

# GryfX: A GPU Gyrofluid Turbulence Code with Kinetic Zonal Flows and Advanced Nonlinear Closures

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# Introduction to Gyrofluid (GF) Model

- Tokamak turbulence can be described by the gyrokinetic approximation:

$$\frac{\omega}{\Omega} \sim \frac{\rho_i}{L_{eq}} \sim k_{\parallel} \rho_i \sim \frac{e\Phi}{T_e} \sim \frac{\delta B}{B_0} \sim O(\epsilon); \quad k_{\perp} \rho_i \sim O(1)$$

- Gyrofluid model derived by taking moments of toroidal gyrokinetic equation (Beer and Hammett, 1996)

$$\frac{\partial}{\partial t} F + \nabla \cdot [F (\mathbf{v}_{\parallel} \hat{\mathbf{b}} + \mathbf{v}_E + \mathbf{v}_d)] + \frac{\partial}{\partial v_{\parallel}} [F (-\frac{e}{m} \hat{\mathbf{b}} \cdot \nabla J_0 \Phi - \mu \hat{\mathbf{b}} \cdot \nabla B + v_{\parallel} (\hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}) \cdot \mathbf{v}_E)] = C(F)$$

- Closures chosen to capture important kinetic effects, most notably Landau damping, toroidal  $\nabla B$  and curvature drifts and associated phase mixing, and toroidal finite Larmor radius (FLR) effects

# Moment definitions and normalizations

$$\delta n = \int \delta f d^3 v$$

$$\delta p_{\parallel} = m \int \delta f v_{\parallel}^2 d^3 v$$

$$\delta q_{\parallel} = -3m v_t^2 n_0 \delta u_{\parallel} + m \int \delta f v_{\parallel}^3 d^3 v$$

$$\delta r_{\parallel, \parallel} = m \int \delta f v_{\parallel}^4 d^3 v$$

$$\delta r_{\perp, \perp} = (m/4) \int \delta f v_{\perp}^4 d^3 v$$

$$\delta s_{\parallel, \parallel} = -15m v_t^4 n_0 \delta u_{\parallel} + m \int \delta f v_{\parallel}^5 d^3 v$$

$$n_0 \delta u_{\parallel} = \int \delta f v_{\parallel} d^3 v$$

$$\delta p_{\perp} = (m/2) \int \delta f v_{\perp}^2 d^3 v$$

$$\delta q_{\perp} = -m v_t^2 n_0 \delta u_{\parallel} + (m/2) \int \delta f v_{\parallel} v_{\perp}^2 d^3 v$$

$$\delta r_{\parallel, \perp} = (m/2) \int \delta f v_{\parallel}^2 v_{\perp}^2 d^3 v$$

$$\delta s_{\perp, \perp} = -2m v_t^2 n_0 \delta u_{\parallel} + (m/4) \int \delta f v_{\parallel} v_{\perp}^4 d^3 v$$

$$\delta s_{\parallel, \perp} = -3m v_t^2 n_0 \delta u_{\parallel} + (m/2) \int \delta f v_{\parallel}^3 v_{\perp}^2 d^3 v$$

$$\left( \frac{\delta n}{n_0}, \frac{\delta u_{\parallel}}{v_t}, \frac{\delta T_{\parallel}}{T_0}, \frac{\delta T_{\perp}}{T_0}, \frac{\delta q_{\parallel}}{n_0 T_0 v_t}, \frac{\delta q_{\perp}}{n_0 T_0 v_t}, \frac{e \delta \Phi}{T_0} \right)$$

$$= \frac{\rho_i}{a} \left( \tilde{n}, \tilde{u}_{\parallel}, \tilde{T}_{\parallel}, \tilde{T}_{\perp}, \tilde{q}_{\parallel}, \tilde{q}_{\perp}, \tilde{\Phi} \right)$$

where  $v_t = \sqrt{T_0/m}$

# Equations: 6 moment Gyrofluid Model

$$\frac{\partial n}{\partial t} + \mathbf{v}_\psi \cdot \nabla n + \left[ \frac{1}{2} \hat{\nabla}_\perp^2 \mathbf{v}_\psi \right] \cdot \nabla T_\perp + B \nabla_\parallel \frac{u_\parallel}{B} - \left( f' + \frac{t'}{2} \hat{\nabla}_\perp^2 \right) i \omega_\star \Psi$$

$$+ \left( 2 + \frac{1}{2} \hat{\nabla}_\perp^2 \right) i \omega_d \Psi + i \omega_d (T_\parallel + T_\perp + 2n) = 0$$

$$\frac{\partial u_\parallel}{\partial t} + \mathbf{v}_\psi \cdot \nabla u_\parallel + \left[ \frac{1}{2} \hat{\nabla}_\perp^2 \mathbf{v}_\psi \right] \cdot \nabla q_\perp + B \nabla_\parallel \frac{T_\parallel}{B} + B \nabla_\parallel \frac{n}{B} + \nabla_\parallel \Psi$$

$$+ \left( T_\perp + n + \frac{1}{2} \hat{\nabla}_\perp^2 \Psi \right) \nabla_\parallel \ln B + i \omega_d (q_\parallel + q_\perp + 4u_\parallel) = 0$$

$$\frac{\partial T_\parallel}{\partial t} + \mathbf{v}_\psi \cdot \nabla T_\parallel + B \nabla_\parallel \frac{q_\parallel + 2u_\parallel}{B} + 2(q_\perp + u_\parallel) \nabla_\parallel \ln B - t' i \omega_\star \Psi + 2i \omega_d \Psi$$

$$+ i \omega_d (6T_\parallel + 2n) + 2|\omega_d| (\nu_1 T_\parallel + \nu_2 T_\perp) = -\frac{2}{3} \nu_{ii} (T_\parallel - T_\perp)$$

# Equations: 6 moment Gyrofluid model (continued)

$$\begin{aligned}
 \frac{\partial T_{\perp}}{\partial t} + \mathbf{v}_{\Psi} \cdot \nabla T_{\perp} + \left[ \frac{1}{2} \hat{\nabla}_{\perp}^2 \mathbf{v}_{\Psi} \right] \cdot \nabla n + \left[ \hat{\nabla}_{\perp}^2 \mathbf{v}_{\Psi} \right] \cdot \nabla T_{\perp} - B \nabla_{\parallel} \frac{u_{\parallel}}{B} \\
 + B^2 \nabla_{\parallel} \frac{q_{\perp} + u_{\parallel}}{B^2} - \left[ \frac{f'}{2} \hat{\nabla}_{\perp}^2 + t' \left( 1 + \hat{\nabla}_{\perp}^2 \right) \right] i \omega_{*} \Psi + \left( 1 + \frac{1}{2} \hat{\nabla}_{\perp}^2 + \hat{\nabla}_{\perp}^2 \right) i \omega_d \Psi \\
 + i \omega_d (4 T_{\perp} + n) + 2 |\omega_d| (\nu_3 T_{\parallel} + \nu_4 T_{\perp}) = \frac{1}{3} \nu_{ii} (T_{\parallel} - T_{\perp})
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial q_{\parallel}}{\partial t} + \mathbf{v}_{\Psi} \cdot \nabla q_{\parallel} + (3 + \beta_{\parallel}) \nabla_{\parallel} T_{\parallel} + \sqrt{2} D_{\parallel} |k_{\parallel}| q_{\parallel} + i \omega_d (-3 q_{\parallel} - 3 q_{\perp} + 6 u_{\parallel}) \\
 + |\omega_d| (\nu_5 u_{\parallel} + \nu_6 q_{\parallel} + \nu_7 q_{\perp}) = -\nu_{ii} q_{\parallel}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial q_{\perp}}{\partial t} + \mathbf{v}_{\Psi} \cdot \nabla q_{\perp} + \left[ \frac{1}{2} \hat{\nabla}_{\perp}^2 \mathbf{v}_{\Psi} \right] \cdot \nabla u_{\parallel} + \left[ \hat{\nabla}_{\perp}^2 \mathbf{v}_{\Psi} \right] \cdot \nabla q_{\perp} + \nabla_{\parallel} \left( T_{\perp} + \frac{1}{2} \hat{\nabla}_{\perp}^2 \Psi \right) \\
 + \sqrt{2} D_{\perp} |k_{\parallel}| q_{\perp} + \left( T_{\perp} - T_{\parallel} + \hat{\nabla}_{\perp}^2 \Psi - \frac{1}{2} \hat{\nabla}_{\perp}^2 \Psi \right) \nabla_{\parallel} \ln B \\
 + i \omega_d (-q_{\parallel} - q_{\perp} + u_{\parallel}) + |\omega_d| (\nu_8 u_{\parallel} + \nu_9 q_{\parallel} + \nu_{10} q_{\perp}) = -\nu_{ii} q_{\perp}
 \end{aligned}$$

## Quasineutrality and definitions

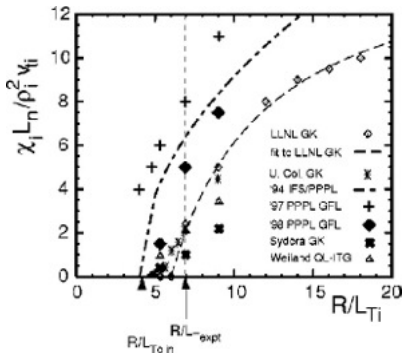
$$n_e = \frac{n}{1 + b/2} - \frac{bT_{\perp}}{2(1 + b/2)^2} + (\Gamma_0 - 1)\Phi$$

When electrons are assumed to be adiabatic, which is the case for all results presented here, we have

$$n_e = \frac{T_{i0}}{T_{e0}}(\Phi - \langle \Phi \rangle)$$

$$\begin{aligned} b &= k_{\perp}^2 \rho_i^2, & \Psi &= \Gamma_0^{1/2}(b)\Phi, & \mathbf{v}_{\Psi} &= \frac{c}{B} \hat{\mathbf{b}} \times \nabla \Psi, & \frac{1}{2} \hat{\nabla}_{\perp}^2 \Psi &= b \frac{\partial \Gamma_0^{1/2}}{\partial b} \Phi, \\ \hat{\nabla}_{\perp}^2 \Psi &= b \frac{\partial^2}{\partial b^2} (b \Gamma_0^{1/2}) \Phi, & \nabla_{\parallel} &= \hat{\mathbf{b}} \cdot \nabla, & i\omega_* &= \frac{-cT_0}{eBn_0} \nabla n_0 \cdot \hat{\mathbf{b}} \times \nabla, \\ i\omega_d &= \frac{cT_0}{eB^2} \hat{\mathbf{b}} \times \nabla B \cdot \nabla, & f' &= \frac{a}{L_N}, & t' &= \frac{a}{L_T} \end{aligned}$$

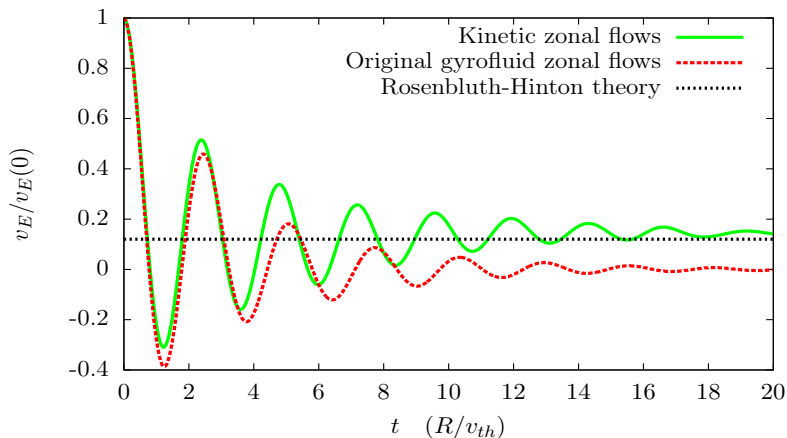
# Need to improve original GF model



"Comparisons and physics basis of tokamak transport models and turbulence simulations", Dimits, et al. Phys. Plasmas, 2000.

- Original toroidal gyrofluid closures missing key physics: linearly undamped zonal flows and nonlinear FLR phase mixing (NLPM)
- Attempts made to correctly model zonal flows by modifying Landau damping closures (Beer, 1998)
- NLPM included in earlier slab-geometry gyrofluid model (Dorland, 1993), but not thought to be as important in toroidal geometry

# Rosenbluth-Hinton residual flow



- Original gyrofluid model does not allow Rosenbluth-Hinton residual flow



# Attempted Zonal Flow Closures (Varena closure)

$$\sqrt{2}D_{\parallel}|k_{\parallel}|q_{\parallel} \rightarrow \sqrt{2}D_{\parallel}|k_{\parallel}|(q_{\parallel} - q_{\parallel}^{(0)})$$

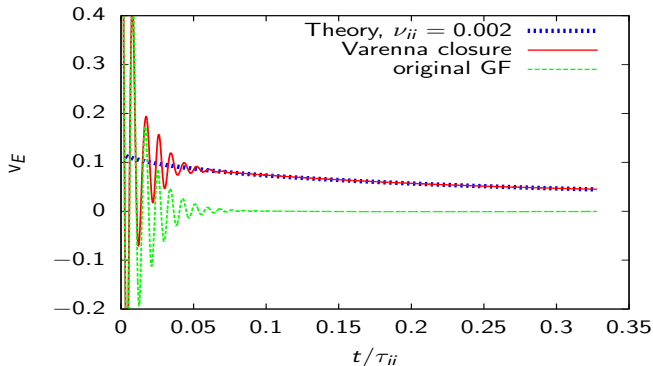
$$\sqrt{2}D_{\perp}|k_{\parallel}|q_{\perp} \rightarrow \sqrt{2}D_{\perp}|k_{\parallel}|(q_{\perp} - q_{\perp}^{(0)})$$

$$q_{\parallel}^{(0)} = 3ik_r\rho_i \frac{qB_0}{\varepsilon B} T_{\parallel}, \quad q_{\perp}^{(0)} = ik_r\rho_i \frac{qB_0}{\varepsilon B} T_{\perp}$$

- GF modified by Beer et al to allow linearly undamped R-H residual flows; published in Varena Proceedings, 1998
- $q_{\parallel}^{(0)}$  and  $q_{\perp}^{(0)}$  are Pfirsch-Schluter heat flows
- New model damps to non-zero equilibrium solution

Beer and Hammett, invited talk, published in Proc. of the Joint Varena-Lausanne Int. Workshop on Theory of Fusion Plasmas (August 1998), p.19 (Varena, Italy 1998).

# Problems with Varena zonal flow closures



- Agreement with Rosenbluth-Hinton theory only good for a small range of low  $k_x$
- Residual still has some damping even without collisions
- Collisional damping of zonal flows not modeled well
- For more exotic geometries, theory of residuals becomes very complicated (Catto & Xiao)

## Even better: Fully kinetic zonal flows

- Evolve  $k_y = 0$  modes with gyrokinetic model (GS2), and all other modes with original gyrofluid model  $\rightarrow$  Spectral representation makes this trivial for linear terms
- Essentially running GS2 with  $N_y=1$  linearly with nonlinear source term from GryfX
- All nonlinear terms calculated in gyrofluid model; in gyrokinetic equation,  $\mathcal{N}_{gk}$  must be reconstructed from the  $k_y = 0$  component of the nonlinear terms of the gyrofluid moments:

$$\begin{aligned}\hat{\mathcal{N}}_{gk} = & \mathcal{N}_n + \frac{v_{\parallel}}{v_t} \mathcal{N}_{u_{\parallel}} + \frac{1}{2} \left( \frac{v_{\parallel}^2}{v_t^2} - 1 \right) \mathcal{N}_{T_{\parallel}} + \frac{1}{2} \left( \frac{v_{\perp}^2}{v_t^2} - 2 \right) \mathcal{N}_{T_{\perp}} \\ & + \frac{1}{2} \left( \frac{v_{\parallel}^3}{3v_t^3} - \frac{v_{\parallel}}{v_t} \right) \mathcal{N}_{q_{\parallel}} + \frac{v_{\parallel}}{v_t} \left( \frac{v_{\perp}^2}{2v_t^2} - 1 \right) \mathcal{N}_{q_{\perp}}\end{aligned}$$

- Consistent with moment definitions, such that  $\mathcal{N}_n = \int \hat{\mathcal{N}}_{gk} d^3v$ ,  $\mathcal{N}_{u_{\parallel}} = \int \hat{\mathcal{N}}_{gk} v_{\parallel} d^3v$ , etc

# Hybrid gyrofluid-gyrokinetic algorithm

GPU (GF)

CPU (GK)

calculate  $\mathcal{N}_{m_i}[t]_{\text{all } k}$

calculate  $\mathcal{L}_{m_i}[t]_{k_y \neq 0}$

estimate  $m_i(t + \Delta t/2)_{k_y \neq 0}$ ,  
 $\Phi(t + \Delta t/2)_{k_y \neq 0}$

copy GPU → CPU

$\mathcal{N}_{m_i}[t]_{k_y=0}$

advance  $g_{k_y=0}$ ,  $\Phi_{k_y=0}$   
 $t \rightarrow t + \Delta t/2$

calculate  $m_i(t + \Delta t/2)_{k_y=0}$

copy CPU → GPU

$m_i(t + \Delta t/2)_{k_y=0}$ ,  $\Phi(t + \Delta t/2)_{k_y=0}$

calculate  $\mathcal{N}_{m_i}[t + \Delta t/2]_{\text{all } k}$

calculate  $\mathcal{L}_{m_i}[t + \Delta t/2]_{k_y \neq 0}$

calculate  $m_i(t + \Delta t)_{k_y \neq 0}$ ,  
 $\Phi(t + \Delta t)_{k_y \neq 0}$

copy GPU → CPU

$\mathcal{N}_{m_i}[t + \Delta t/2]_{k_y=0}$

advance  $g_{k_y=0}$ ,  $\Phi_{k_y=0}$   
 $t + \Delta t/2 \rightarrow t + \Delta t$

calculate  $m_i(t + \Delta t)_{k_y=0}$

copy CPU → GPU

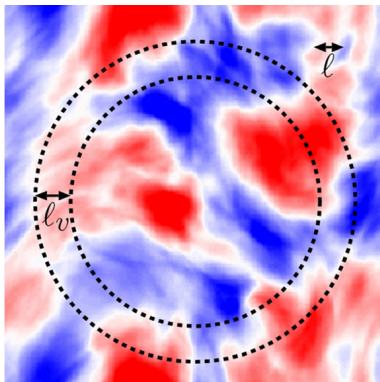
$m_i(t + \Delta t)_{k_y=0}$ ,  $\Phi(t + \Delta t)_{k_y=0}$

# ZF velocity space resolution requirements/questions

- Preliminary simulation results with low velocity space resolution ( $n_{\text{gauss}}=3$ ,  $n_{\text{grid}}=6$ ) give same heat fluxes as higher resolution ( $n_{\text{gauss}}=8$ ,  $n_{\text{grid}}=16$ )
- $n_{\text{theta}}=32$  so really just reducing v-space resolution for passing particles
- How much (non-trapped) velocity space resolution is necessary to capture zonal flow dynamics?
- What is the velocity space structure of the zonal flows in a full GS2 simulation?
- We only have 6 gyrofluid moments ( $\sim 6$  grid points) for non-zonal modes... would having too much velocity space resolution for zonal modes ever become problematic/overdetermined?

# Nonlinear FLR Phase Mixing

- Spread in gyro-averaged  $\mathbf{E} \times \mathbf{B}$  velocities leads to phase mixing in perpendicular direction (analogous to Landau damping in parallel direction)
- Entropy cascade in  $k_{\perp}$  and  $v_{\perp}$
- Damping proportional to  $k_{\perp}^2 |\mathbf{v}_E \cdot \mathbf{k}_{\perp}|$



# 1-D model of NLPM

- Consider a static zonal flow potential  $\Phi = \Phi_{ZF}(x)$  in sheared slab geometry
- Neglecting equilibrium and parallel gradients, the GK equation is

$$\frac{\partial F_1}{\partial t} + J_0 \left( \frac{k_x v_\perp}{\Omega} \right) v_E \frac{\partial F_1}{\partial y} = 0$$

- Consider initial perturbation  $F_1(t=0) = e^{ik_y y} F_M(v)$ , so the solution is

$$F_1(t) = F_M e^{ik_y [y - J_0(k_x v_\perp / \Omega) v_E t]}$$

# 1-D model of NLPM

- Expanding  $J_0$  for small  $k_{\perp}$ , we can integrate analytically to see that all moments of  $F_1$  decay in time:

$$n_{1kin}(t) = \int d^3v F_1 \simeq n_0 e^{ik_y(y-vEt)} \frac{1}{1 - ik_y b v_{Et}/2}$$

$$p_{\perp kin}(t) = \int d^3v \frac{mv_{\perp}^2}{2n_0 v_t^2} F_1 \simeq n_0 e^{ik_y(y-vEt)} \frac{1}{(1 - ik_y b v_{Et}/2)^2}$$

$$T_{\perp kin}(t) = p_{\perp kin} - n_{kin} \simeq n_0 e^{ik_y(y-vEt)} \frac{ik_y b v_{Et}/2}{(1 - ik_y b v_{Et}/2)^2}$$

$$r_{\perp, \perp kin}(t) = \int d^3v \frac{mv_{\perp}^2}{4n_0 v_t^4} F_1 \simeq n_0 e^{ik_y(y-vEt)} \frac{2}{(1 - ik_y b v_{Et}/2)^3}$$

where here  $b = k_x^2 v_t^2 / \Omega^2$ .



# 1-D model of NLPM

- Working backwards, we find that these expressions are exact solutions of the following set of equations:

$$\begin{aligned}\frac{\partial n}{\partial t} + \left(1 - \frac{b}{2}\right) \mathbf{v}_E \cdot \nabla n - \frac{b}{2} \mathbf{v}_E \cdot \nabla T_{\perp} &= 0 \\ \frac{\partial T_{\perp}}{\partial t} + \left(1 - \frac{3b}{2}\right) \mathbf{v}_E \cdot \nabla T_{\perp} - \frac{b}{2} \mathbf{v}_E \cdot \nabla n \\ - \frac{b}{2} \mathbf{v}_E \cdot \nabla (r_{\perp, \perp} - 2n - 4T_{\perp}) &= 0\end{aligned}$$

- Match low- $b$  limit of original gyrofluid nonlinear terms with exception of last term
- For this test problem, an exact solution for  $r_{\perp, \perp}$  can be expressed as

$$r_{\perp, \perp} = \frac{4(n + T_{\perp})}{1 - ik_y b v_{Et}/2} - \frac{2n}{(1 - ik_y b v_{Et}/2)^2}$$

- Can extend to higher  $b$  by taking  $(1 - b/2)\mathbf{v}_E \rightarrow \mathbf{v}_{\Psi}$ ,  $(-b/2)\mathbf{v}_E \rightarrow \frac{1}{2}\hat{\nabla}_{\perp}^2 \mathbf{v}_{\Psi}$

# 1-D model of NLPM

- We want to recover this damping behavior with our closures, so we break the  $r_{\perp,\perp}$  closure into Maxwellian and dissipative pieces

$$r_{\perp,\perp} = 2n + 4T_{\perp} + \frac{\left| \left[ \frac{1}{2} \hat{\nabla}_{\perp}^2 \mathbf{v}_{\psi} \right] \cdot \nabla \right|}{\left[ \frac{1}{2} \hat{\nabla}_{\perp}^2 \mathbf{v}_{\psi} \right] \cdot \nabla} [\mu_1(n + T_{\perp}) + \mu_2 n]$$

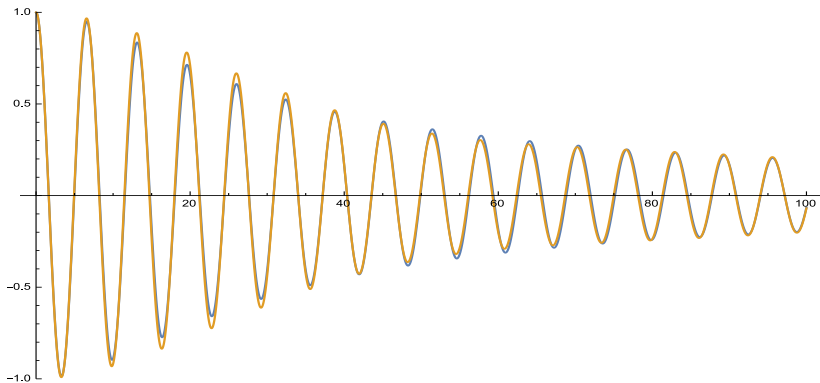
- The closure coefficients contain a dissipative and non-dissipative piece, to try to capture the phase shift

$$\mu = \mu_r + \mu_i \frac{\left| \left[ \frac{1}{2} \hat{\nabla}_{\perp}^2 \mathbf{v}_{\psi} \right] \cdot \nabla \right|}{\left[ \frac{1}{2} \hat{\nabla}_{\perp}^2 \mathbf{v}_{\psi} \right] \cdot \nabla} = (\mu_r, \mu_i)$$

- Find  $\mu$ 's by minimizing difference between fluid and kinetic density responses for  $b = 0.1$

# 1-D model of NLPM

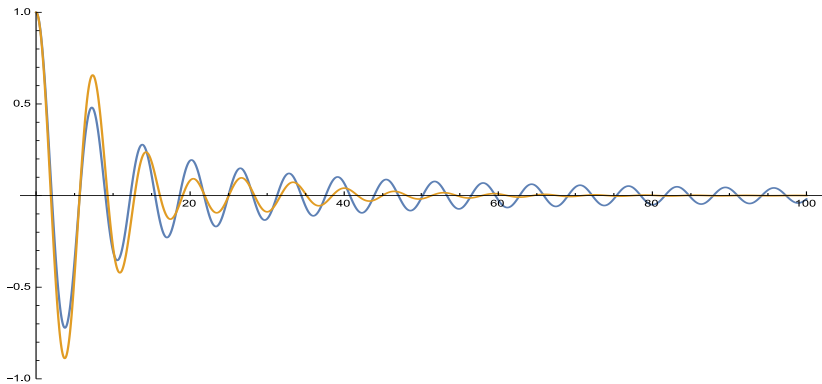
- Good agreement for density response for  $b = 0.1$ .



GryfX in yellow, Kinetic in blue

# 1-D model of NLPM

- Not as good for larger  $b$ ... here  $b = 0.5$ .



GryfX in yellow, Kinetic in blue

# Extending NLPM model to 3-D

- In general 3-D geometry, we currently approximate

$$|\mathbf{v} \cdot \nabla| M \simeq D_{PM} \left( |v_x| \left| \frac{\partial}{\partial x} \right| M + |v_y| \left| \frac{\partial}{\partial y} \right| M \right)$$

- This misses interference terms in the original absolute value
- Adjustable scalar parameter  $D_{PM}$  included to account for this, with  $D_{PM} \lesssim 1$
- Is there a better way to evaluate these terms? Greg suggested

$$|\mathbf{v} \cdot \nabla| M \simeq |k_x| (v_x M - (v_x M)_0) + |k_y| (v_y M - (v_y M)_0)$$

where in equilibrium,  $\frac{\partial}{\partial x} (v_x M)_0 = -\frac{\partial}{\partial y} (v_y M)_0$ , so that

$$\nabla \cdot (\mathbf{v} M)_0 \rightarrow 0$$

# Nonlinear FLR Phase Mixing

- Following a similar procedure for the remaining equations, the final gyrofluid nonlinear terms become

$$\mathcal{N}_n = \mathbf{v}_\psi \cdot \nabla n + \left[ \frac{1}{2} \hat{\nabla}_\perp^2 \mathbf{v}_\psi \right] \cdot \nabla T_\perp$$

$$\mathcal{N}_{u_\parallel} = \mathbf{v}_\psi \cdot \nabla u_\parallel + \left[ \frac{1}{2} \hat{\nabla}_\perp^2 \mathbf{v}_\psi \right] \cdot \nabla q_\perp$$

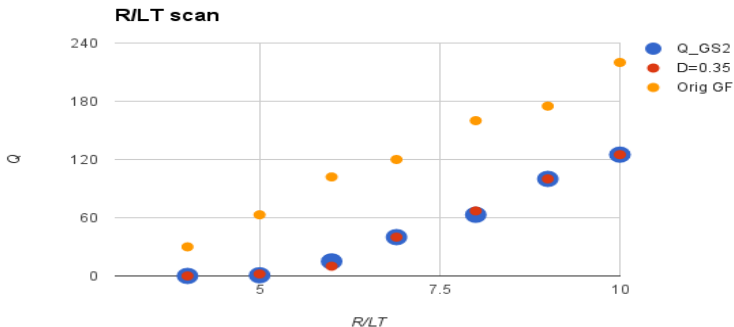
$$\mathcal{N}_{T_\parallel} = \mathbf{v}_\psi \cdot \nabla T_\parallel + \left[ \frac{1}{2} \hat{\nabla}_\perp^2 \mathbf{v}_\psi \right] \cdot \nabla \left| \mu_3 T_\parallel \right|$$

$$\mathcal{N}_{T_\perp} = \mathbf{v}_\psi \cdot \nabla T_\perp + \left[ \frac{1}{2} \hat{\nabla}_\perp^2 \mathbf{v}_\psi \right] \cdot \nabla n + \left[ \hat{\nabla}_\perp^2 \mathbf{v}_\psi \right] \cdot \nabla T_\perp + \left[ \frac{1}{2} \hat{\nabla}_\perp^2 \mathbf{v}_\psi \right] \cdot \nabla \left| (\mu_1 T_\perp + (\mu_1 + \mu_2) n) \right|$$

$$\mathcal{N}_{q_\parallel} = \mathbf{v}_\psi \cdot \nabla q_\parallel + \left[ \frac{1}{2} \hat{\nabla}_\perp^2 \mathbf{v}_\psi \right] \cdot \nabla \left| \mu_3 q_\parallel \right|$$

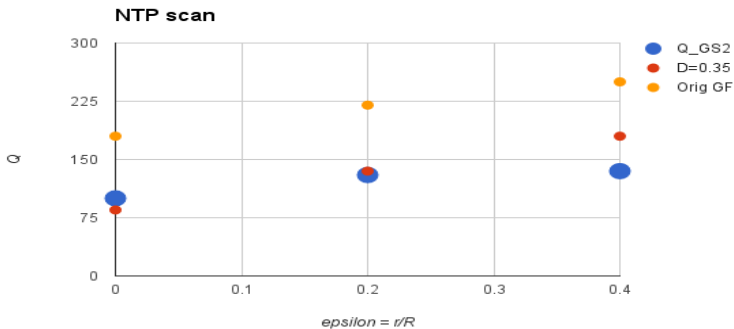
$$\mathcal{N}_{q_\perp} = \mathbf{v}_\psi \cdot \nabla q_\perp + \left[ \frac{1}{2} \hat{\nabla}_\perp^2 \mathbf{v}_\psi \right] \cdot \nabla u_\parallel + \left[ \hat{\nabla}_\perp^2 \mathbf{v}_\psi \right] \cdot \nabla q_\perp + \left[ \frac{1}{2} \hat{\nabla}_\perp^2 \mathbf{v}_\psi \right] \cdot \nabla \left| (\mu_1 q_\perp + (\mu_1 + \mu_2) u_\parallel) \right|$$

# $R/L_T$ scan with NLPM



- Including NLPM and kinetic ZF brings GryfX heat flux predictions into good agreement with GK (GS2)
- $D_{PM} = 0.35$  for all these simulations, `ngauss=3`, `negrid=6`

# NTP scan with NLPM



- Too much NLPM for  $\epsilon = 0$ , too little for  $\epsilon = 0.4$
- $D_{PM}$  could scale with ZF amplitude?
- Or find better way of evaluating  $|\mathbf{v}_E \cdot \nabla|$

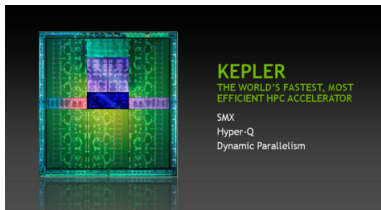


# Why do we need Gyrofluid Codes?

- Transport optimization: Use models coupled to TRINITY transport solver to evaluate transport properties of tokamak design configurations
- To evaluate a configuration, need to vary  $T(a)$ ,  $n(a)$ ,  $Q$ ,  $S$ ,  $\Pi$  independently. With 3 points in each direction,  $3^5 = 243$  TRINITY runs required.
- Configuration parameters to vary include current profile, elongation, triangularity,  $\beta$ , etc. 128 configurations would be a start.
- A single TRINITY + GS2 simulation requires  $O(1M)$  CPU hours, so entire scan would take  $\sim 32,000$  M CPU hours
- Need to find multiple orders of magnitude of acceleration to make this project possible  $\rightarrow$  **gyrofluid, GPUs**

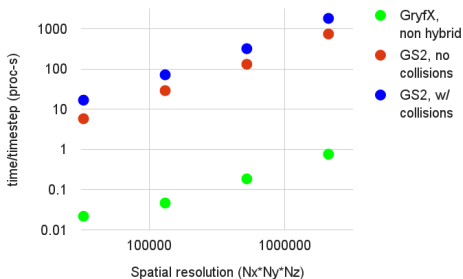
# Introducing GryfX: A GPU-based Gyrofluid Turbulence Code with Kinetic Zonal Flows and Advanced Nonlinear Closures

- Gyrofluid vs Gyrokinetic: 6 moments vs 256 grid points in velocity space  $\sim 40X$  speedup
- Gyrofluid model requires less memory  $\rightarrow$  Able to fit efficiently on GPU
- GPU can achieve 20-30X speedup over CPU
- Total acceleration  $> 1,200 X$

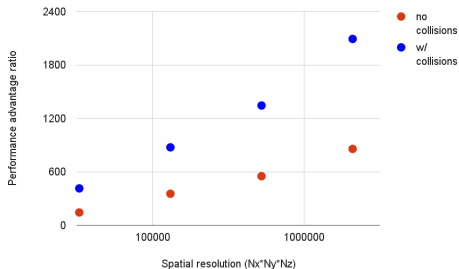


# GryfX (non-hybrid) vs GS2 performance comparison

## Proc-time Comparison

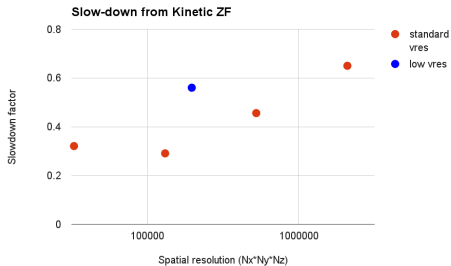
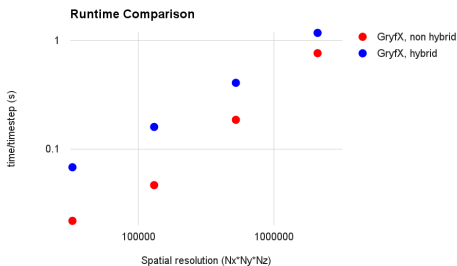


## Performance Advantage of GryfX over GS2



- Total speedup of more than 2,000X over GS2 for large grid

# Cost of kinetic ZF minimized by GPU-CPU concurrency



- GF runs on GPU while GK runs on 16 CPUs: most supercomputer configurations allocate multiple CPUs for each GPU, so additional computation on CPU is essentially free
- Slows down by factor of 1-2, but this is an acceptable trade-off to get zonal flow physics right
- With 'low' velocity space resolution, converged nonlinear simulations in  $\sim 1$  hour (on 1 node = GPU + 16 procs)

# TRINITY + GryfX on Titan, for example

For full design scan:

- GS2 + TRINITY  $\sim$  32,000 M CPU hours
- GryfX (hybrid) + TRINITY  $\sim$  400 M CPU hours
- GryfX (non-hybrid) + TRINITY  $\sim$  25 M CPU hours  
(\*only if charged as 1 GPU-hour = 1 CPU-hour\*)
- Titan (ORNL) has 18,688 nodes, each with 1 GPU and 16 CPUs  $\rightarrow$  7M CPU hours per day  $\rightarrow$  450K GryfX runs per day
- Full tokamak design study could be completed in a month:  
each node is always running **its own** GryfX calculation, so that  
hundreds of configurations can be evaluated **simultaneously**
- Would take years to do this with GS2

# Future Work

- Keep improving gyrofluid NLPM closures
- Use kinetic zonal flows to guide improvements to gyrofluid zonal flow closures
- Implement gyrofluid electron equations (Beer or Snyder)
- Use GryfX for more efficient study of experimental results
- If accurate enough, GryfX can directly aid gyrokinetic simulations (resolution/convergence, pre-conditions, etc)
- DEMO design project: TRINITY + GryfX
  - Scan large parameter space efficiently
  - Find ideal design configurations