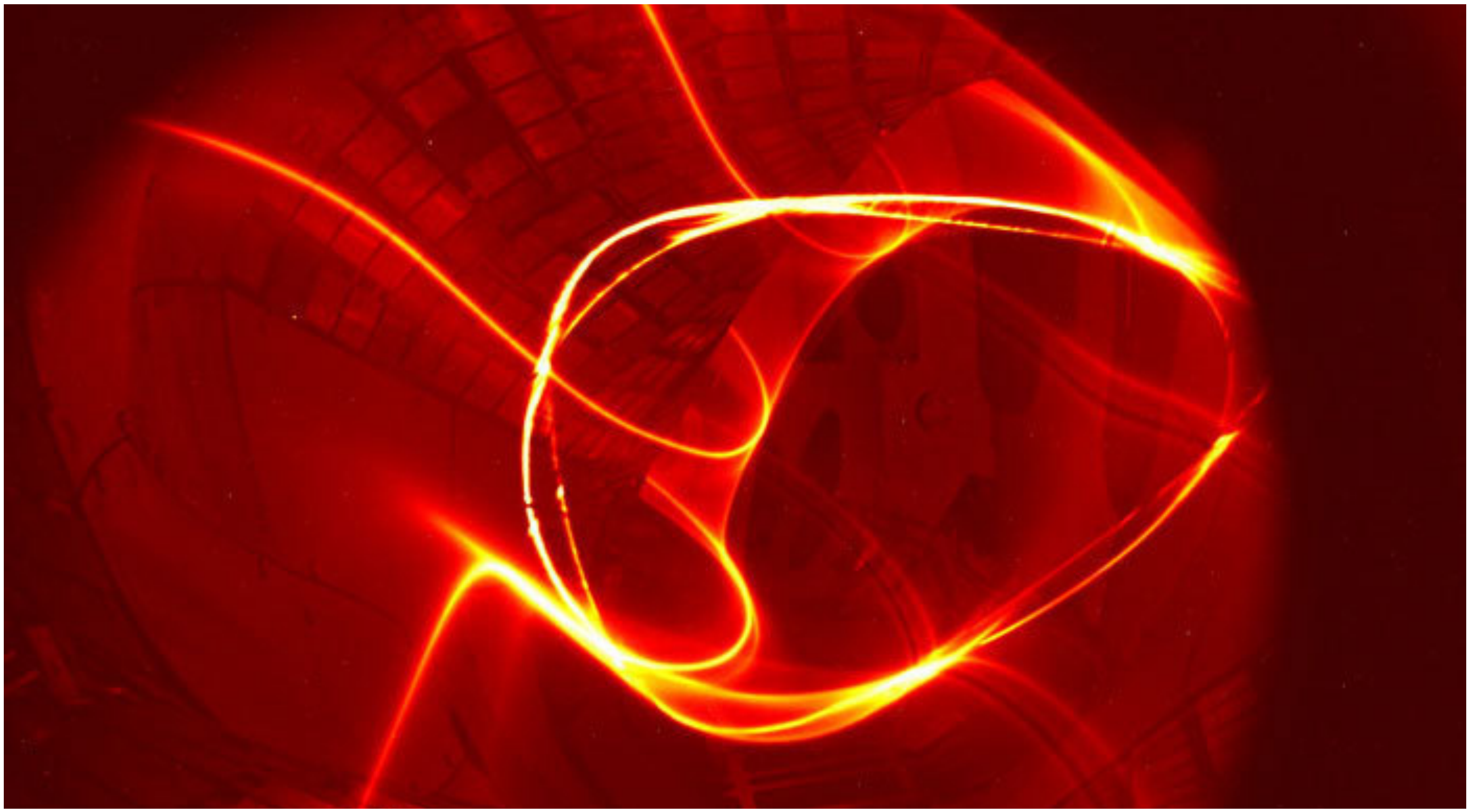


Ida et al., NF 2004



Ida et al., PRL 2012

W7-X just verified that they are island-free… at zero β



Note that island-free 3D devices violate the transport ordering.

Island evolution is determined by two generalized Rutherford equations

- The island width W and its propagation velocity V are governed by:

$$dW/dt = \Delta'(W,V) + \Delta(W, V) \;;$$

$$dV/dt = F(W, V) = 0.$$

- Δ' measures the drive for reconnection (island growth) from currents outside the island, including plasma, wall & RMP.
- Δ & F represent, respectively, the local drive and the forces acting on the island in the binormal direction.
- Classical tearing modes occur when $\Delta'>0$.
- Neoclassical Tearing Modes occur when $\Delta>-\Delta'$.

These equations have their origin in exact conservation laws

- The amplitude equation comes from the flux-surface average of Ohm's law:

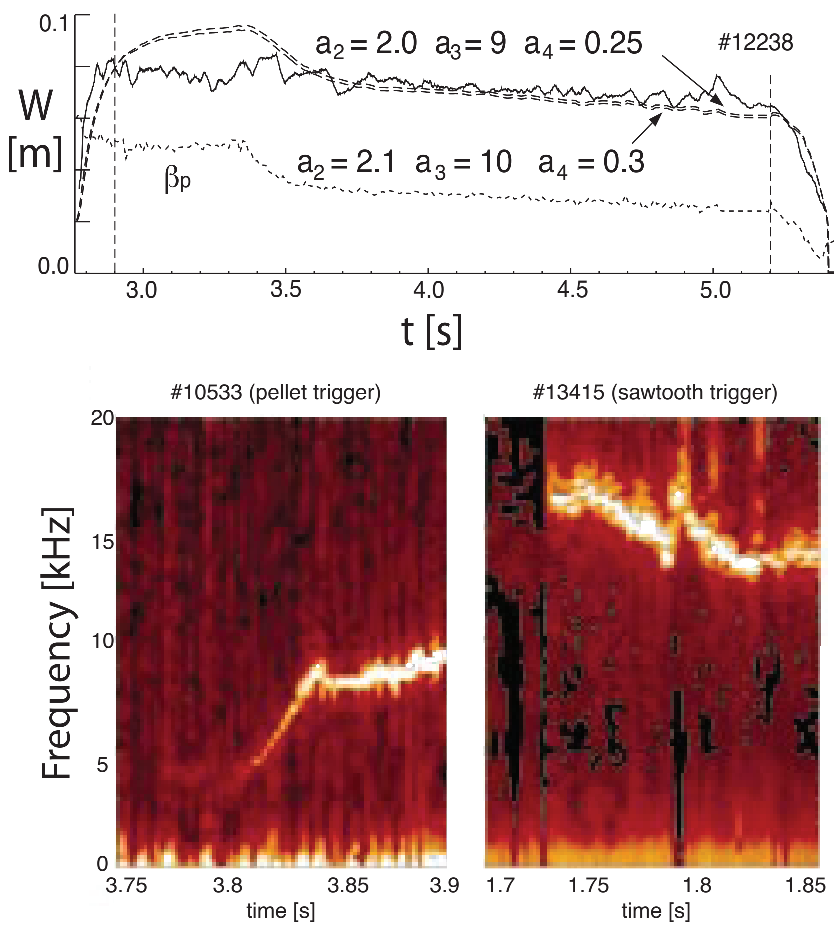
$$\langle \partial_t A \rangle + \langle b \cdot \nabla \cdot \Pi \rangle = \eta \langle J \rangle$$

- The phase velocity equation comes from the flux-surface average of the charge conservation equation

$$\partial_t \langle V_y \rangle + \langle V_r U_\parallel \rangle = \left\langle \left(\nabla \cdot \Pi \right)_y \right\rangle + \langle B_r J_\parallel \rangle + \mu \left\langle \nabla^2 V_y \right\rangle$$

- Full, closed system must be solved

The "lift" and "drag" can be observed



Zohm, NF 2001

- The equations of motion for the island are

$$dW/dt = \Delta' - \Delta(W, V)$$

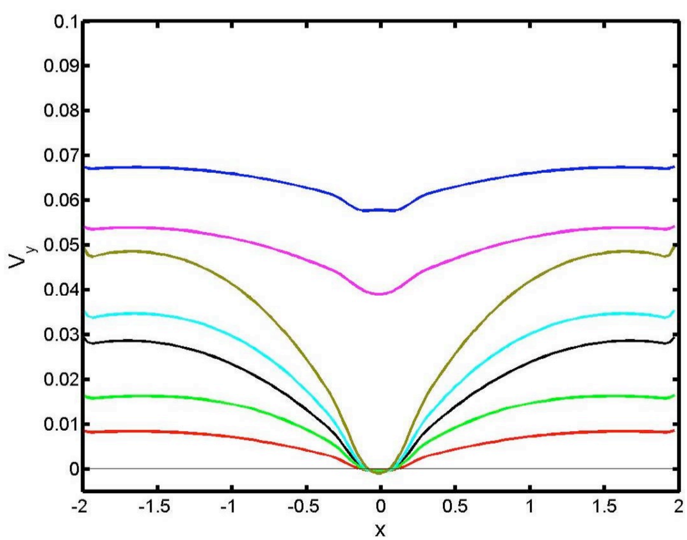
$$dV/dt = F(W, V)$$

- The growth rate for the island is thus a measure of the lift
- The acceleration of the island when the island velocity is swept gives a measure of the binormal force

GYRO simulations of turbulent islands

Long compared to turbulence saturation scale but short compared to confinement time

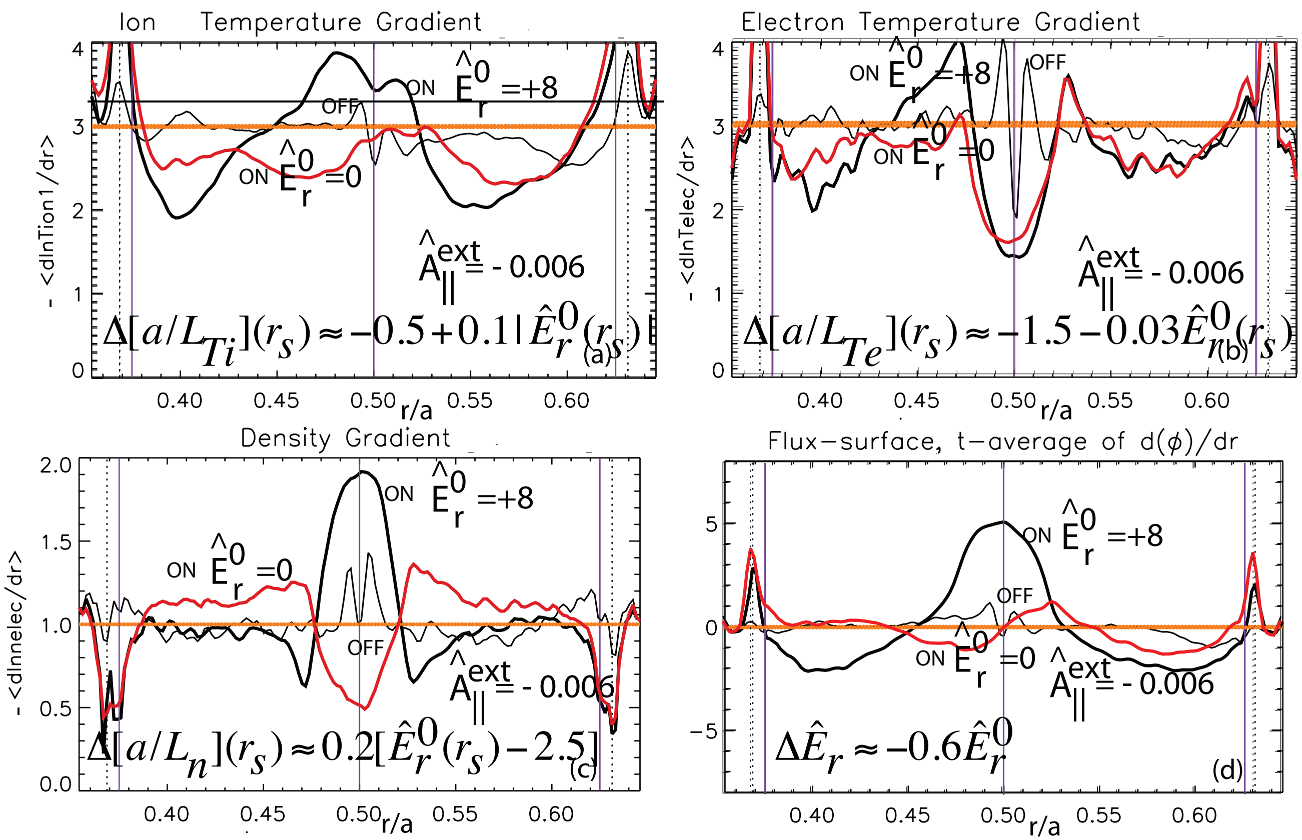
Electrostatic simulations allow the island response to be charted



- Like a channel or wind tunnel measurement, they allow direct calculation of D and F as functions of W and V
- At constant field strength, unlocking occurs when the channel velocity increases above a threshold, here 0.05 v_A (Militello, NF 2009)

Island profile modifications are similar to fluid predictions

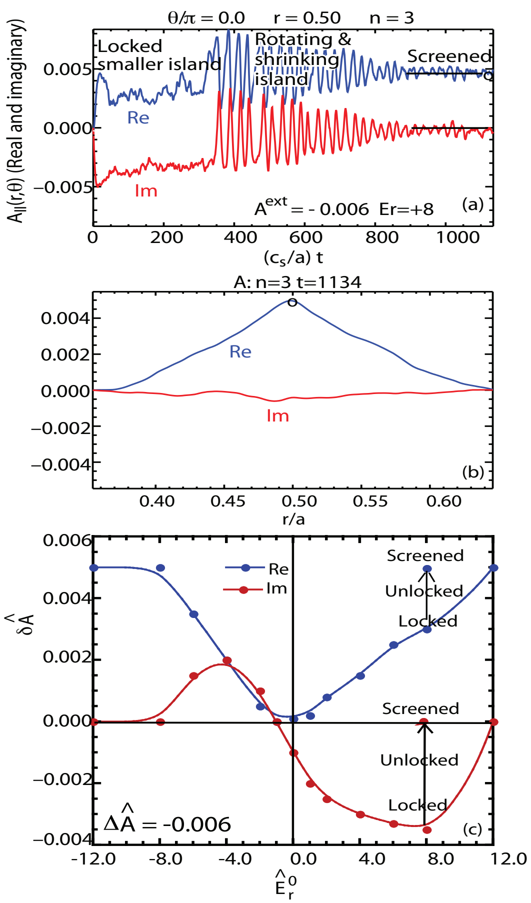
Δn=1 GA-std case: $\hat{E}_r^0 = +8 \,\&\, 0,\ \Delta\hat{A}_{\|}^{ext} = -0.006\ (w/a = 0.05),\ \hat{\gamma}_E = 0.1,\ \hat{\gamma}_\phi = 1.2$



- expected island grad-T_e flattening inside & steepening outside independent of E_r, limited by trapping (cf Park & Chang)
- grad-n close to adia-e $-\partial\Delta n/\partial r \sim n(e/T_e)\partial\Delta\phi/\partial r$
- The lowering of $[a/L_{Ti}]_{ave} = 3 \Rightarrow 2.5$ is a problem.

Waltz & Waelbroeck, PoP 2012

EM Gyrokinetic simulations also show unlocking and suppression



Waltz and Waelbroeck, PoP 2012.

- Island initially shifts its phase to achieve force balance.
- Further phase slippage leads to unlocking and healing.
- Final state is a screened island with the response field canceling the vacuum field at the resonant surface.

Hamiltonian fluid closures

Or how to symplectify your life!

Why fluid closures?

- Outside of turbulent transport theory, a qualitative understanding of many phenomena is lacking.
- Examples abound: saturation and chirping of AE, disruptions, precursor-less sawtooth crashes, snake oscillations, absorption of RF in SOL, ELM trigger and suppression by RMP, NTM etc.
- GK codes are often applicable in principle, but too cumbersome in practice.
- Fluid codes have proven themselves scientifically productive.
- Reduced models are needed to support GK codes in Bayesian inference schemes (UQ).

Why Hamiltonian closures?

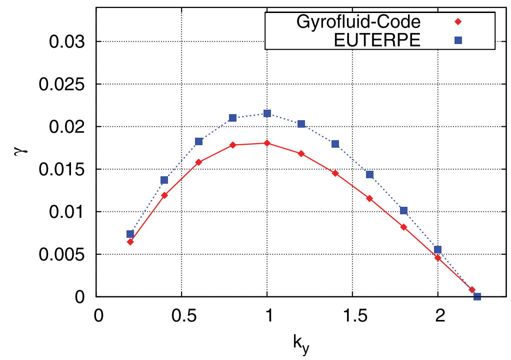
- *The Hamiltonian property guarantees the existence of nontrivial equilibrium solutions.*
- Vlasov-Maxwell is Hamiltonian. Fluid closure introduces known dissipative terms, but the equations without these terms should be Hamiltonian.
- Hamiltonian systems possess infinite family of approximate Poincaré invariants J= $\oint$ p dq.
- They also possess families of exact invariants called Casimirs (e.g. the magnetic flux).
- Lagrangian invariants may determine profiles through equipartition (Naulin, PRL 1998).

Equilibrium integrability

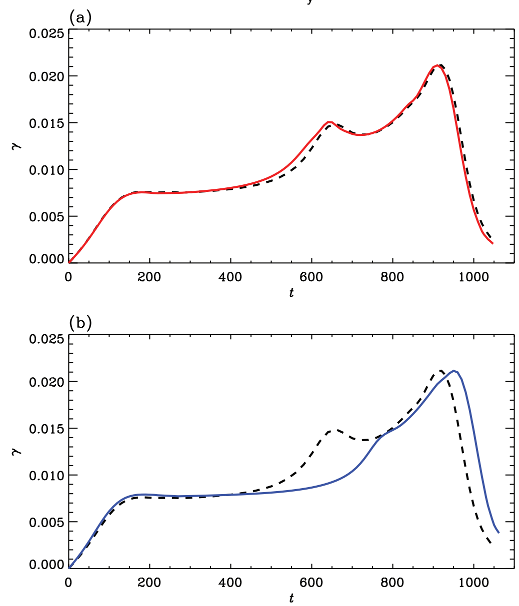
- In MHD, the equilibrium equation $\nabla p = \mathbf{J}\times\mathbf{B}$ determines $\mathbf{J}_\perp = \mathbf{B}\times\nabla p/B^2$.
- Charge conservation requires $\nabla\cdot\mathbf{J} = 0$ or
  $\mathbf{B}\cdot\nabla(J_{||}/B) = -\nabla\cdot(\mathbf{B}\times\nabla p/B^2)$.
- In axisymmetric system, integrating this yields the GS eq.
- In order for the solution to be admissible, it is necessary that $\oint \nabla\cdot\mathbf{J}_\perp \, dl/B = 0$.
- In reduced models, there is a similar constraint for *every moment equation*, sometimes along the field, sometimes along electron or ion streamlines.
- For Hamiltonian models, all these constraints are automatically satisfied and a variational principle yields the integrated equilibrium equations.

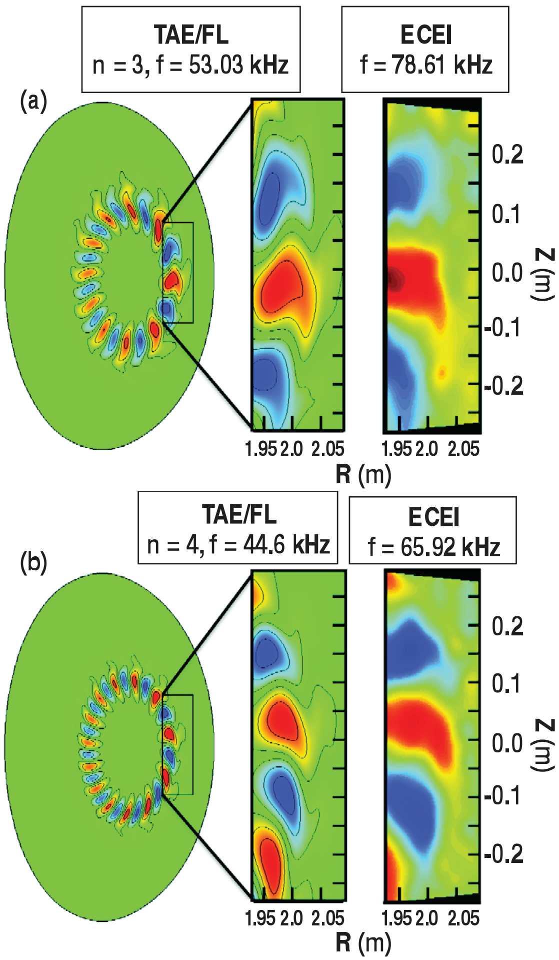
Gyrofluid codes have enabled discoveries in reconnection, TAE theory

Zacharias PoP 2014



Comisso PoP 2013





Tobias PRL'11

IFS

So fine, how do we do this?

- Construct a pseudo-Poisson bracket such that
  $\partial_t \xi = \{\xi, H\}$.
  where H is the Hamiltonian.
- To be a true Poisson bracket, { , } must satisfy the Jacobi identity:
  {a,{b,c}} + cyclic permutations = 0.
- GF models lend themselves surprisingly well to this.
- Nevertheless, you can't make an omelet without breaking eggs.

3+0 Hamiltonian GF model

$D_t n_i = -\nabla_{||} u_i - 2\, u_d\, [x, n_i + \Phi + T];$

$D_t(\Psi + u_i) = -\nabla_{||}(T + n_i) - 4\, u_d\, [x, u_i];$

$D_t T_i = -(\gamma - 1)\nabla_{||} u_i - 2\, u_d\, [x, n_i + \Phi + T];$

$D_t n_e = -\nabla_{||} u_e + 2\, u_d\, [x, n_e - \phi];$

$D_t(\psi - \mu u_e) = -\nabla_{||} n_e/\tau + 2\mu\, u_d\, [x, u_e];$

$n_e = \Gamma_0^{1/2}\, n_i + (\Gamma_0 - 1)\, \phi;$

$2\nabla^2\psi/\tau\, \beta_e = u_e - \Gamma_0^{1/2}\, u_i;$

To satisfy the Jacobi identity, one must take $\gamma=2$.

Keramidas et al., http://arxiv.org/abs/1507.05220

Hamiltonian and conserved quantities

- The conserved energy is

  $H=<n_e^2/\tau+n_i^2+T/(\gamma\text{-}1)+\mu u_e^2+u_i^2+\Phi n_i-\phi n_e$
  $+2|\nabla\psi|^2/\tau\beta_e>$

- Constructing the Casimirs reveals five Lagrangian invariants:

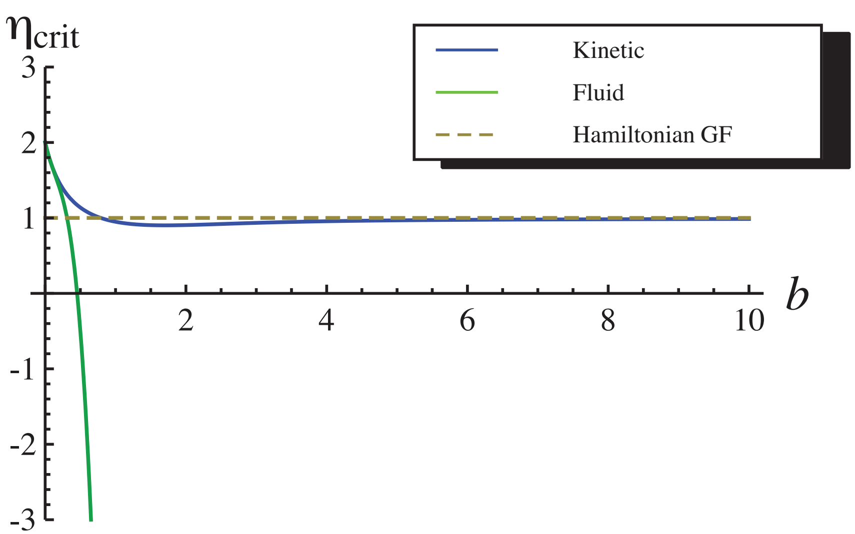
  $V_e=M_e\pm\sqrt{\mu}\, n_e$, where $M_e=\psi-\mu u_e$;

  $V_i=T+n_i\pm\sqrt{2}M_i$, where $M_i=\Psi+u_i$;

  $S_i=T\text{-}n_i$.

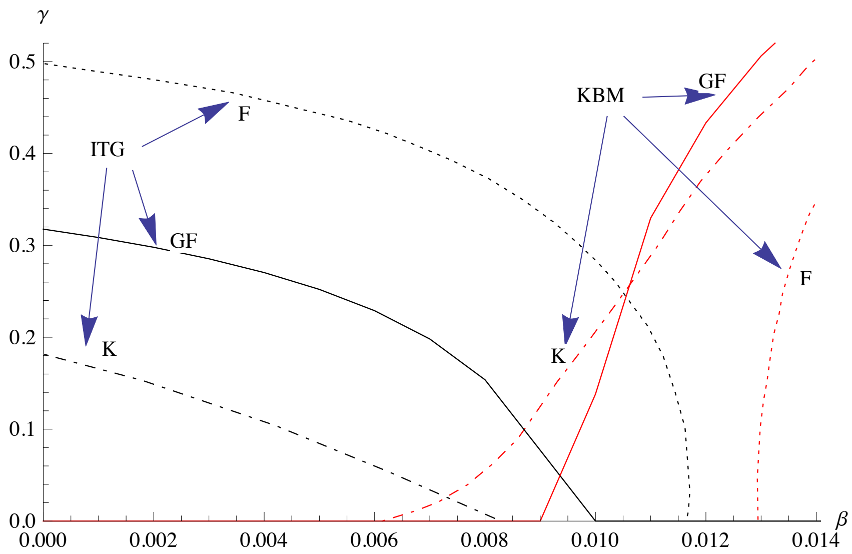
  where I omitted curvature terms for simplicity.

The linear properties are… pretty good

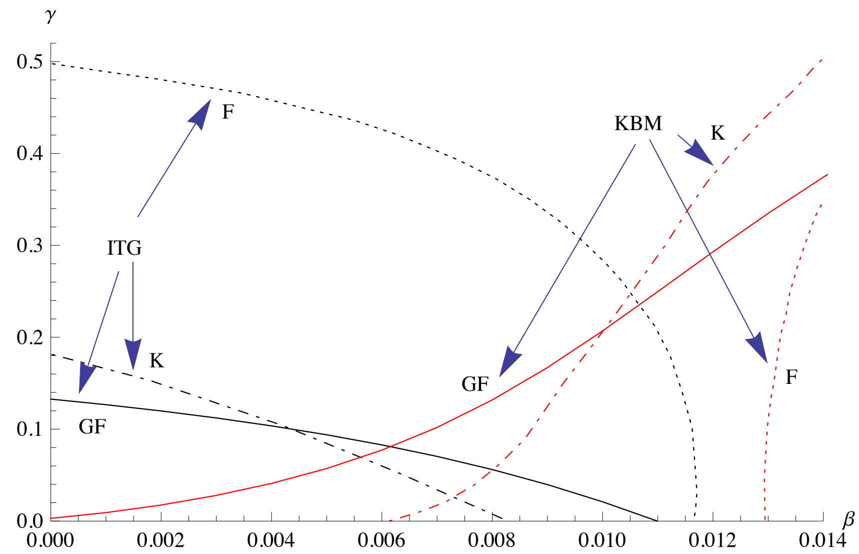


- The marginal stability for the slab ITG have the correct b>>1 behavior but are too small by a factor of 2 at long wavelength.
- This is due to the omission of $T_\perp$ from the system -> No I_1 response.

Ideal response is adequate; inclusion of HP improves it



Ideal local growth rate



local growth rate with HPB damping

Either with or without damping, the model is superior to that used in BOUT++ ELM simulations

Summary

- It has been said that Vienna is a city with a great future behind it.
- By contrast, GF models have a great past to look forward to.