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Magnetic stochasticity and transport due to nonlinear excitation of microtearing modes

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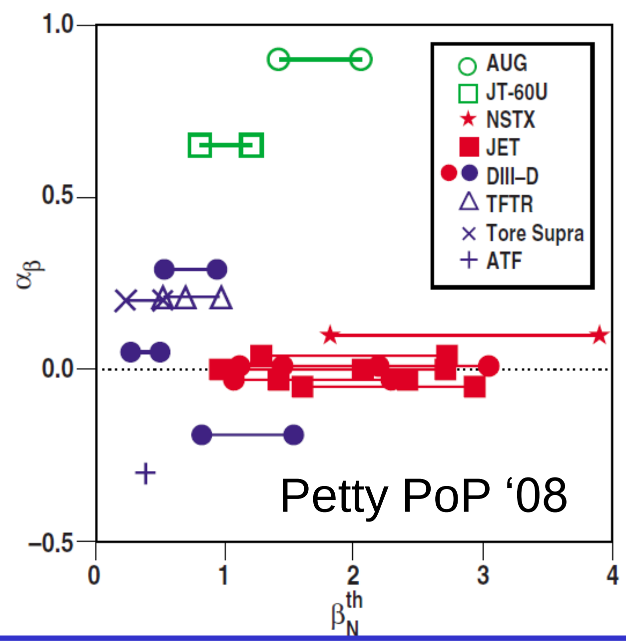
Electromagnetic effects

➔High beta is very desirable for a fusion plasma.

$$\underbrace{nT}\underbrace{\tau_E} = 3\times10^{21} m^{-3} keVs$$

$$\beta = \frac{\boxed{n_{0e}T_{0e}}}{B_0^2/2\mu_0}$$

What is the effect on turbulent transport?



Confinement time:
Wide variation in beta scaling

$$\tau_E \propto \beta^{-\alpha}$$

ITG Turbulence with Electromagnetic Effects

Before nonlinear GK microtearing was tackled . . .

➔basic question

What do electromagnetic effects do in basic ITG/ETG turbulence?

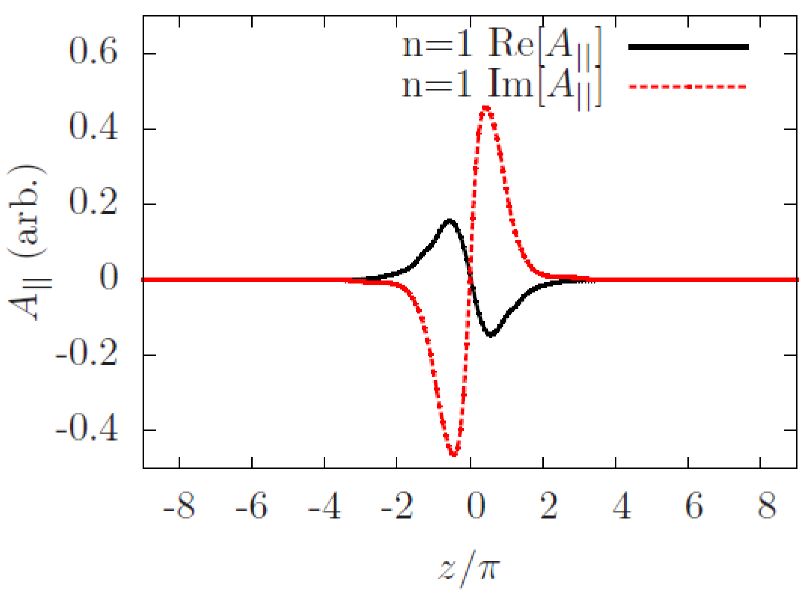
Addressed by several studies in past decade:

Specifically ITG:
Parker PoP ‘04, Candy PoP ‘05, Pueschel PoP ‘08, Waltz PoP ‘10, Nevins PRL ‘11, Wang PoP ‘11

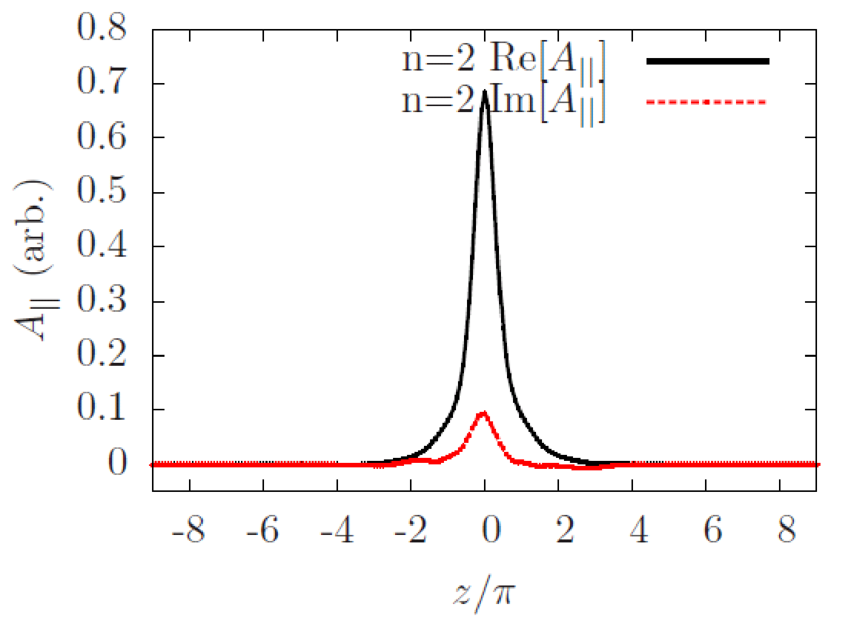
ITG has Ballooning Parity not Tearing Parity

Field following coordinates:
Resonant (tearing) component of $A_{||}$ extracted with an integral along field line.

ITG - Ballooning parity: no resonant component



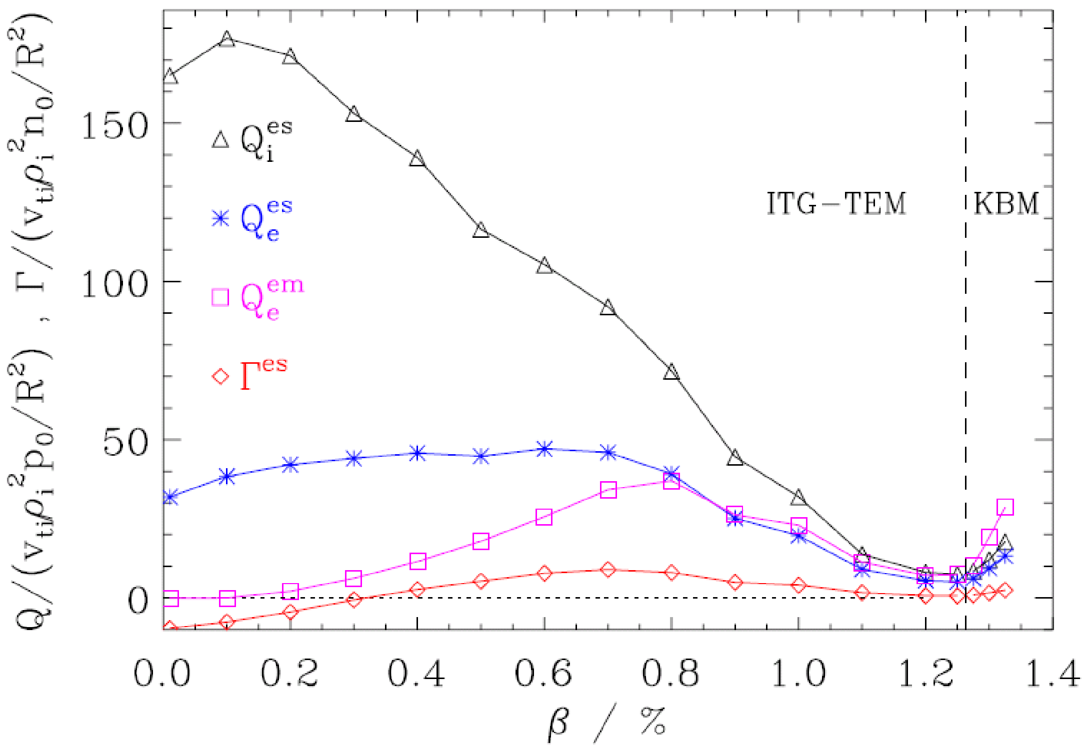
Tearing parity: contains resonant component



Significant Electromagnetic Transport

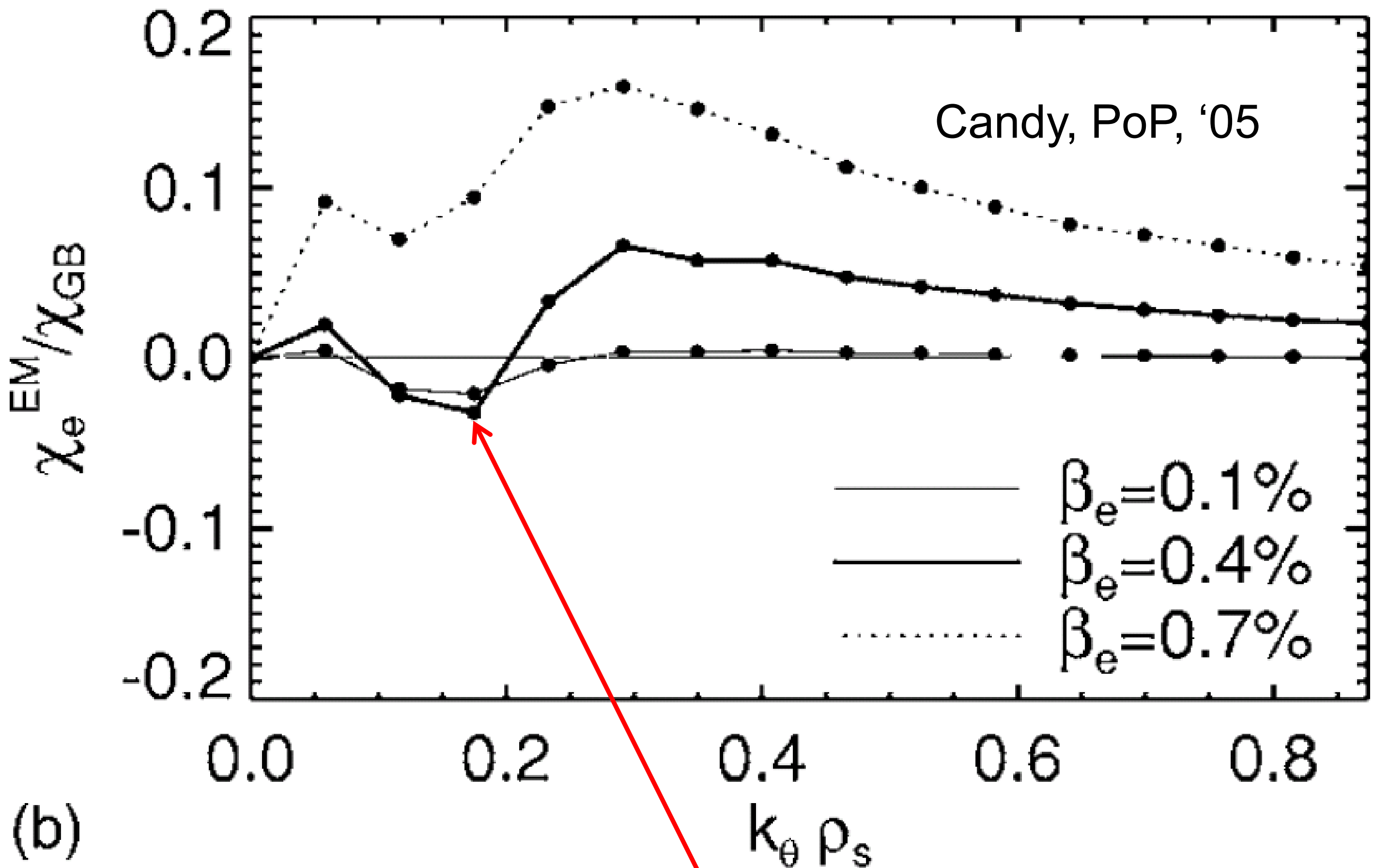
ITG driven turbulence:

Levels of electron electromagnetic heat transport that approach electrostatic transport as beta increases [Candy PoP '05, Pueschel PoP '08].



Pueschel, PoP, '08

Magnetic transport – unusual k_y spectra

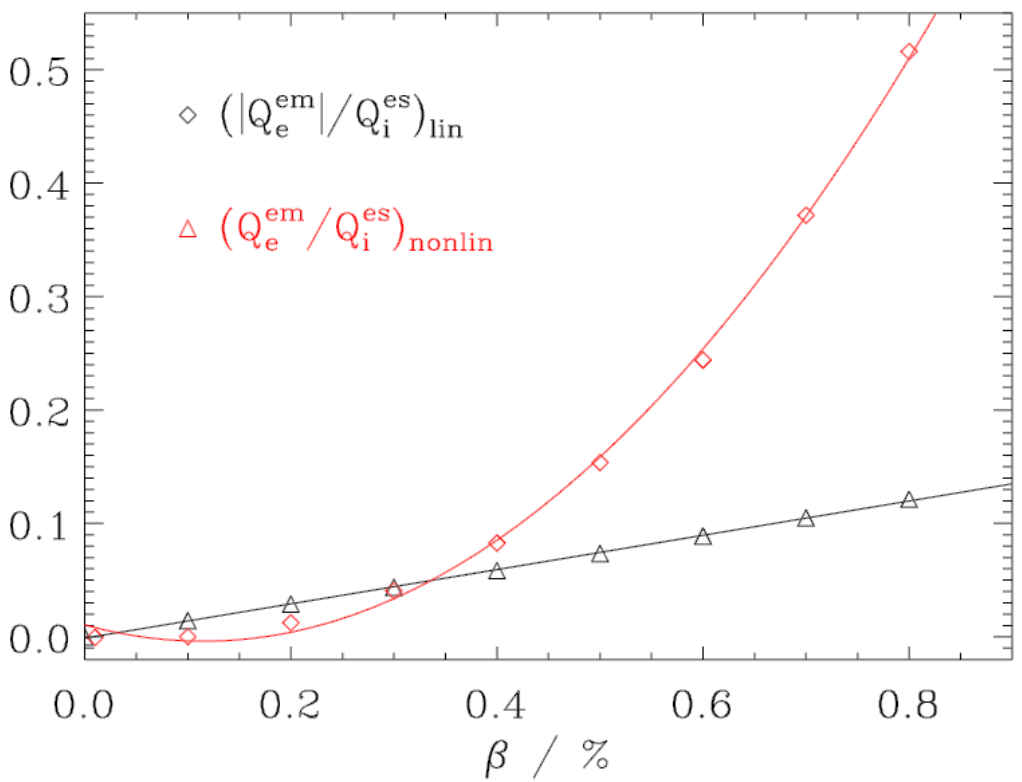


Electromagnetic flux spectrum has "dip" at scales where electrostatic transport peaks.

Anomalous beta scaling

ITG driven turbulence:

Magnetic transport violates quasilinear theory – β^2-scaling.

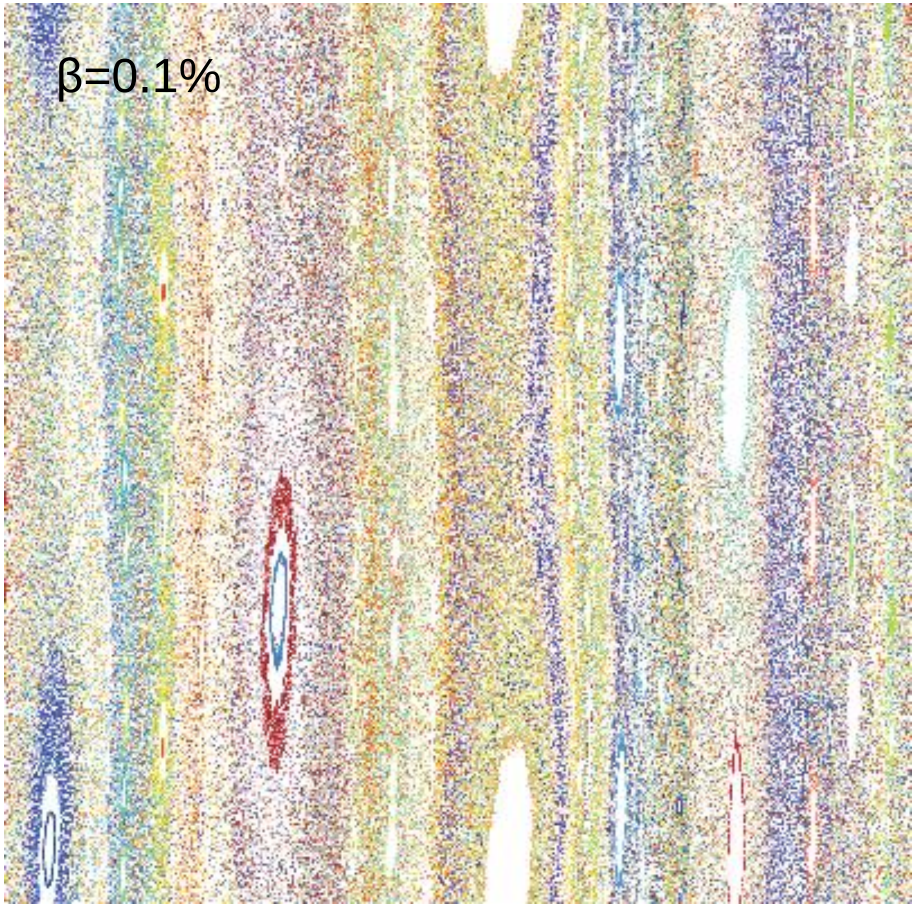


Pueschel PoP '08

Introduction – Recent Results

ITG driven turbulence:

Near-ubiquitous magnetic stochasticity – even at low values of beta [Nevins PRL11, Wang PoP '11]



Nevins PRL, '11

What is the cause?

Observations of electromagnetic effects inconsistent with properties of the driving ITG modes.

Question:

What is the explanation for the observed stochasticity and transport?

Answer:

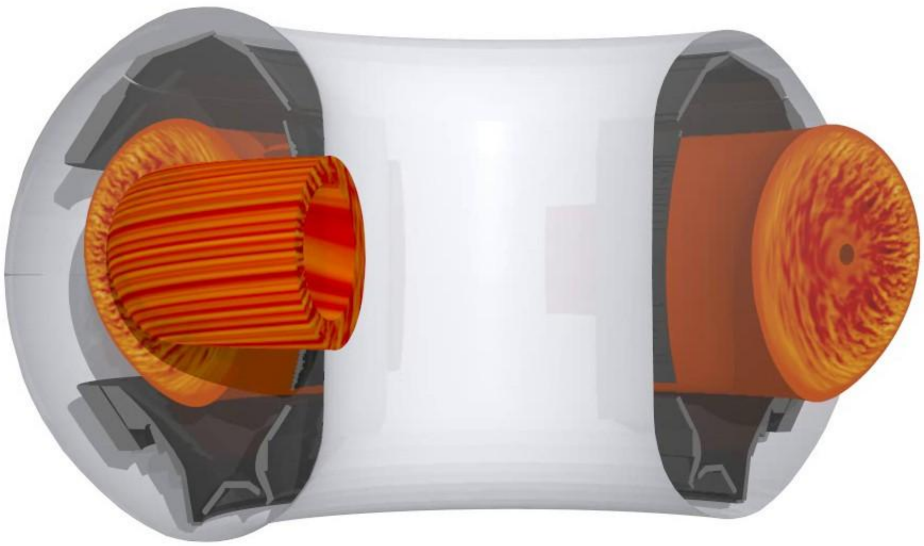
Stochasticity and transport are due to the nonlinearly excitation of damped* microtearing modes.

*Damped eigenmode studies:

Gyrokinetic: Hatch PRL '11, PoP '11

Fluid models: Terry PoP '06, Kim PoP '10, Makwana PoP '11

Description of Simulations



Simulation data from GENE code: http://gene.rzg.mpg.de

Cyclone Base Case parameters plus electromagnetic effects (finite β).

β scan (ranging from electrostatic to 1.2%)

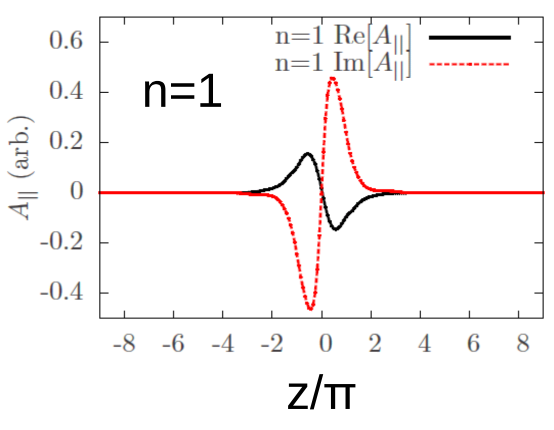
POD* used to separate tearing and ballooning fluctuations

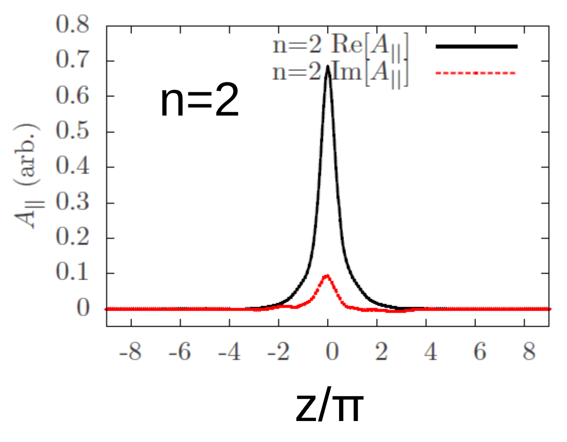
Challenge: isolating two distinct modes operating at the same scales.

Proper Orthogonal Decomposition: $A_{\|k}(z,t) = \sum_n A_{\|k}^{(n)}(z) h_k^{(n)}(t)$

$A^{(n)}_{\|}$, and $h^{(n)}$ are orthogonal and 'optimal' – most efficient decomposition.

n=1 and n=2 modes define a ballooning component and a tearing component.

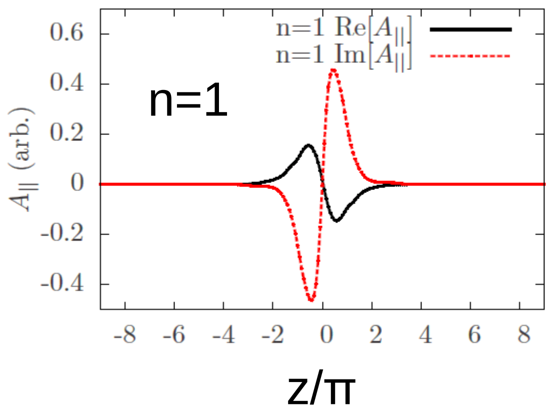


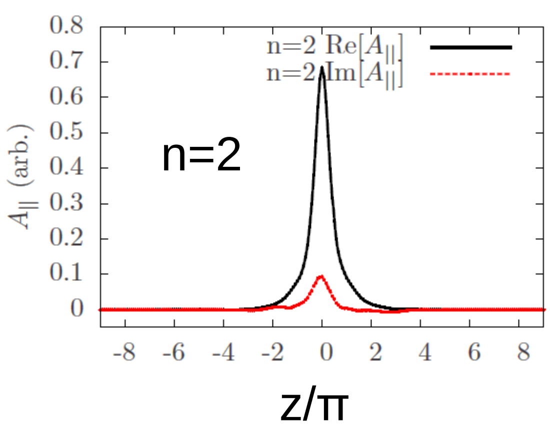


*Selected POD references: Berkooz, Annu. Rev. Fluid Mech. '93, Futatani, PoP, '09

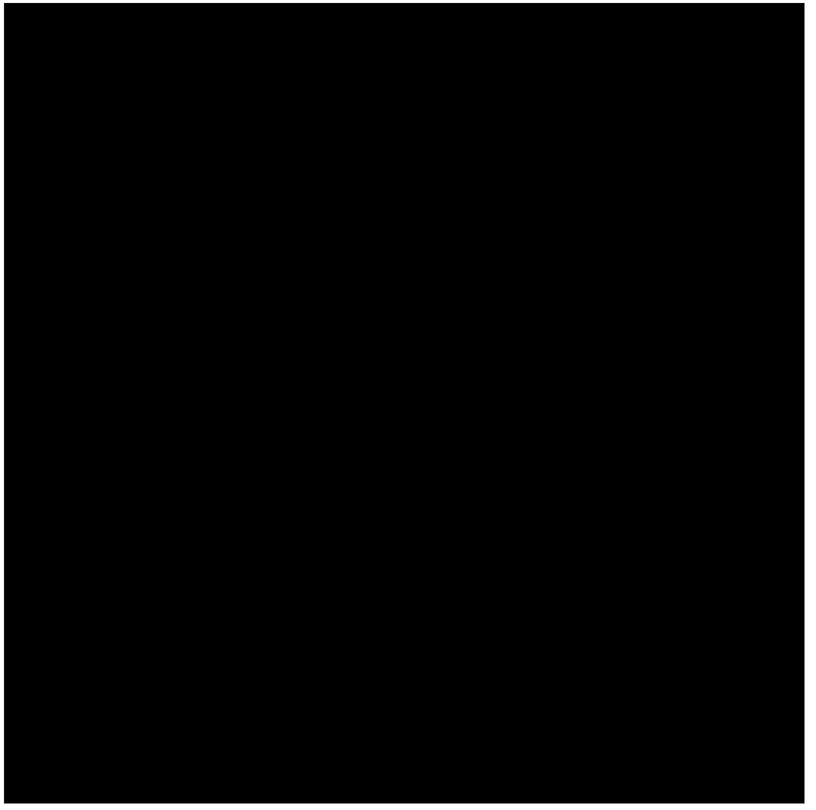
Tearing parity contributes to nonlinear fluctuations

Ballooning





Tearing



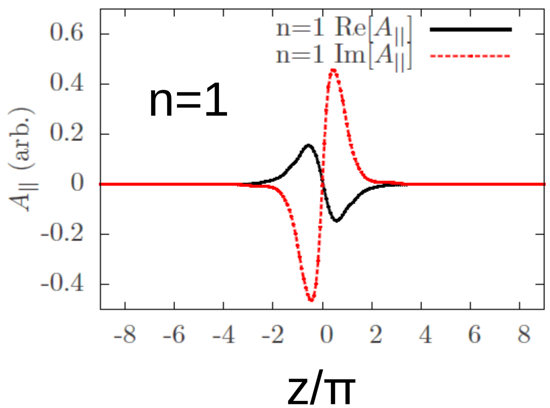
Re[$A_{||}$] fluctuations (k_x=0.0,k_y=0.3)

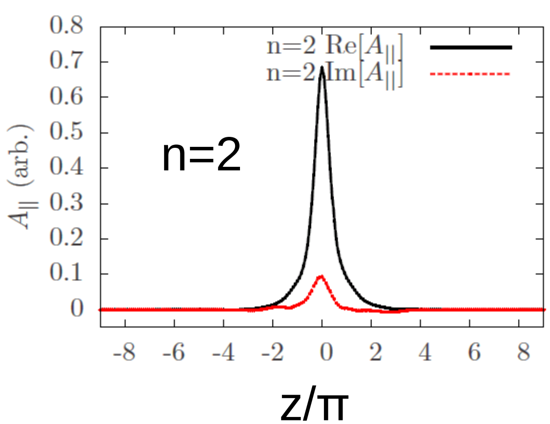
Total Nonlinear

$A_{\parallel}^{(1)} h^{(1)}(t)$

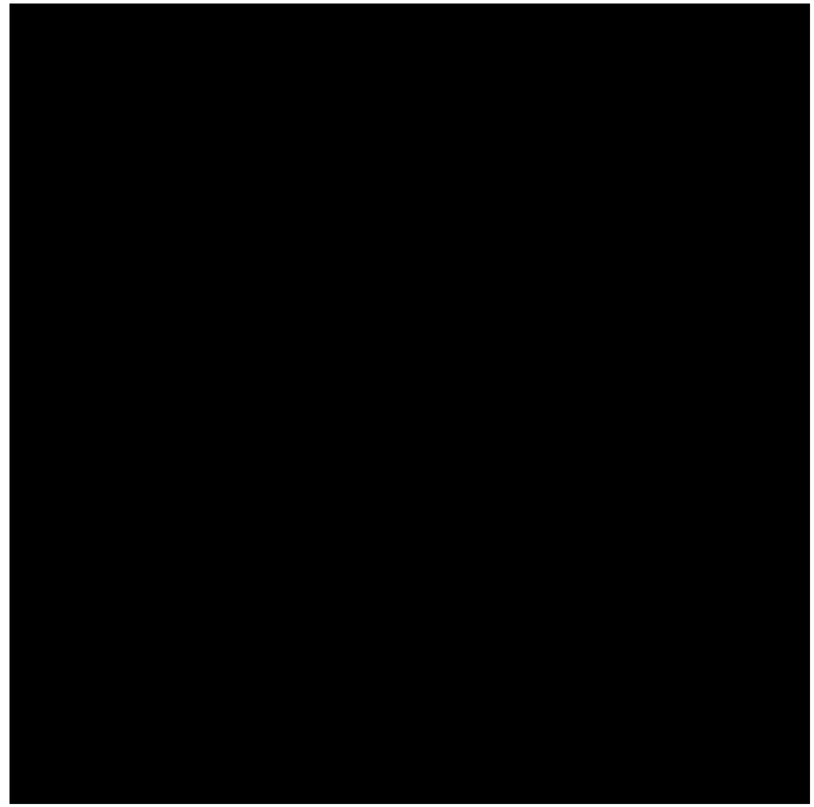
POD ➔ Two modes account for most of nonlinear fluctuations

Ballooning





Tearing

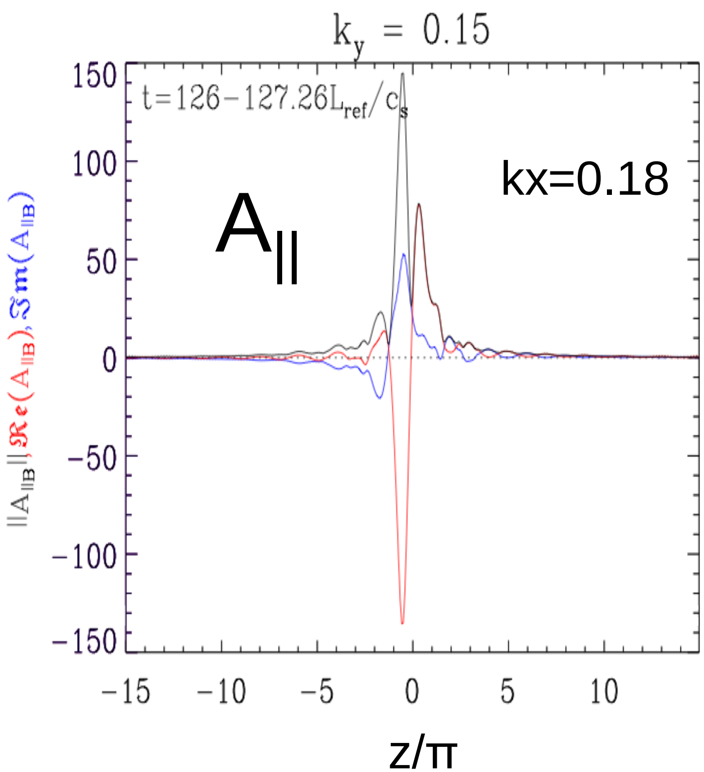


Re[$A_{||}$] fluctuations (k_x=0.0,k_y=0.3)

Total Nonlinear

$A_{\|}^{(1)}h^{(1)}(t) + A_{\|}^{(2)}h^{(2)}(t)$

Perfect ballooning parity no longer enforced at |kx|>0



Even / Odd symmetry is no longer enforced at $k_x \neq 0$. But modes are still "predominantly even" or "predominantly odd".

➔Distinguish with Parity Factor

$$P = \frac{\left| \int dz A_{\|k_x,k_y}(z,t) \right|}{\int dz \left| A_{\|k_x,k_y}(z,t) \right|}$$

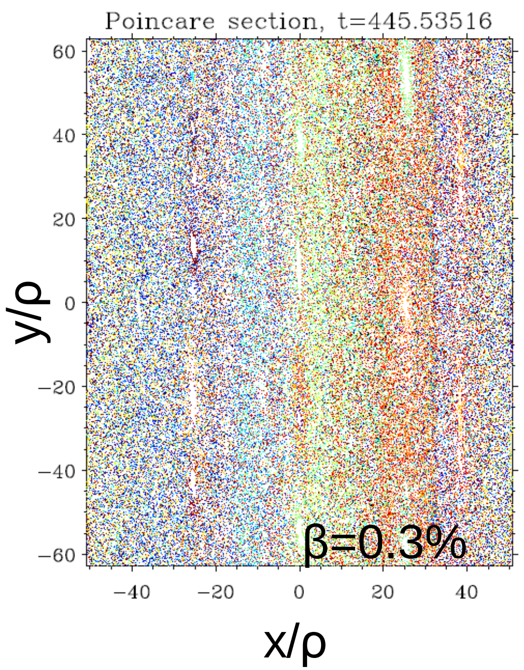
Construct a "tearing-ballooning" decomposition:

$$A_{\|k_x,k_y}(z,t) = A^{(ball)}_{\|k_x,k_y}(z,t) + A^{(tear)}_{\|k_x,k_y}(z,t) + A^{(res)}_{\|k_x,k_y}(z,t)$$

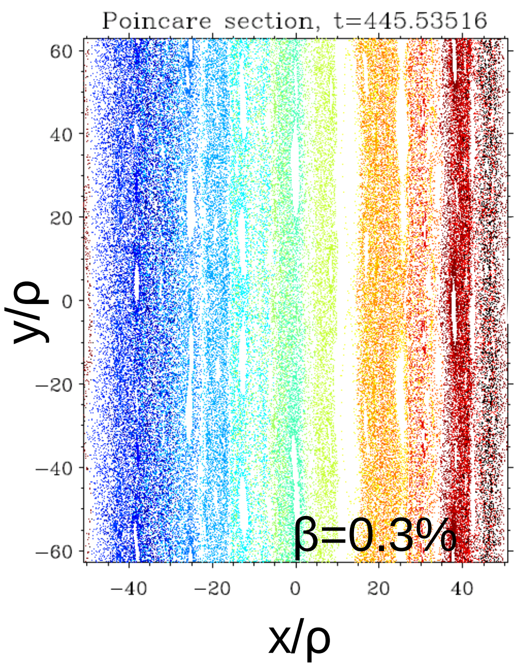
Stochasticity caused by tearing component

$$A_{\|k_x,k_y}(z,t) = A^{(ball)}_{\|k_x,k_y}(z,t) + A^{(tear)}_{\|k_x,k_y}(z,t) + A^{(res)}_{\|k_x,k_y}(z,t)$$

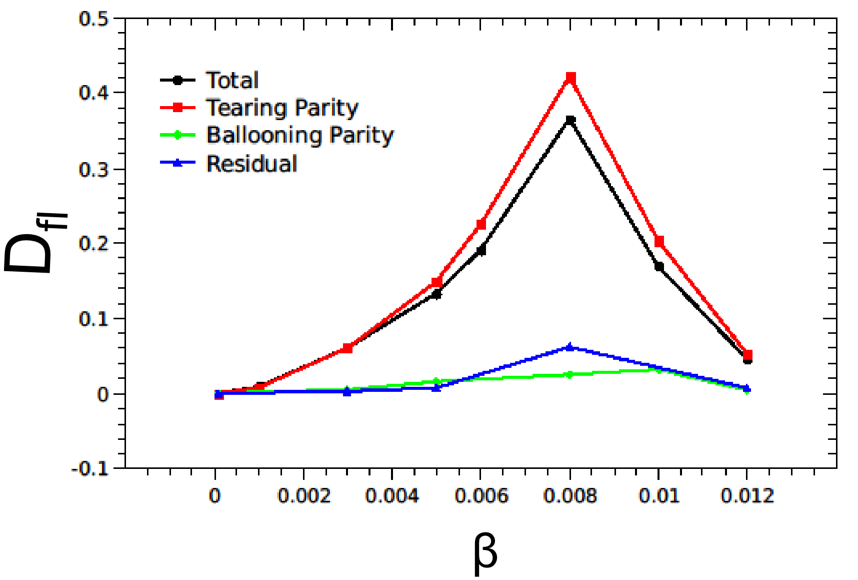
$$A^{(tear)}_{\|k_x,k_y}(z,t)$$



$$A^{(ball)}_{\|k_x,k_y}(z,t)$$

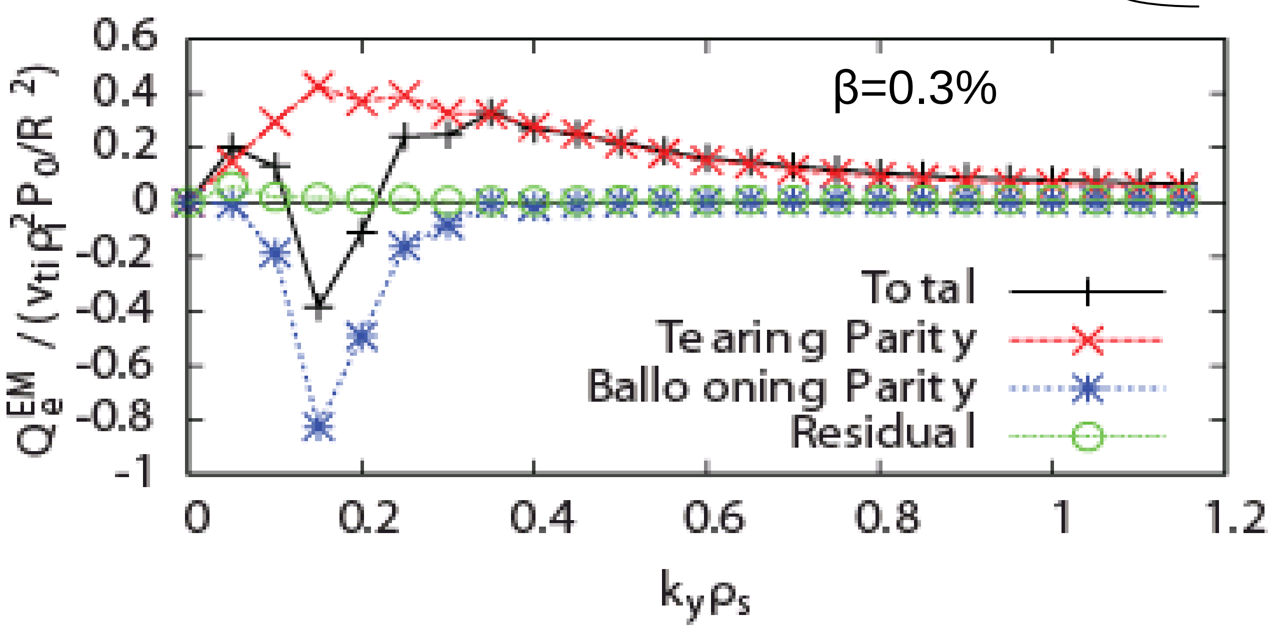


$$D_{fl} = \lim_{l\to\infty} \frac{1}{l} \left\langle \left| r_i(l) - r_i(0) \right|^2 \right\rangle$$

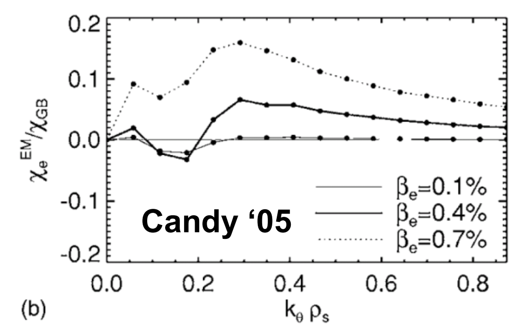


Magnetic transport – superposition of ITG and tearing

$$A_{\parallel k_x,k_y}(z,t) = A^{(ball)}_{\parallel k_x,k_y}(z,t) + A^{(tear)}_{\parallel k_x,k_y}(z,t) + A^{(res)}_{\parallel k_x,k_y}(z,t) \rightarrow \begin{cases} Q_e^{EM\,(tot)} = q_\parallel B_x / B_0 \\ Q_e^{EM\,(tear)} = q_\parallel B_x^{(tear)} / B_0 \\ Q_e^{EM\,(ball)} = q_\parallel B_x^{(ball)} / B_0 \\ Q_e^{EM\,(res)} = q_\parallel B_x^{(res)} / B_0 \end{cases}$$



Recall:

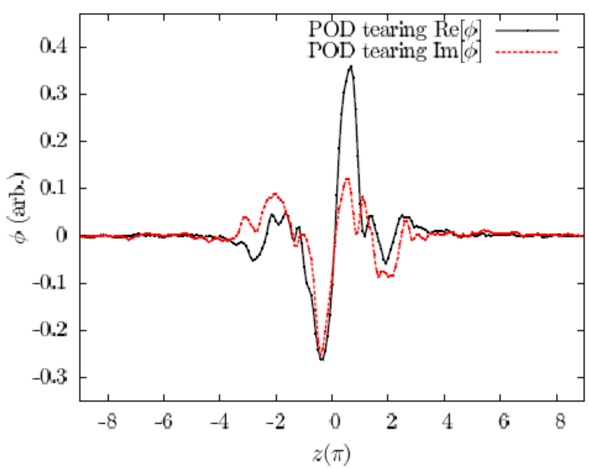


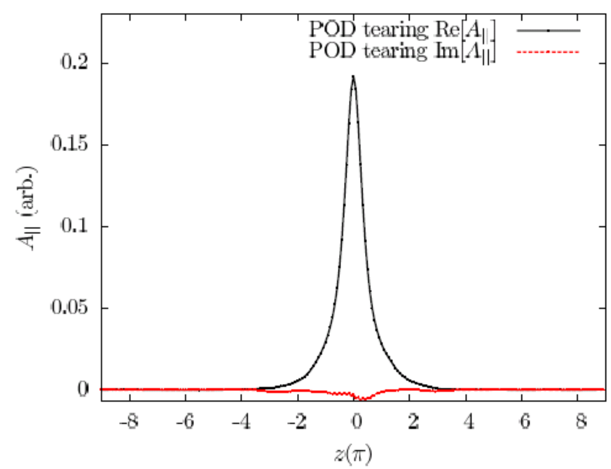
POD of distribution function: Self consistent $A_{||}$, ϕ

Construct POD of distribution function:

$$g_{kx,ky}(z,v_{\|},\mu,t)=\sum_{n} f^{(n)}_{kx,ky}(z,v_{\|},\mu)h^{(n)}_{kx,ky}(t)$$

➔Tearing parity mode with large heat flux (POD n~2-5)

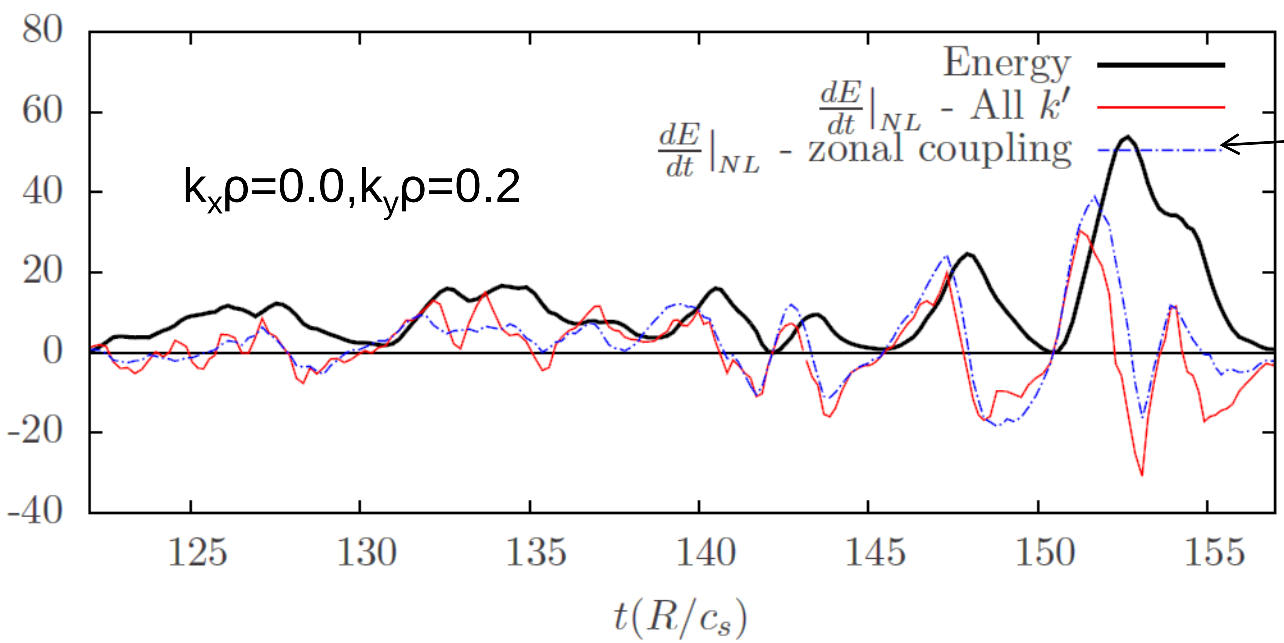




Excitation mechanism: coupling with k_y=0

$$\frac{\partial E_k^{(tear)}}{\partial t} = L[g_k^{(tear)}, g_k] + \sum_{k'_\perp} \boxed{N[g_k^{(tear)}, g_{k'}, g_{k-k'}]} + c.c.$$

Free energy evolution equation
for tearing mode



Energy transfer involving coupling with zonal wavenumbers (k_y=0):

$k_y'\rho = 0.0, k_y''\rho = 0.2$

$k_y'\rho = 0.2, k_y''\rho = 0.0$

Nonlinear mechanism which stabilizes ITG in turn drives microtearing and produces additional transport channel.

Linear spectrum – many modes with tearing parity

Using GENE eigenmode solver.

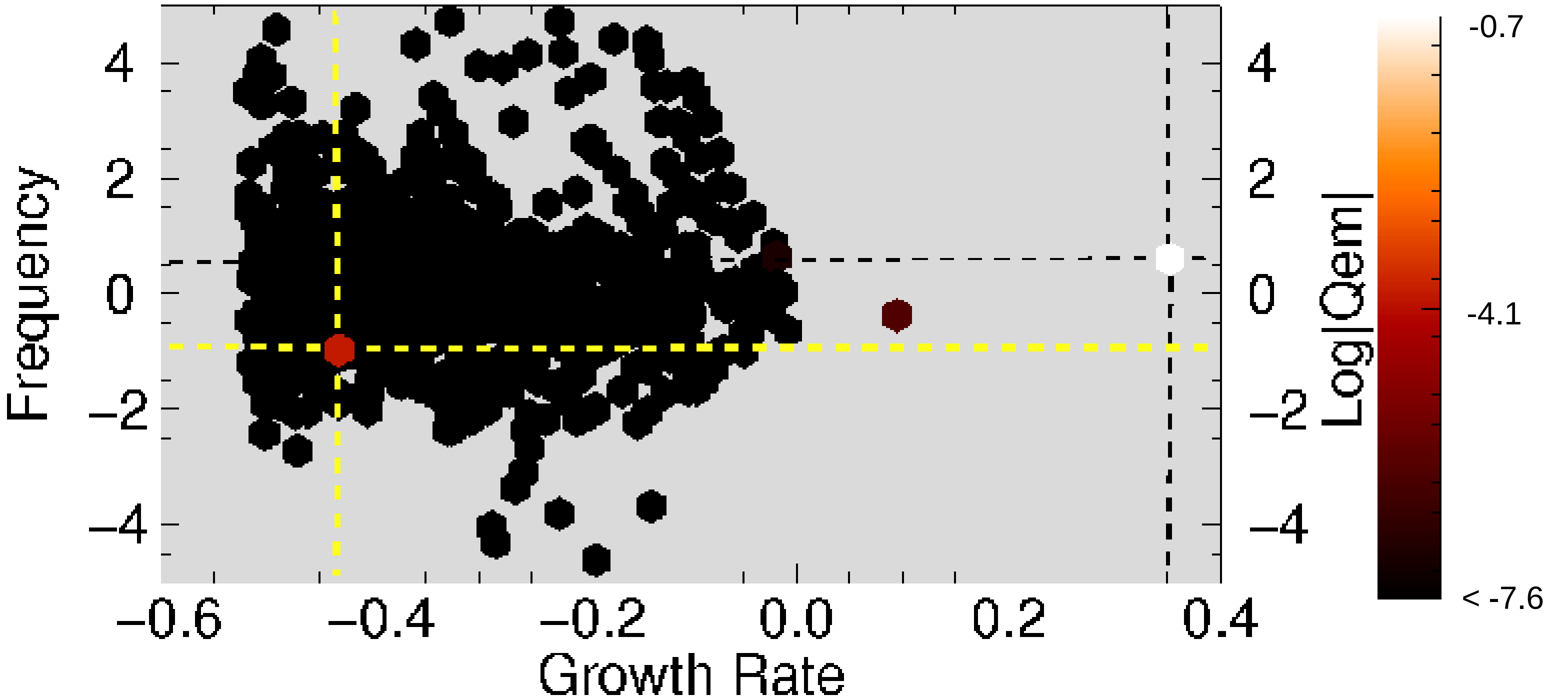
	ITG	TITG	ETG	TETG	Micro Tearing[1]
Tearing Parity		X		X	X
Frequency	+	+	-	-	-
R/Lti Threshold	X	X			
R/Lte Threshold			X	X	X
Low-β Threshold					X
Change when ϕ is deleted	X	X	X	X	

TITG, TETG: Not the cause of the transport.

Transport caused by (some form of) microtearing.

(1) Doerk, PRL, '11,

Microtearing mode responsible for stochasticity and transport



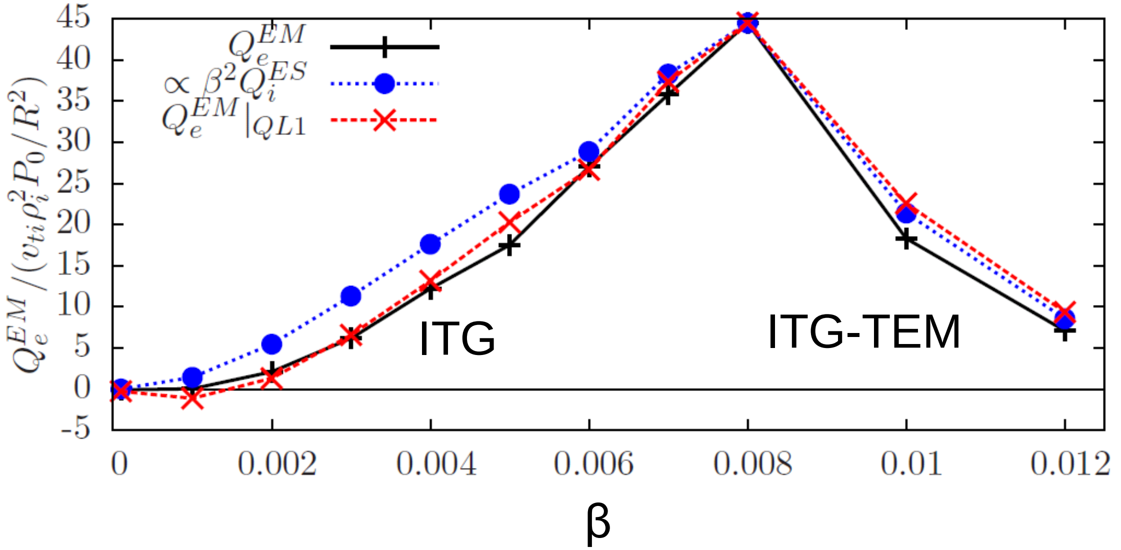
Scalapack eigenmode solver (incorporated in GENE code) solves for all eigenmodes (limited resolution).
Nonlinearly evolved distribution function projected onto 1000 orthogonalized linear eigenmodes ($k_y\rho$=0.2,$k_x\rho$=0.0).
One damped mode produces EM transport, has properties of microtearing mode.

Simple model captures β-dependence

$$Q_e^{EM\,(tear)}(\beta) = C_0 \beta^2 Q_i^{ES}$$ Stochastic tearing transport

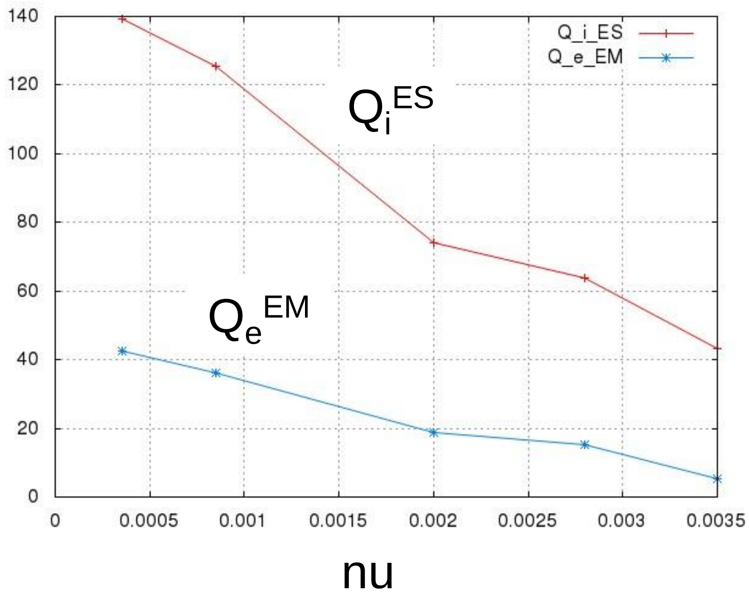
$$Q_e^{EM\,(ball)} = Q_i^{ES} \left\{ \frac{Q_e^{EM}}{Q_i^{ES}} \right\}_{ITG-Lin}$$ ITG – quasilinear contribution

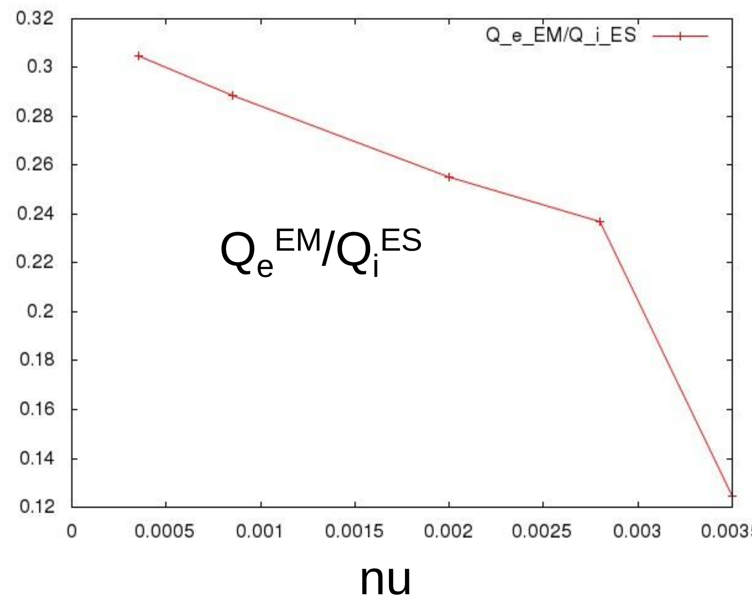
$$Q_e^{EM} = Q_i^{ES} \left(C_0 \beta^2 + \left\{ \frac{Q_e^{EM}}{Q_i^{ES}} \right\}_{ITG-lin} \right)$$ Total



Microtearing tests

Collisionality Dependence

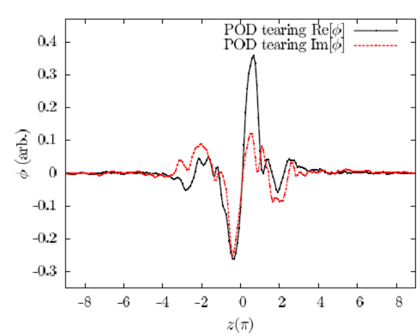


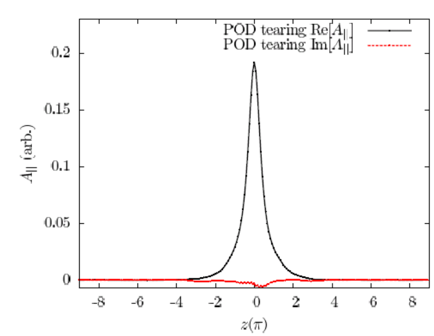


Convergence Tests:

$$k_x^{\max}\rho_s \approx 5.9 \Rightarrow 11.0$$

Qualitatively same EM transport





Also tested: significant increases in z, and phase space resolution

Summary / Conclusions

- GENE code used to study electromagnetic transport in ITG β scan.

- Stochasticity and transport caused by **nonlinear excitation of subdominant microtearing modes**.

- Magnetic transport is superposition of **outward stochastic contribution** from nonlinearly excited microtearing modes and **inward contribution from ITG**.

- Nonlinear excitation mechanism – **coupling with zonal wavenumbers**.

- **Linear mode** has properties of traditional microtearing mode.

- **Simple model captures β dependence** in spite of inapplicability of quasilinear theory.

Backup slides

Nonlinear transfer functions - identify excitation mechanism.

Gyrokinetic free energy:

$$E_k = \sum_j \pi B_0 n_{0j} T_{0j} \int dz dv_\parallel d\mu J(z) \frac{f_j^2}{F_{0j}} + D(k_\perp)\phi^2 + \frac{k_\perp^2}{\beta} A_\parallel^2$$

Energy evolution equation:

$$\frac{\partial E_k}{\partial t} = L[g_k, g_k] + \sum_{k_\perp'} N[g_k, g_{k'}, g_{k-k'}] + c.c.$$

Nonlinear Transfer function:

$$N_{k,k'} = \int dz dv_\parallel d\mu (k_x' k_y - k_x k_y') \left[\frac{q_j F_{oj}}{T_{0j}} \chi_j^*(k) \chi(k') g(k-k') - g_j^*(k) \chi(k-k') g(k') \right]$$

Energy transferred between k and k'

$$N_{k,k'} = -N_{k',k}$$

Contributions to EM Transport

$$Q_e^{EM} = \langle \tilde{q}_{e||} \tilde{B}_x \rangle / B_0$$

$$\tilde{q}_{e||} = -n_{0e}\chi_{e||}\left(\boxed{\frac{d\tilde{T}_{e||}}{dz}} + \frac{\tilde{B}_x}{B_0}\frac{d\tilde{T}_{e||}}{dx} + \boxed{\frac{\tilde{B}_x}{B_0}\frac{dT_{e0}}{dx}}\right)$$

ITG mechanism

Stochastic transport (tearing) mechanism