Slow mode turbulence

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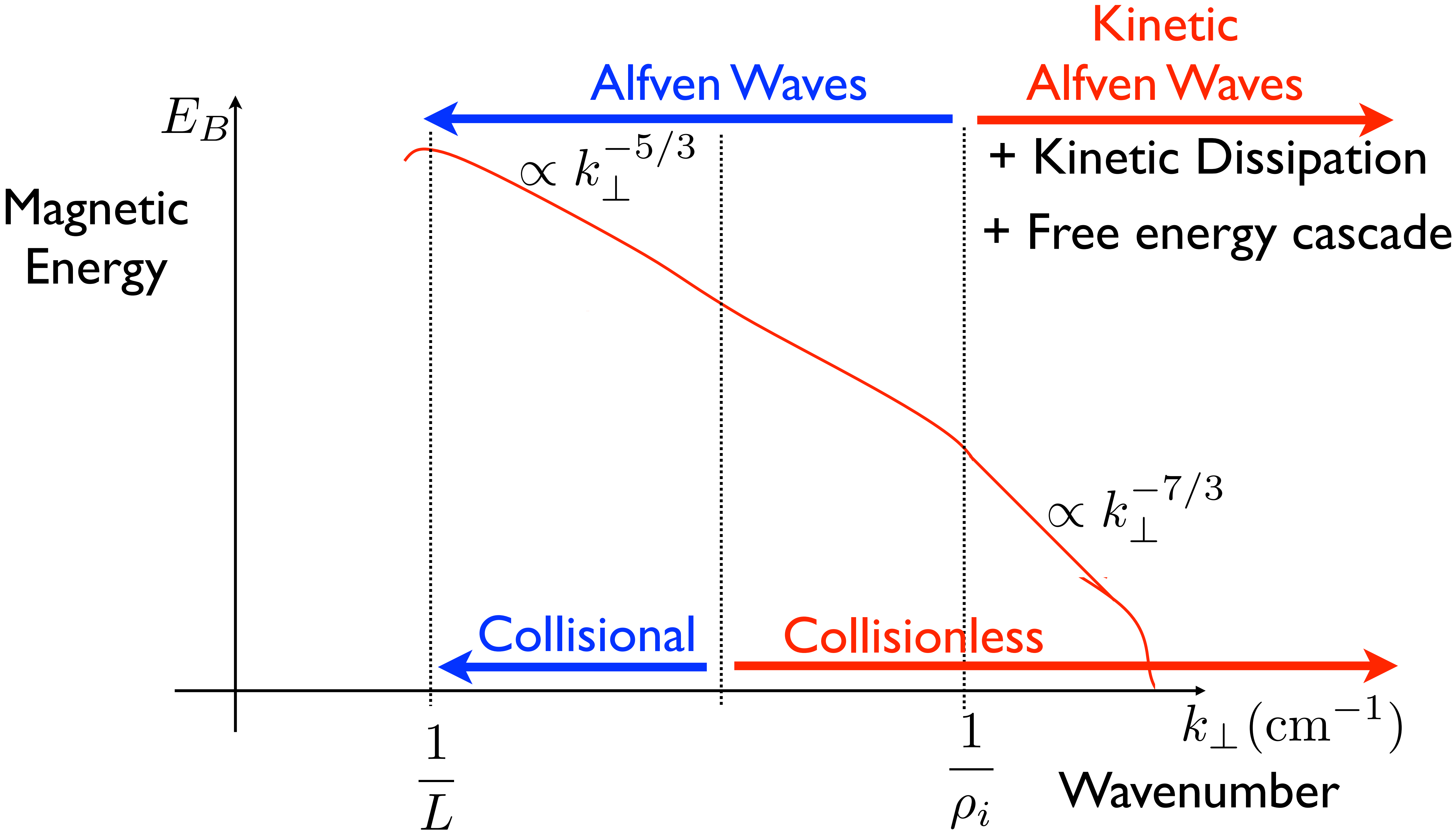
Kinetic
Alfven Waves
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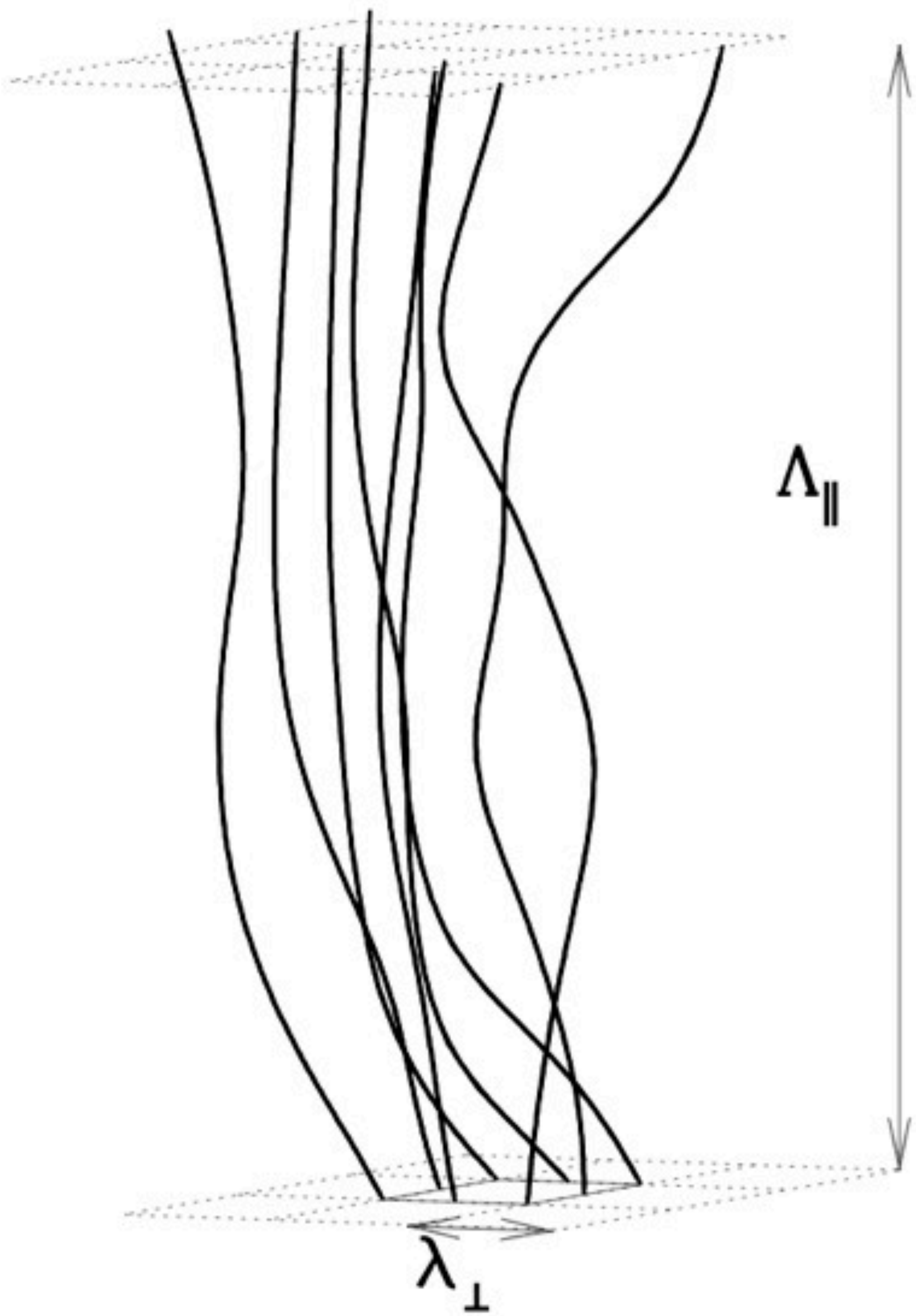
+ Kinetic Dissipation
+ Free energy cascade
E_B
Magnetic
Energy
$\propto k_\perp^{-5/3}$
$\propto k_\perp^{-7/3}$
Collisional
Collisionless
$\frac{1}{L}$
$\frac{1}{\rho_i}$
$k_\perp\,(\mathrm{cm}^{-1})$
Wavenumber



Lithwick and Goldreich 2001, ApJ. 562, 279

Goldreich and Sridhar (1995): Critical Balance

$$\tau_C \sim \tau_A \sim \tau_{nl}$$

+

Kolmogorov : Energy cascade rate independent of length scale

$$v^2_{\lambda_\perp}/\tau_{C,\lambda_\perp} \sim \text{const}$$

↓

$$v_{\lambda_\perp} \sim v_A \left(\frac{\lambda_\perp}{L}\right)^{1/3} \qquad \Lambda_\parallel = \lambda_\perp^{2/3} L^{1/3}$$

Gyrokinetics

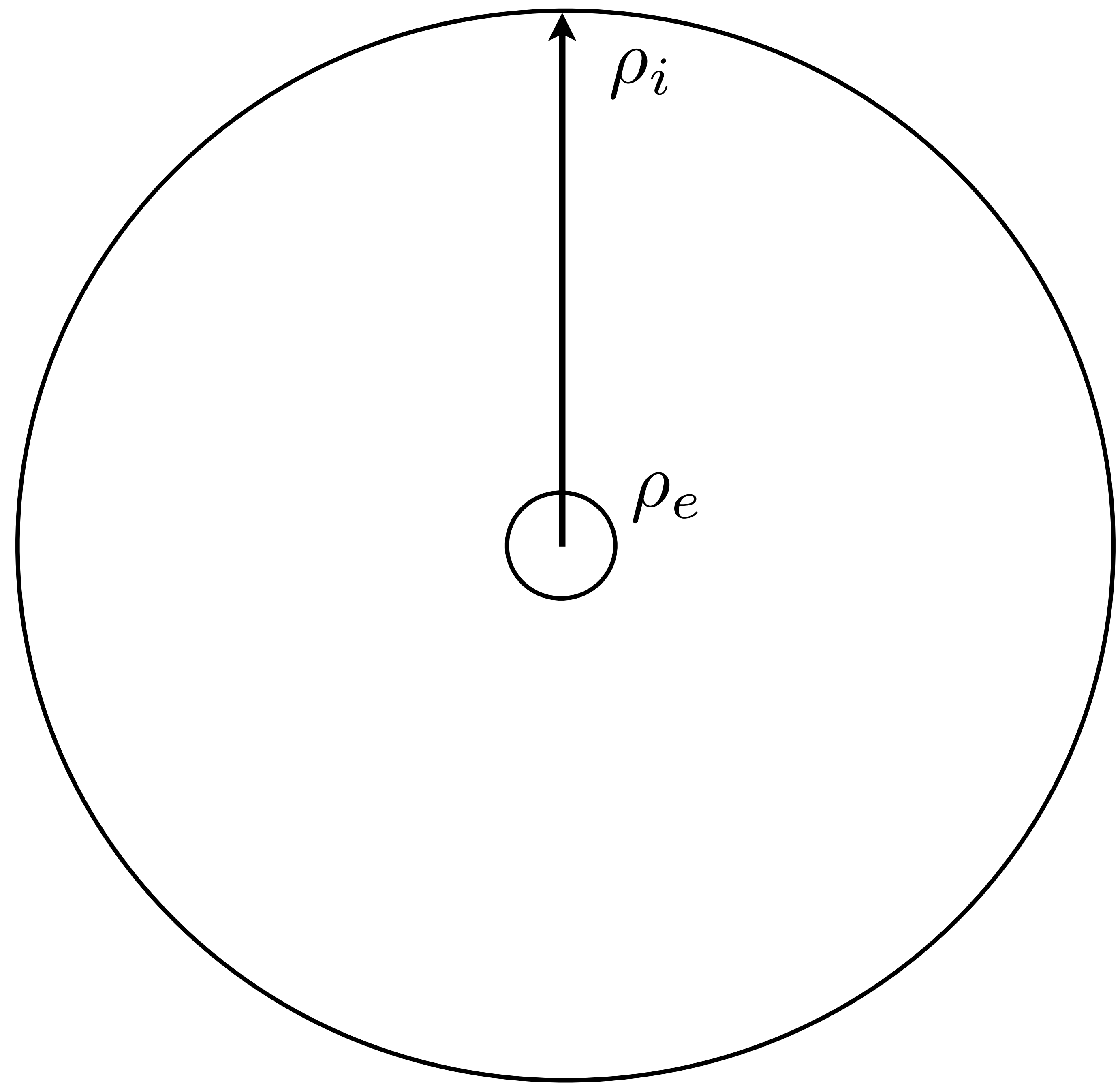
↓

Isothermal electrons

↓

KRMHD

Schekochihin et al., 2009, ApJS, 182, 1



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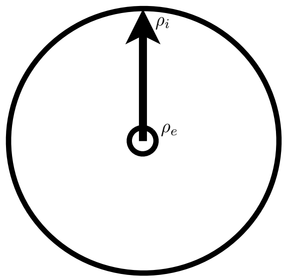

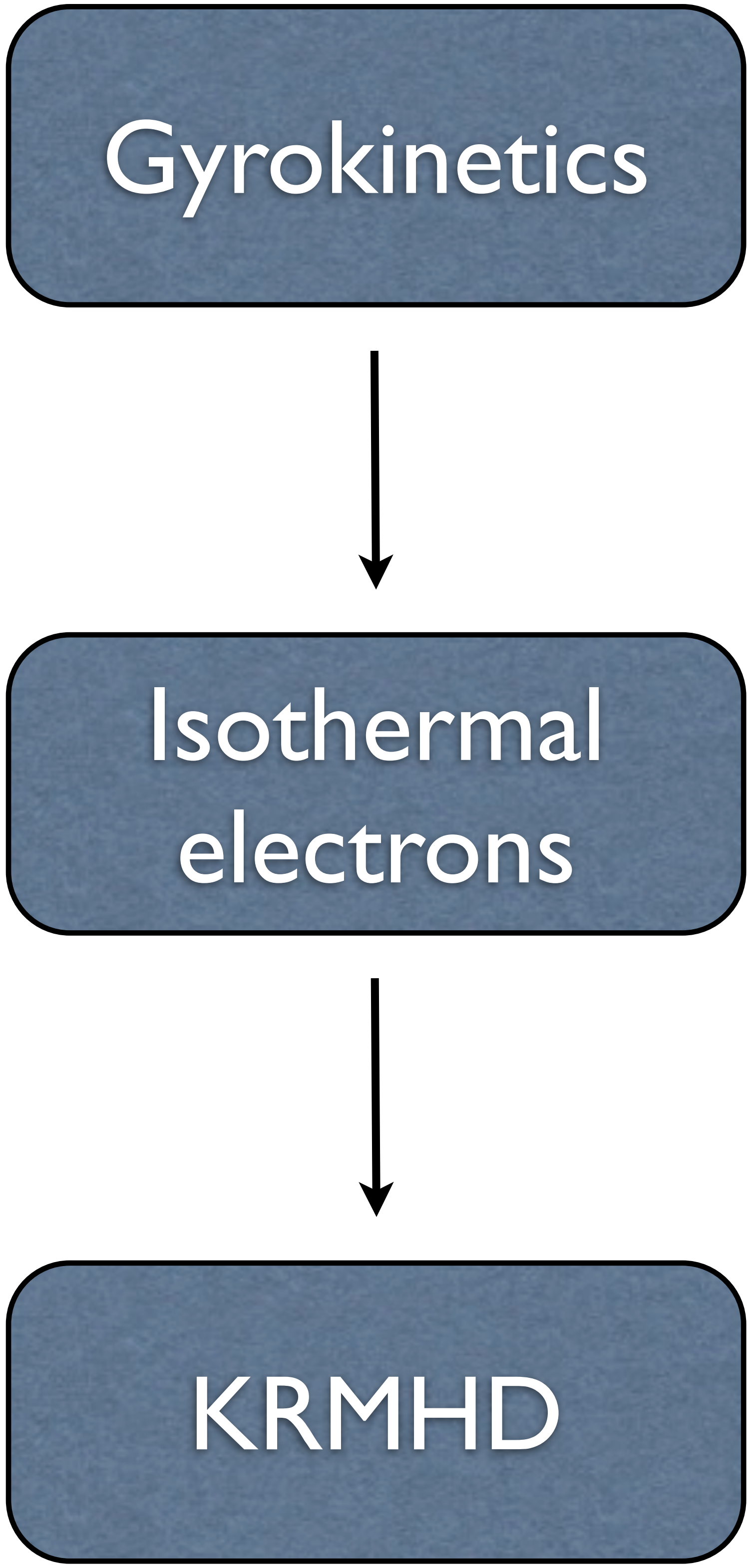

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KRMHD: Alfvénic cascade

$$\frac{\partial \Psi}{\partial t} = v_A \hat{\mathbf{b}} \cdot \nabla \Phi \qquad \frac{d \nabla_\perp^2 \Phi}{dt} = v_A \hat{\mathbf{b}} \cdot \nabla \nabla_\perp^2 \Phi$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \{\Phi, \ldots\}$$

$$\hat{\mathbf{b}} \cdot \nabla = \frac{\partial}{\partial z} + \{\Psi, \ldots\}$$

KRMHD: Alfvénic cascade

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Elsasser variables $\downarrow$ $\zeta^\pm = \Phi \pm \Psi$

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$$\frac{\partial \nabla_\perp^2 \zeta^{\pm}}{\partial t} \mp v_A \frac{\partial \nabla_\perp^2 \zeta^{\pm}}{\partial z} = -\frac{1}{2}\left[\left\{\zeta^+, \nabla_\perp^2 \zeta^-\right\} + \left\{\zeta^-, \nabla_\perp^2 \zeta^+\right\} - \nabla_\perp^2 \left\{\zeta^+, \zeta^-\right\}\right]$$

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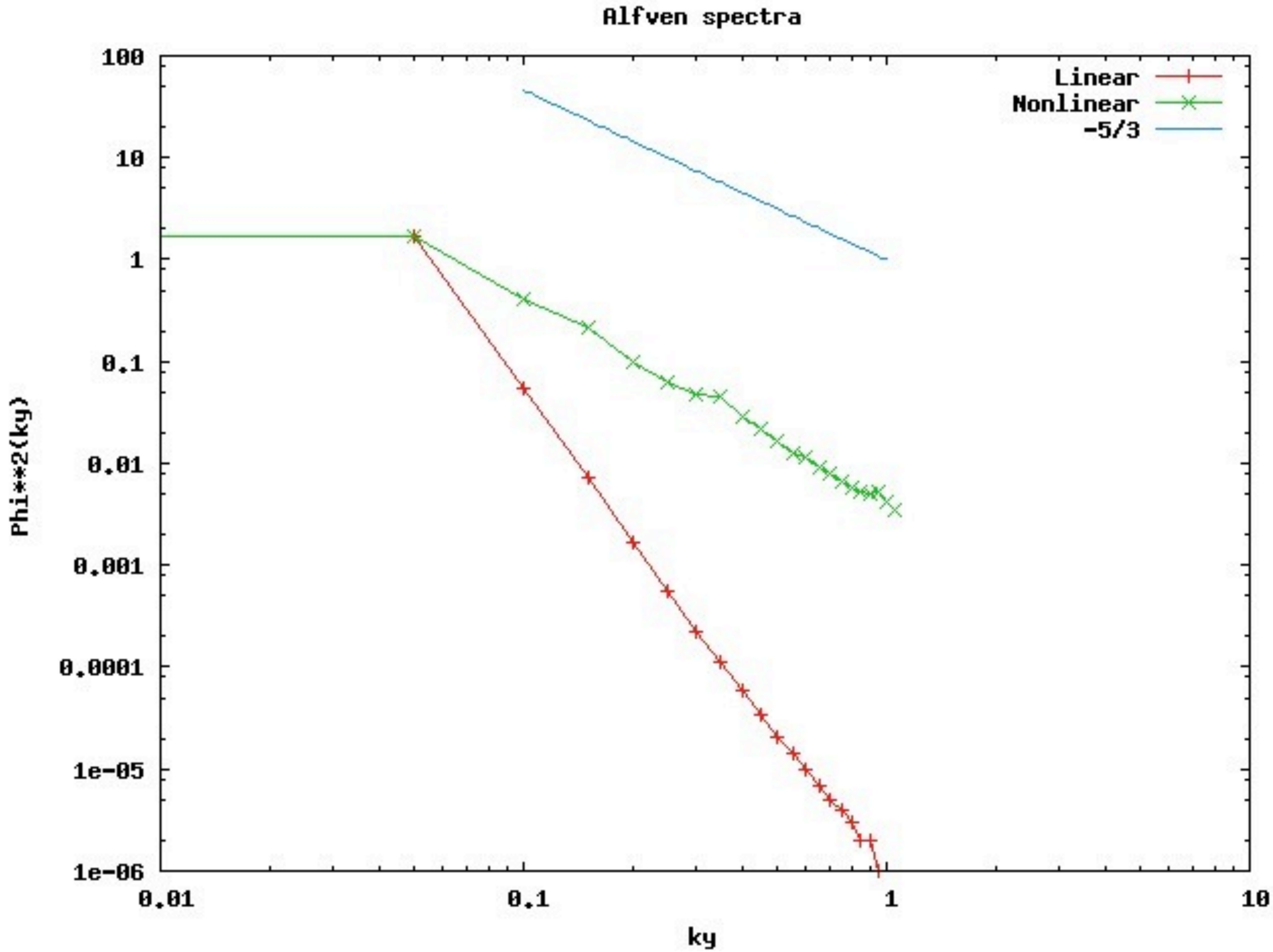
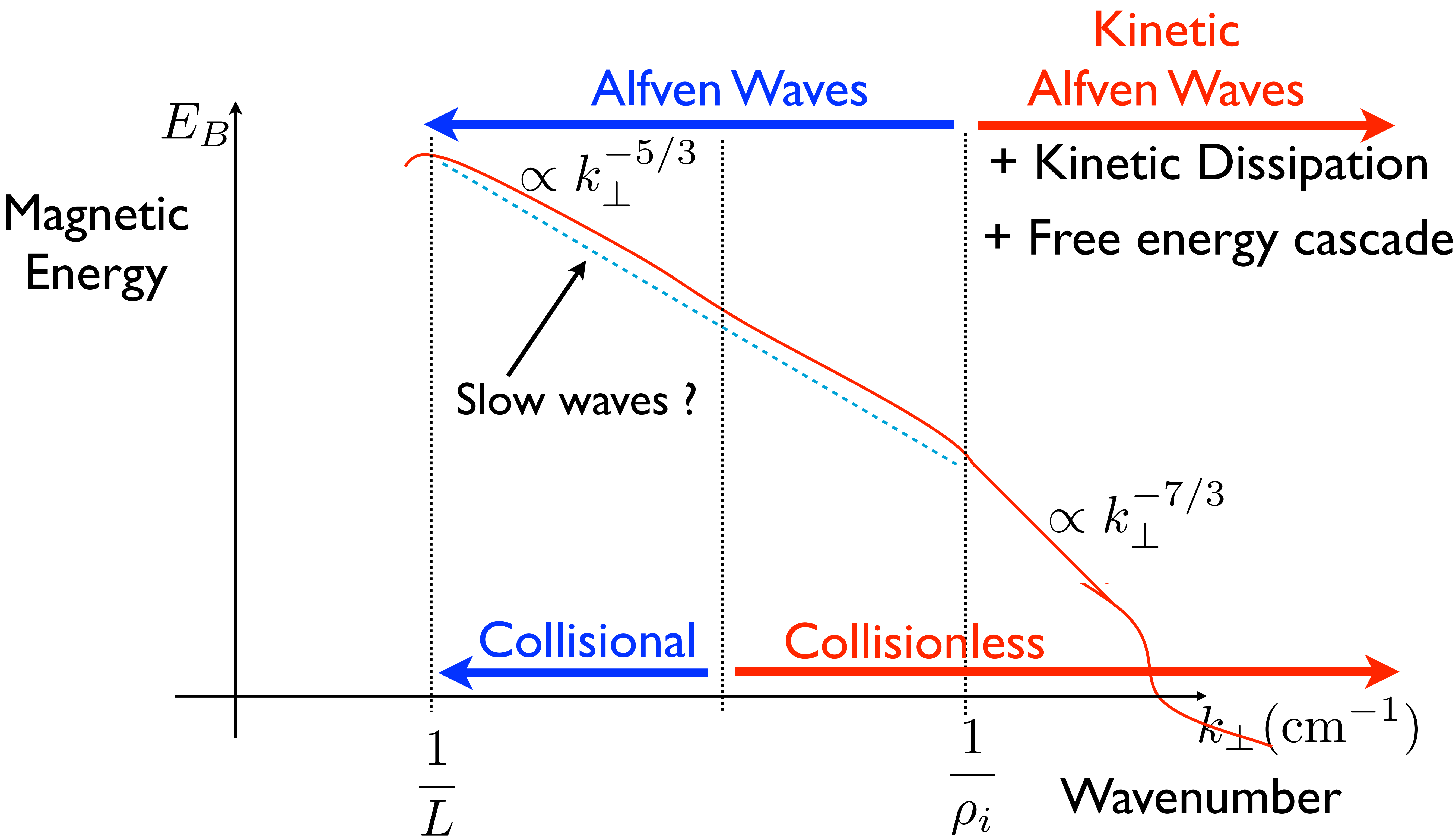
ζ^+ and ζ^- cascade independently

KRMHD code on GPU

- Fully spectral, implicit in time (linear terms)
- Nonlinear terms use RK2 (midpoint method)

Alfven cascade: numerical results

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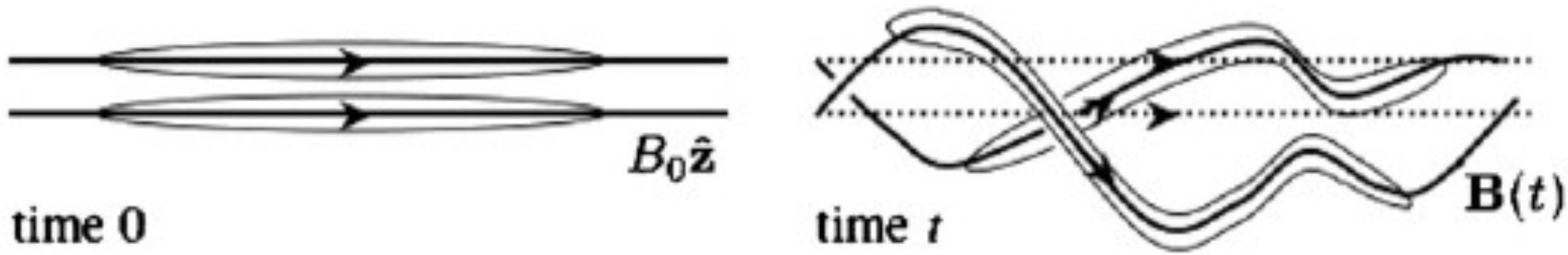
Slow modes

- Slow modes are effectively a passive scalar advected by background Alfvénic turbulence
- They align themselves along the perturbed field lines



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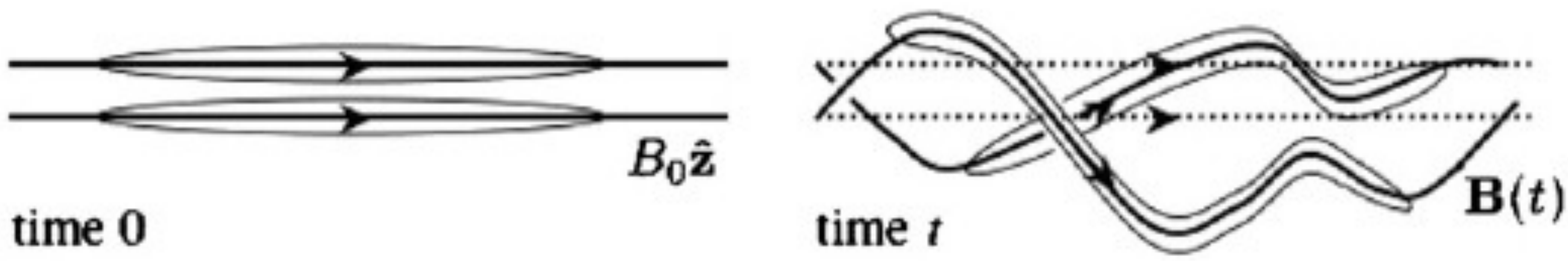

Is there parallel cascade?

KRMHD: Slow modes

$$\frac{dg}{dt} + v_\parallel \hat{\mathbf{b}} \cdot \nabla \left[g + \left(\frac{Z}{\tau} \frac{\delta n_e}{n_{0e}} + \frac{v_\perp^2}{v_{thi}^2} \frac{\delta B_\parallel}{B_0} \right) F_{0i} \right] = \left\langle C_{ii} \left[g + \frac{v_\perp^2}{v_{thi}^2} \frac{\delta B_\parallel}{B_0} F_{0i} \right] \right\rangle$$

$$\frac{\delta n_e}{n_{0e}} = - \left[\frac{Z}{\tau} + 2 \left(1 + \frac{1}{\beta_i} \right) \right]^{-1} \frac{1}{n_{0i}} \int d^3\mathbf{v} \left[\frac{v_\perp^2}{v_{thi}^2} - 2 \left(1 + \frac{1}{\beta_i} \right) \right] g$$

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KRMHD: Slow modes

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$$\int v_\perp dv_\perp \quad \downarrow$$

$$\frac{dG^\pm}{dt} + v_\parallel \hat{\mathbf{b}} \cdot \nabla G^\pm = \frac{v_\parallel F_M(v_\parallel)}{\Lambda^\pm} \hat{\mathbf{b}} \cdot \nabla \int_{-\infty}^{\infty} dv_\parallel' G^\pm \left(v_\parallel' \right)$$

$$G^+ = G_B + \frac{1}{\sigma} \left(1 + \frac{Z}{\tau} \right) G_n$$

$$G^- = G_n + \frac{1}{\sigma} \frac{\tau}{Z} \frac{2}{\beta_i} G_B$$

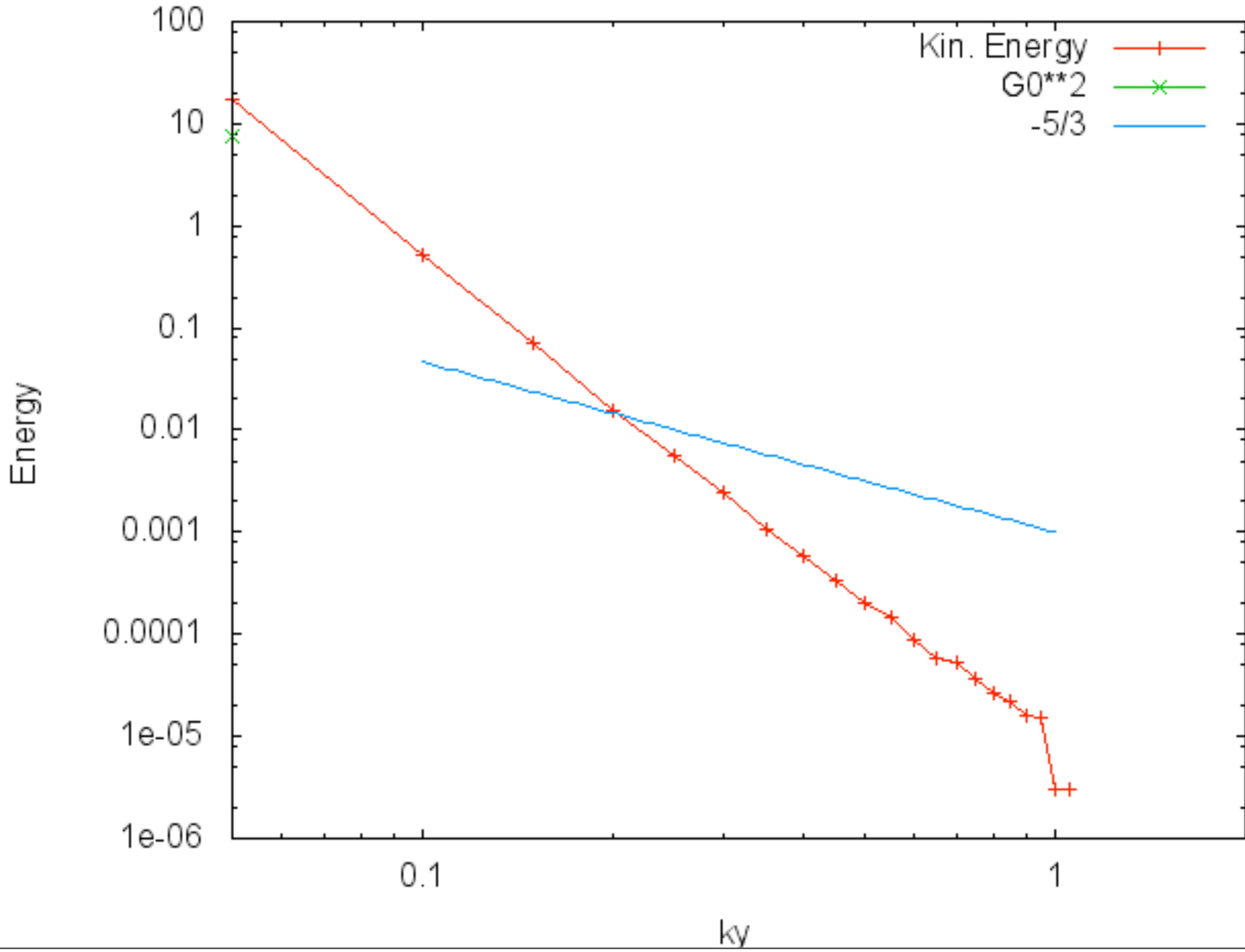
Hermite hierarchy

$$G(v_\parallel) = \sum_{m=0}^{\infty} \frac{H_m\left(\widehat{v_\parallel}\right)}{\sqrt{2^m m!}} \widehat{G}_m \frac{1}{\sqrt{\pi} v_{thi}} e^{-\widehat{v_\parallel}^2}$$

$$\frac{d\widehat{G}_m^{\pm}}{dt} + v_{thi}\hat{\mathbf{b}} \cdot \nabla \left(\sqrt{\frac{m+1}{2}} \widehat{G}_{m+1}^{\pm} + \sqrt{\frac{m}{2}} \widehat{G}_{m-1}^{\pm} \right) = \frac{v_{thi}}{\sqrt{2}\Lambda} \delta_{m,1} \hat{\mathbf{b}} \cdot \nabla \widehat{G}_0$$

- Lenard-Bernstein* collision operator
- Open boundary condition for closure
- Similar to Zocco-Schekochihin (2011) equations

Slow mode cascade: numerical results



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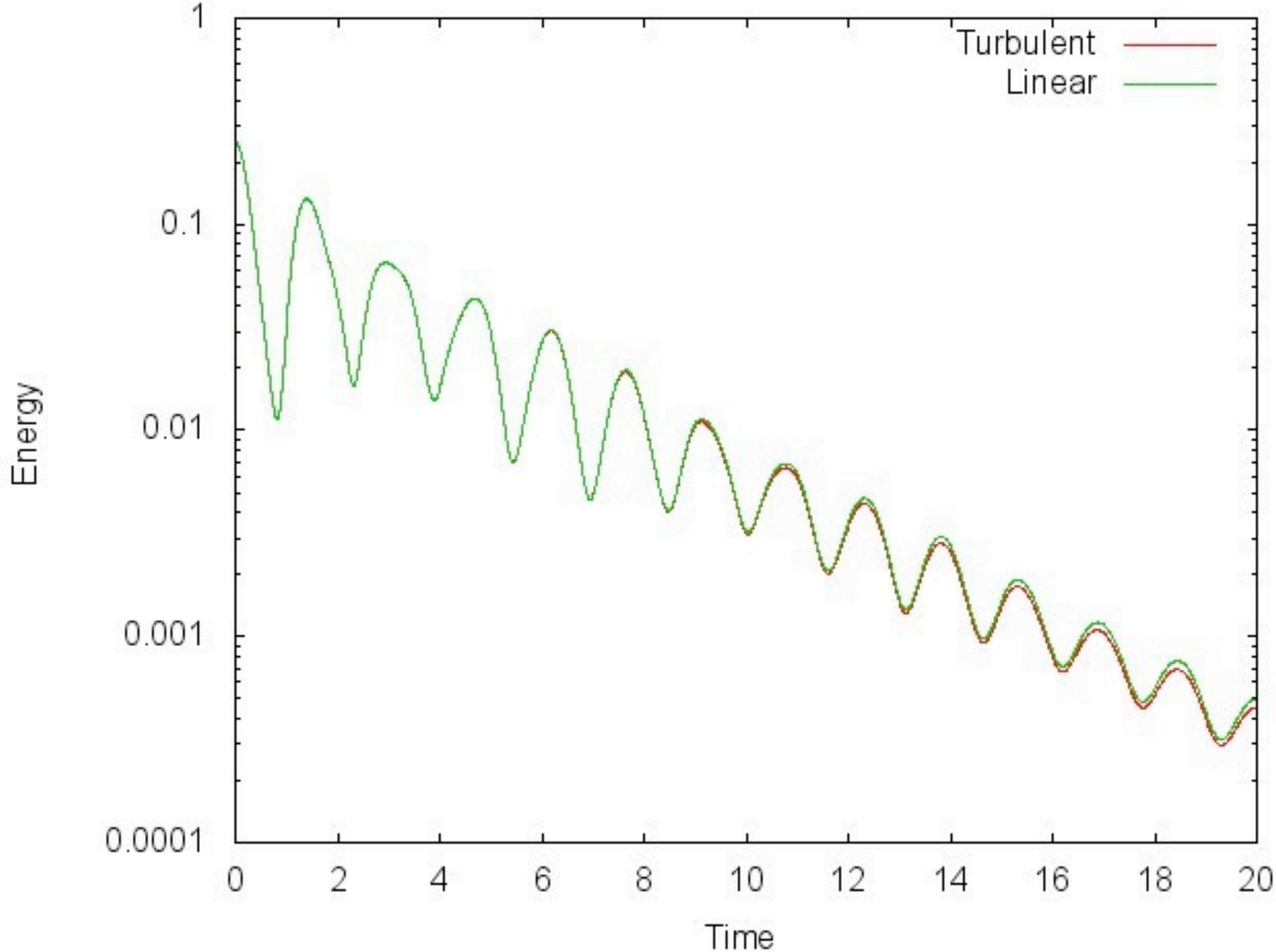

Conclusions & future work

- No parallel cascade for slow modes in the KRMHD limit - possibly a more general result
- Implement a better numerical scheme for slow modes
- AstroGK simulations to study transition at $k_\perp \rho_i \sim 1$

Roots of the linear dispersion relation