Quasi-isodynamic stellarators

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Confinement of circulating particles

The untrapped orbits are automatically well confined, because the drift velocity is

$$\mathbf{v}_d = \frac{v_\|}{B} \nabla \times \left(\frac{v_\| \mathbf{B}}{\Omega} \right)_{\mu, E}$$

and the net radial drift

$$\int (\mathbf{v}_d \cdot \nabla \psi) \frac{dl}{v_\|} = \int \nabla \cdot \left(\frac{v_\| \mathbf{B} \times \nabla \psi}{\Omega} \right) \frac{dl}{B}$$

along a field line many times around the torus vanishes, because

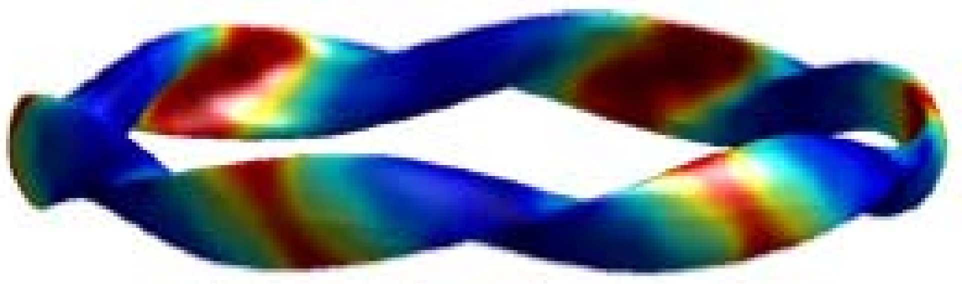
$$\left\langle \nabla \cdot \left(\frac{v_\| \mathbf{B} \times \nabla \psi}{\Omega} \right) \right\rangle = 0$$

Omnigenous configurations

- Omnigenous fields are, by definition, those that confine all collisionless orbits,

$$\int_0^{\tau_b} \mathbf{v}_d \cdot \nabla\psi \, dt = 0$$

- Examples:
  - Quasisymmetric configurations
  - W7-X at high beta
  - More recently found configurations (Subbotin et al 2006)



- Do such configurations have other special properties?

Omnigeneity

- In an exactly omnigenous magnetic field
  - The second adiabatic invariant

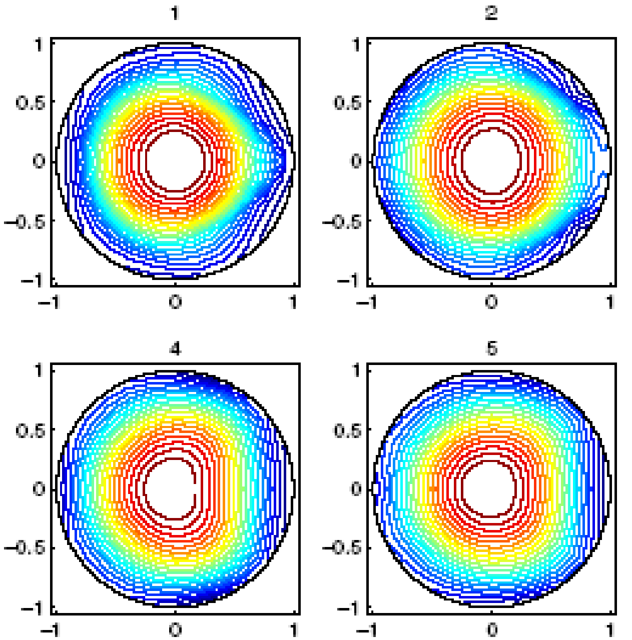
$$J(\psi, \alpha, \lambda) = \int_{l_1}^{l_2} m v_{\|} dl$$

with

$$B(l_1) = B(l_2), \qquad \alpha = \theta - \iota\varphi$$

must be constant on flux surfaces,

$$\frac{\partial J}{\partial \alpha} = 0$$



  - Specifically, for deeply trapped particles *J=0*. Since

$$E = \frac{mv^2}{2} \qquad \text{and} \qquad \mu = \frac{mv_\perp^2}{2B} = \frac{E}{B}$$

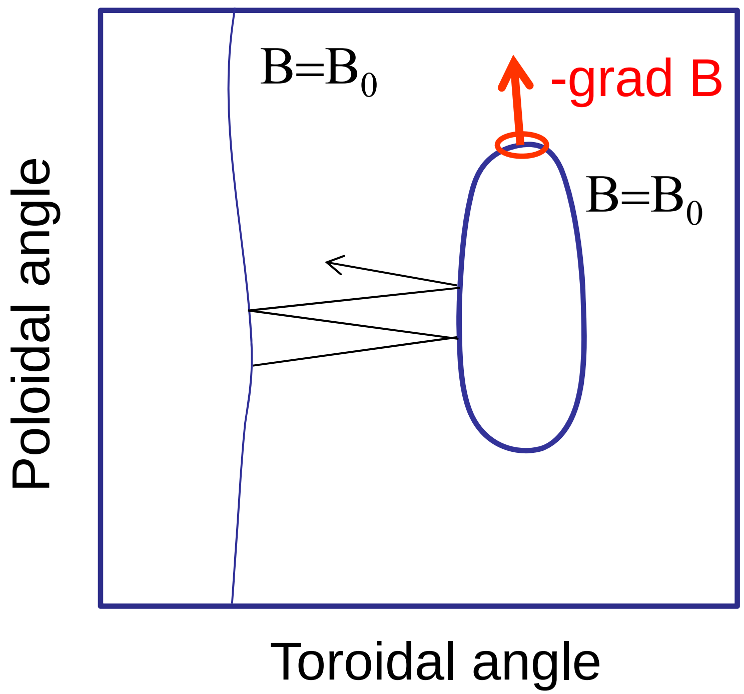
are constant, it follows that <u>the minimum value of B must be same for all field lines on each flux surface</u> (Mynick, Chu and Boozer 1983).

Further properties of omnigenous fields

- Cary and Shasharina (1997) showed that B_{max} is also independent of field line:

$$\frac{\partial B_{\max}}{\partial \alpha} = \frac{\partial B_{\min}}{\partial \alpha} = 0$$

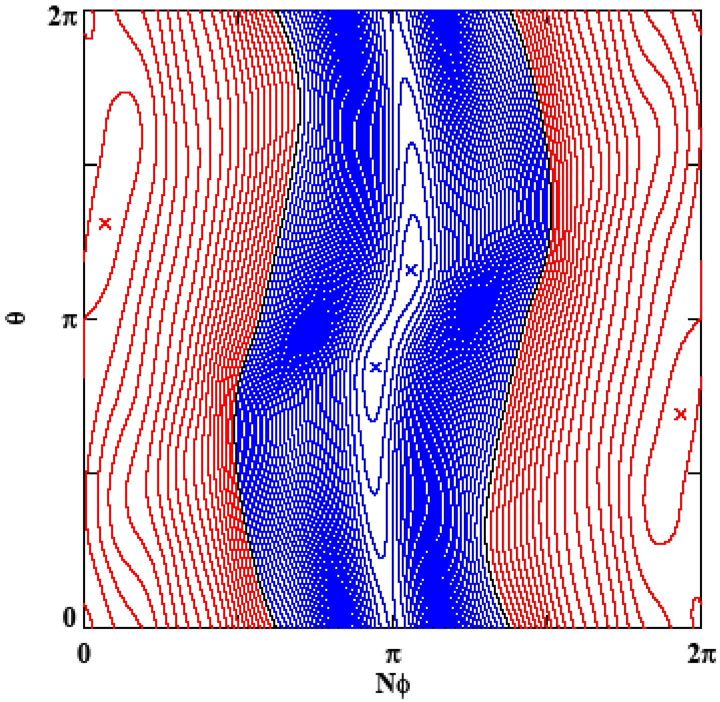
- Otherwise,collisionless detrapping occurs
  - associated with a random radial displacement



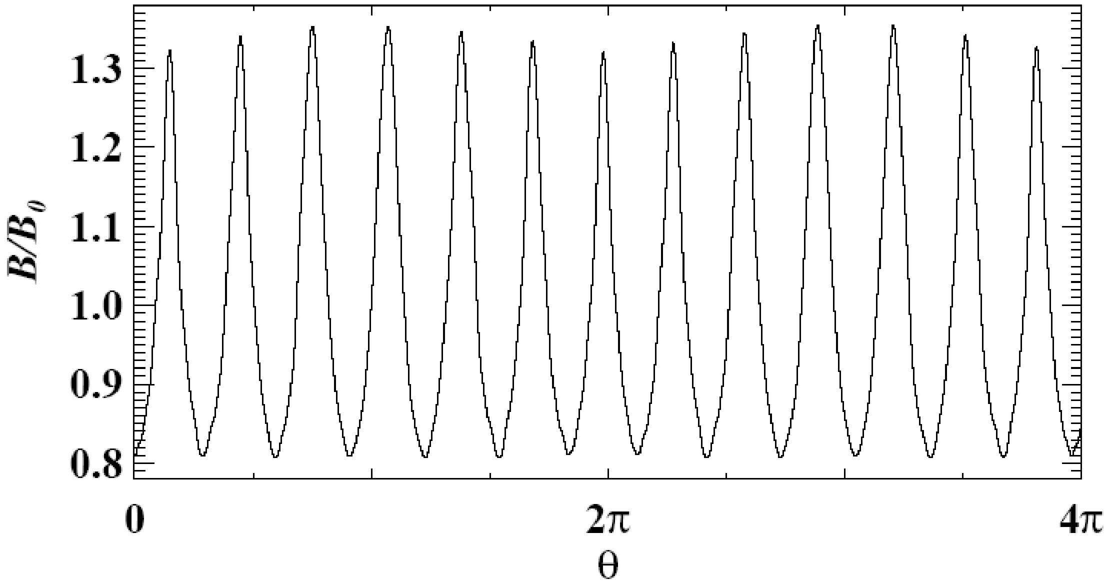
Definition:

quasi-isodynamic = omnigenous with poloidal B-contours

Qi configuration

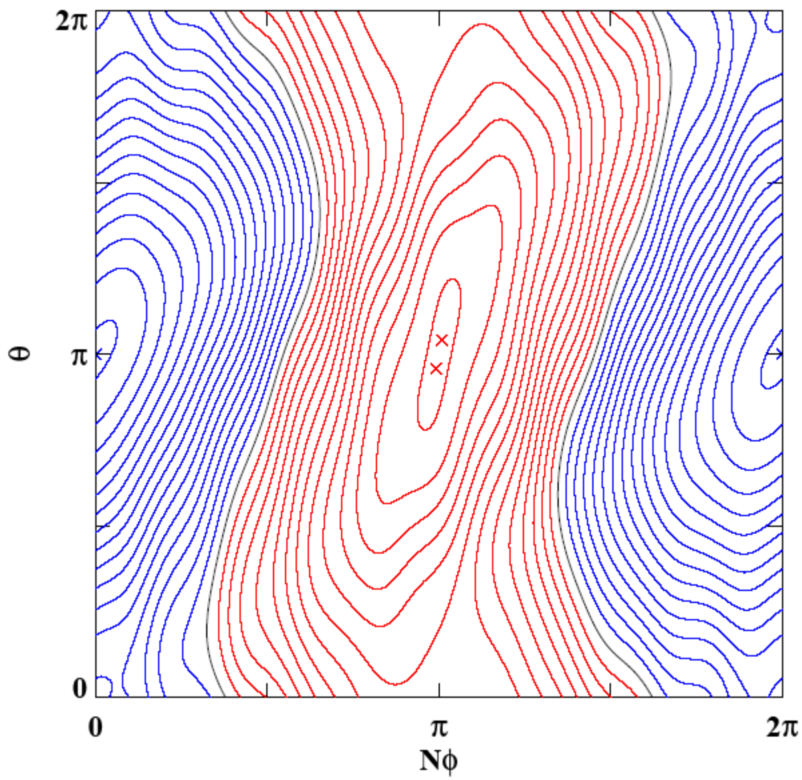


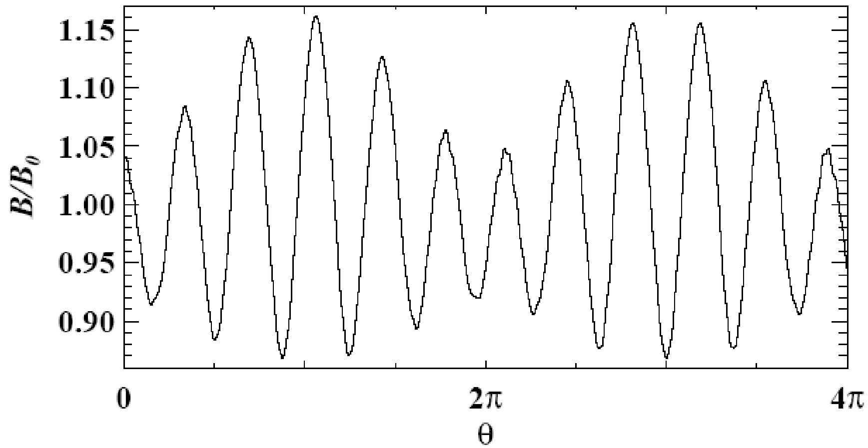
|B| on a flux surface



|B| along a field line

W7-X, high-mirror configuration

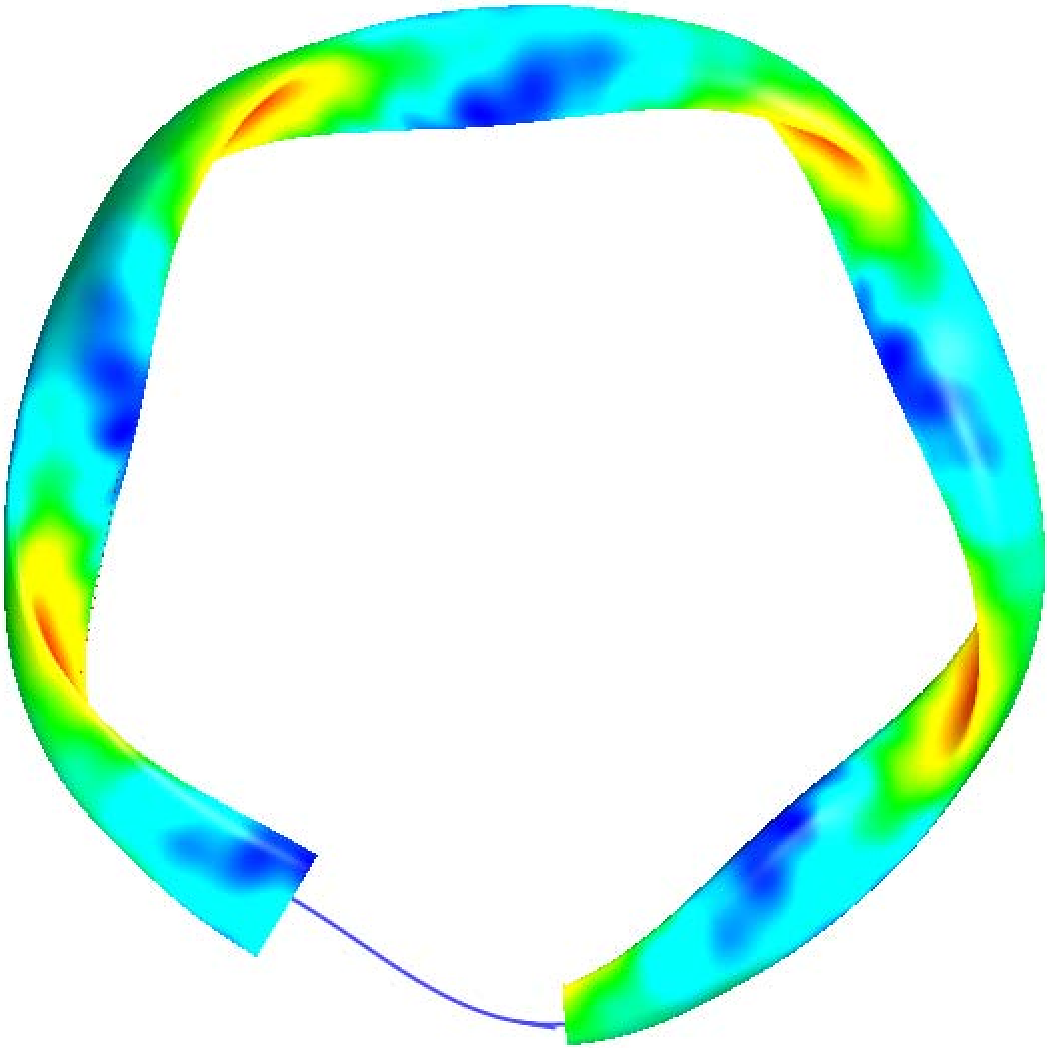




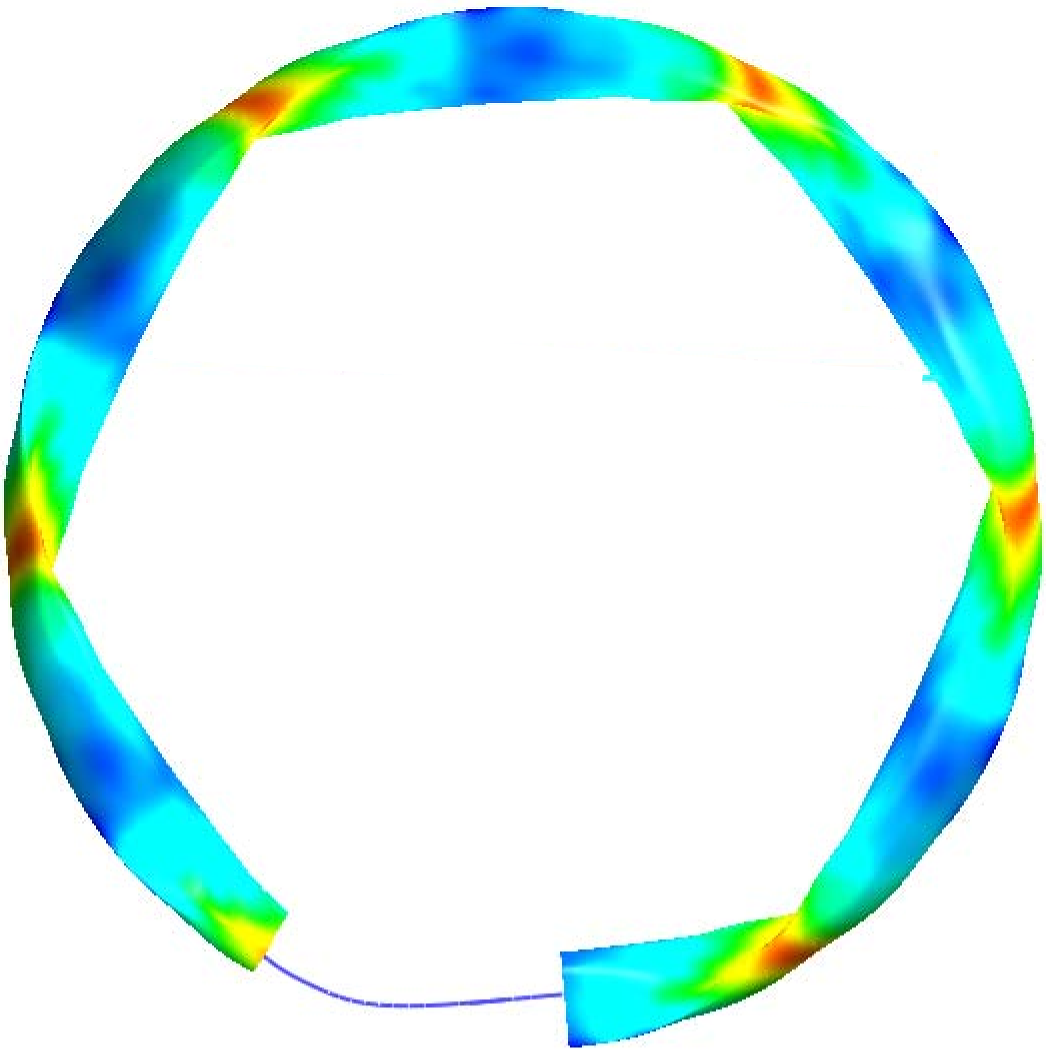
B on a flux surface

B along a field line

W7-X, high-mirror configuration



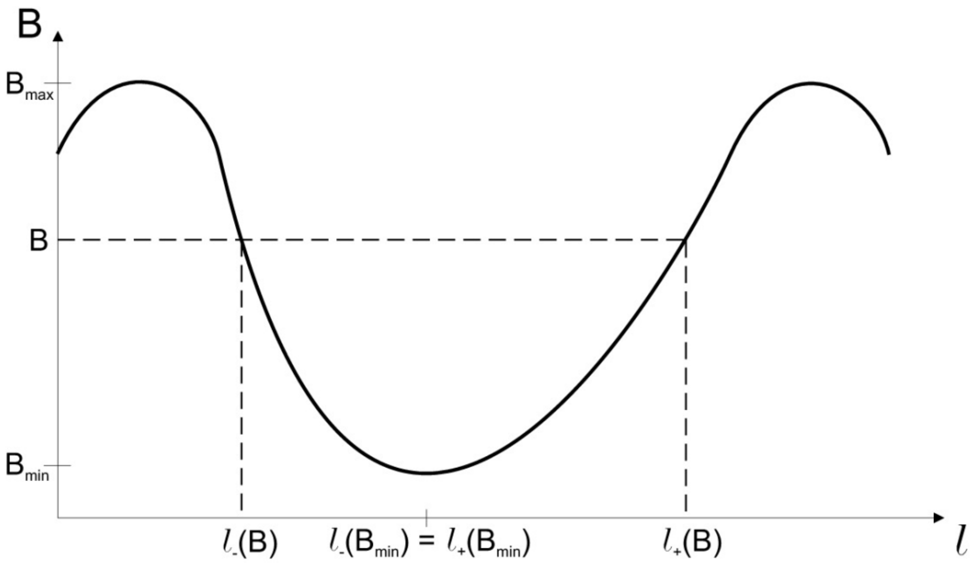
Qi-configuration



Also, for any function $f(\psi, B)$

$$\int_{l_-(\psi,\alpha,B)}^{l_+(\psi,\alpha,B)} f(\psi, B) dl$$

is independent of α.



Radial excursion

$$\Delta\psi = -\frac{\mu_0 J(\psi)}{2\pi}\frac{v_\parallel}{\Omega_a} + \frac{\partial}{\partial\alpha}\int_B^{B_0} h(\psi,\alpha,B)\frac{\partial}{\partial B'}\left(\frac{v_\parallel}{\Omega'_a}\right)dB'$$

$$-\frac{(\mathbf{B}\times\nabla\psi)\cdot\nabla B}{\mathbf{B}\cdot\nabla B} = \frac{\mu_0 J(\psi)}{2\pi} + \frac{\partial h}{\partial\alpha}$$

$$J(\psi) = \text{toroidal current enclosed by } \psi$$

Drift kinetic equation

First-order drift kinetic equation for species a

$$v_\| \nabla_\| f_{a1} + \mathbf{v}_d \cdot \nabla f_{a0} + \frac{e_a v_\| \nabla_\| \phi_1}{T_a} f_{a0} = C_a(f_{a1}) + S$$

Because of omnigeneity, we can write

$$\mathbf{v}_d \cdot \nabla f_{a0} = -v_\| \nabla_\| F_a, \qquad F_a = -\Delta\psi \frac{\partial f_{a0}}{\partial \psi}$$

so that

$$v_\| \nabla_\| (f_{a1} - F_a) = C_a(f_{a1})$$

As usual, expand in the smallness of the collision frequency, $g = g_{a0} + g_{a1} + \dots$

$$\nabla_\| g_{a0} = 0, \qquad v_\| \nabla_\| g_{a1} = C_a(F_a + g_{a0})$$

$$\left\langle \frac{B}{v_\|} C_a(F_a + g_{a0}) \right\rangle = 0$$

The equation we need to solve is

$$\langle C_a(F_a+g_a)\rangle = \oint d\alpha \int_{l_-(B_{\max})}^{l_+(B_{\max})} C_a(F_a+g_a)\frac{dl}{B} \Big/ \oint d\alpha \int_{l_-(B_{\max})}^{l_+(B_{\max})} \frac{dl}{B} = 0$$

But for any function H

$$\oint d\alpha \int_{l_-(\psi,\alpha,B_{\max})}^{l_+(\psi,\alpha,B_{\max})} C_a\left(\frac{\partial H}{\partial \alpha}\right)\frac{dl}{v_\parallel} = \oint d\alpha \frac{\partial}{\partial \alpha}\int_{l_-(\psi,\alpha,B_{\max})}^{l_+(\psi,\alpha,B_{\max})} C_a\left(H\right)\frac{dl}{v_\parallel} = 0$$

so in a currentless stellarator,

$$J(\psi) = 0 \Rightarrow g_a = 0$$

there is no bootstrap current.

Bootstrap current

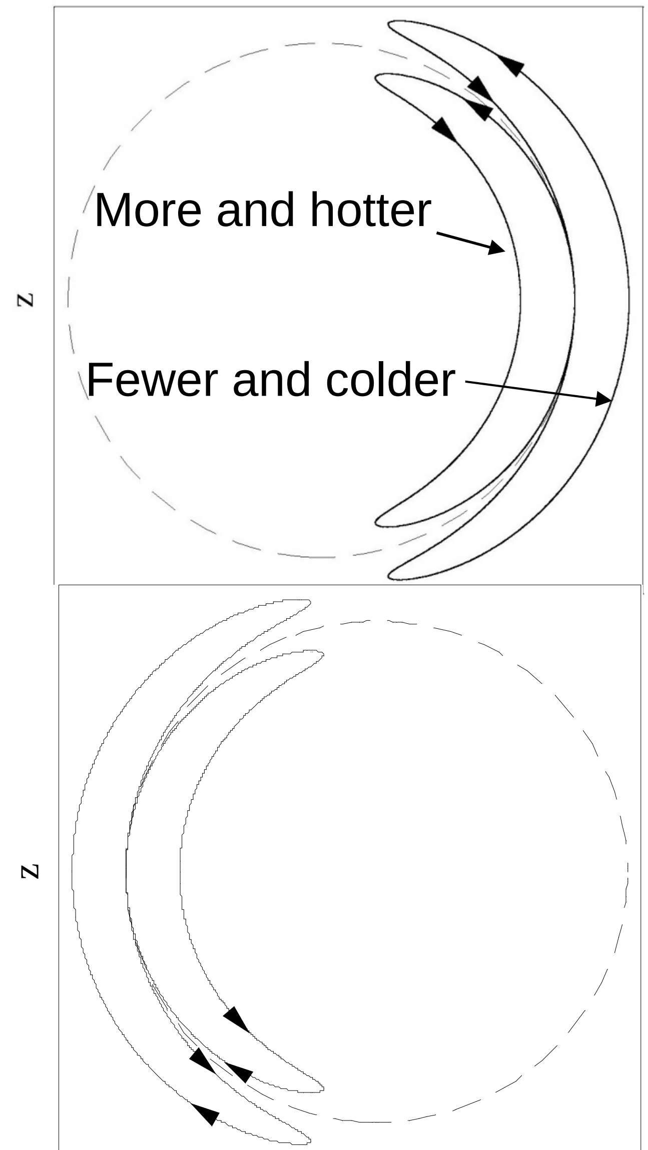
- Diamagnetic effect of banana orbits: on each flux-surface there are
  - more counter-moving trapped electrons than co-moving ones,
  - more co- than counter-moving trapped ions
  - density discrepancy

$$\Delta n_{\rm trapped} \sim -\epsilon^{1/2} \frac{dn}{dr} \Delta r_{\rm banana}$$

- These particles produce a drag on the ciculating ones. Collisional equilibrium occurs when

$$\Delta n_{\rm circ} \sim -\frac{dn}{dr} \Delta r_{\rm banana} \qquad \Rightarrow \qquad j \sim -\frac{\epsilon^{1/2}}{B_p} \frac{dp}{dr}$$

- In stellarators, trapping occurs on the inboard side, too.



Finite collisionality

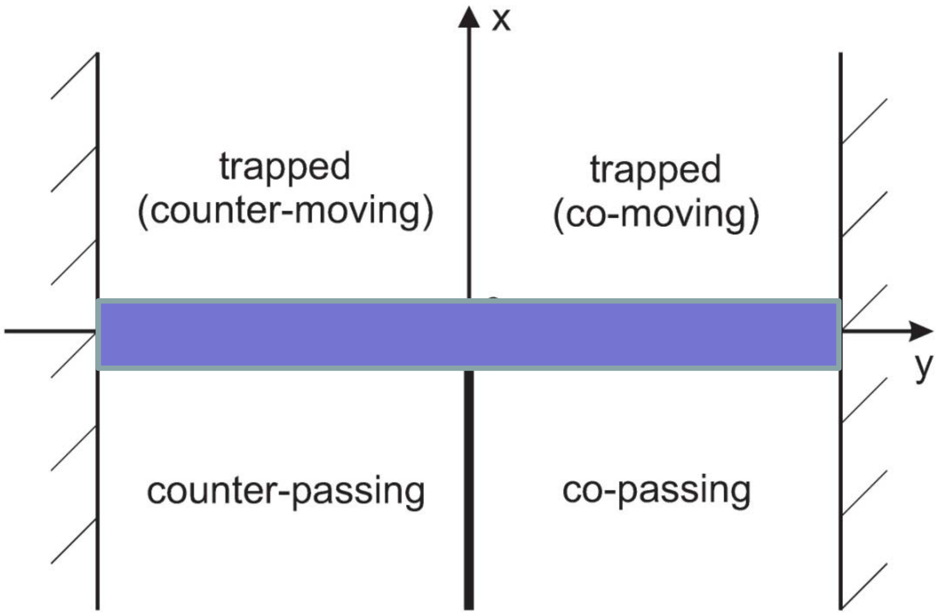
- A finite but small collisionality causes a boundary layer between the trapped and untrapped regions of velocity space.

$$v_{\|} \nabla_{\|} g_a = C_a(g_a + F_a)$$

$$\mathbf{B} \cdot \nabla g_a = \frac{2\nu_D^a}{v} \frac{\partial}{\partial \lambda} \left(\lambda \sqrt{1 - \lambda B} \frac{\partial g_a}{\partial \lambda} \right)$$

$$\frac{\partial g_a}{\partial y} = \frac{\partial^2 g_a}{\partial x^2}$$

$$x(\lambda) = \frac{\lambda B_{\max} - 1}{\sqrt{\nu_{a0}}}$$

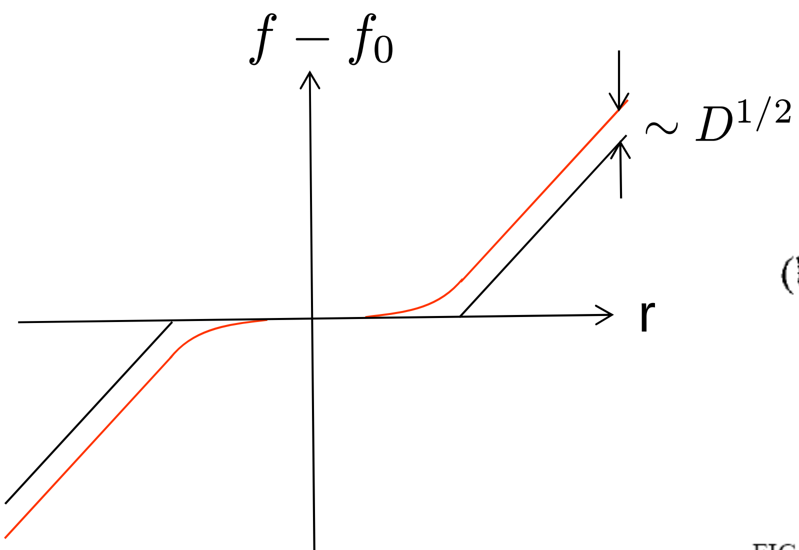


$$y(l) = \sigma \pi \int_{l_{\max}}^{l} \sqrt{1 - b(l')} \frac{dl'}{b(l')} \Big/ \oint \sqrt{1 - b(l')} \frac{dl'}{b(l')}, \qquad b = B/B_{\max}$$

A similar problem

- Simplest transport problem around a magnetic island (Hazeltine, Helander and Catto, 1997)

$$v_{\|}\nabla_{\|}f = \nabla\cdot(D\nabla f)$$



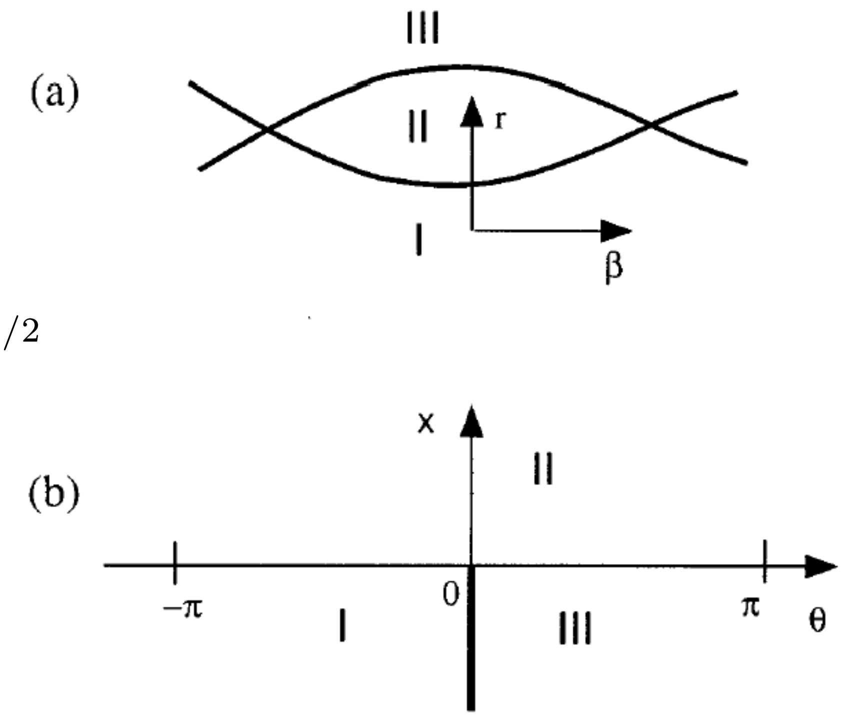


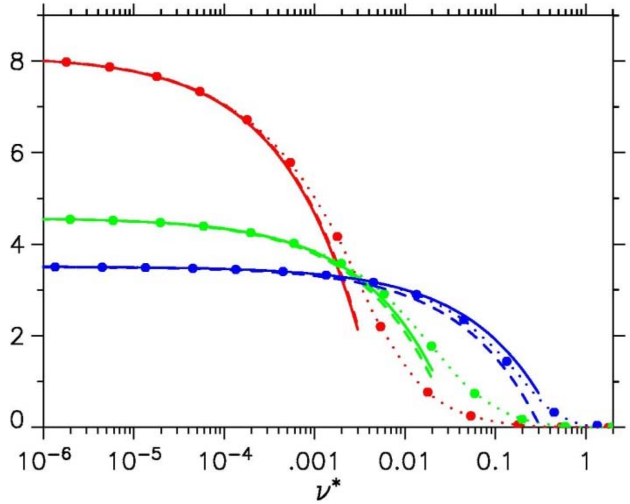
FIG. 1. The original island geometry (a) and its rearrangement (b). Note the regions I, below the island chain and outside its separatrix; II, inside the separatrix; and III, above the island chain and outside its separatrix. The two structures are physically equivalent, provided the thick solid line in (b) is supposed to be impenetrable.

Bootstrap current

- In a tokamak at low collisionality

$$\langle J_{bs} B \rangle = B_0 \left(J_0 - J_1 \nu_*^{1/2} + \cdots \right) \qquad (\nu_* = \text{collisionality})$$

$\langle J_{bs} B \rangle$



3 different flux surfaces in NSTX

- Also true in a quasi-isodynamic stellarator
  - in the absence of current drive

$$J_0 = J_1 = 0$$

Maximum J and stability

- In quasi-isodynamic stellarators, J is a function of ψ, and in fact

$$\frac{\partial J}{\partial \psi} < 0$$

- If $\omega \ll \omega_b$ then

$$J(\psi, E, \mu) = \int_{l_1}^{l_2} m v_{\|} dl = \text{constant}$$

- If an instability moves a particle radially, then

$$\Delta J = \frac{\partial J}{\partial \psi}\Delta\psi + \frac{\partial J}{\partial E}\Delta E = 0 \qquad \Rightarrow \qquad \Delta E = -\frac{\partial J/\partial\psi}{\partial J/\partial E}\Delta\psi$$

so that stability is promoted by

$$\frac{\partial J}{\partial \psi} < 0$$

Drift reversal

If

$$\frac{\partial J}{\partial \psi} < 0$$

the bounce-averaged curvature is favourable, and

$$\overline{\omega}_{da} = k_\alpha \overline{\mathbf{v}_{da} \cdot \nabla \alpha} = -\frac{k_\alpha}{e_a \tau_{ba}} \left(\frac{\partial J}{\partial \psi} \right)_{\mu,E}$$

$$\omega_{*a} = \frac{k_\alpha T_a}{e_a} \frac{d n_a}{d\psi}$$

so that

$$\overline{\omega}_{da} \omega_{*a} = -\frac{k_\alpha^2 T_a}{e_a^2 \tau_{ba}} \frac{\partial J}{\partial \psi} \frac{d n_a}{d\psi} < 0$$

Conclusions

- By definition, quasi-isodynamic configurations
  - confine all collisionless orbits
  - have poloidally closed B-contours
- The bootstrap current vanishes to a high degree of approximation.
- The bounce-averaged curvature is favourable for all orbits.