Many thanks to several
co-workers and collaborators

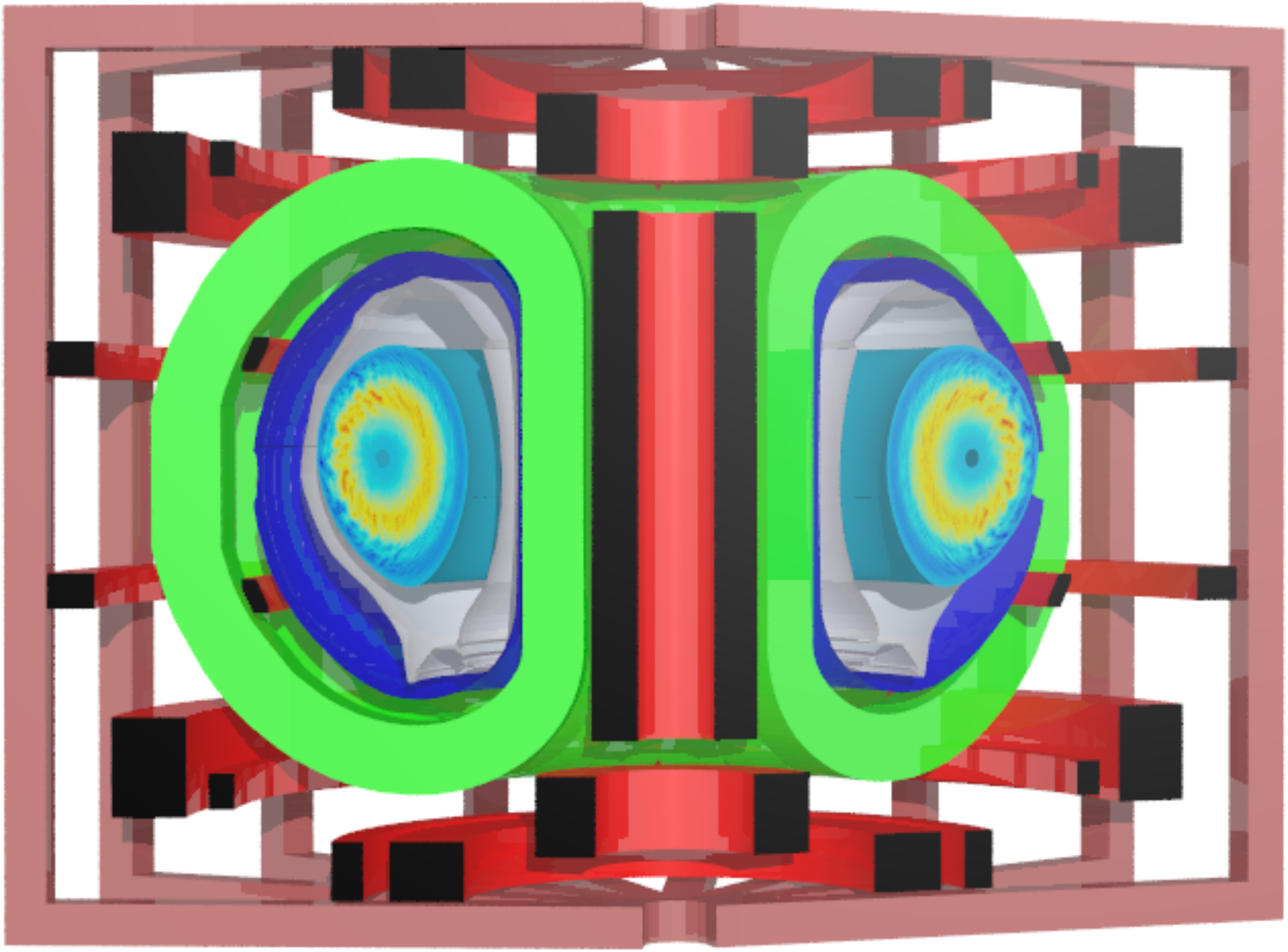
Frank Jenko

Capitalizing on a better understanding of plasma turbulence

Max Planck Institute for Plasma Physics, Garching
Ulm University

Wolfgang Pauli Institute, Vienna
March 21, 2013

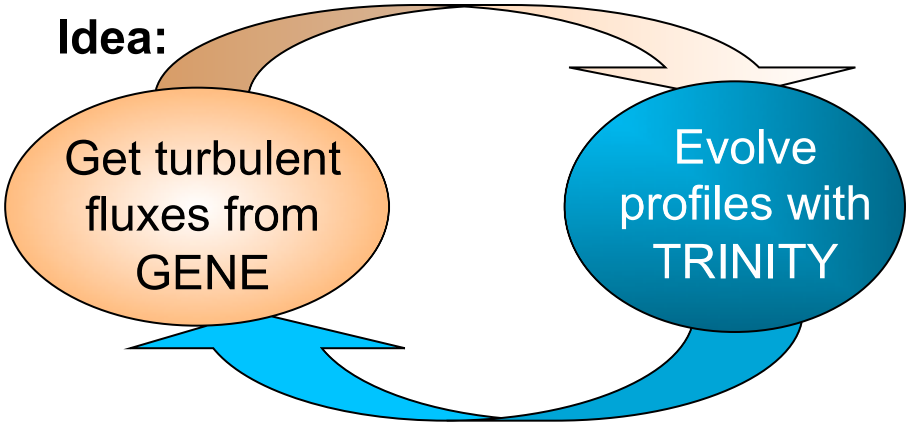
Towards a virtual fusion device?!

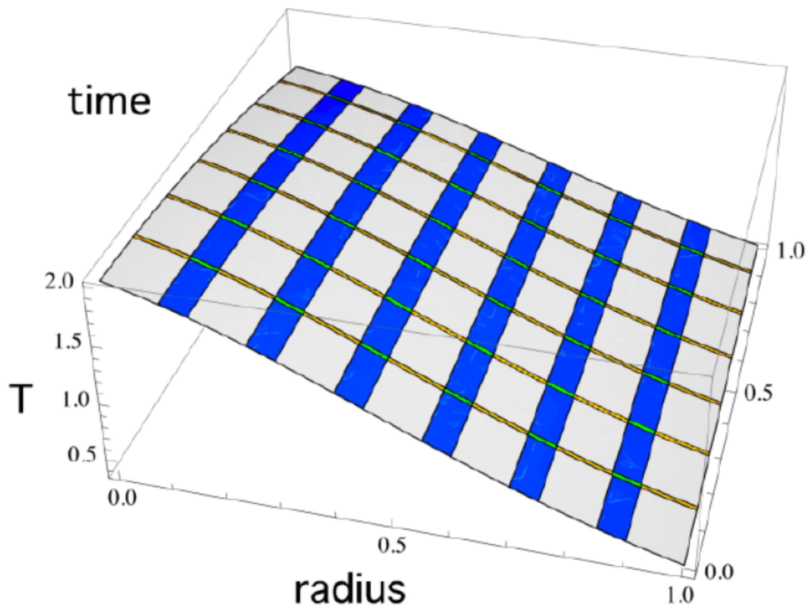


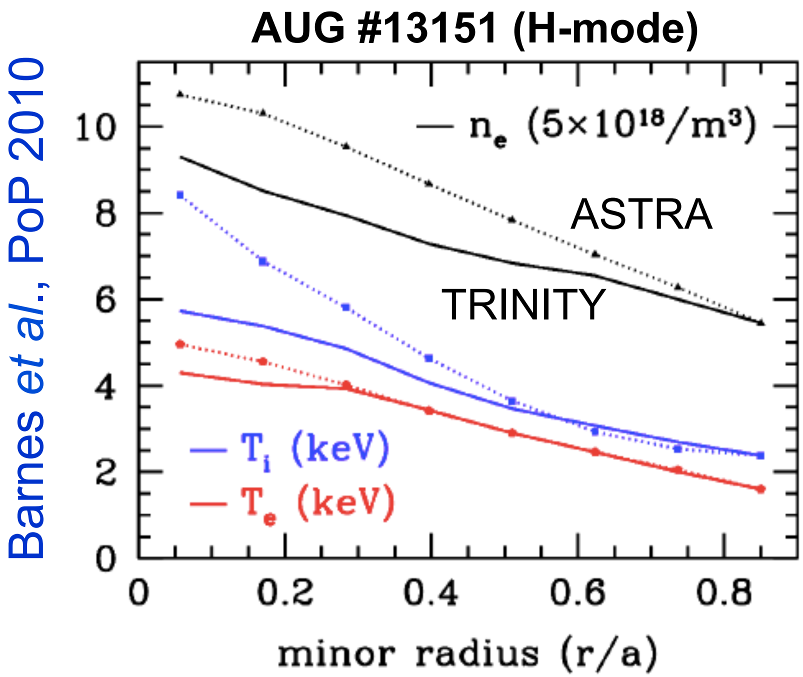
GENE simulation of ASDEX Upgrade

Coupling GENE and TRINITY

Idea:





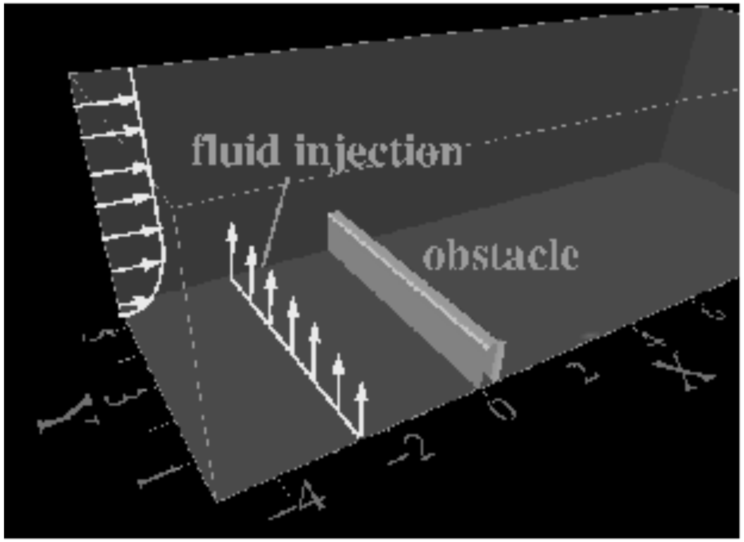


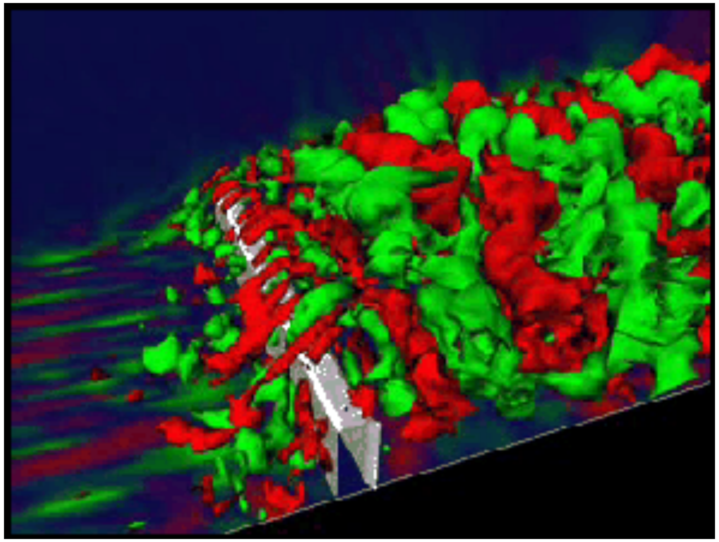
Computational cost much lower than for flux-driven global simulations, but still too high for frequent usage

Dimensional reduction techniques

- Often, one is mainly interested in *large scale dynamics*. There, one finds an interesting interplay between linear (drive) and nonlinear (damping) physics. – Is it possible to remove the small scales?
- Yes: Large Eddy Simulation (LES), POD etc.

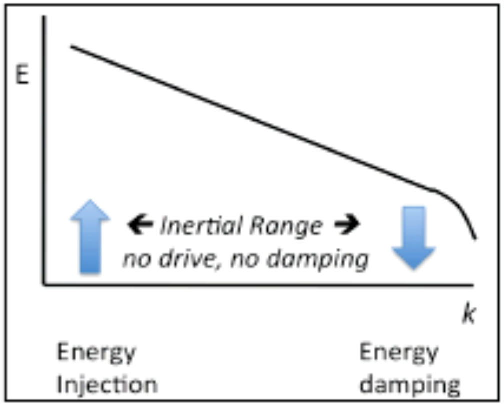
Orellano & Wengle, JT 2001





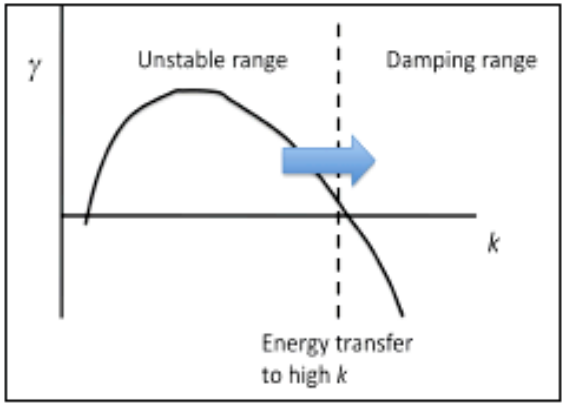
Turbulence in fluids and plasmas – Three basic scenarios

1.Hydrodynamic cascade



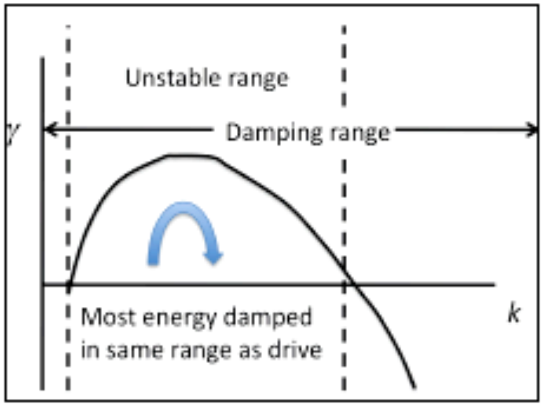
Inertial range
→ no dissipation
→scale invariant dynamics
→power law spectrum

2. Conventional μ-turbulence



Energy transfer to high *k*
like hydro – no inertial range
adjacent unstable,
damping ranges

3. Saturation by damped eigenmode

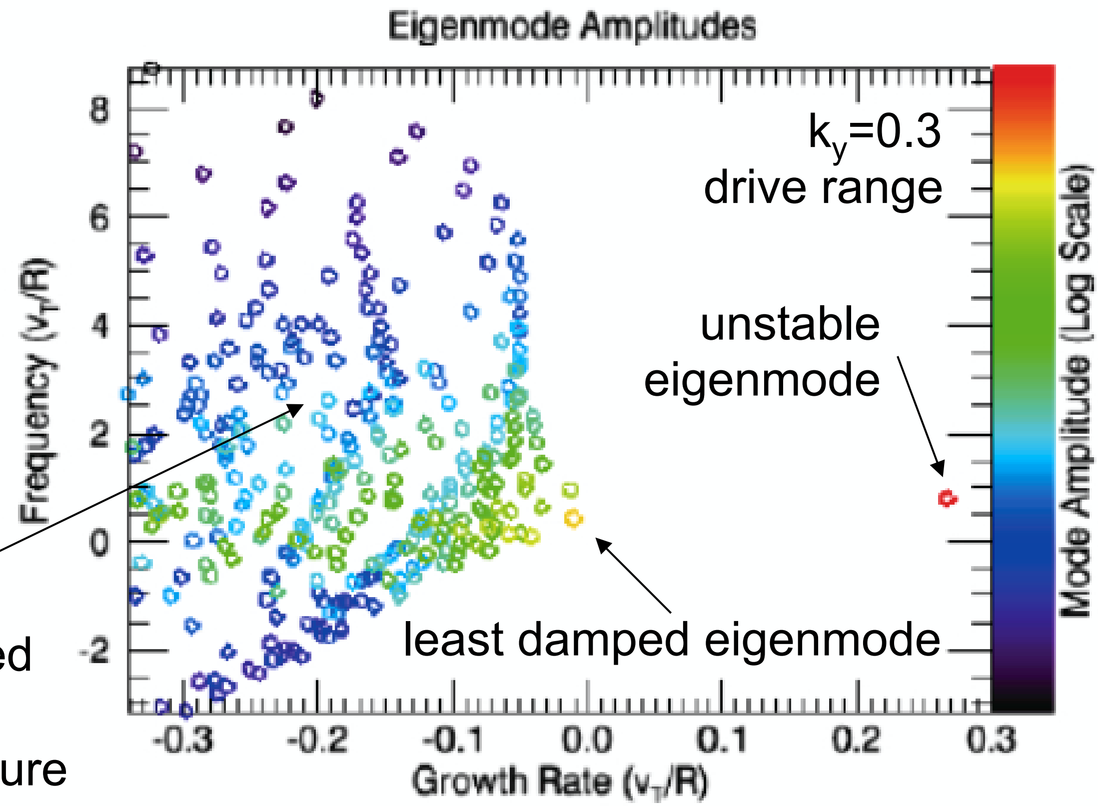


Energy can go to high *k*
but most of it is lost at
low *k* in driving range

…in collaboration with P. W. Terry

Excitation of damped eigenmodes

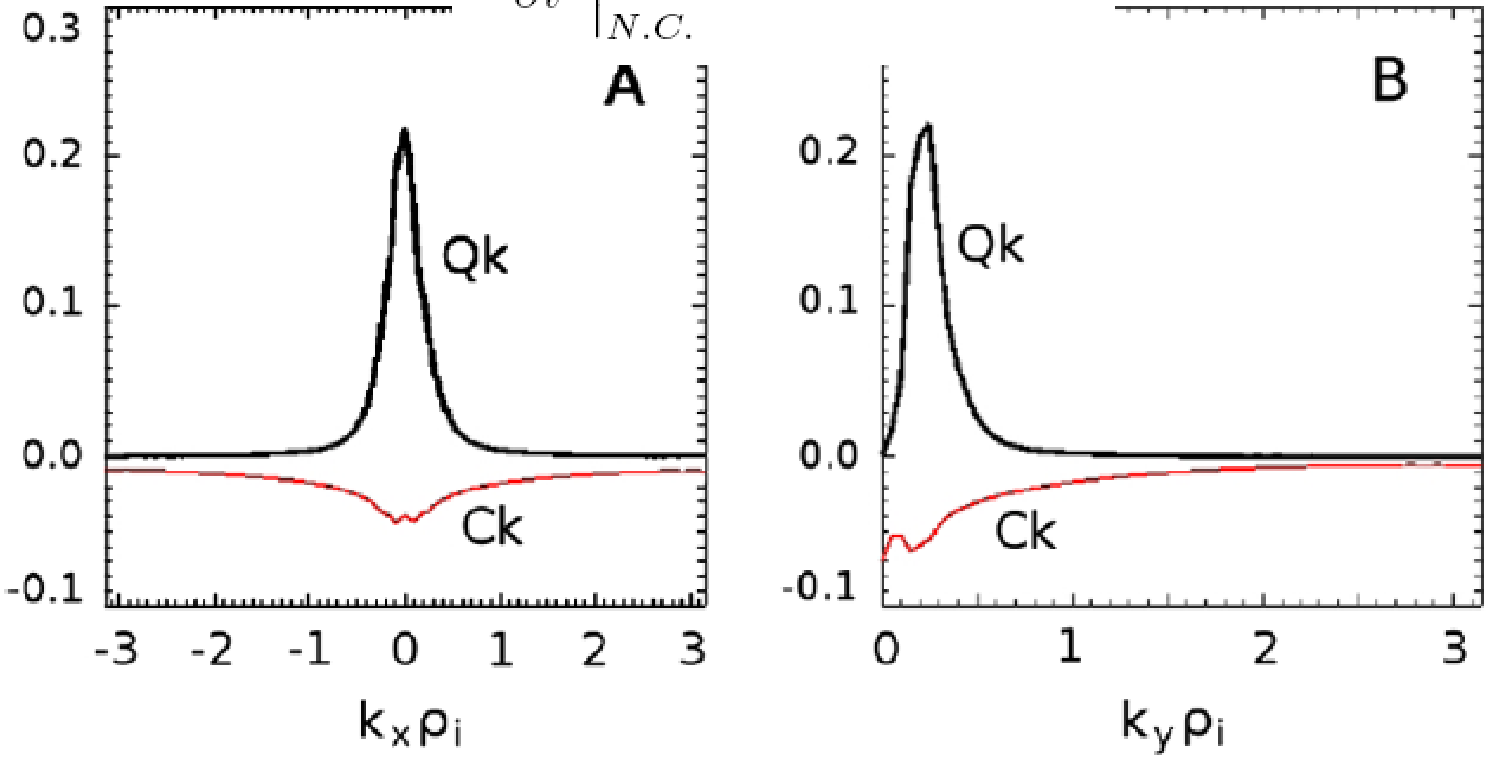
Using GENE as a linear eigenvalue solver to analyze nonlinear ITG runs via projection methods, one finds…



strongly damped eigenmodes (fine-scale structure in $v_\parallel$ and z)

Energetics in wavenumber space

$$\left.\frac{\partial E_k}{\partial t}\right|_{N.C.} = Q_k + C_k + N_k$$



Damped eigenmodes are responsible for significant dissipation in the drive range (!)

Hatch, Terry, Jenko, Merz & Nevins, PRL 2011

Resulting spectrum decays exponentially @lo k, asymptotes to power law @hi k

Spectrum from k space attenuation of $T(k)$ by dissipation $\alpha E(k)$:

nonlinear energy transfer rate

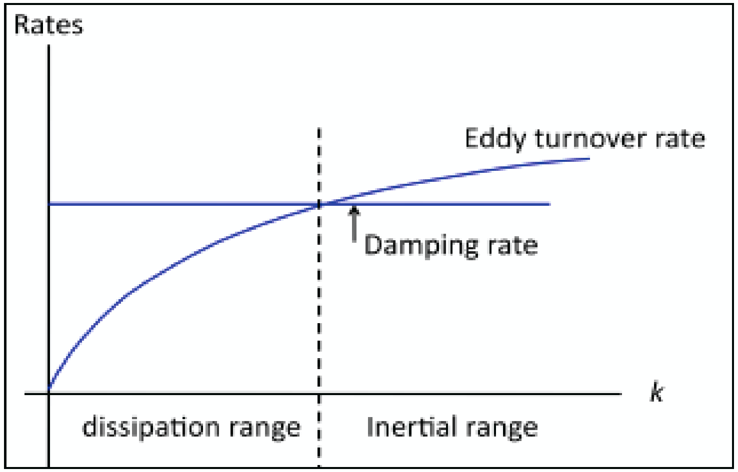
$$\frac{dT(k)}{dk} = \frac{d(v_k^3 k)}{dk} = aE(k)$$

Corrsin closure procedure: $v_k^3 k = v_k^2 \cdot v_k k = E(k)k \cdot \varepsilon^{1/3} k^{-1/3} k$

Solving attenuation ODE:

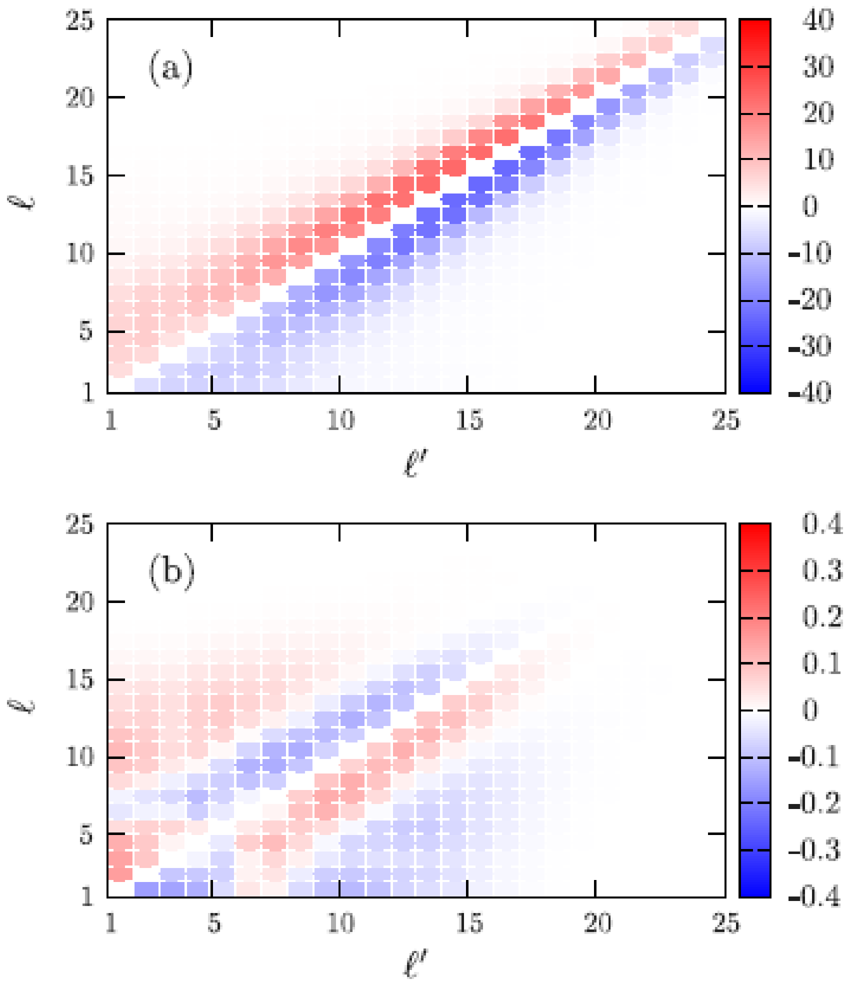
$$E(k) = \beta \varepsilon^{2/3} k^{-5/3} \exp\left[\frac{3}{2}\alpha \varepsilon^{-1/3} k^{-2/3}\right]$$

Spectrum becomes power law in range where eddy turnover rate exceeds constant dissipation rate



Hatch *et al.*, PRL 2011
Terry *et al.*, PoP 2012

Shell-to-shell transfer of free energy



$$\mathcal{E}_f = \sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} \frac{f_j^2}{2},$$

ITG turbulence (adiabatic electrons); logarithmically spaced shells

Entropy contribution dominates; exhibits very local, forward cascade

$$\mathcal{E}_\phi = \sum_j \int d\Lambda q_j \frac{\bar{\phi}_1 f_j}{2}.$$

Bañón Navarro *et al.*, PRL 2011

Application: Gyrokinetic LES models

LES filter in DNS domain: $\partial_t f_{ki} = L[f_{ki}] + N[\phi_k, f_{ki}] - D[f_{ki}]$

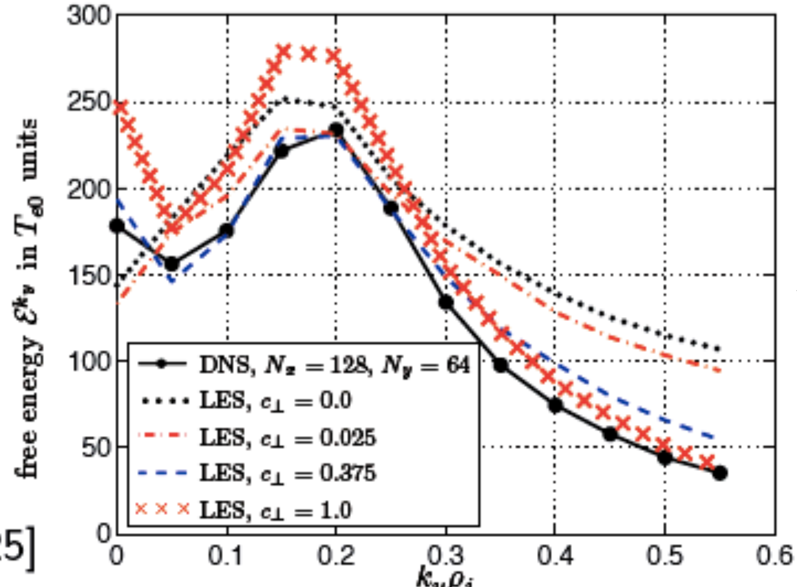
$$\partial_t \overline{f_k} = L[\overline{f_k}] + N[\overline{\phi_k}, \overline{f_k}] + T_{\overline{\Delta}, \Delta^{\mathrm{DNS}}} - D[\overline{f_k}]$$

Sub-grid term: $T_{\overline{\Delta}, \Delta^{\mathrm{DNS}}} = \overline{N}[\phi_k, f_k] - N[\overline{\phi_k}, \overline{f_k}] \approx c_\perp k_\perp^4 h_{ki}$

Free energy spectra vs $c_\perp$:

Cyclone Base Case (ITG)

- ⋆ $c_\perp$ too small
  - ⇒ not enough dissipation
- ⋆ $c_\perp$ too strong
  - ⇒ overestimates injection
- ⋆ $c_\perp = 0.375$ good agreement
  - → "plateau" for $c_\perp \in [0.25, 0.625]$
  - → holds for k_x



Morel *et al.*, PoP 2011

Substantial savings in computational cost: Here, a factor of 20

Self-adjustment of model parameters

Test filter in DNS domain: $\partial_t \widehat{f}_k = L[\widehat{f}_k] + N[\widehat{\phi_k, f_k}] - D[\widehat{f}_k] + T_{\widehat{\Delta},\Delta^{\mathrm{DNS}}}$

Test filter in LES domain: $\partial_t \widehat{f}_k = L[\widehat{f}_k] + \widehat{N}[\overline{\phi_k}, \overline{f_k}] - D[\widehat{f}_k] + \widehat{T}_{\overline{\Delta},\Delta^{\mathrm{DNS}}}$

$\widehat{\overline{\cdots}} = \widehat{\cdots}$ ...for the Fourier cut-off filters used here

One thus obtains the (Germano) identity:

$$T_{\widehat{\Delta},\Delta^{\mathrm{DNS}}} = \widehat{T}_{\overline{\Delta},\Delta^{\mathrm{DNS}}} + \widehat{N}[\overline{\phi_k}, \overline{f_k}] - N[\widehat{\phi_k}, \widehat{f_k}] = \widehat{T}_{\overline{\Delta},\Delta^{\mathrm{DNS}}} + T_{\widehat{\Delta},\overline{\Delta}}$$

Approximate sub-grid terms and minimize error:

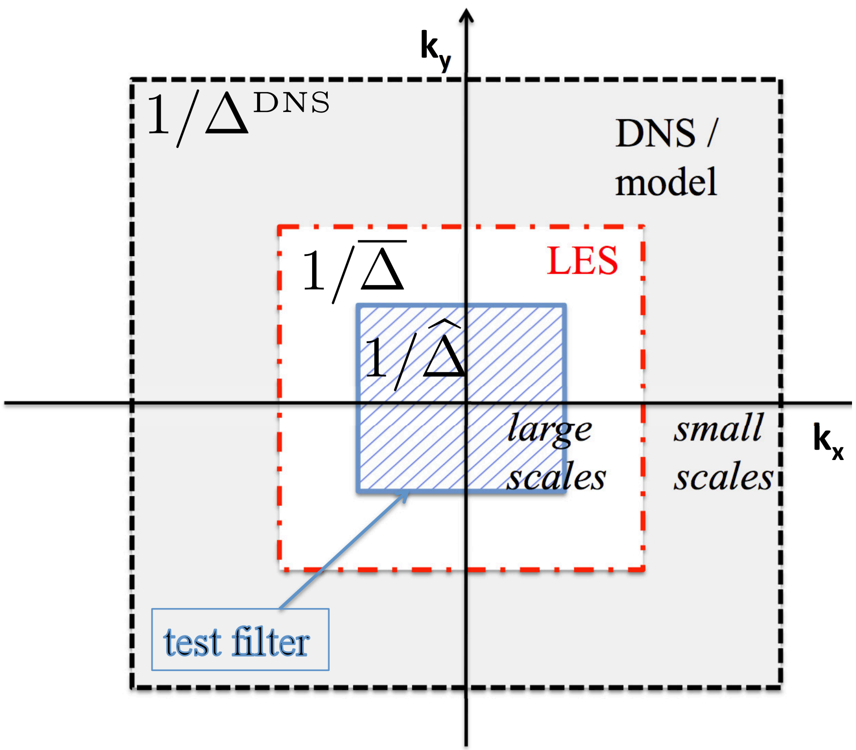
$$T_{\widehat{\Delta},\Delta^{\mathrm{DNS}}} \approx M_{\widehat{\Delta}} \quad ; \quad T_{\overline{\Delta},\Delta^{\mathrm{DNS}}} \approx M_{\overline{\Delta}} \qquad M_{\widehat{\Delta}} \approx \widehat{M}_{\overline{\Delta}} + T_{\widehat{\Delta},\overline{\Delta}}$$

$$d^2 = \left\langle \left(T_{\widehat{\Delta},\overline{\Delta}} + \widehat{M}_{\overline{\Delta}} - M_{\widehat{\Delta}} \right)^2 \right\rangle_\Lambda$$

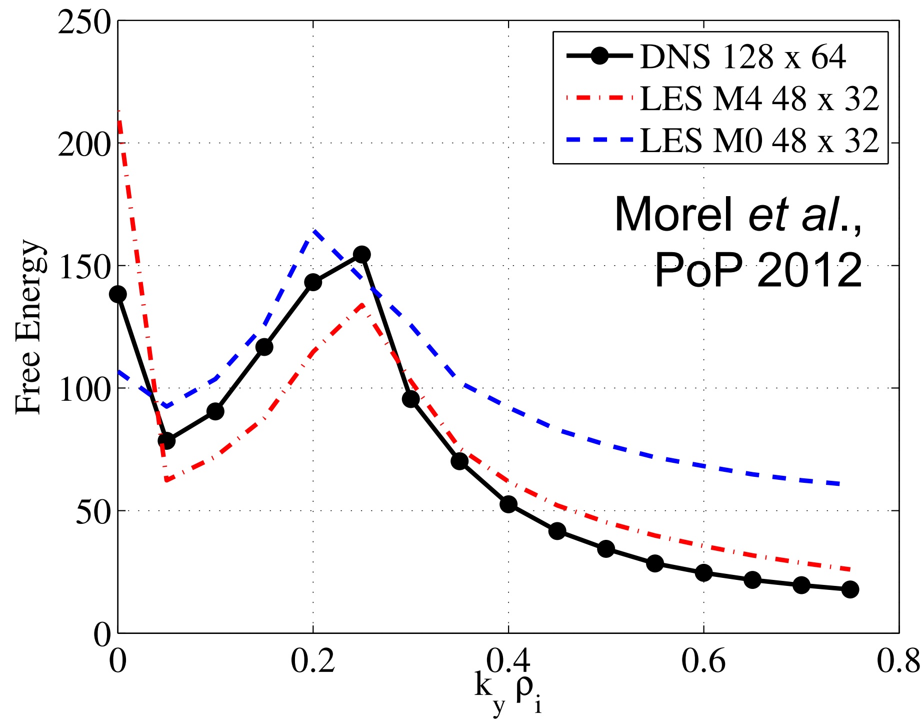
...this procedure yields explicit expressions for the model parameter(s)

The "dynamic procedure" in practice

Schematic of „dynamic procedure"



Free energy spectra
(w/ and w/o model)



LES techniques are likely to reduce the simulation effort substantially without introducing many free parameters. This offers an interesting perspective…

Summary and perspectives

Trying to tackle plasma turbulence

Ab initio simulations will remain very challenging (although invaluable), despite continuing growth in computer power

Quasilinear models can be extremely useful but fail to capture important nonlinear effects; thus, they must sometimes be complemented (or replaced) by nonlinear simulations

This motivates the search for reliable but minimal models; Large Eddy Simulations represent one such line of research

In general, we are in need of a still better understanding of plasma turbulence in order to model it efficiently