Linear microtearing instability in tokamaks

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Summary

- Microtearing (MT) modes predicted unstable in:
  - STs (NSTX, MAST) – candidate to explain $\Omega\tau_E \sim \nu_*^{-1}$ scaling
  - Tokamaks (ASDEX-UG, DIII-D) – candidate to explain β degradation
  - RFPs (RFX, MST)
- Recent nonlinear simulations predict transport follows linear trends ($\chi_e \sim \nu_e$, β_e, a/L_{Te})
  - Useful to better characterize linear thresholds and scaling
  - What can we do to minimize microtearing (if in fact important)?

- Many linear MT scalings in tokamaks predicted from sheared slab theory with time-dependent thermal force (e.g. Gladd et al., 1980)
  - Non-monotonic dependence with collisionality (ν_e/ω) – generally requires kinetic treatment, especially for "weakly-collisional" ($\nu_e/\omega \leq 1$)
  - Non-monotonic dependence with magnetic shear (s/q) – cannot assume "constant ψ" for weak shear (s/q<2)
- No theories comprehensively treat toroidal effects:
  - Toroidal drifts ∇B, κ
  - Trapping (previously treated ad hoc, but regime of relevance too narrow to explain GK results)
  - Non-uniform, strongly ballooning $A_{||}(\theta)$
- Need brilliant theorists (you!) to improve theory for quantitatively accurate predictions useful for modeling, etc…

A lot has been done for microtearing simulation and theory – references:

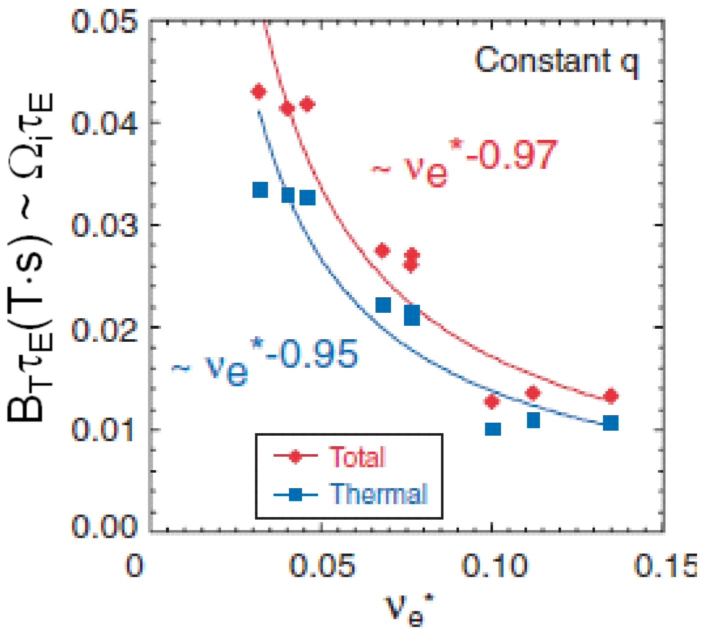
Gyrokinetic simulation

- [7] M. Kotschenreuther et al., Nucl. Fusion **40**, 677 (2000).
- [8] H.R. Wilson et al., Nucl. Fusion **44**, 917 (2004).
- [9] M.H. Redi et al., 30st EPS (St. Petersburg), P-4.94 (2003).
- [10] M.H. Redi, et al., 31st EPS (London), P-2.162 (2004).
- [11] M.H. Redi, et al., 32nd EPS (Tarragona), P-5.041 (2005).
- [12] D.J. Applegate et al., Phys. Plasmas **11**, 5085 (2004).
- [13] C.M. Roach et al., Plasma Phys. Control. Fusion **47**, B233 (2005).
- [14] J.W. Connor et al.,, IAEA-FEC, TH/P2-2 (2006).
- **[15] D. Applegate, Ph.D. Thesis, Imperial College London (2006).**
- **[16] D.J. Applegate et al., Plasma Phys. Control. Fusion 49, 1113 (2007).**
- [17] F.M. Levinton et al., Phys. Plasmas **14**, 056119 (2007).
- [18] K.L. Wong et al., Phys. Rev. Lett. **99**, 135003 (2007).
- [19] K.L. Wong et al., Phys. Plasmas **15**, 056108 (2008).
- [38] L. Vermare et al., J. Physics, Conference Series **123**, 012040 (2008).
- [39] D. Told et al., Phys. Plasmas **15**, 102306 (2008).
- [41] I. Predebon et al., Phys. Rev. Lett. **105**, 195001 (2010).
- [37] D.R. Smith et al., Plasma Phys. Control. Fusion **53**, 035013 (2011).
- [40] H. Doerk et al., Phys. Rev. Lett. **102**, 155003 (2011).
- [20] W. Guttenfelder et al., Phys. Rev. Lett. **106**, 155004 (2011).
- **[x] W. Guttenfelder et al., Phys. Plasmas 19, 022506 (2012).**

Analytic/slab theory

- **[23] R.D. Hazeltine, D. Dobrott, T.S. Wang, Phys. Fluids 18, 1778 (1975).**
- **[24] J.F. Drake and Y.C. Lee, Phys. Fluids 20, 1341 (1977).**
- [25] J.F. Drake, N.T. Gladd, C.S. Liu and C.L. Chang, Phys. Rev. Lett. **44**, 994 (1980).
- [26] M. Rosenberg, R.R. Dominguez, W. Pfeiffer, R.E. Waltz, Phys. Fluids **23**, 2022 (1980).
- **[27] N.T. Gladd, J.F. Drake, C.L. Chang, C.S. Liu, Phys. Fluids 23, 1182 (1980).**
- [28] D'Ippolito, Y.C. Lee, J.F. Drake, Phys. Fluids 23, 771 (1980).
- [29] R.R. Dominguez, M. Rosenberg, C.S. Chang, Phys. Fluids **24**, 472 (1981).
- [30] C.S. Chang, R.R. Dominguez, R.D. Hazeltine, Phys. Fluids **24**, 1655 (1981).
- [31] A.B. Hassam, Phys. Fluids **23**, 2493 (1980).
- [32] A.B. Hassam, Phys. Fluids **23**, 2493 (1980).
- [33] L. Chen, P.H. Rutherford, W.M. Tang, Phys. Rev. Lett. **39**, 460 (1977).
- [34] P.J. Catto, M.N. Rosenbluth, Phys. Fluids **24**, 243 (1981).
- [35] J.W. Connor, S.C. Cowley and R.J. Hastie, Plasma Physics Control. Fusion **32**, 799 (1990).

Experimental motivation - strong collisionality scaling in STs



NSTX (Kaye et al., Nucl. Fusion 2007)

$$\Omega\tau_E^{th} \sim \nu_{*e}^{-0.95}$$

MAST (Valovič et al., Nucl. Fusion 2011)

$$\Omega\tau_E^{th} \sim \nu_{*e}^{-0.82}$$

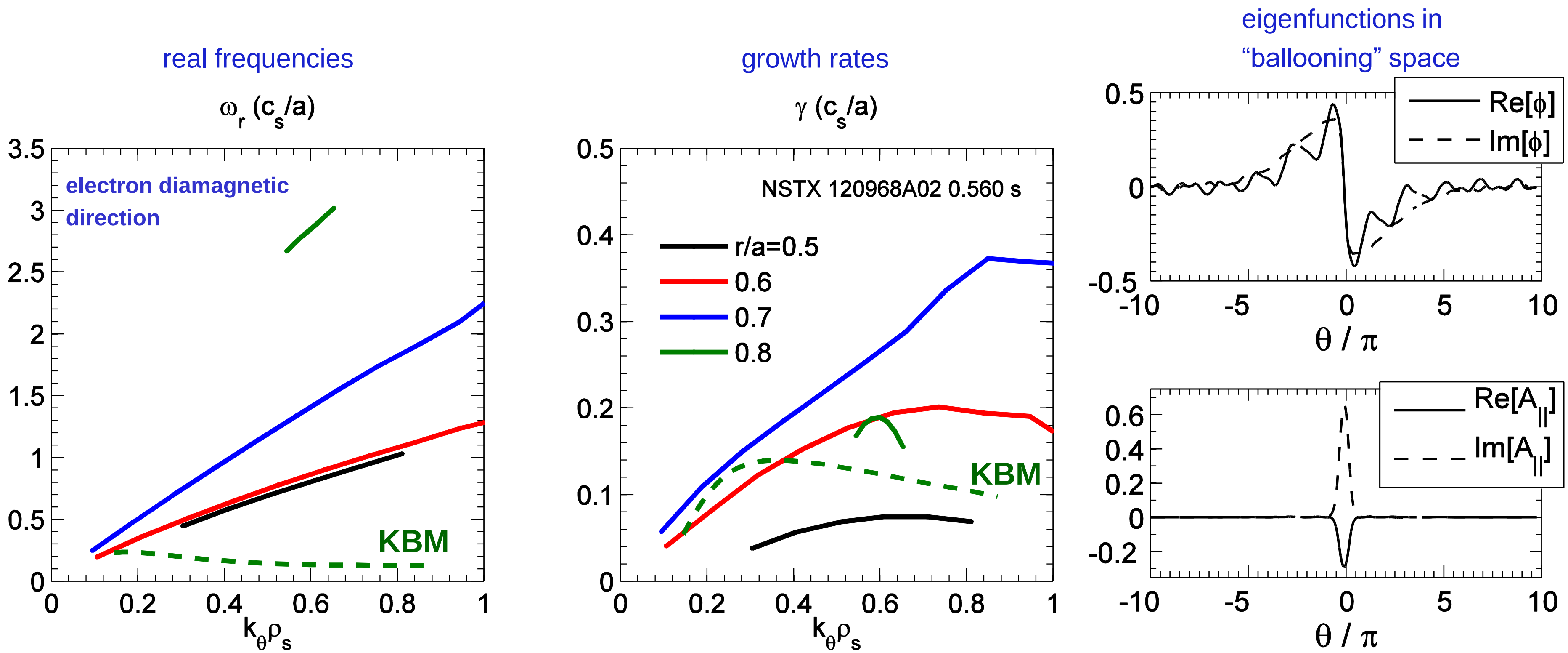
ITER (PIPB, Doyle et al., Nucl.Fusion 2007)

$$\Omega\tau_E^{th,04(2)} \sim \nu_{*e}^{-0.2}$$

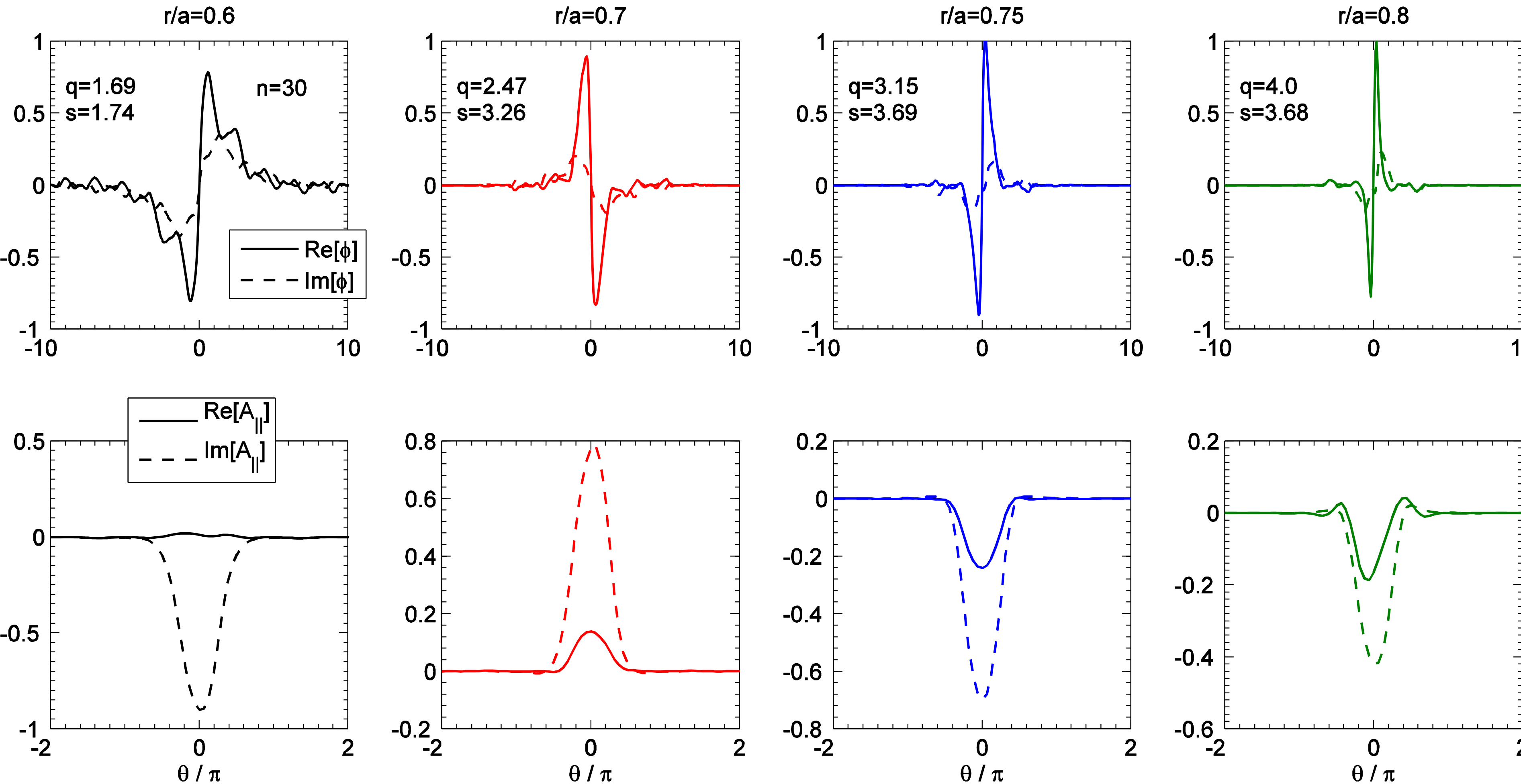
- Ion transport is neoclassical, consistent with strong toroidal flow and flow shear
- What is the cause of anomalous electron thermal transport?
- Will favorable τ_E scaling hold at lower ν_* envisioned for next generation ST (high heat flux, CTF, …)?

Microtearing modes found to be unstable in many high ν_* discharges

- Microtearing dominates over r/a=0.5-0.8, $k_\theta\rho_s$<1 (n≈5-70)
- Real frequencies in electron diamagnetic direction, $\omega \approx \omega_{*e} = (k_\theta\rho_s)\cdot(a/L_n+a/L_{Te})\cdot(c_s/a)$
- ETG mostly stable due to larger Z_{eff}≈3, $(R/L_{Te})_{crit,ETG} \sim (1+Z_{eff}T_e/T_i)$



Many variation in eigenfunctions – partially coupled to changing magnetic shear



Conceptual picture of linear microtearing instability

- High-m tearing mode around a rational $q(r_0)=m/n$ surface ($k_{||}(r_0)=0$)
  - Classic tearing mode stable for large m, $\Delta' \approx -2m/r<0$
- Need instability drive from something else, e.g. mechanism to drive parallel current from gradients:

- Imagine helically resonant (q=m/n) δB_r perturbation $$\delta B_r \sim \cos(m\theta - n\varphi)$$
- ∇T_e projected onto field line gives parallel gradient $$\tilde{\nabla}_{\parallel} T_{e0} = \frac{\vec{B}\cdot\nabla T_{e0}}{B} = \frac{\delta B_r}{B}\underline{\nabla T_{e0}}$$
- Parallel thermal force* drives parallel electron current that reinforces δB_r via Amperes's law $$k_\perp^2 \rho_s^2 \hat{A}_\parallel = \underline{\frac{\beta_e}{2}} \hat{j}_\parallel \quad , \quad B_r = i k_\theta A_\parallel$$
  - Requires e-i collisions
  - Time dependence important $$R_{T\parallel} \sim -[1+\underline{\alpha(\omega)}] n_e \nabla T_e$$

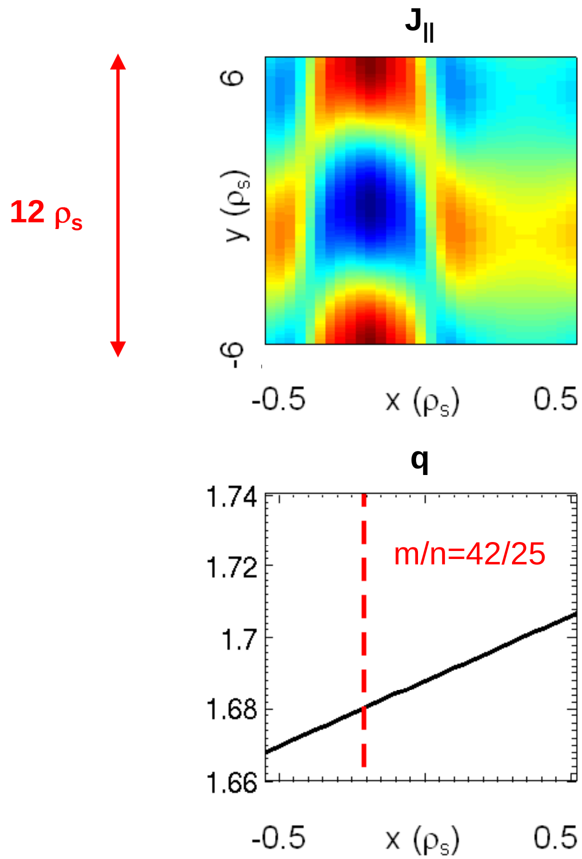
- Instability requires sufficient ∇T_e, β_e, $\nu^{e/i}$, and finite frequency (ω)

*e.g. Hazeltine et al., Phys. Fluids 18, 1778 (1975); Gladd et al., Phys. Fluids 23, 1182 (1980); D'Ippolito et al., Phys. Fluids 23, 771 (1980); M. Rosenberg et al., Phys. Fluids 23, 2022 (1980).

Linear mode structure in perpendicular plane illustrates key microtearing mode features

- Narrow resonant current channel (≈0.3ρ_s≈1.4 mm) centered on rational surface

x-y perpendicular plane (θ=0)

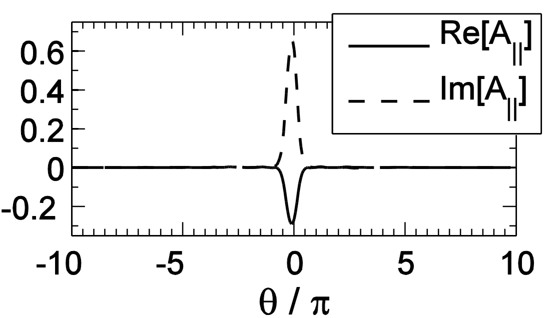


Linear mode structure in perpendicular plane illustrates key microtearing mode features

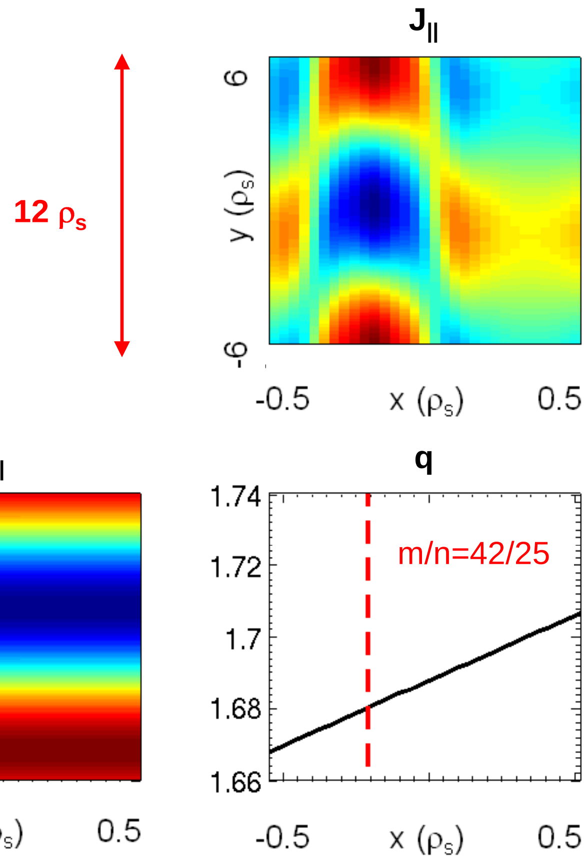
- Narrow resonant current channel (≈$0.3\rho_s$≈1.4 mm) centered on rational surface
- Finite $\langle A_{||} \rangle_\theta$ (resonant tearing parity), *strongly ballooning*

"ballooning" space

$$k_r(\theta) = \hat{s} k_\theta (\theta - \theta_0)$$



x-y perpendicular plane (θ=0)

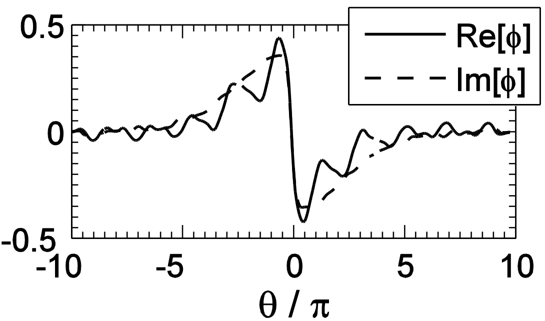


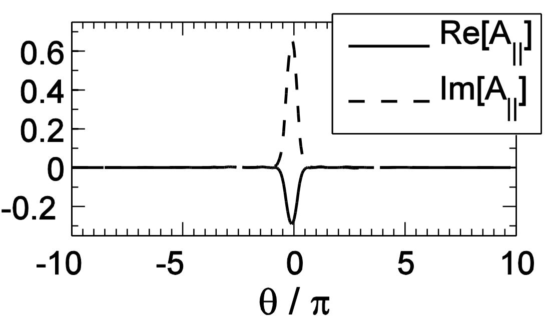
Linear mode structure in perpendicular plane illustrates key microtearing mode features

- Narrow resonant current channel ($\approx 0.3\rho_s \approx 1.4$ mm) centered on rational surface
- Finite $\langle A_{||} \rangle_\theta$ (resonant tearing parity), *strongly ballooning*
- Narrow n_e & T_e perturbations
- Nearly unmagnetized/adiabatic ion response $\Rightarrow \frac{\tilde{n}}{n_0} \approx -Z_{eff}\left(\frac{e\tilde{\varphi}}{T_i}\right)$

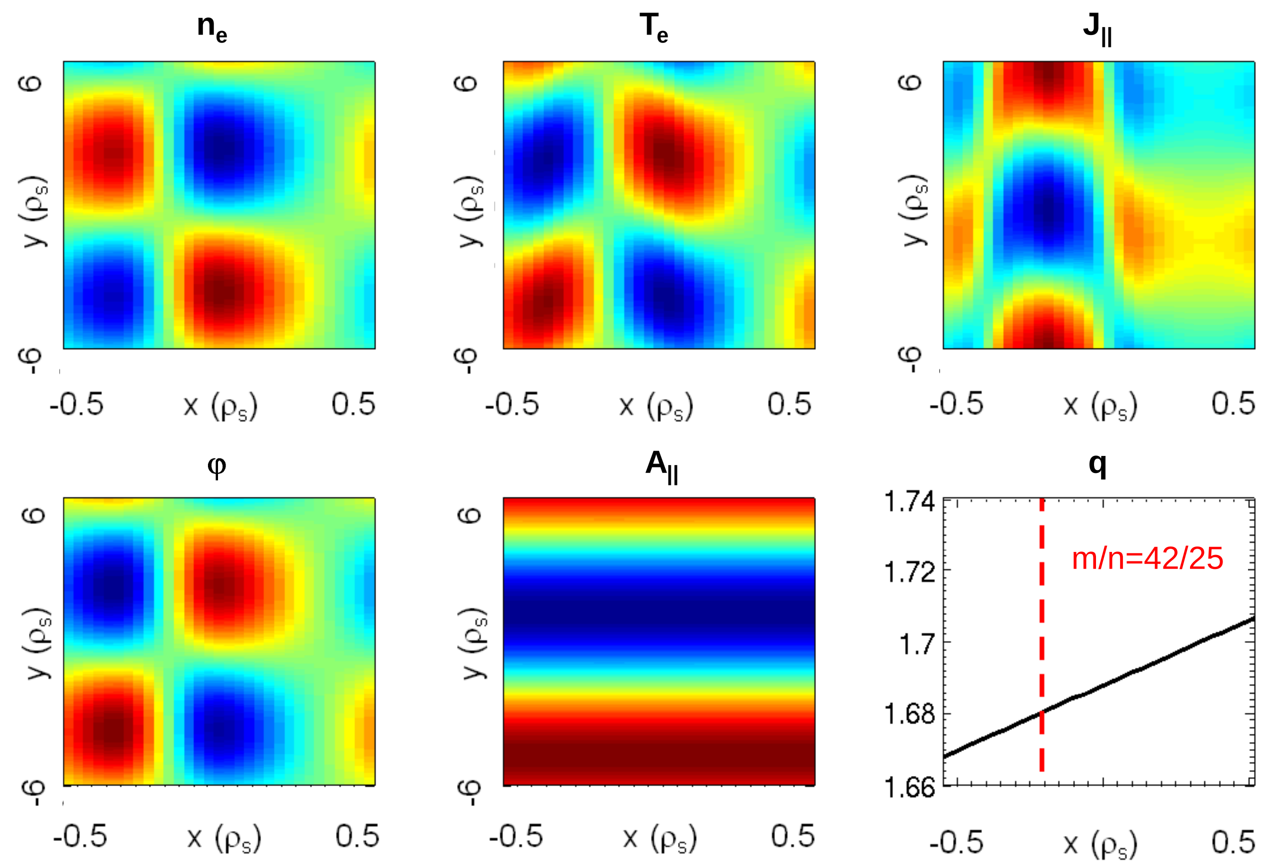
"ballooning" space

$$k_r(\theta) = \hat{s}k_\theta(\theta - \theta_0)$$





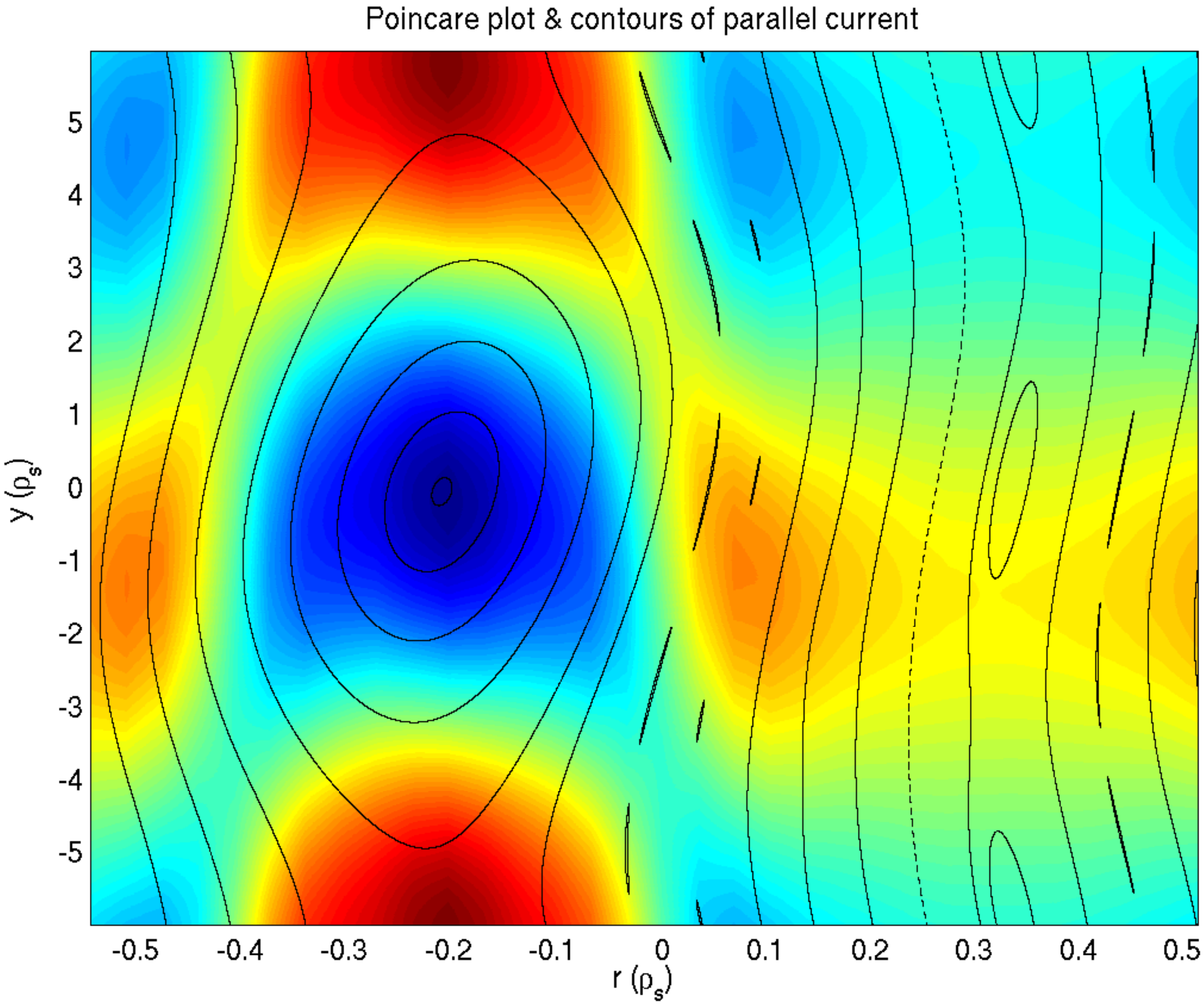
x-y perpendicular plane (θ=0)



Resonant current, tearing parity $\langle A_{||}\rangle>0$, leads to island growth

- δB_r leads to radially perturbed field line, finite island width

$$w = 4\left(\frac{\delta B_r}{B}\frac{rR}{n\hat{s}}\right)^{1/2}$$



NSTX microtearing instability is an electromagnetic ($A_{||}$) electron drift wave (∇T_e)

For this "flavor" of NSTX microtearing mode:

- Instability remains if adiabatic ions enforced or ∇T_i=0
- Real frequency follows $\omega \sim \omega_{*e} = (k_\theta \rho_s)\cdot(a/L_n + a/L_{Te})\cdot(c_s/a)$

} electron drift wave

- Instability disappears when $\delta A_{||}$=0 enforced
- Instability remains when $\delta\varphi$=0 enforced (usually gets stronger)

} electromagnetic

- Instability remains if δf_{trap}=0 enforced (no trapped particles)
  $\rightarrow$ passing electrons most important

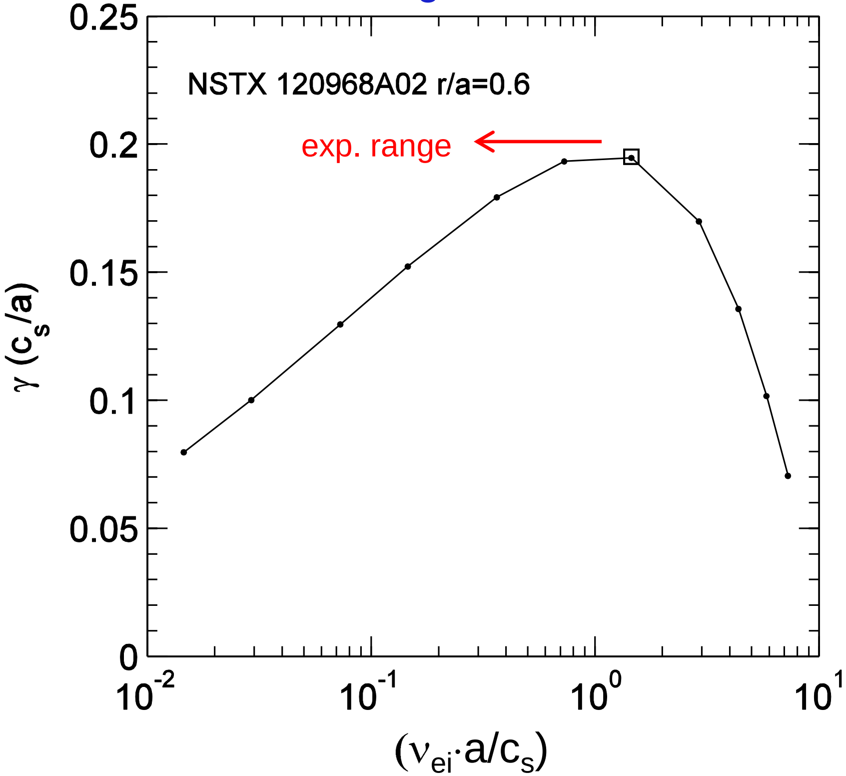
- Instability remains if $v_{\nabla B/\kappa}\cdot\nabla$=0 enforced
  $\rightarrow$ toroidal drifts not critical

Many similarities to MAST GS2 analysis reported by Applegate et al. (2007)

A distinguishing feature of the microtearing mode is the non-monotonic dependence on ν_{ei}/ω

- Peak γ occurs for ν_{ei}/ω ~ 4, similar to slab kinetic calculations [Gladd et al., 1980]

linear growth rates



- γ decreases with ν_e in experimental range, qualitatively consistent with confinement scaling

Collisionality scaling consistent with time-dependent thermal force (TDTF)

$$R_T = -\alpha_T \cdot n_e \nabla T_e$$

- Braginskii, $\omega << k_{||}^2 v_{Te}^2/\nu_e$ $\qquad \alpha_T=0.71\text{-}1.5\ (Z=1\rightarrow\infty)$

- "Semi-collisional" limit of Drake & Lee (1977), Hassam fluid (2nd order Chapman-Enskog)

  $\omega/\nu_e<1,\ k_{||}\lambda_{mfp}<<1$ $\qquad \alpha_T(\omega) \sim 1+i\alpha_1(\omega/\nu_e)$

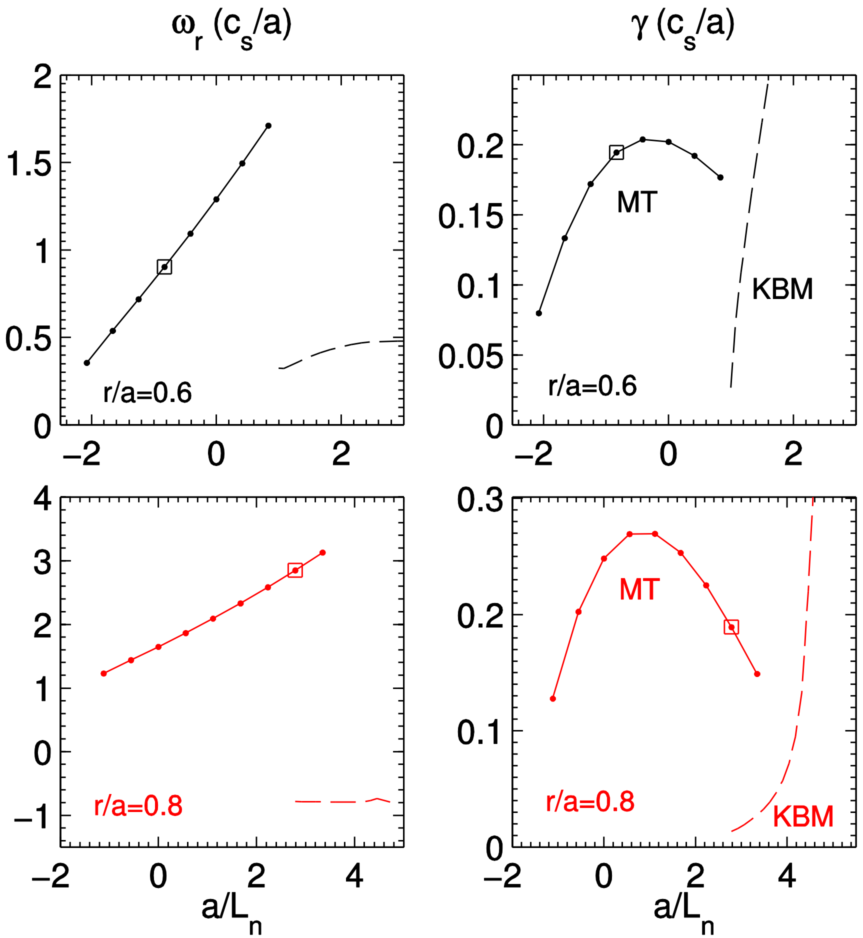
- Fully kinetic (Hazeltine et al., 1975; Gladd et al., 1980; D'Ippolito et al., 1980; Rosenberg et al., 1980)

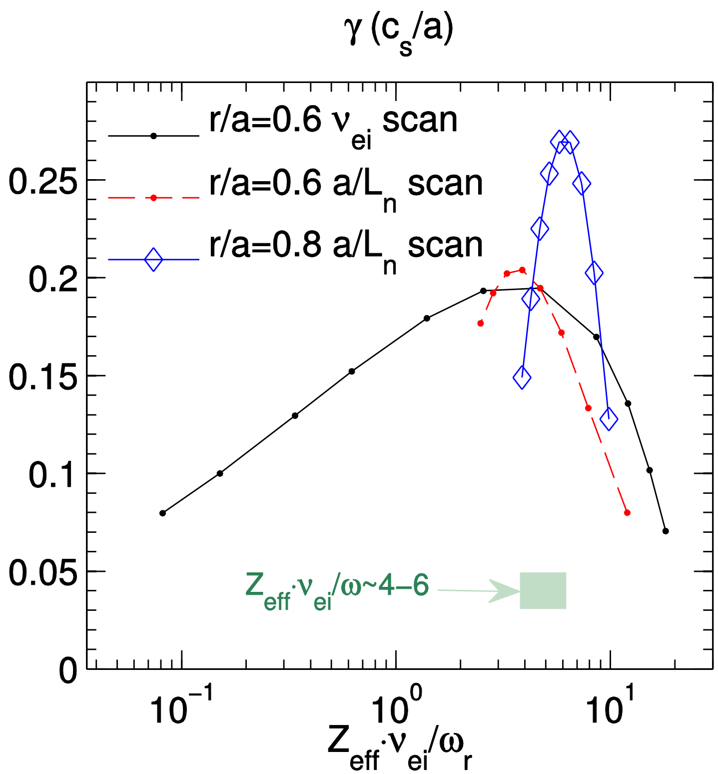
  $0<\omega/\nu_e<\infty$ $\qquad \alpha_T(\omega) = 0.8\dfrac{1+i\cdot 0.54(\omega/\nu_e)}{1+0.29(\omega/\nu_e)^2}$

- Removing trapped particles ($\delta f_{trap}\rightarrow 0$ in GYRO, $r/R\rightarrow 0$ in GS2 [Applegate, 2007]) leaves instability – trapped/passing boundary layer effects not critical [Chen et al,. 1977; Catto & Rosenbluth, 1980; Conner et al., 1990], but still influence quantitative growth rate

Dependence of growth rate with density gradient consistent with TDTF

- Dependence on a/L_n partially due to variation in $\omega \sim \omega_{*e} = (k_\theta \rho_s)\cdot(a/L_n + a/L_{Te})\cdot(c_s/a)$
- Peak γ occurs for $\nu^{e/i}/\omega \sim 1\text{-}6$
- $\nu^{e/i}/\omega$ window of instability much smaller with large $|a/L_n|$, additional stabilizing effects?

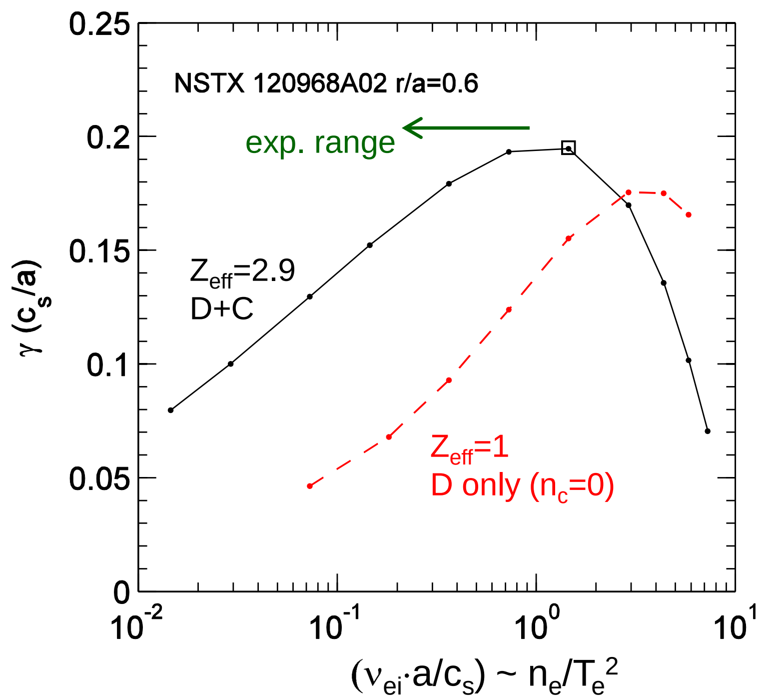




Increased impurity content (shielding of potential) can be destabilizing

- In addition to shifting peak in $\nu^{e/i}/\omega$, Z_{eff} can *enhance instability* through shielding potential from adiabatic ion response, $\delta n_i \sim -Z_{eff}\delta\varphi/T_i$
- Almost always the case for "small" current widths ($\Delta_j \leq 0.4\ \rho_s$), which here tends to correspond to "weakly-collisional" regime, $\nu_{ei}/\omega < 1$
- Opposite to slab (Gladd et al., 1980, semi-collisional $\nu_{ei}/\omega > 1$) and MAST (Applegate et al., 2007, wider current layer, $\Delta_j \sim 0.8\ \rho_s$, lower shear, s=0.29, towards core)

growth rates



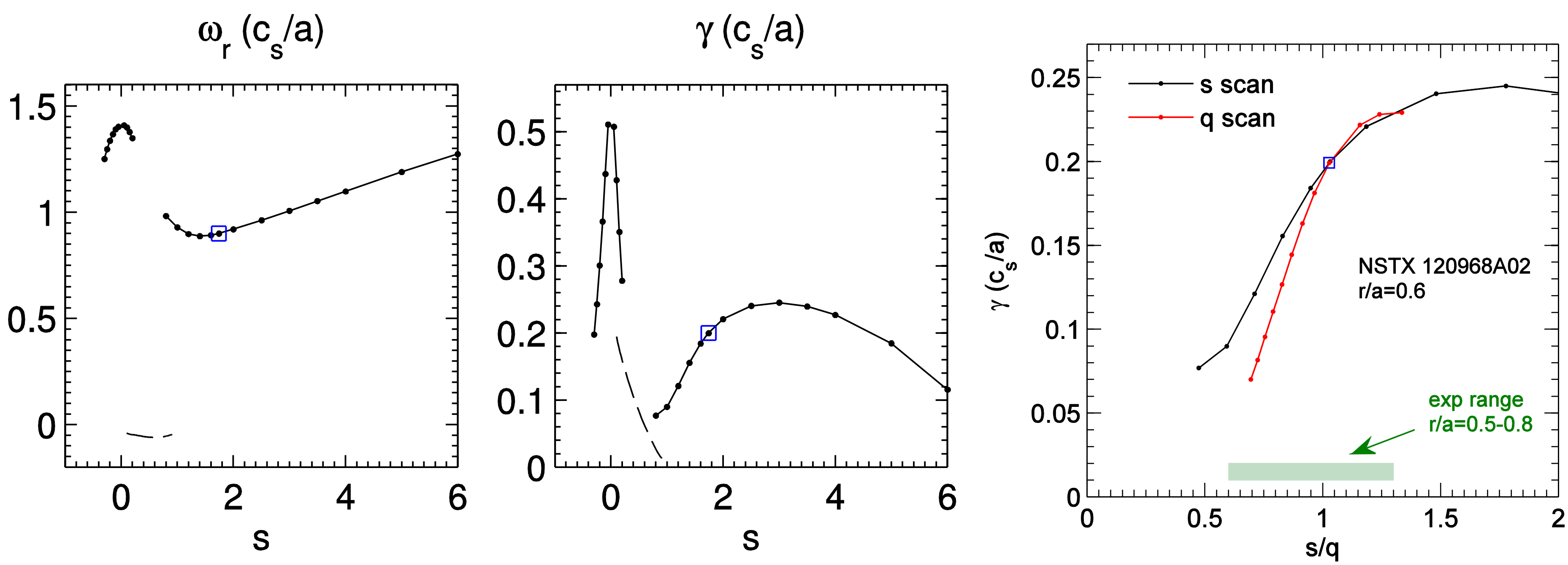
* Guttenfelder et al., Phys. Plasmas **19**, (Oct, 2011)

Non-monotonic dependence on magnetic shear (s/q)

(i) field-line bending in inner resistive layer at high shear (Δ_j narrow enough that "constant-ψ" valid), $\gamma_{damp} \sim \Delta'/L_s \sim -(s/q)\cdot 2k_\theta$

(ii) field-line bending outside inner resistive layer at low shear ("constant-ψ" no longer valid) [Gladd et al., 1980]

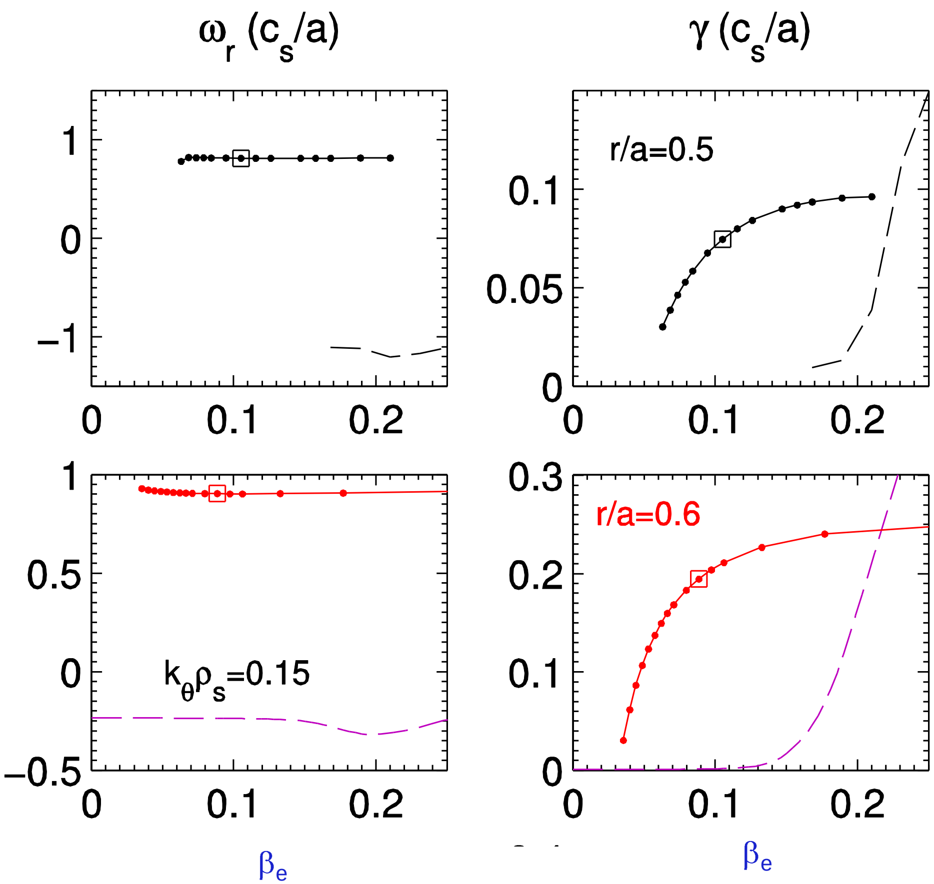
- Gladd et al. estimate criteria for constant-ψ as $\Delta A_{||}/A_{||} \ll 1$ $\quad (R/L_{Te})^2 (q/s)^2 \beta_e \ll 1$

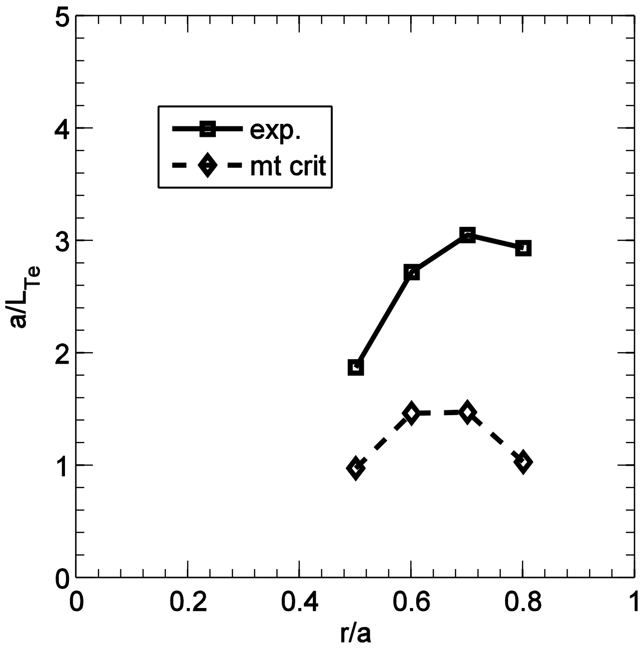


- MT returns at near-zero magnetic shear (noted by Redi et al., EPS 2005), and also for electron scale MT modes ($k_\theta \rho_s \approx 3\text{-}15$, Smith et al., 2011)

Thresholds in a/L_{Te}, β_e

- Clear threshold in a/L_{Te} and β_e
- Not well described by semi-collisional slab estimate [D'Ippolito et al., 1980] $R/L_{Te} > \left(\frac{0.3}{\mu} \frac{Z_{eff} \hat{\nu}_{ei}}{\beta_e} \frac{\hat{s}}{q} \frac{a}{R} \right)^{1/2}$



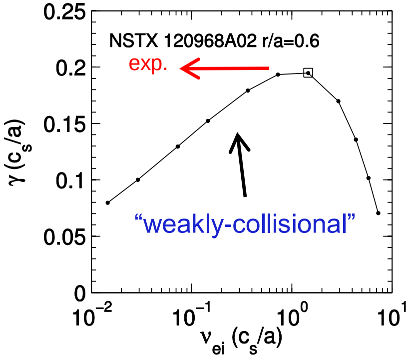


Summary: Improvements in analytic/reduced microtearing theory could be useful for developing reduced transport models

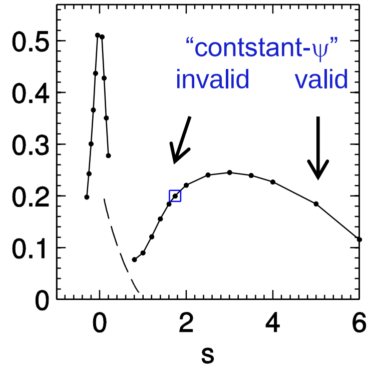
<u>Wish for improved linear stability theory with quantitative accuracy:</u>

- Arbitrary collisionality ($\nu^{e/i}/\omega$) and magnetic shear (s/q)
- Account for ballooning $A_{||}(\theta)$, toroidicity and trapped particles
- Influence of electrostatic potential is unclear (shielding through Z_{eff} + adiabatic response)
- Prediction of linear thresholds $(a/L_{Te})_{crit}$, $(\beta_e)_{crit}$

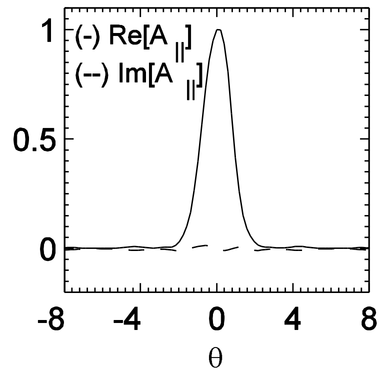
Weakly-collisional
$\nu^{e/i}/\omega \leq 1$



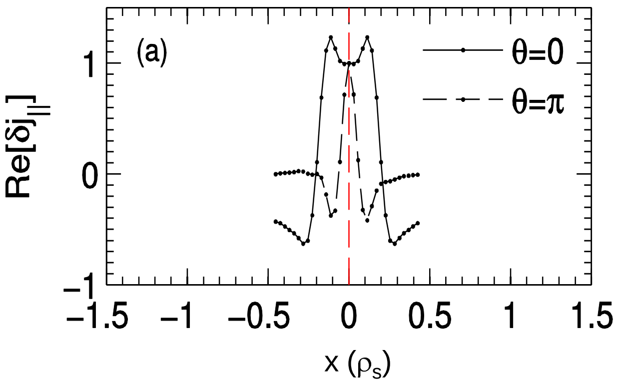
Weak magnetic shear
$s/q \leq 2$



Strongly ballooning
$A_{||}(\theta=0) >> A_{||}(\theta=\pi)$

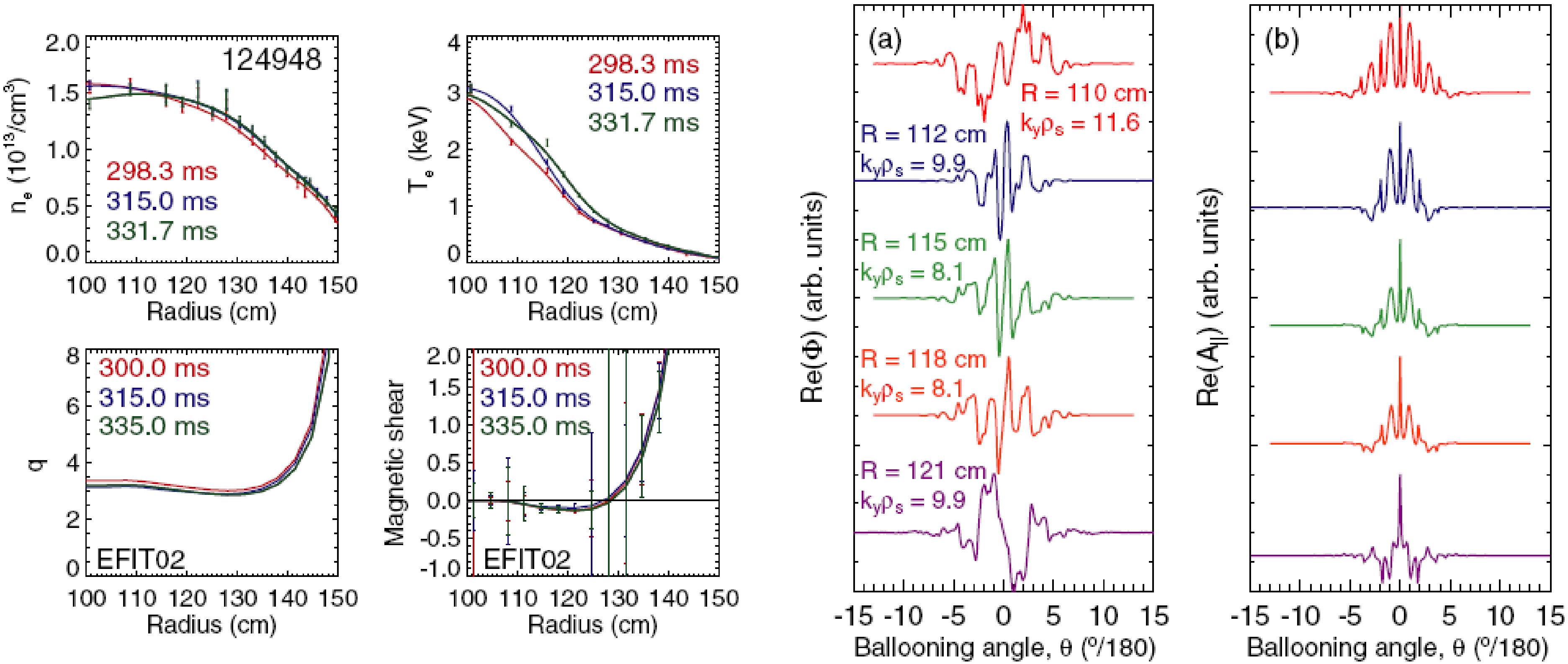


Narrow current layer
$\Delta_j \leq \rho_s$



Electron scale microtearing mode (Smith et al., 2011)

- RF heated discharges, relatively low density and beta, larger Te and ∇Te
- Unstable for very low magnetic shear (s = -0.15 - 0.06)
- Microtearing modes unstable for $k_\theta \rho_s \approx 3\text{-}15$



GYRO* used for gyrokinetic simulations

- Eulerian solver of gyrokinetic-Maxwell equations, evolving $\delta f(E,\lambda,r,\alpha,\theta)$

$$\delta f \sim \delta\hat{f}(r,\theta)e^{-in\alpha} \qquad \alpha = \phi + \nu(r,\theta) \approx \phi - q(r)\theta$$

$$k_\theta \doteq \frac{nq}{r}$$

High-aspect ratio, low β limit

parallel
binormal
radial

- Kinetic ions (D+C) and electrons, general equilibrium
- Fully collisional & electromagnetic ($\delta A_{||}$, $\delta B_{||}$) (both important in NBI heated ST)
- ~~Freedom to include toroidal flow and flow shear (important in NBI heated ST)~~
- ~~Can use experimental profile variations, T(r), n(r), q(r), etc… (likely important in ST, ρ_s/a~1/100, ρ_s/L~1/40)~~

- *All following <u>linear</u> calculations performed in the local flux-tube limit (periodic BC's) without toroidal flow & shear*

*J. Candy & R.E. Waltz, Phys. Rev. Lett. **91**, 045001 (2003); J. Comp. Physics **186**, 545 (2003); https://fusion.gat.com/theory/Gyro
J. Candy, Phys. Plasmas Control. Fusion **51**, 105009 (2009); E.A. Belli & J. Candy, Phys. Plasmas **17**, 112314 (2010).