Zonal flows in stellarators at finite ambient radial electric field

A. Mishchenko

Motivation

- Mictroturbulence may affect stellarator confinement
- Zonal flows affect the microturbulence
- Zonal flow is a linear plasma mode (which can be nonlinearly dirven)
- Study of linear zonal-flow evolution helps to develop gyrokinetic codes (Rosenbluth-Hinton test in tokamaks)
- Certain physical intuition can result already from a linear analysis
  (Examples:
  Dimits shift with and without collisions;
  correlation between value of Dimits shift and linear residual potential)

Zonal flow in stellarators

- Importance of locally-reflected particles for the residual potential (Sugama et al, 2006, LHD; Mynick 2006)
- Importance of ambient electric field for the residual potential (Sugama et al, 2009, LHD)
- Linear zonal flow is oscillatory in stellarators (Mishchenko et al, 2008)
- The oscillations are damped (no electric field; Helander et al, 2010)
- Global linear PIC simulations (Kleiber et al, 2010)
- Flux-tube nonlinear Eulerian simulations (Xanthopoulos et al, 2011)

Physical "ingridients"

- Particle orbits: passing, toroidicity-reflected (trapped), locally-reflected
- Bounce-averaged motion: toroidal precession, radial drifts
- Radial bounce-averaged drifts $\Rightarrow$ non-ambipolar currents
- Non-ambipolar currents $\Rightarrow$ ambient radial electric field
- Ambient radial electric field $\Rightarrow$ particle drift orbits changed

All the "ingridients" depend on macroscopic properties of plasma (magnetic configuration, density, temperature etc) and affect the zonal flow evolution

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Solution of kinetic-quasineutrality equations

IPP

- **Solve linearized gyrokinetic equation ...**

$$\frac{\partial f_1}{\partial t} + v_\parallel \nabla_\parallel f_1 + \vec{v}_d \cdot \nabla f_1 + \vec{v}_E \cdot \nabla f_1 = -f_0\, \vec{v}_d \cdot \nabla \langle \hat{\phi} \rangle\ , \quad \hat{\phi} = \frac{e\phi}{T} \tag{1}$$

- **... employing Boozer coordinates, $h = f_1 + f_0 \langle \hat{\phi} \rangle$ and $\Omega_E = \Phi'/\psi_0$ ...**

$$\frac{\partial h_k}{\partial t} + \frac{v_\parallel B}{I} \frac{\partial h_k}{\partial \zeta} + \left(\Omega_E - \frac{v_\parallel B}{\psi_0} \frac{\partial \rho_\parallel}{\partial s} \right) \frac{\partial h_k}{\partial \alpha} + i\omega_r h_k = \frac{\partial \phi_k}{\partial t} J_{0g} \tag{2}$$

- **... using an eikonal approximation for radial direction ...**

$$\frac{\partial h_k}{\partial t} + \frac{v_\parallel B}{I} \frac{\partial h_k}{\partial \zeta} + \left(\Omega_E - \frac{v_\parallel B}{\psi_0} \frac{\partial \rho_\parallel}{\partial s} \right) \frac{\partial h_k}{\partial \alpha} + i\omega_r h_k = \frac{\partial \phi_k}{\partial t} J_{0g} \tag{3}$$

Solution of kinetic-quasineutrality equations

- **Derive the lowest-order bounce-averaged kinetic equations**

$$\frac{\partial \hat{h}_k^{(0)}}{\partial t} + \Omega_E \frac{\partial \hat{h}_k^{(0)}}{\partial \alpha} + i \langle \omega_r \rangle_b \hat{h}_k^{(0)} = \frac{\partial \phi_k}{\partial t} J_{0g} J_{0b} \tag{4}$$

- **Solve it introducing coordinates**

$$\eta = \alpha - \Omega_E t \; , \quad \tau = t \tag{5}$$

- **The solution is [here $\Delta_r(\tau) = \int_0^\tau \langle v_r \rangle_b(\tau) \mathrm{d}\tau'$ is a radial width of drift orbit]**

$$\hat{h}_k^{(0)} = \hat{h}_k^{(0)}(\tau = 0) + \int_0^\tau \mathrm{d}\tau' \frac{\partial \phi_k}{\partial \tau'} J_{0g} J_{0b} \exp\{i\, k_r [\Delta_r(\tau') - \Delta_r(\tau)]\} \tag{6}$$

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Solution of kinetic-quasineutrality equations

- **Substitute the solution to the quasineutrality condition**

$$-n_0\phi_k + \left\langle \int \mathrm{d}^3v\, h_k f_0 J_{0g} \right\rangle^{(ions)} = 0 \tag{7}$$

- **The resulting equation determines the evolution of the zonal flow.**

$$-n_0\phi_k + \langle k_r^2\rho_i^2\rangle n_0\phi_0 + \int_0^{\tau} \mathrm{d}\tau' \, \frac{\partial \phi_k}{\partial \tau'} \left\{ J_{0g}^2\, J_{0b}^2 \exp\left(ik_r[\Delta_r(\tau') - \Delta_r(\tau)]\right) \right\} = 0$$

- **Approximate the bounce-averaged drift velocity $\langle v_r\rangle_b = V_r(s)\sin\alpha$. Then**

$$-n_0\phi_k + \langle k_r^2\rho_i^2\rangle n_0\phi_0 + \int_0^{\tau} \mathrm{d}\tau' \, \frac{\partial \phi_k}{\partial \tau'} \left\{ J_{0g}^2\, J_{0b}^2\, J_0\Big[K(\tau-\tau')\, d_r\Big] \right\} = 0$$

$$K(\tau) = 2k_r \sin\left(\frac{\Omega_E \tau}{2}\right) \ , \quad d_r = V_r/\Omega_E$$

Solution of kinetic-quasineutrality equations

- **Include the toroidal precession drift** $\Omega_E \Rightarrow \Omega_E + \bar{\omega}_\alpha(v)$; $\bar{\omega}_\alpha(v) \sim v^2$

- **Find the solution using Laplace transform (assume $k_r d_r \ll 1$)**

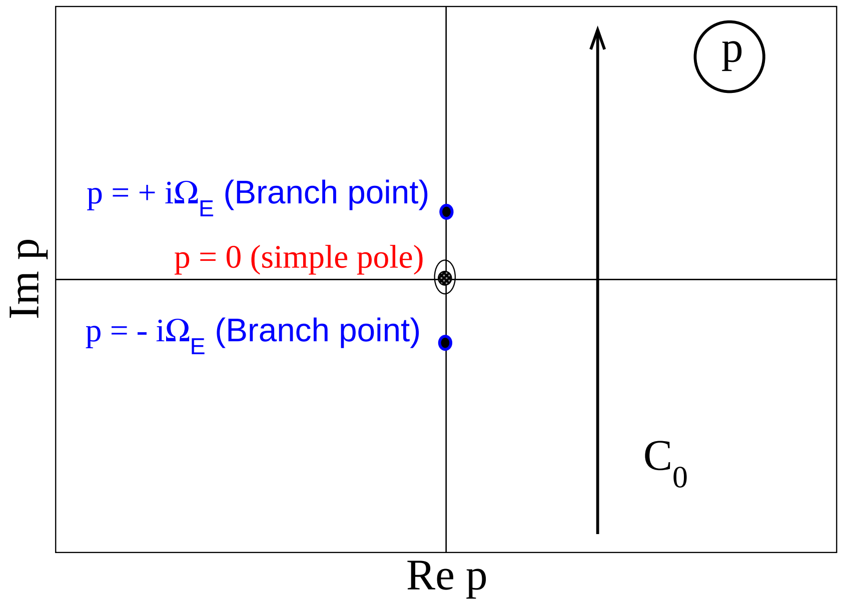
$$\Phi(p) = \frac{\phi_0}{p}\left[1 + \frac{\{\delta_r^2\}}{\langle\rho_i^2\rangle} + \frac{1}{\langle\rho_i^2\rangle}\left\langle \int \frac{\Omega_E^2\, d_r^2\, f_{0i}}{p^2 + [\Omega_E^2 + \bar{\omega}_\alpha(v)]^2}\right\rangle \mathrm{d}^3 v\right]^{-1} \tag{8}$$

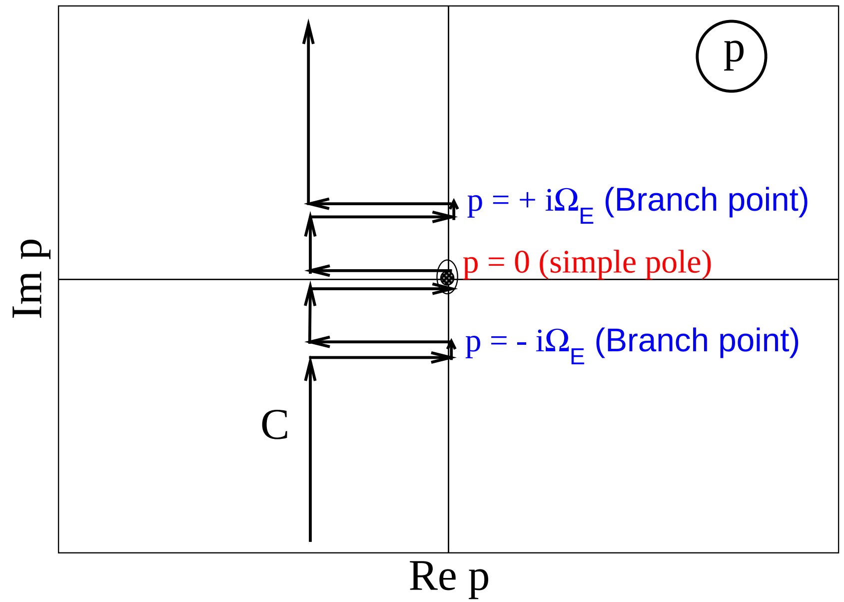
- **Following velocity integral appears:**

$$I = \int_0^\infty \frac{v^6\, e^{-v^2}\, \mathrm{d}v}{(\omega_\alpha v^2 + \Omega_E + ip)\,(\omega_\alpha v^2 + \Omega_E - ip)} \tag{9}$$

- **The integrand has poles** $v_{1,2} = \pm\,(\Omega_E/\omega_\alpha + i\,p/\omega_\alpha)^{1/2}$

Inverse Laplace transform



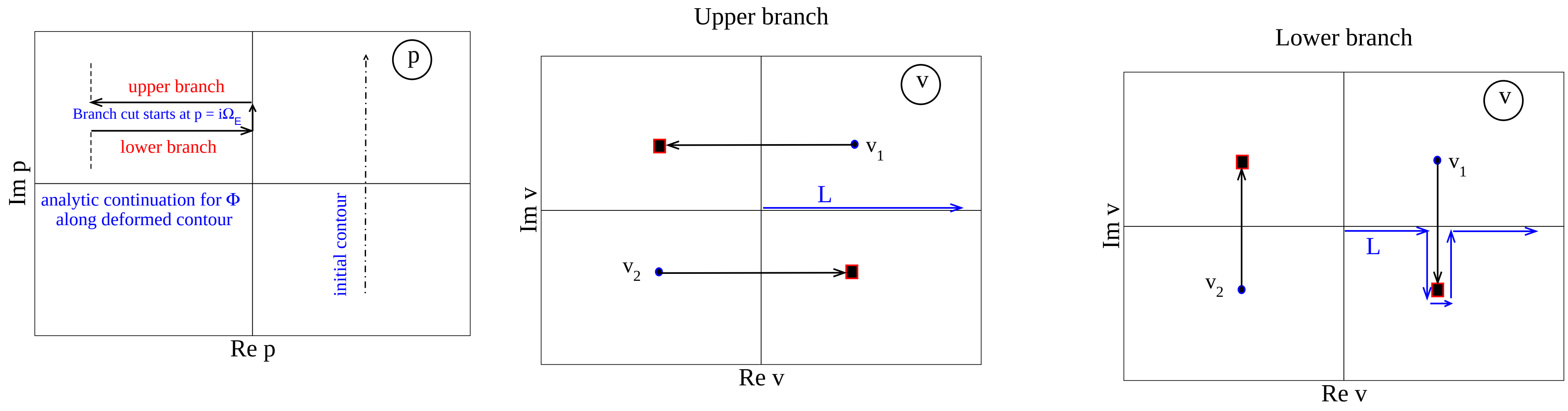


$$\phi(t) = \frac{1}{2\pi i} \int_C \Phi(p)\, e^{pt}\, \mathrm{d}p \tag{10}$$

Modify the integration contour $C_0 \Rightarrow C$;

Pole and branch cuts must be taken into account

Analytic continuation for $\Phi(p)$



$$\Phi = \frac{\phi_0}{p}\left[c_0 + c_1\left\langle \int_{trapped} d\lambda \int_0^\infty \frac{x^6 e^{-x^2} dx}{p^2 + (\Omega_E + \omega_\alpha x^2)^2}\right\rangle\right]^{-1}, \quad x = v/v_{\mathrm{th}} \qquad (11)$$

Poles in v-space move when p-contour is shifted;

Analytic continuation for $\Phi(p)$ to $Rep \le 0$ must account for that

(this explains the branch cuts at $p = \pm i\,\Omega_E$)

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Long-time linear evolution of zonal flow

IPP

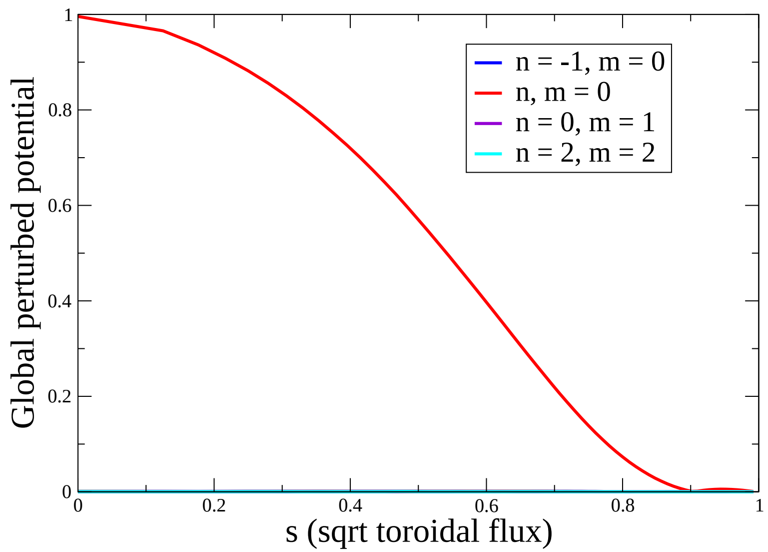
- Pole in complex p plane gives the residual potential
- Branch cuts give the algebraically-damped oscillations (phase mixing at large times)

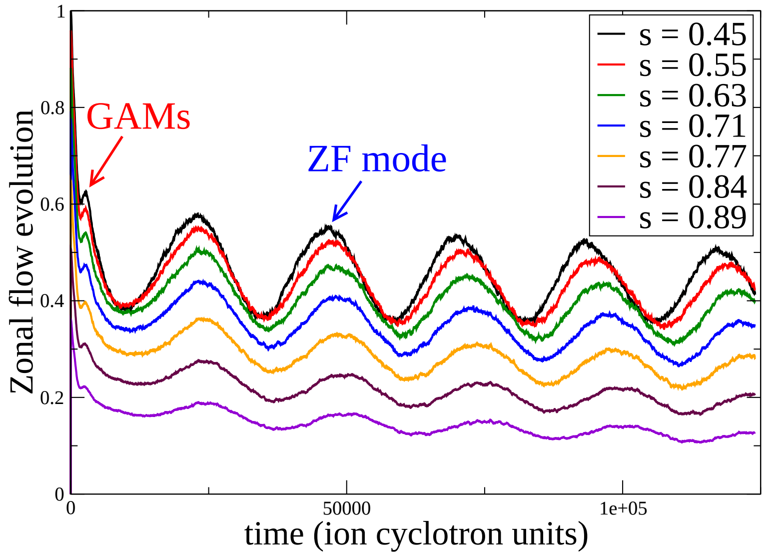
$$\phi(t) = \underbrace{\frac{\langle \rho_i^2 \rangle}{\langle \rho_i^2 \rangle + \{\delta_r^2\} + \{\bar{d}_r^2\}}}_{residual\ \ flow} + \underbrace{\frac{105\sqrt{\pi}}{32Q^2} \left\langle \frac{\bar{d}_r^2}{\langle \rho_i^2 \rangle} \frac{1}{(\omega_\alpha t)^{7/2}} \right\rangle \cos\left(\Omega_E t - \pi/4\right)}_{damped\ \ oscillations} \quad (12)$$

$$Q = 1 + \frac{\{\delta_r^2\}}{\langle \rho_i^2 \rangle} + \frac{3\sqrt{\pi}}{16} \frac{\Omega_E}{\langle \rho_i^2 \rangle} \left\langle \frac{\bar{d}_r^2}{\omega_\alpha} \right\rangle \quad (13)$$

Algebraic damping is a consequence of phase mixing in velocity space caused by precessional drift of locally-trapped particles

PIC simulations (EUTERPE)





- **Global PIC simulations (EUTERPE)**, **standard W7-X, adiabatic electrons;**
- **Initial condition** $\delta f(t=0) \sim \cos(\pi s^2/2)$ **(axially symmetric)**
- **Constant (radially) ambient electric field with** $\Omega_E/\omega_{ci} = 3.996 \times 10^{-5}$

All points oscillate with the same frequency Ω_E. Damping is very week.

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Qualitative properties of ZF evolution (analytics)

- **Properties of residual flow**

$$\phi_{\mathrm{res}}(t) = \frac{\langle \rho_i^2 \rangle}{\langle \rho_i^2 \rangle + \{\delta_r^2\} + \{V_r^2/\Omega_E^2\}} \tag{14}$$

  – Residual potential increases with ambient electric field

  – Residual potential does not depend on k_r (neither does oscillatory part) (both do in the absence of ambient electric field)

- **Properties of damped oscillations**

$$\phi_{\mathrm{osc}}(t) \sim 1/\Omega_E^2 \times \cos\left(\Omega_E t - \pi/4\right)/(\omega_\alpha t)^{7/2} \tag{15}$$

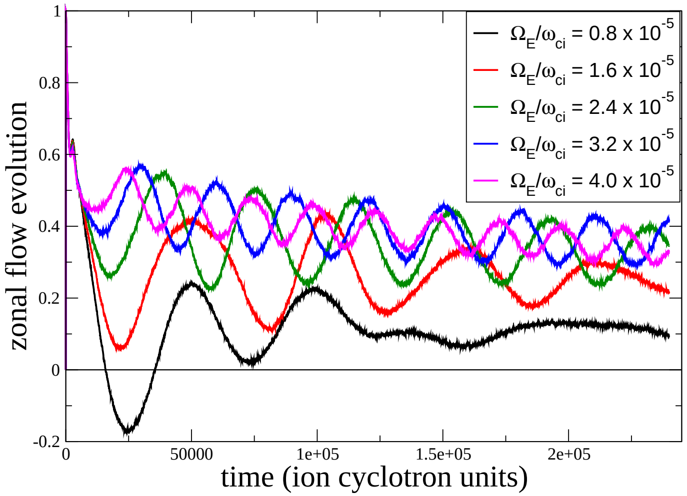
  – Oscillation frequency is given by the ambient field

  – Oscillation amplitude decreases with ambient electric field

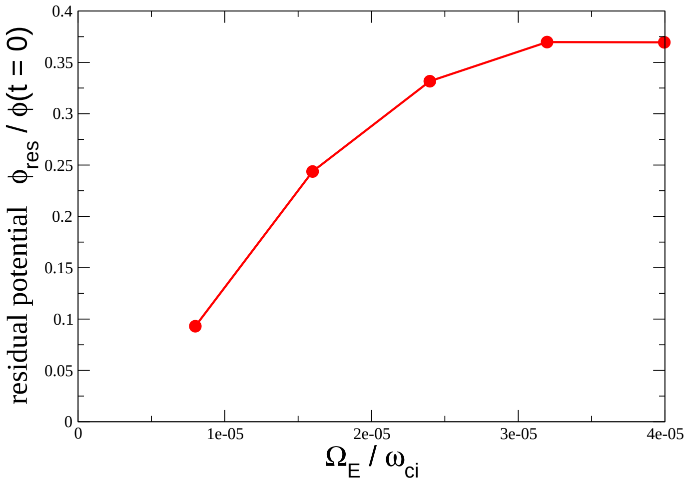
  – Algebraic damping is week since ω_α (toroidal precession) is small

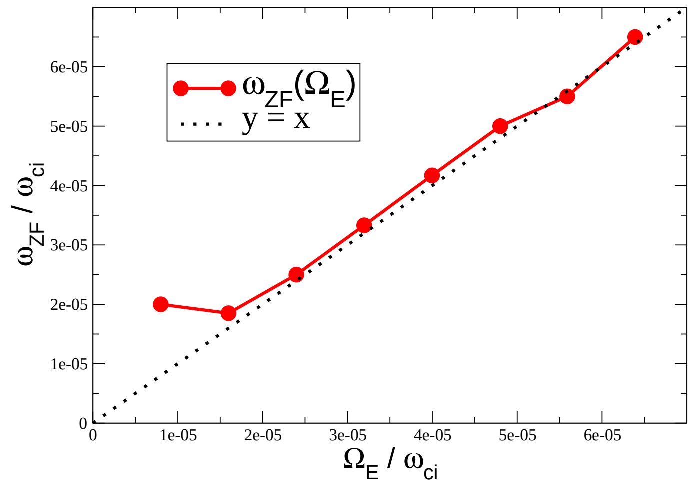
Qualitative properties of ZF evolution (simulations)

ZF at position s = 0.55



Residual potential





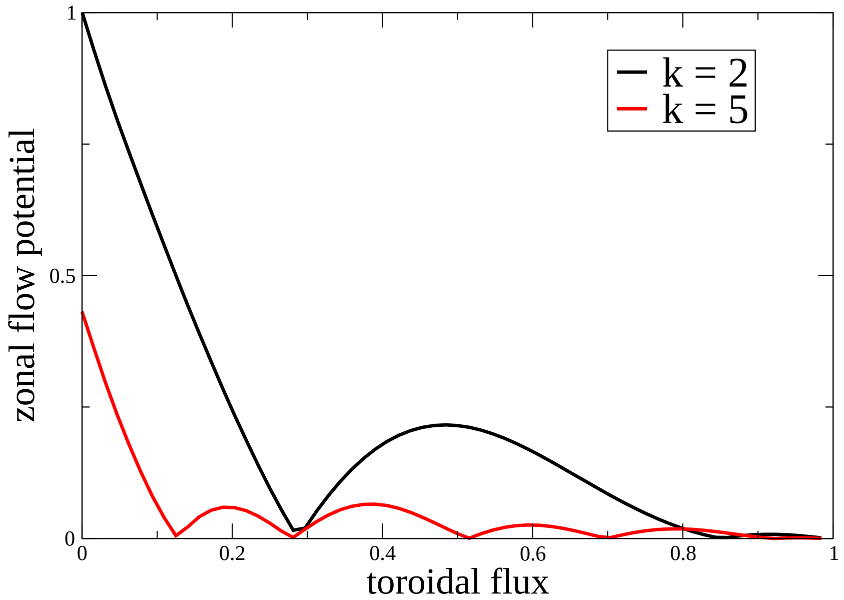
Residual flow increases with ambient electric field (to the "tokamak" asymptotic value)

ZF oscillation frequency is closely related to Ω_E (this relation is violated at small Ω_E)

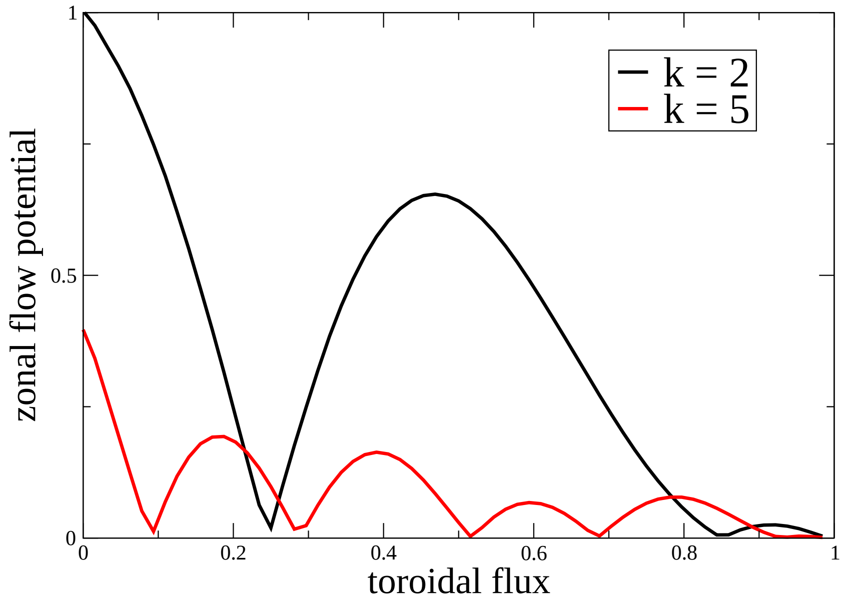
ZF ocsillation amlitude decreases with Ω_E

Qualitative properties of ZF evolution (simulations)

Large electric field $\Omega_E/\omega_{ci} = 4 \times 10^{-5}$



Small electric field $\Omega_E/\omega_{ci} = 0.8 \times 10^{-5}$

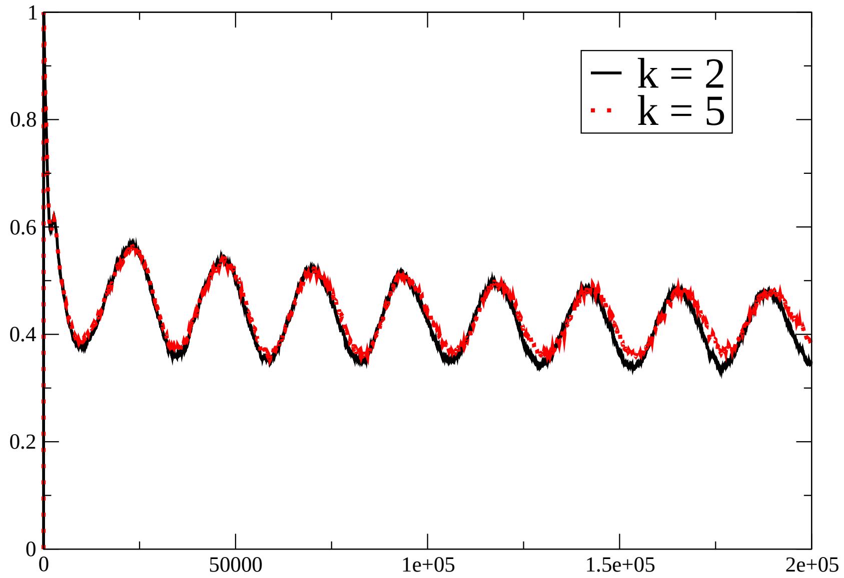


Simulations initialized with $\delta f(t=0) \sim \underline{\cos(k\Psi_{tor}/\Psi_0)}$

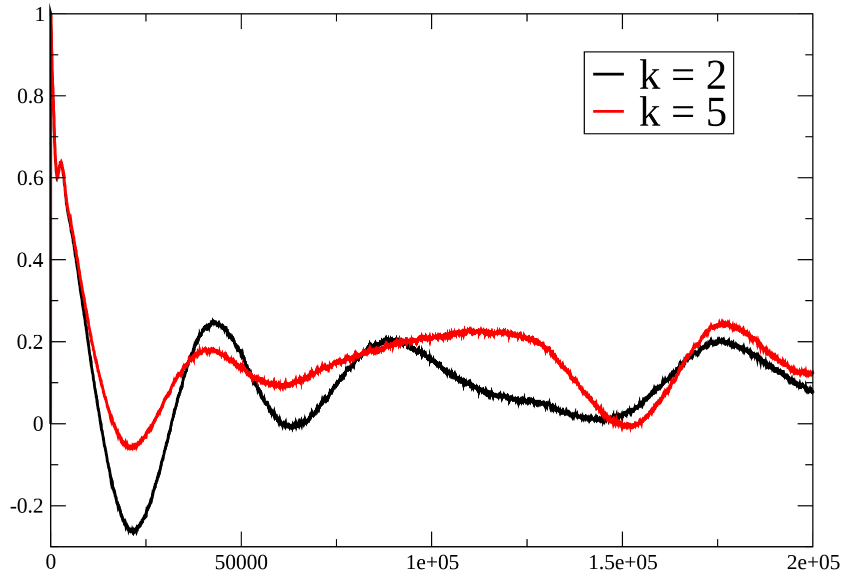
Compare evolition at different Mach numbers (ambient electric fields)
Look for dependence on the radial "wave number"
(related to initialization parameter k)

Qualitative properties of ZF evolution (simulations)

Large electric field $\Omega_E/\omega_{ci} = 4 \times 10^{-5}$



Small electric field $\Omega_E/\omega_{ci} = 0.8 \times 10^{-5}$



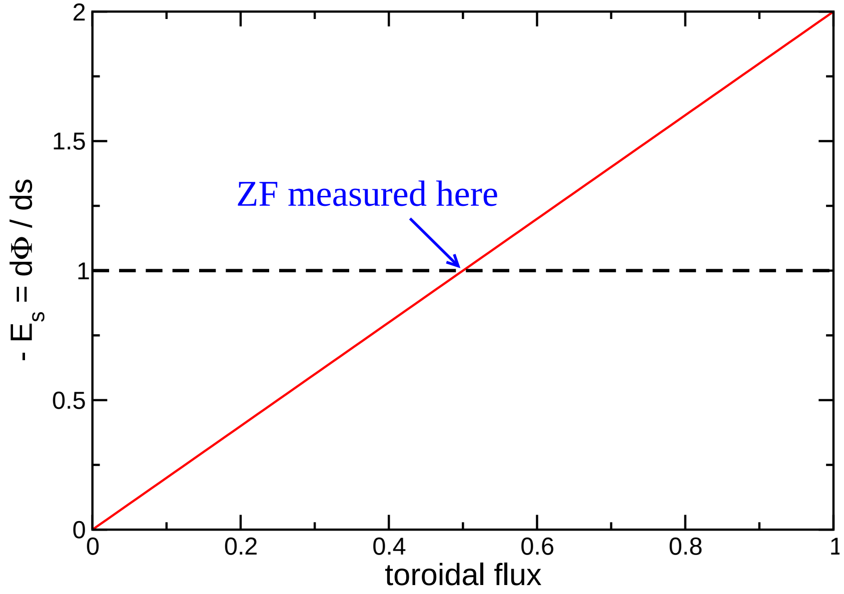
Simulations initialized with $\delta f(t=0) \sim \underline{\cos(k\Psi_{\rm tor}/\Psi_0)}$

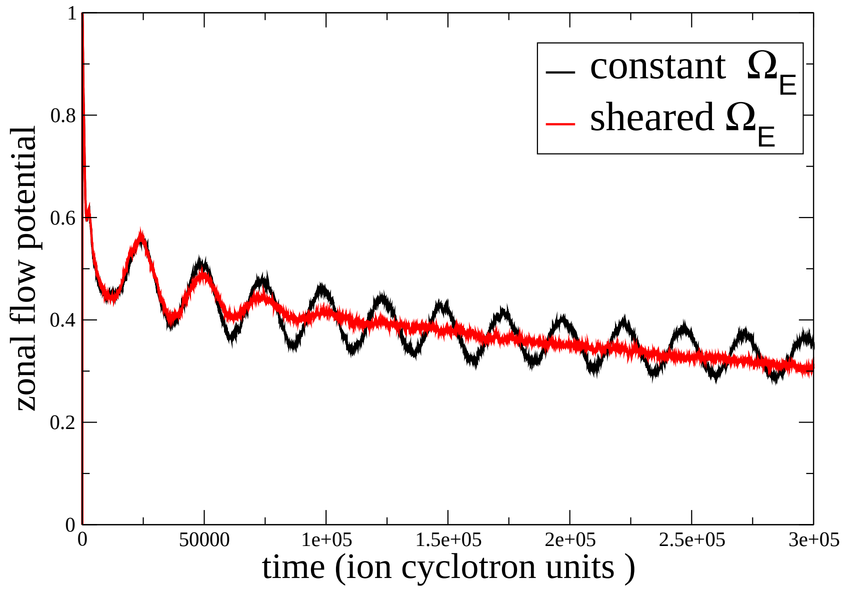
No radial wave-number dependence unless Ω_E is not too small!

("large" and "small" Mach numbers: $u_E^{(1)}/v_{\rm th\it i} = 0.018$, $u_E^{(2)}/v_{\rm th\it i} = 0.0036$)

Qualitative difference in evolution pattern at small ambient electric fields

Non-local effects in ZF evolution





Compare simulations with constant and sheared ambient electric field

Continuum damping of ZF mode at finite shear of Ω_E!

(“large” Mach number $u_E^{(1)}/v_{\mathrm{th}i} = 0.018$)

Stellarator ZF is sensitive to both value and shear of ambient electric field

Analityc approach to continuum damping

- Consider sheared ambient electric field $\Omega_E(r) = \Omega_E' \, r$

- Employ eikonal and write the quasineutrality equation in the form:

$$\frac{\mathrm{d}^2\bar{\Phi}(p,r)}{\mathrm{d}r^2} + \underline{k_r^2}\left[1 + \frac{\{\delta_r^2\}}{\langle\rho_i^2\rangle} + \frac{1}{\langle\rho_i^2\rangle}\left\langle\int\frac{\Omega_E^2(r)\, d_r^2\, f_{0i}}{p^2 + [\Omega_E^2(r) + \bar{\omega}_\alpha(v)]^2}\right\rangle \mathrm{d}^3v\right]\bar{\Phi}(p,r) =$$

$$= k_r^2\,\frac{\phi_0(r)}{p}\,e^{i\,k_r\,r}\,, \quad \bar{\Phi}(p,r) = \Phi(p,r)\,e^{i\,k_r\,r}$$

- This can be solved analytically assuming $k_r\,r_a \gg 1$ (WKB, Green functions)

- Transition points appear in position $p = \pm i\Omega_E \Rightarrow \sqrt{g_1(p)},\ \log[g_2(p)]\ldots$

- Inverse Laplace transform + Contour deformation + Watson Lemma $(t \to \infty) \Rightarrow$ continuum damping

Summary

- Zonal flows in stellarators are sensitive with respect to ambient electric field
- Linear zonal flow consists off a constant (residual) and oscillatory parts
- Residual part increases with electric field to some asimptotic value ("equivalent-tokamak")
- ZF oscillation is closely related (equal) to the ExB time scale
- ZF dynamics is qualitatively different at small Mach numbers
- Non-local effects (continuum damping) are important for ZF oscillations

Interpretations and Speculations

- Radial non-ambipolar currents play key role in stellarator ZF
- These currents make the linear ZF response non-static:
  phase mixing becomes important (both in velocity and coordinate spaces)
- Affecting non-ambipolar currents, one affects ZF $\Rightarrow$ anomalous transport
  (reminiscent to GAM/fast-particle idea in tokamaks)
- ZF is sensitive to magn. geometry (orbits) /collisionality . . . (electric field)
- Electrons can be important (their contribution to non-ambipolar current,
  phase mixing caused by electron precession etc)