Trapped-particle instabilities in quasi-isodynamic stellarators

Josefine H. E. Proll, Per Helander, Jack W. Connor, Gabriel G. Plunk

IPP Greifswald
Stellarator Theory

Vienna, 19 March 2012
Introduction

- optimised stellarators: reduced neoclassical transport
- How does this influence microinstabilities (and their turbulence)?

quasi-isodynamic stellarator, $\beta \approx 4\%$, figure courtesy of Y. Turkin
Introduction

Electrostatic microinstabilities

- ion temperature gradient mode (ITG)
- electron temperature gradient mode (ETG)
- trapped-particle modes
  - trapped-electron mode (TEM)
    - collisionless TEM - CTEM
    - dissipative TEM - DTEM
  - trapped-ion mode (TIM)
    - collisionless TIM - CTIM or CTPM (collisionless trapped-particle mode)
    - dissipative TIM - DTIM or TIM
- other (e.g. ubiquitous mode)
Introduction

- optimised stellarators: reduced neoclassical transport
- How does this influence microinstabilities (and their turbulence)?

We will demonstrate:
Quasi-isodynamic stellarators are immune to the collisionless trapped-particle instability and the ordinary TEM.
Quasi-isodynamic stellarators: reduced transport for both neoclassical and turbulent channels

Quasi-isodynamic stellarator, $\beta \approx 4\%$, figure courtesy of Y. Turkin
Outline

Introduction

Properties of quasi-isodynamic stellarators

The energy budget of the instability

$g_a$ for low frequencies

Practical implications

Conclusions and Outlook
Outline

Introduction

Properties of quasi-isodynamic stellarators

The energy budget of the instability

$g_a$ for low frequencies

Practical implications

Conclusions and Outlook
Properties of quasi-isodynamic stellarators needed

- bounce averaged radial drift vanishes

\[
\frac{1}{\tau_b} \int_0^{\tau_b} v_d \cdot \nabla \psi \, dt = 0.
\]

(with bounce time \( \tau_b \) and drift velocity \( v_d \))

- parallel adiabatic invariant \( J \) constant on flux-surfaces

- maximum-\( J \) configuration: \( J \) has a maximum on the magnetic axis and \( \frac{\partial J}{\partial \psi} < 0 \), beneficial for stability

- direction of the precessional drift for max-\( J \) configuration

\[
\omega_{*a} \cdot \overline{\omega_{da}} < 0
\]

with the magnetic drift frequency \( \omega_{da} = \mathbf{k}_\perp \cdot \mathbf{v}_{da} \) and the drift wave frequency \( \omega_{*a} = (T_a k_\alpha / e_a) d \ln n_a / d\psi \)
Outline

Introduction

Properties of quasi-isodynamic stellarators

The energy budget of the instability

$g_a$ for low frequencies

Practical implications

Conclusions and Outlook
The set of equations

1. **Electrostatic collisionless gyro-kinetic equation in ballooning space**

   \[ iv_{||} \nabla_{||} g_a + (\omega - \omega_{da}) g_a = \frac{e_a \phi}{T_a} J_0 \left( k_{\perp} v_{\perp} / \Omega_a \right) \left( \omega - \omega_{*a}^T \right) f_{a0} \]

   with

   \[ f_a = f_{a0} - \frac{e_a \phi}{T_a} f_{a0} + \hat{g}_{a0} \]

   \[ \eta_a = \frac{d \ln T_a}{d \ln n_a} \]

   \[ \omega_{*a}^T = \omega_{*a} \left[ 1 + \eta_a (x^2 - 3/2) \right], \quad x^2 = m_a v^2 / 2 T_a \]
The set of equations

- Electrostatic collisionless gyro-kinetic equation in ballooning space

\[ iv_\parallel \nabla_\parallel g_a + (\omega - \omega_{da}) g_a = \frac{e_a \phi}{T_\alpha} J_0 \left( k_\perp v_\perp / \Omega_a \right) \left( \omega - \omega^*_a \right) f_a \]

with

\[ f_a = f_a^0 - \frac{e_a \phi}{T_\alpha} f_a^0 + \hat{g}_a^0 \]

\[ \eta_a = d \ln T_a / d \ln n_a \]

\[ \omega^*_a = \omega^*_a [1 + \eta_a (x^2 - 3/2)], \quad x^2 = m_a v^2 / 2 T_a \]

- Close the system with the quasi-neutrality equation

\[ \sum_a n_a e_a^2 \phi = \sum_a e_a \int g_a J_0 d^3 v \]
The energy budget of the system

- local change of energy of the particle guiding centres species $a$

$$\frac{dE_a}{dt} = - \frac{e_a}{2} \int d^3v \left[ \hat{g}_a(R, t) \left( v_\parallel \hat{b} + v_{da} \right) \cdot \nabla \langle \hat{\phi}^* \rangle_R + c.c. \right]$$
The energy budget of the system

- local change of energy of the particle guiding centres species $a$

\[
\frac{dE_a}{dt} = -\frac{e_a}{2} \int d^3v \left[ \hat{g}_a(R, t) \left( v_\parallel \hat{b} + v_{da} \right) \cdot \nabla \langle \hat{\phi}^* \rangle_R + c.c. \right]
\]

- use eikonal representation with $\nabla_\parallel S = 0$ and $\nabla_\perp S = k_\perp$

\[
\langle \hat{\phi} \rangle_R \approx \left\langle \phi(R)e^{i(S(R+\rho)-\omega t)} \right\rangle_R = \phi(R)e^{i(S(R)-\omega t)} J_0 \left( k_\perp v_\perp / \Omega_a \right)
\]

\[
\hat{g}_a(R, t) = g_a(R)e^{i(S(R)-\omega t)}
\]
The energy budget of the system

- local change of energy of the particle guiding centres species \( a \)

\[
\frac{dE_a}{dt} = -\frac{e_a}{2} \int d^3v \left[ \hat{g}_a(R, t) \left( v_\parallel \hat{b} + v_{da} \right) \cdot \nabla \langle \hat{\phi}^* \rangle_R + c.c. \right]
\]

- use eikonal representation with \( \nabla_\parallel S = 0 \) and \( \nabla_\perp S = k_\perp \)

\[
\langle \hat{\phi} \rangle_R \approx \left\langle \phi(R) e^{i(S(R+\rho) - \omega t)} \right\rangle_R = \phi(R) e^{i(S(R) - \omega t)} J_0 \left( k_\perp v_\perp / \Omega_a \right)
\]

\[
\hat{g}_a(R, t) = g_a(R) e^{i(S(R) - \omega t)}
\]

- Integrate along the field line \( \int \frac{dl}{B} \), average in time over one period and obtain with \( \{ \ldots \} = \int \frac{dl}{B} \int d^3v (\ldots) \)

\[
P_a = \frac{\Omega_a}{2\pi} \int_0^{2\pi/\Omega_a} dt \int \frac{dl}{B} \frac{dE_a}{dt} = e_a \text{Im} \left\{ (iv_\parallel \nabla_\parallel g_a - \omega_{da} g_a) \phi^* J_0 \right\}
\]
Energy budget obtained from the GK equation

- obtain $P_a$ from gyro-kinetic equation:
  - multiply by $e_a J_0 \phi^*$ and sum over all species
  - integrate over velocity space and along the field line
  - take the imaginary part
  - define $\omega = \omega_r + i\gamma$
Energy budget obtained from the GK equation

- obtain $P_a$ from gyro-kinetic equation:
  - multiply by $e_a J_0 \phi^*$ and sum over all species
  - integrate over velocity space and along the field line
  - take the imaginary part
  - define $\omega = \omega_r + i \gamma$

- obtain a relation that describes the energy budget of the fluctuations

$$- \sum_a P_a = \gamma \sum_a \frac{n_a e_a^2}{T_a} \int \frac{dl}{B} (1 - \Gamma_0) |\phi|^2$$

where

$$\Gamma_0(b) = n_a^{-1} \int J_0^2 f_{a0} d^3 v < 1$$

and

$$b = k_\perp (T_a / m_a)^{1/2} / \Omega_a.$$
Outline

Introduction

Properties of quasi-isodynamic stellarators

The energy budget of the instability

$g_a$ for low frequencies

Practical implications

Conclusions and Outlook
For low frequencies $\omega \ll \omega_{ba}$

- consider $\omega \ll \omega_{ba}$, e.g.
  - electrons in the case of ordinary TEMs
  - both species for the collisionless trapped-ion modes ('collisionless trapped-particle instability')
For low frequencies $\omega \ll \omega_{ba}$

- consider $\omega \ll \omega_{ba}$, e.g.
  - electrons in the case of ordinary TEMs
  - both species for the collisionless trapped-ion modes ('collisionless trapped-particle instability')
- Expanding the distribution function, $g_a = g_{a0} + g_{a1} + \cdots$, gives

\[
g_{a0} = \frac{e_a J_0 \phi \omega - \omega_{*a}}{T_a} \frac{\omega - \bar{\omega}_{da}}{\omega} f_{a0}
\]

\[
iv_{\parallel} \nabla_{\parallel} g_{a1} = (\omega - \omega_{*a}) \frac{e_a}{T_a} \left( J_0 \phi - \frac{\omega - \omega_{da}}{\omega - \bar{\omega}_{da}} J_0 \phi \right) f_{a0}
\]

with the bounce average $\tau_b(\ldots) = \int (\ldots) \frac{dl}{v_{\parallel}}$
\( P_a \) for low frequencies \( \omega \ll \omega_{ba} \)

- obtain for the energy transfer

\[
P_a = \frac{e_a^2}{T_a} \text{Im} \left\{ (\omega - \omega_{*a}) \left( \frac{|J_0\phi|^2}{\omega - \overline{\omega}_{da}} - \frac{\omega |J_0\phi|^2}{\omega - \overline{\omega}_{da}} \right) f_{a0} \right\}
\]
\( P_a \) for low frequencies \( \omega \ll \omega_{ba} \)

- obtain for the energy transfer

\[
P_a = \frac{e_a^2}{T_a} \text{Im} \left\{ (\omega - \omega_{*a}) \left( \frac{|J_0 \phi|^2}{\omega} - \frac{\omega |J_0 \phi|^2}{\omega - \omega_{da}} \right) f_a \right\}
\]

- approach marginal stability \( \gamma \to 0^+ \):

\[
P_a = \frac{\pi e_a^2}{T_a} \left\{ \delta (\omega - \overline{\omega}_{da}) \overline{\omega}_{da} (\overline{\omega}_{da} - \omega_{*a}) |J_0 \phi|^2 f_a \right\}
\]
For low frequencies $\omega \ll \omega_{ba}$

- obtain for the energy transfer

$$P_a = \frac{e_a^2}{T_a} \text{Im} \left\{ (\omega - \omega_{*a}) \left( \frac{|J_0\phi|^2}{\omega - \overline{\omega}_{da}} - \frac{\omega |J_0\phi|^2}{\omega - \overline{\omega}_{da}} \right) f_{a0} \right\}$$

- approach marginal stability $\gamma \to 0^+$:

$$P_a = \frac{\pi e_a^2}{T_a} \left\{ \delta(\omega - \overline{\omega}_{da}) \overline{\omega}_{da} (\overline{\omega}_{da} - \omega_{*a}) |J_0\phi|^2 f_{a0} \right\}$$

- assumptions:
  - low temperature gradients $0 < \eta_a < 2/3$
  - quasi-isodynamic configurations $\omega_{*a} \overline{\omega}_{da} < 0$

$\Rightarrow P_a > 0$ for $\omega \ll \omega_{ba}$
**$P_a$ for low frequencies $\omega \ll \omega_{ba}$**

- obtain for the energy transfer

\[
P_a = \frac{e_a^2}{T_a} \text{Im} \left\{ \left( \omega - \omega_{T,a} \right) \left( \left| J_0 \phi \right|^2 - \frac{\omega \left| J_0 \phi \right|^2}{\omega - \bar{\omega}_{da}} \right) f_{a0} \right\}
\]

- approach marginal stability $\gamma \to 0+$:

\[
P_a = \frac{\pi e_a^2}{T_a} \left\{ \delta(\omega - \bar{\omega}_{da}) \bar{\omega}_{da}(\bar{\omega}_{da} - \omega_{T,a}) \left| J_0 \phi \right|^2 f_{a0} \right\}
\]

- assumptions:
  - low temperature gradients $0 < \eta_a < 2/3$
  - quasi-isodynamic configurations $\omega_{T,a} \bar{\omega}_{da} < 0$

  \[\Rightarrow P_a > 0 \text{ for } \omega \ll \omega_{ba}\]

- energy flows from the electric field fluctuations to plasma species $a$
Resilience against C-TPM and TEM

- for $\omega \ll \omega_{bi}, \omega_{be}$
- both $P_i, P_e > 0$
Resilience against C-TPM and TEM

- for $\omega \ll \omega_{bi}, \omega_{be}$
  - both $P_i, P_e > 0$
  - remember $\sum_a P_a \propto \gamma = 0$
Resilience against C-TPM and TEM

- for $\omega \ll \omega_{bi}, \omega_{be}$
  - both $P_i, P_e > 0$
  - remember $\sum_a P_a \propto \gamma = 0$
  - **no** point of marginal stability for the C-TPM
Resilience against C-TPM and TEM

- for $\omega \ll \omega_{bi}, \omega_{be}$
  - both $P_i, P_e > 0$
  - remember $\sum_a P_a \propto \gamma = 0$
  - no point of marginal stability for the C-TPM

- for $\omega_{bi} \sim \omega \ll \omega_{be}$
  - only $P_e > 0$: electrons are stabilizing
Resilience against C-TPM and TEM

- for $\omega \ll \omega_{bi}, \omega_{be}$
  both $P_i, P_e > 0$
  remember $\sum_a P_a \propto \gamma = 0$
  no point of marginal stability for the C-TPM

- for $\omega_{bi} \approx \omega \ll \omega_{be}$
  only $P_e > 0$: electrons are stabilizing
  no ordinary TEM, only ion-driven instabilities
Resilience against C-TPM and TEM

- for $\omega \ll \omega_{bi}, \omega_{be}$
  - both $P_i, P_e > 0$
  - remember $\sum_a P_a \propto \gamma = 0$
  - no point of marginal stability for the C-TPM
- for $\omega_{bi} \simeq \omega \ll \omega_{be}$
  - only $P_e > 0$: electrons are stabilizing
  - no ordinary TEM, only ion-driven instabilities

Extension of an old result by Rosenbluth [Phys. Fluids 11, 869 (1968)] to

- an arbitrary number of particle species,
- finite $k_\perp \rho_a$,
- finite $\eta_a < 2/3$
- finite $\omega/\omega_{da}$
Outline

Introduction

Properties of quasi-isodynamic stellarators

The energy budget of the instability

$g_a$ for low frequencies

Practical implications

Conclusions and Outlook
Ideal quasi-isodynamic stellarators

- for one of the most recent optimised stellarator configurations [Subbotin et al., Nucl. Fusion 46, (2006)]
- $\omega_a \cdot \overline{\omega_{da}} < 0$ basically everywhere
- TEMs should be stable (awaiting confirmation through full flux surface GENE simulations, PhD project Greifswald)

$\overline{\omega_{de}}$ of the quasi-isodynamic stellarator, $\beta \approx 4\%$, figure courtesy of Y. Turkin
Ideal quasi-isodynamic stellarators

- for one of the most recent optimised stellarator configurations [Subbotin et al., Nucl. Fusion 46, (2006)]
- \( \omega_{*a} \cdot \overline{\omega_{da}} < 0 \) basically everywhere
- TEMs should be stable (awaiting confirmation through full flux surface GENE simulations, PhD project Greifswald)

\( \overline{\omega_{de}} \) of the quasi-isodynamic stellarator, \( \beta \approx 2\% \), figure courtesy of Y. Turkin
Ideal quasi-isodynamic stellarators

- for one of the most recent optimised stellarator configurations [Subbotin et al., Nucl. Fusion 46, (2006)]
- \( \omega_{*a} \cdot \omega_{da} < 0 \) basically everywhere
- TEMs should be stable (awaiting confirmation through full flux surface GENE simulations, PhD project Greifswald)

\( \bar{\omega}_{de} \) of the quasi-isodynamic stellarator, \( \beta \approx 2\% \), figure courtesy of Y. Turkin
Ideal quasi-isodynamic stellarators

- for one of the most recent optimised stellarator configurations [Subbotin et al., Nucl. Fusion 46, (2006)]
- $\omega_a \cdot \bar{\omega} < 0$ basically everywhere
- TEMs should be stable (awaiting confirmation through full flux surface GENE simulations, PhD project Greifswald)

$B$ of the quasi-isodynamic stellarator, $\beta \approx 2\%$, figure courtesy of Y. Turkin
Wendelstein 7-X

- Wendelstein 7-X: not perfectly quasi-isodynamic (see Per’s talk)
- improves for higher $\beta$-values
- awaiting full flux surface GENE simulations

$\bar{\omega_{de}}$ of SC- W7-X with $\beta = 0\%$, figure courtesy of Y. Turkin
Wendelstein 7-X

- Wendelstein 7-X: not perfectly quasi-isodynamic (see Per’s talk)
- improves for higher $\beta$-values
- awaiting full flux surface GENE simulations

$\bar{\omega}_{de}$ of SC- W7-X with $\beta = 2\%$, figure courtesy of Y. Turkin
Wendelstein 7-X

- Wendelstein 7-X: not perfectly quasi-isodynamic (see Per’s talk)
- improves for higher $\beta$-values
- awaiting full flux surface GENE simulations

$\bar{\omega}_{de}$ of SC-W7-X with $\beta = 4\%$, figure courtesy of Y. Turkin
Wendelstein 7-X

- Wendelstein 7-X: not perfectly quasi-isodynamic (see Per’s talk)
- improves for higher $\beta$-values
- awaiting full flux surface GENE simulations

$\overline{\omega_{de}}$ of SC-W7-X with $\beta = 4\%$, figure courtesy of Y. Turkin
Wendelstein 7-X

- Wendelstein 7-X: not perfectly quasi-isodynamic (see Per’s talk)
- improves for higher $\beta$-values
- awaiting full flux surface GENE simulations

$B$ of SC- W7-X with $\beta = 4\%$, figure courtesy of Y. Turkin
Outline

Introduction

Properties of quasi-isodynamic stellarators

The energy budget of the instability

$g_a$ for low frequencies

Practical implications

Conclusions and Outlook
Conclusions and Outlook

Conclusions:
- for low frequencies, both the electrostatic collisionless trapped-particle instability and the ordinary TEM are stable in quasi-isodynamic configurations
- and more generally, in any omnigeneous, maximum-$J$ configuration
- quasi-isodynamic stellarators: expect reduced neoclassical and TEM turbulent transport

Outlook:
- GENE simulations
- maybe some instability due to some paths with $\omega e\omega_{de} > 0$
- consider ion-driven instabilities with frequencies $\omega \simeq \omega_{bi}$