Sawtooth Period Prediction

Influence of neo-classical resistivity on scaling from JET to ITER.

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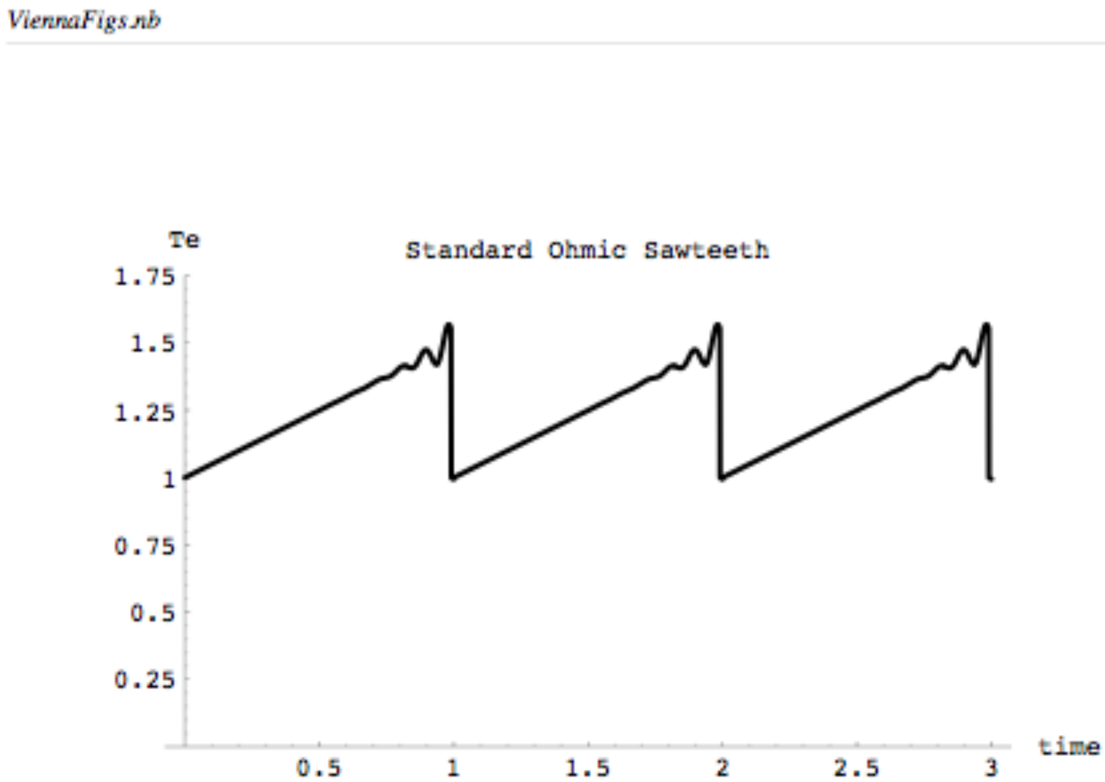
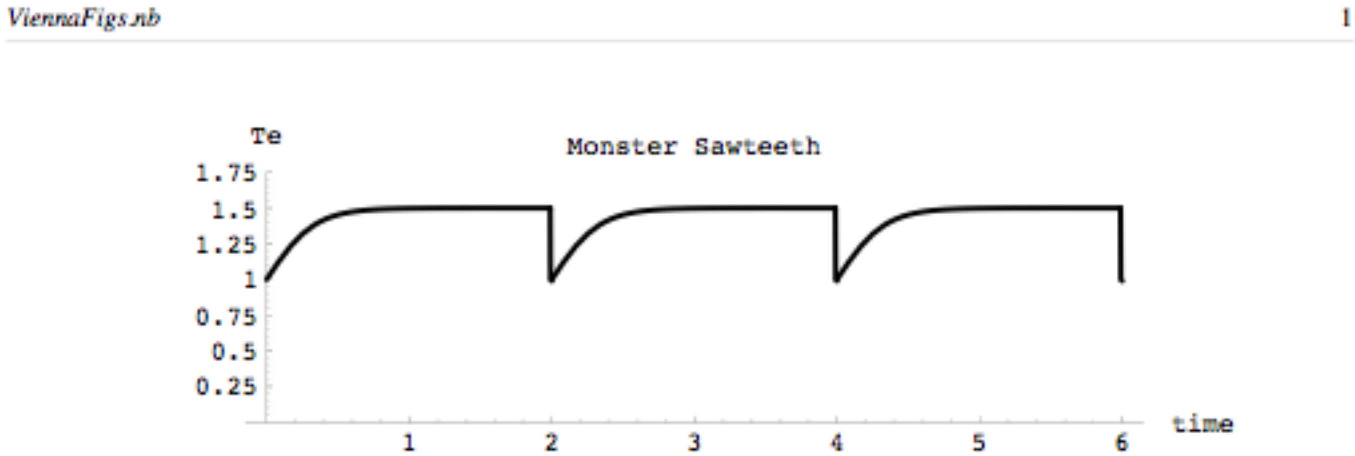
Content.

- Sawteeth: some general features.
- Modelling (a) Direct non-linear 3D
- (b) $1^1/_2$ D transport code
- Role of η; neo-classical versus Spitzer
- Toy model numerical solutions for the Ramp.
- Some questions & conclusions.

Sawteeth in Ohmic Tokamaks were like this:



Sawteeth in Large Tokamaks with auxiliary heating:



During flat top, only q evolves.
What duration in ITER?

Sawtooth Period Prediction

- τ_s ~ 100sec. Is too demanding for global kinetic simulation.
- Must resort to modelling by

- (a) employing dissipative fluid eqs. (as in Park-Monticello, Nucl. Fus. (1990) and Halpern et al. PPCF (2011))

OR

- (b) breaking cycle into 3 separate phases:

  Trigger → Collapse → Ramp,

  (as in Porcelli-Boucher-Rosenbluth, 1996)

Role for gyro-kinetics in Trigger & Collapse.

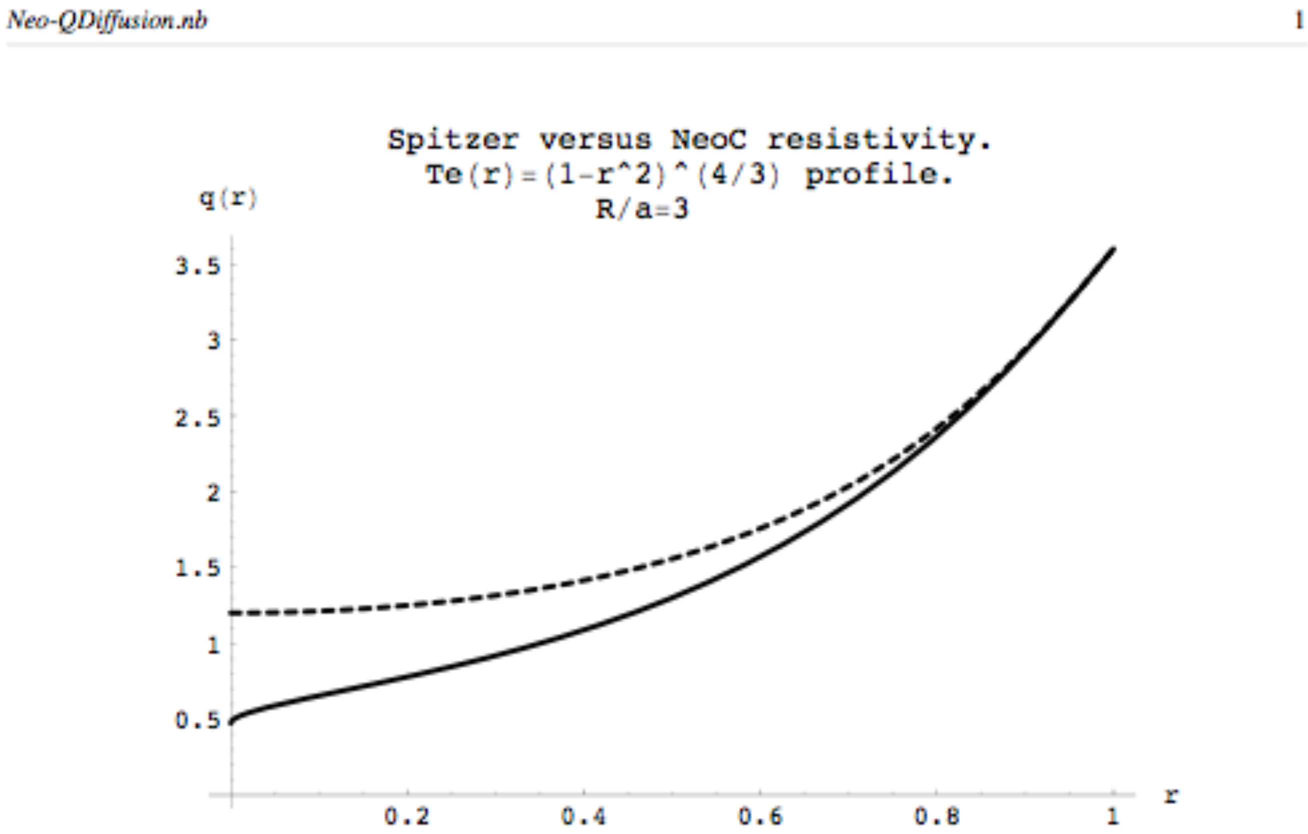
(a) Global, long duration, fluid simulations

- For recent toroidal simulations with 2-fluid effects see F.D.Halpern et al. PPCF 53(1) 015011 (2011). XTOR-2F code.
- Realistic simulation: with diamagnetic stabilisation effects giving long quiescent ramp and fast crash. But Spitzer η.
- W.Park & D.Monticello (1990). Single fluid, but investigated effect of Neo-classical η in such simulations.

(b) $1^1/_2$D Neoclassical η drives discharges towards 1/1 Instability.

- η(Neo) $\geq$ 4 η(Spitzer) for $r \sim a$
- η(Neo) ~ η(Spitzer) for $r \rightarrow 0$
- η(Neo) causes stronger peaking of J(r)
- And therefore lower q(0).

Steady state q(r)



- - - - Spitzer: ——— Neoclassical

Note "cuspy structure" in neo case.

3D Sawtooth simulation of Park-Monticello

- Following a Sawtooth collapse, neoclassical resistivity controls the subsequent q(r,t) evolution during quiescent ramp.
- W. Park & D.A. Monticello Nucl.Fus. (1990), 2413 --- MH3D simulation. Concluded:-
- Collapse like Kadomtsev reconnection;
- And large drop in q_0 during subsequent ramp, $\delta q \sim 0.2$, caused by neoclassical correction to η near r~0.

Ramp evolution of q(r,t) with $\eta \approx \eta_{Sp} (1-\sqrt{(r/R)})^{-2}$

- Neoclassical correction is weak at r~0, but generates singularity in the diffusion equation as r→0.

$$\begin{aligned} \frac{\partial B_\theta}{\partial t} &= -(\nabla \times \mathbf{E})_\theta, \\ &= \frac{\partial}{\partial r}[\frac{\eta}{\mu_0 r}\frac{\partial}{\partial r}(rB_\theta)] \end{aligned} \quad (1)$$

or equivalently:

$$\begin{aligned} \frac{\partial}{\partial t}(\frac{1}{q}) &= \frac{1}{r}\frac{\partial}{\partial r}[\frac{\eta}{\mu_0 r}\frac{\partial}{\partial r}(\frac{r^2}{q})], \\ \frac{\partial q}{\partial \tau} &= -4q^2\frac{\partial}{\partial x}\hat{\eta}(x)\frac{\partial}{\partial x}(\frac{x}{q}), \\ \tau &= \frac{t}{\tau_\eta}, \qquad \tau_\eta = \frac{\mu_0 a^2}{\eta(0)}, \qquad x = \frac{r^2}{a^2} \end{aligned} \quad (2)$$

Regularity is restored by axial "collisional boundary layer"

- Transition out of Banana regime. Hirshman, Hawryluk & Birge (1977).

$$[1-(\frac{r}{R})^{1/2}] \quad \rightarrow \quad [1-(\frac{a}{R})^{1/2}\frac{r^2}{r^{3/2}+\nu_*}], \qquad (1)$$

$$\nu_* \quad = \quad \frac{\nu_e R q}{(\frac{a}{R})^{3/2} V_{Te}} \qquad (2)$$

Applied on axis, this predicts

$$\frac{\partial q_0}{\partial t} \sim -\frac{8(\frac{a}{R})^{1/2}}{\nu_* \tau_\eta}, \qquad (1)$$

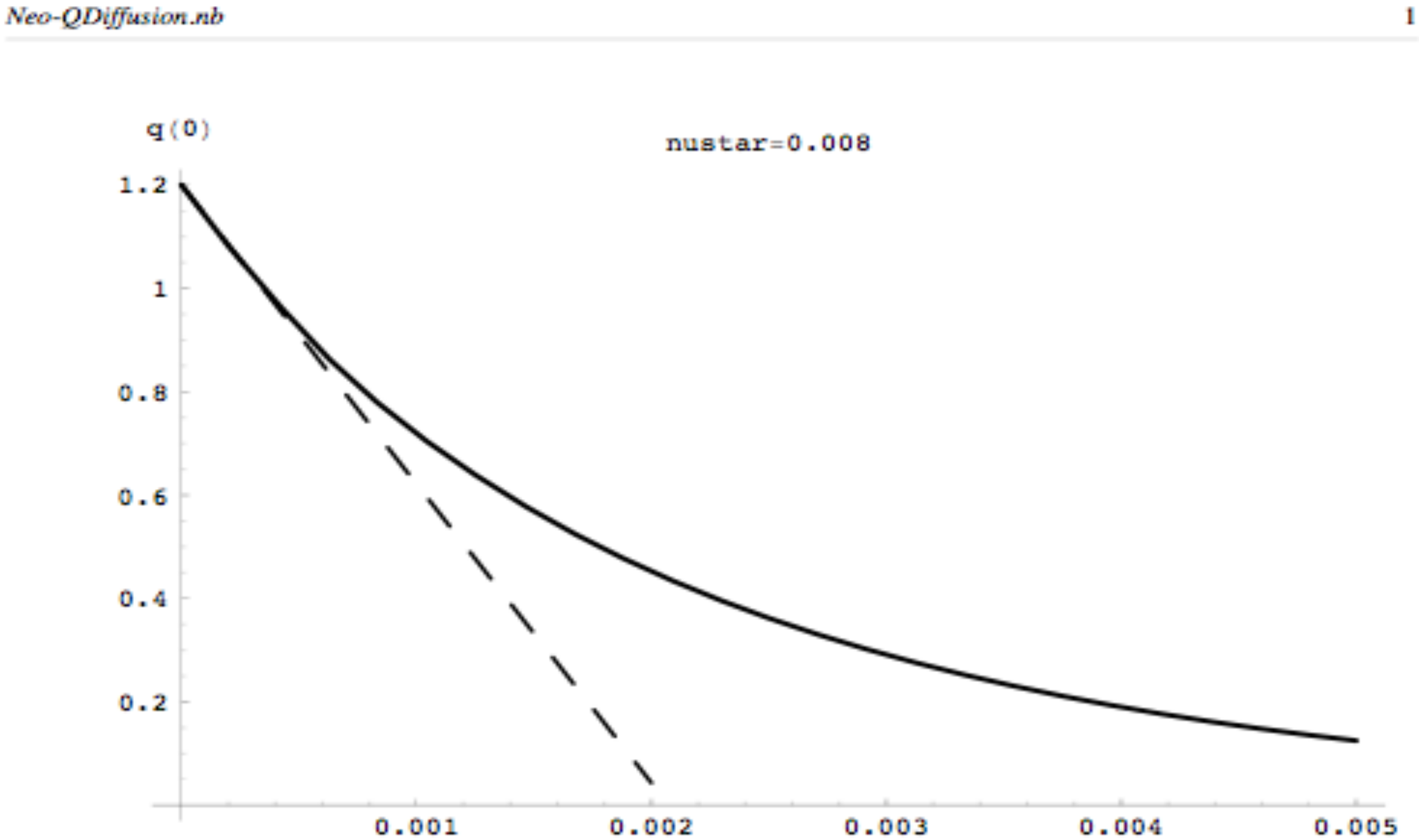
Implies "fast" linear decrease of $q_0 = 1 - t/\tau_0$

Neoclassical Evolution of q(r)

- If the local "cuspy" structure of J(r) around magnetic axis is destroyed in the collapse (as in the Kadomtsev model), then fast downward evolution of q_0 ensues.
- Timescale is
- $\tau_0 \sim (\tau_\eta \nu_* / 8\sqrt{\varepsilon}) \propto (a^{3/2} R^{3/2} N / T^{1/2})$

  $\ll \tau_\eta \propto a^2 T^{3/2}$

Toy cylinder model: Neoclassical evolution of $q_0(t)$



Large drop after 0.1% of resistive diffusion time.

- - - Initial analytic: ———— Numerical evolution.

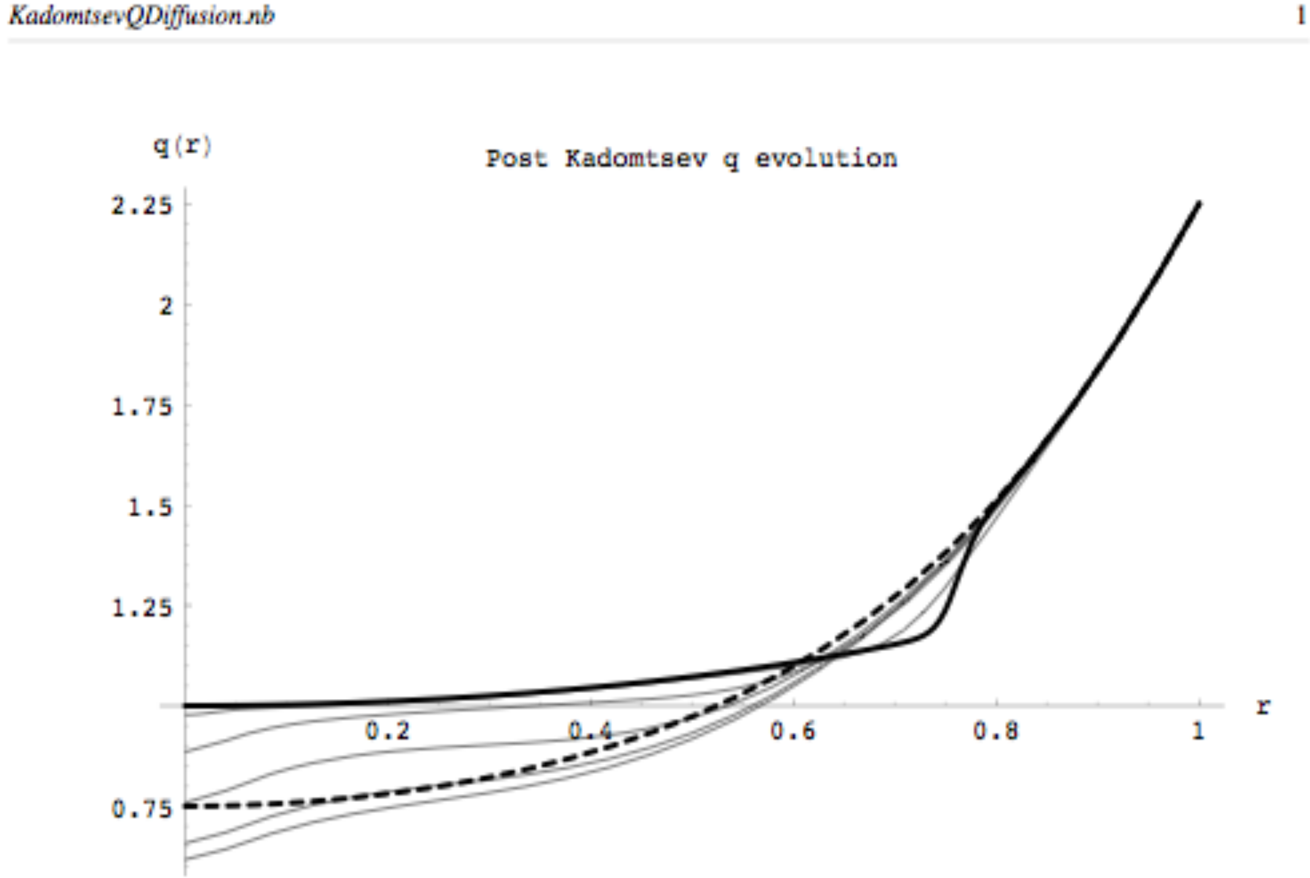
Porcelli-Boucher-Rosenbluth (1996)

- $1\,^{1/2}$ D modelling, predicted Sawtooth period in ITER of ~ 150secs. for Kadomtsev model;
- Consistent with $\tau_s \propto T^{1.5} a^2$ scaling from JET.
- Park-Monticello (after comparison of many devices) suggest:

$$\tau_s = 9\ T^{1.5} R^2 \text{ ms.} = 0.0016\ \tau_\eta$$

- $\Rightarrow$ Same scaling.
- But is this scaling valid at very high T_e?

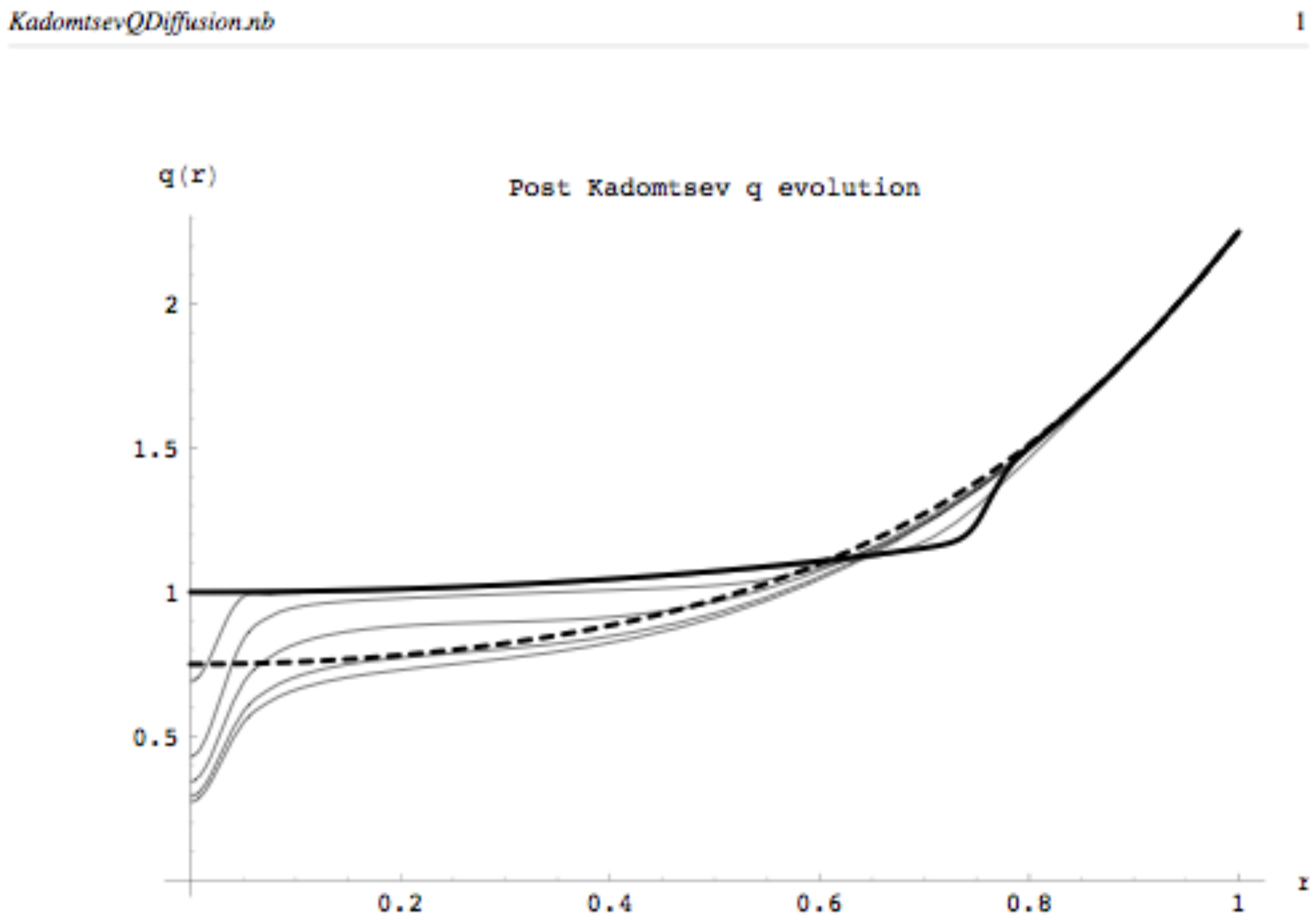
Example: equilibrium q evolution after a Kadomtsev reconnection. 40ms. to 4sec.



JET at 4keV $\nu_* \sim 0.01$

---- Pre-crash, ——————Post-crash

ITER q from 2.5s. to 250s.



ITER at 25keV $\nu_* \sim 0.0006$

Does ν_* matter?

- Key trigger parameter may be the shear at q=1, s_1. (P-B-R, 1996, C-H-Z, 2012)

- $s_1(t)$ is influenced by 2 conflicting scalings.

$\tau_\eta \propto T^{1.5}\ a^2$ global resistive diffusion of q.

$\tau_0 \propto\ a^3\ N\ /\ T^{1/2}$ from axial influence of ν_*

Conclusions: (1) Trigger

- Trigger - Collapse - Ramp Modelling.

- Progress on analytic trigger criteria (Connor, Hastie & Zocco, PPCF, 54, p035003, 2012)
- Shows importance of β/s^2, η_e and η_i as well as $\Delta'_{1/1}$
- Provides framework to compare with linear gyro-kinetic codes.
- Toroidal Δ' code for 1/1 mode, with α-particles, is required. Work in progress.

Conclusions: (2) Collapse

- Can Gyro-kinetics simulate collapse for ~100 μs. or more?
- Is local axial structure of J(r) destroyed during turbulent collapse?
- Which collapse model is most realistic?
- Can fluid codes like XTOR-2F generate a new collapse model for use in $1^1/_2$ D codes?
- ECE imaging is also providing invaluable insights.

Recent ECE data: Hyeon Park

Compares with Sawtooth Collapse models:

Kadomtsev reconnection; Kadomtsev

Quasi-Interchange; Wesson

Ideal Ballooning; Bussac&Pellat, Cowley&Wilson

" No model is completely correct."

"No model is totally wrong."

H.Park(2012)

Conclusions: (3) Ramp simulation

- If axial structure of J(r) is destroyed during collapse, is q(r,t) evolution influenced by ν_* as predicted in the Toy model?
- What timescale for, $s_1(t)$, shear at r_1 ?
- Do other pitch-angle scattering mechanisms broaden the axial "layer" in high T_e tokamaks?

Stochasticity during Collapse

- From Halpern, Leblond, Lutjens & Luciani. PPCF, 53(1), 015011, (2011)
- The following Poincare plots show initial growth of a 1/1 Kadomtsev island, followed by secondary instability and spread of stochasticity. Could explain 1980 observations of precursor oscillation and crash on TFR.
- Foundation for a new collapse model?

Magnetic field cross sections