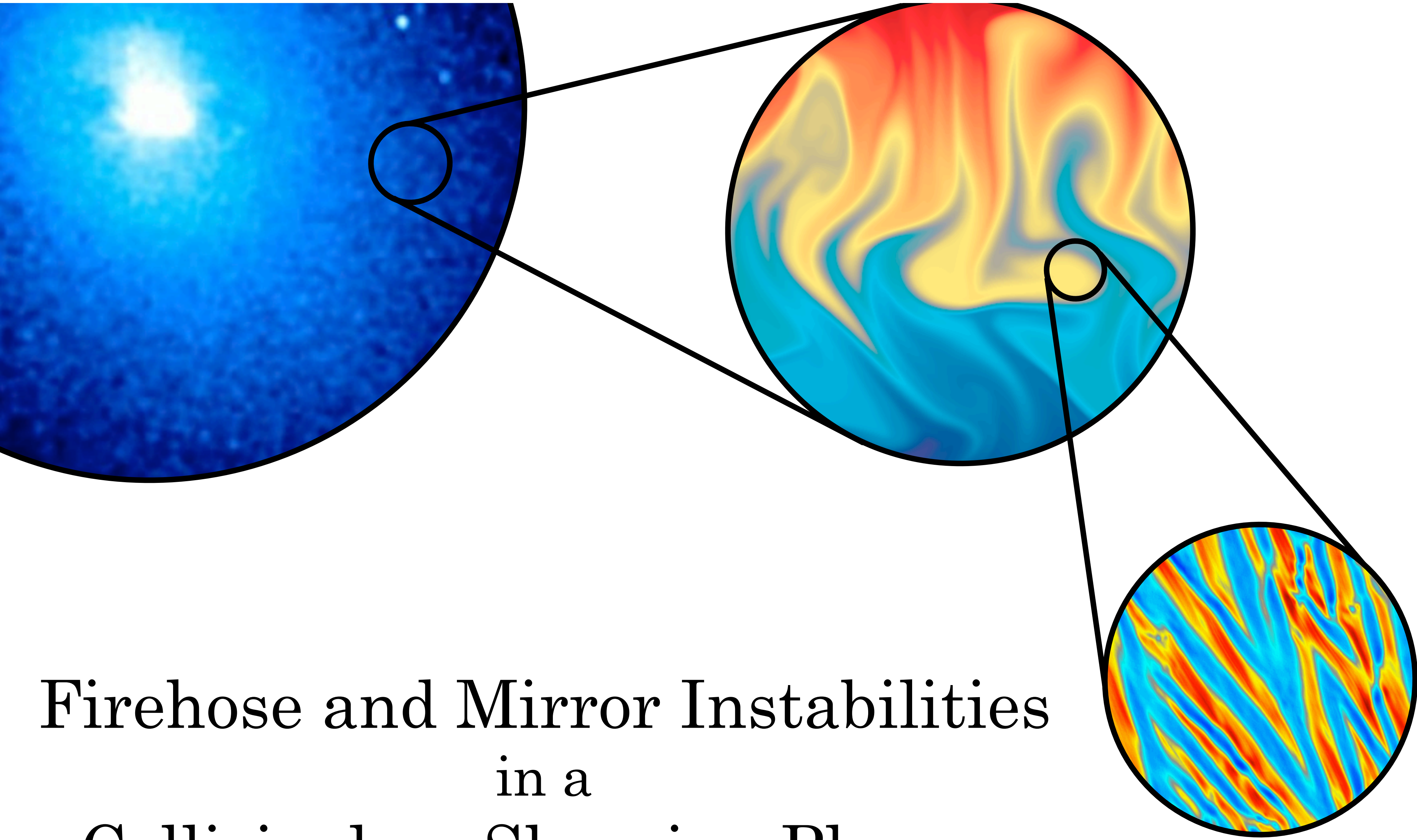
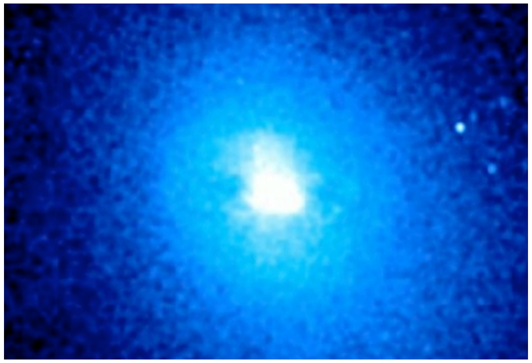


Firehose and Mirror Instabilities
in a
Collisionless Shearing Plasma

Matthew Kunz, Alex Schekochihin, Jim Stone

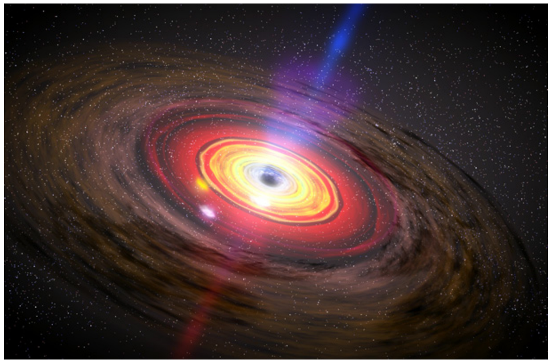


ICM @ $\mathrm{R}_{\mathrm{cool}}$

$$n \sim 10^{-2}\ \mathrm{cm}^{-3}$$

$$T \sim 10\ \mathrm{keV}$$

$$B \sim 1\ \mu\mathrm{G}$$

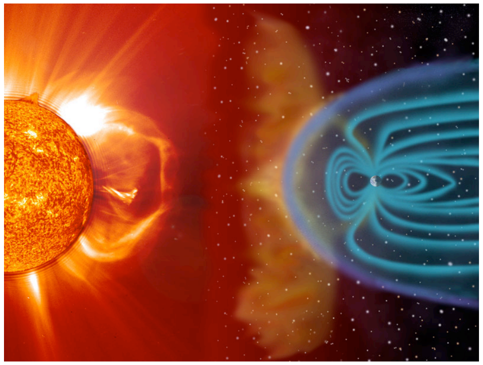


Sgr A* @ $\mathrm{R}_{\mathrm{Bondi}}$

$$n \sim 100\ \mathrm{cm}^{-3}$$

$$T \sim 2\ \mathrm{keV}$$

$$B \sim 1\ \mathrm{mG}$$



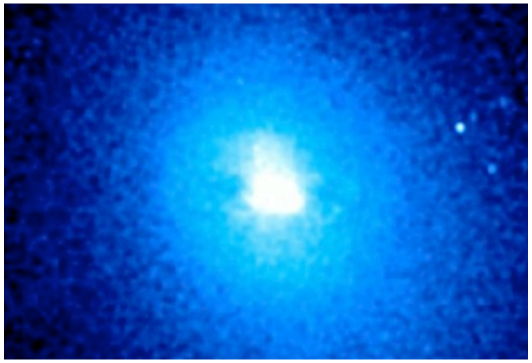
solar wind @ $\mathrm{d}_{\oplus}$

$$n \sim 10\ \mathrm{cm}^{-3}$$

$$T \sim 1\ \mathrm{eV}$$

$$B \sim 20\ \mu\mathrm{G}$$

weakly collisional → collisionless

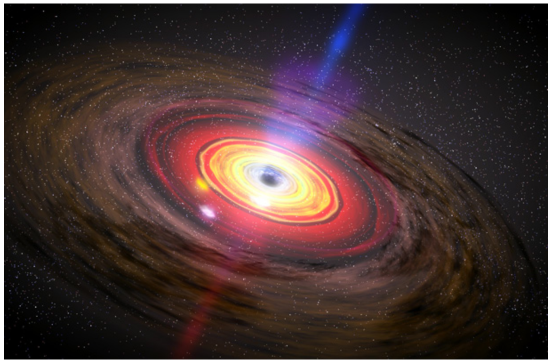


ICM @ $\mathrm{R}_{\mathrm{cool}}$

$L \sim 100$ kpc

$\lambda_{\mathrm{mfp}} \sim 1$ kpc

$\rho_{\mathrm{i}} \sim 1$ npc

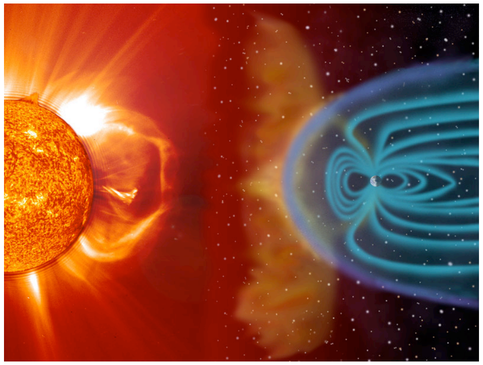


Sgr A* @ $\mathrm{R}_{\mathrm{Bondi}}$

$L \sim 0.1$ pc

$\lambda_{\mathrm{mfp}} \sim 0.01$ pc

$\rho_{\mathrm{i}} \sim 1$ ppc

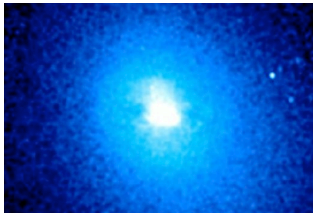


solar wind @ $\mathrm{d}_{\oplus}$

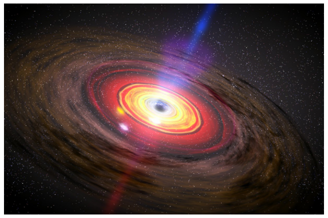
$L \sim 1$ au

$\lambda_{\mathrm{mfp}} \sim 1$ au

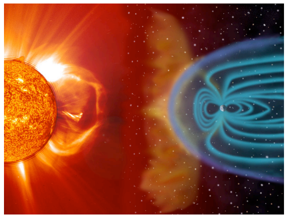
$\rho_{\mathrm{i}} \sim 10^{-6}$ au



intracluster medium of galaxy clusters

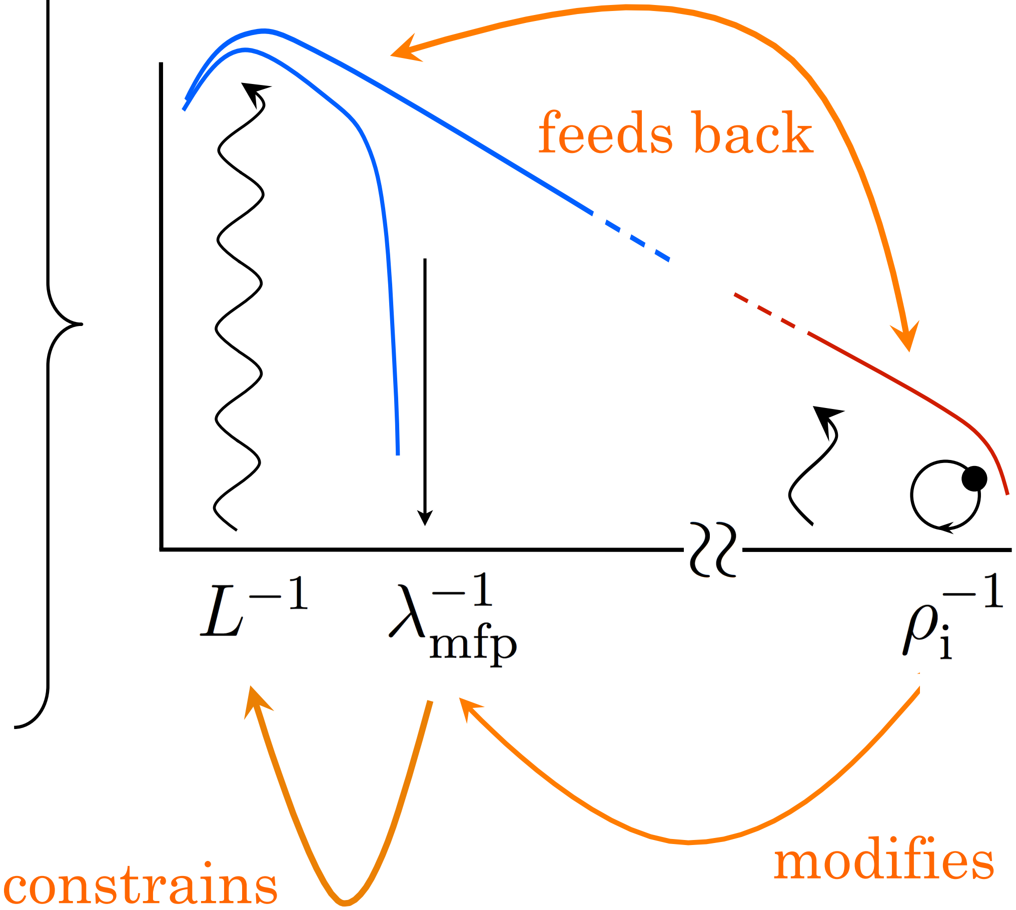


radiatively inefficient accretion flows

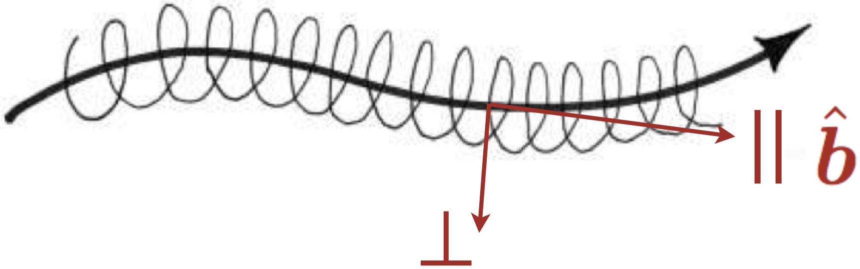


solar wind

underlying universal physics



$L, \lambda_{\mathrm{mfp}} \gg \rho_{\mathrm{i}}$



Why pressure anisotropy?

$$\frac{\mathrm{d}}{\mathrm{d}t} \oint \boldsymbol{p} \cdot \mathrm{d}\boldsymbol{q} \simeq 0$$

magnetic field introduces periodic motion

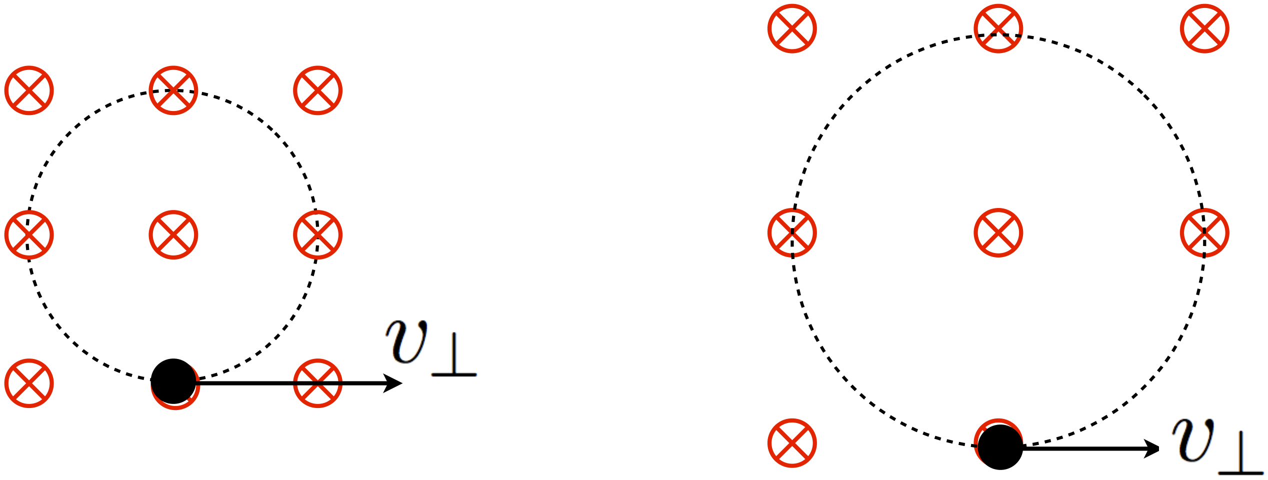
Why pressure anisotropy?

$$\frac{\mathrm{d}}{\mathrm{d}t}\oint \boldsymbol{p}\cdot \mathrm{d}\boldsymbol{q} \simeq 0$$

1. Magnetic moment: conservation of angular momentum

$$\mu \equiv \frac{m v_\perp'^2}{2B} \sim \mathrm{const} \qquad m v_\perp' \rho = \frac{m v_\perp'^2}{\Omega} \propto \frac{m v_\perp'^2}{B}$$

sum over particles: $\left(\int \mathrm{d}^3\boldsymbol{v}'\mu f = \frac{p_\perp}{B}\right) \times \text{volume} \rightarrow \boxed{\frac{p_\perp}{nB} \sim \mathrm{const}}$



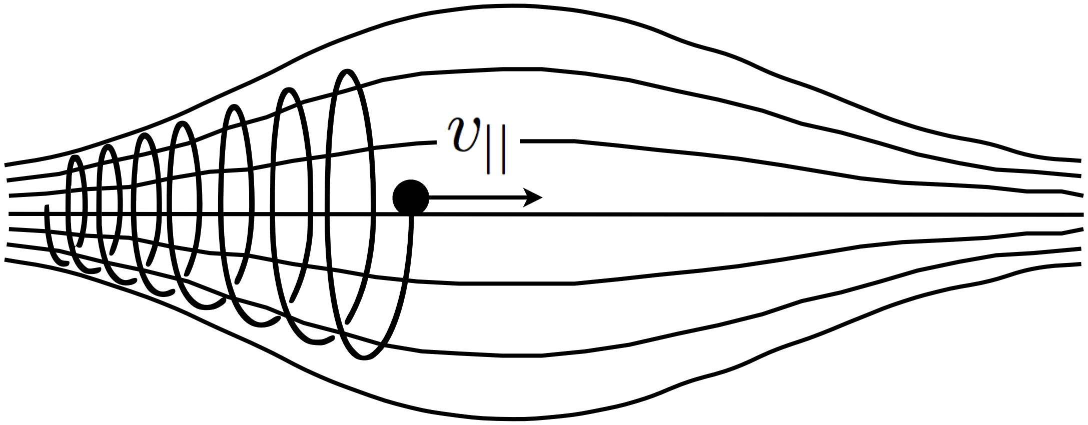
Why pressure anisotropy?

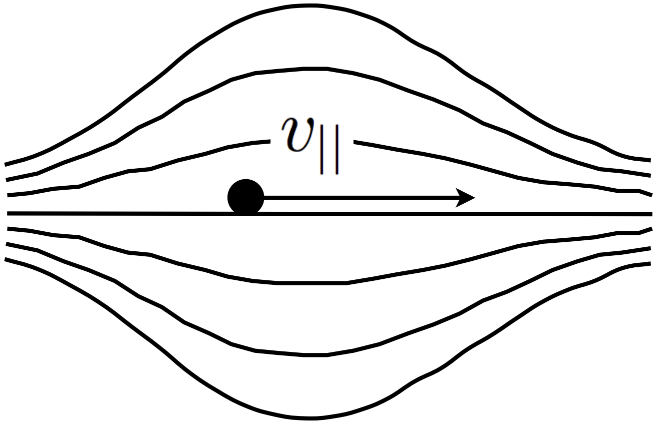
$$\frac{\mathrm{d}}{\mathrm{d}t}\oint \boldsymbol{p}\cdot\mathrm{d}\boldsymbol{q}\simeq 0$$

2. Bounce invariant: conservation of linear momentum

$$J\equiv\oint mv'_{||}\mathrm{d}\ell\sim\text{const}$$

sum over particles: $\left(\int \mathrm{d}^3\boldsymbol{v}' Jf=\frac{p_{||}B^2}{n^2}\right)\times \text{volume}\rightarrow \boxed{\frac{p_{||}B^2}{n^3}\sim\text{const}}$

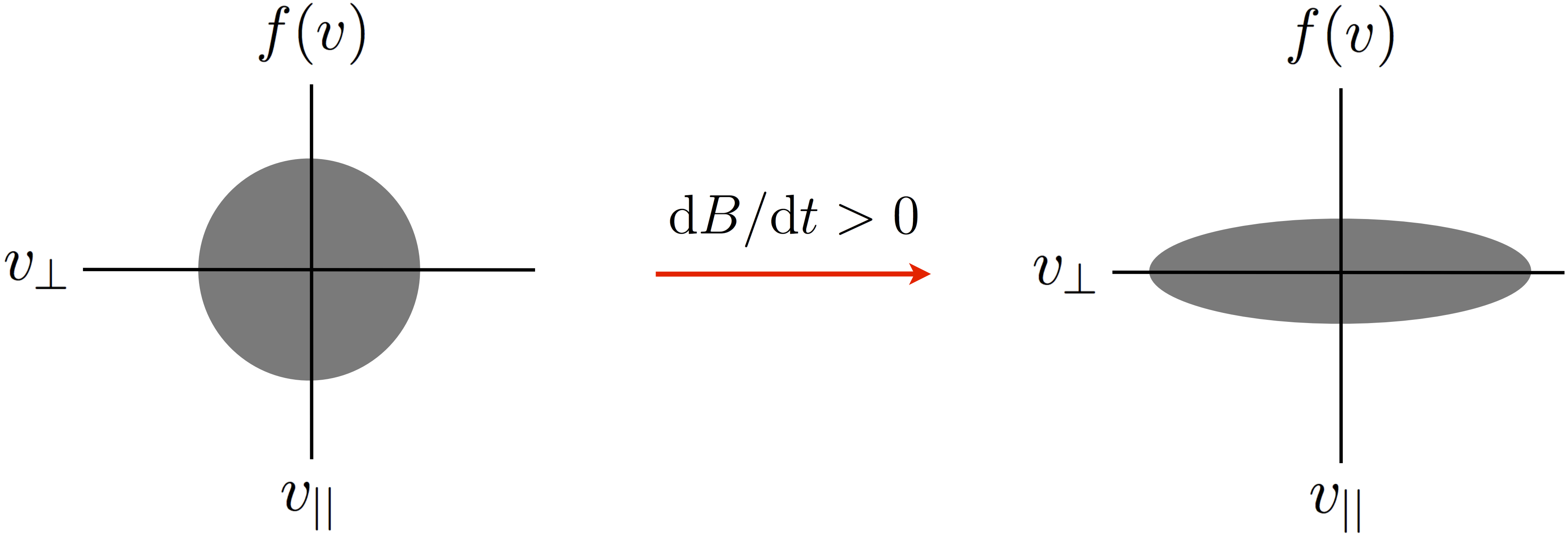




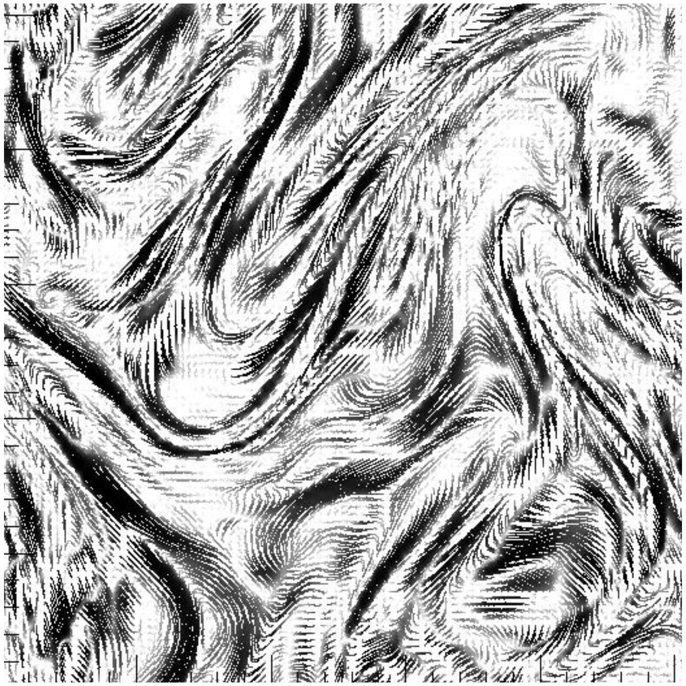
Why pressure anisotropy?

adiabatic evolution:

$$p \propto n^{5/3} \longrightarrow \quad p_\perp \propto nB \quad \text{and} \quad p_{||} \propto \frac{n^3}{B^2}$$

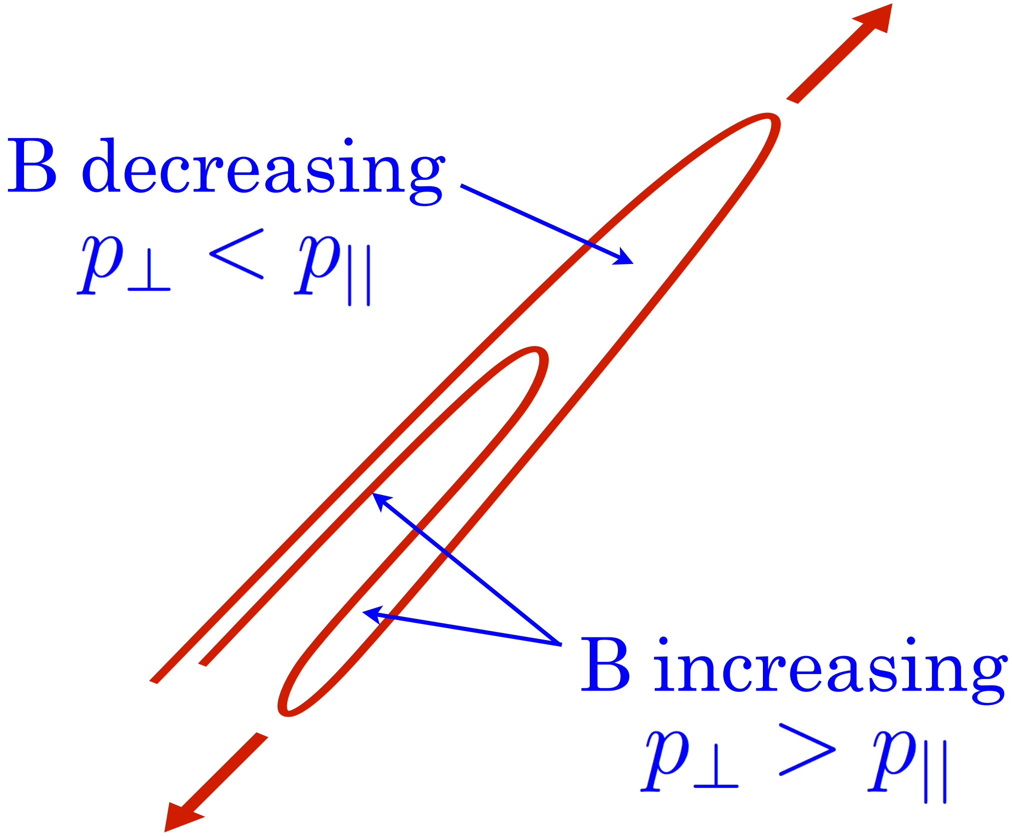


Where pressure anisotropy?



typical structure of magnetic fields generated by turbulence from MHD simulations with (isotropic) Pm >> 1 (Schekochihin+ 2004)

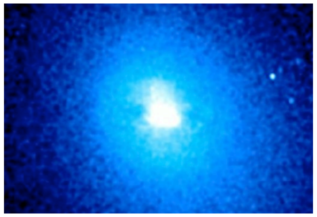
$$\ell_\perp \ll \ell_{||} \sim \ell_{\mathrm{visc}}$$

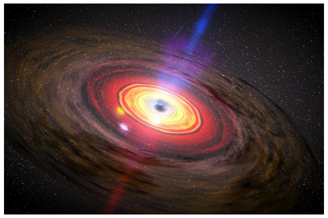


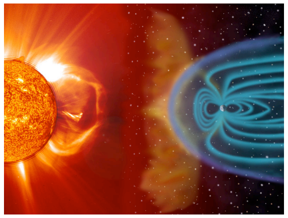
B decreasing
$p_\perp < p_{||}$

B increasing
$p_\perp > p_{||}$

How much pressure anisotropy?


intracluster medium of galaxy clusters


radiatively inefficient accretion flows


solar wind

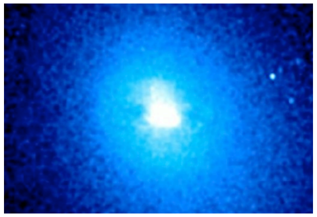
include collisions ...
then on scales larger than λ_{mfp} ,

Braginskii 1965

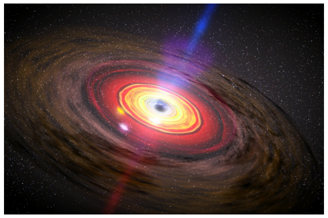
$$\frac{p_\perp - p_{||}}{p} \sim \boxed{\frac{1}{\nu_{\mathrm{ii}}}}\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\ln\frac{B^3}{n^2}}$$

collisional relaxation

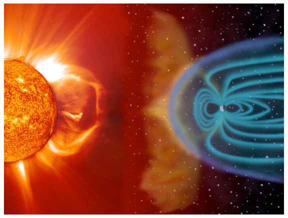
adiabatic production



intracluster medium of
galaxy clusters



radiatively inefficient
accretion flows



solar wind

$$\frac{p_\perp - p_{||}}{p} \sim \frac{\overset{\overset{\sim 0.3\ (?)}{\downarrow}}{u}}{v_{\rm th}} \frac{\overset{\overset{\sim 0.05\ (?)}{\swarrow}}{\lambda_{\rm mfp}}}{\ell}$$

$$\sim \text{few} \times 10^{-2} \gtrsim \frac{1}{\beta}$$

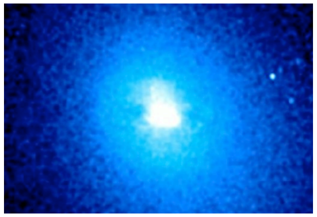
Why important?

Kunz 2011:
Kunz+ 2012:

$$\frac{\Delta T}{T} \propto \frac{\delta B_{||}}{B} \propto \frac{\delta p_\perp - \delta p_{||}}{p}$$

modifies convection
linearly & non-linearly

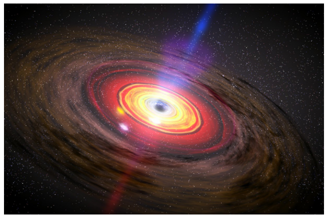
Kunz+ 2011: $Q^+ \propto p\nu_{\rm ii} \left(\frac{p_\perp - p_{||}}{p}\right)^2$



intracluster medium of galaxy clusters

$$\frac{p_\perp - p_{||}}{p} \sim \frac{\mathrm{d}\Omega}{\mathrm{d}\ln R}\,t$$

$$\gtrsim \frac{1}{\beta}$$ in less than an orbit



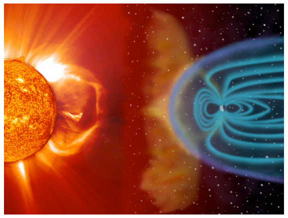
radiatively inefficient accretion flows

Why important?

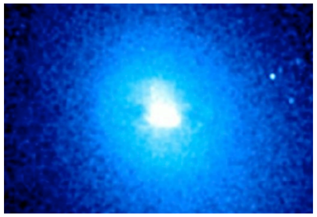
Sharma et al 2006, 2007:

$$T_{R\phi} \approx -\frac{\delta B_R \delta B_\phi}{4\pi}\left(1 + \frac{p_\perp - p_{||}}{B^2}\right)$$

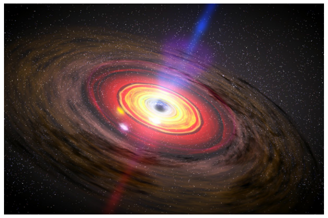
$$Q^+ \propto \frac{\mathrm{d}\Omega}{\mathrm{d}\ln R}(p_{||} - p_\perp)\frac{\delta B_R \delta B_\phi}{B^2}$$



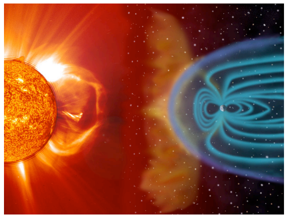
solar wind



intracluster medium of
galaxy clusters



radiatively inefficient
accretion flows



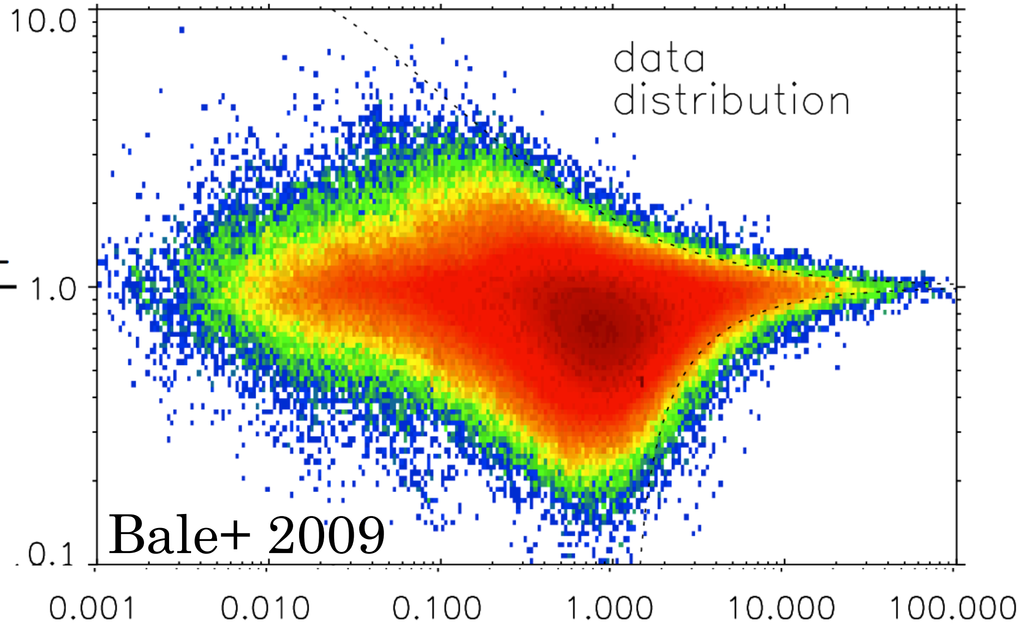
solar wind

adiabatic expansion: $\frac{T_\perp}{T_{||}} \propto r^{-2}$

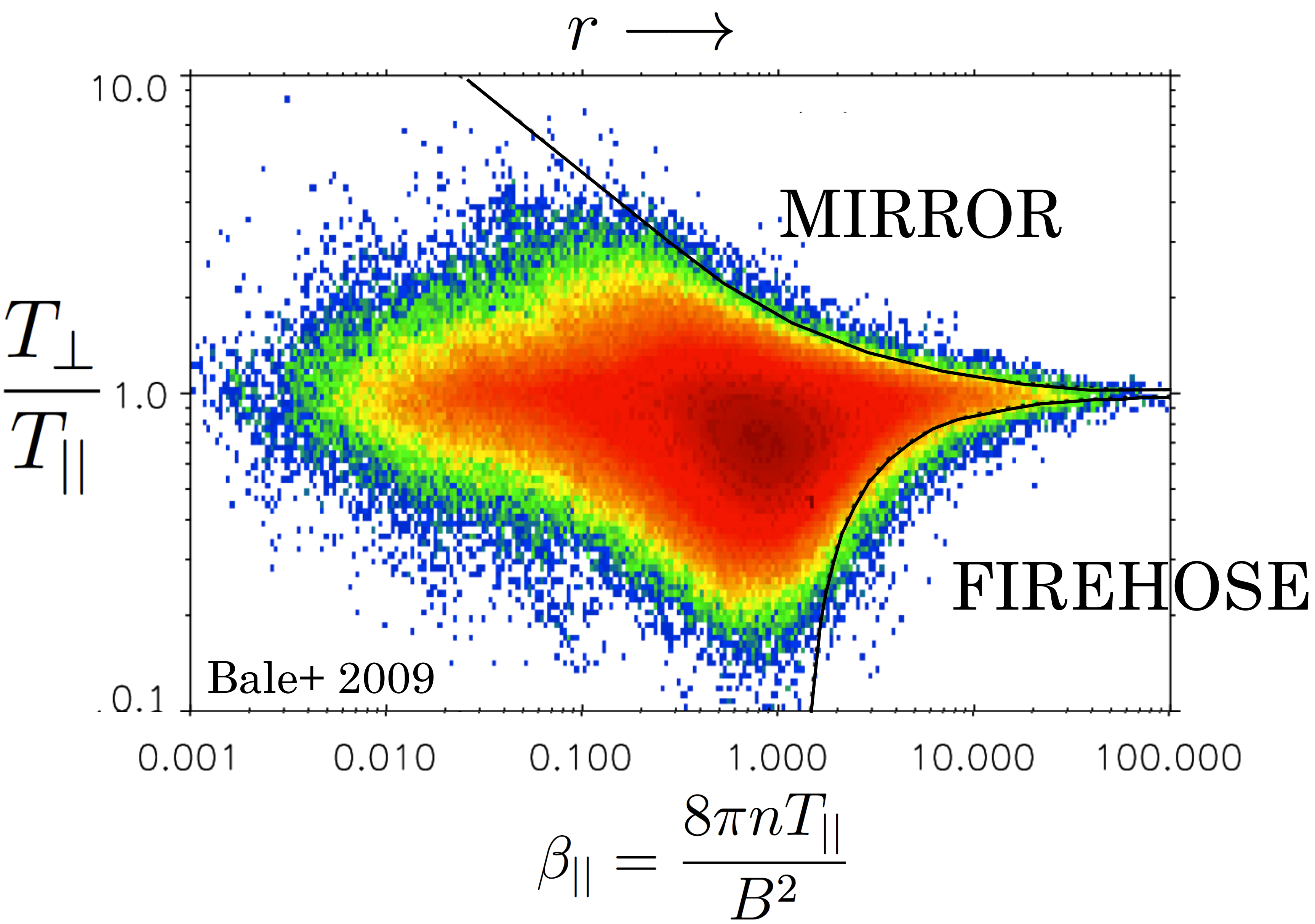
can be observed

$r \longrightarrow$

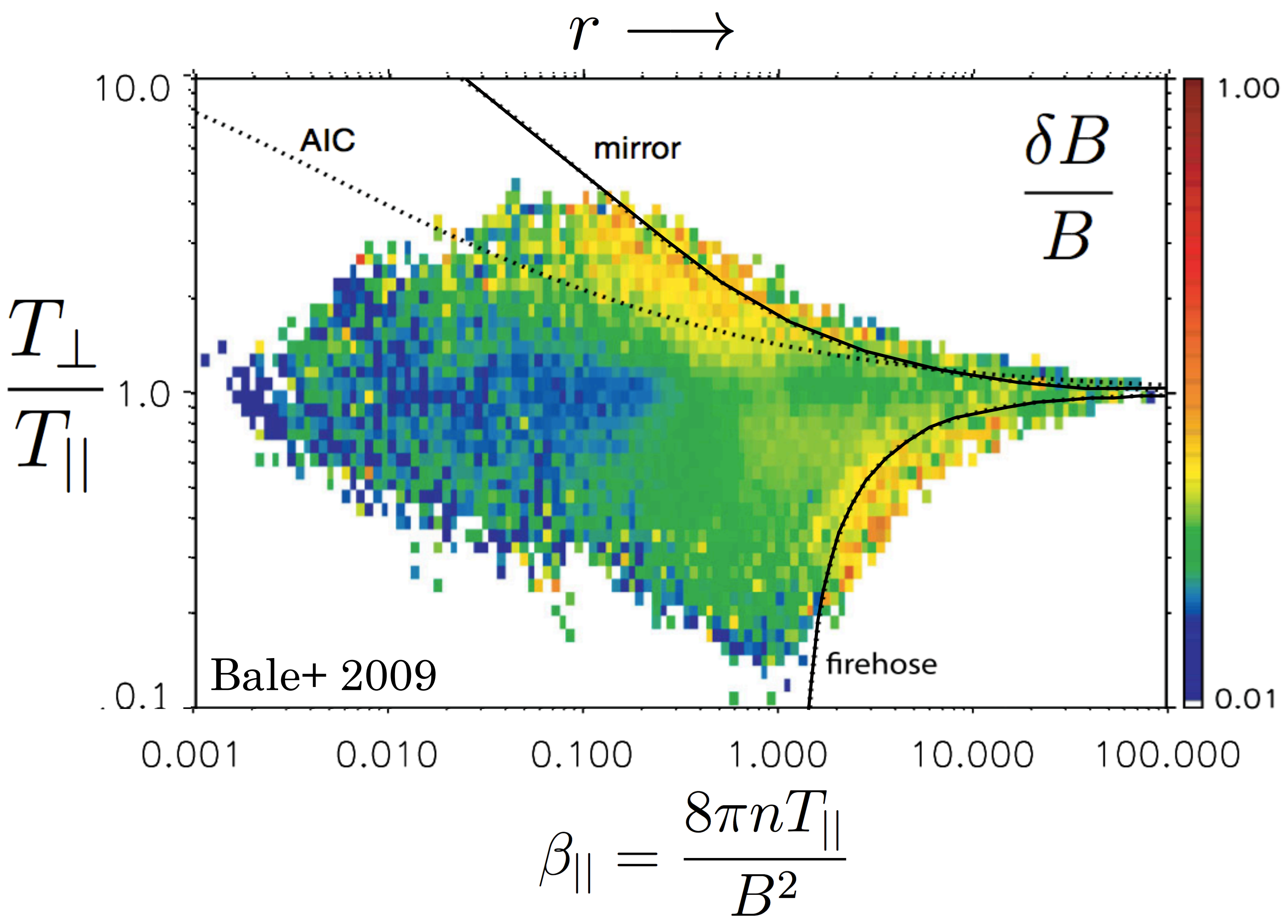
$\frac{T_\perp}{T_{||}}$



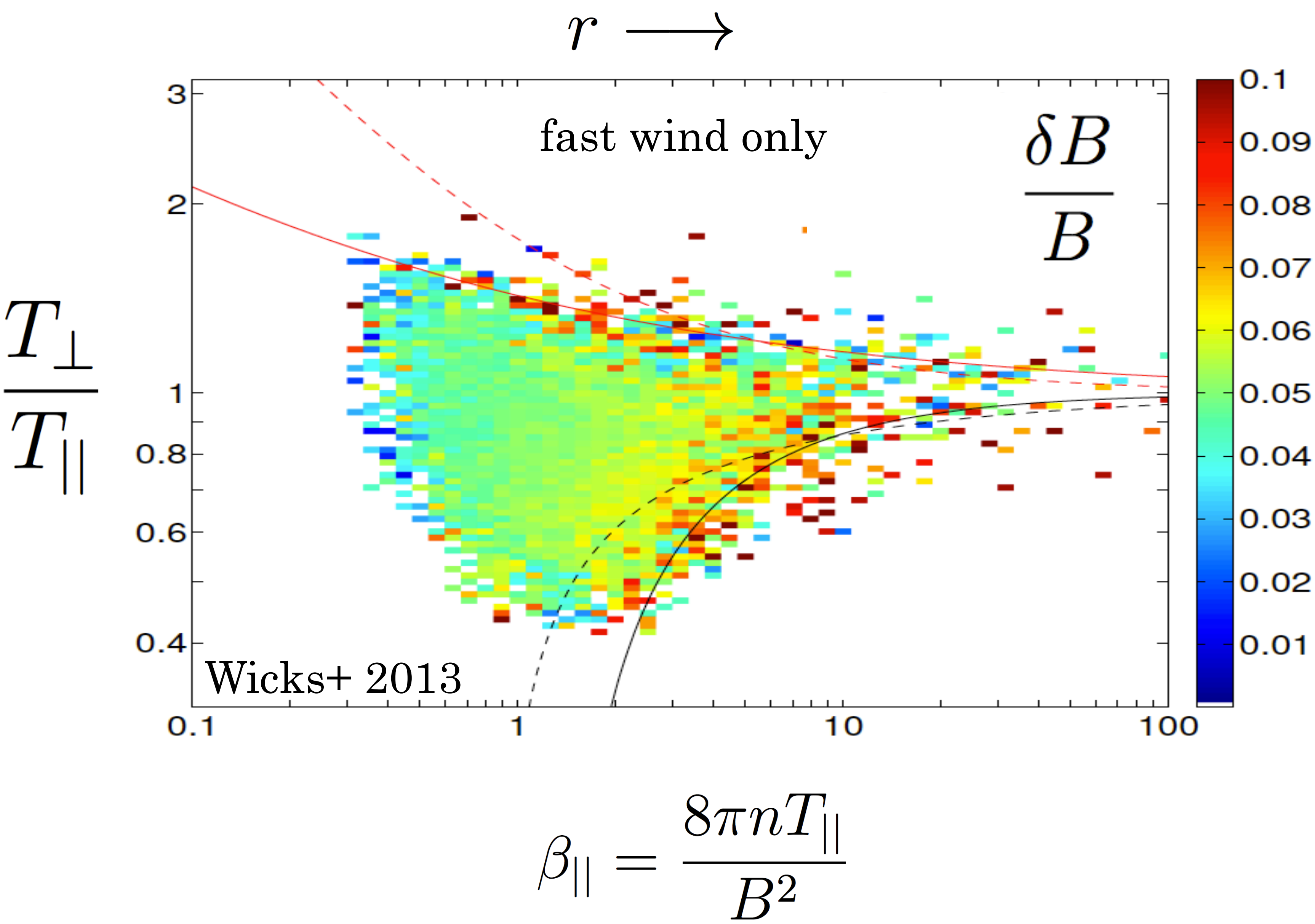
$$\beta_{||} = \frac{8\pi n T_{||}}{B^2}$$



solar wind obeys stability thresholds ...
expect ICM and hot accretion flows to do the same

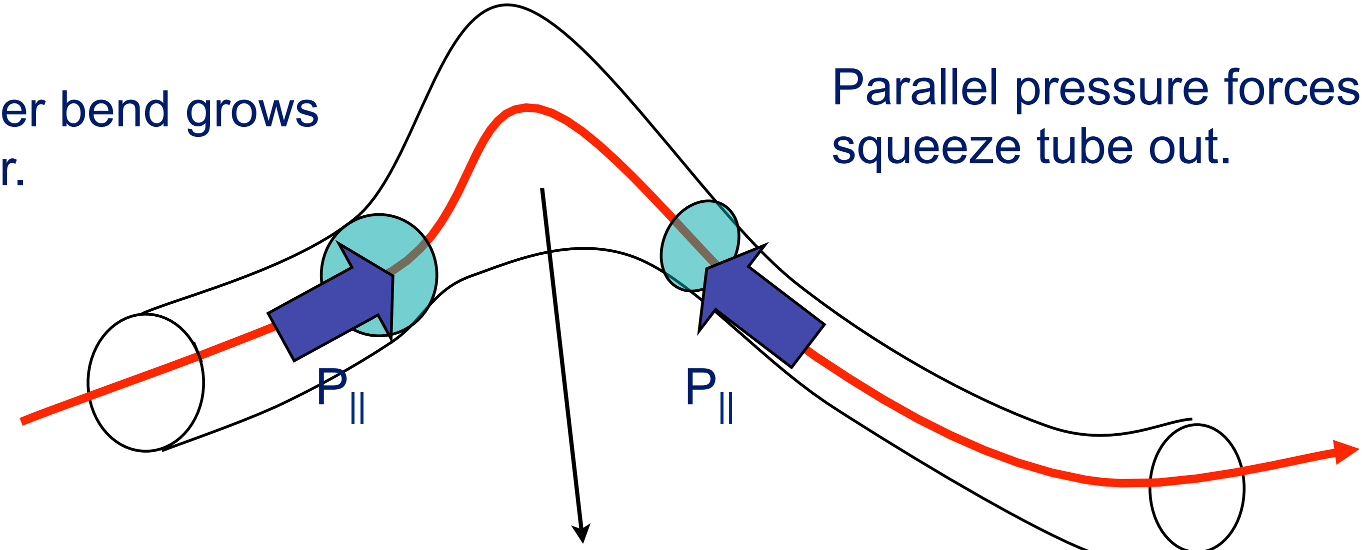


fluctuations pronounced at boundaries ...
what is their role in regulating the pressure anisotropy?



fluctuations pronounced at boundaries ...
what is their role in regulating the pressure anisotropy?

firehose instability



Tighter bend grows faster.

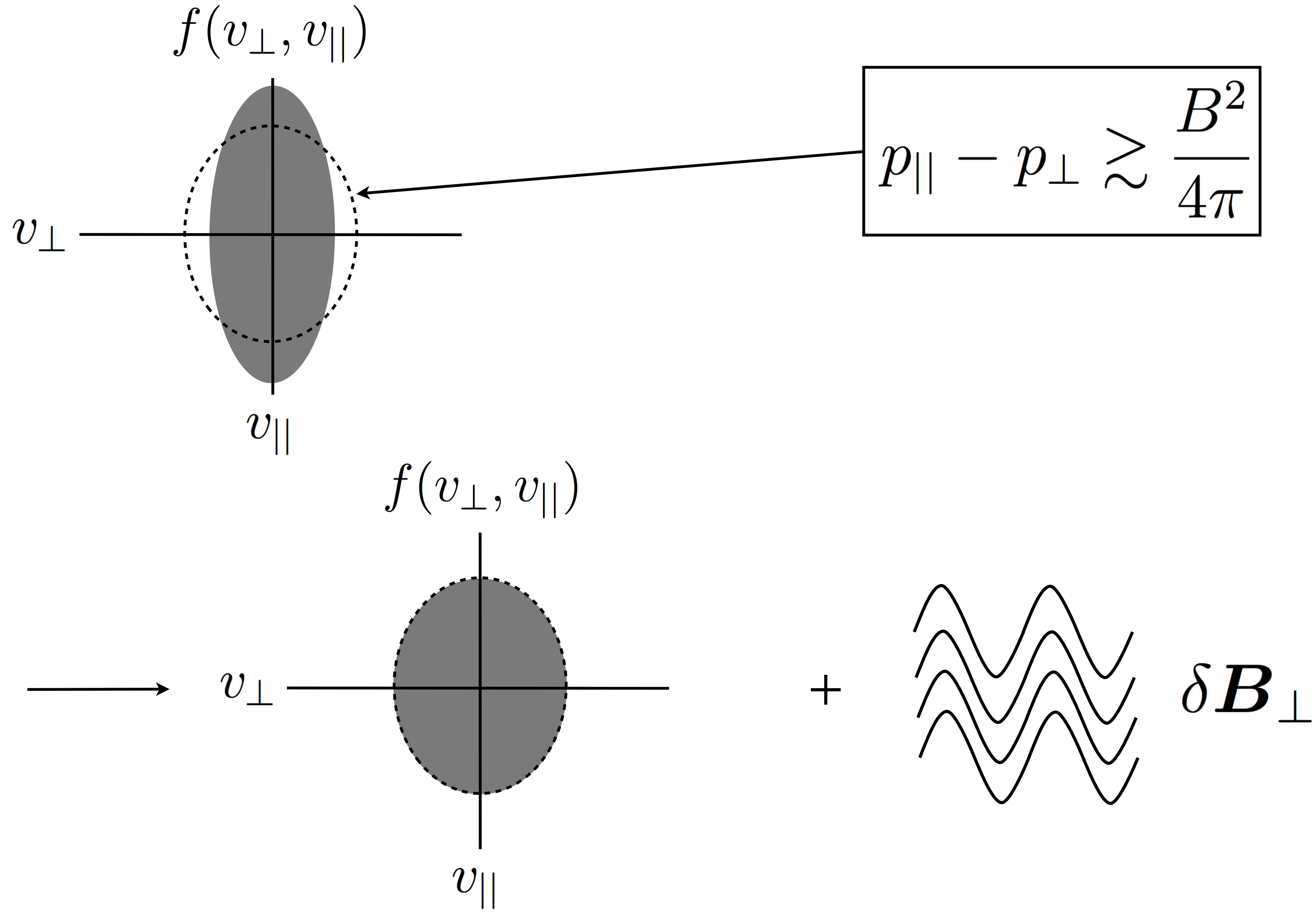
Parallel pressure forces squeeze tube out.

tension force

Rosenbluth 1956
Southwood and Kivelson 1993

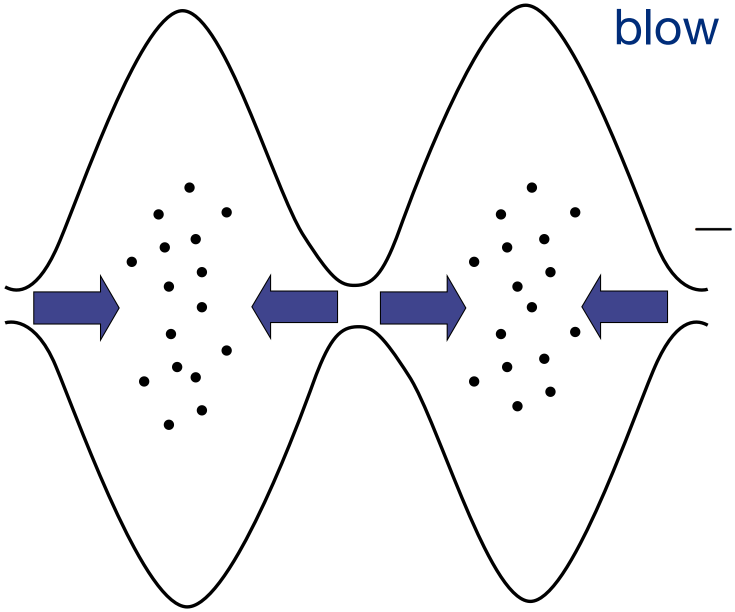
$$p_{||} - p_{\perp} \gtrsim \frac{B^2}{4\pi}$$

quasi-linear theory: firehose instability



$$\boxed{p_{||} - p_\perp \gtrsim \frac{B^2}{4\pi}}$$

mirror instability



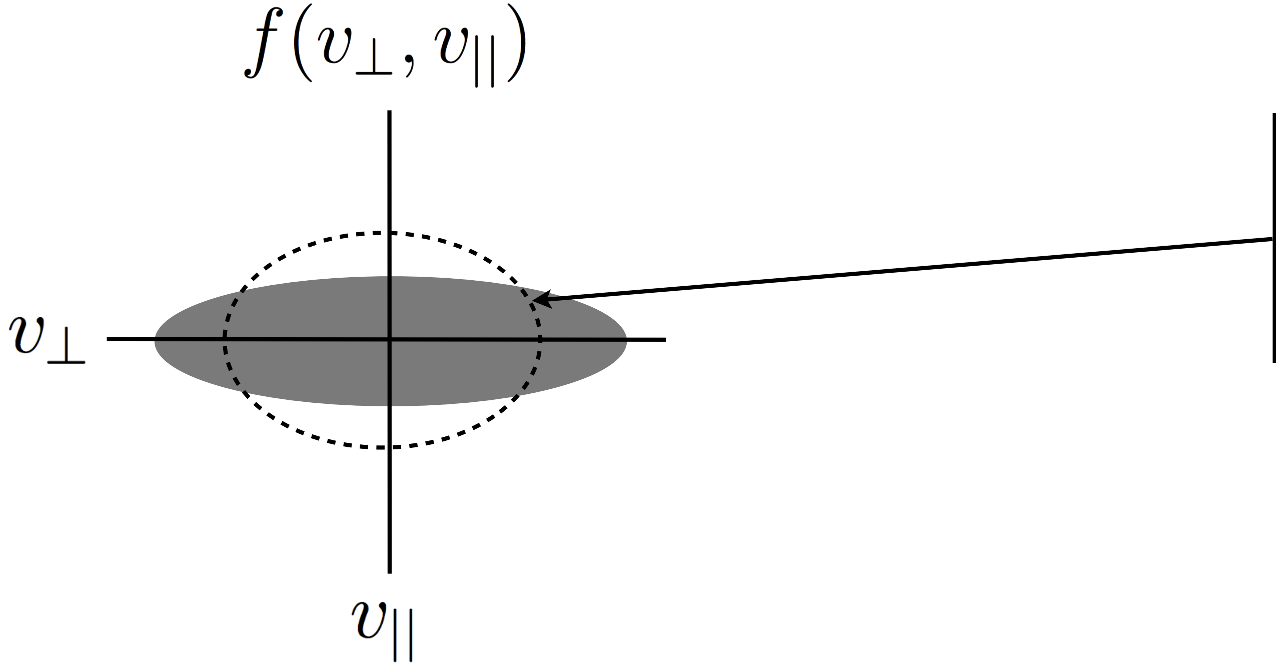
Perpendicular pressure forces
blow out field lines

$$-\hat{\boldsymbol{b}}\,(p_\perp - p_{||})\nabla_{||}\delta B_{||}$$

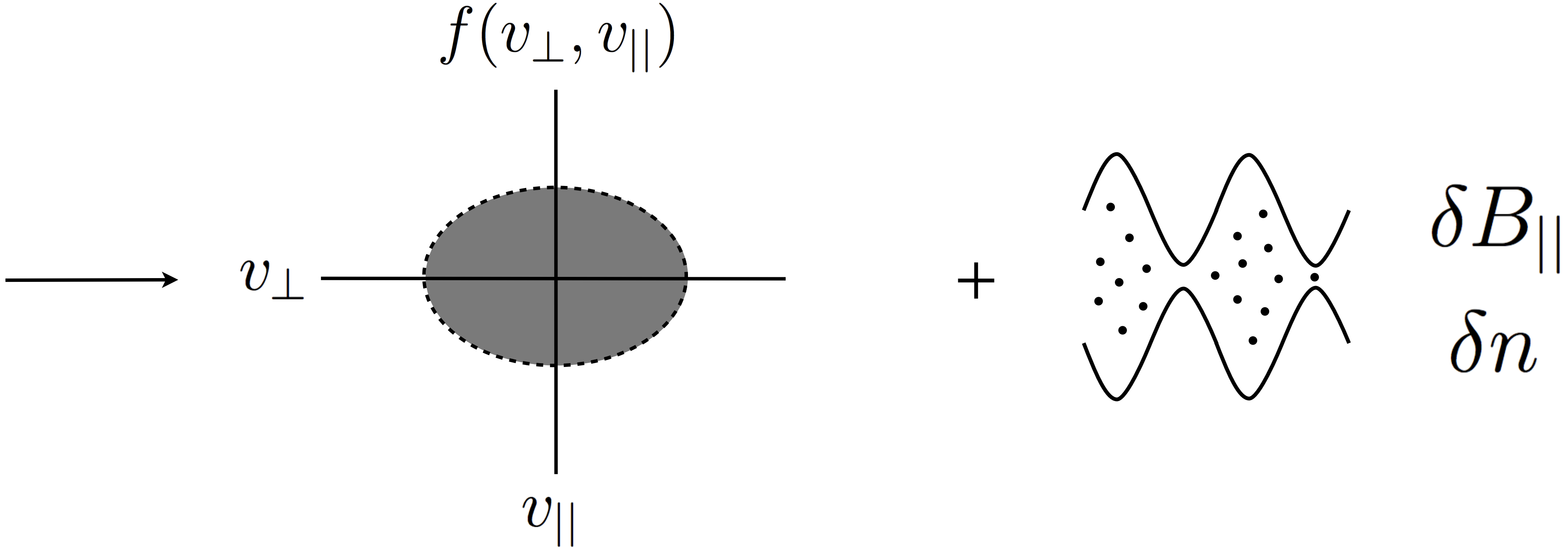
Rudakov & Sagdeev 1958
Shapiro & Shevchenko 1964

$$p_\perp - p_{||} \gtrsim \frac{B^2}{8\pi}$$

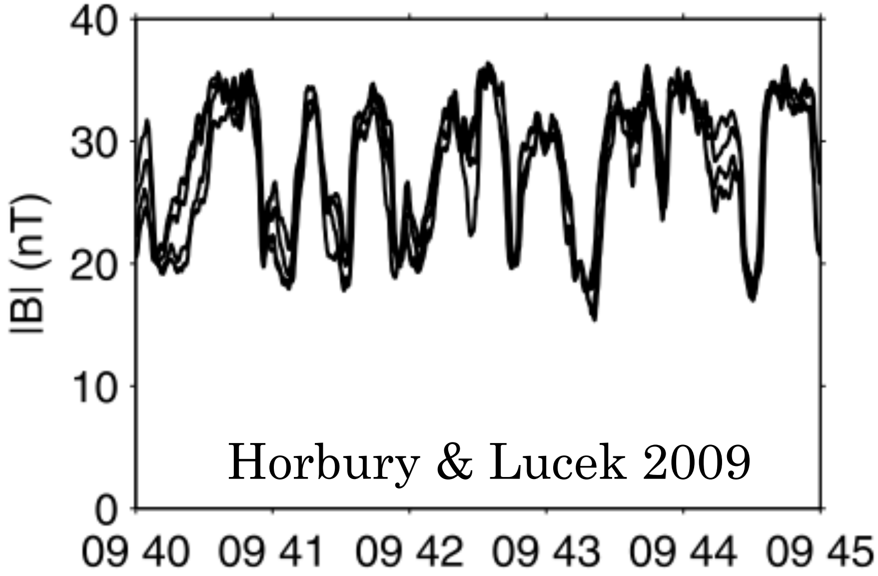
quasi-linear theory: mirror instability



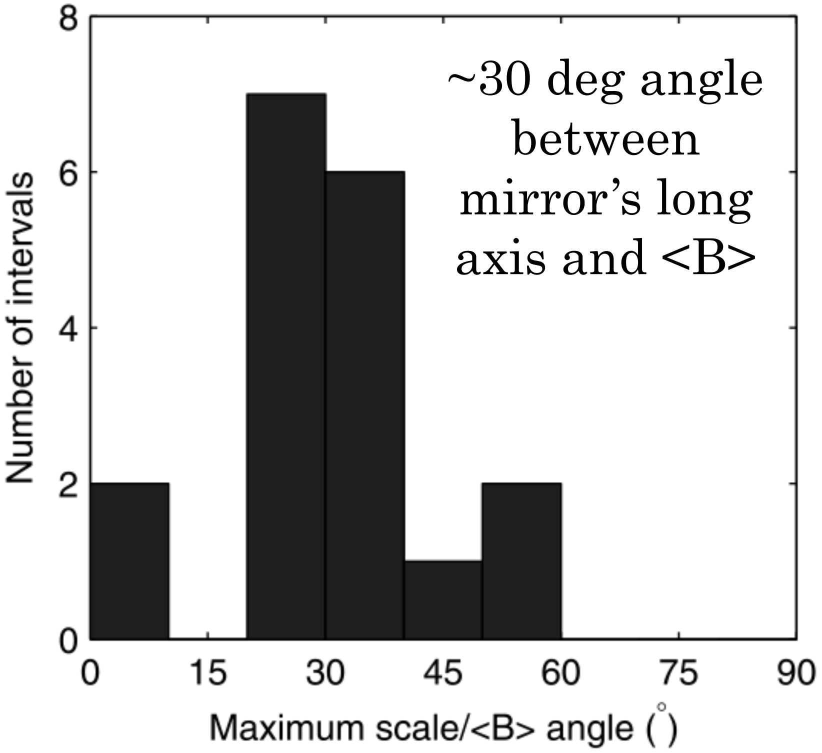
$$\boxed{p_\perp - p_{||} \gtrsim \frac{B^2}{8\pi}}$$



mirror modes observed in magnetosheath

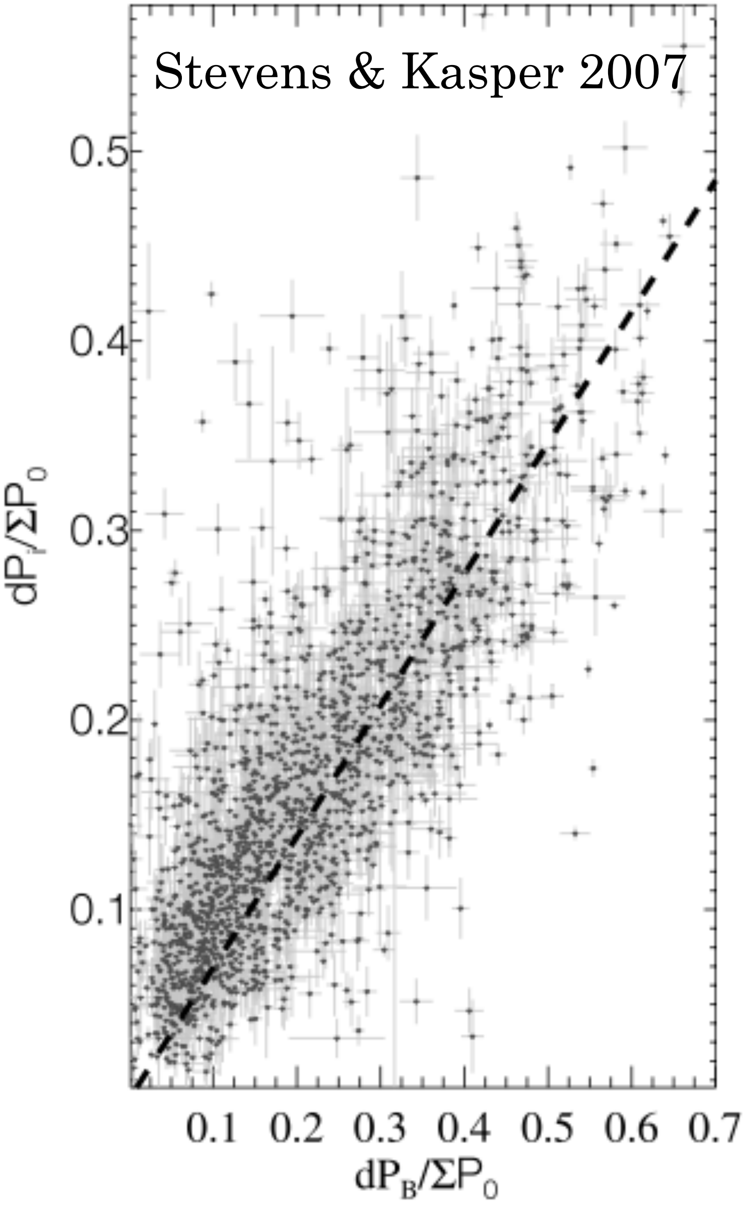


Horbury & Lucek 2009



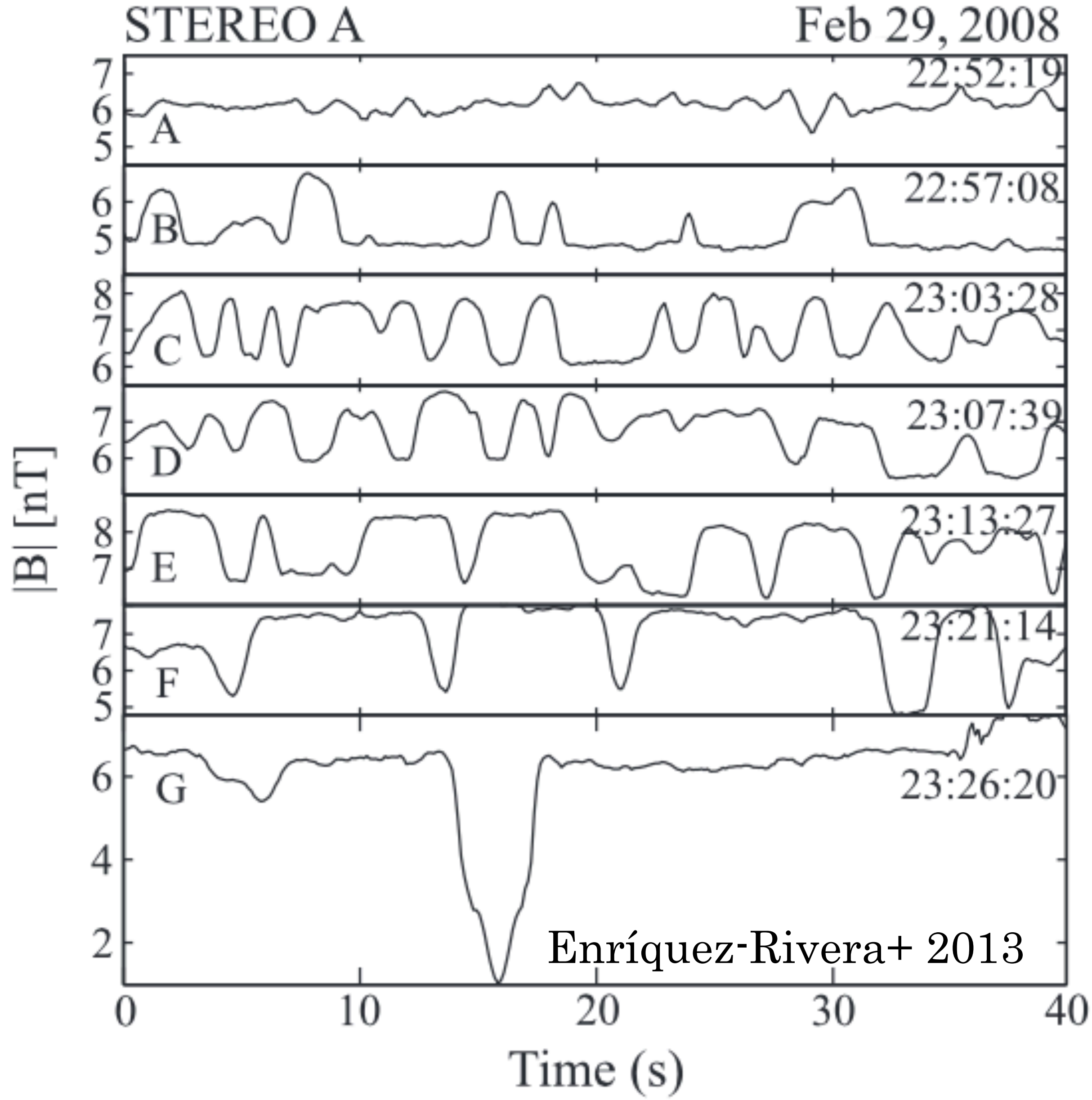
~30 deg angle between mirror's long axis and <B>

pressure-supported structures



Stevens & Kasper 2007

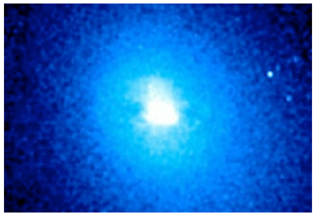
mirror-mode "storms" in stream-interaction regions

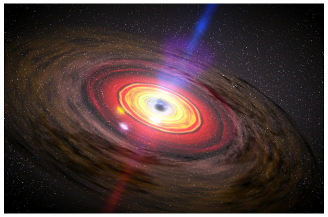


Enríquez-Rivera+ 2013

what if pressure anisotropy is driven instead of initially imposed?

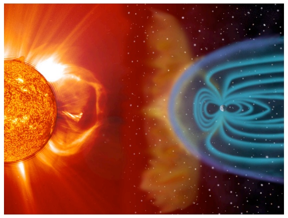
(as it is in nature)





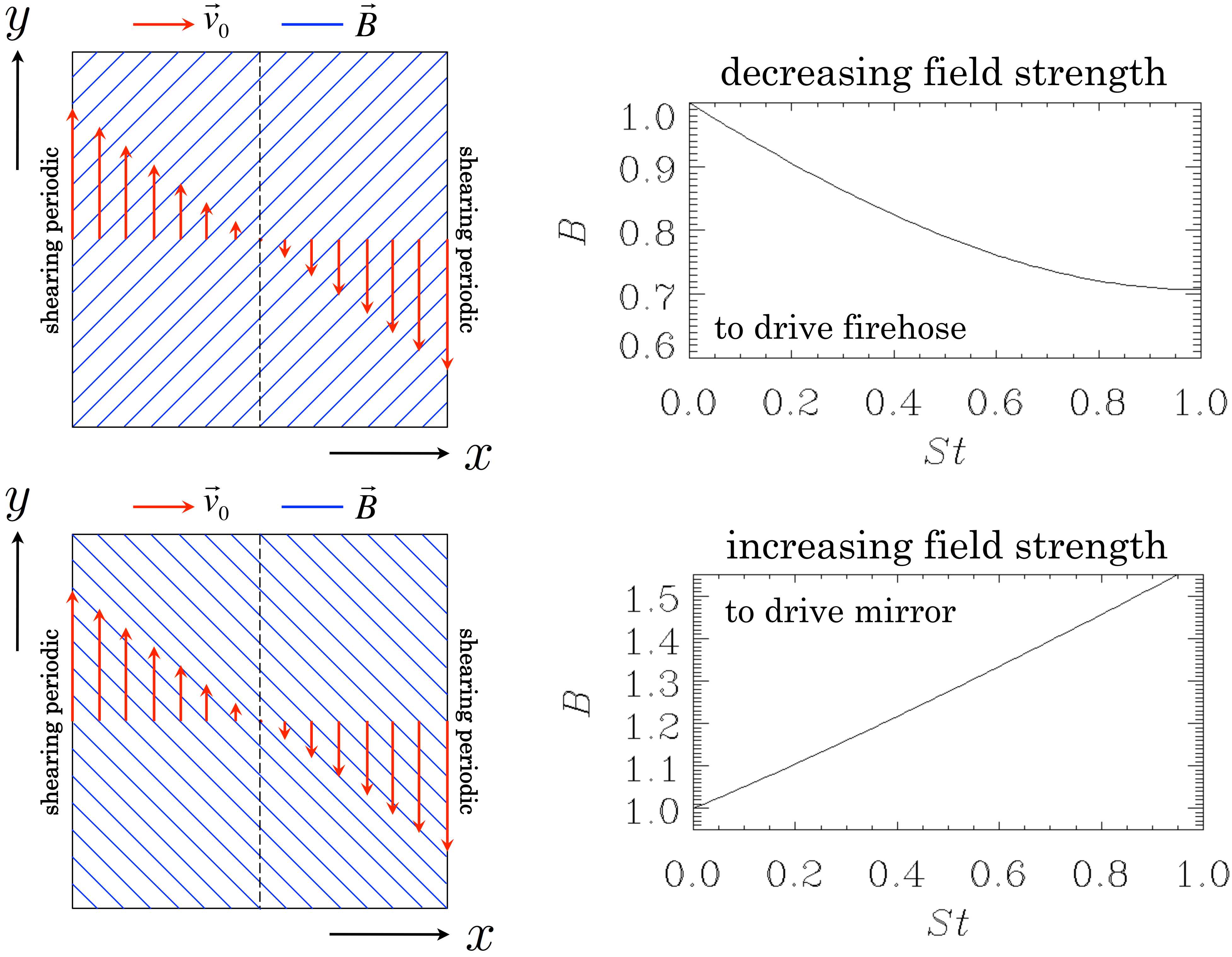
turbulent stirring

shearing-sheet model
this work



radial expansion

expanding-box model
Matteini+ 2006
Hellinger & Trávnícek 2008


y
$\vec{v}_0$
$\vec{B}$
shearing periodic
shearing periodic
x
decreasing field strength
B
1.0
0.9
0.8
0.7
0.6
to drive firehose
0.0
0.2
0.4
0.6
0.8
1.0
St
y
$\vec{v}_0$
$\vec{B}$
shearing periodic
shearing periodic
x
increasing field strength
B
1.5
1.4
1.3
1.2
1.1
1.0
to drive mirror
0.0
0.2
0.4
0.6
0.8
1.0
St

kinetic ions: $$\left(\frac{\partial}{\partial t} - Sx\frac{\partial}{\partial y}\right) f_{\mathrm{i}} + \boldsymbol{v}\boldsymbol{\cdot}\boldsymbol{\nabla} f_{\mathrm{i}} + \left[\frac{Ze}{m_{\mathrm{i}}}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\right) + Sv_x\hat{\boldsymbol{y}}\right]\boldsymbol{\cdot}\frac{\partial f_{\mathrm{i}}}{\partial \boldsymbol{v}} = 0$$

Faraday: $$\left(\frac{\partial}{\partial t} - Sx\frac{\partial}{\partial y}\right)\boldsymbol{B} = -c\boldsymbol{\nabla}\times\boldsymbol{E} - SB_x\hat{\boldsymbol{y}}$$

massless fluid electrons: $$\boldsymbol{E} = -\frac{\boldsymbol{u}_{\mathrm{i}}\times\boldsymbol{B}}{c} + \frac{(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi Zen_{\mathrm{i}}} - \frac{T_{\mathrm{e}}\boldsymbol{\nabla}n_{\mathrm{i}}}{en_{\mathrm{i}}}$$

quasi-neutrality:

$$n_{\mathrm{e}} = Zn_{\mathrm{i}} \equiv Z\int \mathrm{d}^3\boldsymbol{v}\, f_{\mathrm{i}}$$

ion pressure tensor:

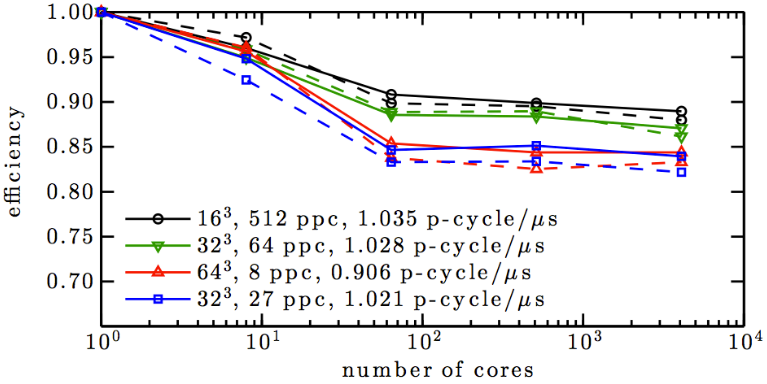
$$\mathsf{P}_{\mathrm{i}} \equiv \int \mathrm{d}^3\boldsymbol{v}\, m_{\mathrm{i}}\boldsymbol{v}\boldsymbol{v}\, f_{\mathrm{i}}$$

Pegasus

A 6-D hybrid-kinetic PIC code for astrophysical plasma dynamics

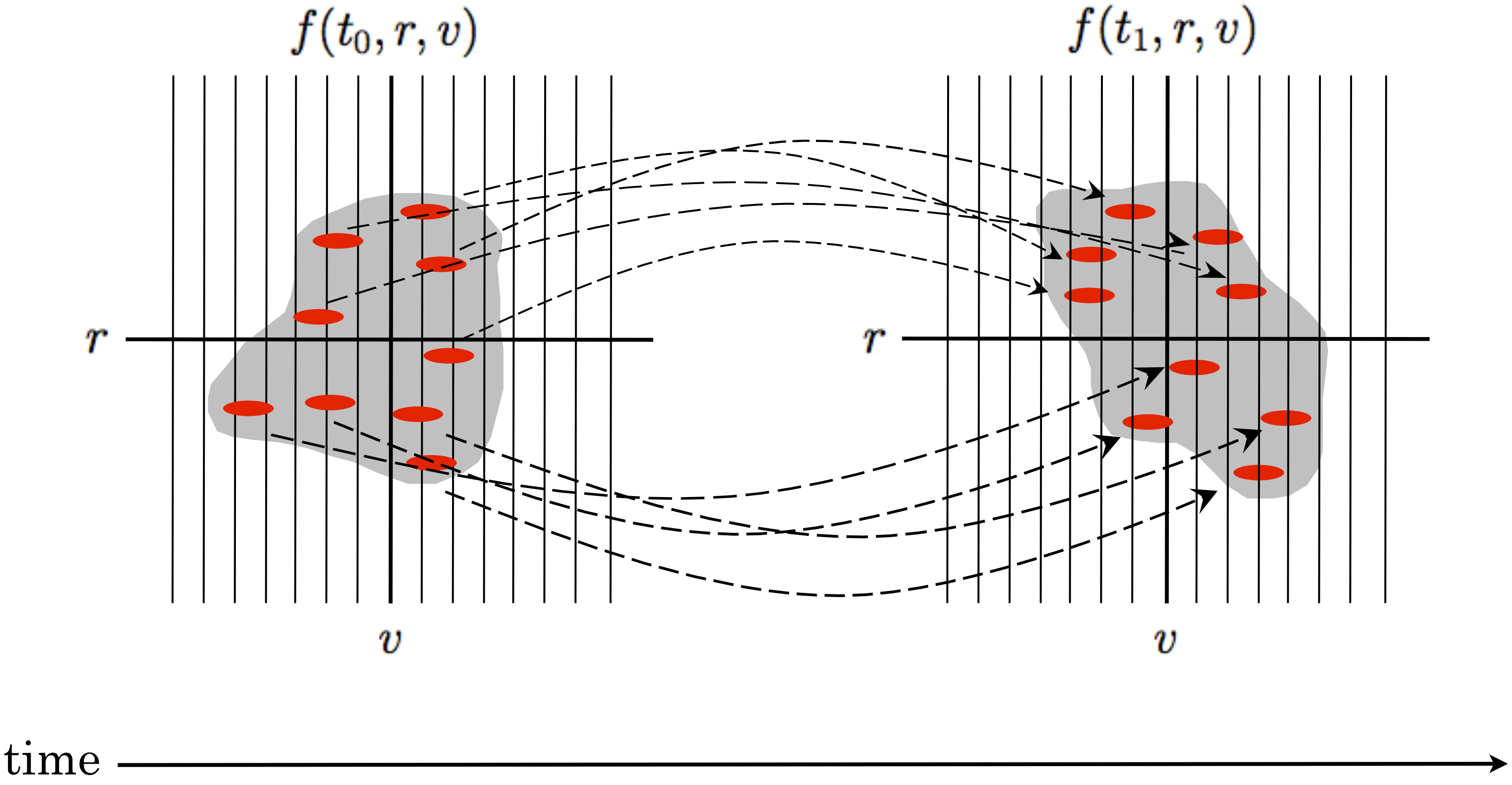
2nd-order-accurate iterative Crank-Nicholson algorithm with:

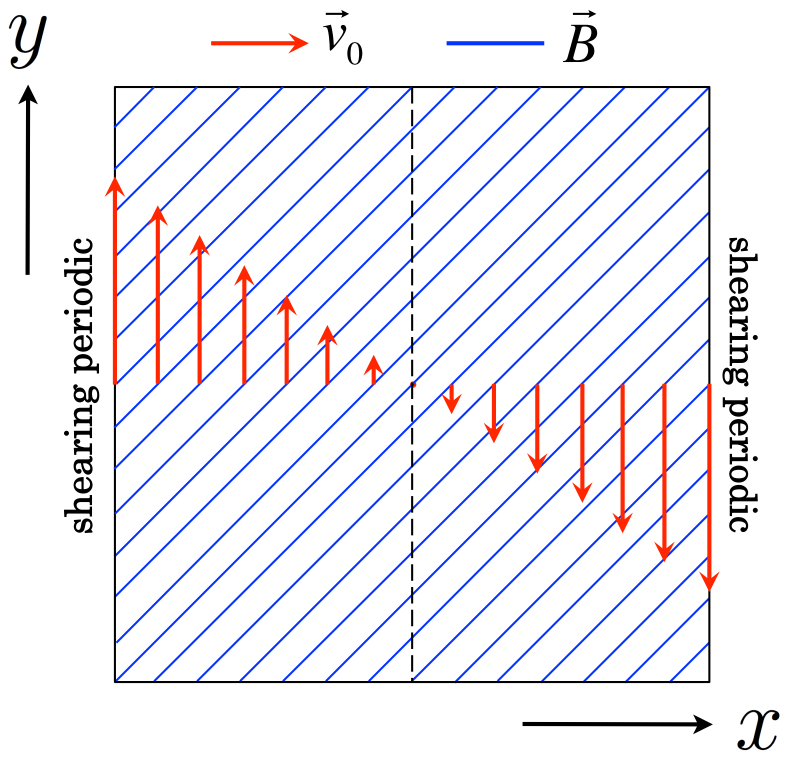
- symplectic solver for ions
- constrained transport for B
- delta-f or full-F methods
- shearing box with FARGO
- variety of boundary conditions
- well-tested
- efficiently parallelized with MPI




Kunz et al. 2014, JCoPh

how particle-in-cell works





shearing sheet

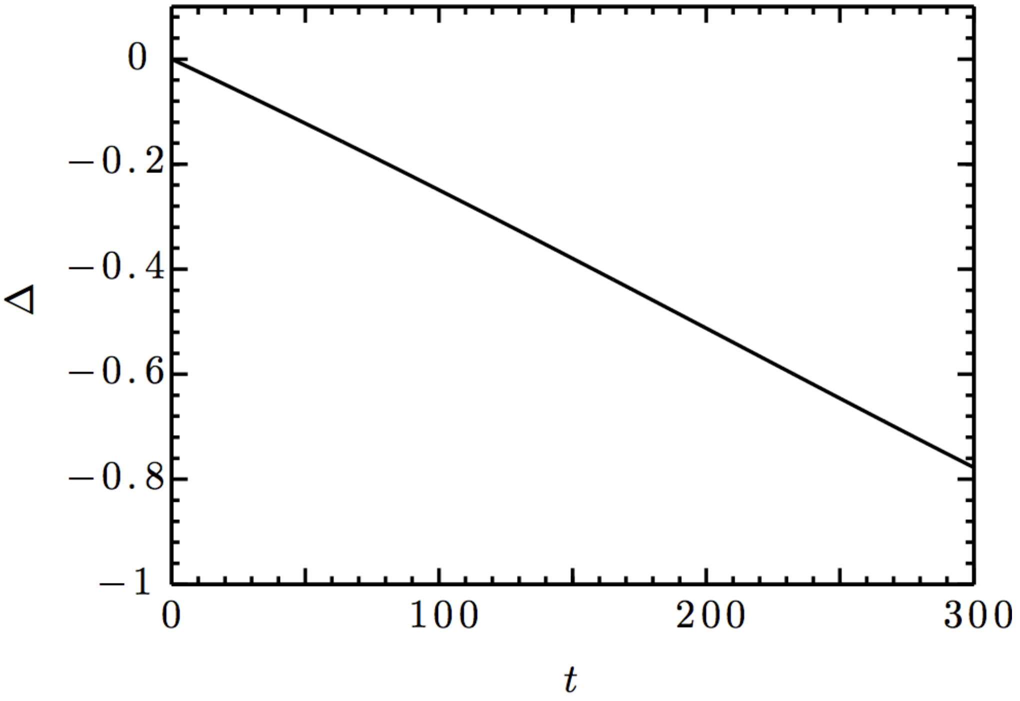
$$\boldsymbol{v}_0 = -Sx\,\hat{\boldsymbol{y}}$$

field unwraps

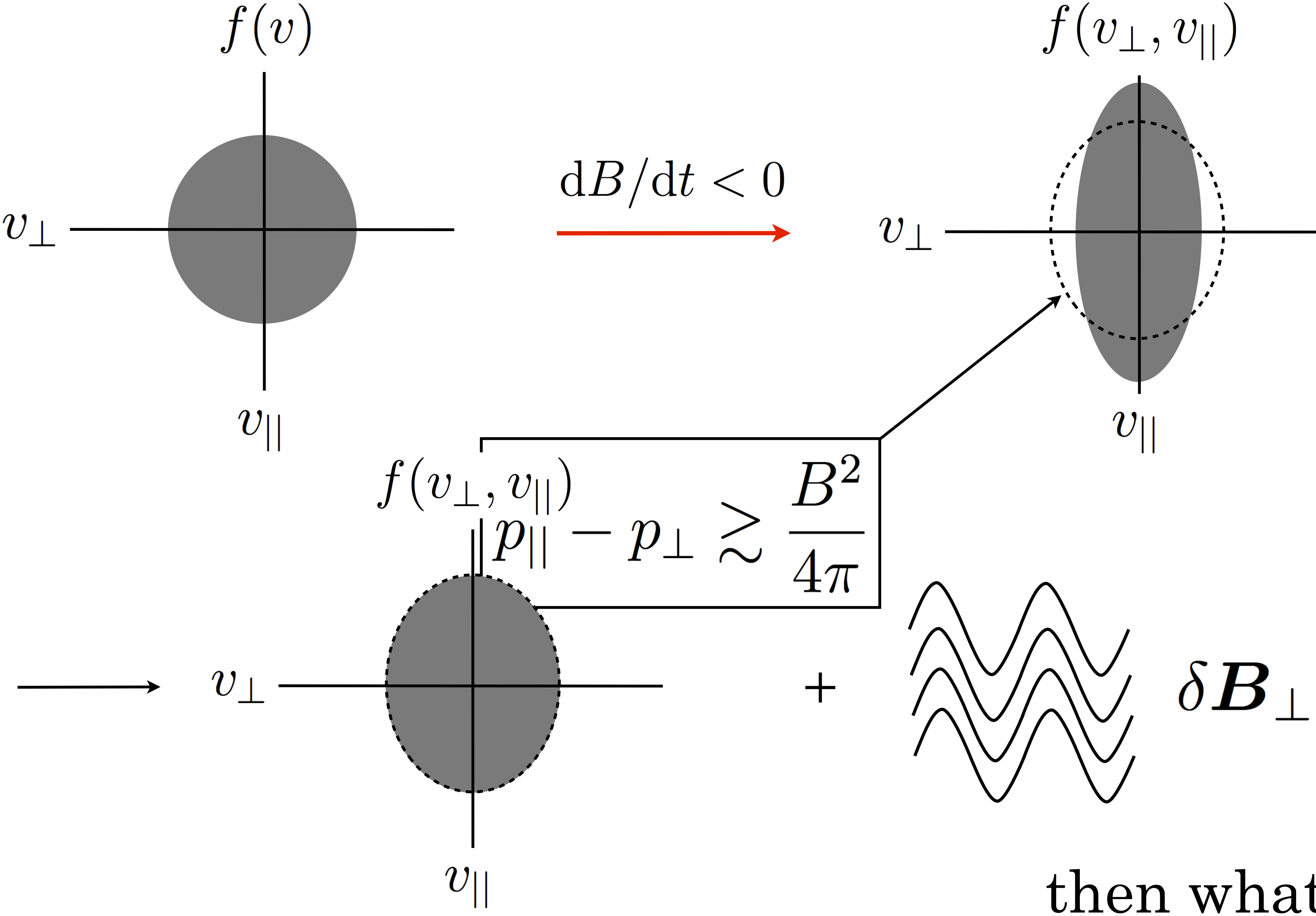
drives $p_{||} > p_{\perp}$

goes firehose unstable

$$\Delta \equiv \frac{p_\perp - p_{||}}{p} \longrightarrow$$



firehose instability



$$p_{||} - p_\perp \gtrsim \frac{B^2}{4\pi}$$

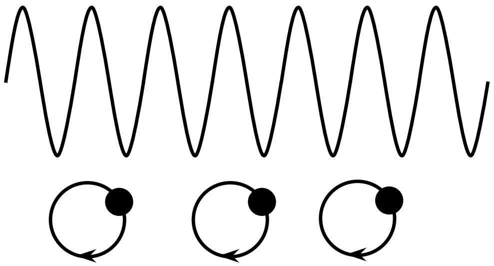
then what?

either:

the plasma effectively increases its collisionality

increase ν_{i}

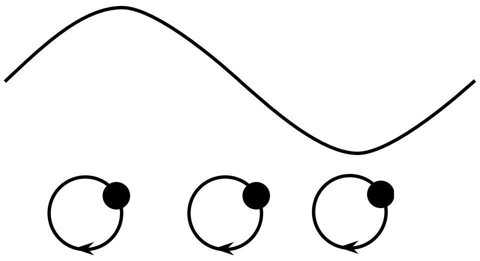
i.e. break μ



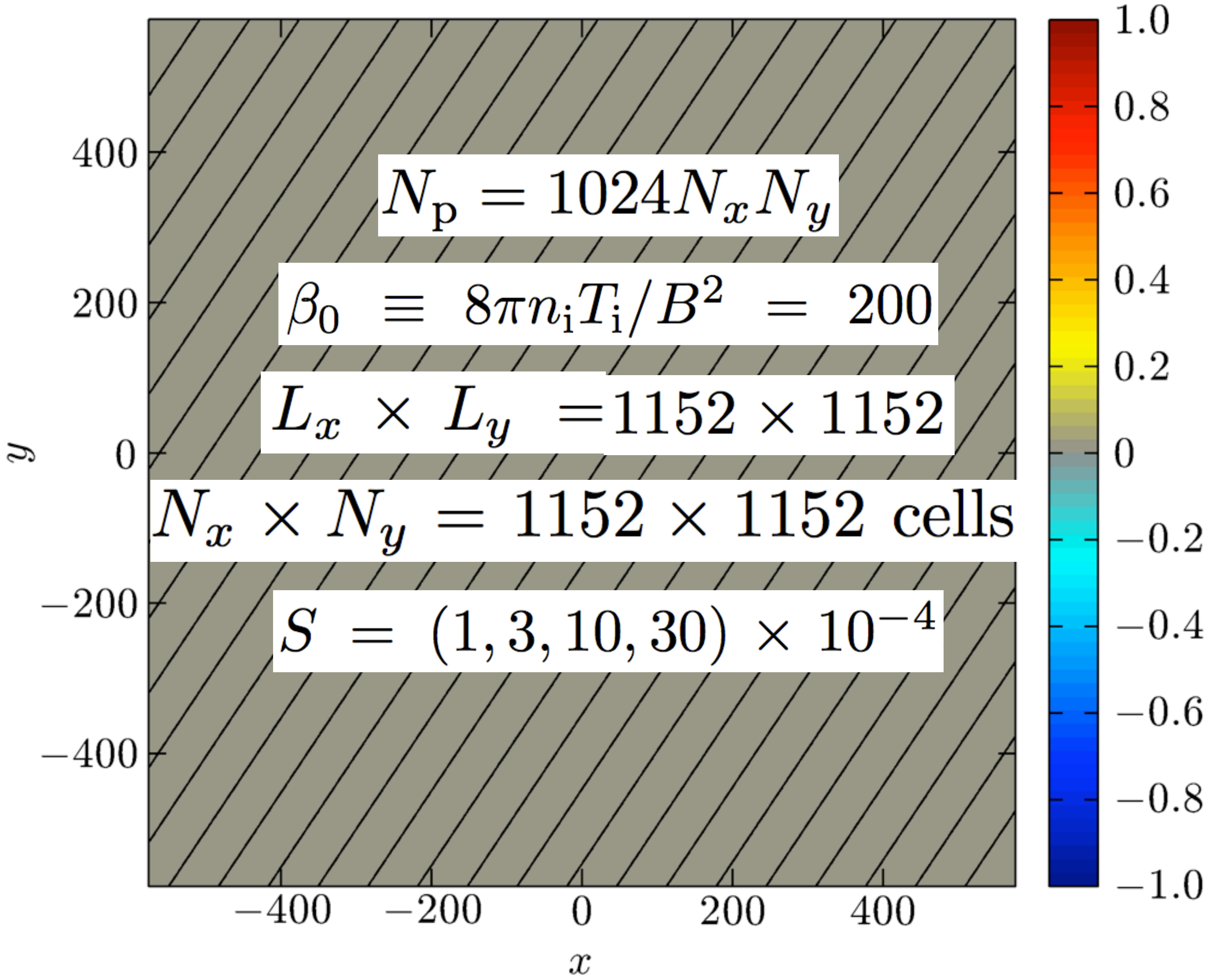
the plasma finds a way of not producing pressure anisotropy

minimize $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla}\boldsymbol{v}$

i.e. regulate $\mathrm{d}\ln B/\mathrm{d}t$



what is the effective Reynolds number?



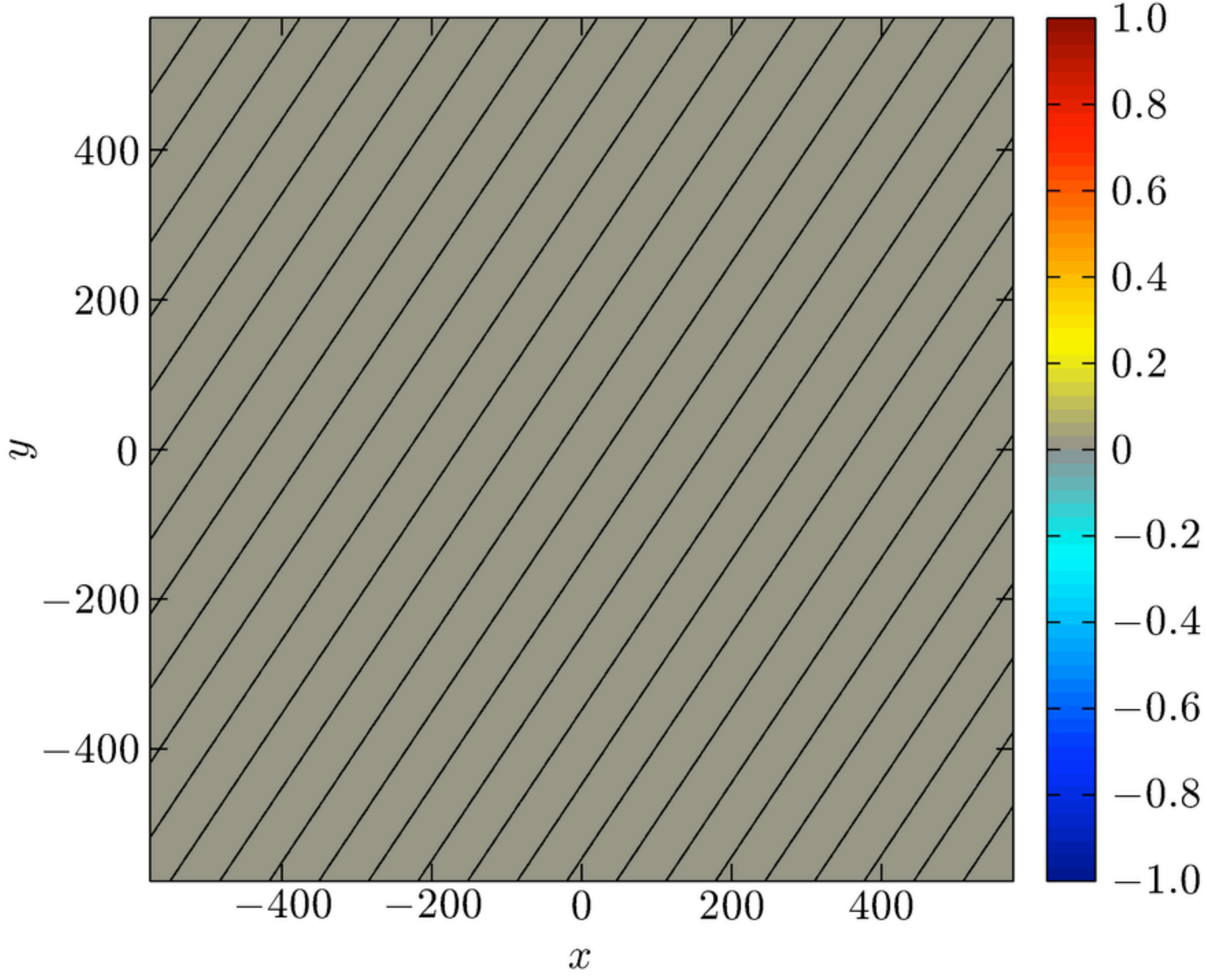
$N_\mathrm{p} = 1024 N_x N_y$

$\beta_0 \equiv 8\pi n_\mathrm{i} T_\mathrm{i} / B^2 = 200$

$L_x \times L_y = 1152 \times 1152$

$N_x \times N_y = 1152 \times 1152$ cells

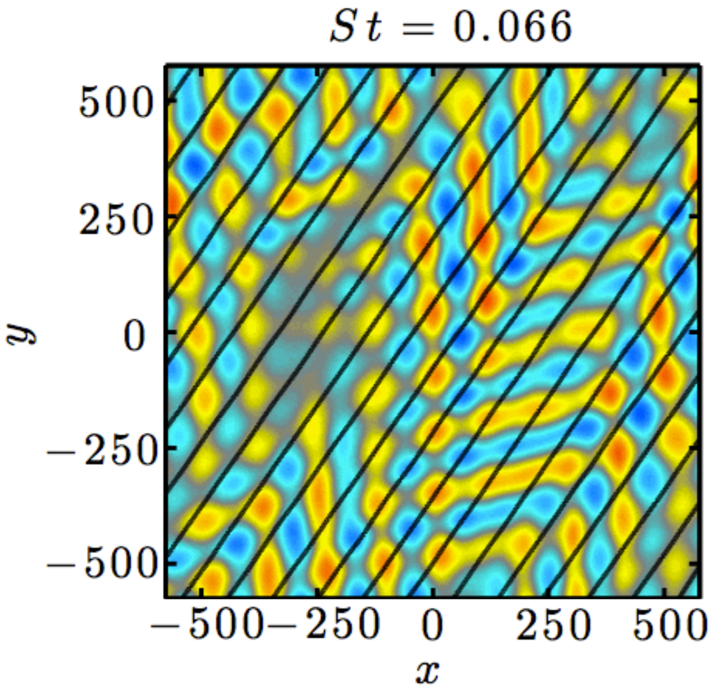
$S = (1, 3, 10, 30) \times 10^{-4}$

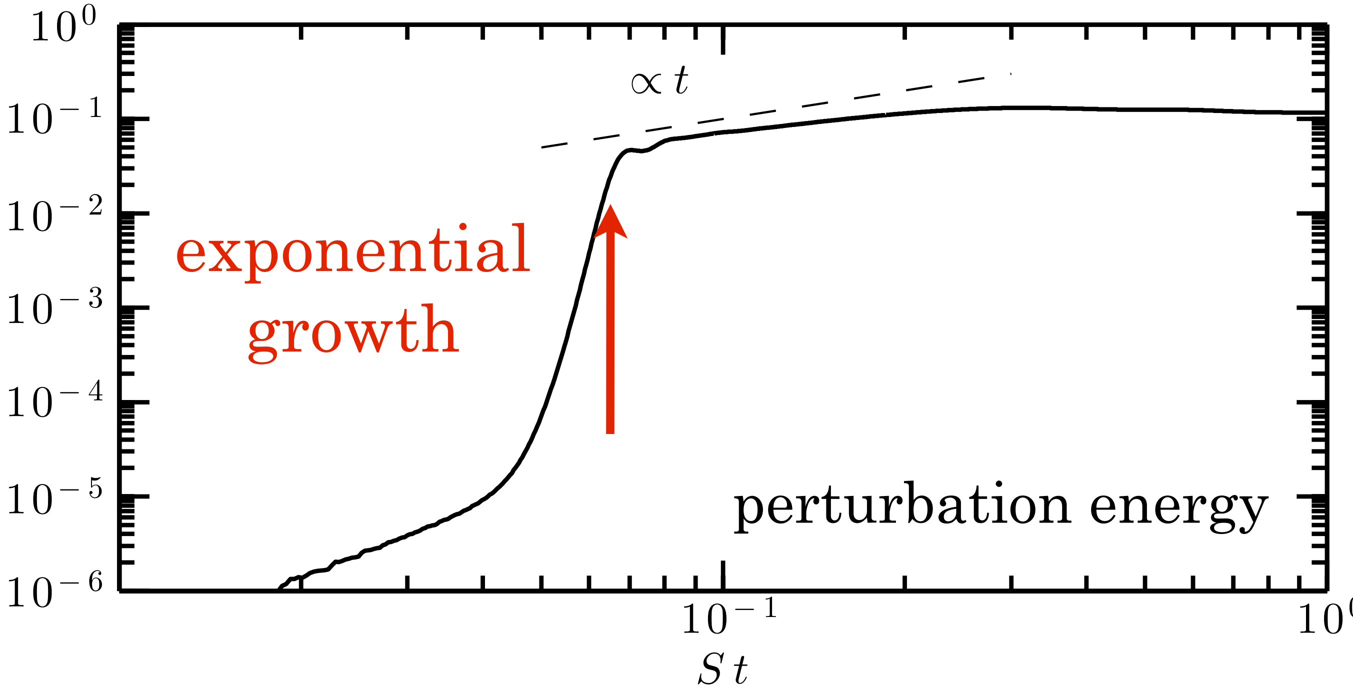


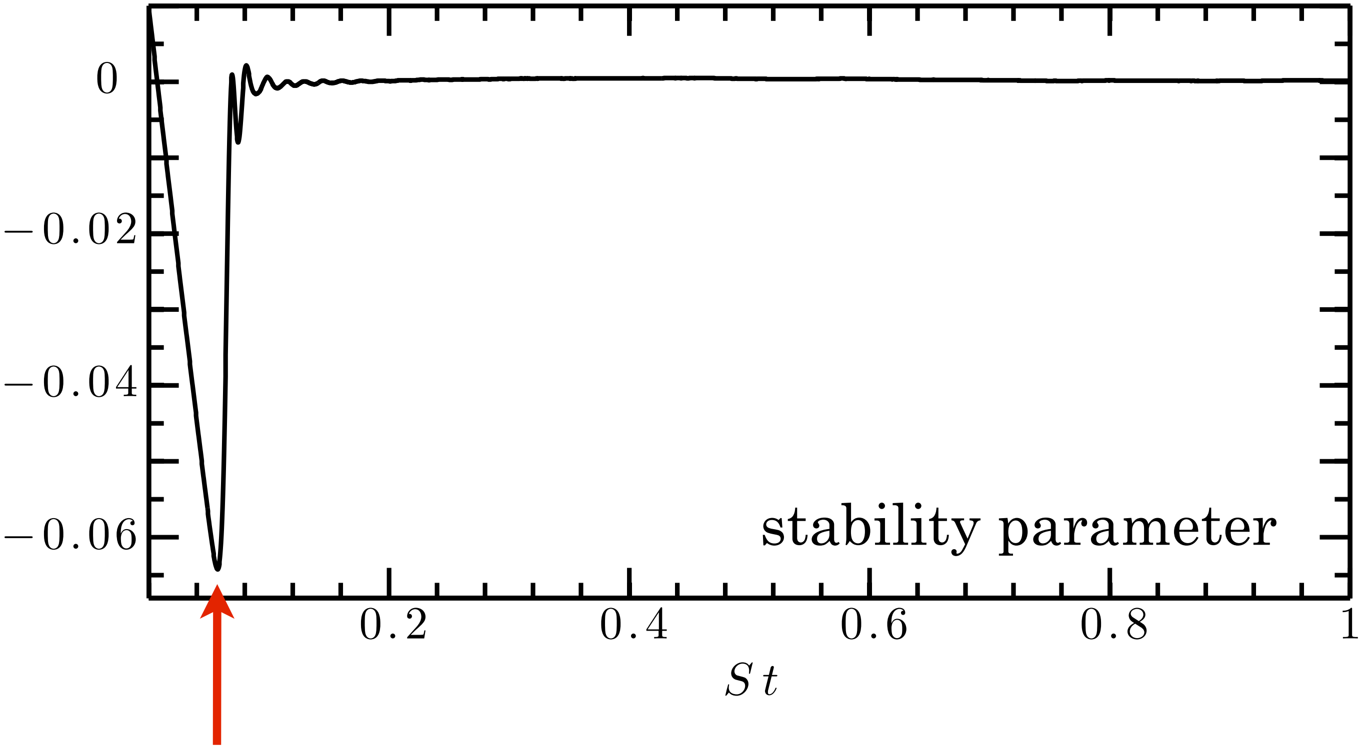
400
200
0
−200
−400
y
−400
−200
0
200
400
x
1.0
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1.0

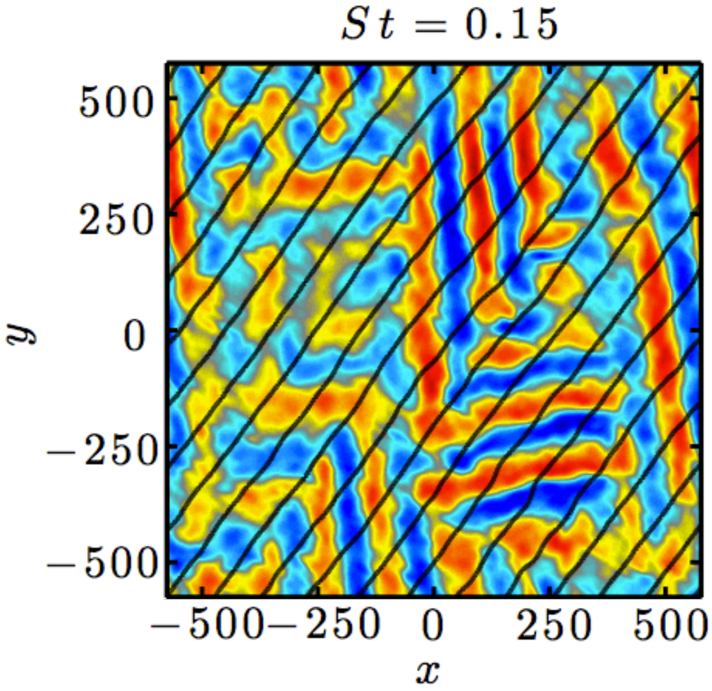
oblique modes

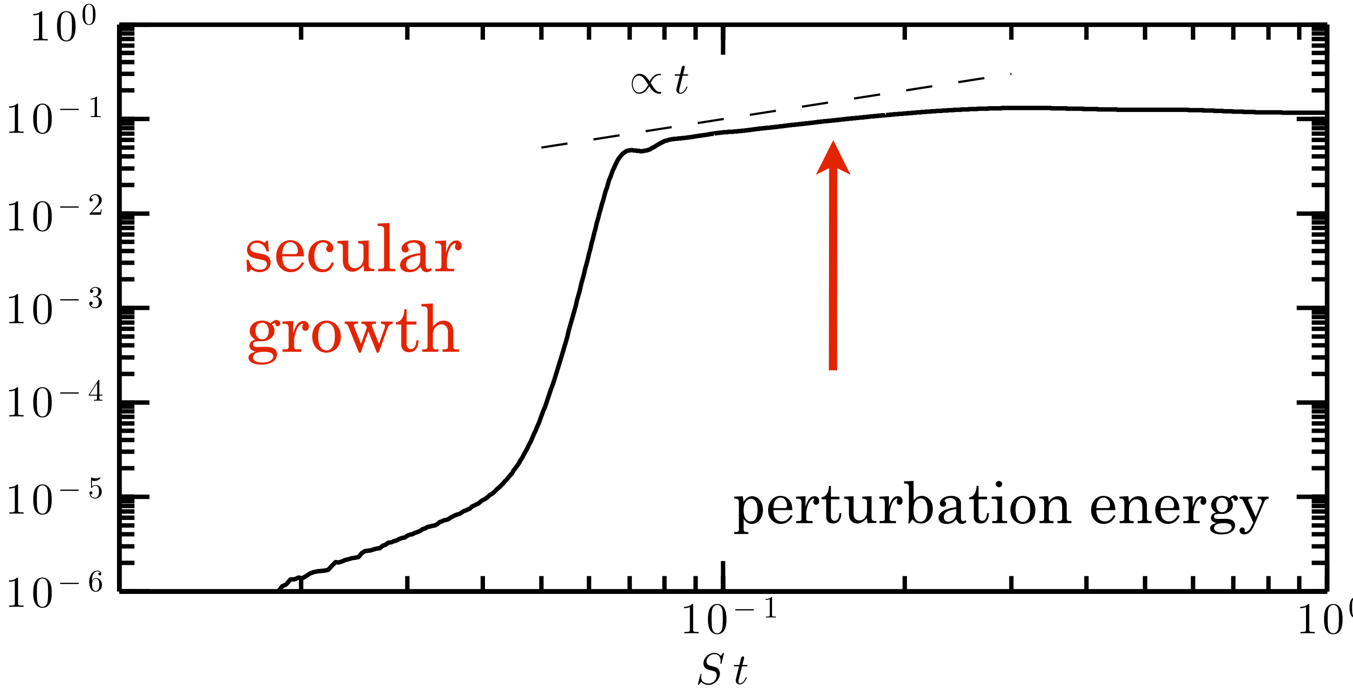
$$k_{||}\rho_{\mathrm{i}} \approx k_{\perp}\rho_{\mathrm{i}} \approx 0.4$$



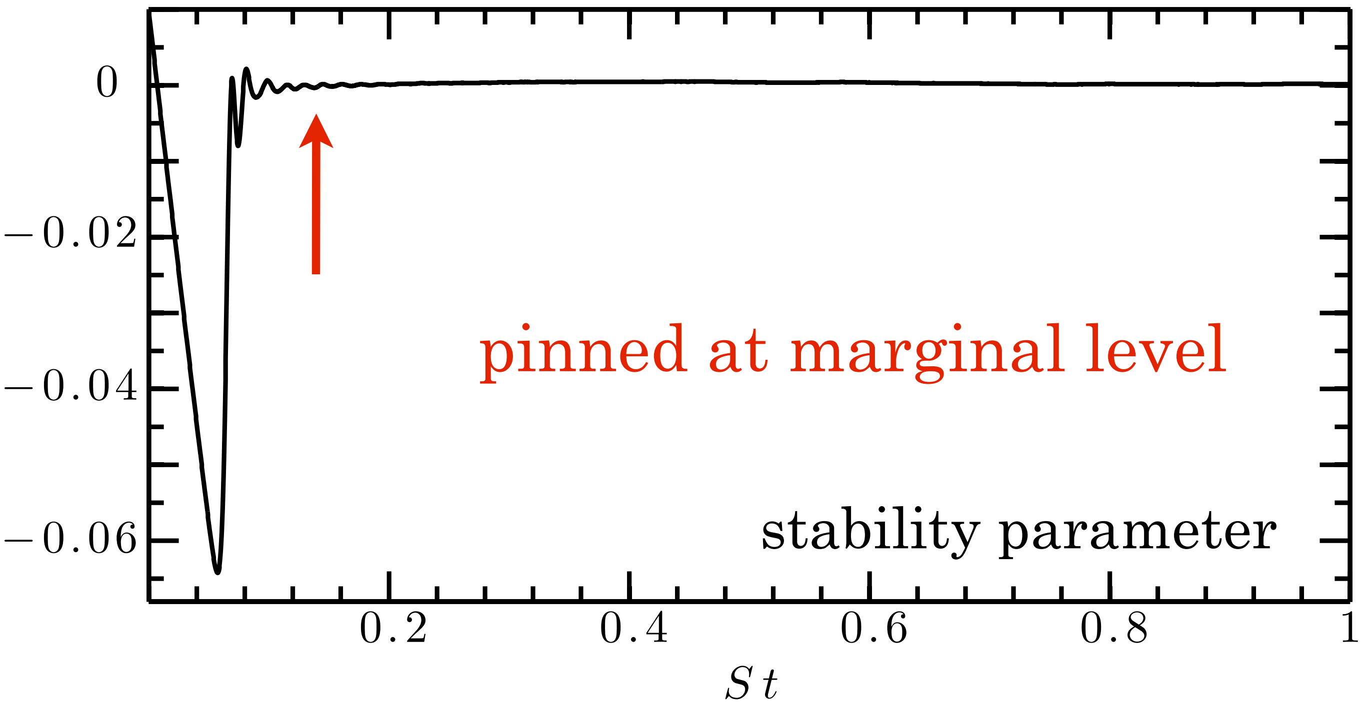








secular growth



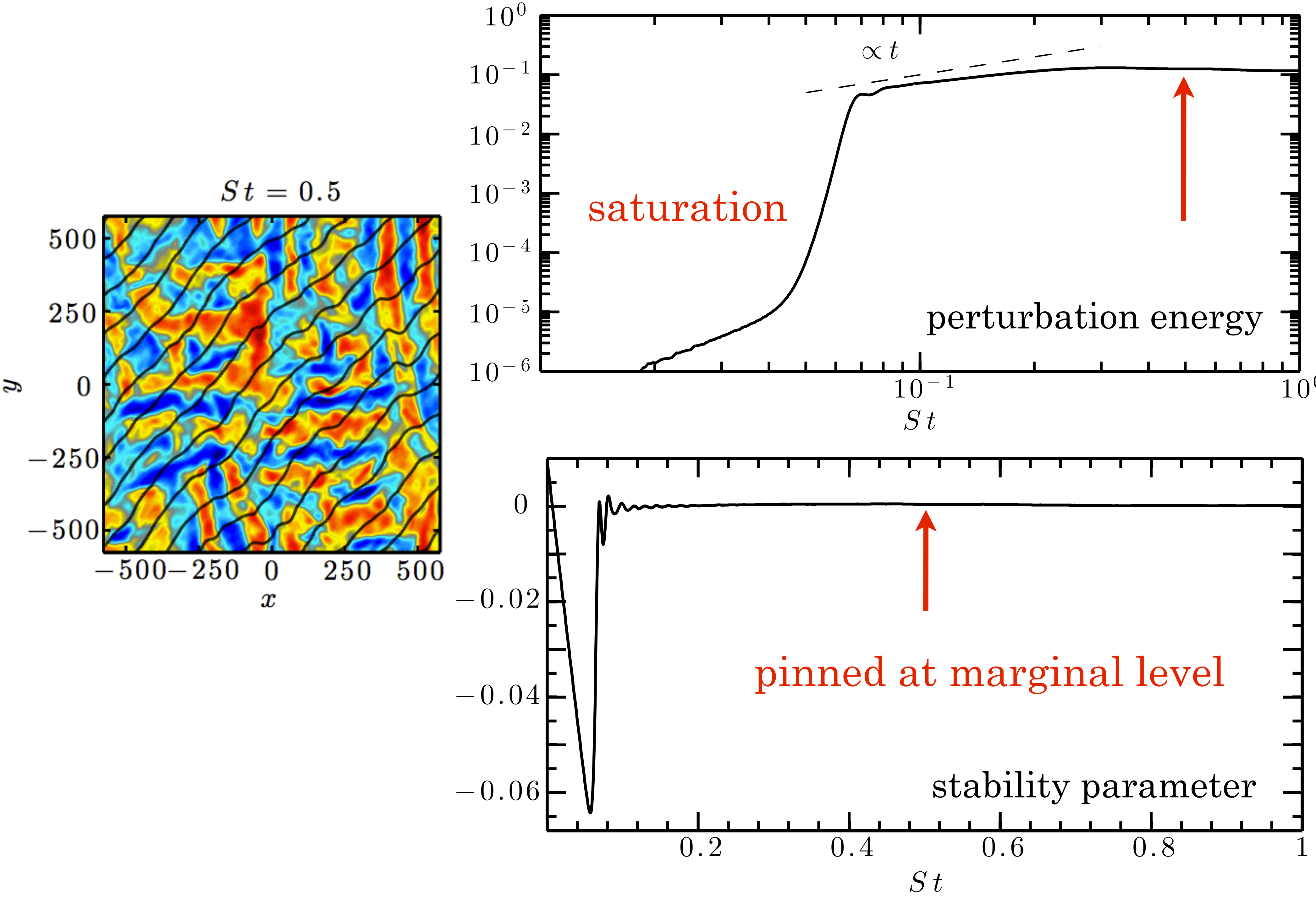
pinned at marginal level

secular growth comes from minimizing $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\boldsymbol{:}\boldsymbol{\nabla}\boldsymbol{v}$

Schekochihin et al 2005; Rosin et al 2011

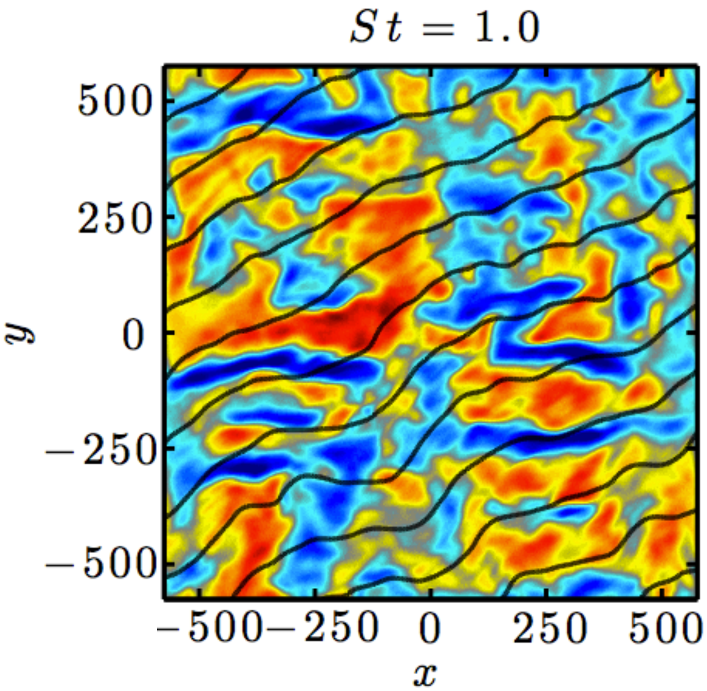
$$\underbrace{\frac{3}{2}\frac{\overline{|\delta\boldsymbol{B}_\perp(t)|^2}}{B_0^2}}_{\substack{\text{pressure}\\ \text{anisotropy}\\ \text{driven by FH}}} = \underbrace{3S\int^t \hat{b}_x(t')\hat{b}_y(t')\,dt'}_{\substack{\text{pressure}\\ \text{anisotropy}\\ \text{driven by shear}}} - \frac{2}{\beta(t)}$$

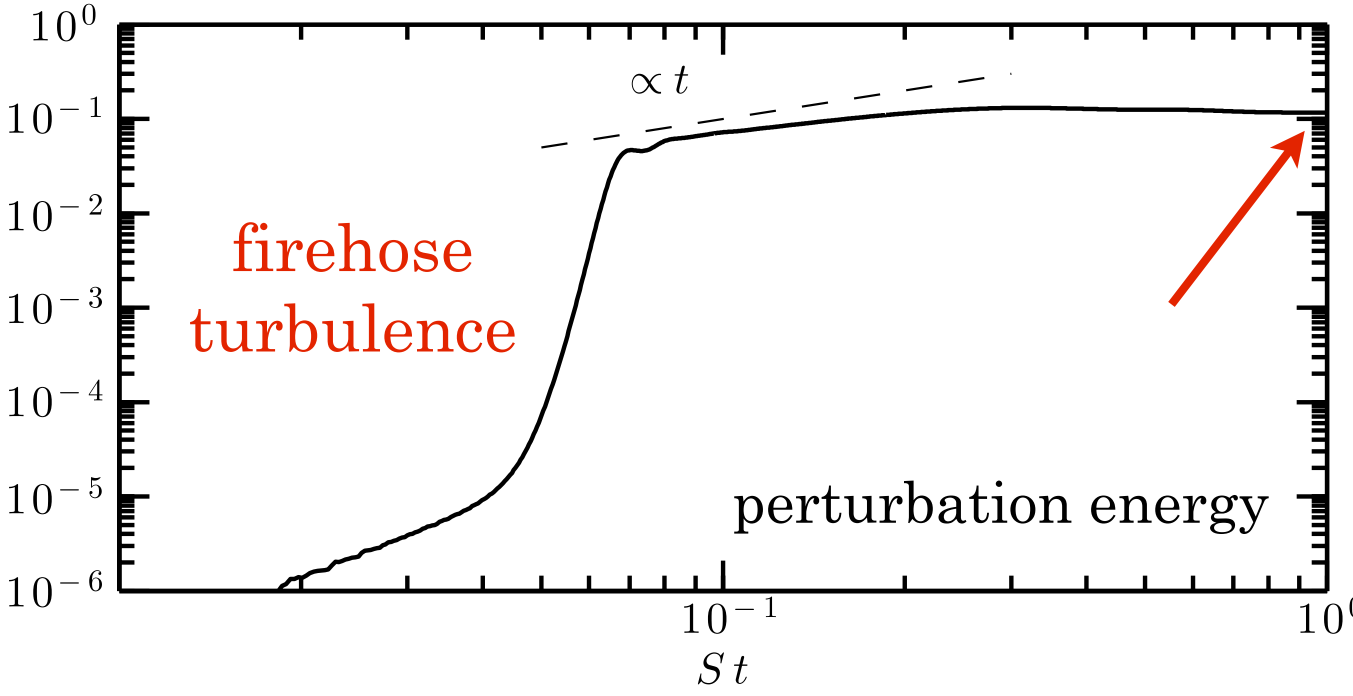
$$\longrightarrow |\delta\boldsymbol{B}_\perp|^2 \propto St$$



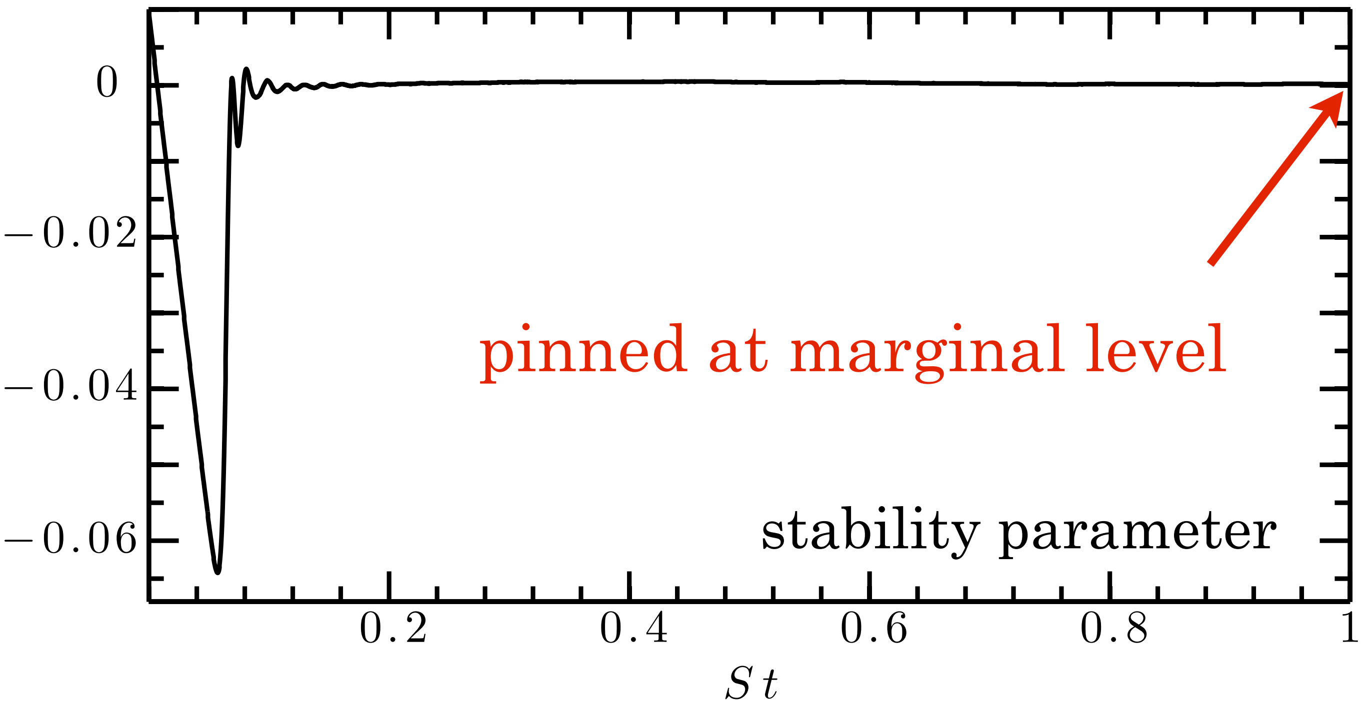
saturation

pinned at marginal level

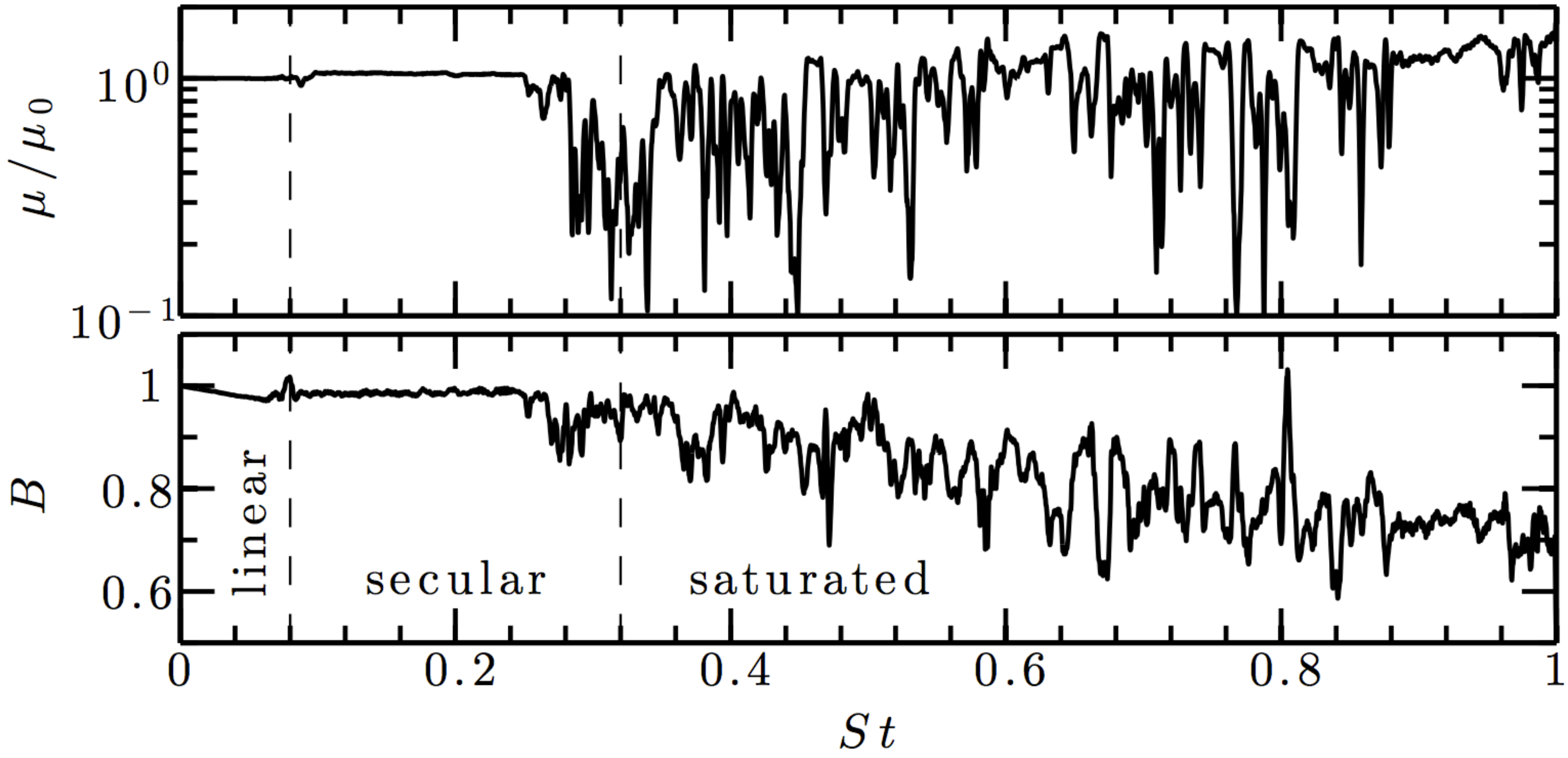




firehose turbulence



pinned at marginal level



pressure anisotropy ultimately regulated by breaking μ

vary shear rate...

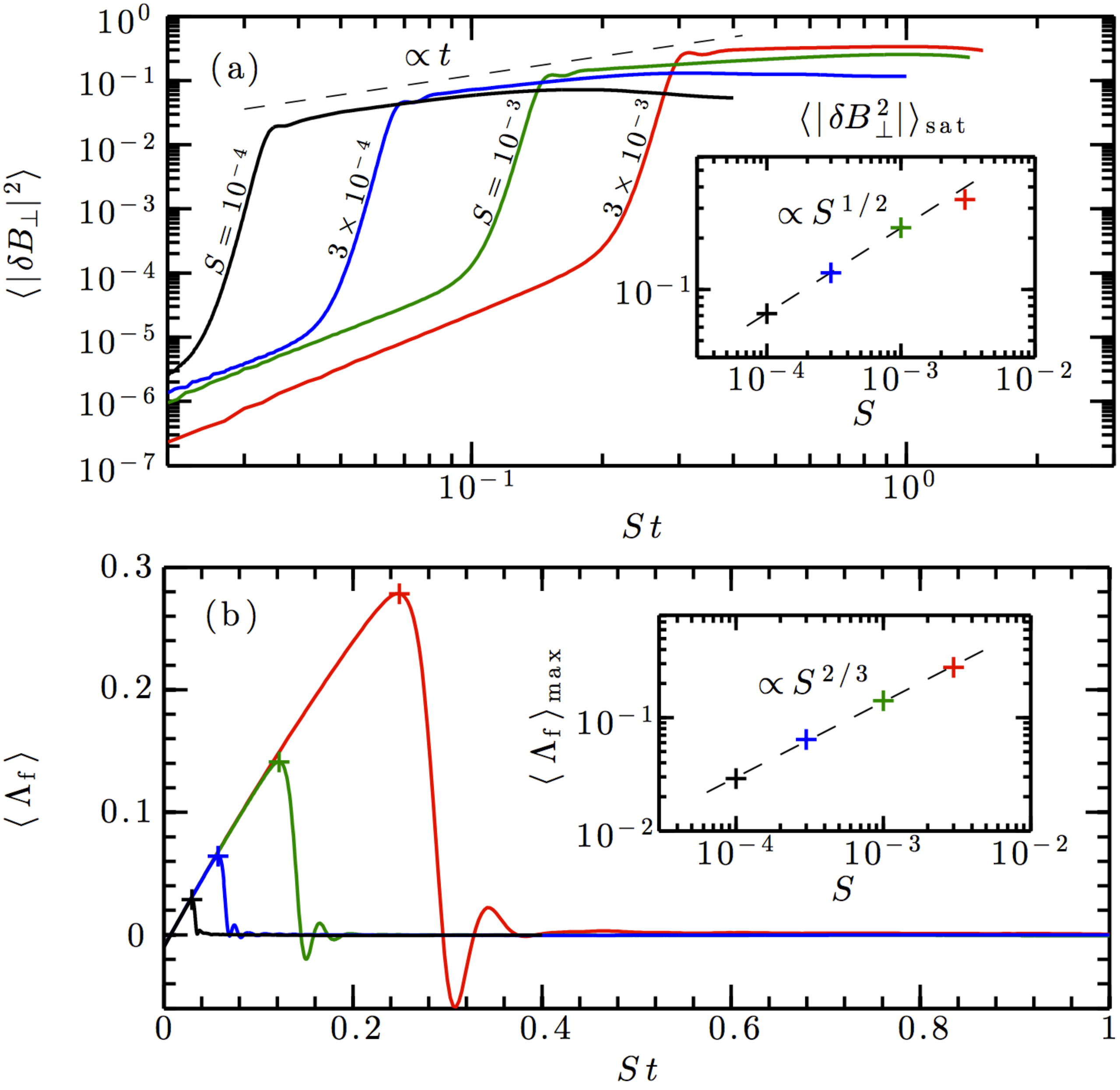
$$|\delta \boldsymbol{B}_\perp|^2 \propto S^{1/2}$$

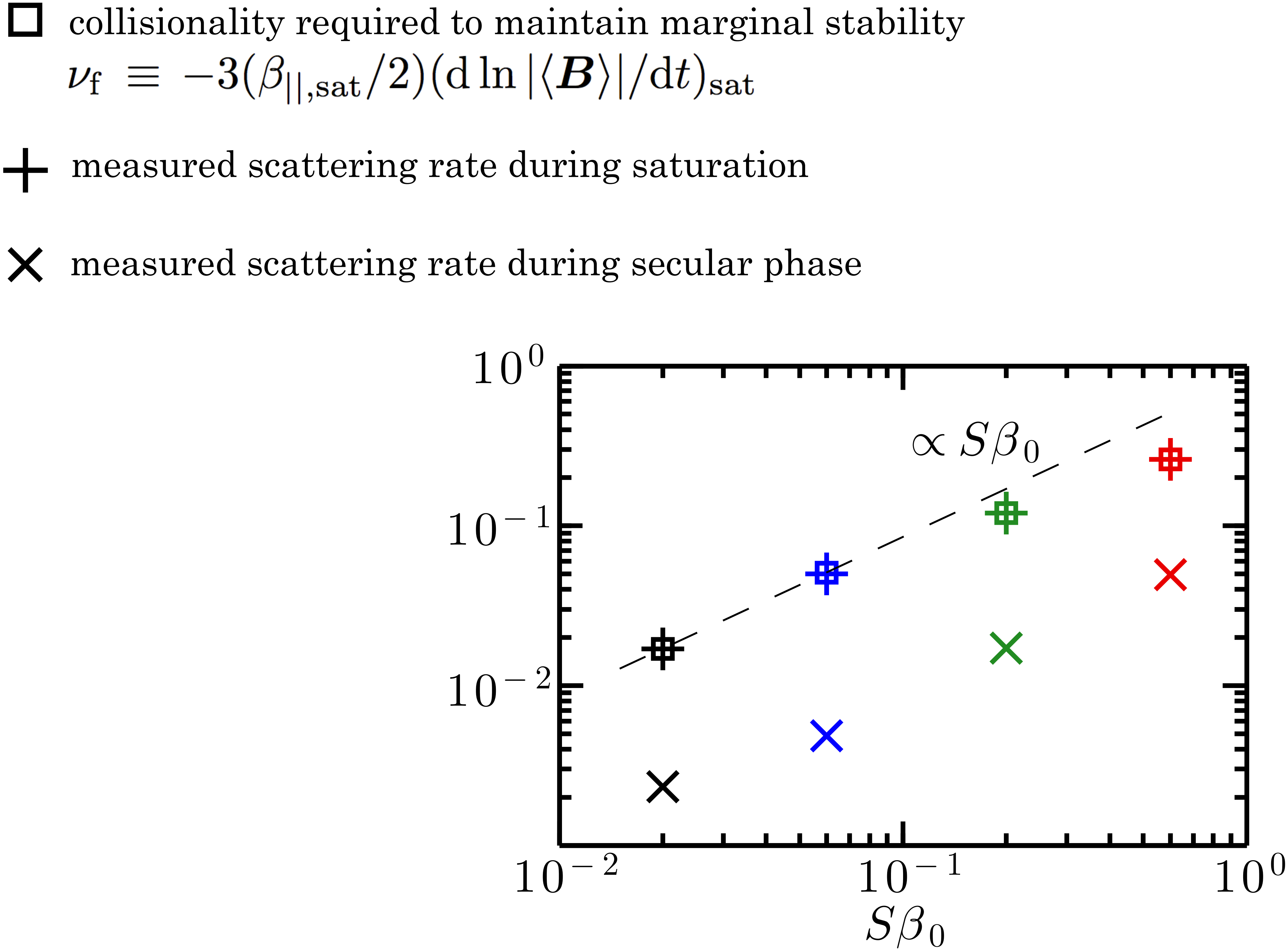
at saturation

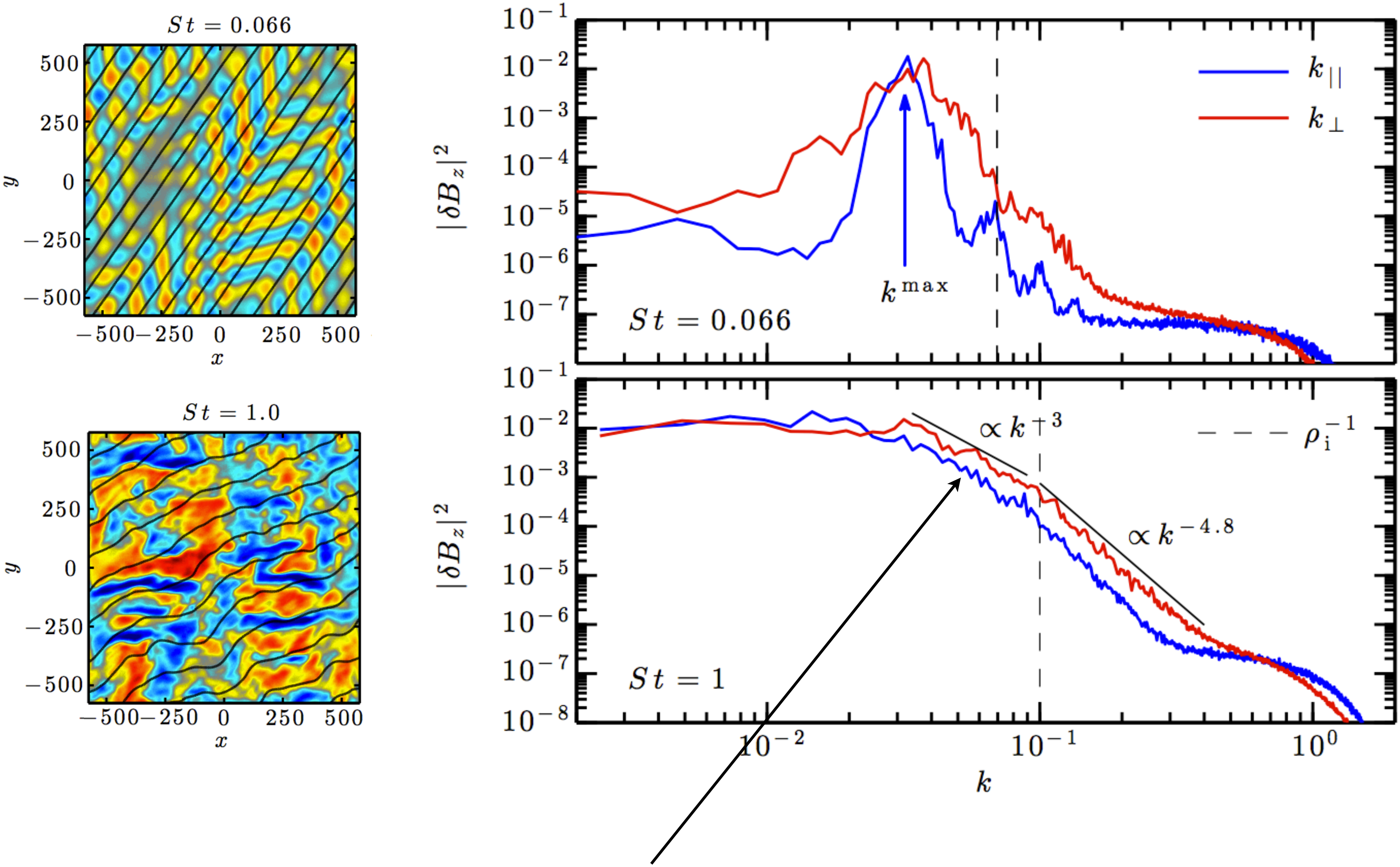
$$S \ll \Omega_{\rm i}$$

↓

small-amplitude firehose turbulence




□ collisionality required to maintain marginal stability
$\nu_{\mathrm{f}} \equiv -3(\beta_{||,\mathrm{sat}}/2)(\mathrm{d}\ln|\langle \boldsymbol{B}\rangle|/\mathrm{d}t)_{\mathrm{sat}}$
+ measured scattering rate during saturation
× measured scattering rate during secular phase
$\propto S\beta_0$
10^0
10^{-1}
10^{-2}
10^{-2}
10^{-1}
10^0
$S\beta_0$



firehose turbulence

energy-containing mode during secular phase has

$$\gamma_{\mathrm{peak}} \sim \Lambda_{\mathrm{f}} \sim 1/t \qquad\qquad k_{||,\mathrm{peak}} \sim \Lambda_{\mathrm{f}}^{1/2} \sim 1/t^{1/2}$$

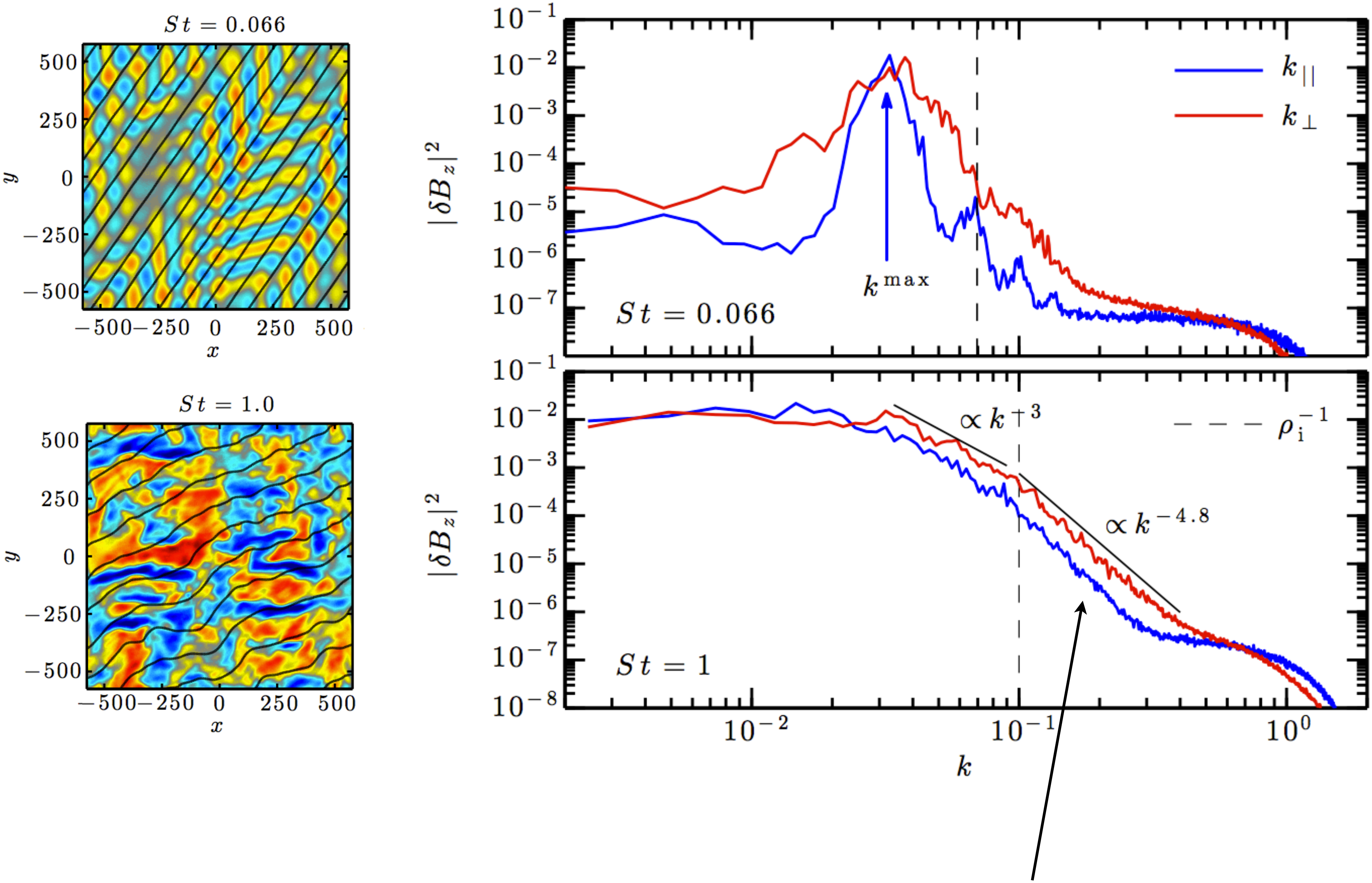
and we know

$$\sum_{k_{||}} |\delta \boldsymbol{B}_{\perp,k_{||}}|^2 \sim St$$

Suppose $|\delta \boldsymbol{B}_{\perp,k_{||}}|^2 \sim k_{||}^{-\alpha}$; then

$$\sum_{k_{||}} |\delta \boldsymbol{B}_{\perp,k_{||}}|^2 \sim k_{||,\mathrm{peak}}^{1-\alpha} \sim t^{-(1-\alpha)/2}$$

$$\longrightarrow \alpha = 3$$

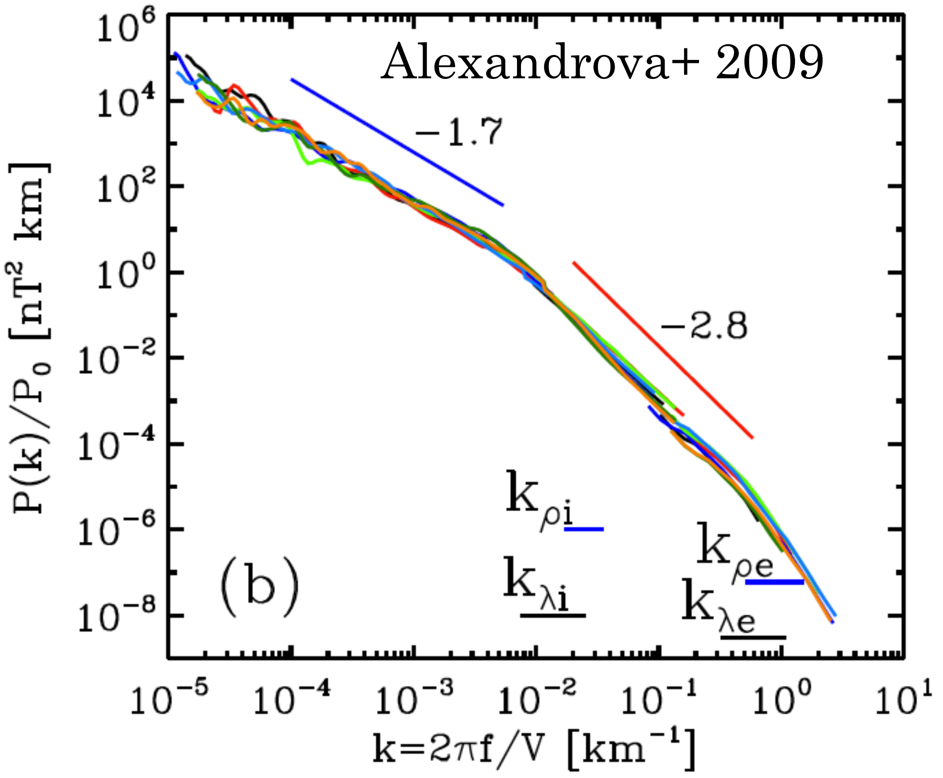

$St = 0.066$
$St = 1.0$
y
x
500
250
0
−250
−500
$|\delta B_z|^2$
$k_{||}$
$k_\perp$
$k^{\max}$
$St = 0.066$
$St = 1$
$\propto k^{-3}$
$\propto k^{-4.8}$
ρ_i^{-1}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{0}
k

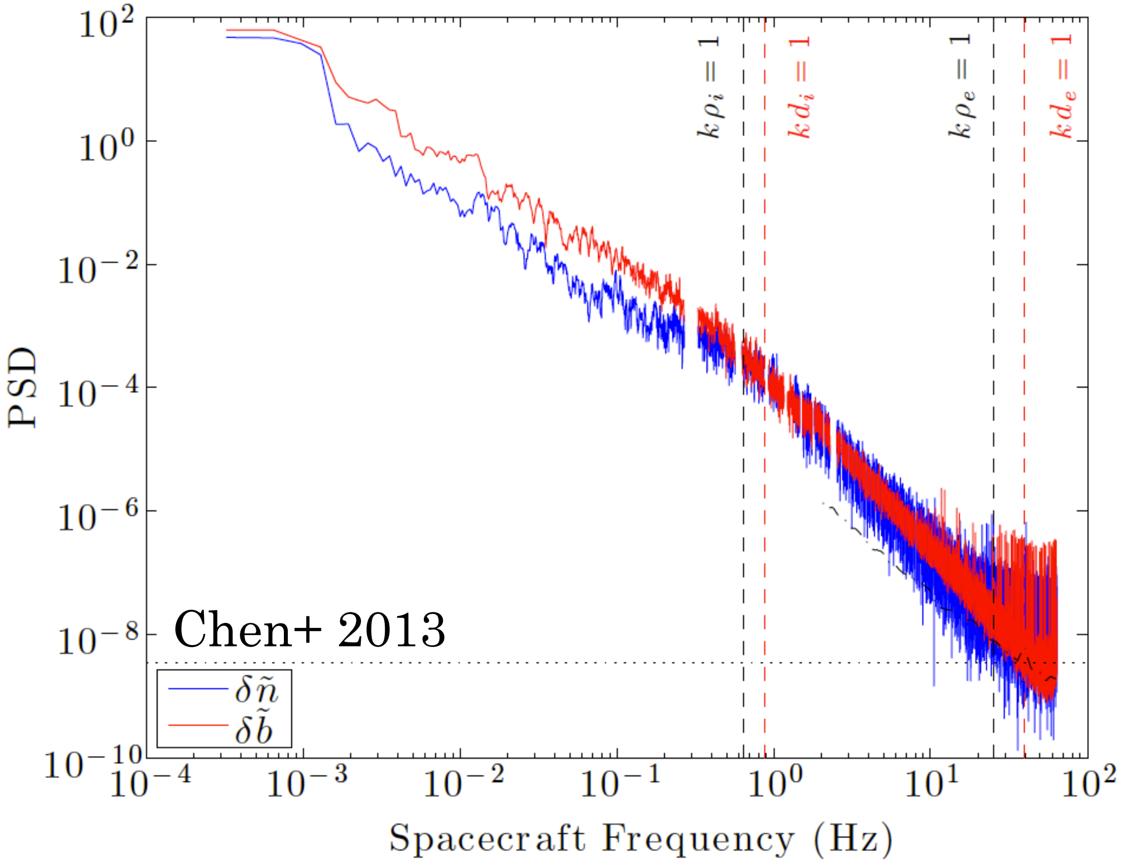
KAW cascade

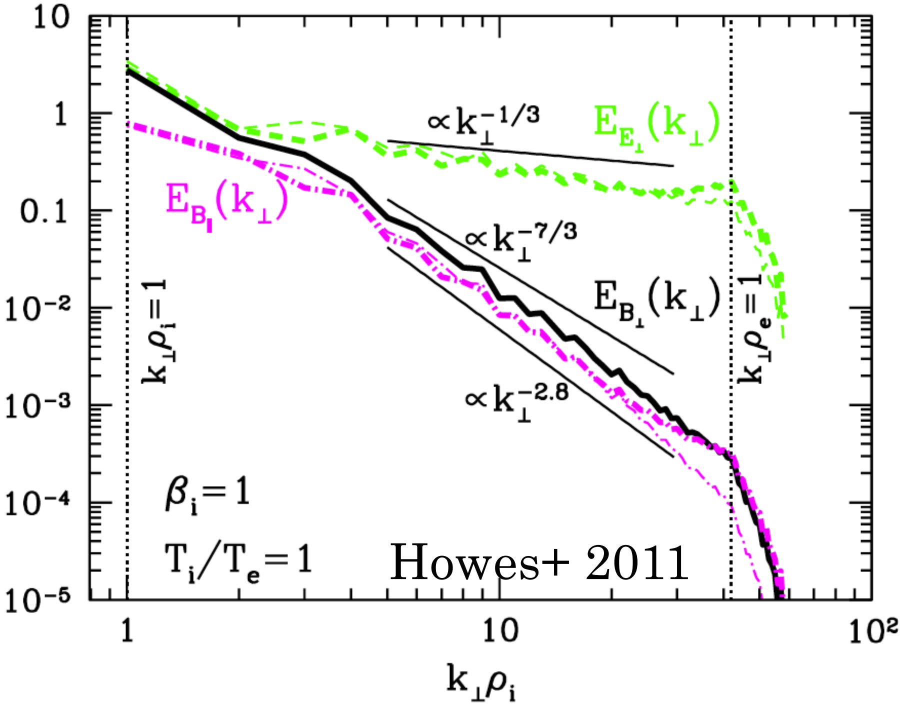
KAW cascade

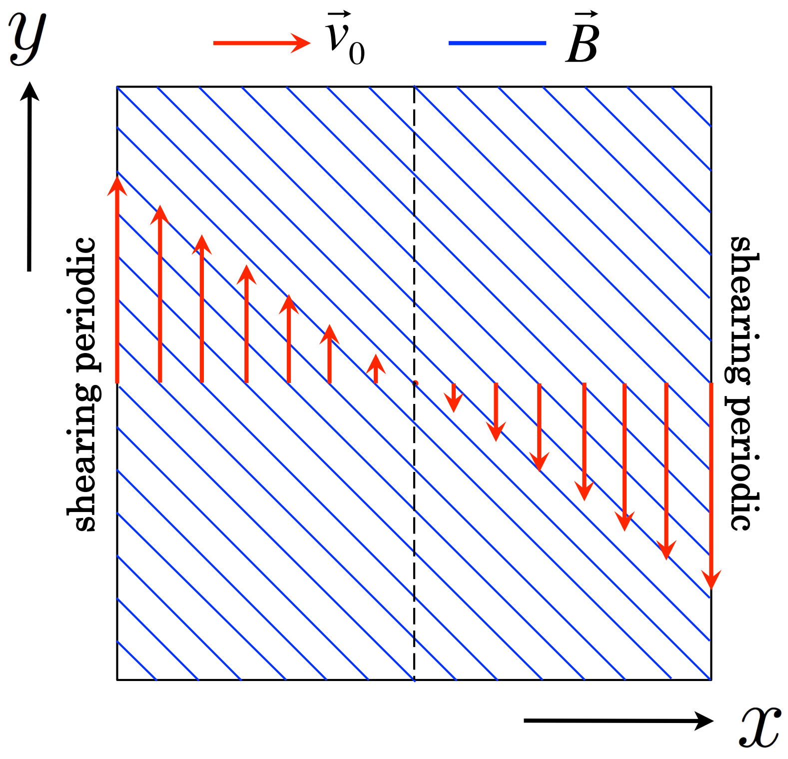
$\alpha \approx 4.8$

observed in SW
simulated using GK







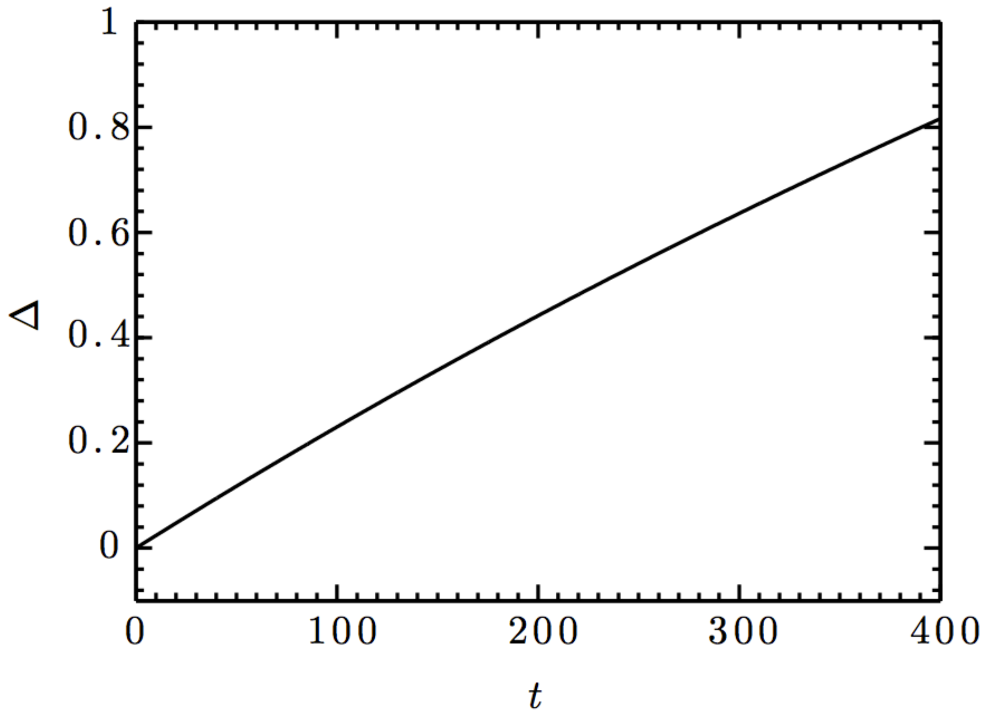


shearing sheet

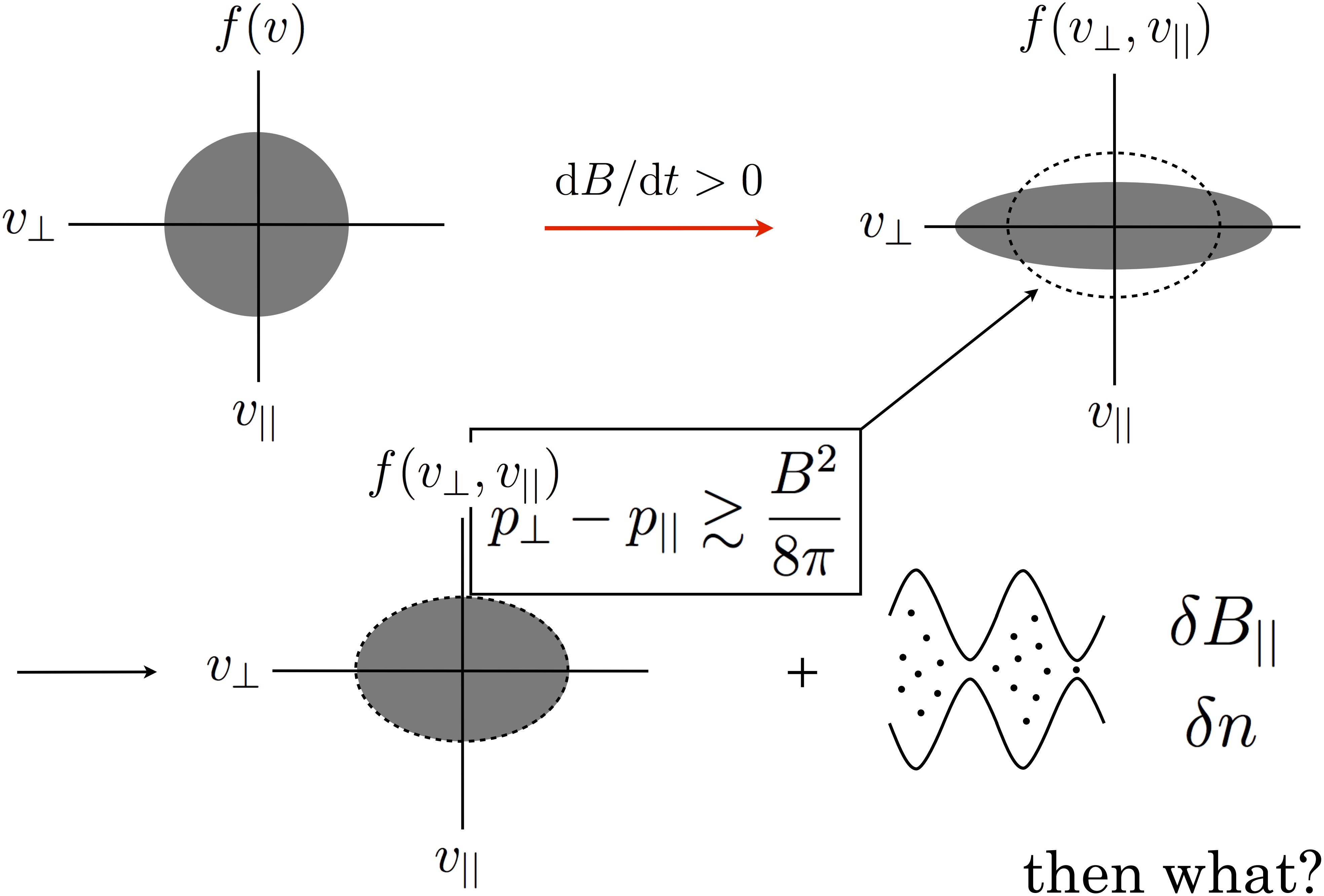
$$\boldsymbol{v}_0 = -Sx\,\hat{\boldsymbol{y}}$$

field wraps up
drives $p_\perp > p_{||}$
goes mirror unstable

$$\Delta \equiv \frac{p_\perp - p_{||}}{p} \longrightarrow$$

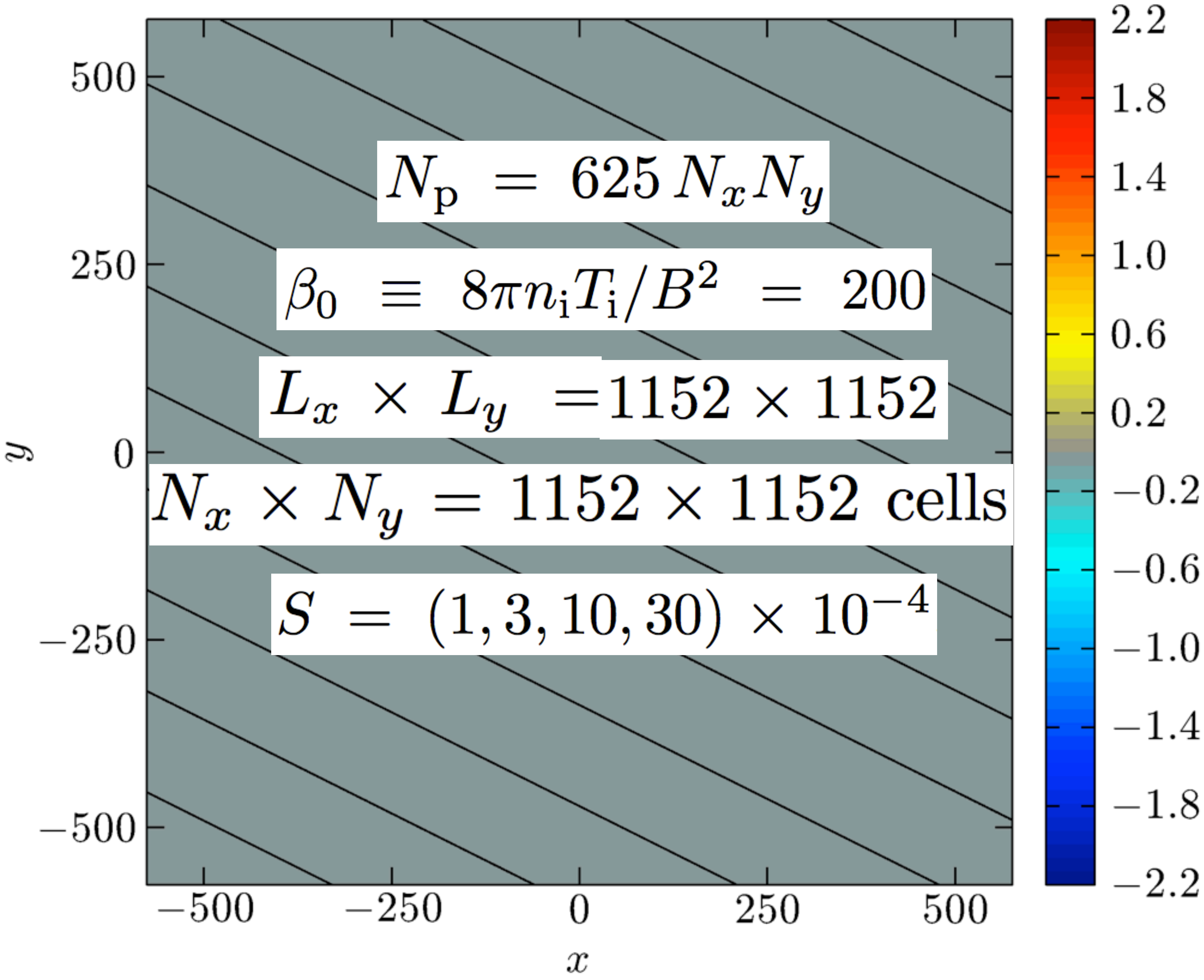


mirror instability



$$\boxed{p_\perp - p_{||} \gtrsim \frac{B^2}{8\pi}}$$

then what?



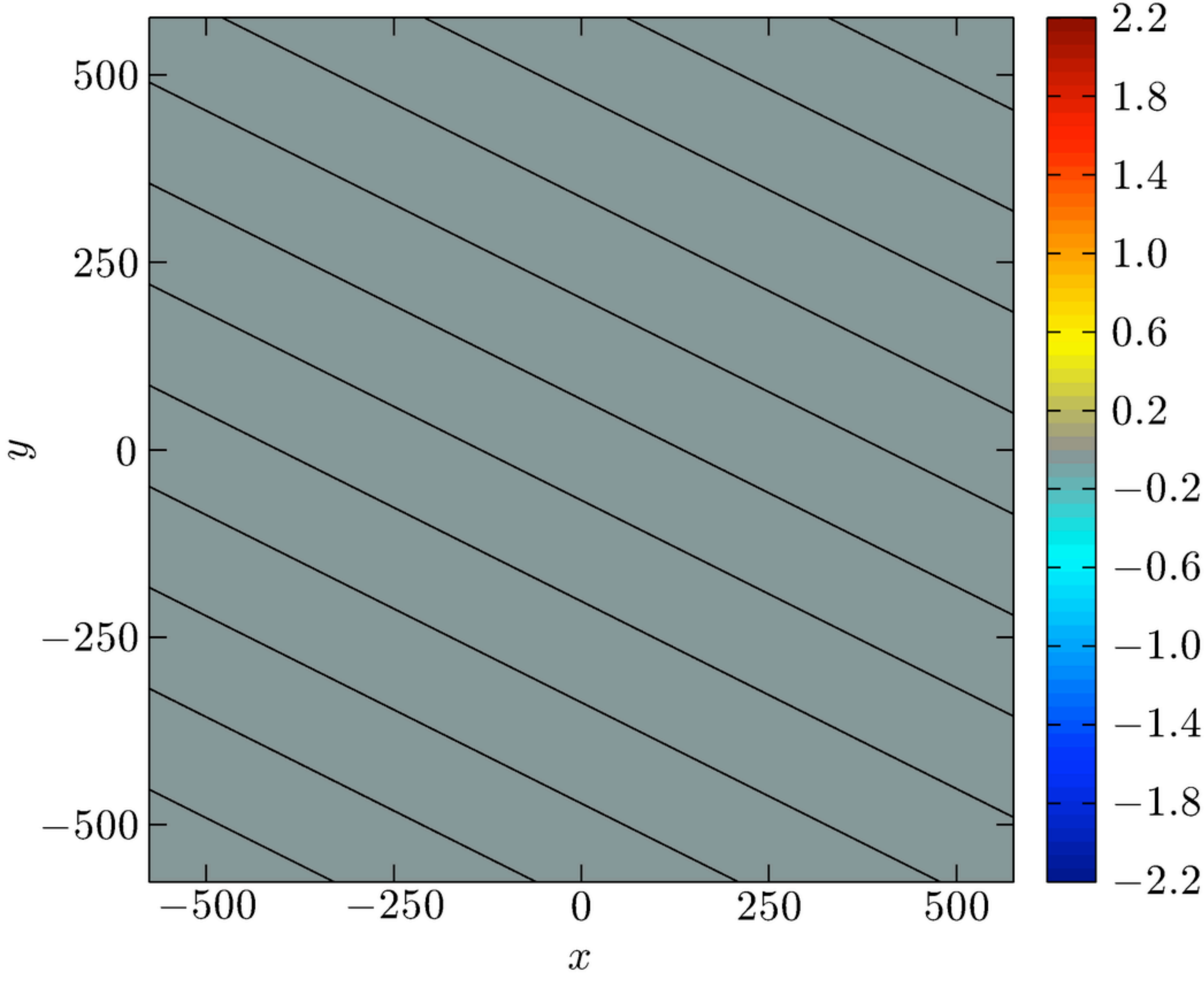
$$N_{\mathrm{p}} = 625\, N_x N_y$$

$$\beta_0 \equiv 8\pi n_{\mathrm{i}} T_{\mathrm{i}}/B^2 = 200$$

$$L_x \times L_y = 1152 \times 1152$$

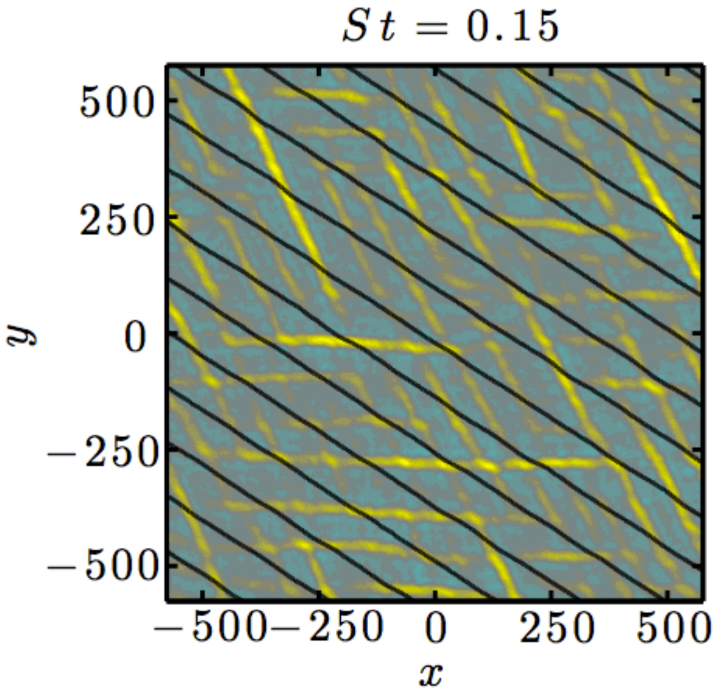
$$N_x \times N_y = 1152 \times 1152 \text{ cells}$$

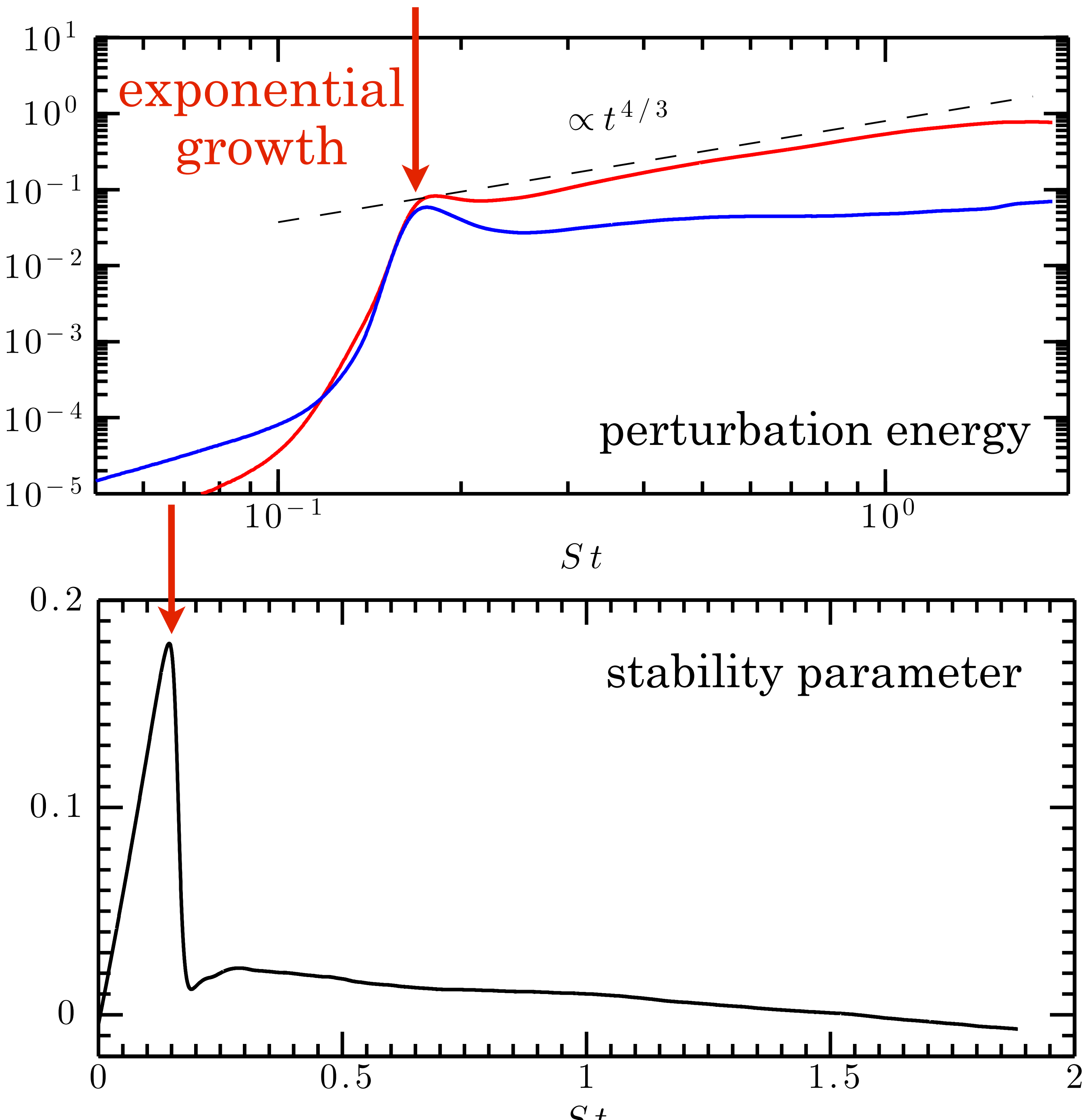
$$S = (1, 3, 10, 30) \times 10^{-4}$$

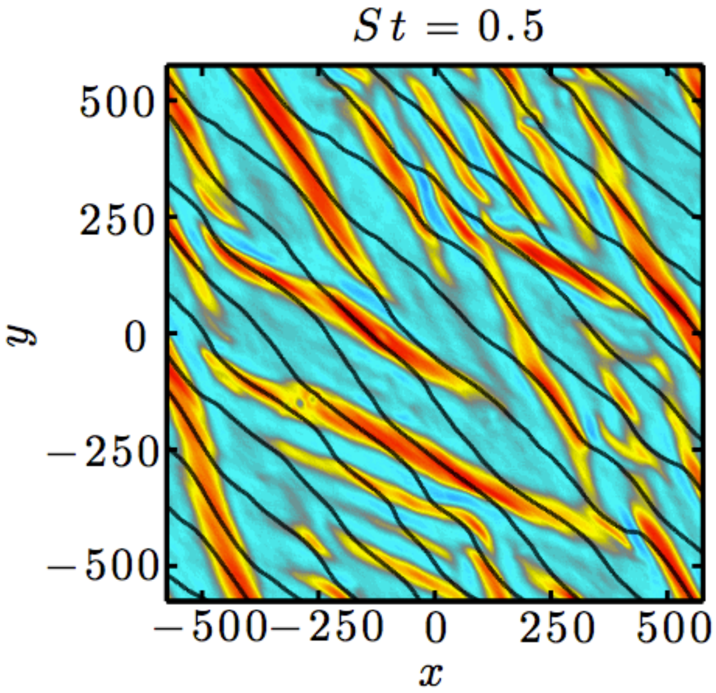


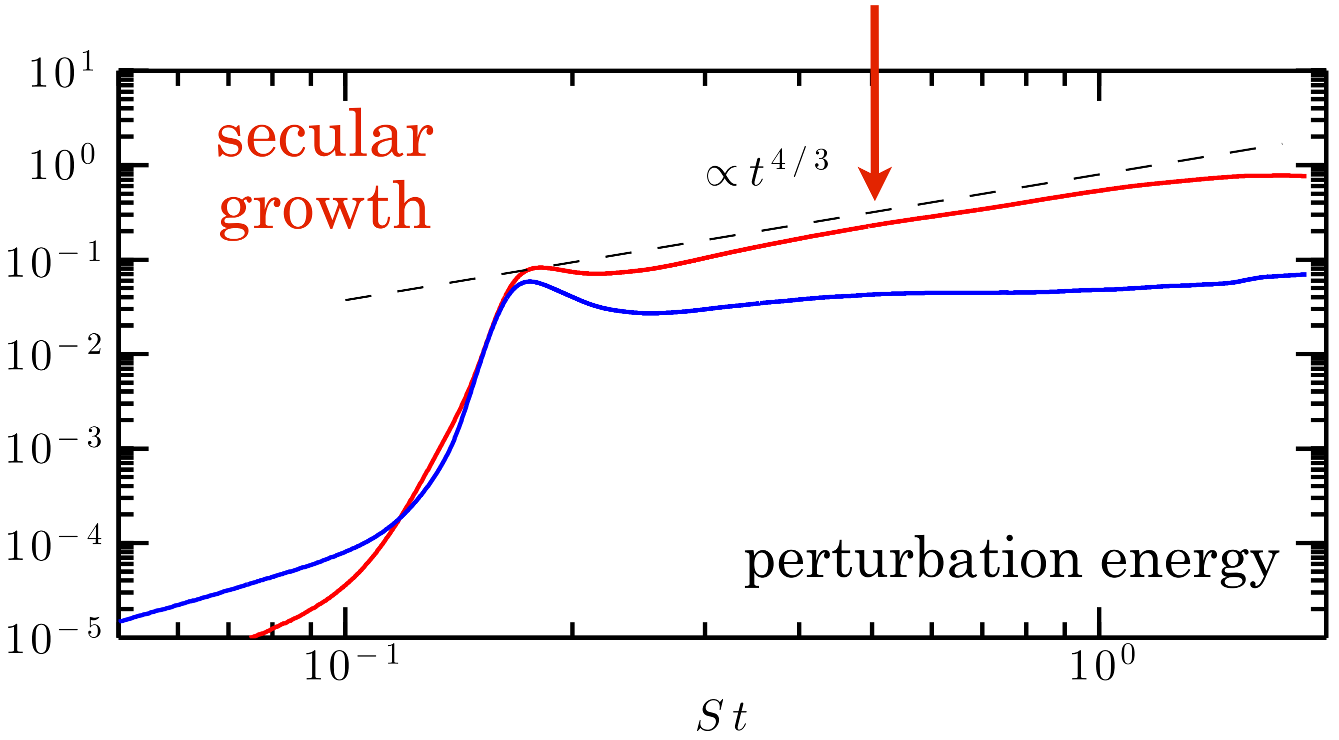
y
500
250
0
−250
−500
x
−500
−250
0
250
500
2.2
1.8
1.4
1.0
0.6
0.2
−0.2
−0.6
−1.0
−1.4
−1.8
−2.2

oblique modes $k_\perp / k_{||} \sim \Lambda_{\rm m}^{1/2}$

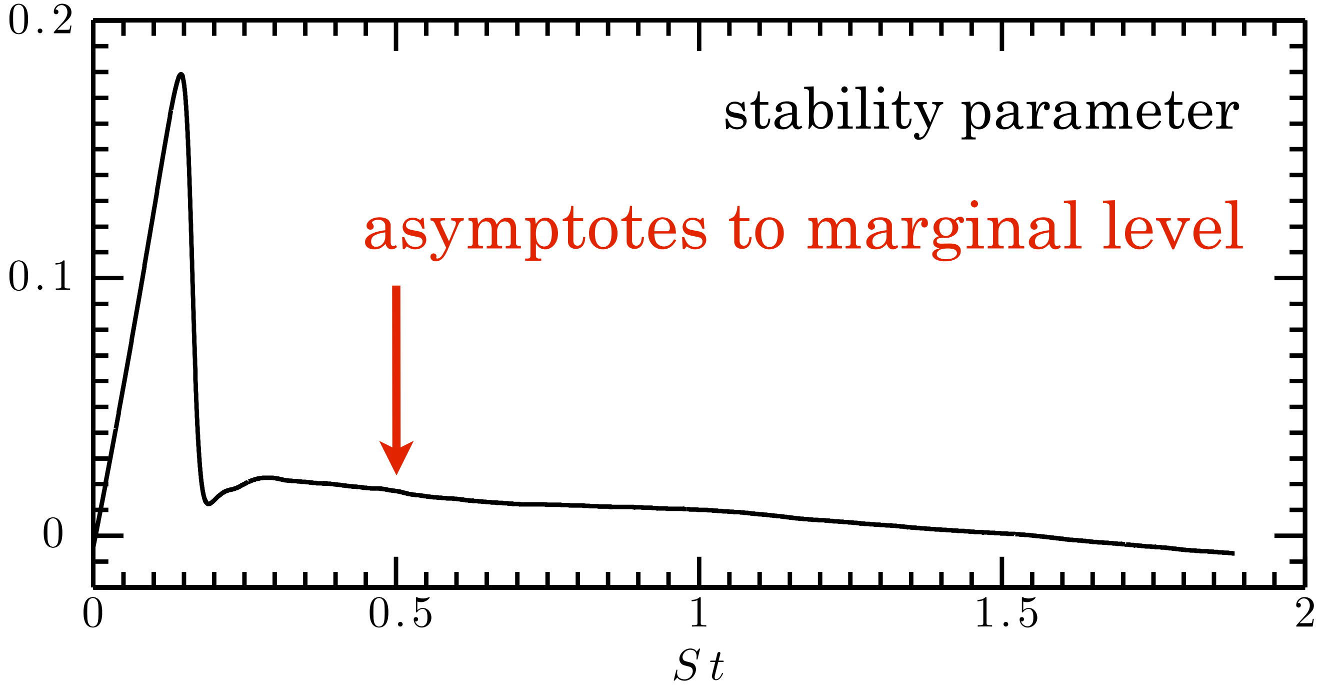





$St = 0.5$


secular growth

perturbation energy


stability parameter

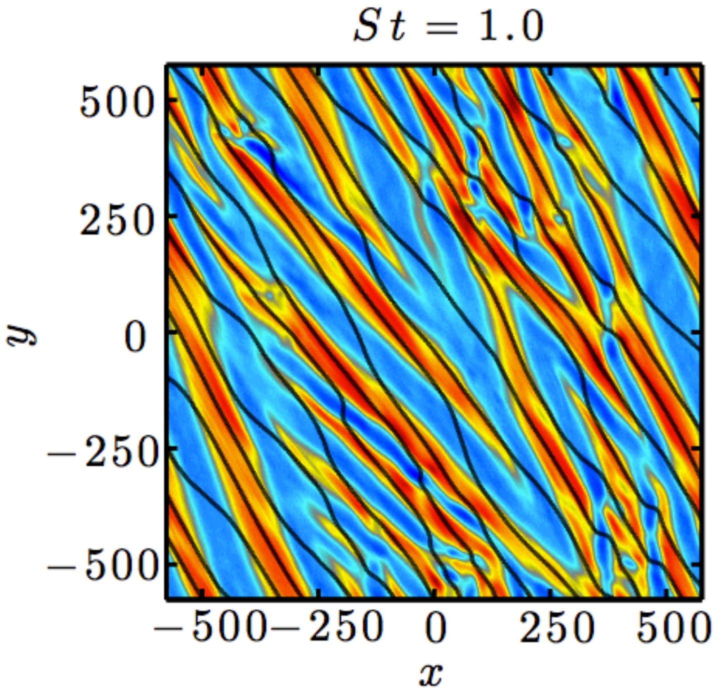
asymptotes to marginal level

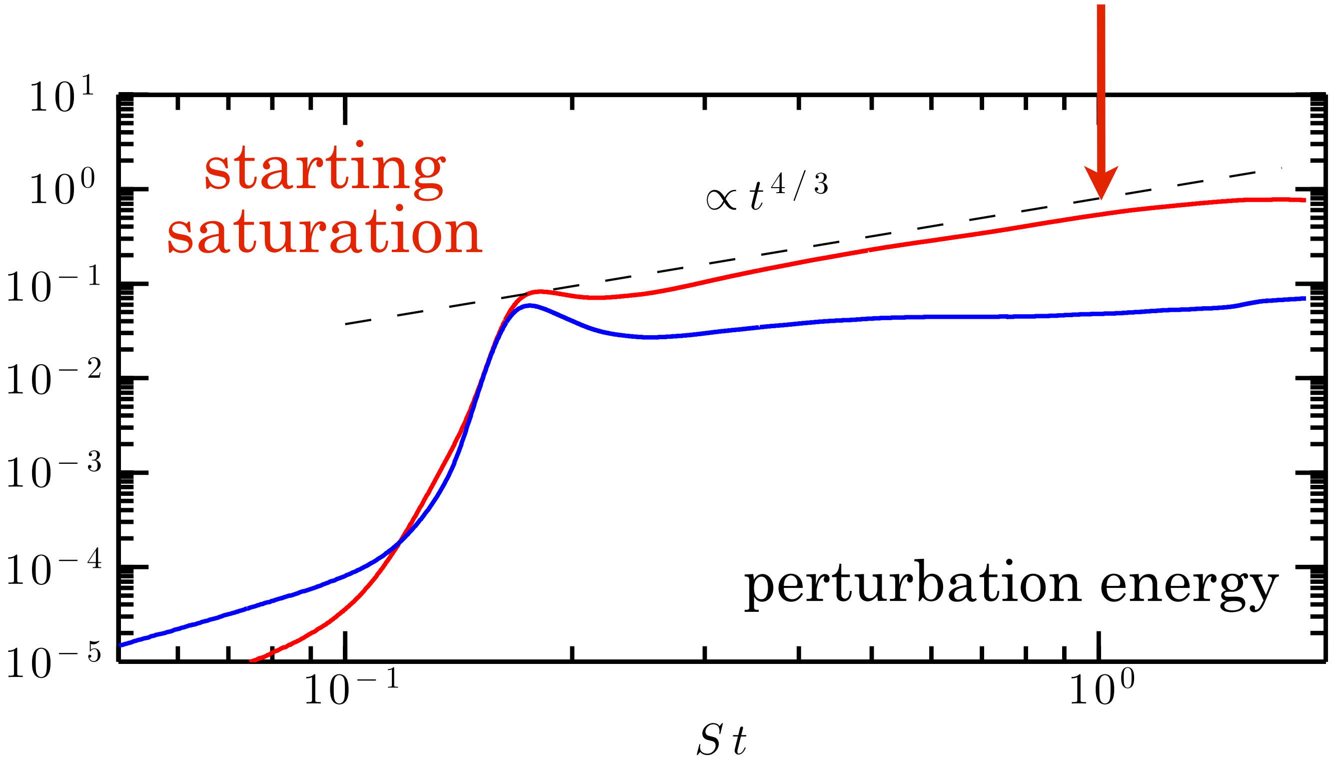
secular growth comes from minimizing $\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}:\boldsymbol{\nabla}\boldsymbol{v}$

Schekochihin et al 2005

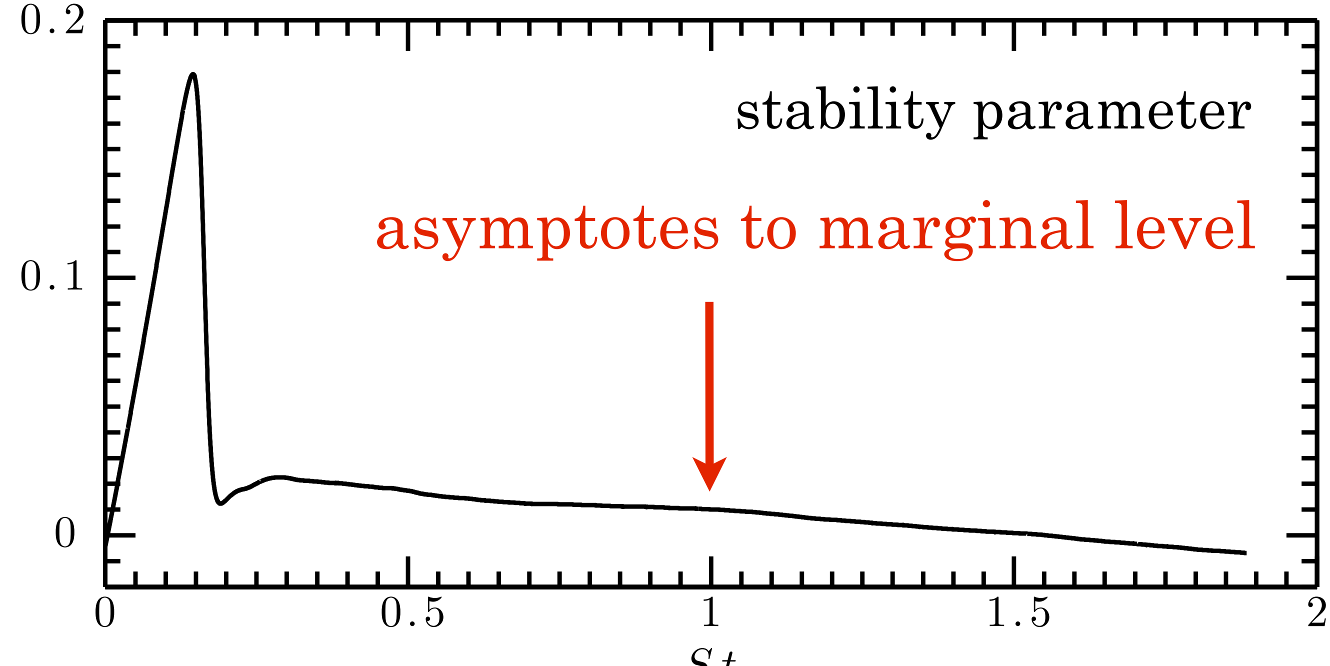
$$\underbrace{\frac{3}{2}\left(\frac{\overline{|\delta B_{||}|}}{B_0(t)}\right)^{3/2}}_{\substack{\text{pressure}\\\text{anisotropy}\\\text{driven by mirror}}} = \underbrace{3S\int^t \hat{b}_x(t')\hat{b}_y(t')\,dt'}_{\substack{\text{pressure}\\\text{anisotropy}\\\text{driven by shear}}} + \frac{1}{\beta(t)}$$

$$\longrightarrow (\delta \boldsymbol{B}_{||})^2 \propto (St)^{4/3}$$

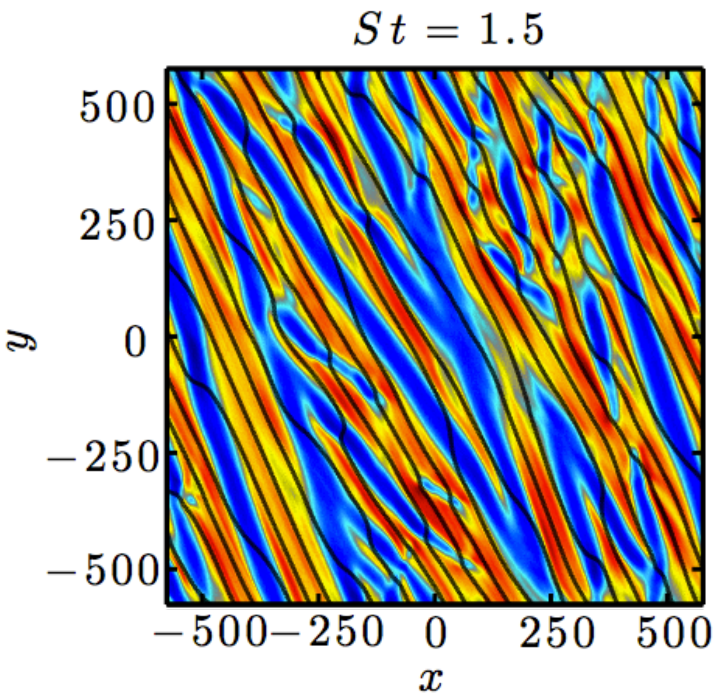


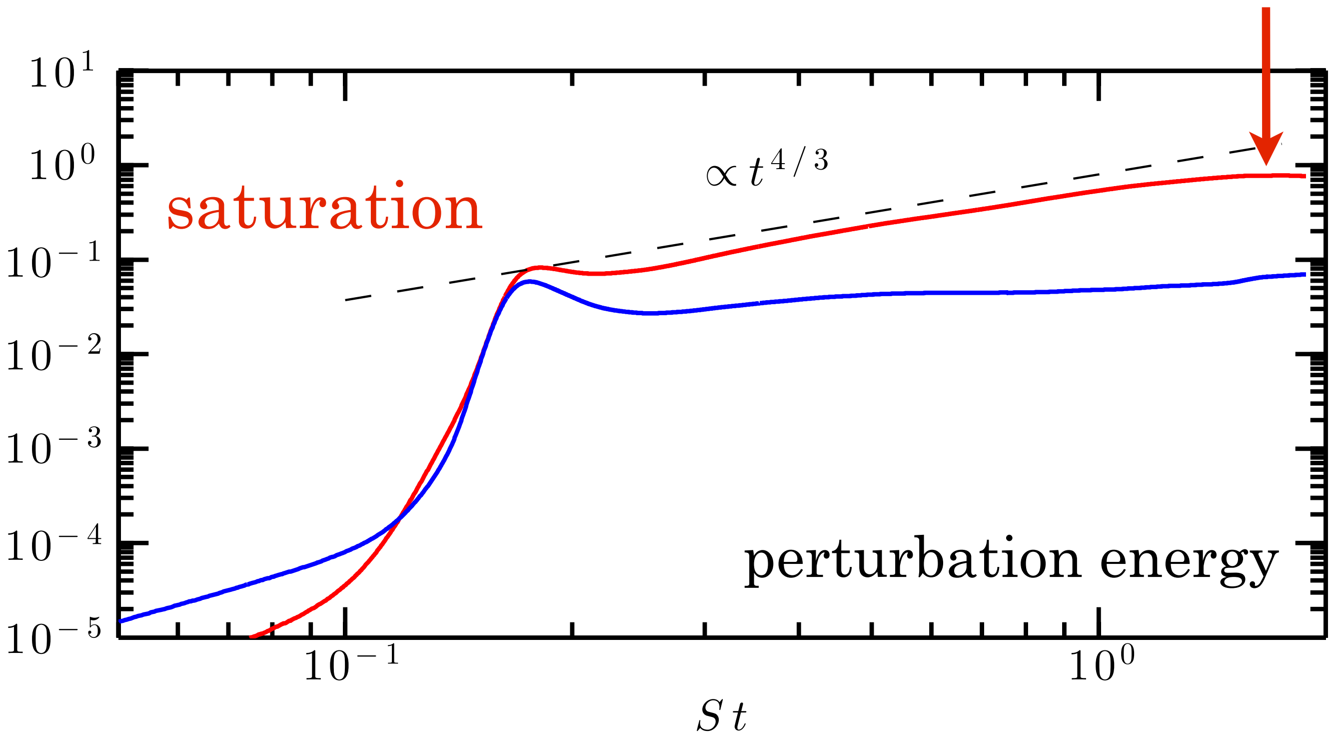

starting saturation

perturbation energy


stability parameter

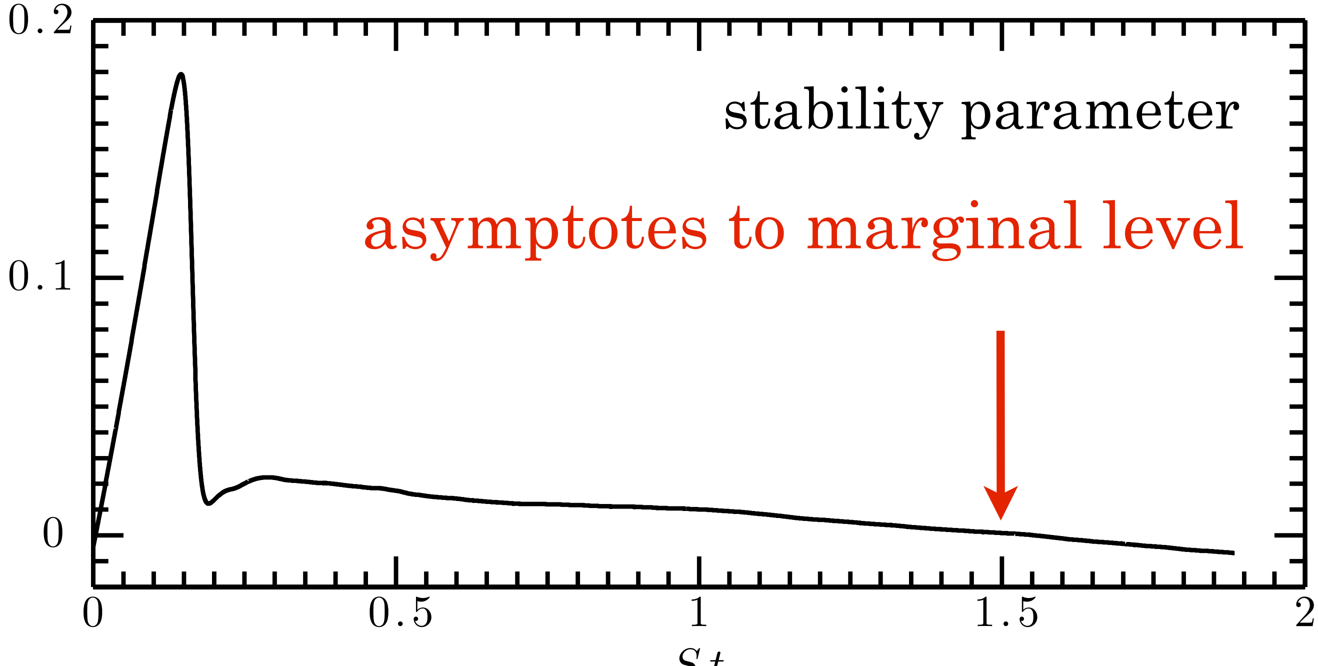
asymptotes to marginal level





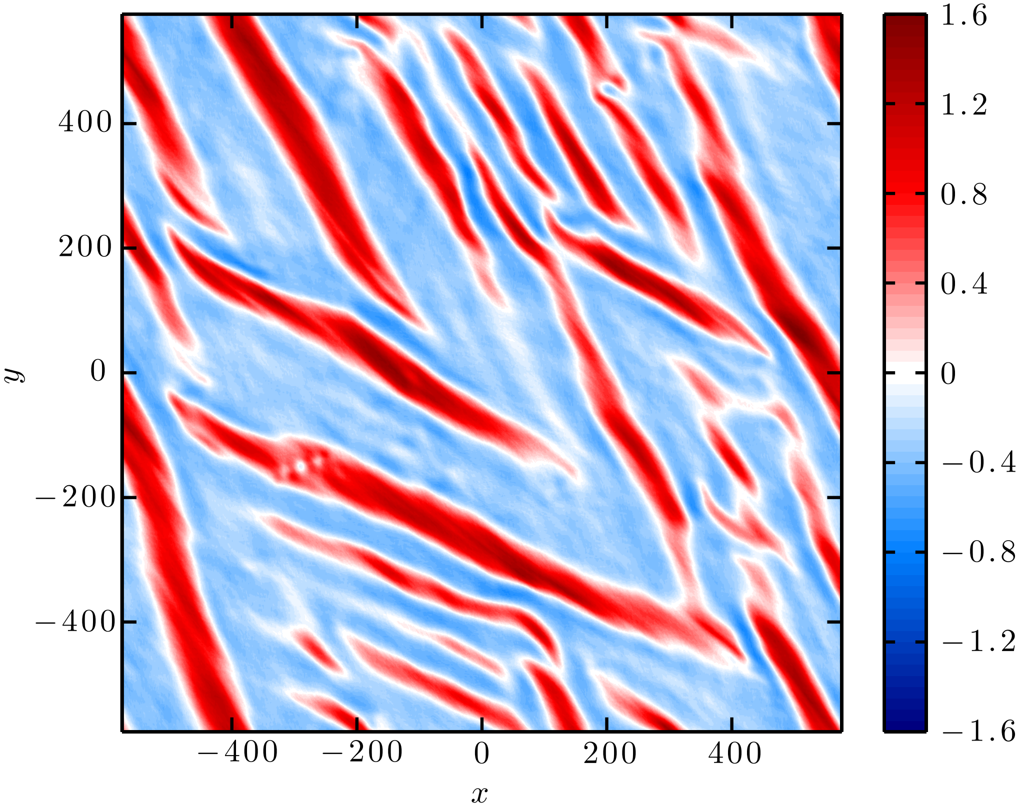
saturation

perturbation energy

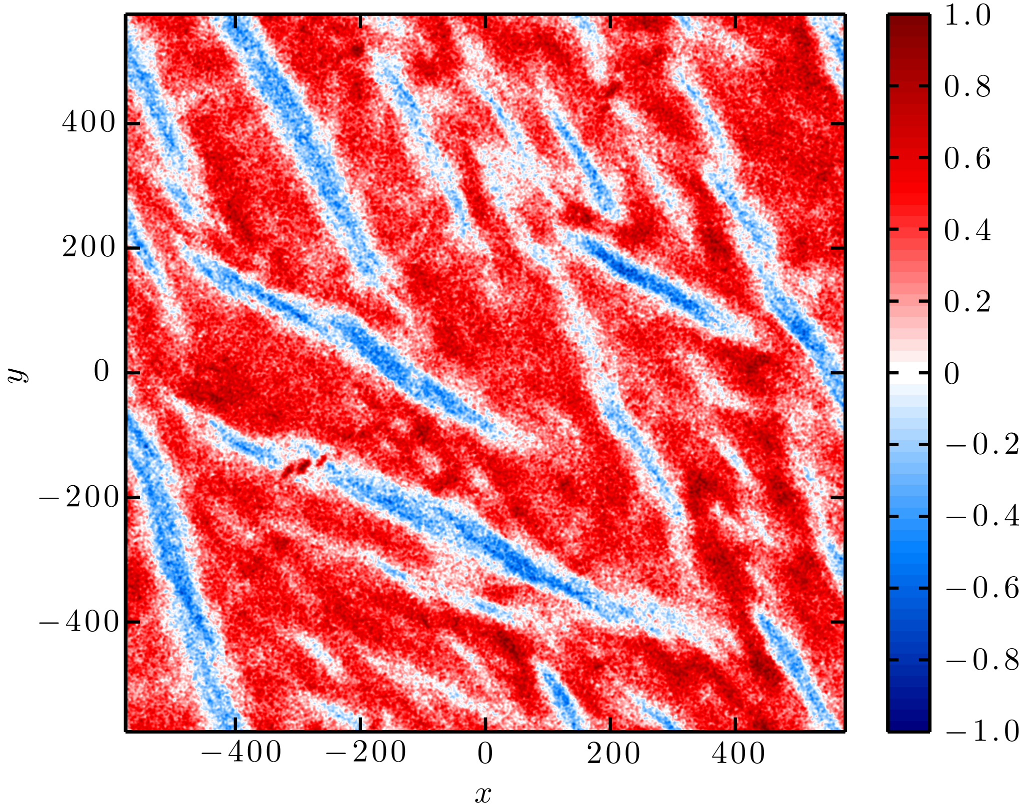


stability parameter

asymptotes to marginal level

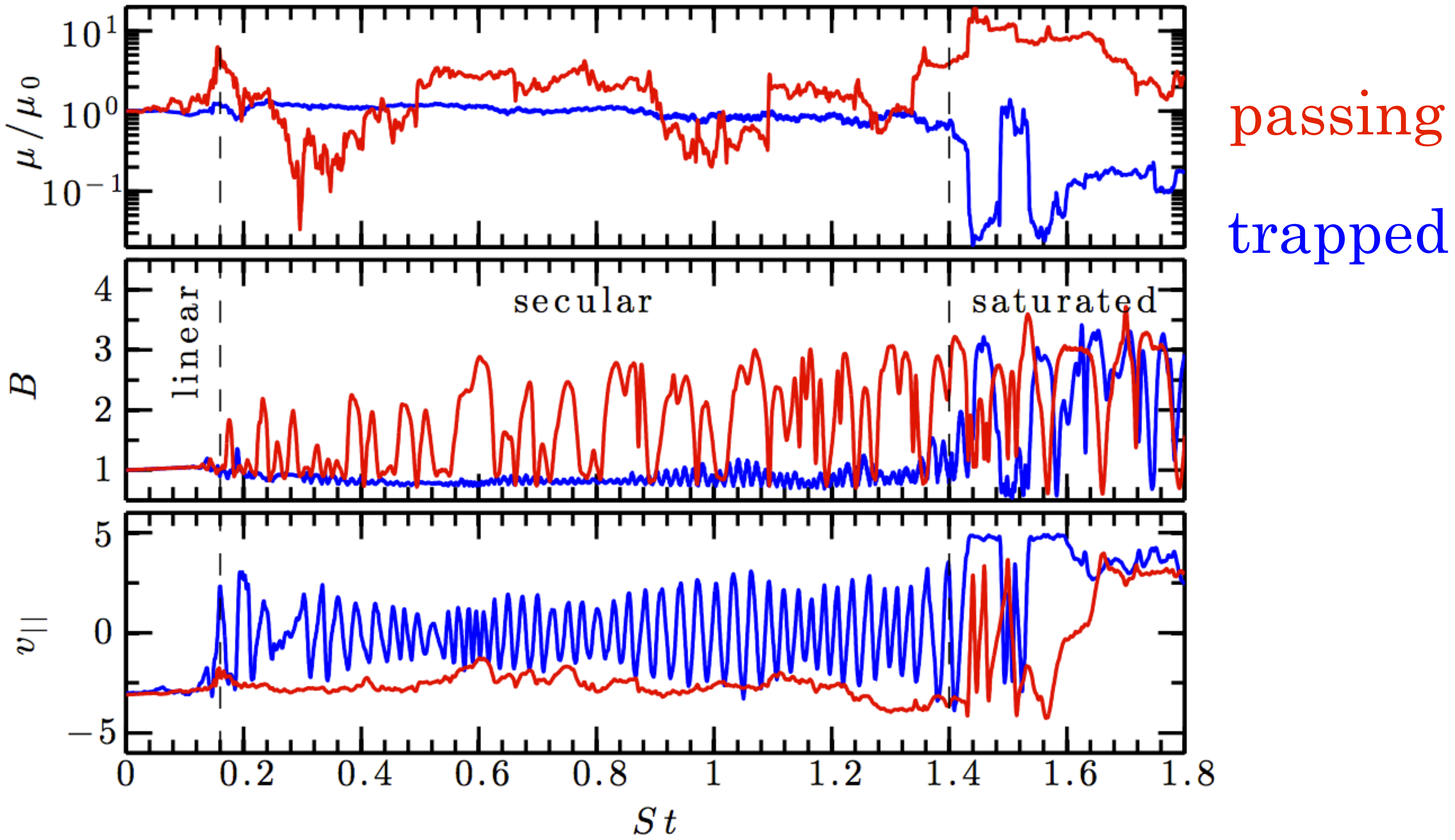


$\delta B_{||}$



δn x 100

pressure anisotropy regulated by (majority) trapped particles sampling regions where dlnB/dt ~ 0



pressure anisotropy regulated by (majority) trapped particles sampling regions where dlnB/dt ~ 0

vary shear rate...

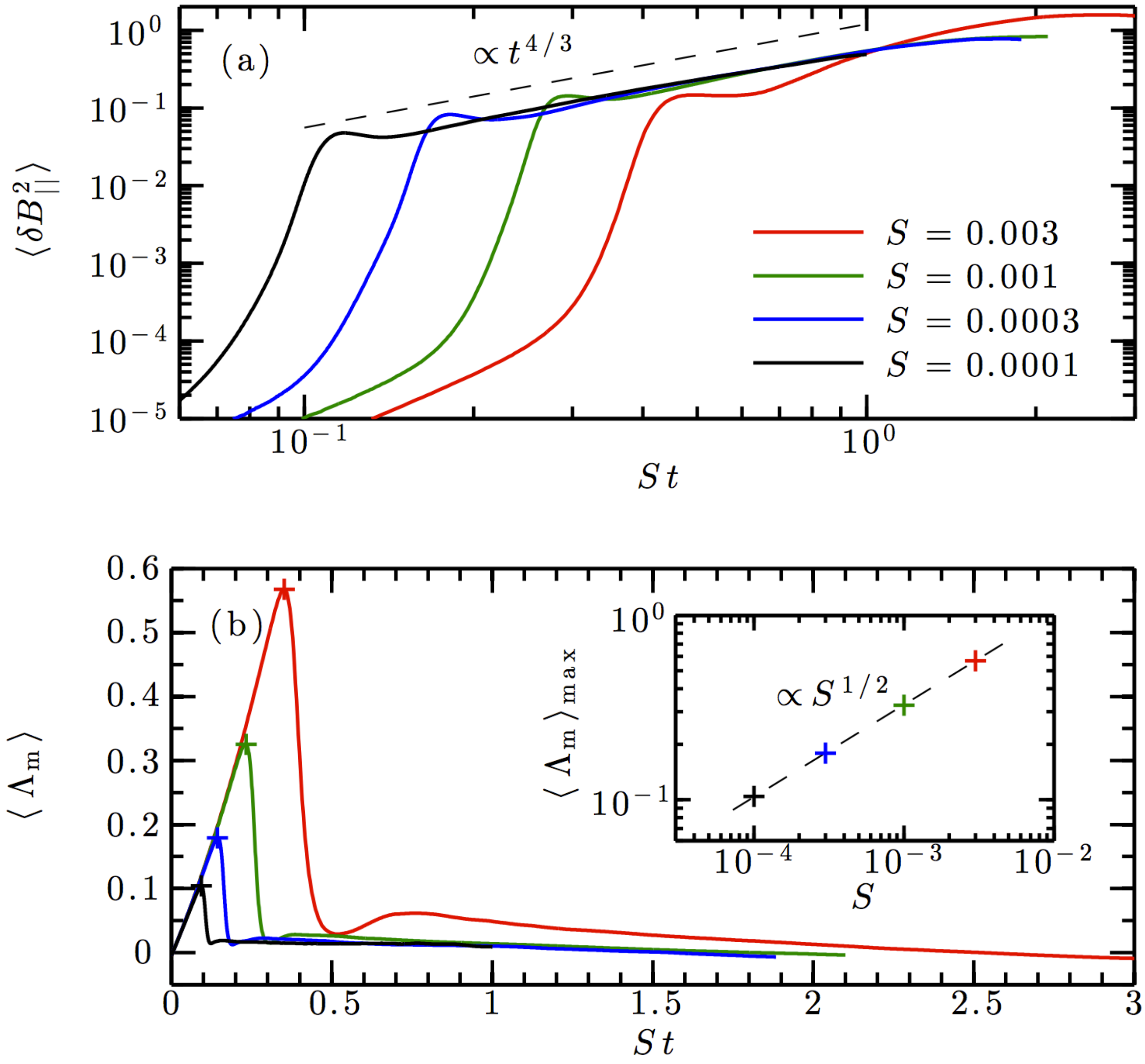
$$\delta B_{||}^2 \sim 1$$

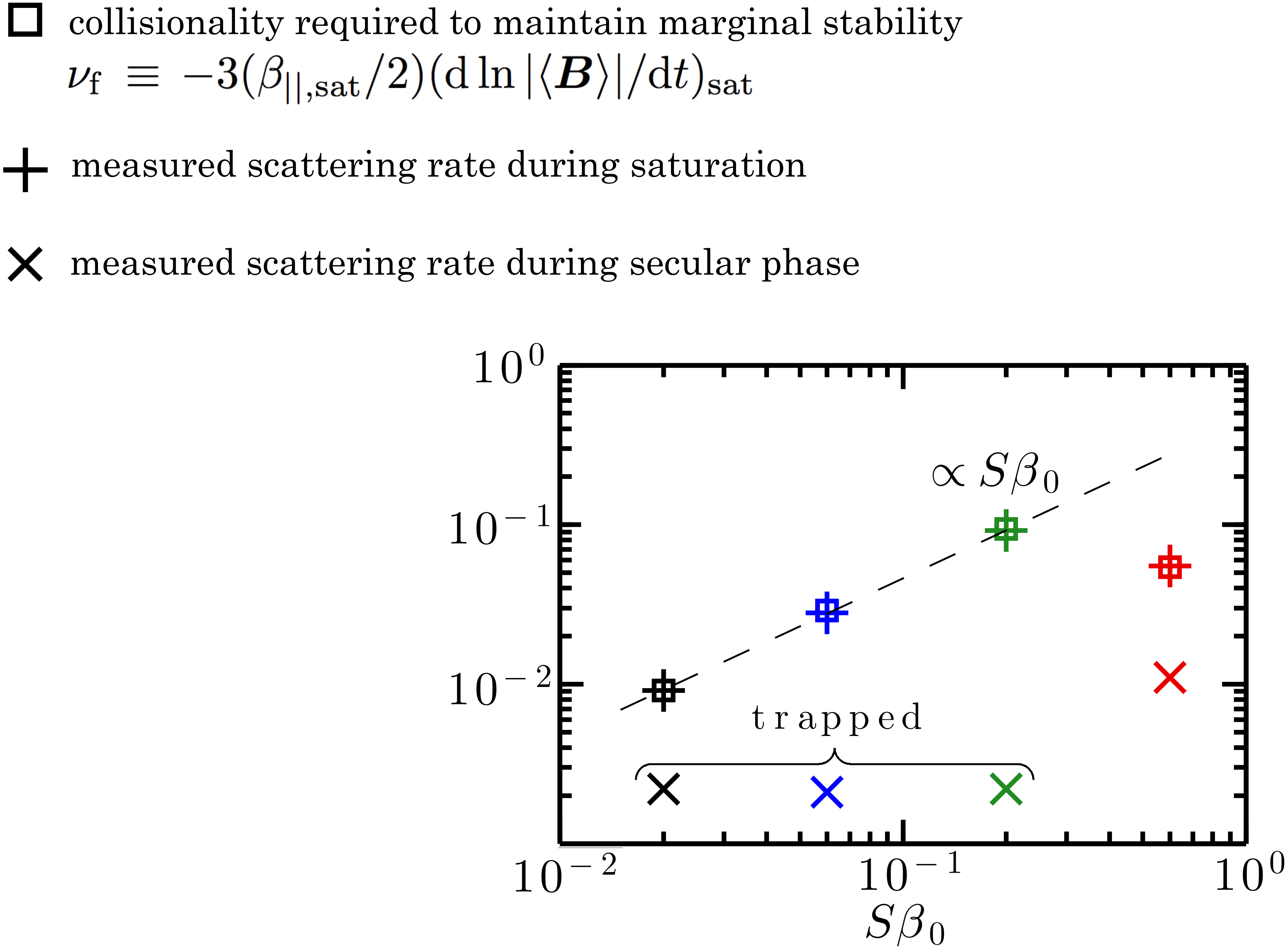
at saturation

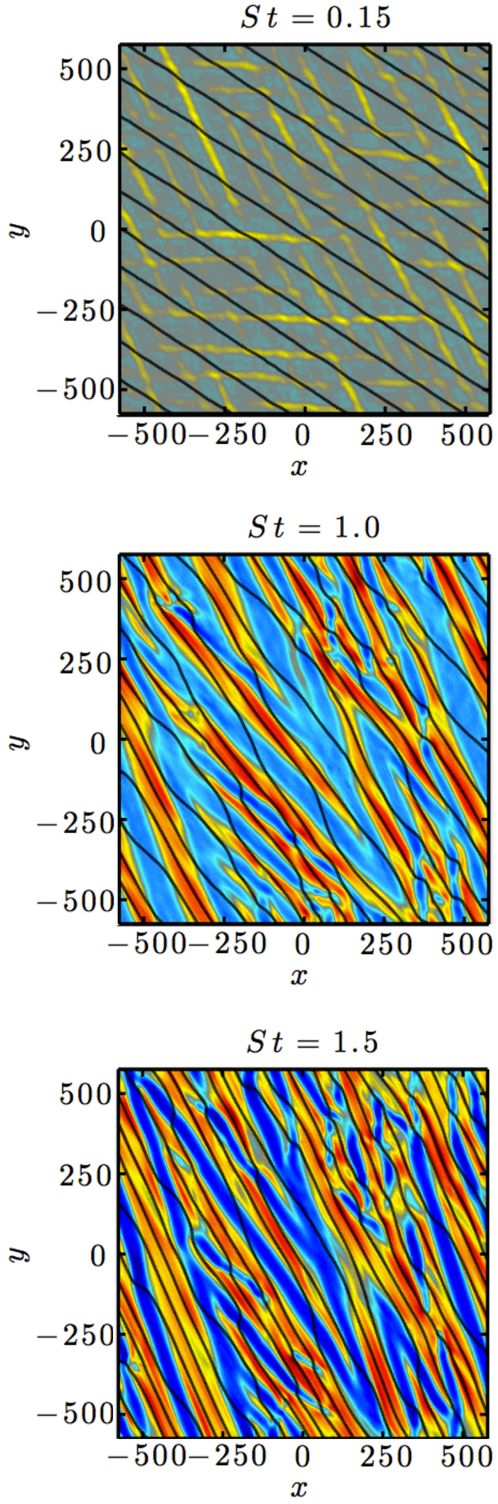
$$S \ll \Omega_{\mathrm{i}}$$

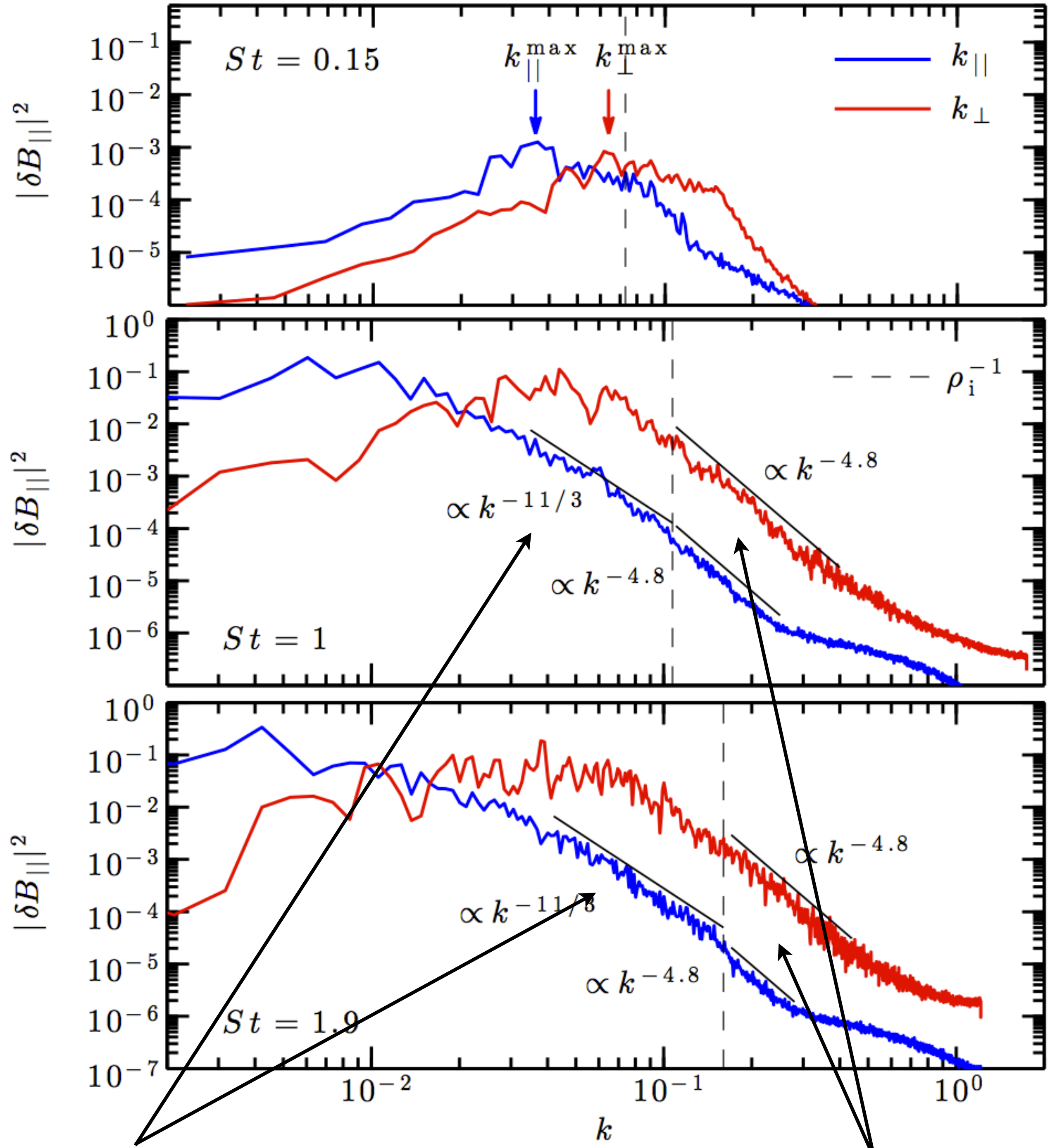
↓

universal behavior



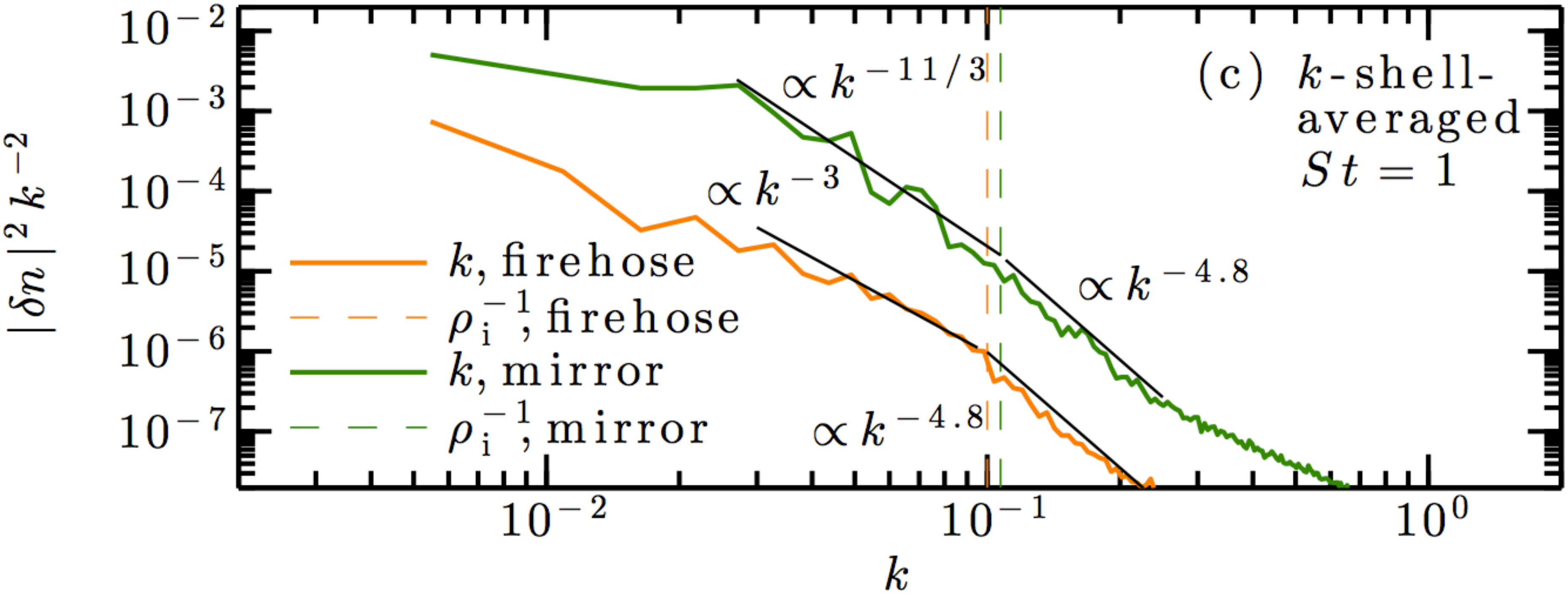

□ collisionality required to maintain marginal stability
$\nu_{\rm f} \equiv -3(\beta_{||,\rm sat}/2)(\mathrm{d}\ln|\langle \boldsymbol{B}\rangle|/\mathrm{d}t)_{\rm sat}$
+ measured scattering rate during saturation
× measured scattering rate during secular phase
10^{0}
10^{-1}
10^{-2}
$\propto S\beta_0$
trapped
10^{-2}
10^{-1}
10^{0}
$S\beta_0$





mirror turbulence

KAW cascade



$$\delta n \sim \beta^{-1}\,\delta B_{||}$$

energy-containing mode during secular phase has

$\gamma_{\rm peak} \sim \Lambda_{\rm m}^2 \sim 1/t$ $\qquad$ $k_{||,\rm peak} \sim \Lambda_{\rm m} \sim 1/t^{1/2}$

and we know

$$\sum_{k_{||}} |\delta B_{||,k_{||}}|^2 \sim (St)^{4/3}$$

Suppose $|\delta B_{||,k_{||}}|^2 \sim k_{||}^{-\alpha}$; then

$$\sum_{k_{||}} |\delta B_{||,k_{||}}|^2 \sim k_{||,\rm peak}^{1-\alpha} \sim t^{-(1-\alpha)/2}$$

$$\longrightarrow \alpha = 11/3$$

Summary

exponential growth, secular evolution, marginal stability...

- firehose: ...maintained (independent of S) by particle scattering;
- mirror: ...maintained by μ-conserving trapped particles trapped in regions of $\delta B_{||} < 0$;

and saturation...

- firehose: ...by scattering with $|\delta \boldsymbol{B}_{\perp}|^2 \propto S^{1/2} \ll 1$.
- mirror: ...by scattering with $(\delta B_{||})^2 \sim 1$.

power-law spectra for firehose and mirror (observable?)

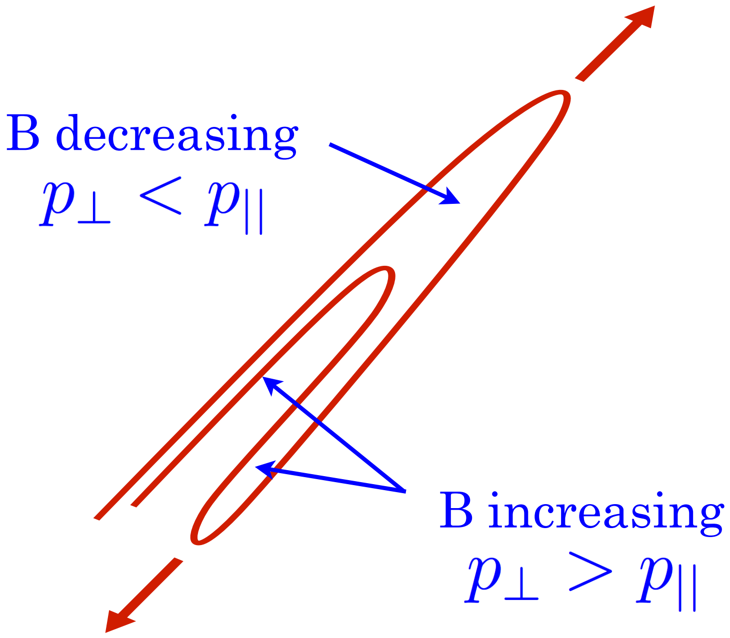
microscale energy injection drives KAW cascade (observed)

sub-grid model suggested for firehose; more difficult for mirror

Outlook: transport coefficients

effectively no magnetic tension in regions of decreasing B

maintained by $\lambda_{\mathrm{mfp}} \sim \rho_{\mathrm{i}}$



B decreasing
$p_\perp < p_{||}$

B increasing
$p_\perp > p_{||}$

parallel rate-of-strain regulated in regions of increasing B:
energy diverted into producing microscale mirrors

how do electrons interact with spectrum of ion-scale mirrors?

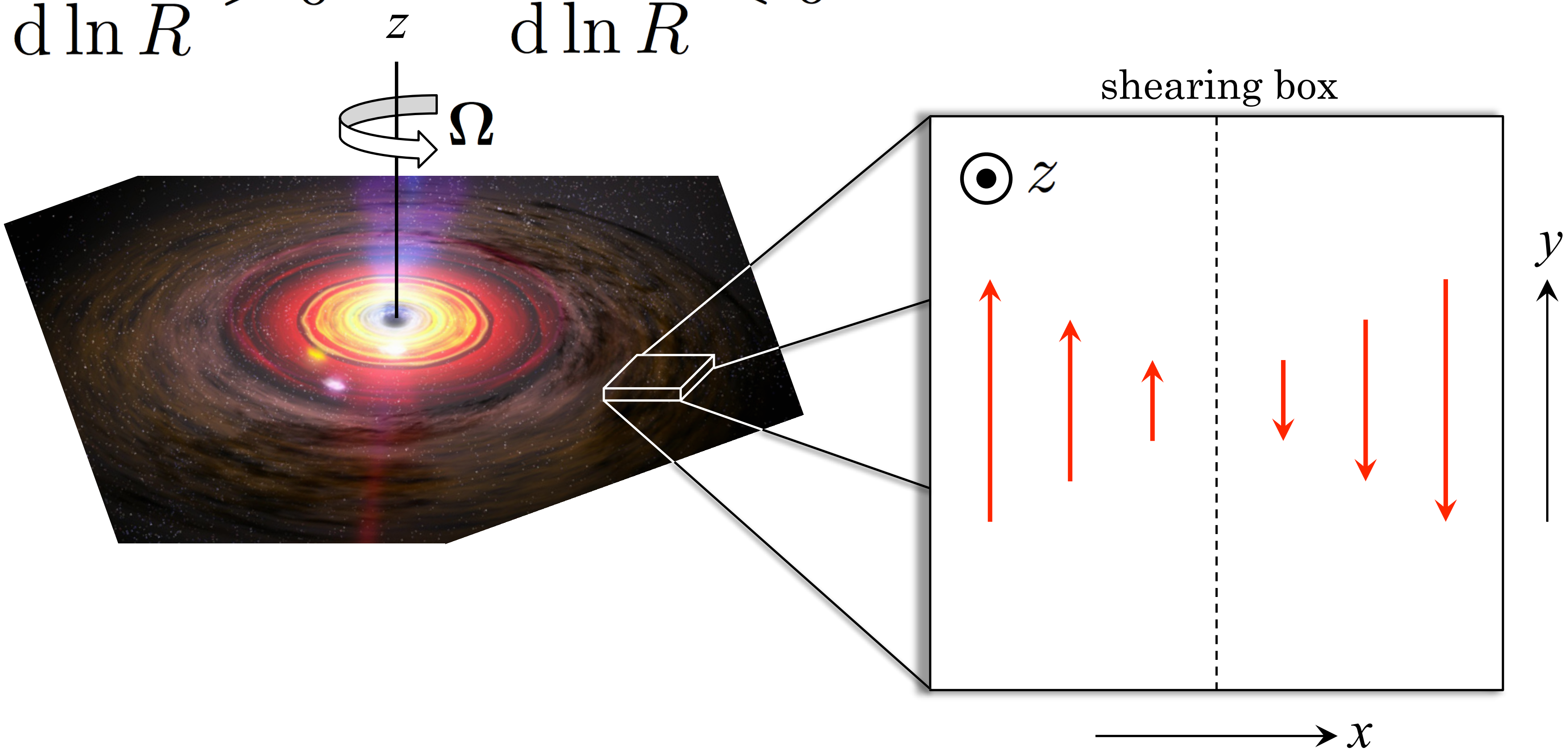
$$\kappa \sim v_{\mathrm{th,e}}\,\lambda_{\mathrm{mfp}} \quad \longrightarrow \quad \kappa \sim v_{\mathrm{th,e}}\,\ell_B \sim v_{\mathrm{th,e}}\,\rho_{\mathrm{i}}$$

Kinetic MRI slides

how do these microinstabilities affect mesoscale evolution?

look at accretion disk:

$$\frac{\mathrm{d}\ell^2}{\mathrm{d}\ln R} > 0 \qquad \frac{\mathrm{d}\Omega^2}{\mathrm{d}\ln R} < 0$$



what you get is the kinetic MRI

(Quataert, Dorland & Hammett 2002)

tidal

Coriolis

$$\ddot{x}\boxed{-2\Omega\dot{y}}+\boxed{\frac{\mathrm{d}\Omega^2}{\mathrm{d}\ln R}\,x}=\boxed{-K_x^2\,x}$$

$$\ddot{y}\boxed{+2\Omega\dot{x}}=\boxed{-K_y^2\,y}$$

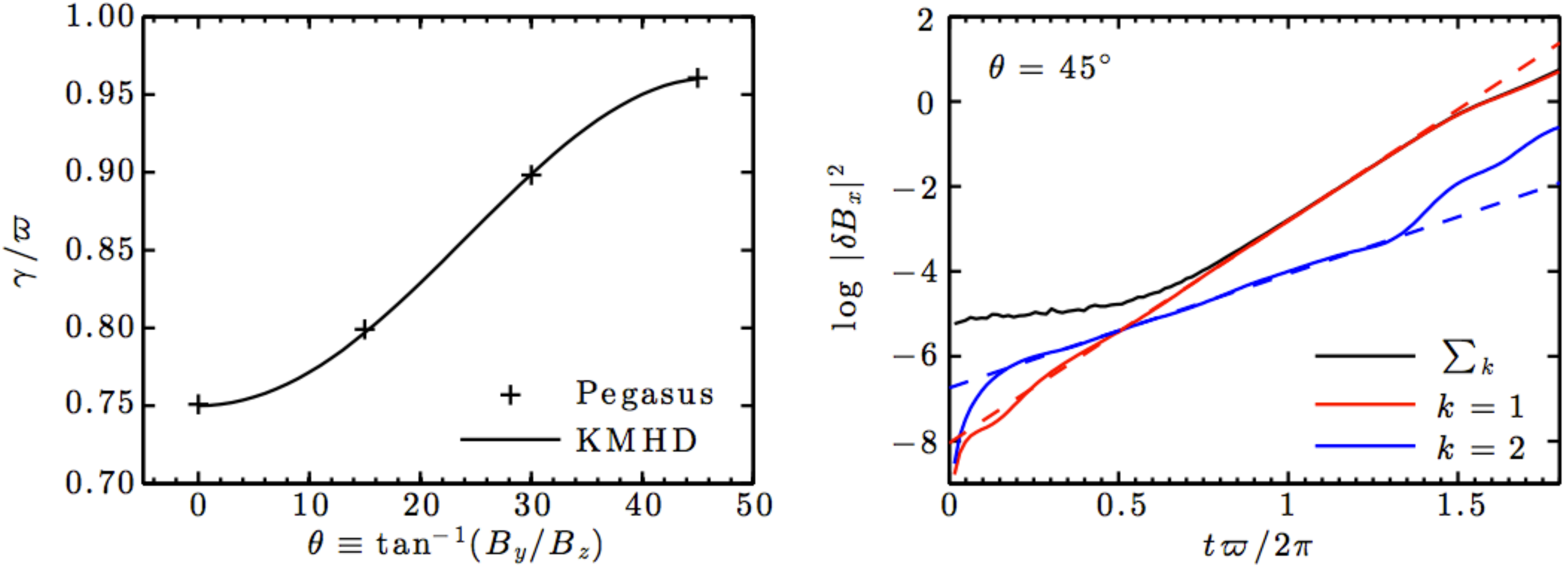
anisotropic spring

$$x, y \sim \exp\left(-\mathrm{i}\omega t\right)$$

$$\omega^2\left(\omega^2-\frac{1}{R^4}\frac{\mathrm{d}\ell^2}{\mathrm{d}\ln R}\right)=K_x^2\left(\omega^2-K_y^2\right)+K_y^2\left(\omega^2-\frac{\mathrm{d}\Omega^2}{\mathrm{d}\ln R}\right)$$

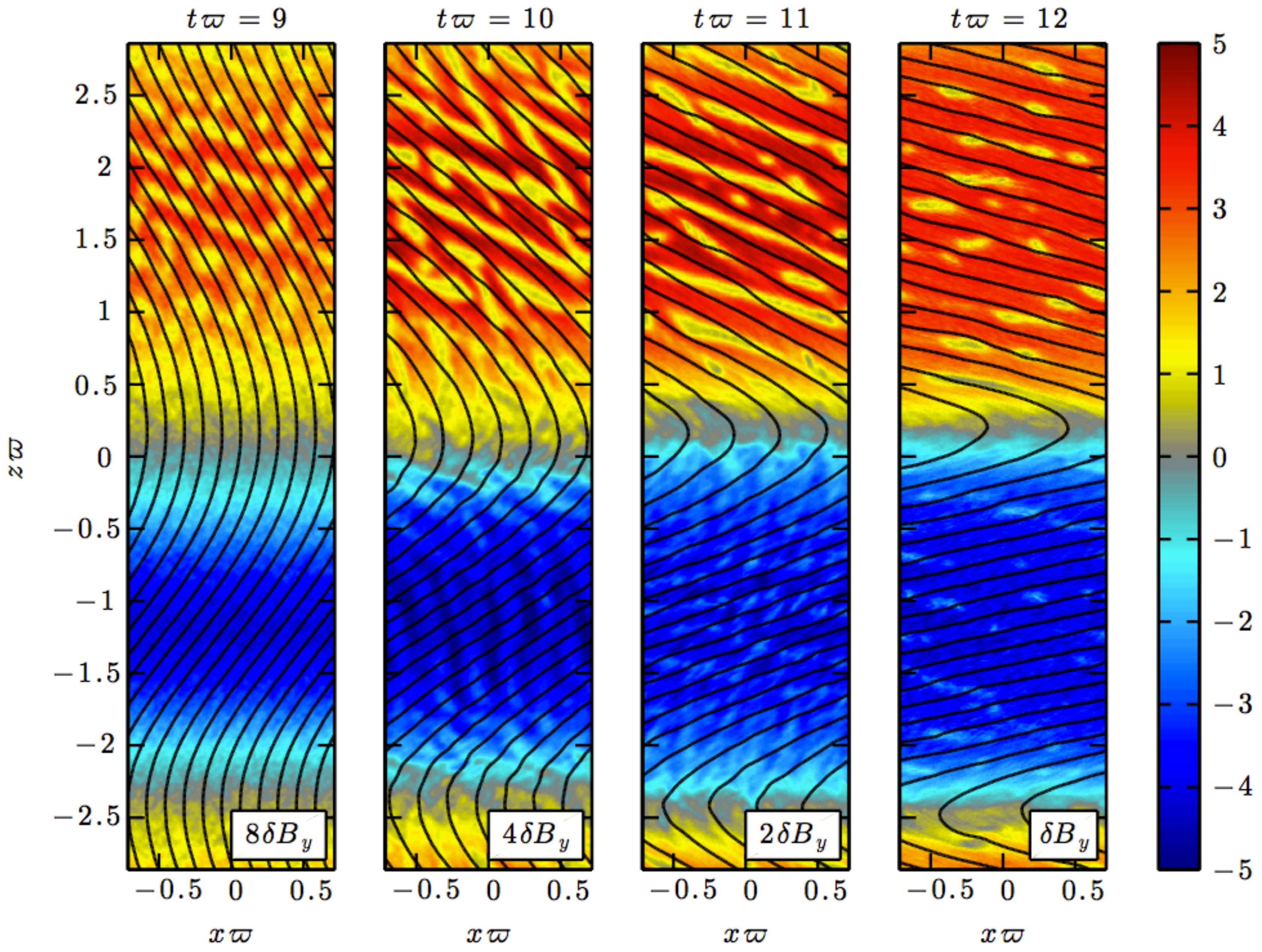
what you get is the kinetic MRI

(Quataert, Dorland & Hammett 2002)

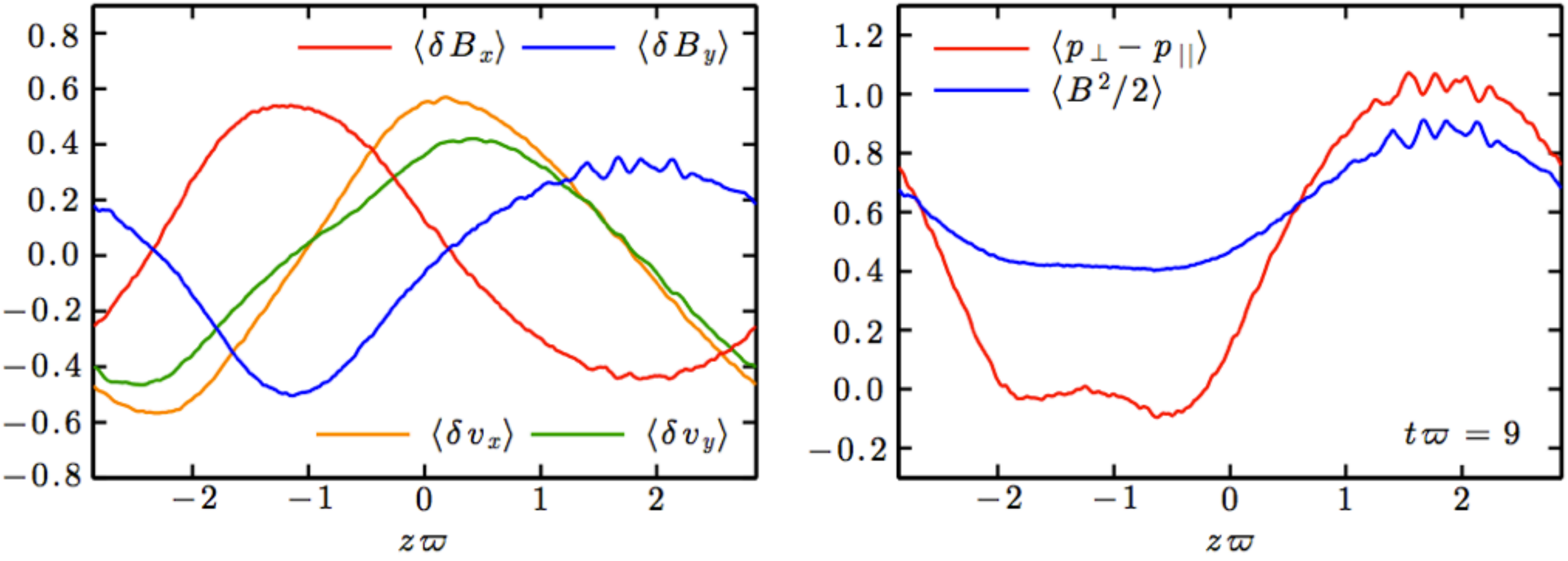


Kunz+ 2014

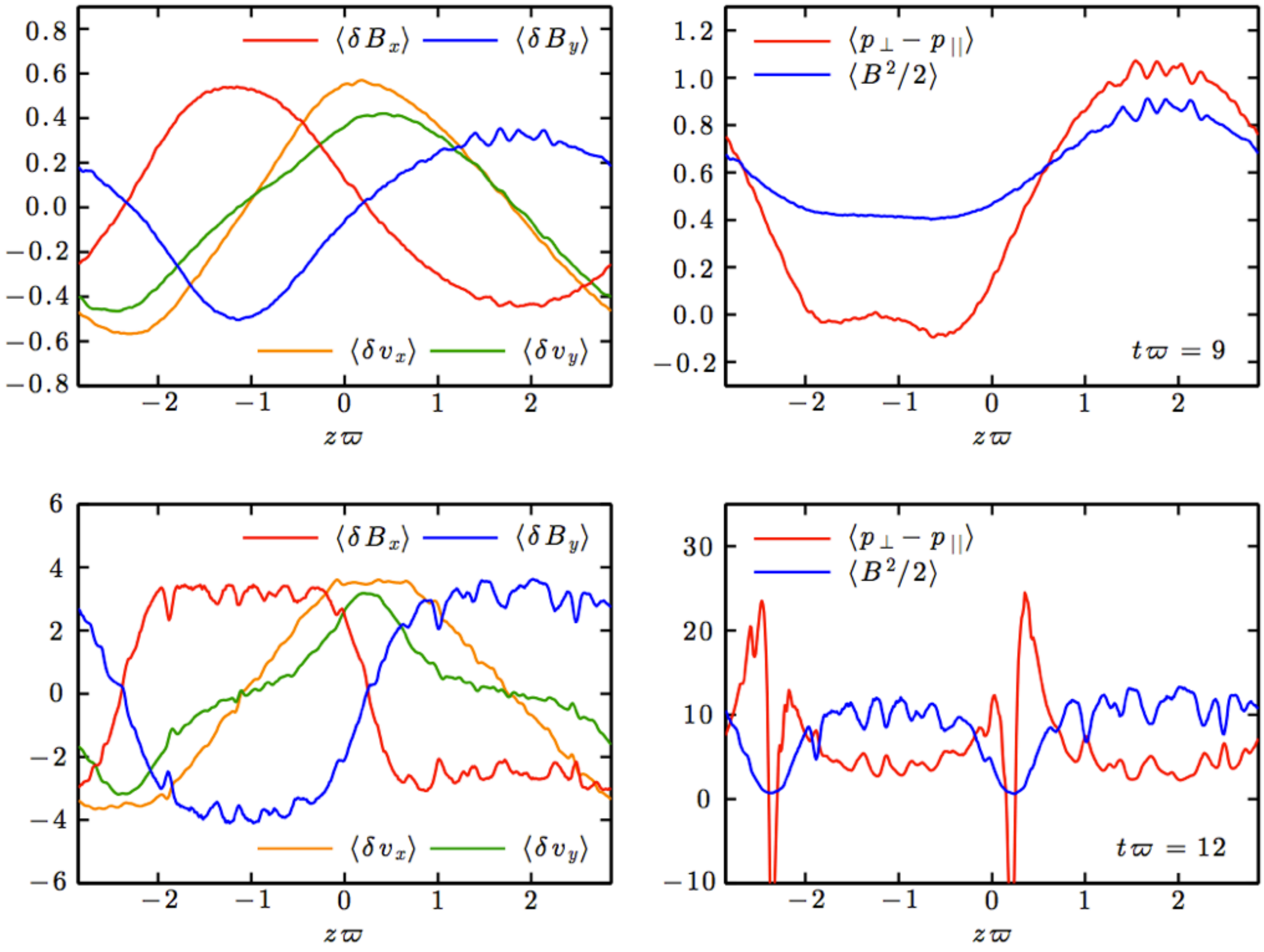
in axisymmetry with net flux



Kunz+ 2014

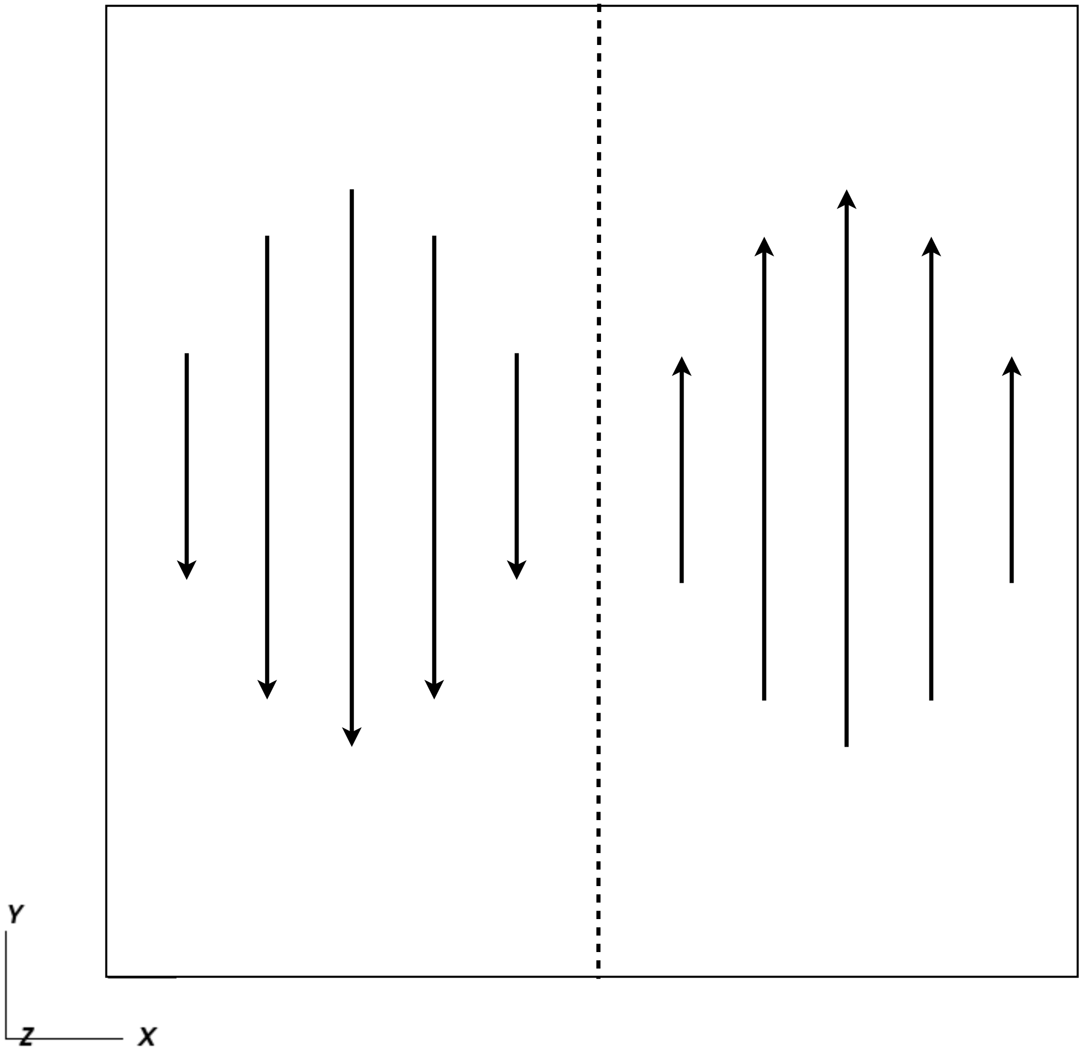

$\langle \delta B_x \rangle$
$\langle \delta B_y \rangle$
$\langle \delta v_x \rangle$
$\langle \delta v_y \rangle$
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−2
−1
0
1
2
$z\varpi$
$\langle p_\perp - p_{||} \rangle$
$\langle B^2/2 \rangle$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
$t\varpi = 9$
−2
−1
0
1
2
$z\varpi$

Kunz+ 2014


$\langle \delta B_x \rangle$
$\langle \delta B_y \rangle$
$\langle \delta v_x \rangle$
$\langle \delta v_y \rangle$
$\langle p_\perp - p_{||} \rangle$
$\langle B^2/2 \rangle$
$t\varpi = 9$
$t\varpi = 12$
$z\varpi$
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
1.2
1.0
6
4
2
0
−2
−4
−6
30
20
10
−10
−2
−1
1

Kunz+ 2014

in axisymmetry with zero net flux



$$\beta_{\mathrm{i}} = 25 \quad \Omega_{\mathrm{rot}} = 0.02$$

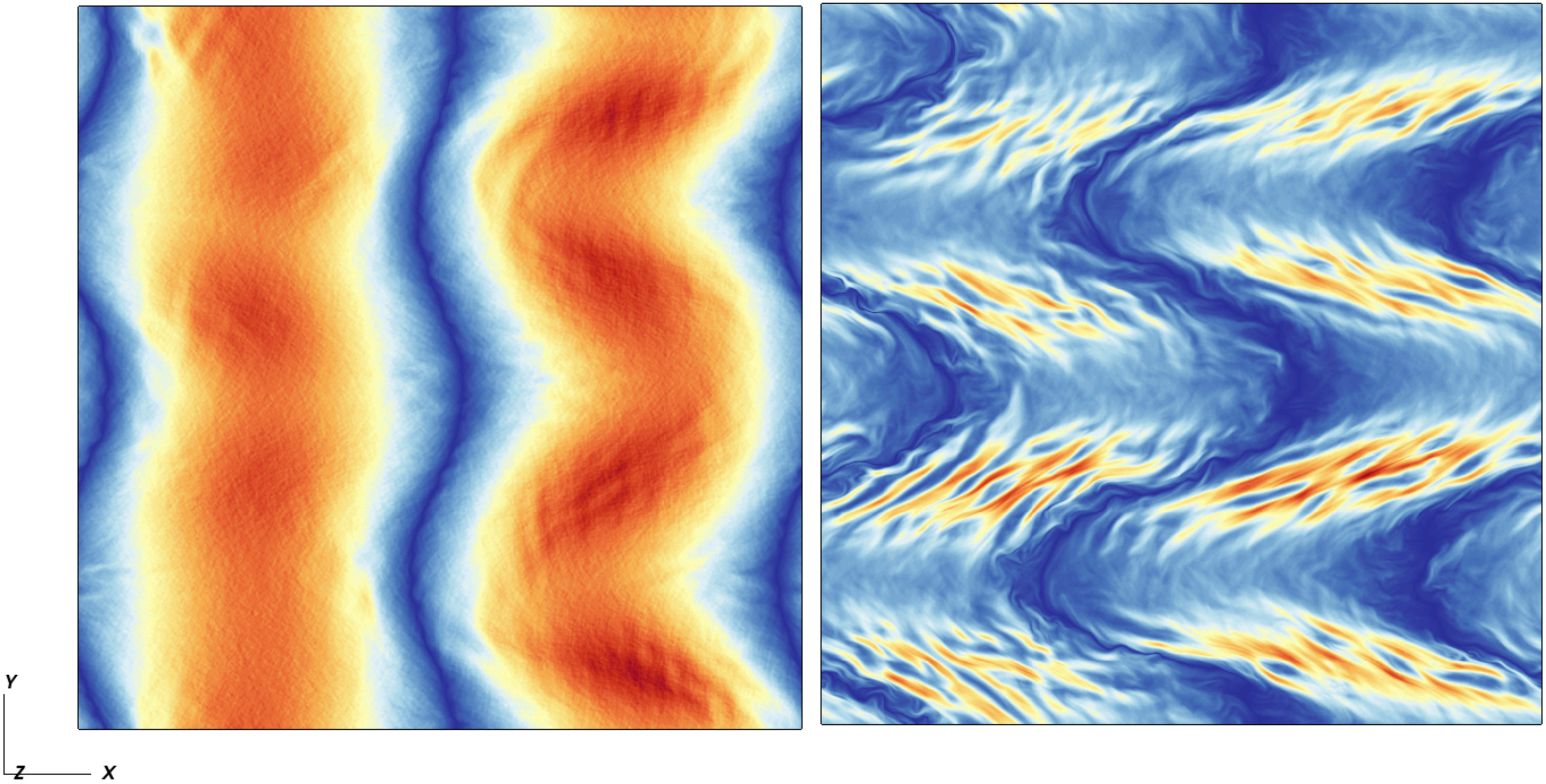
$$128\rho_{\mathrm{i}} \times 128\rho_{\mathrm{i}}$$

$$\mathrm{ppc} = 1024$$

$$1280 \times 1280 \text{ cells}$$

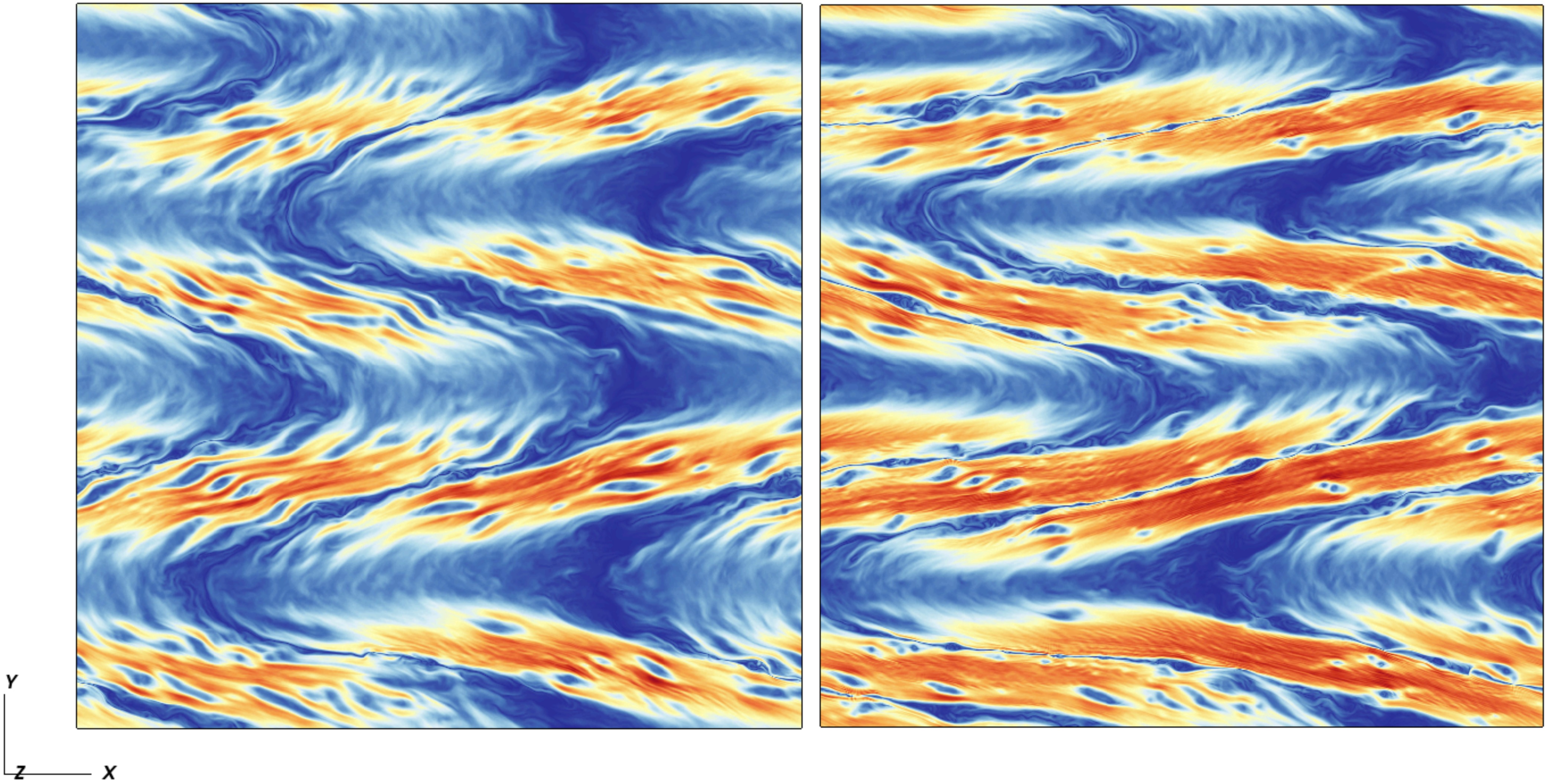
color: magnetic-field strength

time →

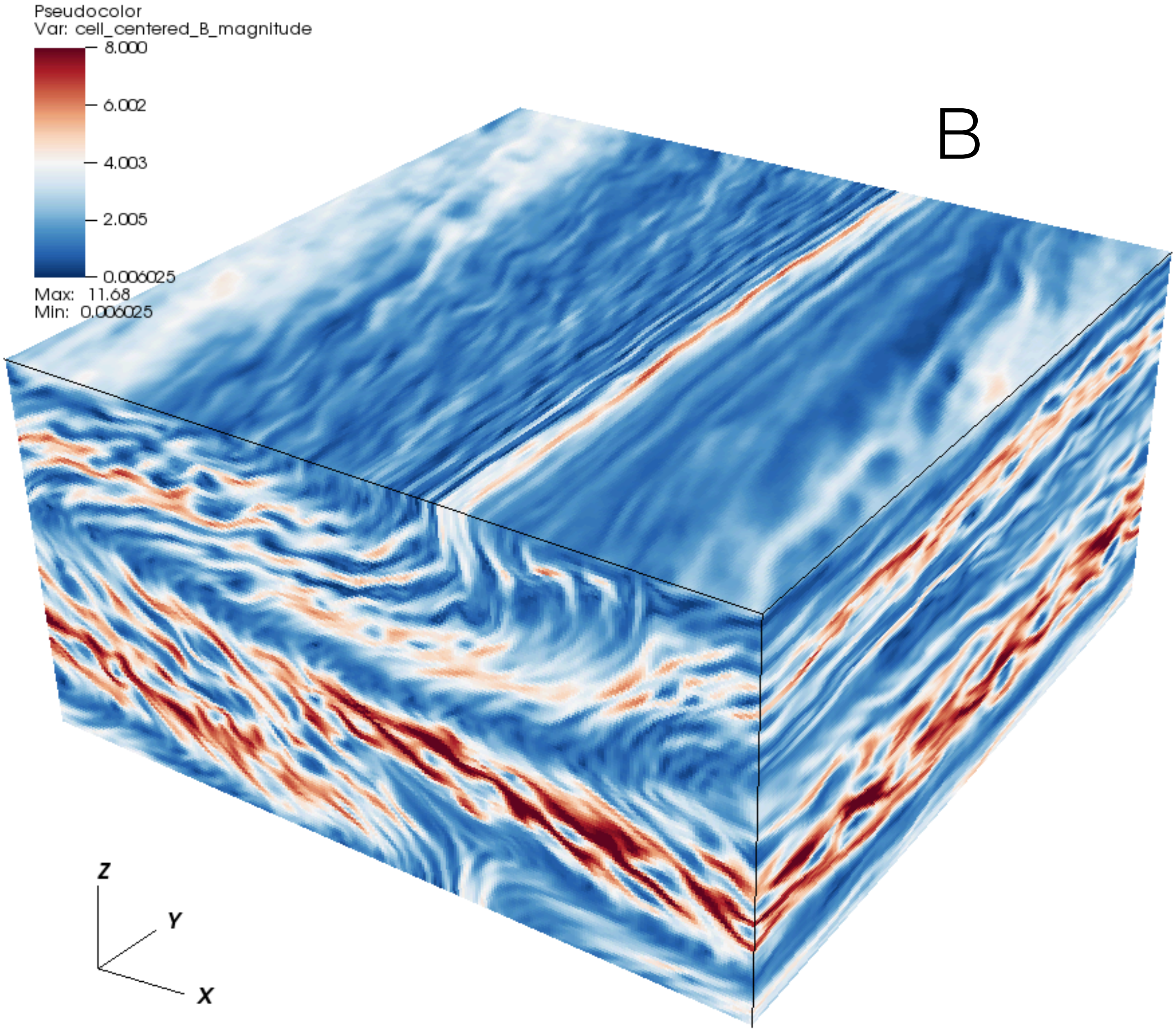


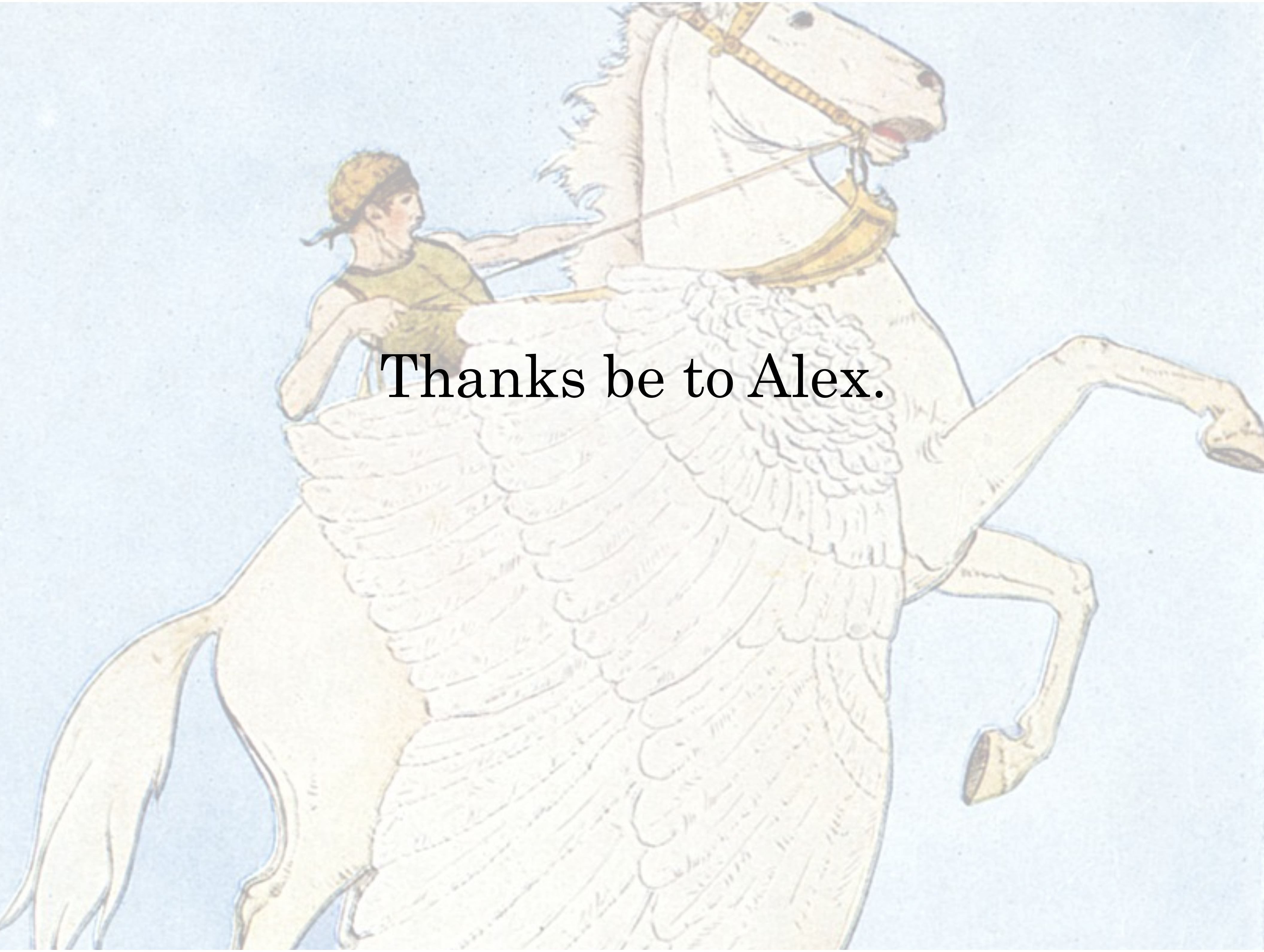
color: magnetic-field strength

time →



full 3d-3v being pursued...





Thanks be to Alex.