3D Braginskii simulations of Turbulence in the tokamak edge, part 1: the SOL

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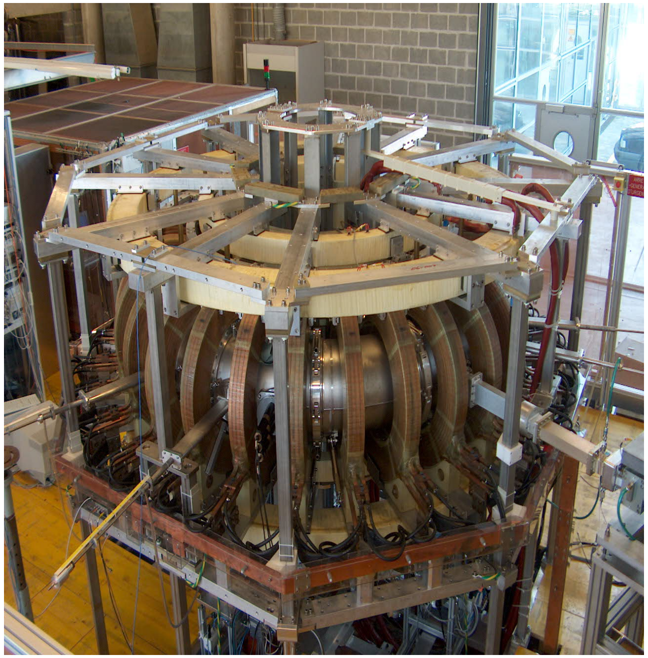
How do global fluid simulations of the edge differ from local (flux tube) simulations? When are global effects important?

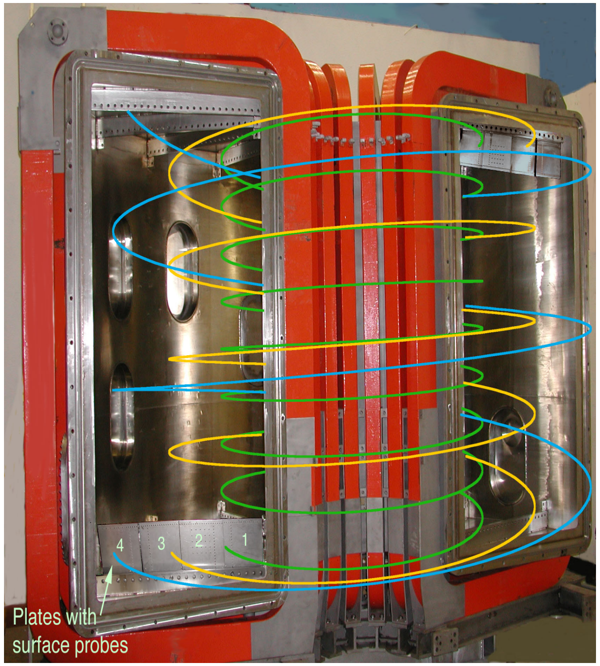
What can global simulations teach about transport, H and L pedestal structure, the LH transition, density limit, etc?

Can we predict the plasma profiles given the source strengths and plasma parameters?

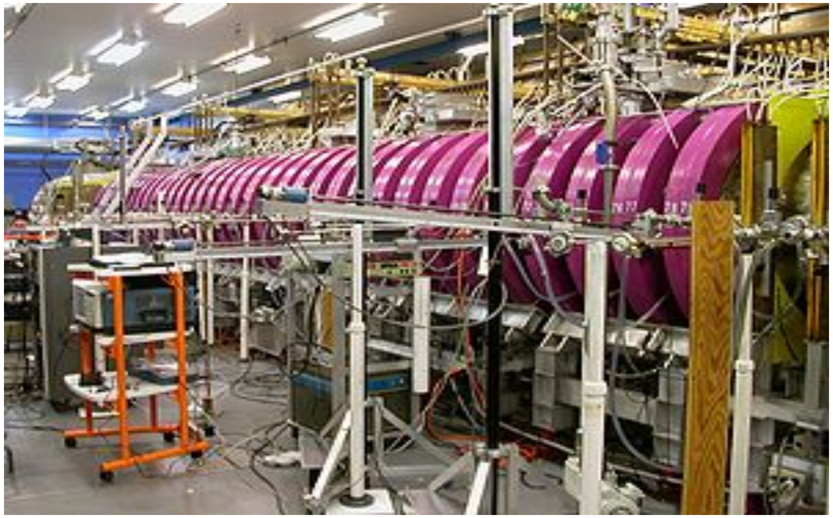
Torpex (EPFL-CRPP)



Helimac (U. of Texas)



LAPD: (UCLA)



3D Braginskii Equations (T_i=0)

$$\frac{d\nabla_\perp^2\phi}{dt} = -V_{||i}\frac{\partial\nabla_\perp^2\phi}{\partial z} + \frac{2B}{cm_iRn}\frac{\partial p_e}{\partial y} + \frac{m_i\Omega_{ci}^2}{e^2n}\frac{\partial j_\|}{\partial z}$$

$$\frac{dV_{||i}}{dt} = -V_{||i}\frac{\partial V_{||i}}{\partial z} - \frac{1}{nm_i}\frac{\partial p_e}{\partial z}$$

$$m_en\frac{dV_{||e}}{dt} = -m_eV_{||e}\frac{\partial V_{||e}}{\partial z} - \frac{T_e}{n}\frac{\partial n}{\partial z} + e\frac{\partial\phi}{\partial z} - 1.71\frac{\partial T_e}{\partial z} + \frac{e}{\sigma_\|}j_\|$$

$$\frac{dn}{dt} = \frac{2c}{eRB}\left(\frac{\partial p_e}{\partial y} - en\frac{\partial\phi}{\partial y}\right) - \frac{\partial(nV_{||e})}{\partial z} + S_n \longleftarrow \text{Source}$$

$$\frac{dT_e}{dt} = \frac{4cT_e}{3eRB}\left(\frac{7}{2}\frac{\partial T_e}{\partial y} + \frac{T_e}{n}\frac{\partial n}{\partial y} - \frac{\partial\phi}{\partial y}\right) + \frac{2}{3}\frac{T_e}{en}0.71\frac{\partial j_\|}{\partial z}$$

$$-V_{||e}\frac{\partial T_e}{\partial z} - \frac{2}{3}T_e\frac{\partial V_{||e}}{\partial z} + S_T \longleftarrow \text{Source}$$

$Parallel\ BCs\ at\ sheath\ edges\ (z = \pm L_z/2):$

$$V_{||i} = \pm c_s\ , \quad c_s = \sqrt{T_e/m_i}$$

$$V_{||e} = \pm c_s \exp\left(\Lambda - e\phi/T_e\right)\ , \quad \Lambda = \ln\sqrt{m_i/(2\pi m_e)} \simeq 3$$

Resistive Ballooning Mode

$$\frac{\partial \nabla_\perp^2 \phi}{\partial t} = \frac{2B}{cm_i Rn}\frac{\partial p_e}{\partial y} + \frac{4\pi V_A^2}{c^2}\frac{\partial j_\parallel}{\partial z} \to -\gamma k_y^2 \tilde{\phi} = \frac{2B}{cm_i Rn} ik_y \tilde{p}_e + \frac{4\pi V_A^2}{c^2} ik_\parallel \tilde{j}_\parallel$$

$$\frac{\partial p_e}{\partial t} = \frac{c}{B}\left[\phi, p_e\right] \to \gamma \tilde{p}_e = -ik_y p'_{e0} c\tilde{\phi}/B$$

$$\eta_\parallel j_\parallel = -\frac{\partial \phi}{\partial z} \to \tilde{j}_\parallel = -ik_\parallel \tilde{\phi}/\eta_\parallel$$

These give:

$$\gamma = \frac{\gamma_b^2}{\gamma} - \gamma_b \frac{k_b^2}{k_y^2} \quad \text{where} \quad \gamma_b^2 = \frac{2c_s^2}{RL_p}\ , \quad k_b^2 = \frac{4\pi V_A^2 k_\parallel^2}{\eta_\parallel c^2 \gamma_b}\ , \quad k_\parallel \sim 1/(qR)$$

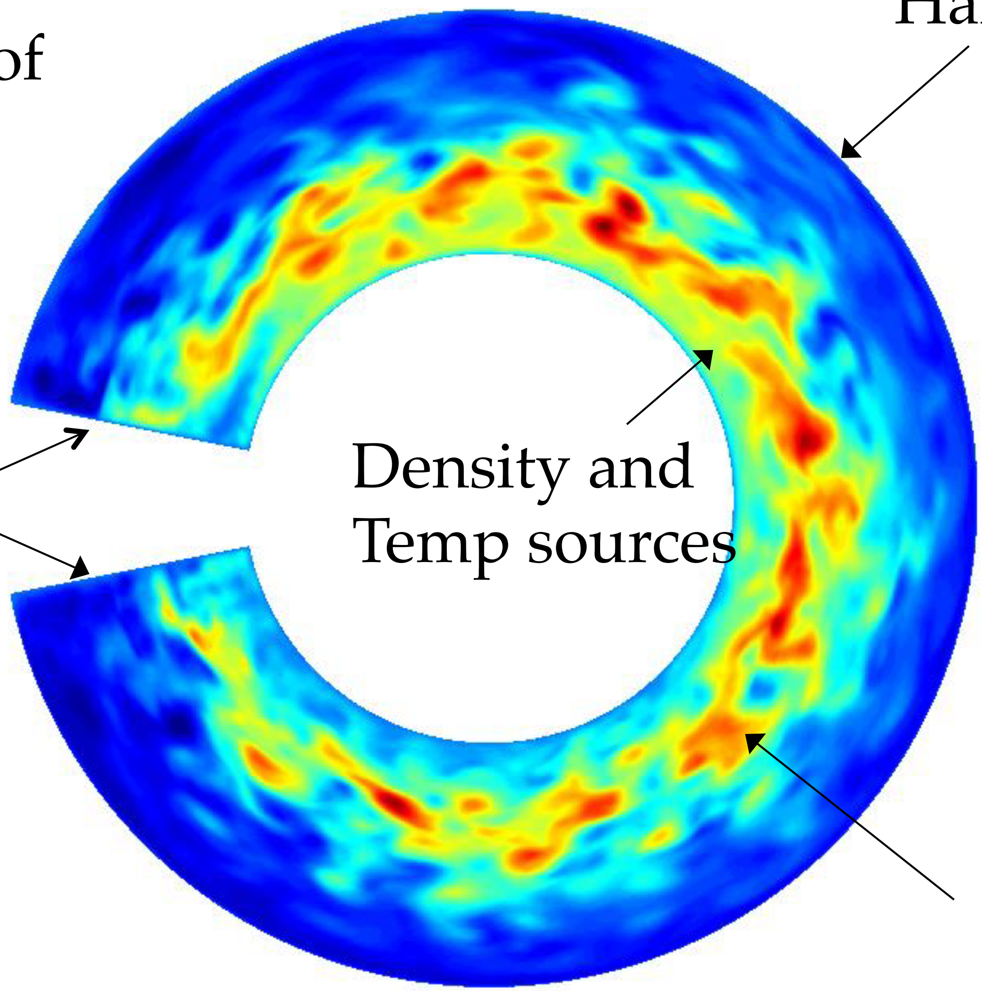
So: $\gamma \simeq \gamma_b$ for $k_y > k_b$, $\gamma \simeq (k_y^2/k_b^2)\gamma_b$ for $k_y < k_b$

For later: $Max\{\gamma/k_y\} = (0.6)\gamma_b/k_b$ for $k_y = (0.9)k_b$

Radial mode width: $\sigma_x \sim \sqrt{L/k_y} \ll L$ for $k_y L \gg 1$

q=4 : full system

Poloidal slice of 3D toroidal simulation



'Hard' wall

Limiter

Density and Temp sources

Resistive B-mode turbulence

Nonlinear saturation mechanisms

1. KH: $\partial_t W \sim \gamma W \sim [\phi, W] \sim k_x k_y \phi W \;\rightarrow\; \phi \sim \gamma/(k_x k_y)$

2. Gradient Removal: $dp_1/dx \sim k_x p_1 \sim dp_0/dx \;\rightarrow p_1 \sim p_0'/k_x$

But: $\partial_t p \sim [\phi, p] \;\rightarrow\; p_1 \sim k_y \phi p_0'/\gamma$

and combining this with previous result gives the KH estimate:

$$p_1 \sim k_y \phi p_0'/\gamma \sim p_0'/k_x \;\rightarrow \phi \sim \gamma/(k_x k_y)$$

Thus we expect KH and gradient removal mechanisms to be potentially competitive, *provided that KH is unstable*

Why would it matter if KH is unstable, if gradient removal is a possibility? It matters because of k_x.

$$\partial_t W \sim [\phi, W] \sim k_x k_y \phi W \;\rightarrow\; \phi \sim \gamma/(k_x k_y)$$

$$dn_1/dx \sim k_x n_1 \sim n_0' \;\rightarrow\; n_1 \sim n_0'/k_x \sim n_0/(k_x L)$$

Thus consider the particle flux and diffusivity:

$$\Gamma_x \sim V_{Ex} n_1 \sim k_y \phi n_1 \sim n_0 \gamma/(k_x^2 L)$$

$$D \sim \Gamma_x/n_0' \sim \gamma/k_x^2$$

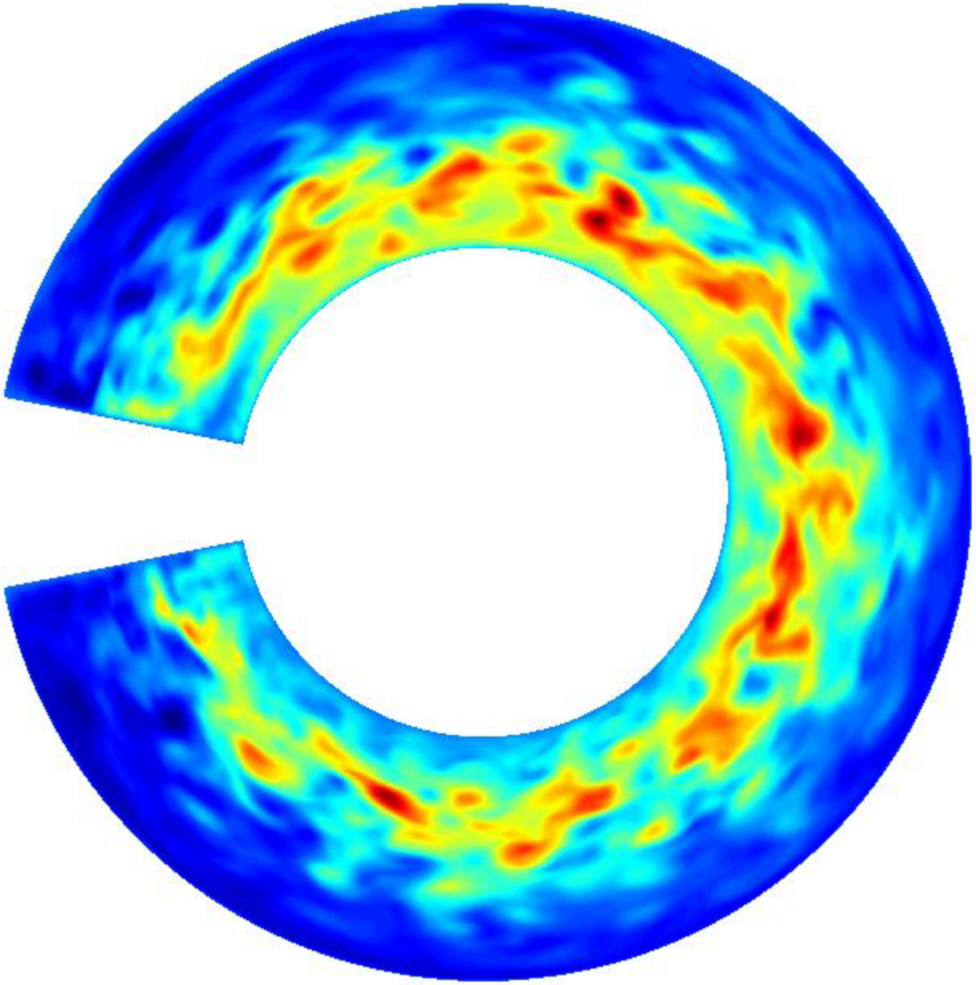
Now, if KH is unstable, it typically breaks up eddies with $k_x \sim k_y$

But if KH is stable, smallest $k_x \sim 1/\sigma_x \sim \sqrt{k_y/L} \sim k_y/\sqrt{k_y L}$

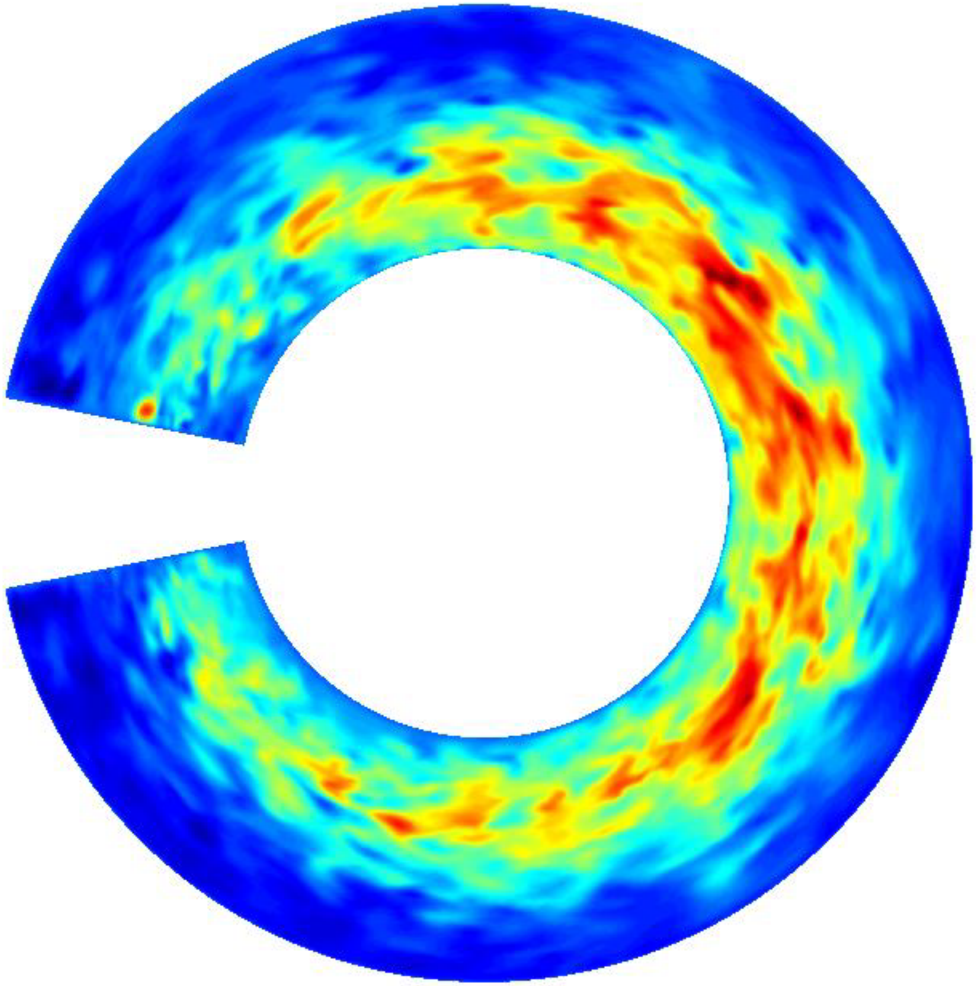
Thus: $D_{KH}/D_{no\ KH} \sim 1/(k_y L) \ll 1$

Gyro-Bohm vs Bohm for driftwaves

q=4 : full system

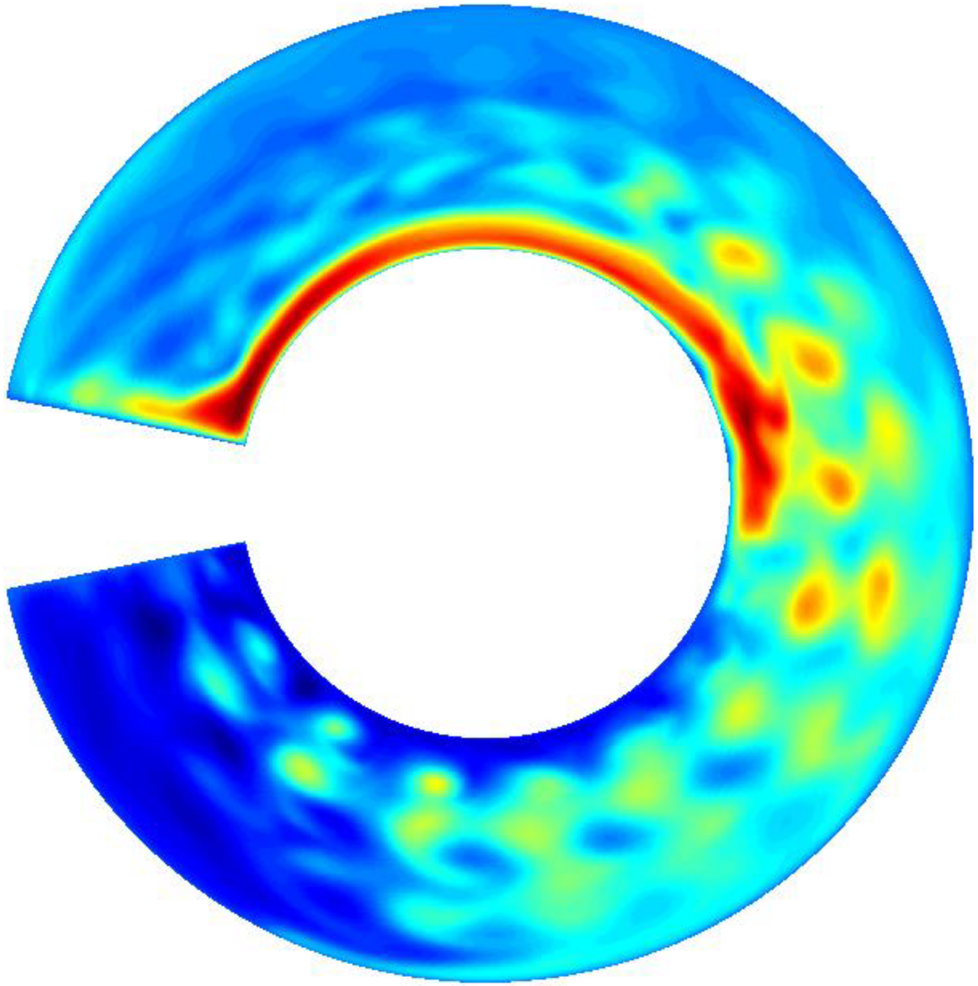


q=4, no KH: $\partial_t W + [\phi, W] \;\rightarrow\; \partial_t W + [\langle\phi\rangle_y, W]$

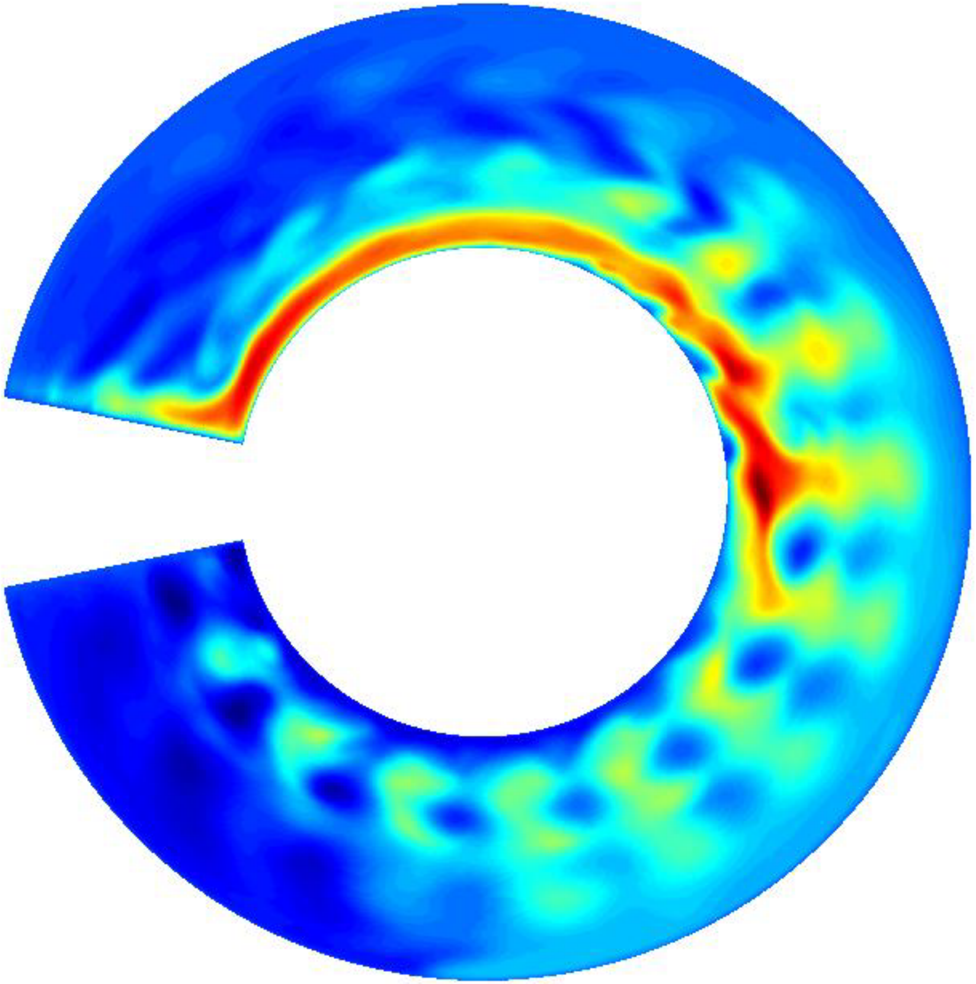


Very little changes in plasma profiles, etc…

q=16 : full system



q=16: no KH



Elimination of KH leads to streamers and flatter profiles

Why is KH stable at low q but not higher q?
At low q, the eddies are too short to be KH unstable:

Typical KH growth rate (sinusoidal flows, max for k_x=(0.6)k_y)

$$\gamma_{KH} \sim (0.3) k_y V_{Ex} \qquad (k_x \simeq 0.6 k_y)$$

Max eddy turnover time:

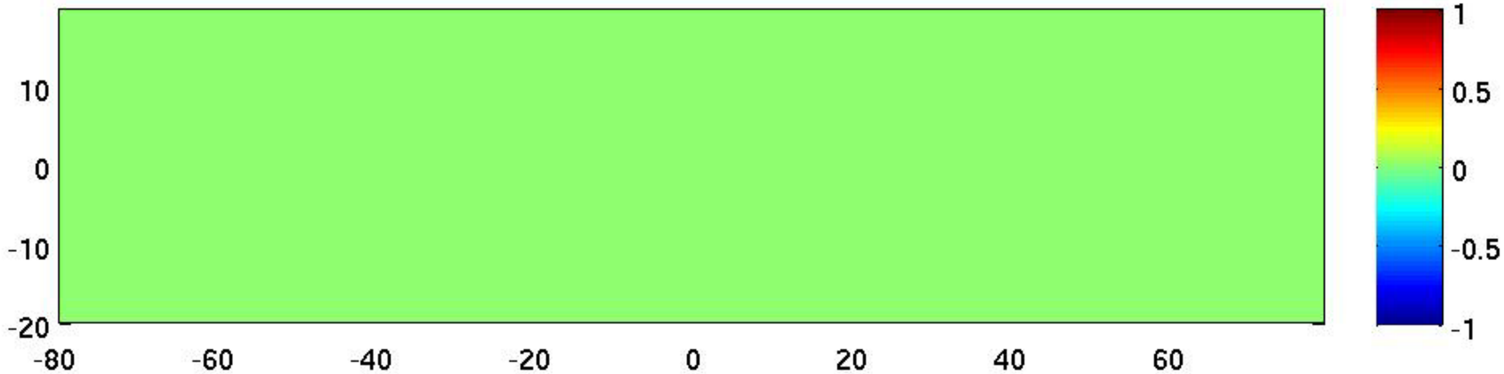
$$\tau \sim \sigma_x / V_{Ex} = \sqrt{L/k_y} / V_{Ex}$$

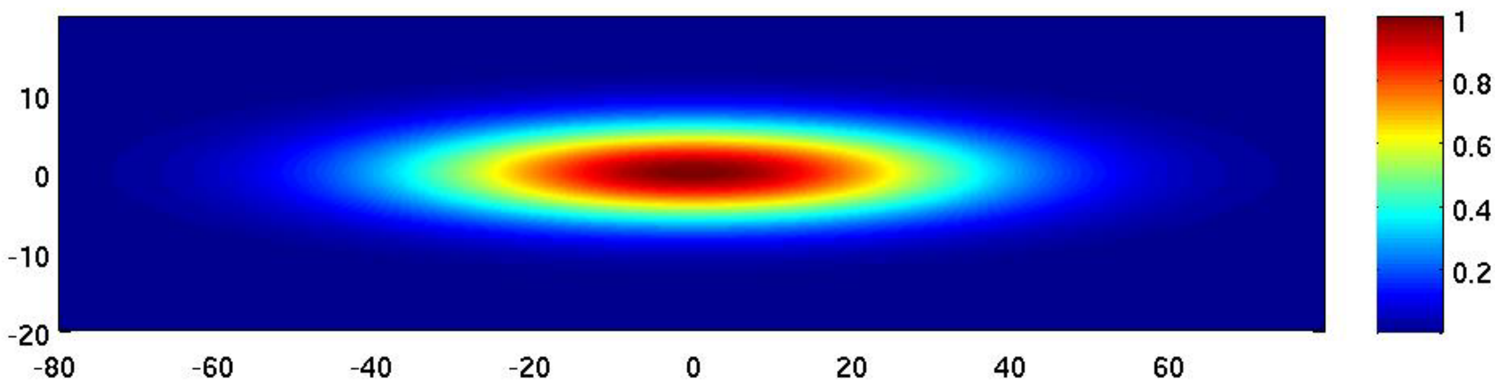
So for KH instability, need:

$$\gamma_{KH} \tau > 1 \;\rightarrow\; \sqrt{k_y L} > 3$$

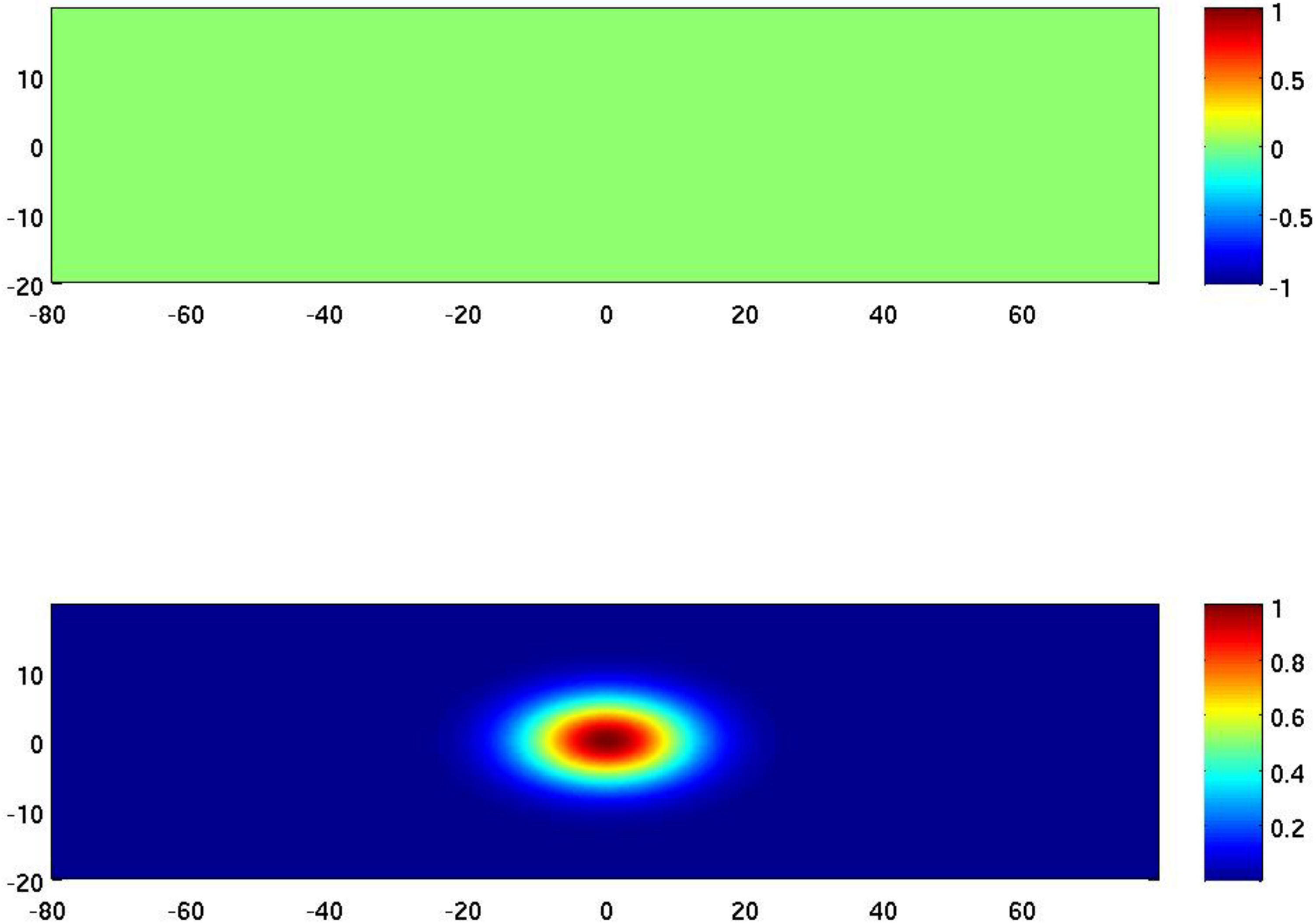
Gaussian eddy, $L_x/L_y=6$

(with $L_y \sim \lambda/2$: $k_y L_x \sim 6\pi$)

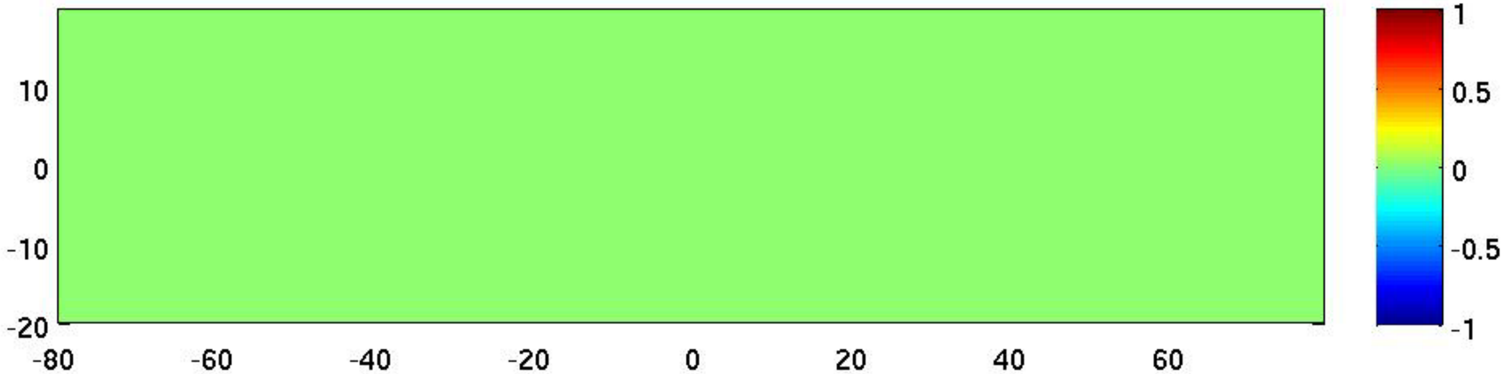


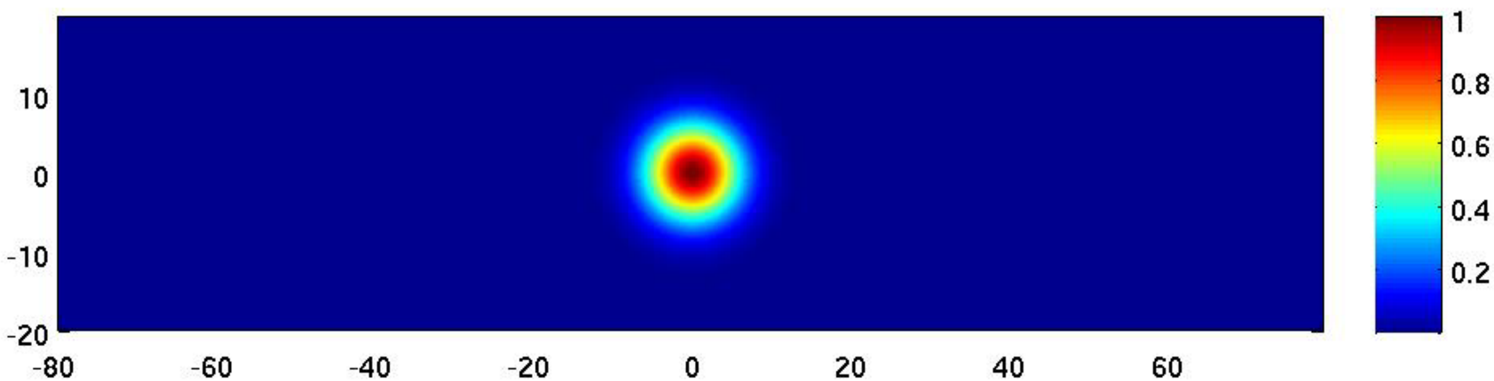


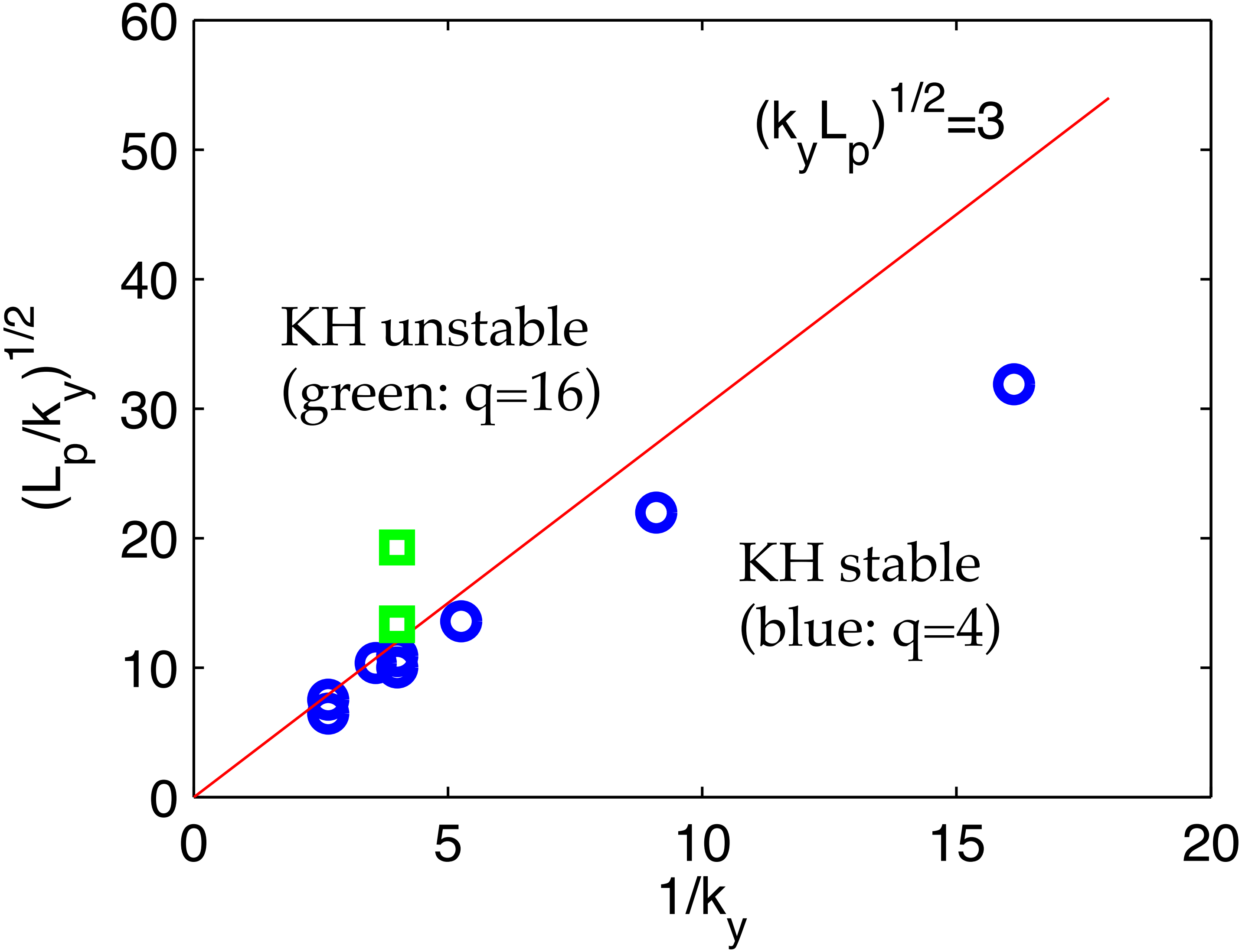
Gaussian eddy, Lx/Ly=2 ($k_y L_x \sim 2\pi$)



Gaussian eddy, Lx/Ly=1



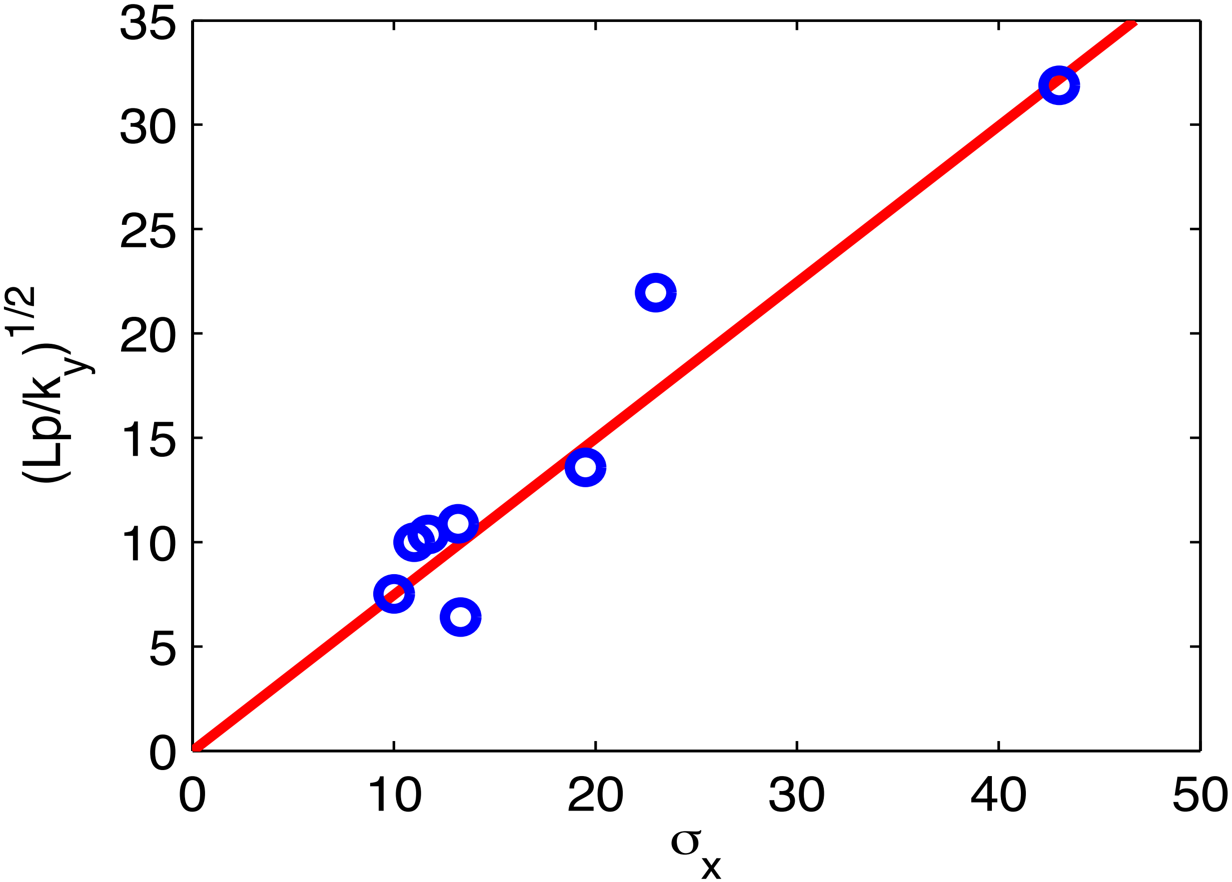




60
50
40
30
20
10
0
0
5
10
15
20
$1/k_y$
$(L_p/k_y)^{1/2}$
$(k_y L_p)^{1/2}=3$
KH unstable
(green: q=16)
KH stable
(blue: q=4)

Radial eddy length exhibits expected scaling

$$\sigma_x \sim \sqrt{L/k_y}$$



Transport and profile scaling for KH stable cases

Consider density as an example:

$$\partial_t n + \nabla \cdot (n\vec{V}_E) = S_n$$

Take flux surf and time average, assume parallel losses dominate:

$$d\Gamma_x/dx \sim \Gamma_x/L \sim n_0 c_s/(qR) \text{ where } \Gamma_x \sim n_1 V_{Ex}$$

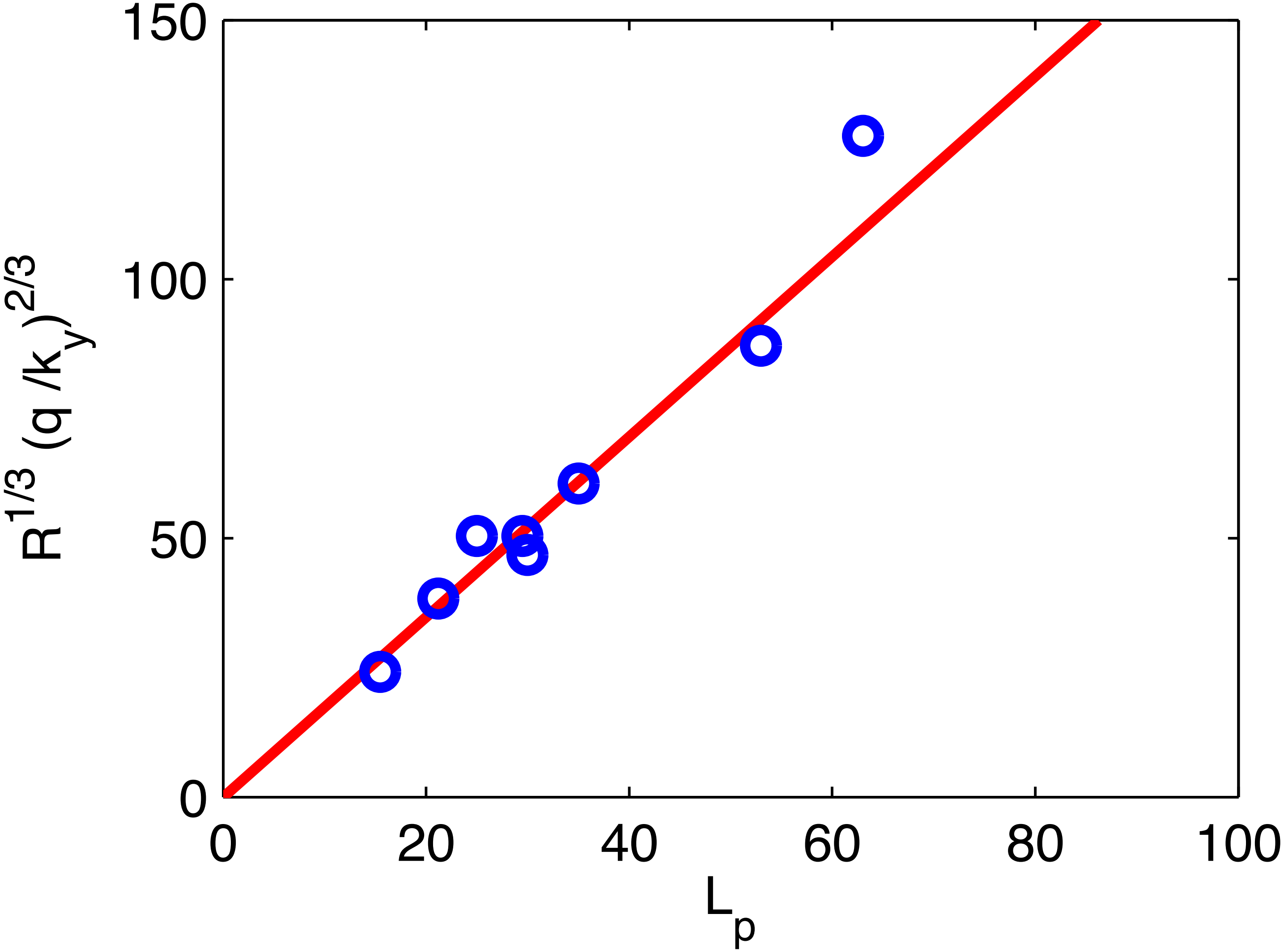
Recall previous result for Γ_x and assume $k_x \sim \sqrt{k_y/L}$

$$\Gamma_x \sim n_0\gamma/(Lk_x^2) \sim n_0\gamma/k_y \sim n_0\gamma_b/k_y \ , \quad k_y \sim k_b$$

Thus:

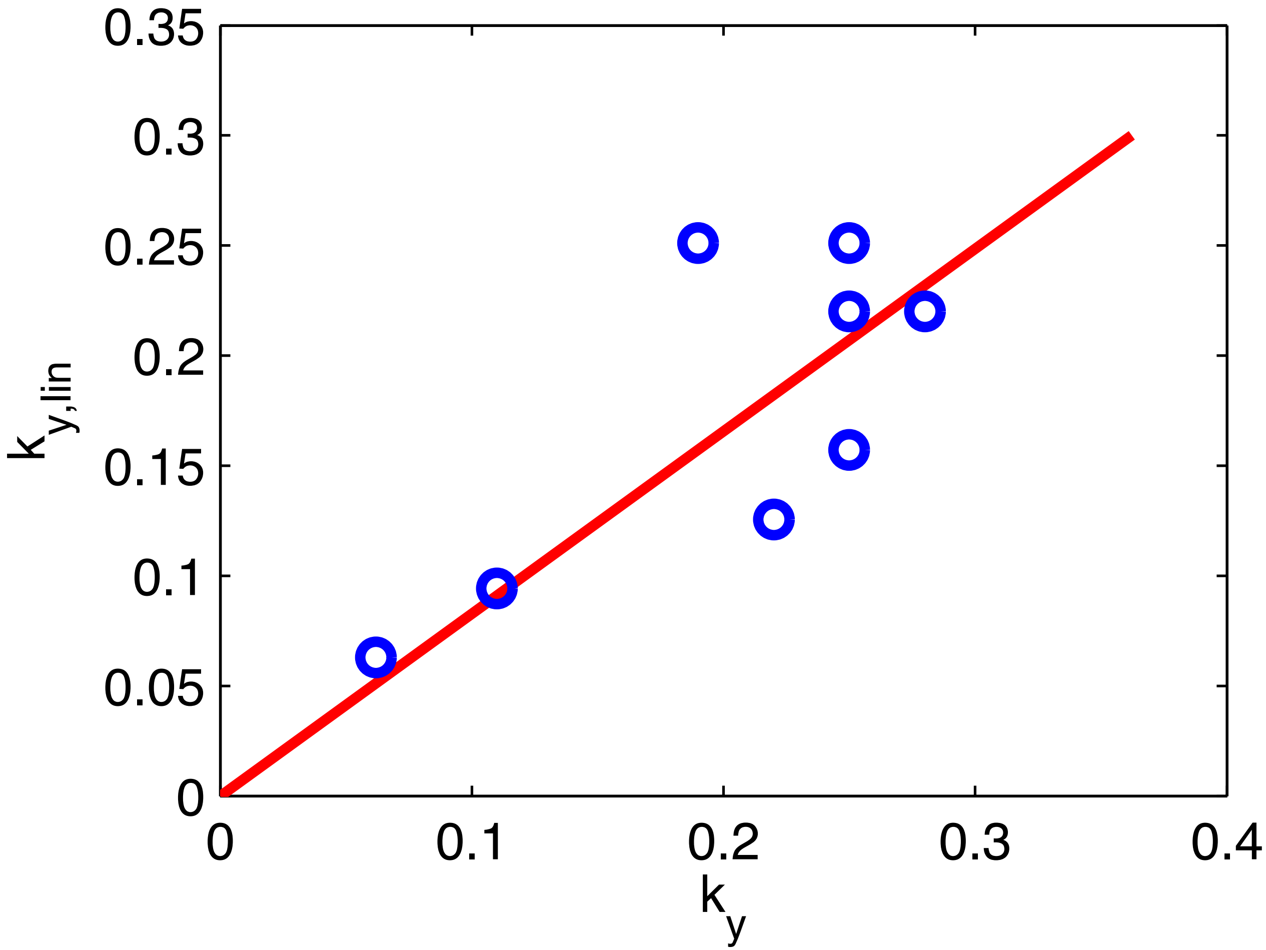
$$n_0 c_s/(qR) \sim \Gamma_x/L \sim n_0\gamma_b/(k_y L) \sim n_0[c_s/\sqrt{RL}]/(k_y L)$$

Comparing first and last: $L \sim R^{1/3}(q/k_y)^{2/3} \ , \quad k_y \sim k_b$



150
100
50
0
0
20
40
60
80
100
L_p
$R^{1/3}(q/k_y)^{2/3}$

Observed k_y vs predicted k_y $(Max\{\gamma/k_y\})$



But we are not quite done. From before:

$$L \sim R^{1/3}(q/k_y)^{2/3} \ , \quad k_y \sim k_b$$

Now use

$$k_b^2 = \frac{4\pi V_A^2 k_\|^2}{\eta_\| c^2 \gamma_b} \ , \qquad \gamma_b^2 = \frac{2c_s^2}{RL_p} \ , \qquad k_\| \sim 1/(qR)$$

to get

$$L_{eq} \sim R^{5/7} q^{8/7} \left(\sqrt{2} c_s \eta_\| c^2/(4\pi V_A^2)\right)^{2/7} = R^{1/3} \left(q/k_{b,eq}\right)^{2/3}$$

or $\quad k_{b,eq} L_{eq} \sim \left[4\pi V_A^2 R q^3/(\sqrt{2} c_s \eta_\| c^2)\right]^{1/7}$

Final condition for KH instability: $\quad \sqrt{k_{b,eq} L_{eq}} > 3$

Summary

- For the cases considered, KH is near marginal stability, with gradient removal mechanism dominant. The transport increases strongly at this point due to an elongation of the eddies:

$$k_x \sim k_y \ \rightarrow \ k_x \sim \sqrt{k_y/L}$$

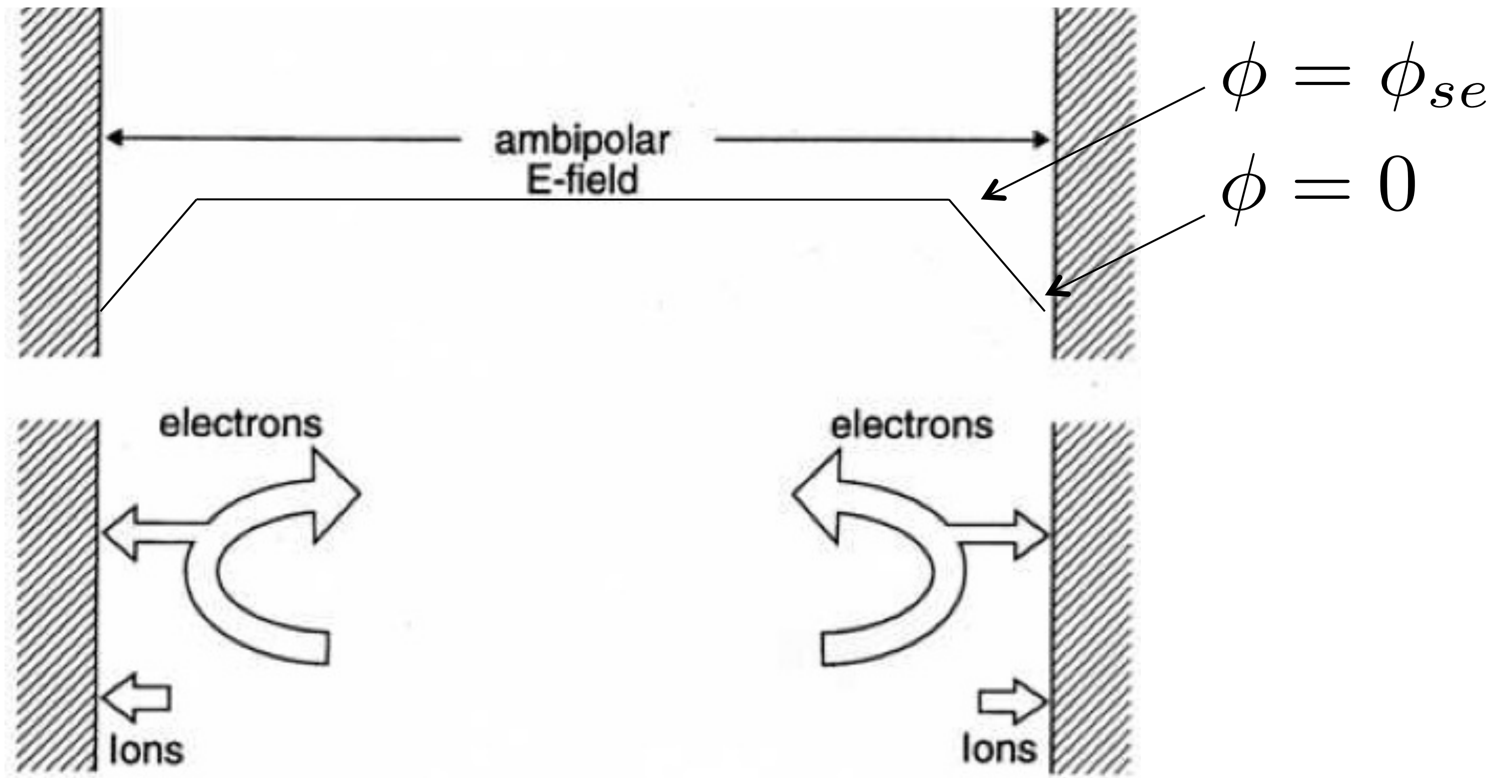
- The SOL equilibrium scale length:

$$L_{sol} \sim R^{1/3}(q/k_{b,eq})^{2/3}$$

is a geometric mean of large and small scales

Sheath Physics

In an open fieldline system, the plasma develops a positive potential relative to the wall until the electron and ion outflows along B balance:



Electrons dynamics in sheath region

$$f_e = n_{se}\left[m_e/(2\pi T_e)\right]^{3/2}\exp\left\{-\left[(1/2)m_e v^2 - e(\phi-\phi_{se})\right]/T_e\right\}$$

$$n = \int f_e d^3v = n_{se}\exp\left\{e(\phi-\phi_{se})/T_e\right\} \quad (= n_{se} \text{ for } \phi = \phi_{se})$$

$$\Gamma_{\|e} = \int_{v_z>0} f_e v_z d^3v = n_{se}\sqrt{T_e/(2\pi m_e)}\exp\left\{e(\phi-\phi_{se})/T_e\right\}$$

$$= n_{se}c_s\exp\left\{\Lambda + e(\phi-\phi_{se})/T_e\right\}$$

$$\text{where } c_s = \sqrt{T_e/m_i}\ ,\ \Lambda = \ln\sqrt{m_i/(2\pi m_e)}$$

So electron flux to wall where $\phi = 0$:

$$\Gamma_{\|e,wall} = n_{se}c_s\exp\left\{\Lambda - e\phi_{se}/T_e\right\}$$

Continuity across sheath :

$$n_{se}V_{\|e,se} = \Gamma_{\|e,wall} \quad \text{so} \quad \boxed{V_{\|e,se} = c_s\exp\left\{\Lambda - e\phi_{se}/T_e\right\}}$$

Ion dynamics at edge of sheath

Continuity :

$$\partial_z \left(nV_{\|i}\right) = 0 \quad \text{so} \quad n\partial_z V_{\|i} = -V_{\|i}\partial_z n$$

Momentum (isothermal for simplicity) :

$$m_i n V_{\|i}\partial_z V_{\|i} = -T_e \partial_z n$$

Combining these gives :

$$-m_i V_{\|i}^2 \partial_z n = -T_e \partial_z n \quad \text{or} \quad \boxed{V_{\|i,se} = c_s \ , \quad c_s = \sqrt{T_e/m_i}}$$

Electrons : $\quad V_{\|e,se} = c_s \exp\left\{\Lambda - e\phi_{se}/T_e\right\}$

$$\boxed{\text{Bottom Line : in equilibrium need } V_{\|i} \simeq V_{\|e} \to \phi \simeq \Lambda T_e/e}$$

Resistive Driftwaves

Neglecting the curvature terms, soundwaves, and m_e :

$$\nu_e k_y^2 \rho_s^2 \gamma^2 + k_{||}^2 c_s^2 (1 + 2.94 k_y^2 \rho_s^2)\gamma + i k_{||}^2 c_s^2 \omega_* = 0 \qquad (\nu_e = e^2 n \eta_{\|} / m_i)$$

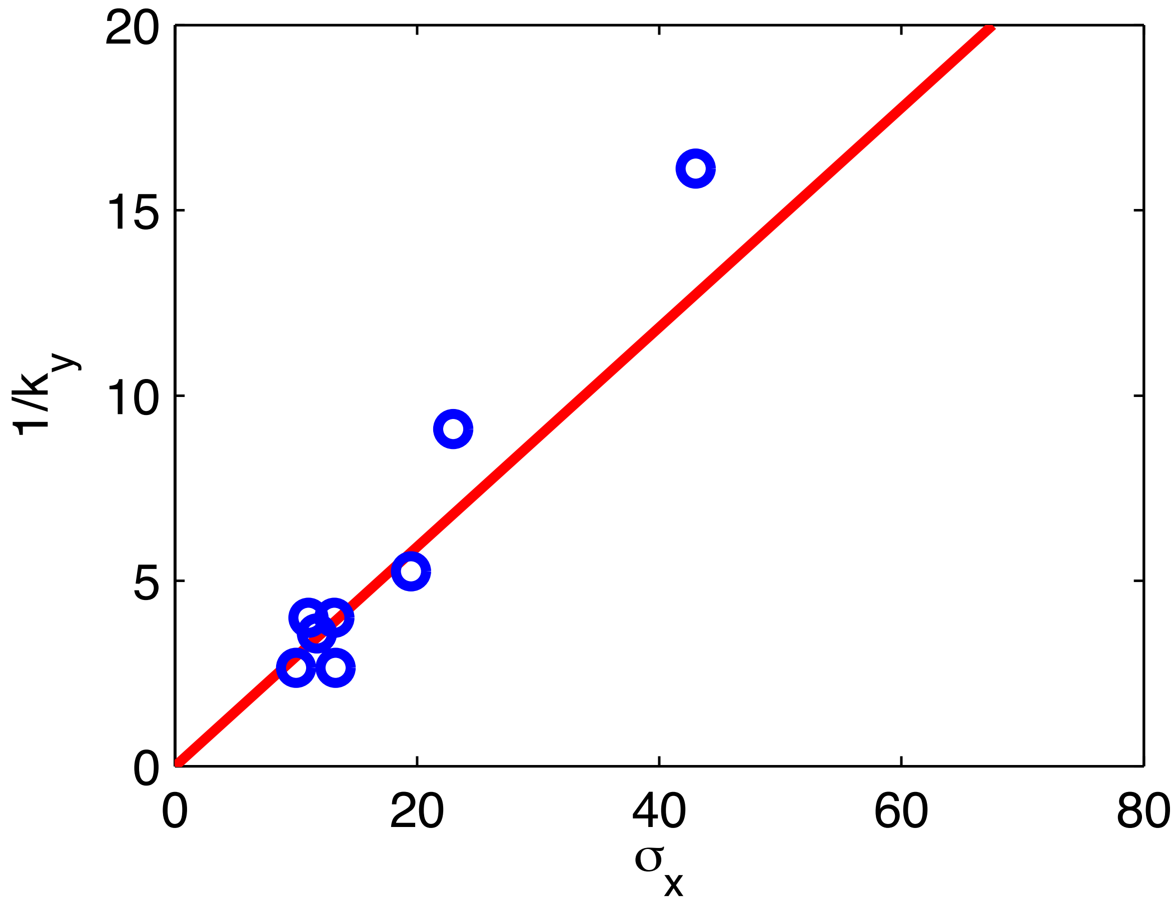
Fastest mode :

$$\gamma_{dw} \simeq 0.1 c_s / L_p \quad \text{for} \quad k_\perp \rho_s \simeq 0.5, \quad k_{\|} \simeq 0.2 \sqrt{\nu_e / (c_s L_p)}$$

Define driftwave regime as :

$$\gamma_{dw} > \gamma_I \text{ where } \gamma_I \simeq c_s / \sqrt{R L_p}$$

$$\boxed{\text{Thus for DWs need } L_p / R < 0.01}$$



20
15
10
5
0
1/k_y
0
20
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σ_x