The Zero-Turbulence Manifold in a Toroidal Plasma

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Outline

- Turbulence and Sheared Flows
- Zero Magnetic Shear
- Subcritical Turbulence and Transient Fluctuations
- The Zero-Turbulence Manifold

Turbulence

- Turbulent transport of heat limits the performance of fusion devices.

Large background gradients

↓

Unstable linear eigenmodes

↓

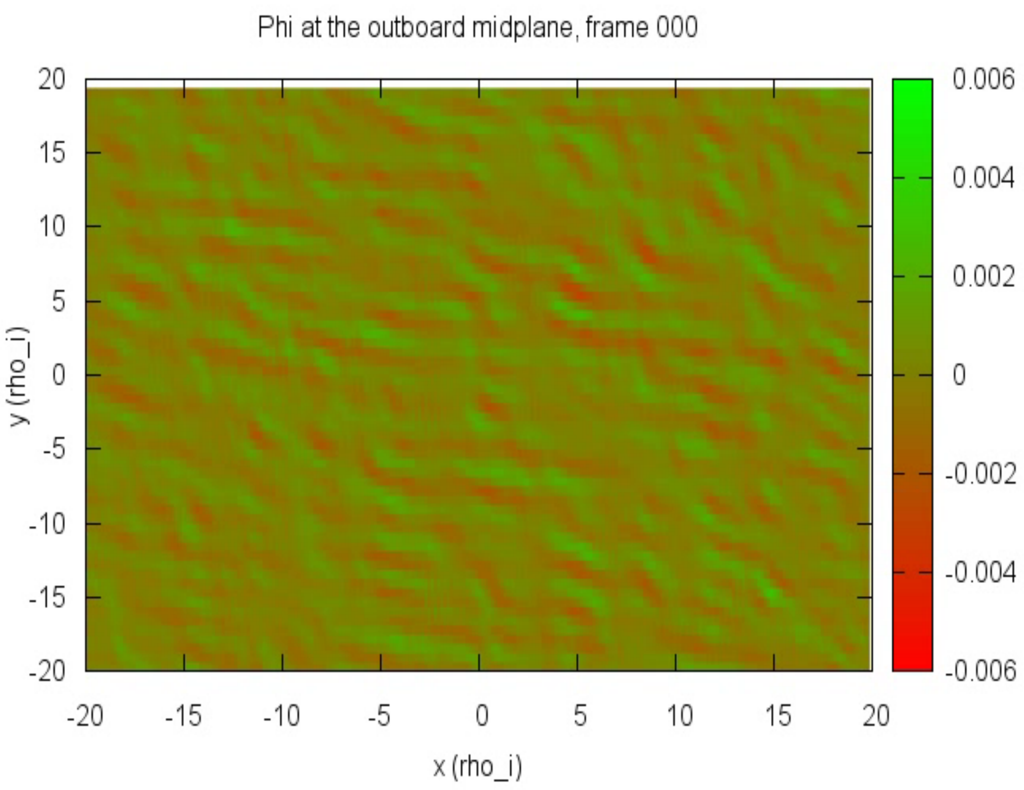
Nonlinear interaction

↓

Formation of eddies

↓

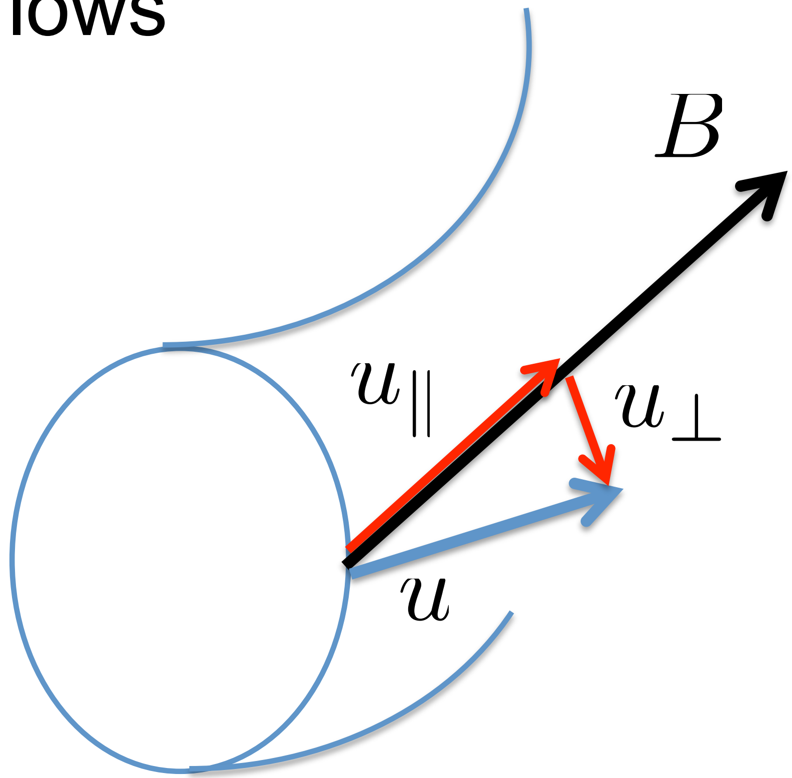
Transport of Heat, Particles and Momentum



Sheared Flows

- Consider turbulence in a toroidal device with a strong background sheared flow.
- A gradient of the perpendicular component of the flow can suppress turbulence.
- A gradient of the *parallel* component of the flow can drive an instability. Catto et al., 1973
  - Increase the turbulence
- In a torus with toroidal flow:

$$\frac{\nabla u_{||}}{\nabla u_{\perp}} \text{ is fixed}$$

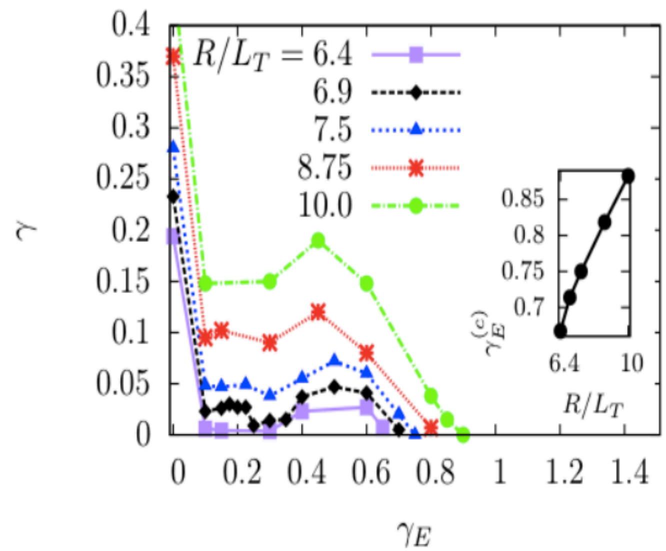


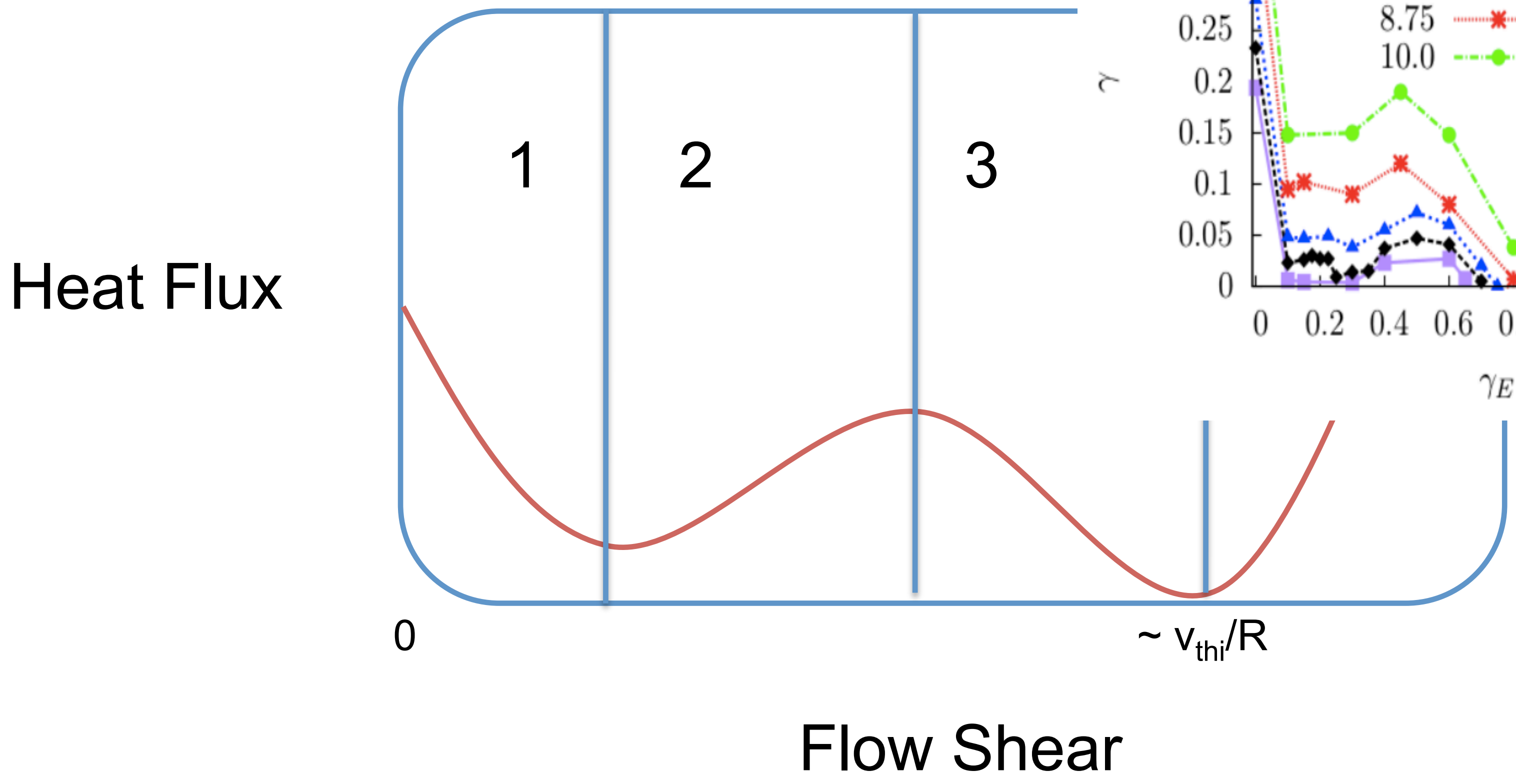
Equations and Numerics Summary

- Use gyrokinetic nonlinear flux tube code GS2. Dorland & Jenko 2000
- Solve the δf gyrokinetic equation. Friemen & Chen 1982
- Use a first principles treatment of the equilibrium flow shear. Sugama & Horton 1998, Hammett et. al. 2006
- Use Cyclone Base Case parameters (core turbulence). Dimits et. al. 2000
  - Varying temperature gradient, flow shear, magnetic shear
- Published:
  - Highcock et. al., PRL,105, 215003 (2010)
  - Barnes et. al., PRL 106, 175004 (2011)
  - Parra et. al., PRL 106, 115004 (2011)
- Full details, equations, and references:
  - arXiv.org: 1105.5750

The Effects of Flow Shear at Constant R/L_T

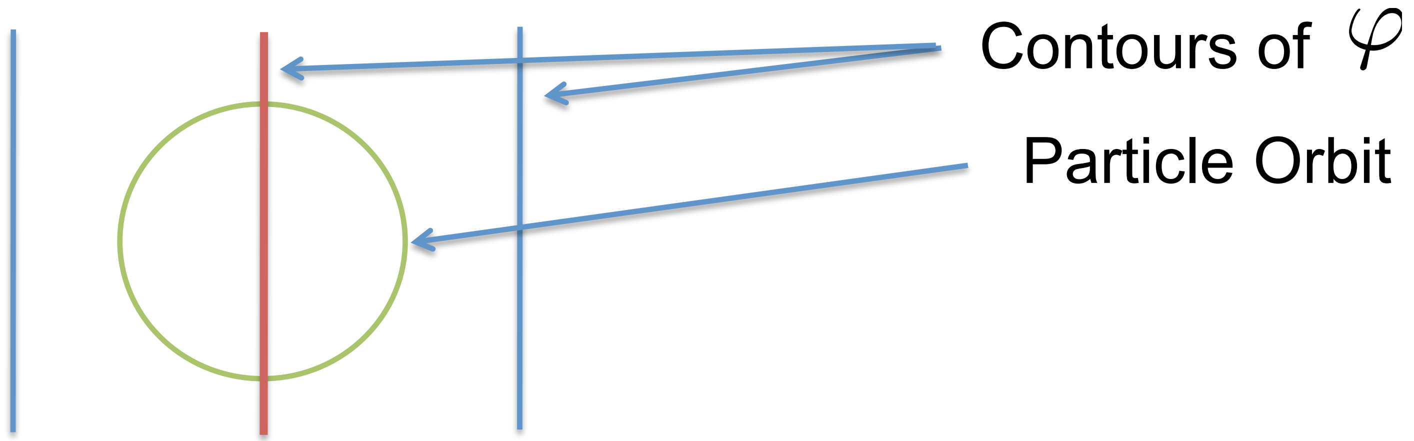
- Cartoon: 4 regions.
  - Actual data in Barnes et. al. PRL 2011





Effect of Perpendicular Flow Shear

- The perturbed potential experienced by the particle is averaged over an orbit.



- Long wavelength potential perturbation: particle sees strong field.

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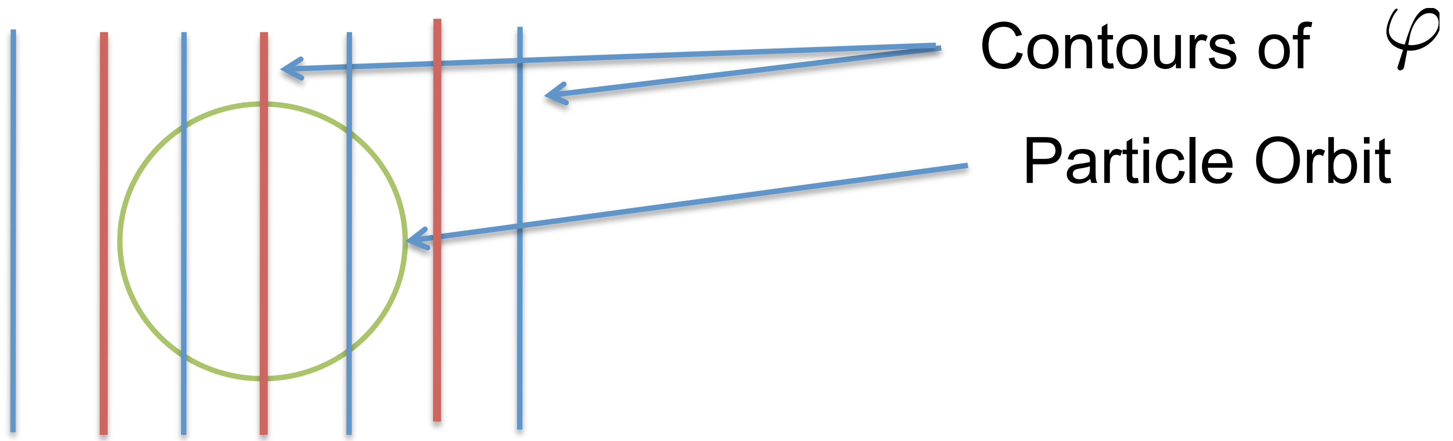
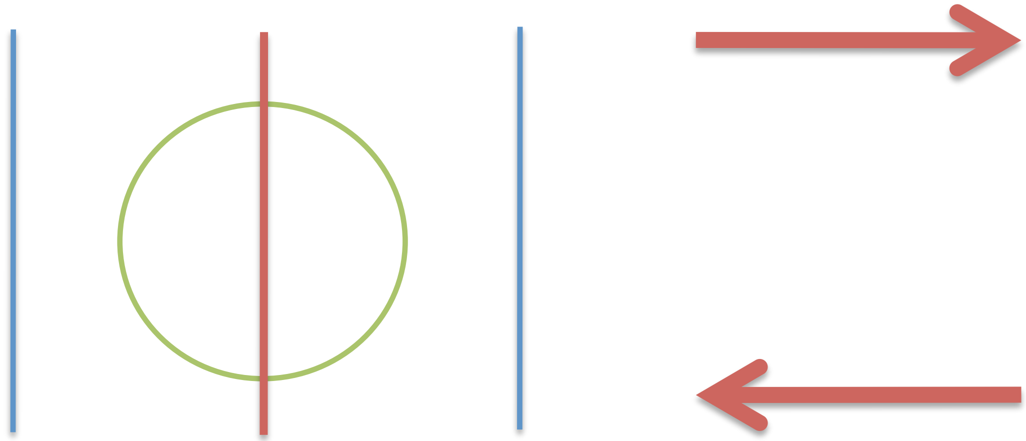


- Short wavelength potential perturbation: field averages to zero.

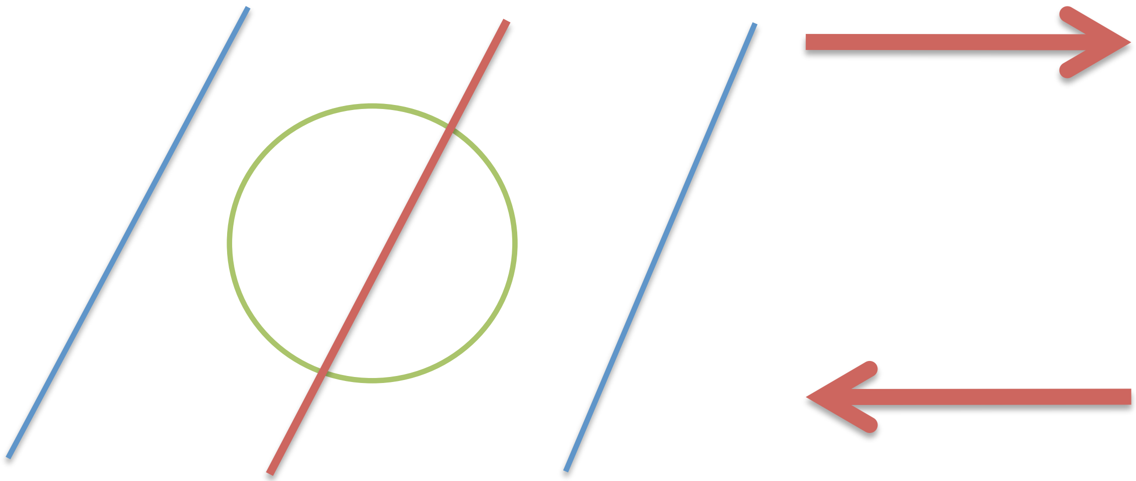
Effect of Perpendicular Flow Shear

- Start with long wavelength and a flow with a perpendicular gradient.



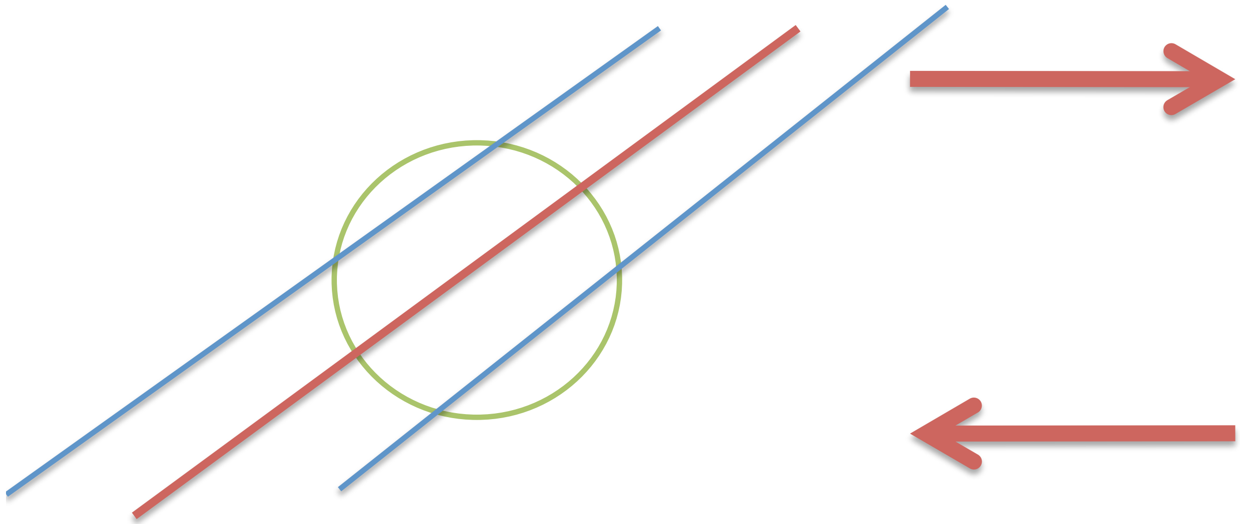
Effect of Perpendicular Flow Shear

- Flow shears the field, increasing its wavenumber.



Effect of Perpendicular Flow Shear

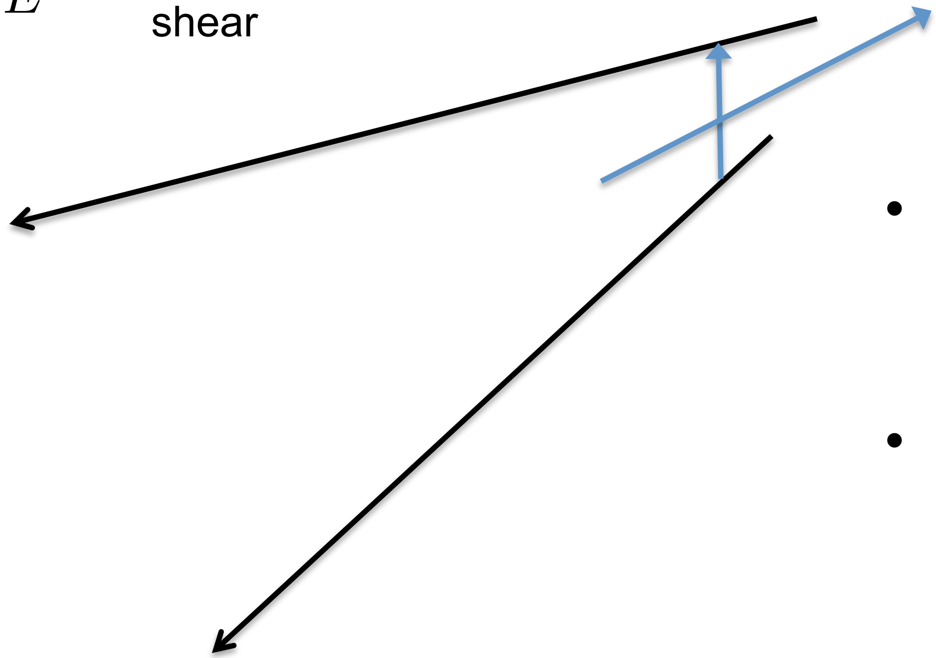
- Eventually particle sees zero averaged field.



The Effect of Magnetic Shear

$\hat{s} =$ inverse magnetic shear scale length

$\gamma_E =$ perpendicular flow shear



- Velocity shear cancelled if the mode moves with speed

$$\frac{\gamma_E}{\hat{s}}$$

Waltz et. al. 1994
Newton et. al. 2010

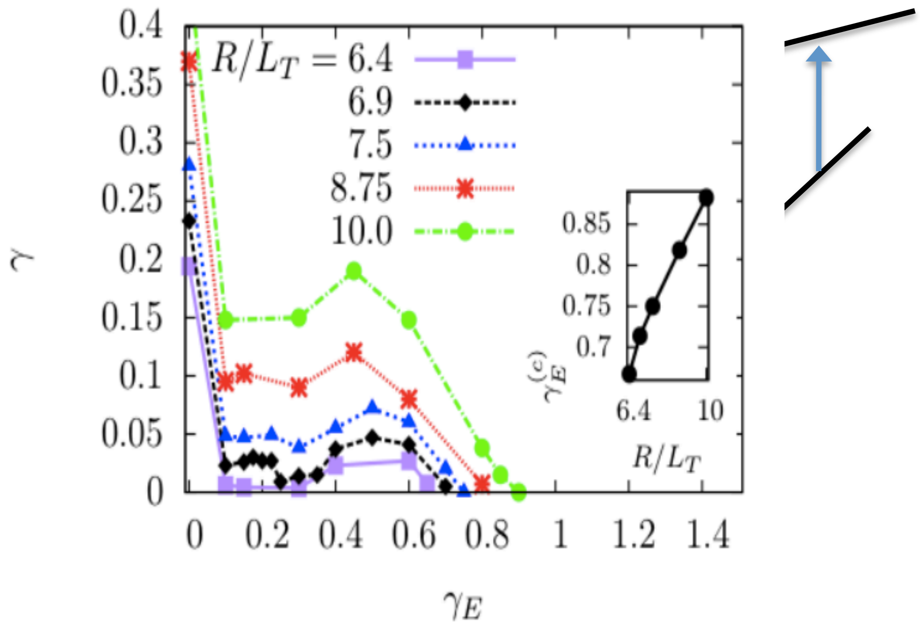
- => Linear modes possible for low γ_E and finite $\hat{s}$
  - Regions 1 & 2
- No linear modes for

$$\frac{\gamma_E}{\hat{s}} \gg c_s$$

Newton et. al. 2010

  - Region 3

The Effect of Magnetic Shear



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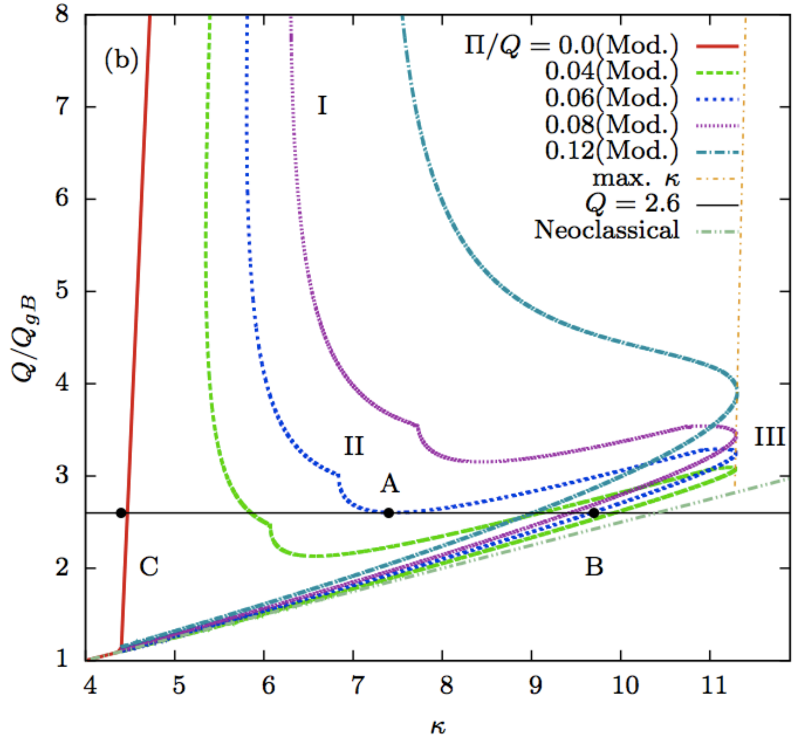
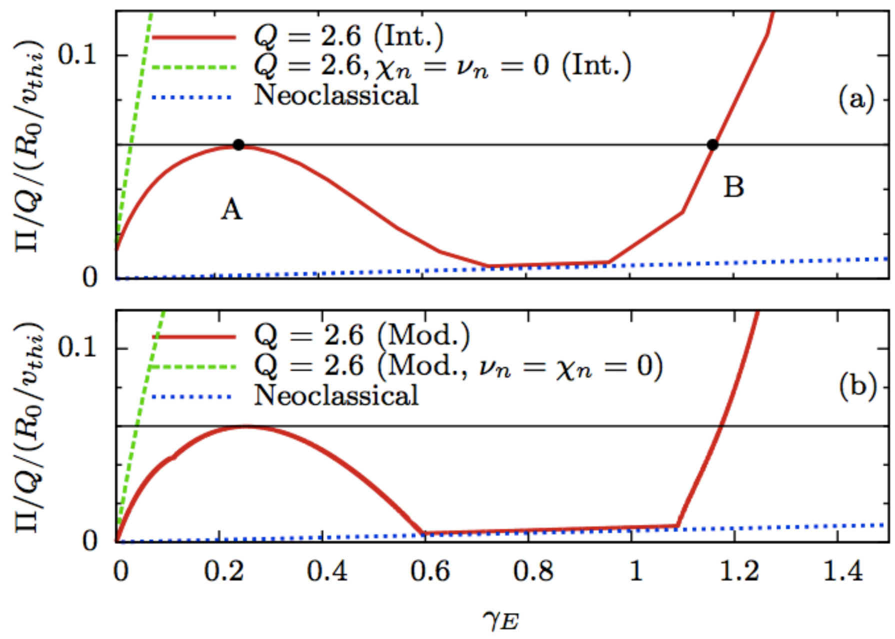
The Effects of Flow Shear at Constant R/L$_T$

- Region 4: Strong PVG driven turbulence
- No linearly growing modes



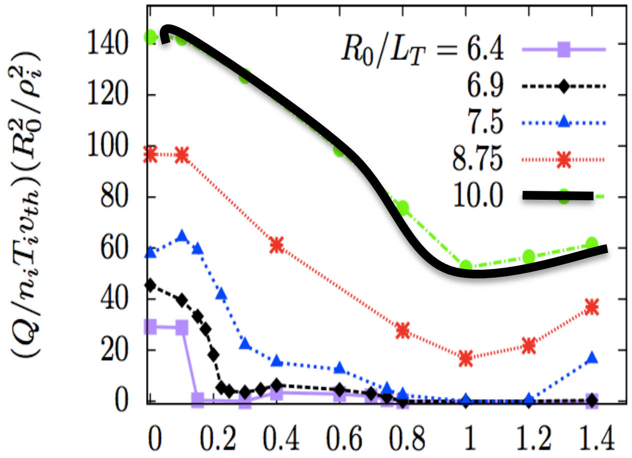


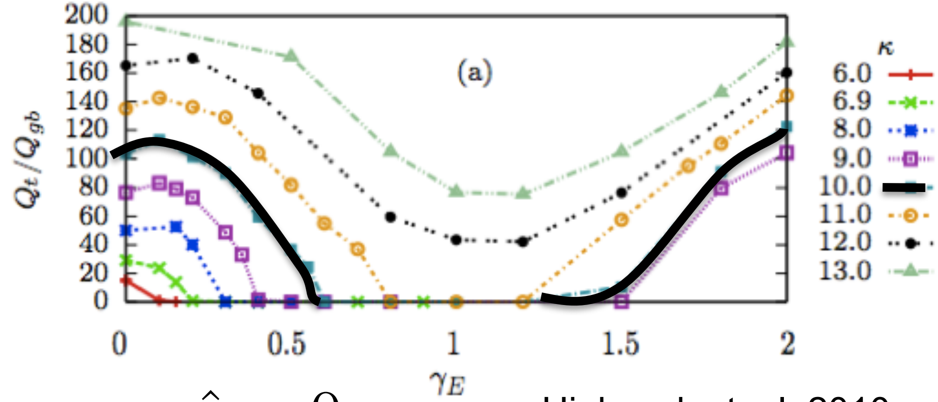
Zero Magnetic Shear

Barnes et. al. 2011

$\hat{s} = 0.8$

- $\dfrac{\gamma_E}{\hat{s}}$ is infinite.
- *The turbulence is subcritical for all non-zero flow shear*
- No growing linear modes exist.
- Regions 1 & 2 disappear
- Heat flux is lower
- Turbulence stabilised more rapidly by the flow shear.
- Much larger region where there is no turbulence at all.



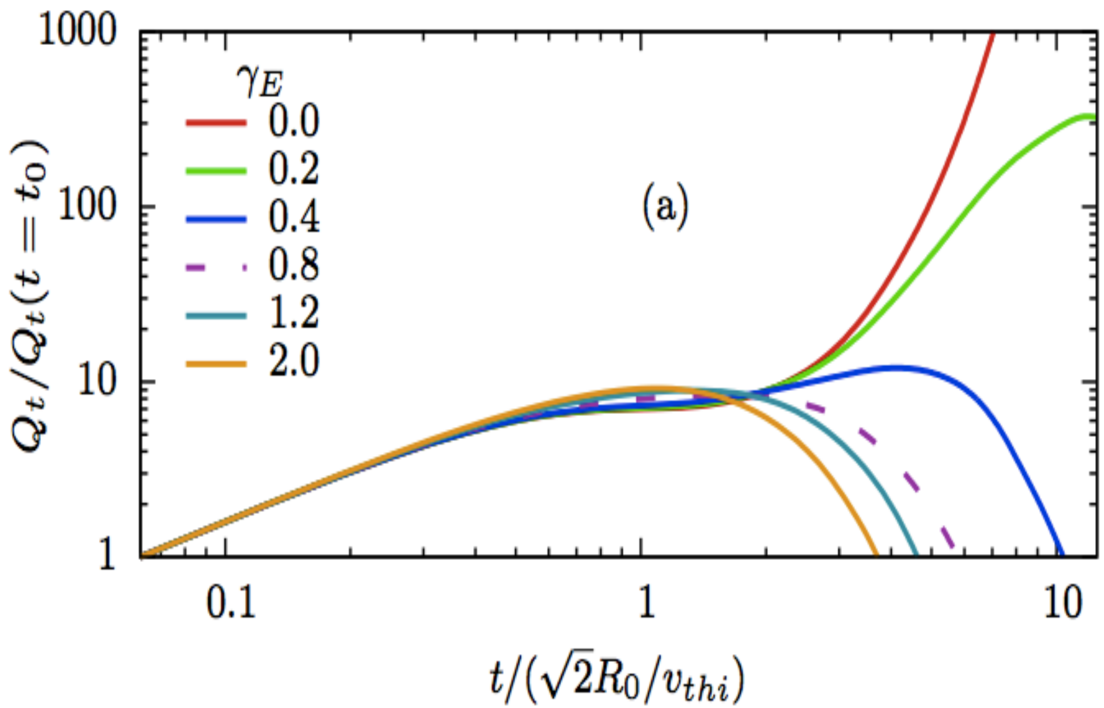


$\hat{s} = 0$

Highcock et. al. 2010

Subcritical Turbulence

- No linearly growing modes
- Transient growth only
- Required to sustain turbulence:
  - sufficient initial amplitude
  - sufficient amplification
  - non-linear interaction
- Many well known neutral fluid examples

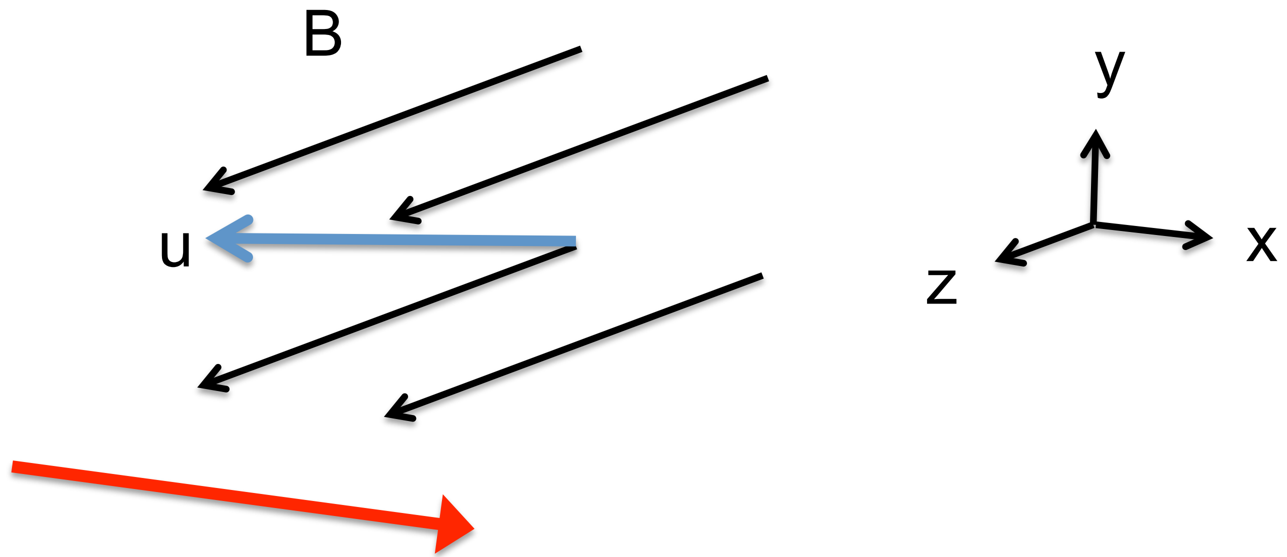


Trefethen et. al 1993
Bagget et. al 1995
Kerswell 2005

Subcritical Turbulence

- Subcritical turbulence occurs at high flow shear…
  - We would like to eliminate it!
- What are the criteria which determine when subcritical turbulence can be sustained?
- No longer a linear growth rate
  - How can we use linear theory to understand nonlinear results?
  - How can we make an estimate of where subcritical turbulence can and cannot be sustained?

Zero Magnetic Shear Slab

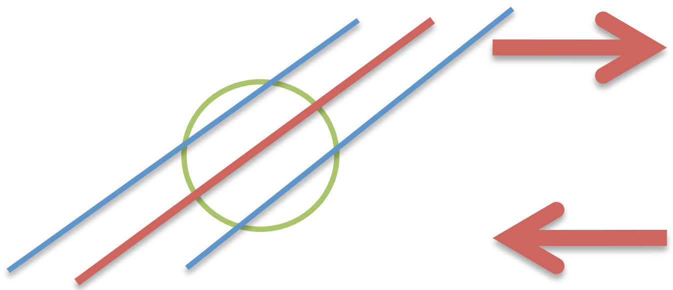


$$\nabla T, \nabla u, \nabla n$$

Shearing Box

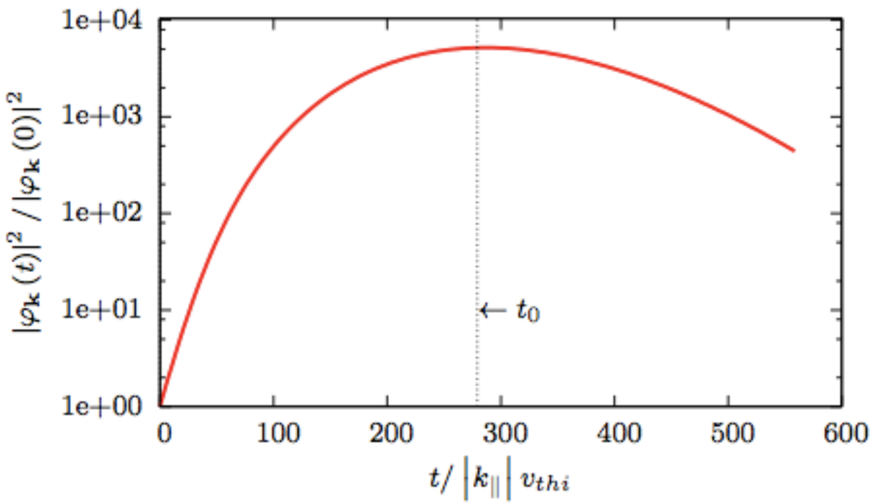
$$t' = t, \quad x' = x, \quad y' = y - Sxt, \quad z' = z$$

$$k_\perp^2 = \left(k_x^{'} - Sk_y^{'}t\right)^2 + k_y^{'2}$$



Transient Growth

- 2 Cases: PVG, ITG-PVG
- 3 Stages:
  - Short time limit – unsheared growth
  - Intermediate time – growth turns to decay
  - Long time – decay



Intermediate Time

- In the limit

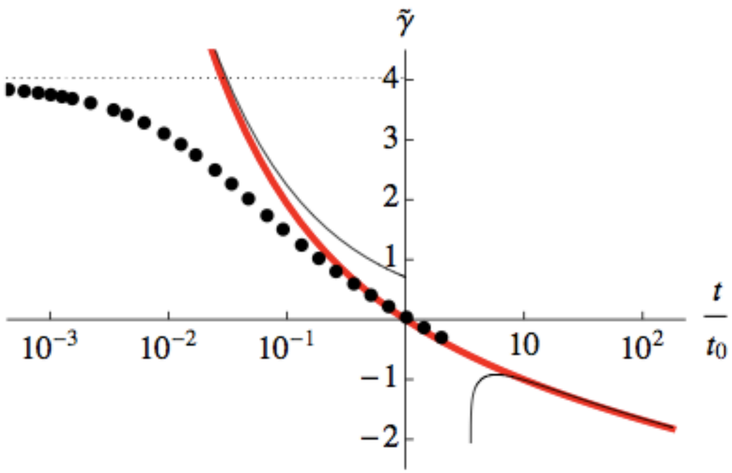
$$St \gg 1, Stk_y\rho_i \gg 1$$

  - Expand Bessel functions in $1/t$
  - Solve to find the point at which $\gamma(t) = 0$

$$t_0 = \frac{(q/\epsilon)\bar{\omega}_S - 1}{(1+\tau/Z)\sqrt{\pi}(\bar{\omega}_S^2+1)|\bar{\omega}_S|}$$

  - Write time dependent dispersion relation in terms of t_0

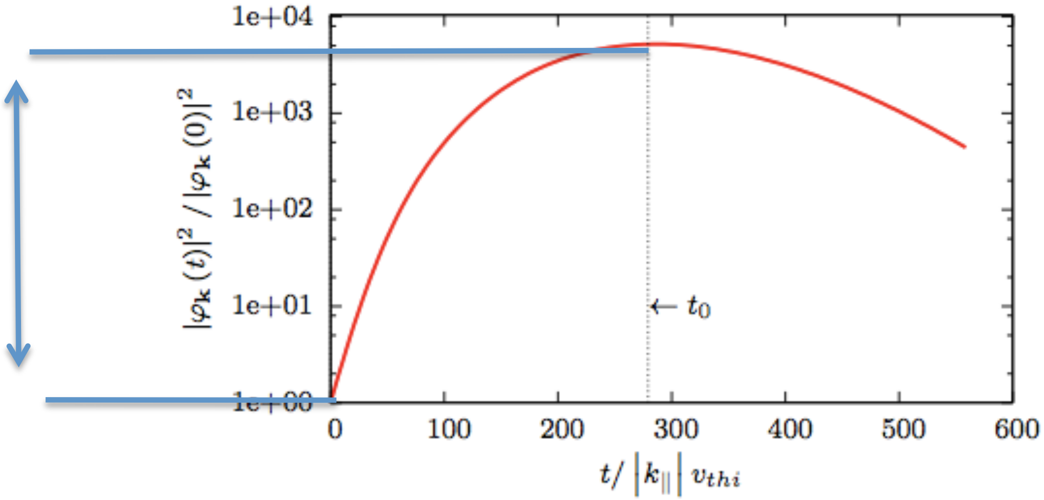
$$\frac{t}{t_0} = 1 + i\tilde{\gamma}\, Z(i\tilde{\gamma})$$



Calculate Exponentiation

$$N = \int_0^{t_0} dt\bar{\gamma}(t)$$

$$N = \frac{1}{2}\ln\left(\frac{\varphi^2(t_0)}{\varphi^2(0)}\right)$$

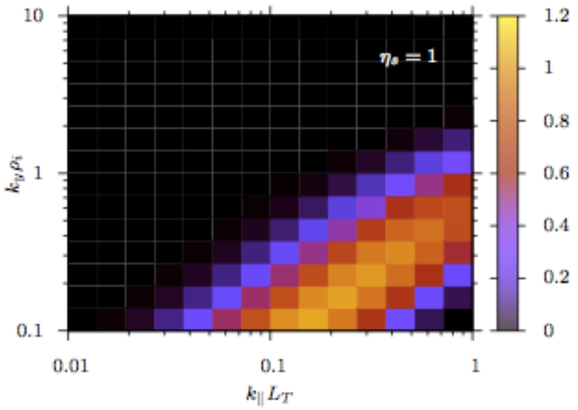


Maximum Exponentiation

PVG limit

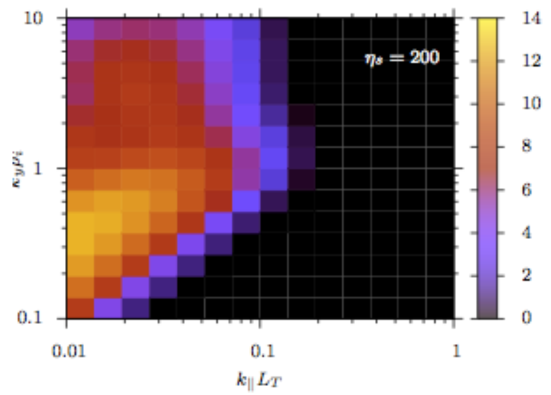
$$N_{\max} \approx 0.45 \frac{q/\epsilon}{1+\tau/Z}$$

for $$k_y \rho_i \approx \left(\frac{\epsilon}{q}\right)^{1/3} \frac{k_{\|} v_{\mathrm{th}i}}{S}$$



ITG limit

$$N_{\max} \sim N \sim 0.057 \frac{\kappa/S}{1+\tau/Z}$$

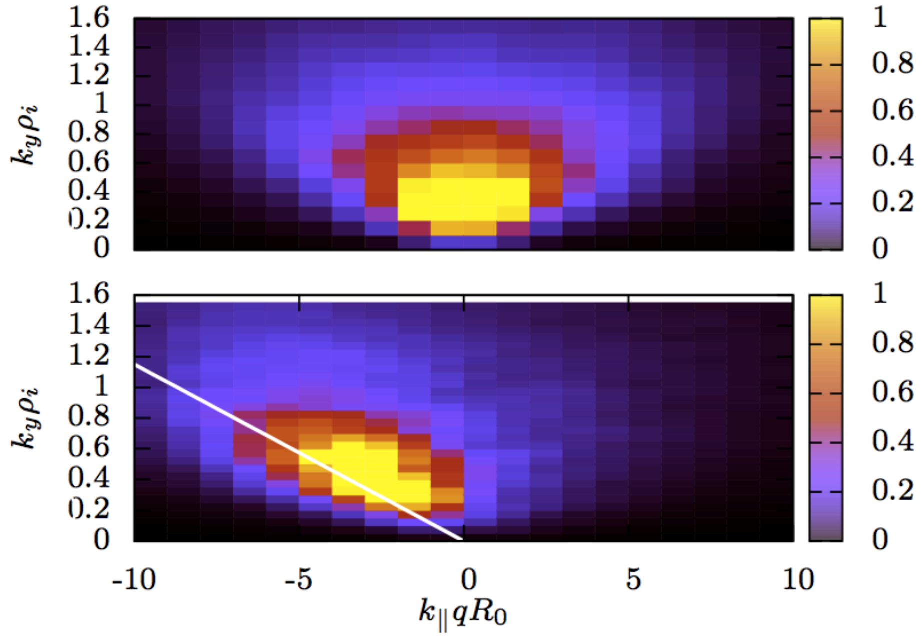


Motivation For Nmax

In Region 4:

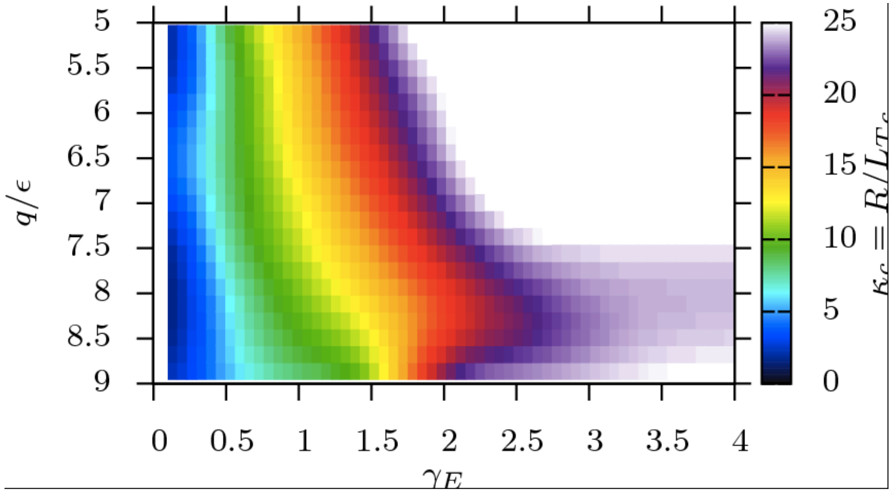
- Subcritical turbulence driven by PVG.
- Spectrum follows the line of maximal amplification

$$k_y \rho_i \simeq \left(\frac{r_0}{R_0 q_0}\right)^{1/3} \frac{k_{\|} R_0}{\sqrt{2}\gamma_E}$$



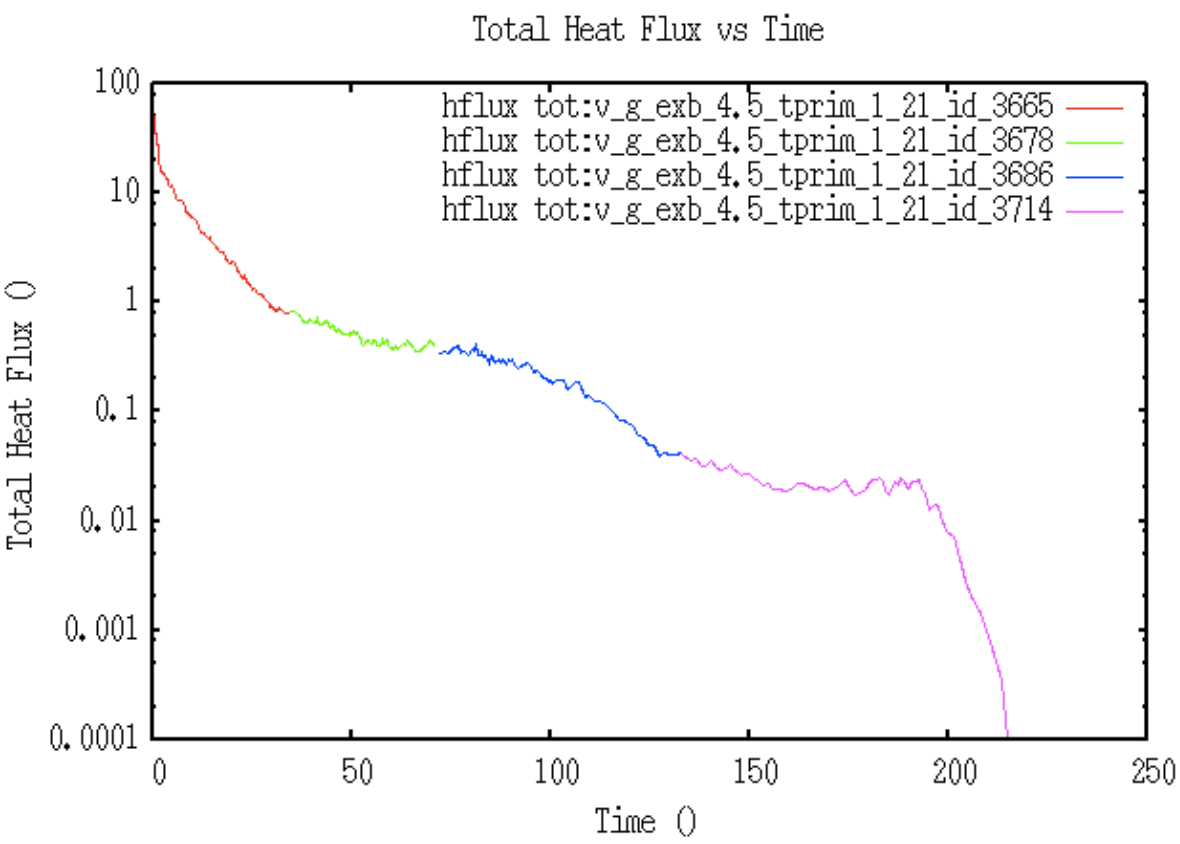
Linear Prediction

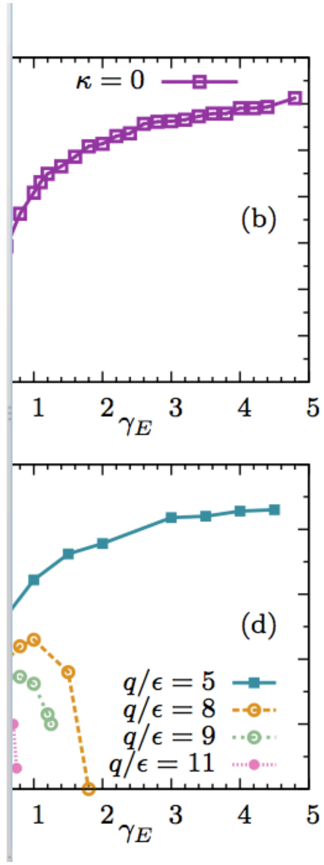
- Assume $N_{\max} \sim 1$ for turbulence to be sustained.
- How can we maximise the temperature gradient without generating turbulence.
- Linear prediction:
  - Minimise q/ϵ, the ratio of the magnetic safety factor to the inverse aspect ratio
  - Maximise the perpendicular flow shear.



Nonlinear Scan

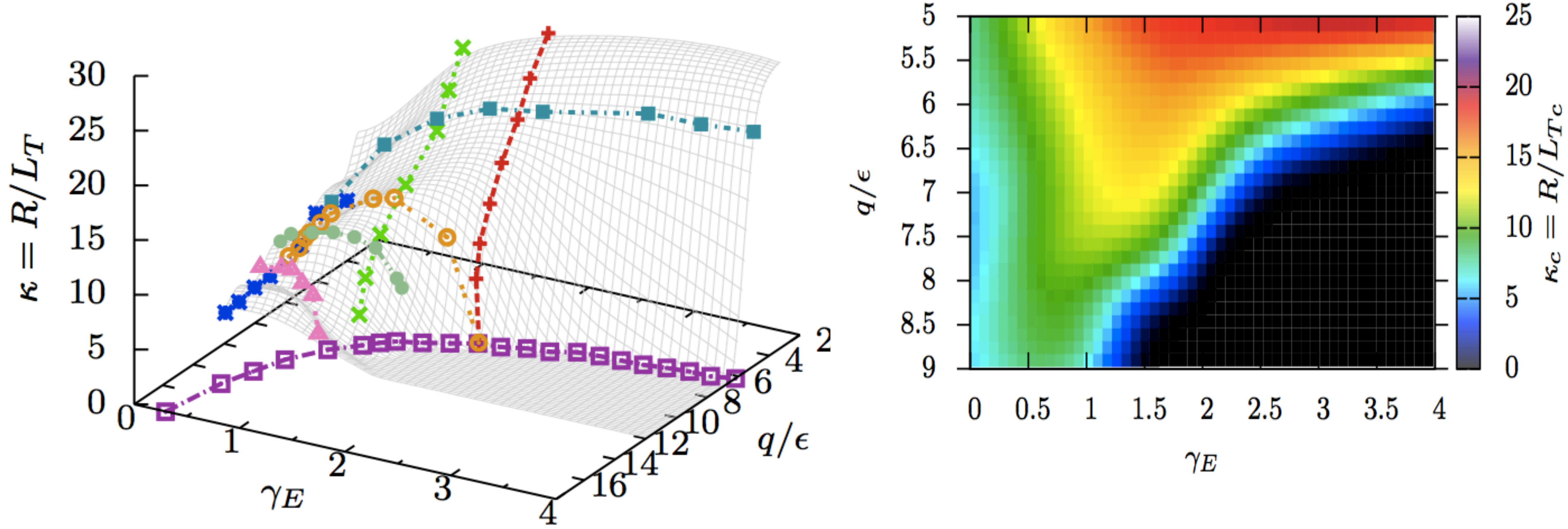
- >1500 r simulati
- For eac perpend either tl gradien value o below w decays





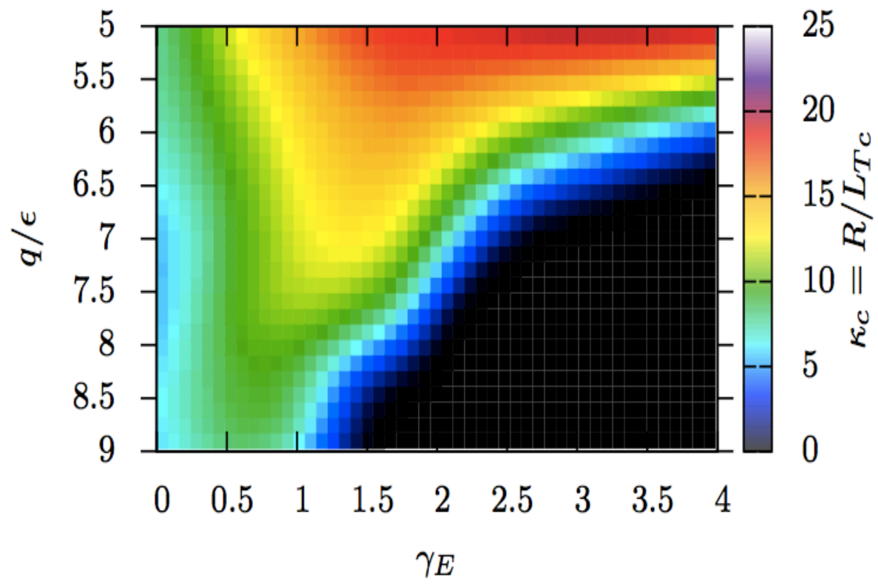
The Zero Turbulence Manifold

- Interpolate to get the zero turbulence manifold:



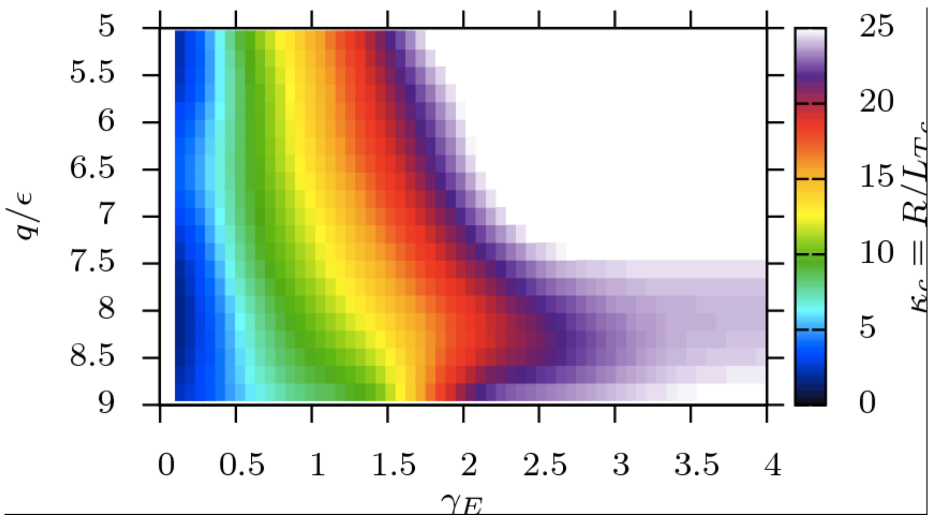
Maximising the Temperature Gradient

- Still find lowest value of q/ϵ
- Now need to find *optimum* value of flow shear (rather than maximising it).

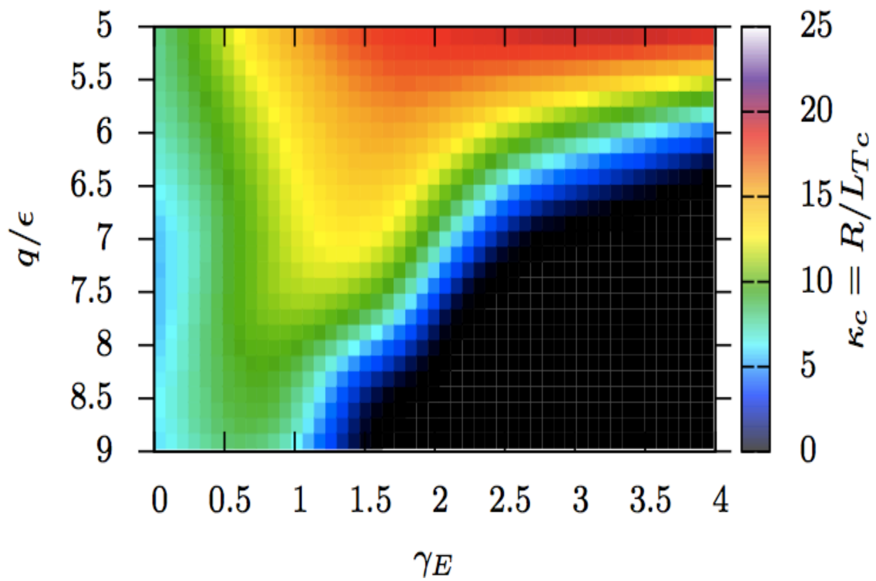


Compare with Prediction using $N_{\max} \sim 1$

- Linear prediction



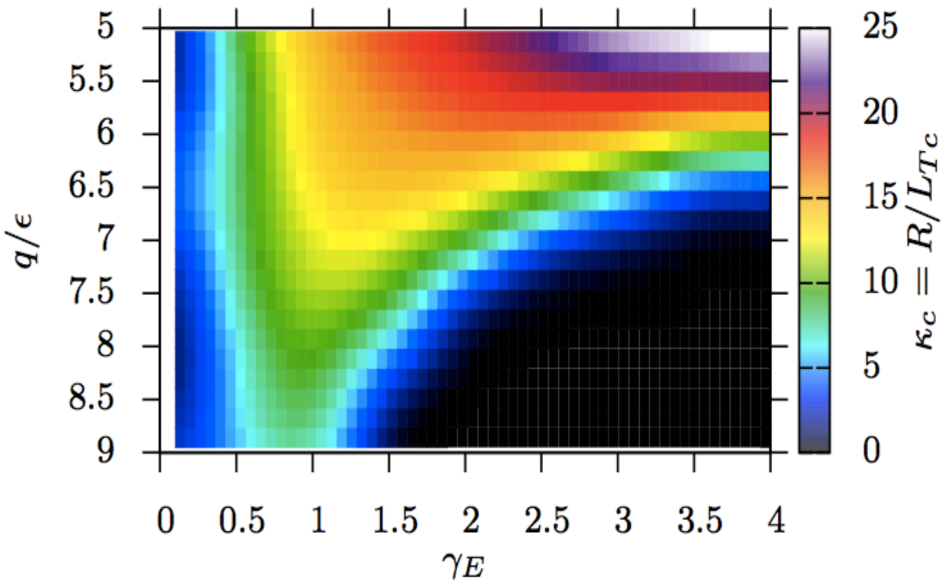
- Nonlinear Manifold



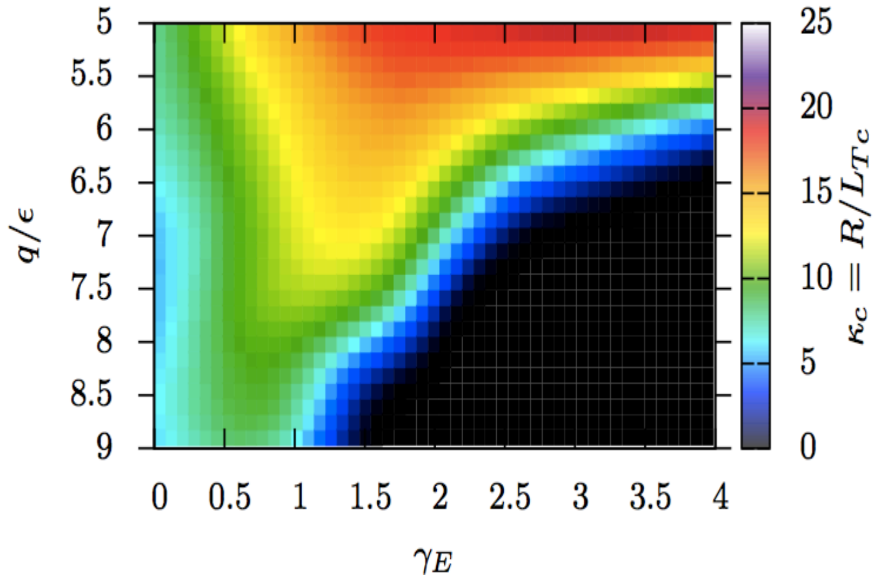
Using fitting formula

- Linear Prediction

$$N_{\max} \sim \begin{cases} 1.0 & \gamma_E \leq 1 \\ \frac{1.0}{\gamma_E} & \gamma_E > 1 \end{cases}$$



- Nonlinear Manifold



Summary

- At zero magnetic shear, considered subcritical turbulence that occurs at non-zero flow shear.
- Nonlinear scan of the 3-way interaction of ion temperature gradient, parallel flow shear and perpendicular shear flow.
- Calculated the manifold that divides the regions in the parameter space of those three parameters where turbulence can and cannot be sustained.
- To get the highest temperature gradient without turbulence:
  - Minimise q/ϵ
  - Find the optimum value of perpendicular flow shear
- Can achieve R/LT > 20
- Linear predictions need a little work!
-