Trapped-particle instabilities in quasi-isodynamic stellarators

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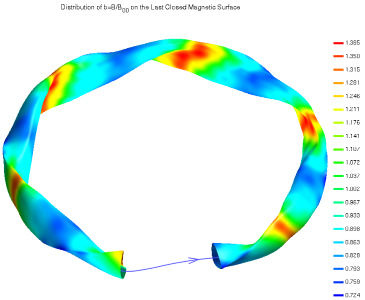
IPP Greifswald
Stellarator Theory

Vienna, 19 March 2012

Introduction

- optimised stellarators: reduced neoclassical transport
- How does this influence microinstabilities (and their turbulence)?



quasi-isodynamic stellarator, $\beta \approx 4\%$, figure courtesy of Y. Turkin

Introduction

Electrostatic microinstabilities

- ion temperature gradient mode (ITG)
- electron temperature gradient mode (ETG)
- trapped-particle modes
  - trapped-electron mode (TEM)
    - collisionless TEM - CTEM
    - dissipative TEM - DTEM
  - trapped-ion mode (TIM)
    - collisionless TIM - CTIM or CTPM (collisionless trapped-particle mode)
    - dissipative TIM - DTIM or TIM
  - other (e.g. ubiquitous mode)

Introduction

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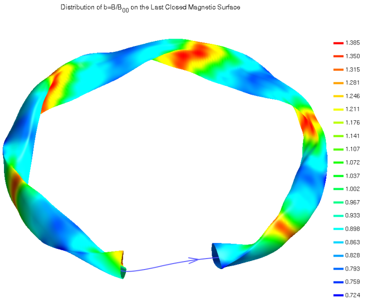
- How does this influence microinstabilities (and their turbulence)?

We will demonstrate:
Quasi-isodynamic stellarators are immune to the collisionless trapped-particle instability and the ordinary TEM.
Quasi-isodynamic stellarators: reduced transport for both neoclassical and turbulent channels



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Outline

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Properties of quasi-isodynamic stellarators needed

- bounce averaged radial drift vanishes

$$\frac{1}{\tau_b}\int_0^{\tau_b} \mathbf{v}_d \cdot \nabla\psi \, dt = 0.$$

(with bounce time τ_b and drift velocity $\mathbf{v}_d$)

- parallel adiabatic invariant J constant on flux-surfaces
- maximum-J configuration: J has a maximum on the magnetic axis and $\partial J/\partial\psi < 0$, beneficial for stability
- direction of the precessional drift for max-J configuration

$$\omega_{*a} \cdot \overline{\omega_{da}} < 0$$

with the magnetic drift frequency $\omega_{da} = \mathbf{k}_\perp \cdot \mathbf{v}_{da}$ and the drift wave frequency $\omega_{*a} = (T_a k_\alpha/e_a) d \ln n_a/d\psi$

Outline

The set of equations

- Electrostatic collisionless gyro-kinetic equation in ballooning space

$$iv_{\|}\nabla_{\|}g_a + (\omega - \omega_{da})g_a = \frac{e_a\phi}{T_a}J_0(k_\perp v_\perp/\Omega_a)\left(\omega - \omega_{*a}^T\right)f_{a0}$$

with

$$f_a = f_{a0} - \frac{e_a\phi}{T_a}f_{a0} + \hat{g}_{a0}$$

$$\eta_a = d\ln T_a/d\ln n_a$$

$$\omega_{*a}^T = \omega_{*a}[1+\eta_a(x^2-3/2)], x^2 = m_a v^2/2T_a$$

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- Close the system with the quasi-neutrality equation

$$\sum_a \frac{n_a e_a^2}{T_a}\phi = \sum_a e_a \int g_a J_0 \mathrm{d}^3 v$$

The energy budget of the system

- local change of energy of the particle guiding centres species a

$$\frac{\mathrm{d}E_a}{\mathrm{d}t} = -\frac{e_a}{2}\int \mathrm{d}^3v \left[\hat{g}_a(\mathbf{R},t)\left(v_{\|}\hat{\mathbf{b}} + \mathbf{v}_{da}\right)\cdot\nabla\langle\hat{\phi}^*\rangle_{\mathbf{R}} + c.c.\right]$$

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$$\langle\hat{\phi}\rangle_{\mathbf{R}} \approx \left\langle \phi(\mathbf{R}) e^{i(S(\mathbf{R}+\boldsymbol{\rho})-\omega t)}\right\rangle_{\mathbf{R}} = \phi(\mathbf{R}) e^{i(S(\mathbf{R})-\omega t)} J_0\left(k_{\perp} v_{\perp}/\Omega_a\right)$$

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- Integrate along the field line $\int \frac{\mathrm{d}l}{B}$, average in time over one period and obtain with $\{\ldots\} = \int\frac{\mathrm{d}l}{B}\int \mathrm{d}^3v(\ldots)$

$$P_a = \frac{\Omega_a}{2\pi}\int_0^{2\pi/\Omega_a}\mathrm{d}t\int\frac{\mathrm{d}l}{B}\frac{\mathrm{d}E_a}{\mathrm{d}t} = e_a\mathrm{Im}\left\{(iv_\parallel\nabla_\parallel g_a - \omega_{da}g_a)\phi^* J_0\right\}$$

Energy budget obtained from the GK equation

- obtain P_a from gyro-kinetic equation:
  - multiply by $e_a J_0 \phi^*$ and sum over all species
  - integrate over velocity space and along the field line
  - take the imaginary part
  - define $\omega = \omega_r + i\gamma$

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  - define $\omega = \omega_r + i\gamma$
- obtain a relation that describes the energy budget of the fluctuations

$$-\sum_a P_a = \gamma \sum_a \frac{n_a e_a^2}{T_a} \int \frac{\mathrm{d}l}{B} (1 - \Gamma_0) |\phi|^2$$

where

$$\Gamma_0(b) = n_a^{-1} \int J_0^2 f_{a0} d^3 v < 1$$

and

$$b = k_\perp (T_a/m_a)^{1/2} / \Omega_a.$$

Outline

For low frequencies $\omega \ll \omega_{ba}$

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  - electrons in the case of ordinary TEMs
  - both species for the collisionless trapped-ion modes ('collisionless trapped-particle instability')

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  - electrons in the case of ordinary TEMs
  - both species for the collisionless trapped-ion modes ('collisionless trapped-particle instability')
- Expanding the distribution function, $g_a = g_{a0} + g_{a1} + \cdots$, gives

$$g_{a0} = \frac{e_a \overline{J_0 \phi}}{T_a} \frac{\omega - \omega_{*a}^T}{\omega - \overline{\omega}_{da}} f_{a0}$$

$$i v_\parallel \nabla_\parallel g_{a1} = (\omega - \omega_{*a}^T) \frac{e_a}{T_a} \left(J_0 \phi - \frac{\omega - \omega_{da}}{\omega - \overline{\omega}_{da}} \overline{J_0 \phi} \right) f_{a0}$$

with the bounce average $\tau_b \overline{(\ldots)} = \oint (\ldots) \frac{\mathrm{d}l}{v_\parallel}$

P_a for low frequencies $\omega \ll \omega_{ba}$

- obtain for the energy transfer

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- assumptions:
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- energy flows from the electric field fluctuations to plasma species a

Resilience against C-TPM and TEM

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Extension of an old result by Rosenbluth [Phys. Fluids **11**, 869 (1968)] to

- an arbitrary number of particle species,
- finite $k_\perp \rho_a$,
- finite $\eta_a < 2/3$
- finite ω/ω_{da}

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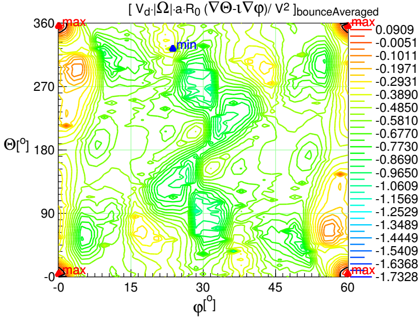
Outline

Ideal quasi-isodynamic stellarators

- for one of the most recent optimised stellarator configurations [Subbotin *et al.*, Nucl. Fusion **46**, (2006)]
- $\omega_{*a} \cdot \overline{\omega_{da}} < 0$ basically everywhere
- TEMs should be stable (awaiting confirmation through full flux surface GENE simulations, PhD project Greifswald)



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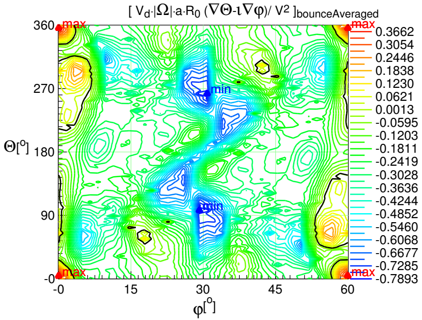

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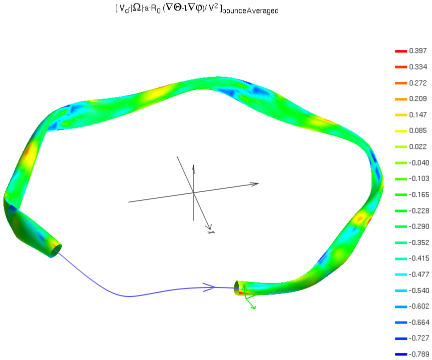

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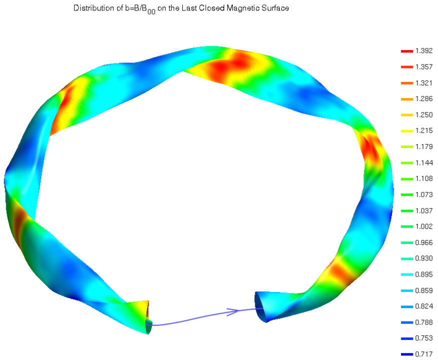

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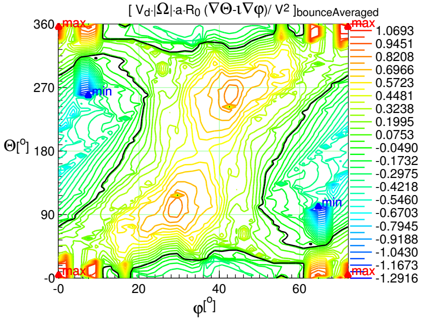

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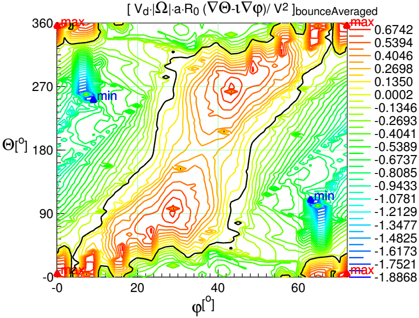

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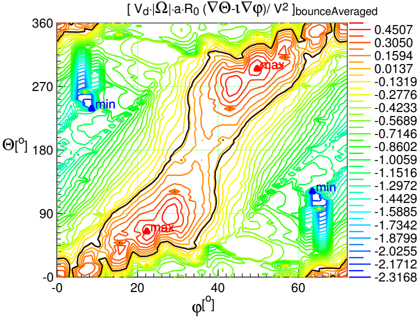

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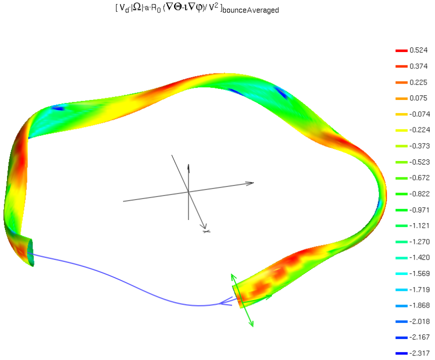

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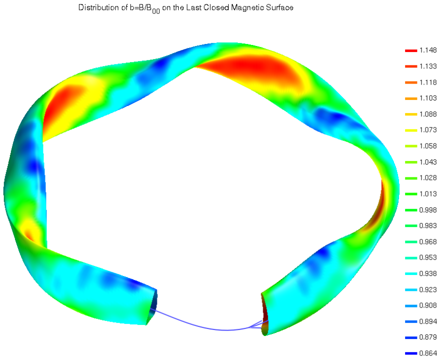

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Outline

Conclusions and Outlook

Conclusions:

- for low frequencies, both the electrostatic collisionless trapped-particle instability and the ordinary TEM are stable in quasi-isodynamic configurations
- and more generally, in any omnigeneous, maximum-J configuration
- quasi-isodynamic stellarators: expect reduced neoclassical and TEM turbulent transport

Outlook:

- GENE simulations
- maybe *some* instability due to some paths with $\omega_{*e}\overline{\omega_{de}} > 0$
- consider ion-driven instabilities with frequencies $\omega \simeq \omega_{bi}$