The Role of Alfven Wave Collisions in Governing the Dynamics of Plasma Turbulence

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NASA and White House Office of Science and Technology Policy
PECASE Award

National Science Foundation CAREER Award

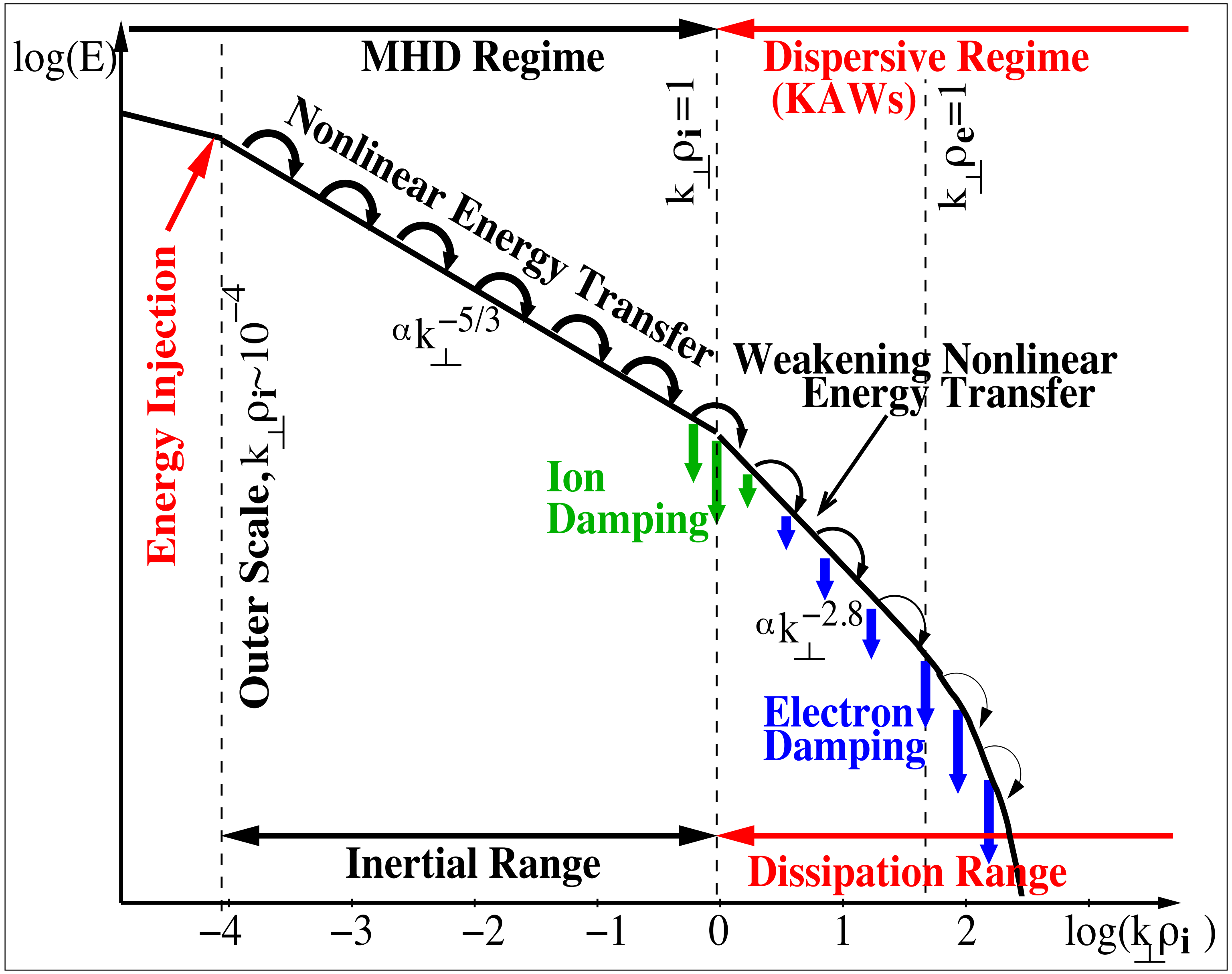
National Science Foundation XSEDE Program

Department of Energy INCITE Program

Outline

- Basic Picture of Plasma Turbulence
  - Key Questions about Plasma Turbulence
- Alfven Wave Collisions: The Fundamental Building Block of Plasma Turbulence
- Asymptotic Analytical Solution
  - The Physics of Turbulent Energy Transfer
- Gyrokinetic Numerical Validation
- Experimental Verification
- Alfven Wave Collisions Generate Current Sheets
- Conclusions

Basic Picture of Plasma Turbulence



Key Properties: 1) Nonlinear Energy Transfer

2) Collisionless Damping and Dissipation

3) Coherent Structures

Key Questions about Plasma Turbulence

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Important Questions about the Properties of Plasma Turbulence

Q1) What is the nature of Nonlinear Energy Transfer?

a) In the MHD Regime $k_\perp \rho_i \ll 1$

b) In the Dispersive Regime $k_\perp \rho_i \gtrsim 1$

Q2) What physical mechanism governs the Dissipation of the Turbulence?

Q3) How do Coherent Structures arise from and/or affect both the nonlinear energy transfer and dissipation?

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Nonlinear Energy Transfer in MHD Regime

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- The Incompressible MHD Equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho}\nabla(p + \frac{B^2}{8\pi}) + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi\rho} \qquad \nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times B) \qquad \nabla \cdot \mathbf{B} = 0$$

- The Incompressible MHD Equations in Elsasser Form:

Elsasser Variables

$$\mathbf{z}^{\pm} = \mathbf{u} \pm \delta \mathbf{B}/\sqrt{4\pi\rho}$$

Linear Term — Nonlinear Term

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} \mp \mathbf{v}_A \cdot \nabla \mathbf{z}^{\pm} + \mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} = -\nabla p$$

Linear Term: Requires parallel dimension

Nonlinear Term: Requires 2-D in perpendicular plane

MHD Plasma Turbulence Is Inherently 3D!

(Howes et al. 2011, Phys. Rev. Lett., **107**:035004, Howes 2013, ArXiv 1306.4589)

This qualitative property is expected to persist beyond the MHD Regime, even for kinetic turbulence in the Dispersive Regime!

Nonlinear Energy Transfer in MHD Regime

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Incompressible MHD Equations in symmetric Elsasser form:

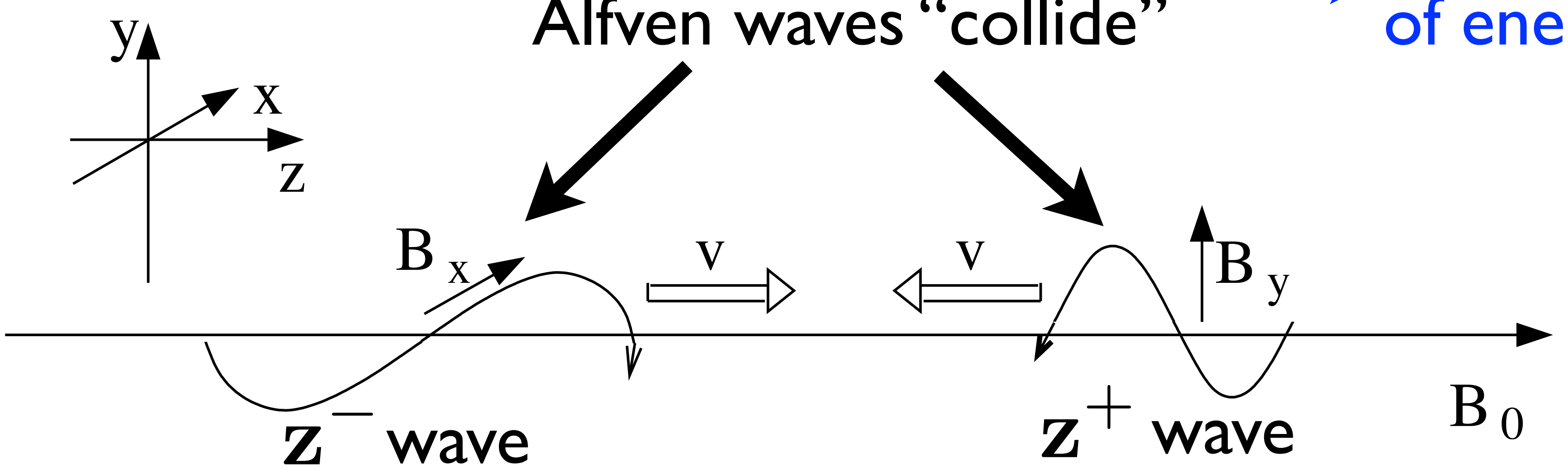
Linear Term →

$\mathbf{z}^+$	travels down $\mathbf{B}_0$ field
$\mathbf{z}^-$	travels up $\mathbf{B}_0$ field

$$\frac{\partial \mathbf{z}^\pm}{\partial t} \mp \mathbf{v}_A \cdot \nabla \mathbf{z}^\pm + \mathbf{z}^\mp \cdot \nabla \mathbf{z}^\pm = -\nabla p$$

Nonlinear Term → Responsible for "collisions" between oppositely propagating Alfven waves

Counterpropagating Alfven waves "collide" → Nonlinear term leads to transfer of energy to higher wavenumber



Alfven wave collisions are the fundamental building block of plasma turbulence!

(Kraichnan, 1965)

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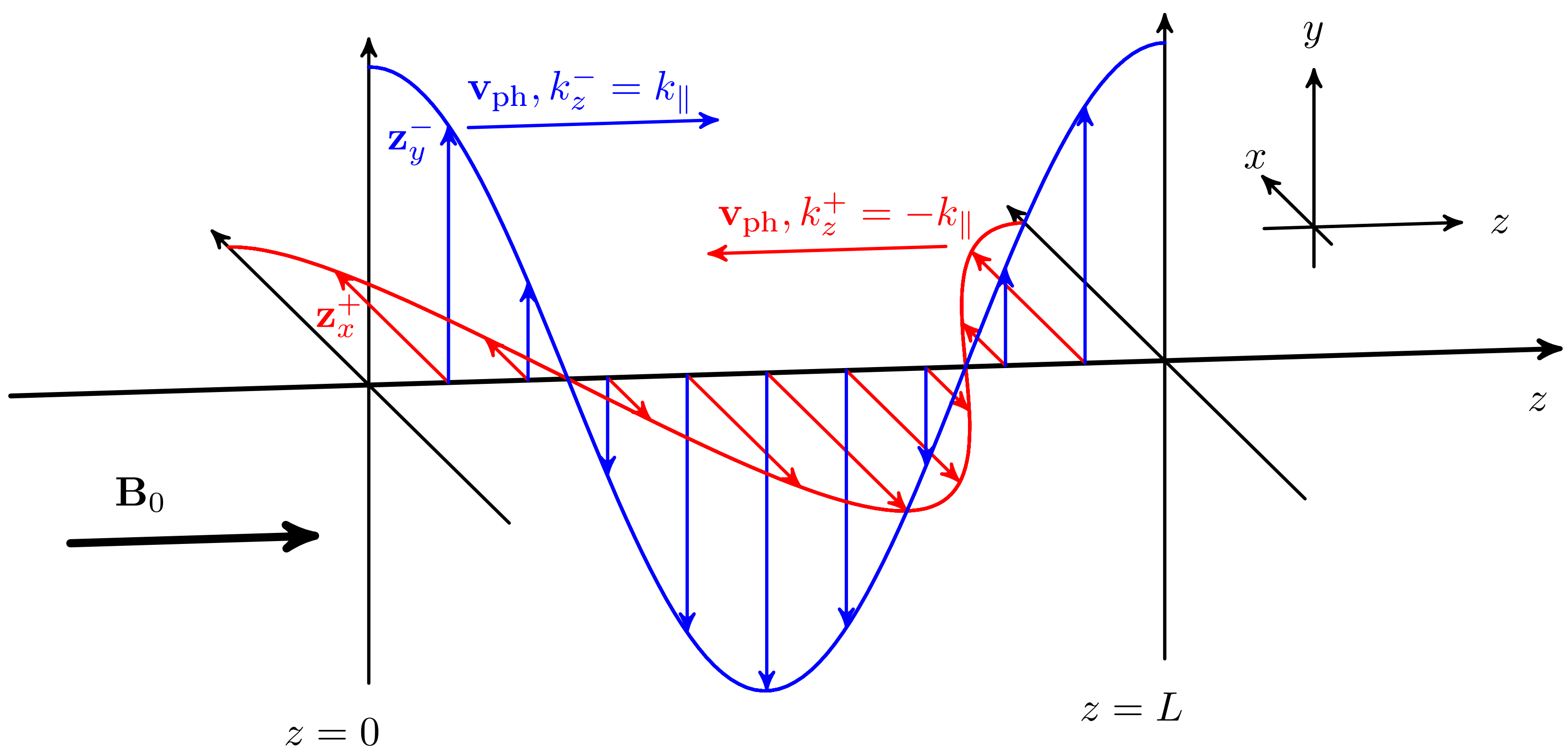
Setup for Alfven Wave Collision Problem

We can solve asymptotically for the nonlinear evolution of Alfven wave collisions in the weakly nonlinear limit. (Howes & Nielson 2013, Phys. Plasmas **20**:072302)

Initial Conditions:

Two perpendicularly polarized, counterpropagating plane Alfven waves



$$\mathbf{z}^+ = z_+ \cos(k_\perp x - k_\parallel z - \omega_0 t)\hat{\mathbf{y}}$$
$$\mathbf{z}^- = z_- \cos(k_\perp y + k_\parallel z - \omega_0 t)\hat{\mathbf{x}}$$

Reduced MHD in Elsasser Potential Form

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} \mp \mathbf{v}_A \cdot \nabla \mathbf{z}^{\pm} + \mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm} = -\nabla p$$

- Incompressible MHD includes both Alfven and pseudo-Alfven wave dynamics, but in the anisotropic limit, $k_{\parallel} \ll k_{\perp}$, the Alfven dynamics decouples from pseudo-Alfven wave dynamics

Elsasser Potentials $\mathbf{z}^{\pm} = \hat{z} \times \nabla_{\perp} \zeta^{\pm}$

- The Elsasser Potential Equations for Reduced MHD:

$$\frac{\partial \nabla_{\perp}^2 \zeta^+}{\partial t} - v_A \frac{\partial \zeta^+}{\partial z} = -\frac{1}{2}\left[\{\zeta^+, \nabla_{\perp}^2 \zeta^-\} + \{\zeta^-, \nabla_{\perp}^2 \zeta^+\} - \nabla_{\perp}^2 \{\zeta^+, \zeta^-\}\right]$$

$$\frac{\partial \nabla_{\perp}^2 \zeta^-}{\partial t} + v_A \frac{\partial \zeta^-}{\partial z} = -\frac{1}{2}\left[\{\zeta^+, \nabla_{\perp}^2 \zeta^-\} + \{\zeta^-, \nabla_{\perp}^2 \zeta^+\} + \nabla_{\perp}^2 \{\zeta^+, \zeta^-\}\right]$$

(Schekochihin et al. 2009)

Linear Terms

Nonlinear Terms

where the Poisson bracket is defined by

$$\{f, g\} = \hat{z} \cdot (\nabla_{\perp} f \times \nabla_{\perp} g)$$

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Method of Characteristics

We can make use of Method of Characteristics for linear terms

$$\phi_+ = z + v_A t \qquad z = \frac{\phi_+ + \phi_-}{2}$$

$$\phi_- = z - v_A t \qquad t = \frac{\phi_+ - \phi_-}{2v_A}$$

Reduces the Partial Differential Equation to an Ordinary Differential Equation:

$$\frac{\partial \nabla_\perp^2 \zeta^+}{\partial t} - v_A \frac{\partial \nabla_\perp^2 \zeta^+}{\partial z} = -2v_A \frac{\partial \nabla_\perp^2 \zeta^+}{\partial \phi_-}$$

$$\frac{\partial \nabla_\perp^2 \zeta^-}{\partial t} + v_A \frac{\partial \nabla_\perp^2 \zeta^-}{\partial z} = 2v_A \frac{\partial \nabla_\perp^2 \zeta^-}{\partial \phi_+}$$

Asymptotic Weak Turbulence Solution

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$$\mathbf{z}^+ = z_+ \cos(k_\perp x - k_\parallel z - \omega_0 t)\hat{\mathbf{y}}$$
$$\mathbf{z}^- = z_- \cos(k_\perp y + k_\parallel z - \omega_0 t)\hat{\mathbf{x}}$$

We can solve for the evolution of this system asymptotically in the weak turbulence limit

$$\frac{z_+}{v_A} \sim \frac{z_-}{v_A} \sim \epsilon \ll 1$$

-Expand and solve equations order by order

$$\zeta^+ = \zeta_0^+ + \epsilon\zeta_1^+ + \epsilon^2\zeta_2^+ + \epsilon^3\zeta_3^+ + \cdots$$

$$\zeta^- = \zeta_0^- + \epsilon\zeta_1^- + \epsilon^2\zeta_2^- + \epsilon^3\zeta_3^- + \cdots$$

Equations at Each Order

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At first order $\mathcal{O}(\epsilon)$, $\quad \frac{\partial \nabla_\perp^2 \zeta_1^+}{\partial \phi_-} = 0 \qquad \frac{\partial \nabla_\perp^2 \zeta_1^-}{\partial \phi_+} = 0$

At second order $\mathcal{O}(\epsilon^2)$,

$$\frac{\partial \nabla_\perp^2 \zeta_2^+}{\partial \phi_-} = \frac{1}{4v_A}\left[\{\zeta_1^+, \nabla_\perp^2 \zeta_1^-\} + \{\zeta_1^-, \nabla_\perp^2 \zeta_1^+\} - \nabla_\perp^2 \{\zeta_1^+, \zeta_1^-\}\right]$$

$$\frac{\partial \nabla_\perp^2 \zeta_2^-}{\partial \phi_+} = -\frac{1}{4v_A}\left[\{\zeta_1^+, \nabla_\perp^2 \zeta_1^-\} + \{\zeta_1^-, \nabla_\perp^2 \zeta_1^+\} + \nabla_\perp^2 \{\zeta_1^+, \zeta_1^-\}\right]$$

At third order $\mathcal{O}(\epsilon^3)$,

$$\begin{aligned}\frac{\partial \nabla_\perp^2 \zeta_3^+}{\partial \phi_-} &= \frac{1}{4v_A}\left[\{\zeta_1^+, \nabla_\perp^2 \zeta_2^-\} + \{\zeta_2^+, \nabla_\perp^2 \zeta_1^-\} + \{\zeta_1^-, \nabla_\perp^2 \zeta_2^+\}\right. \\ &\quad \left. + \{\zeta_2^-, \nabla_\perp^2 \zeta_1^+\} - \nabla_\perp^2 \{\zeta_1^+, \zeta_2^-\} - \nabla_\perp^2 \{\zeta_2^+, \zeta_1^-\}\right]\end{aligned}$$

$$\begin{aligned}\frac{\partial \nabla_\perp^2 \zeta_3^-}{\partial \phi_+} &= -\frac{1}{4v_A}\left[\{\zeta_1^+, \nabla_\perp^2 \zeta_2^-\} + \{\zeta_2^+, \nabla_\perp^2 \zeta_1^-\} + \{\zeta_1^-, \nabla_\perp^2 \zeta_2^+\}\right. \\ &\quad \left. + \{\zeta_2^-, \nabla_\perp^2 \zeta_1^+\} + \nabla_\perp^2 \{\zeta_1^+, \zeta_2^-\} + \nabla_\perp^2 \{\zeta_2^+, \zeta_1^-\}\right]\end{aligned}$$

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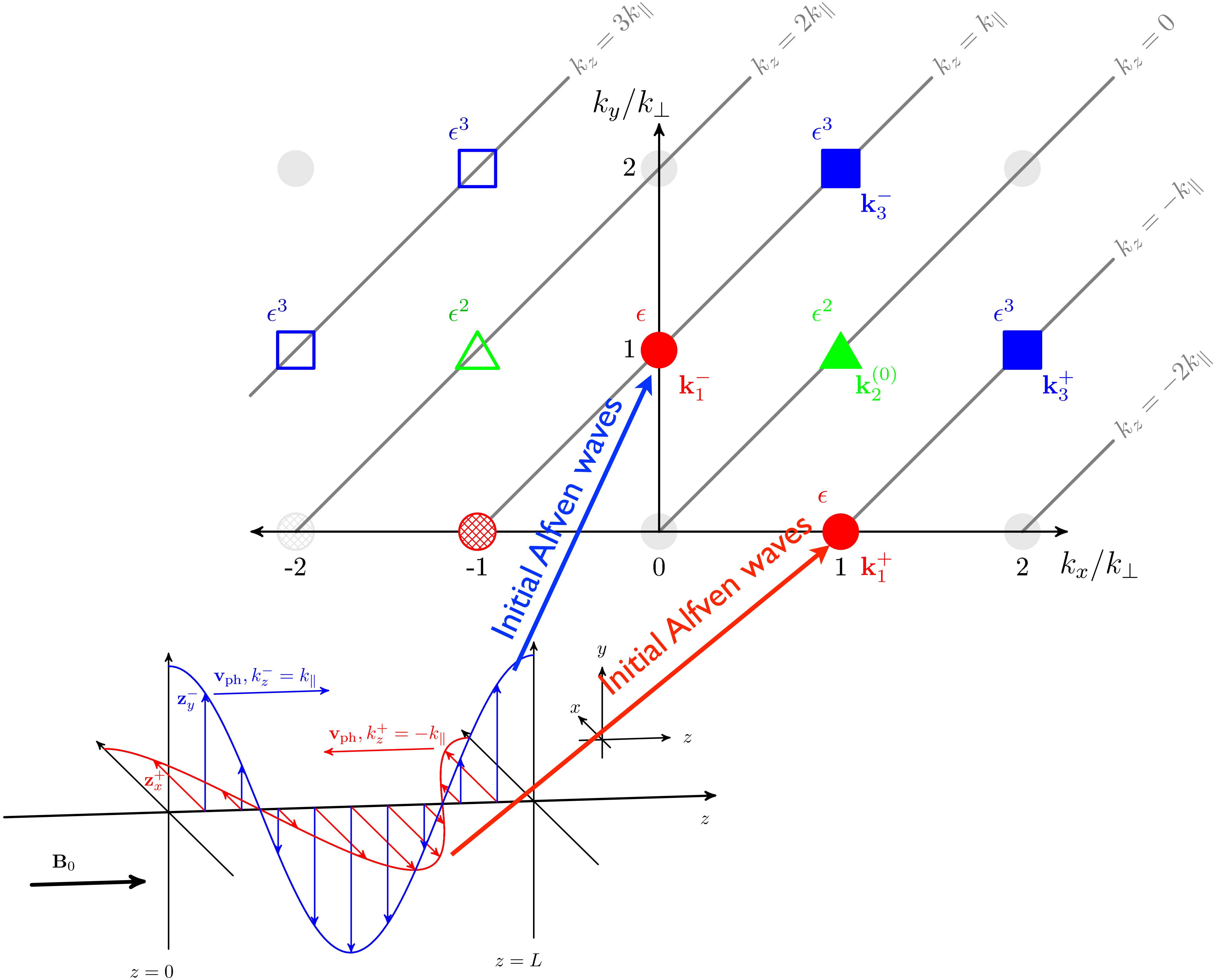
Nonlinear Energy Transfer by Alfven Wave Collisions



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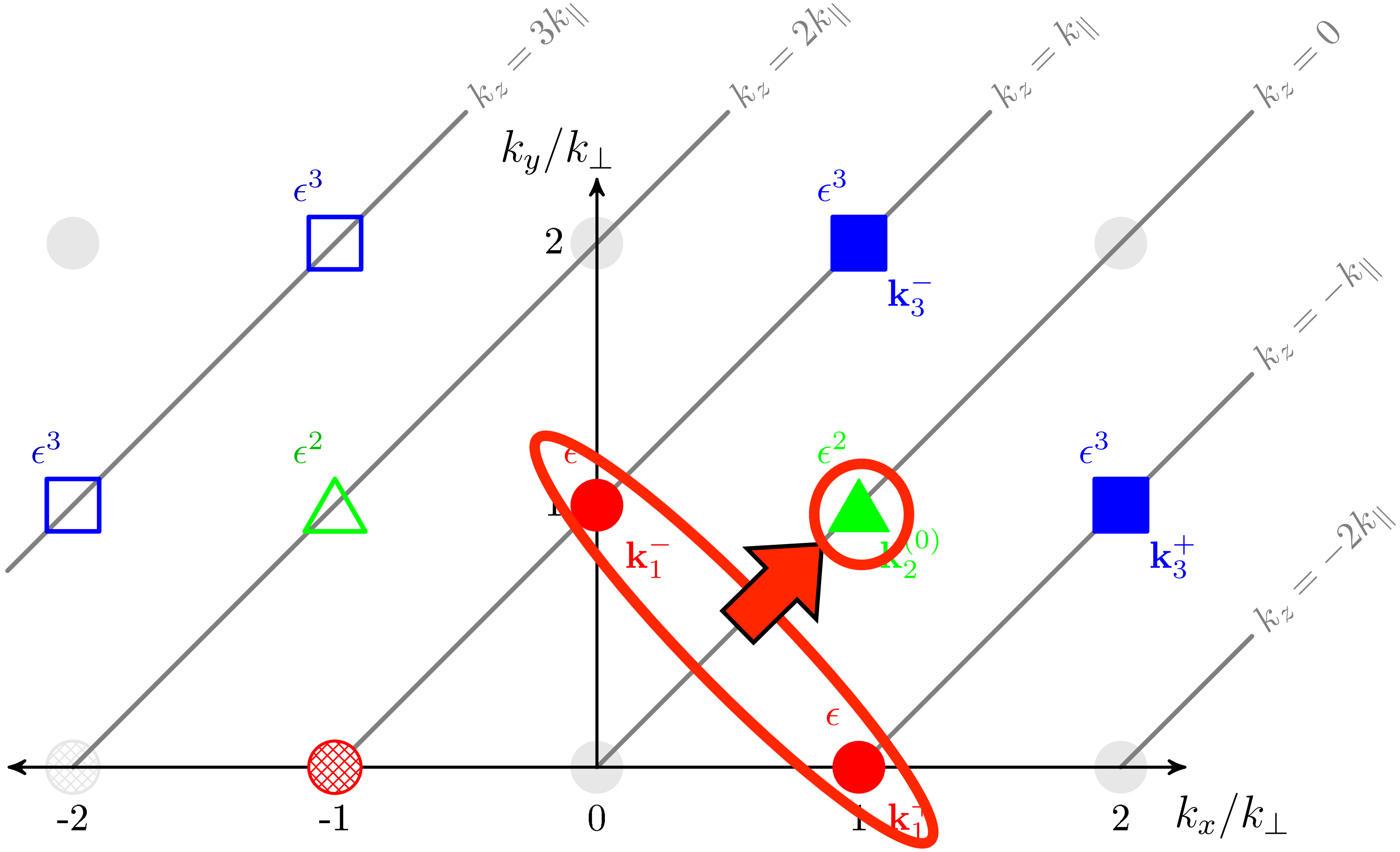
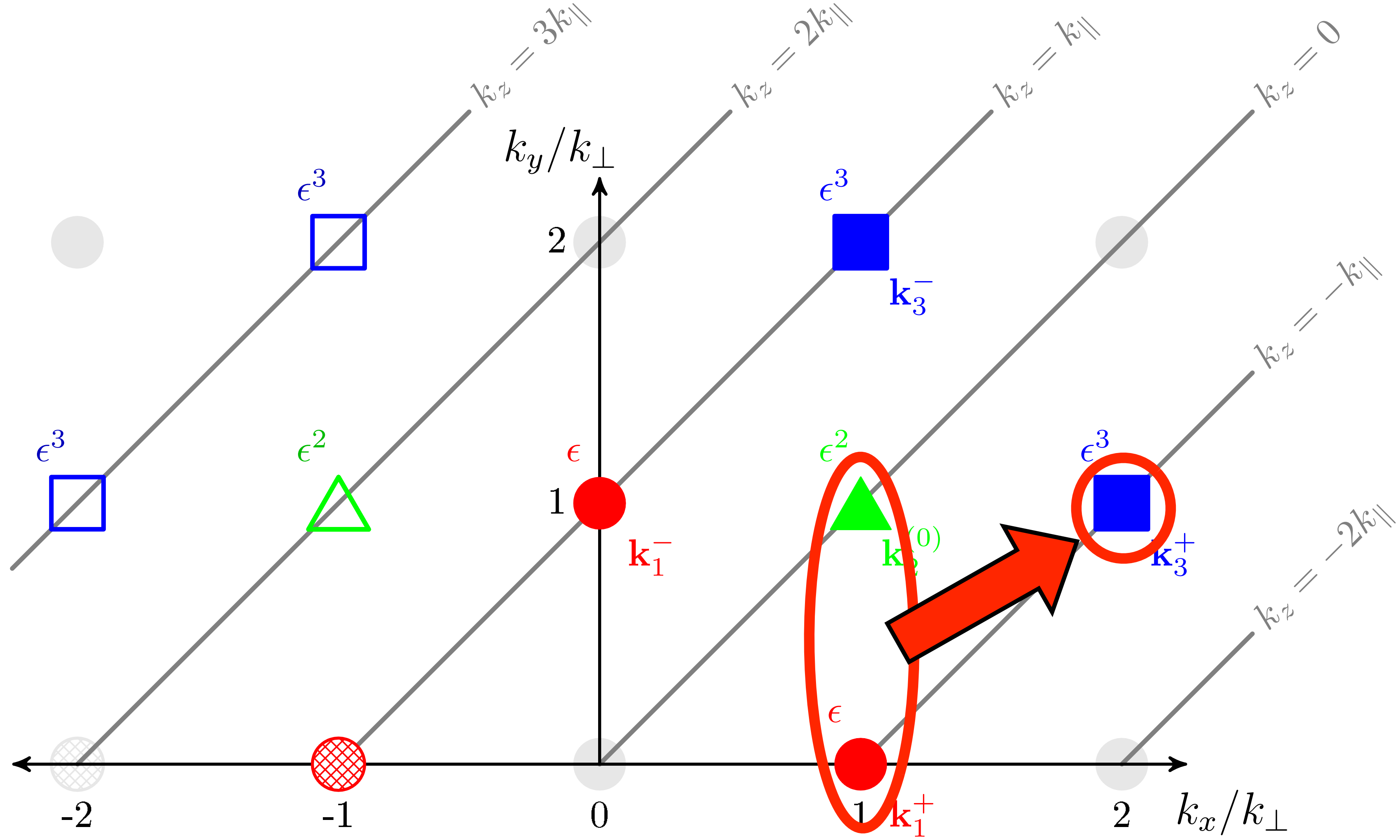


- Two Primary Alfven waves interact

  $$\mathcal{O}(\epsilon^2): \quad \mathbf{k}_1^+ + \mathbf{k}_1^- = \mathbf{k}_2^{(0)}$$

- Generate a Secondary, inherently nonlinear mode
  - Purely magnetic
  - $k_\parallel = 0$
  - $\omega = 2\omega_0$

Nonlinear Energy Transfer by Alfven Wave Collisions



- Primary Alfven wave and Secondary mode interact

$$\mathcal{O}(\epsilon^3): \quad \mathbf{k}_1^\pm + \mathbf{k}_2^{(0)} = \mathbf{k}_3^\pm$$

- Generate a Tertiary Alfven wave
  - Secular transfer of energy
  - Increased $k_\perp$, Same $k_\parallel$

This is the turbulent transfer of energy to small scales!

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Gyrokinetic Simulation of Alfven Wave Collisions

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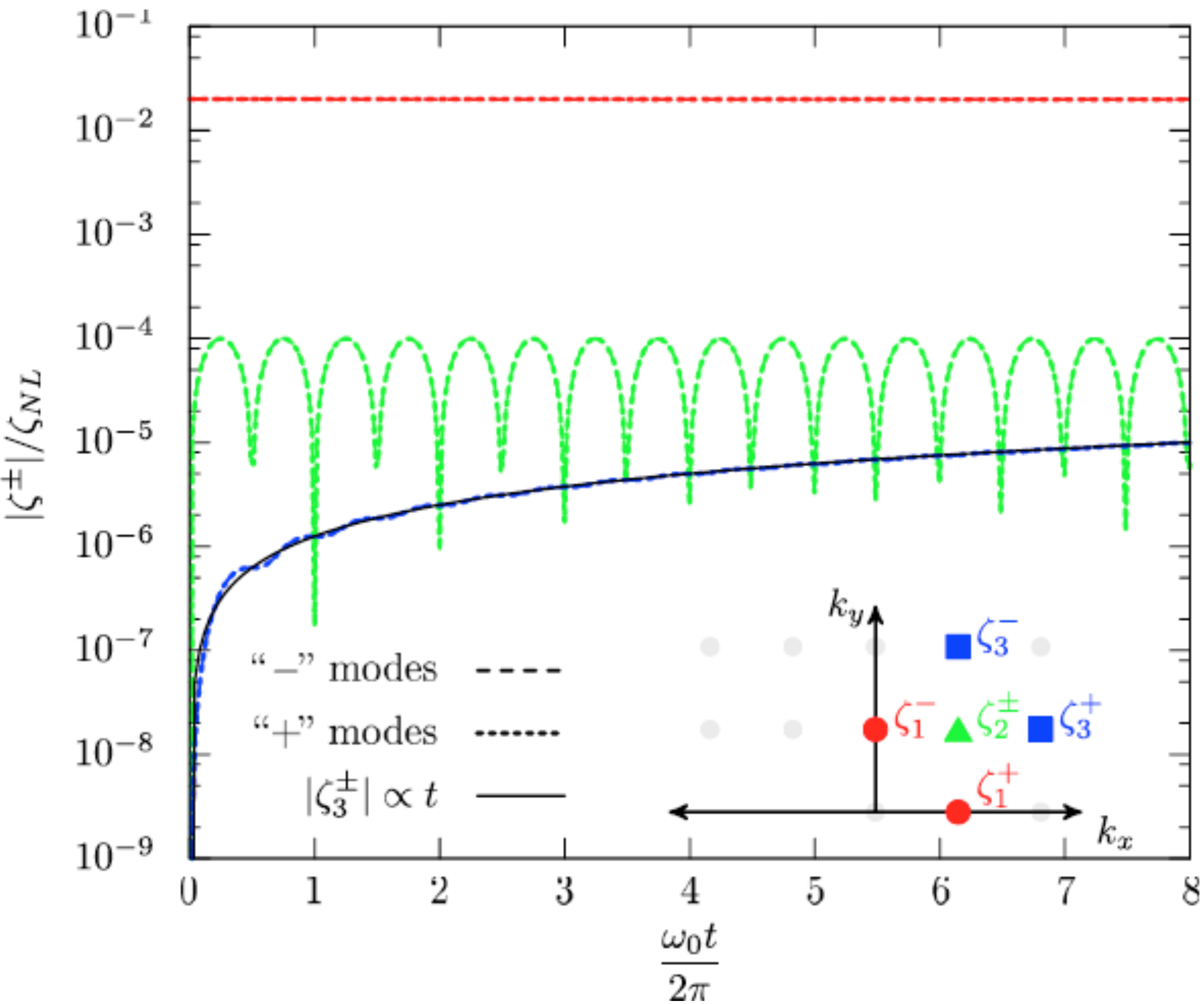


(Nielson, Howes, & Dorland 2013, Phys. Plasmas **20**:072303)

- Generates secular transfer of energy from

$\zeta_1^{\pm}$ Parent Alfven Wave

to

$\zeta_3^{\pm}$ Daughter Alfven Wave

via the nonlinearly generated $\zeta_2^{\pm}$ mode.

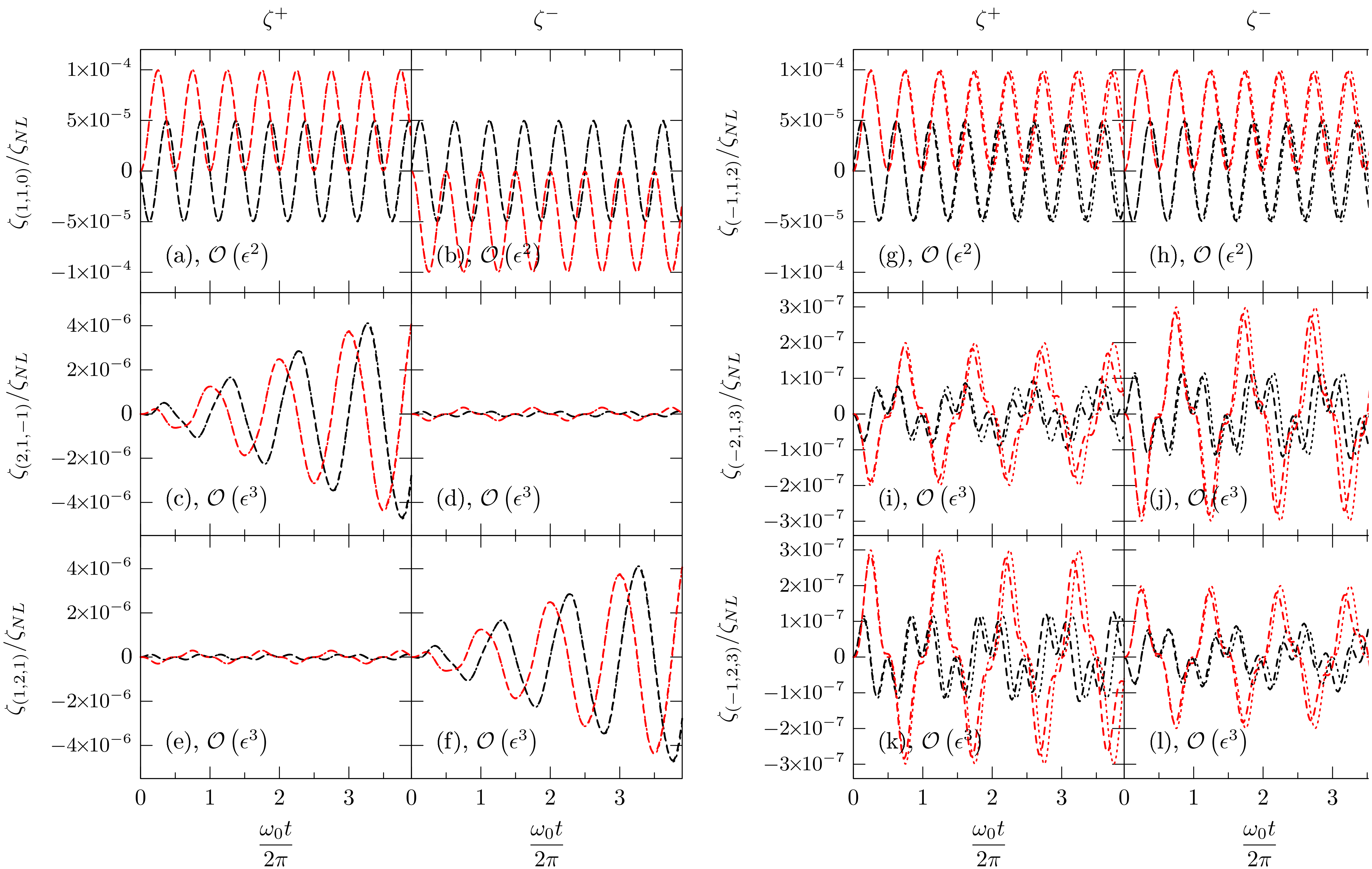
Numerical Simulation of Alfven Wave Collisions

Analytical solutions of incompressible MHD Alfven wave collisions

have been verified by

nonlinear gyrokinetic simulations using AstroGK

(Nielson, Howes, & Dorland 2013, Phys. Plasmas **20**:072303)



Implications of Numerical Validation

- Validation of incompressible MHD solution with gyrokinetic code:

  - Properties of incompressible MHD solution persist even under the weakly collisional plasma conditions relevant to space and astrophysical plasmas

  - Dynamical behavior of the turbulent cascade, in particular the nonlinear energy transfer, is adequately described by the simplified framework of incompressible MHD

Fluid nature of the nonlinear energy transfer!

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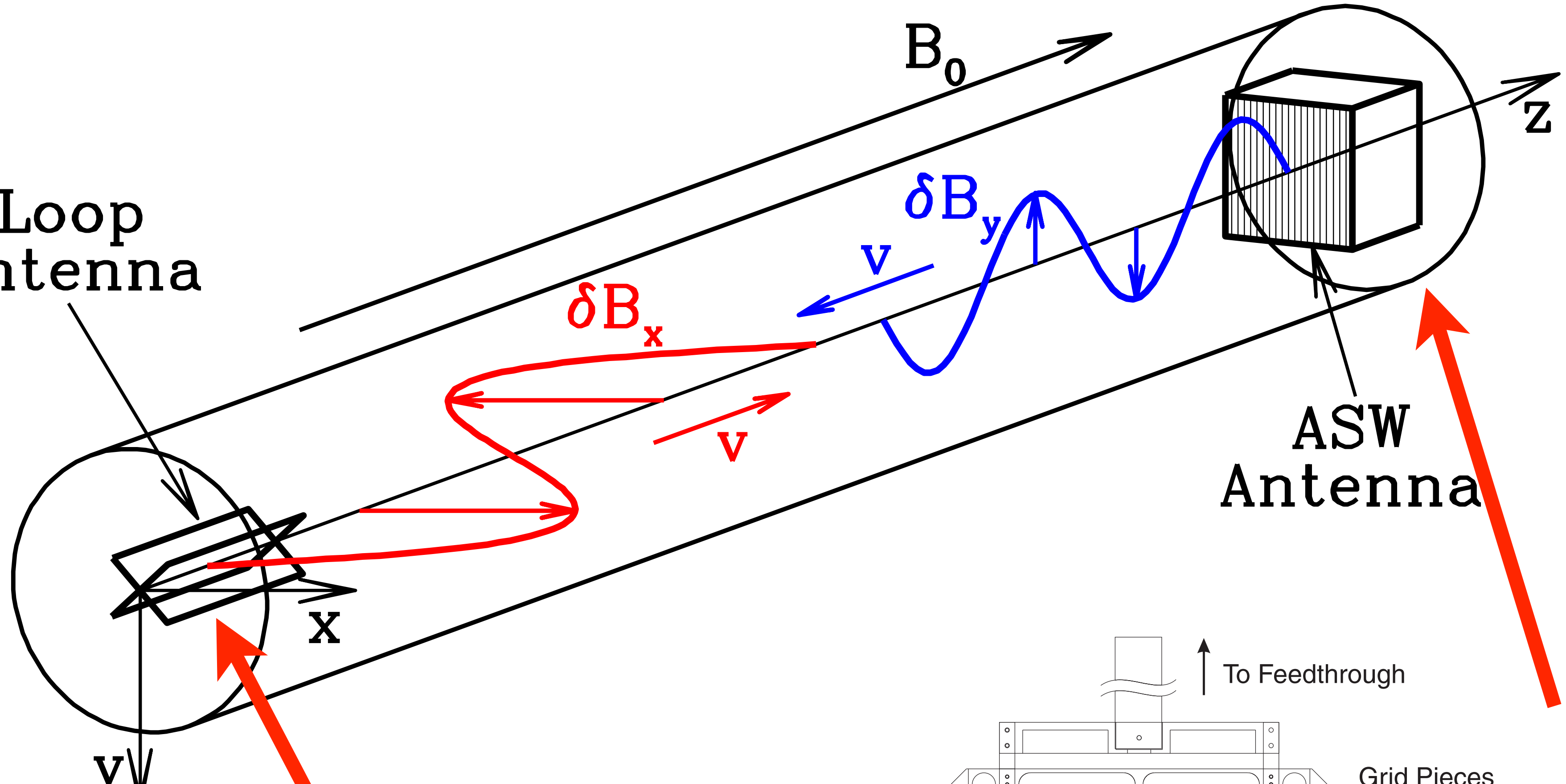
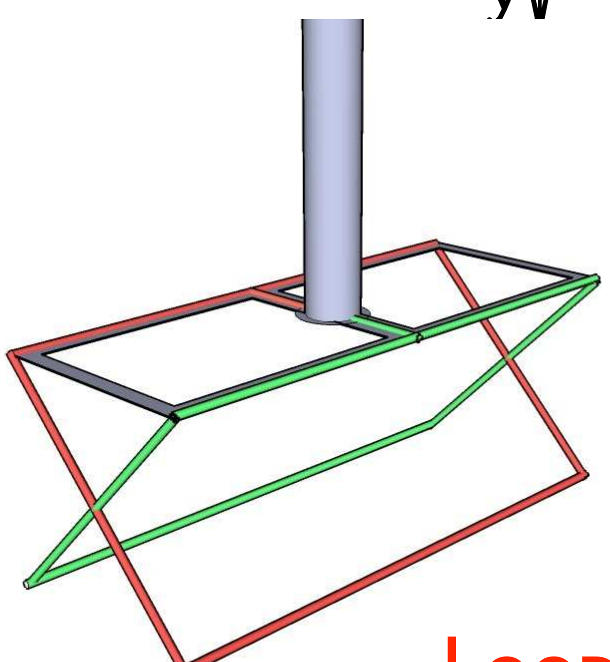
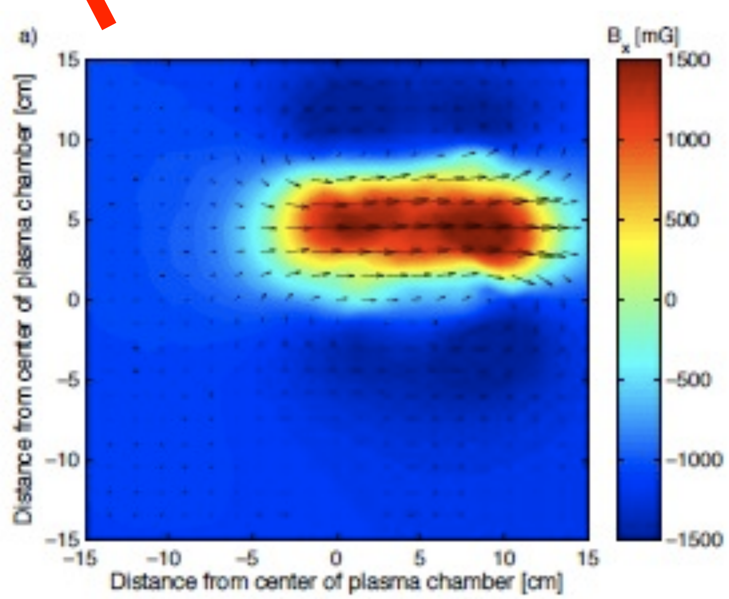
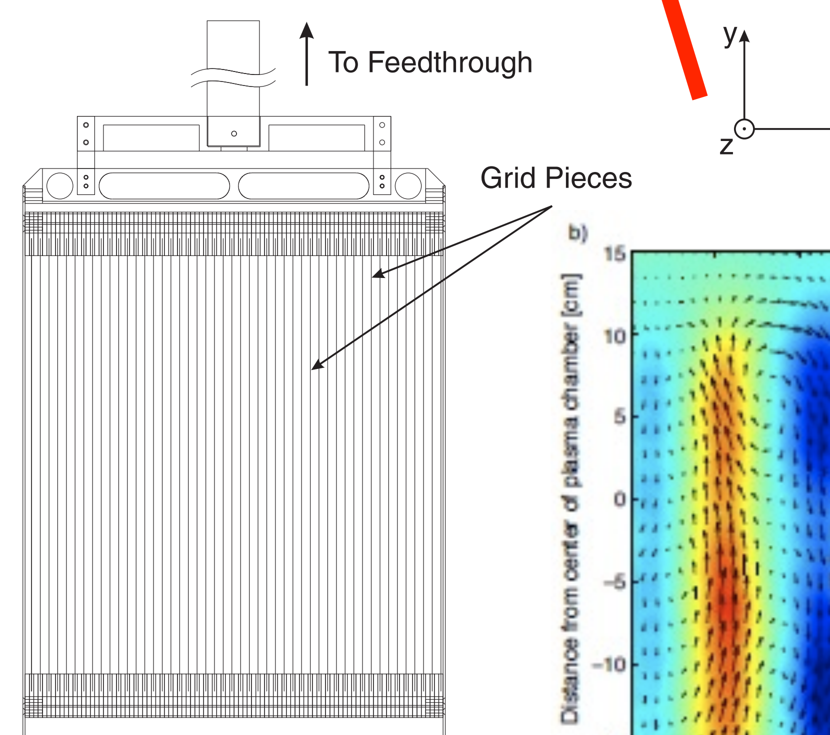
Experimental Verification of Alfven Wave Collisions

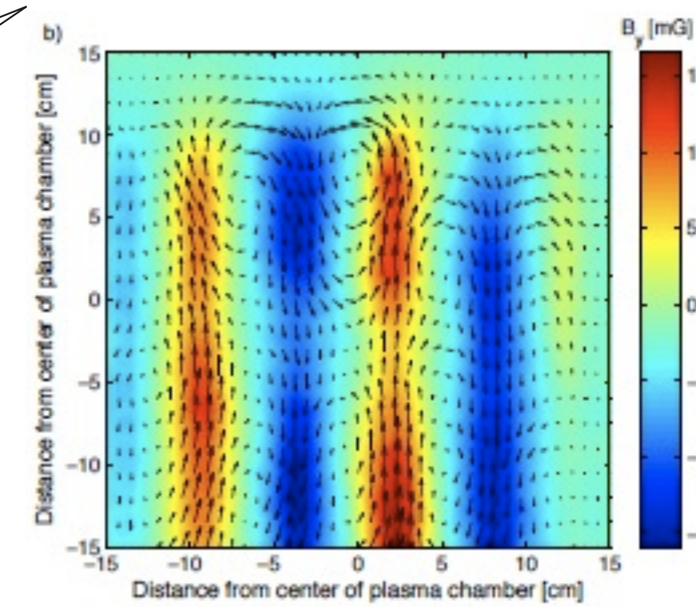
Basic Experiment of Alfven Wave Collisions on the LAPD











Loop Antenna
(Carter, UCLA)

ASW Antenna
(Kletzing & Skiff, Univ. Iowa)

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Experimental Verification of Alfven Wave Collisions

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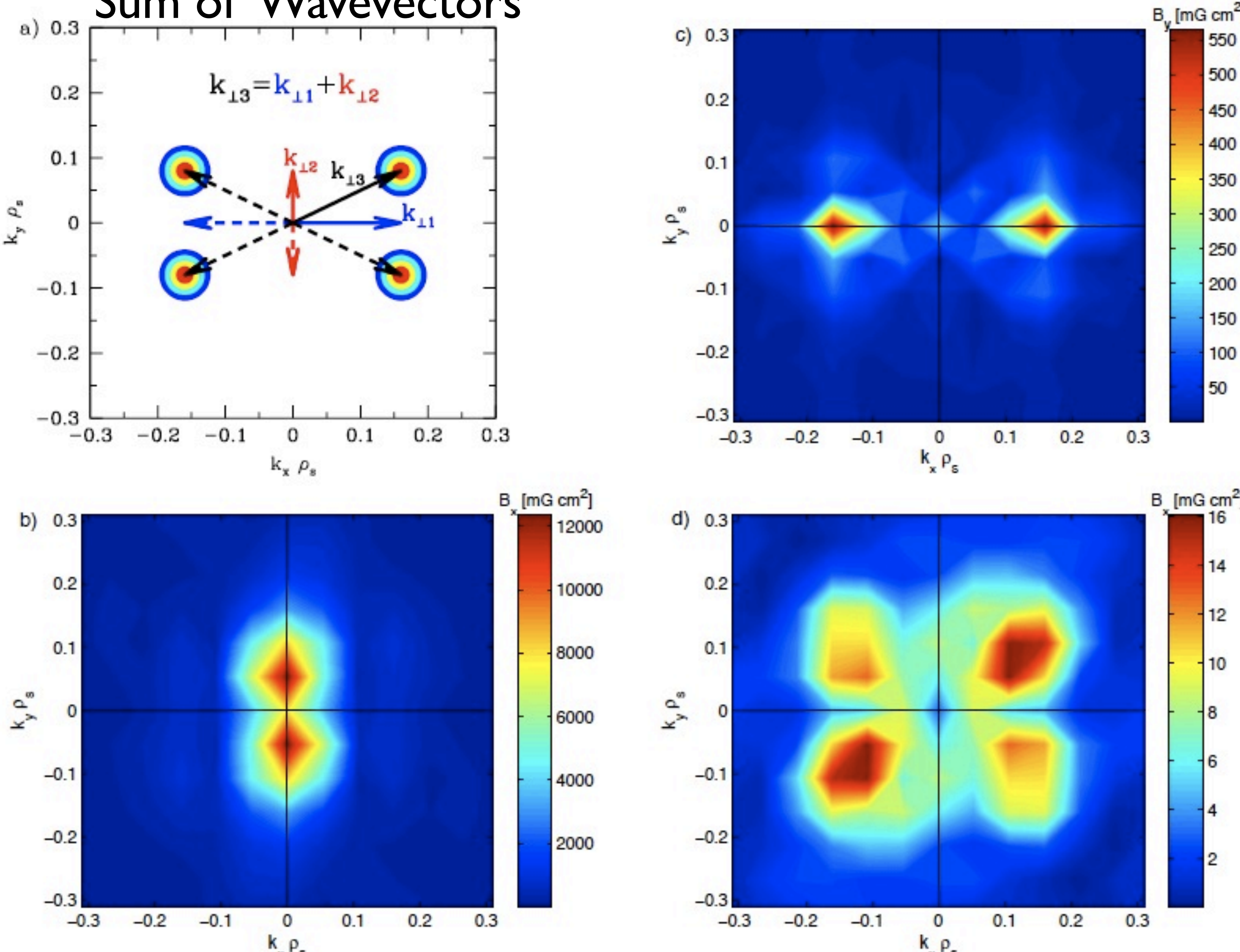
Sum of Wavevectors



ASW Antenna Alfven Wave

Nonlinearly produced daughter Alfven wave

Loop Antenna Alfven Wave

(Howes et al. 2012, Phys. Rev. Lett., **109**:255001, Drake et al. 2013, Phys. Plasmas, **20**:072901)

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Alfven Wave Collisions Generate Current Sheets

Please contact Greg Howes (gregory-howes@uiowa.edu) if you would like to see this section of the talk.

Conclusions

- "Collisions" between counterpropagating Alfven waves govern the nonlinear transfer of energy from large to small scales
  - Primary Alfven waves first generate a purely magnetic fluctuation
  - These primary waves then interact with this nonlinear fluctuation to transfer energy secularly to smaller scale Alfven waves
- This property persists even under the weakly collisional conditions of solar wind plasmas
- We have analytically, numerically, and experimentally verified the physics of Alfven wave collisions
- The amplitude and phase relationships, determined by the nonlinear interaction between counterpropagating Alfven waves, determines two important observed properties of plasma turbulence
  - Development of coherent structures, such as current sheets, at small scales in plasma turbulence
  - Dynamic alignment of velocity and magnetic field fluctuations

END