Electromagnetic Turbulence

Steve Cowley

UK Atomic Energy Authority and Imperial College

Bill Dorland, Alessandro Zocco, Sarah Newton, etc.

Introduction

Electrostatic turbulence is the current paradigm.

- As β increases the magnetic field component increases:

$$\frac{|\delta \mathbf{B}|}{B} \propto \beta \frac{e\phi}{T}$$

- Electron transport along stochastic field could get large.

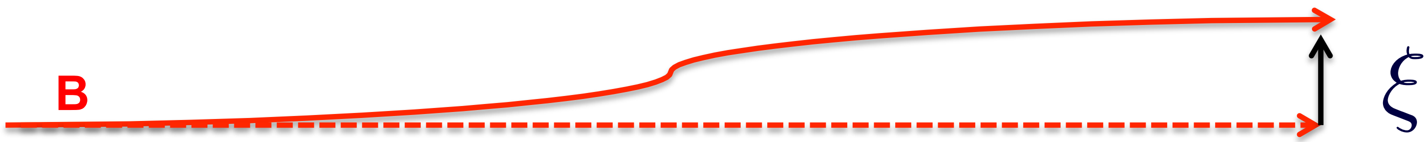
Could limit achievable β in tokamaks.

- Does it?
- Is it micro-tearing or EM Ion Temperature Gradient driven modes?

Rough Estimate of Diffusion

Random Step going a correlation length l_c along the field line.

$$\xi \sim l_c \frac{\delta B_\perp}{B}$$



Diffusion of field lines

$$d_M = \lim_{l \to \infty} \frac{< r(l) - r(0) >^2}{l} \sim \frac{\xi^2}{l_c}$$

Spatial diffusion of an electron moving at v_{the} along field

$$\chi = v_{the} \frac{\xi^2}{l_c} = v_{the} l_c \frac{\delta B_\perp^2}{B^2}$$

ITG with δB

W. M. Nevins,[1] E. Wang,[1] and J. Candy[2] PRL **106,** 065003 (2011)

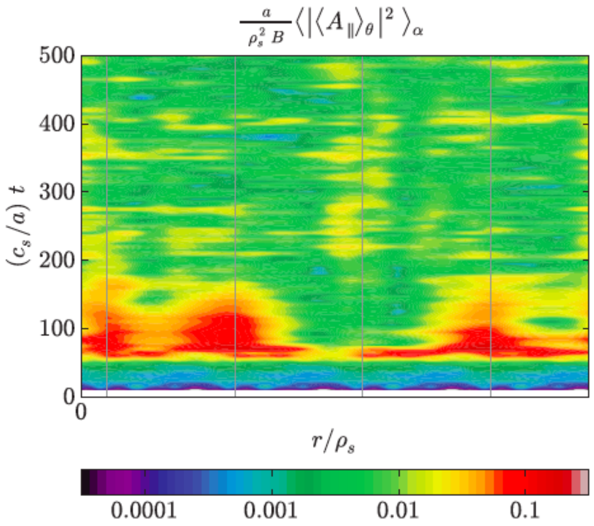


FIG. 1 (color). The resonant magnetic intensity from a GYRO simulation at $\beta = 0.1\%$. Vertical lines show the fundamental rational surfaces at $r = 2.97\rho_s$, $17.86\rho_s$, $32.74\rho_s$, and $47.62\rho_s$.

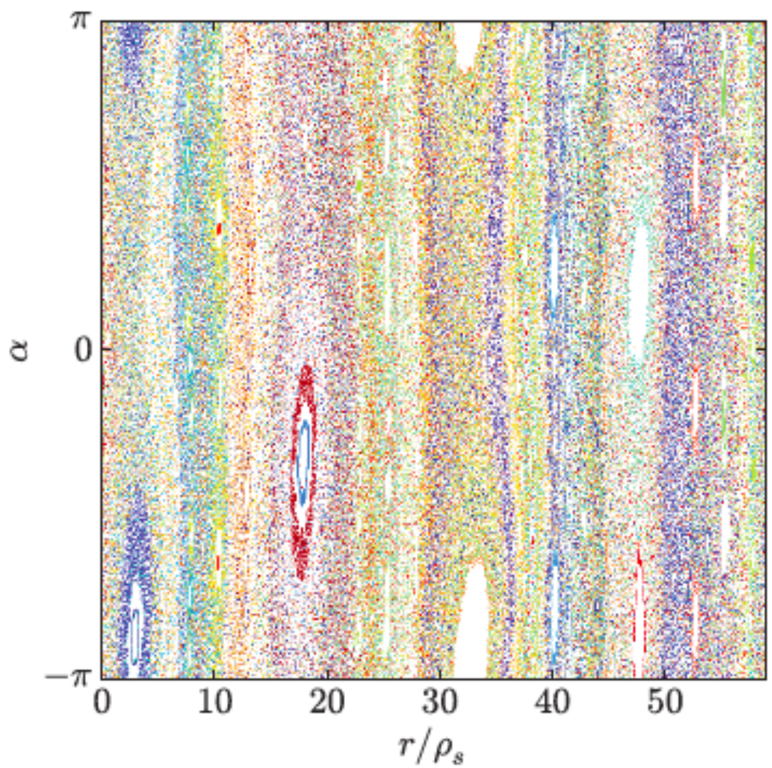


FIG. 2 (color). A Poincaré surface-of-section plots for the GYRO simulation at $\beta_e = 0.1\%$ and $t = 250$, where individual field lines are denoted by their color.

Field line diffusion – Electron diffusion

W. M. Nevins,[1] E. Wang,[1] and J. Candy[2]

$$d_m = \lim_{\ell \to \infty} \frac{\langle [r_i(\ell) - r_i(0)]^2 \rangle}{2\ell}$$

Field-line diffusion

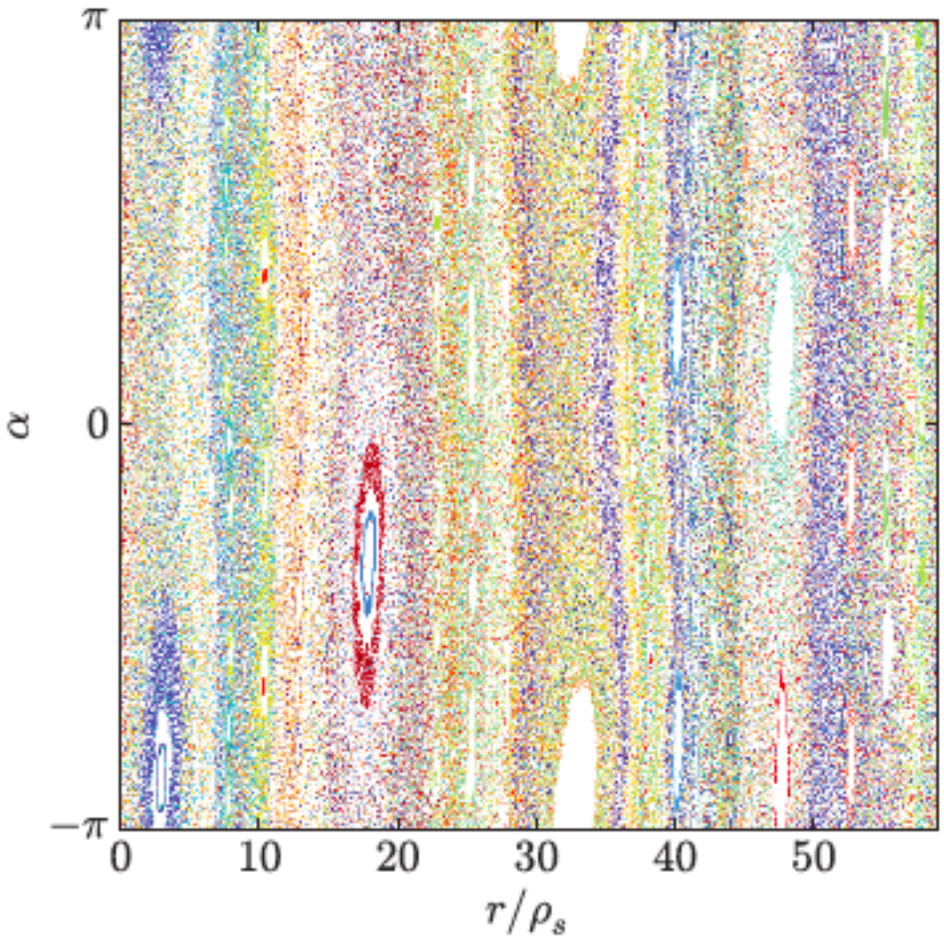
$$\chi_e^{st} = 2 f_p \sqrt{2/\pi}\, v_{te} d_m$$

Electron heat diffusion

$$f_p \approx 1 - \sqrt{r/R}$$

Fraction of passing particles

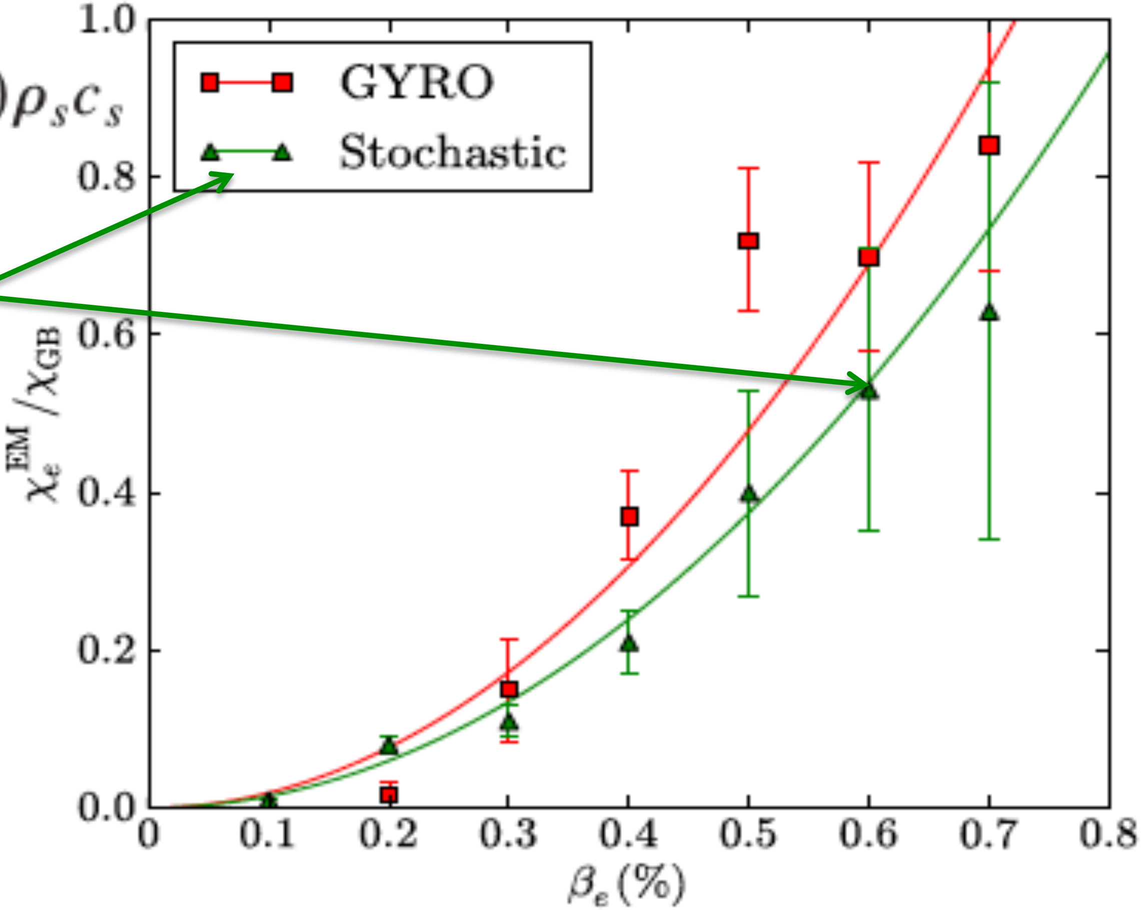
Electron thermal velocity



EM transport

$$\chi_e^{EM} \approx \underline{1.9 \times 10^4 \beta_e^2} (\rho_s/a)\rho_s c_s$$

$$\chi_e^{st} = 2 f_p \sqrt{2/\pi} v_{te} d_m$$



$$\chi_e^{ES} = (2-3)\frac{\rho_s}{a}\rho_s c_s$$

Electrostatic

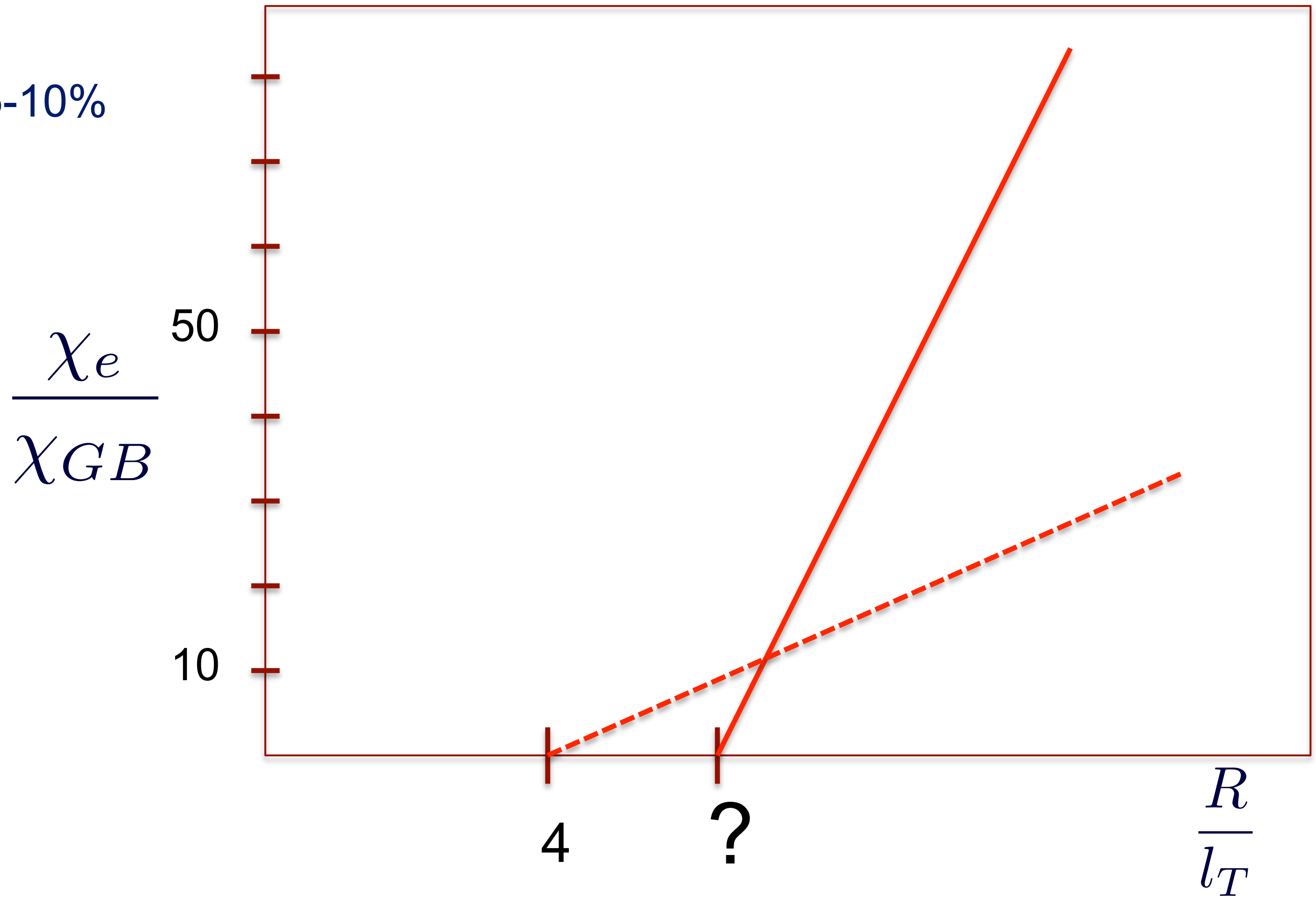
$$\chi_i = (6-8)\frac{\rho_s}{a}\rho_s c_s = (6-8)\chi_{GB}$$

ITER – β_e ~ 5-10%

EM transport

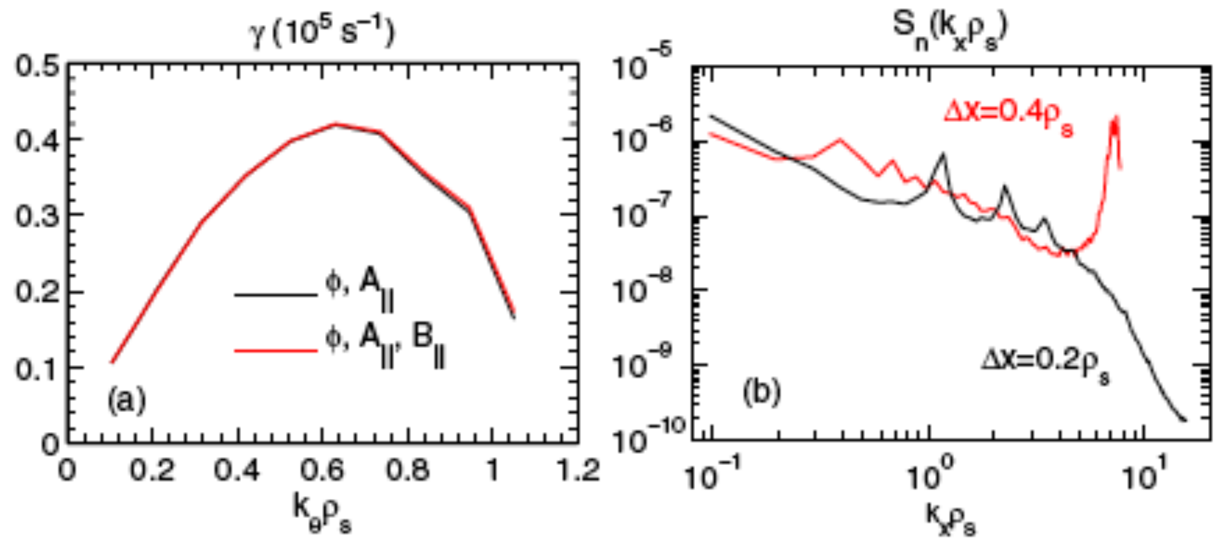
$$\chi_e^{EM} \approx 1.9 \times 10^4 \beta_e^2 \, (\rho_s/a)\rho_s c_s \, F\left(\frac{R}{L_T} - \left[\frac{R}{L_T}\right]_{crit}\right)$$

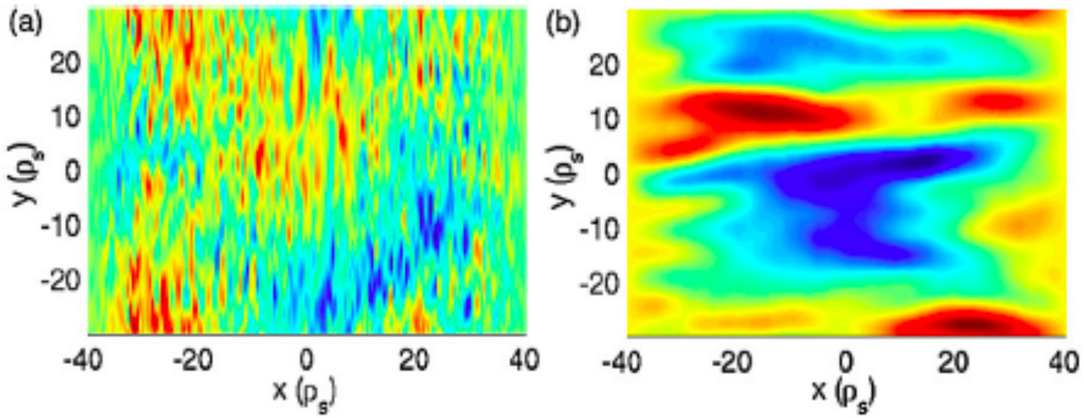
ITER – β_e ~ 5-10%



Micro-tearing

W. Guttenfelder,[1] PRL **106**, 155004 (2011)





$$\chi_e \approx 1.2\rho_s^2 c_s/a$$

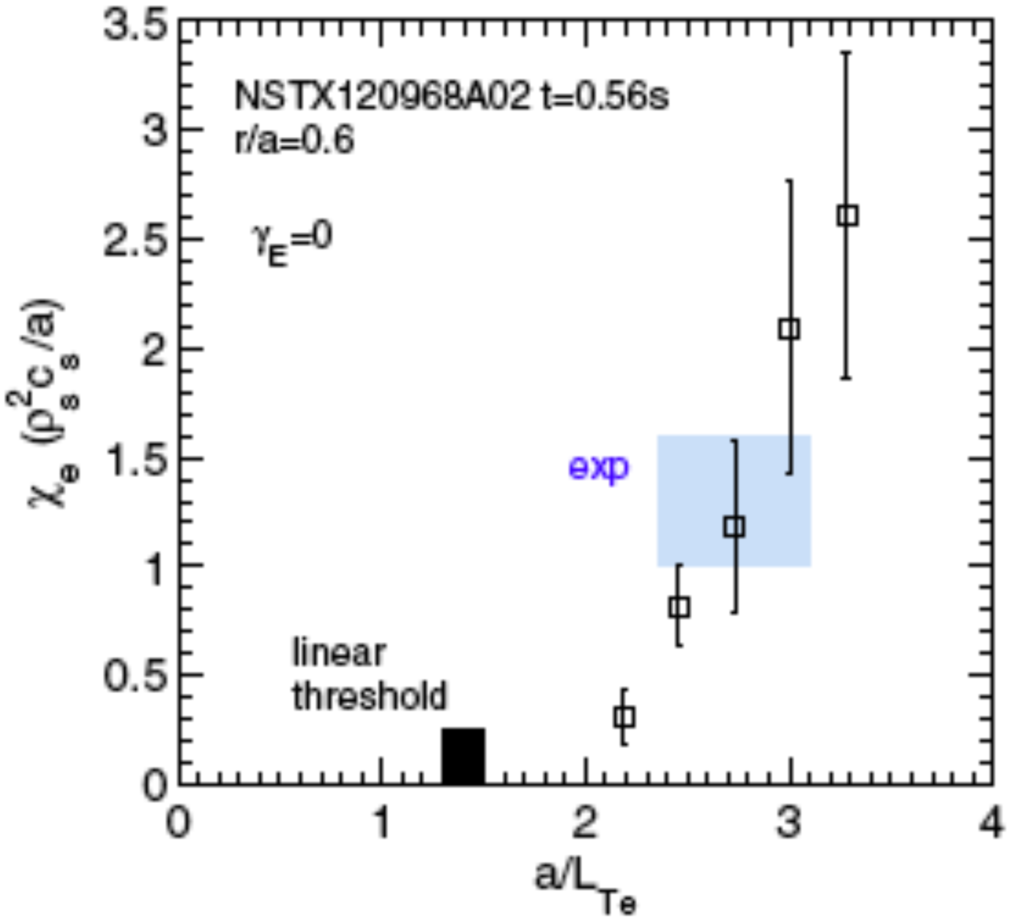
All Electromagnetic

$\beta = 9\%$

FIG. 2 (color online). Contour plots of (a) δn and (b) $\delta A_{\|}$ perturbations at a snapshot in time.

Micro-tearing

W. Guttenfelder,[1] PRL **106,** 155004 (2011)



$$\cancel{\chi_e^{\mathrm{EM}} \approx 1.9 \times 10^4 \beta_e^2 (\rho_s/a)\rho_s c_s}$$

Drake Scaling

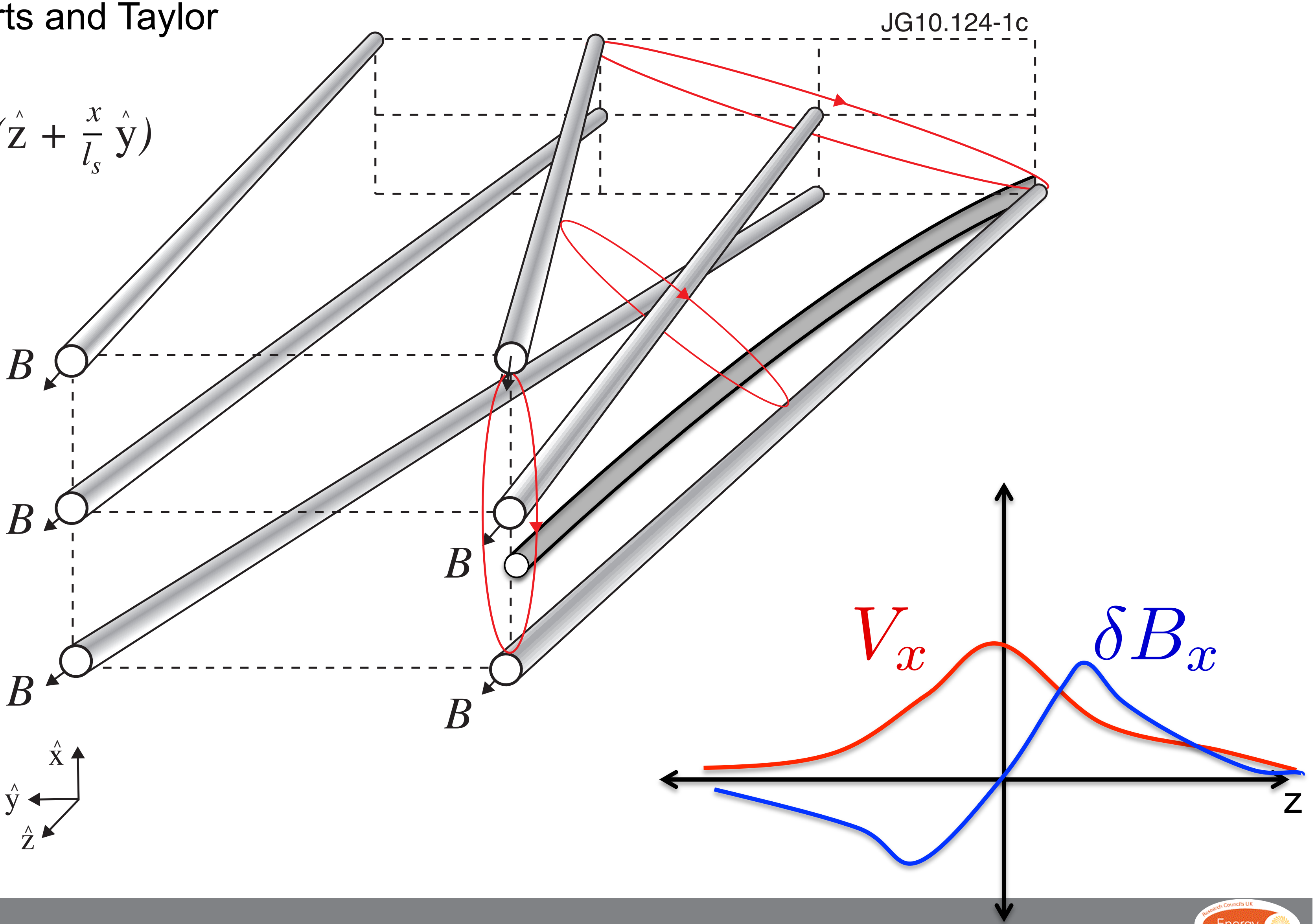
$$\frac{\delta B_\perp}{B} = \frac{\rho_e}{l_T}$$

$$\chi = v_{the} l_c \frac{\delta B_\perp^2}{B^2} = l_c v_{the} \frac{\rho_e^2}{l_T^2}$$

Interchange Parity

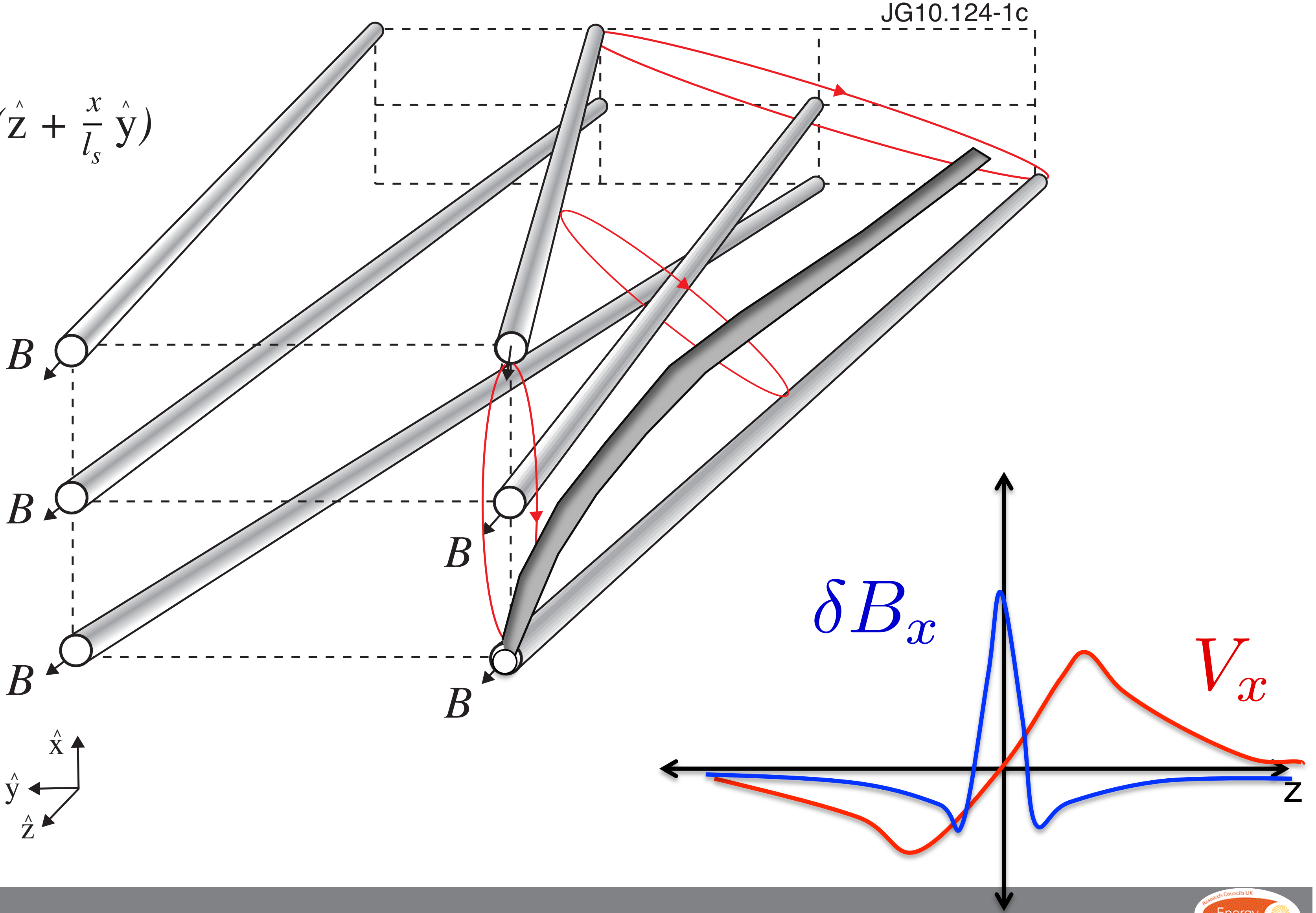
Roberts and Taylor

$$B = B_0(\hat{z} + \frac{x}{l_s}\hat{y})$$



Micro-tearing

$$B = B_0(\hat{z} + \frac{x}{l_s}\hat{y})$$



"Frozen Field"

- Ian Abel has shown that where: $k_\perp \rho_e (\frac{\nu_e}{\omega})^{1/2} \ll 1$

(collisional limit)

$E_{||} = \mathbf{b} \cdot \nabla \psi$ and velocity of field lines becomes

$$\mathbf{v}_{frozen} = c\frac{(\mathbf{E} - \nabla\psi) \times \mathbf{B}}{B^2}$$

In the slab $\psi = \frac{T\delta n_e}{e n_0}$

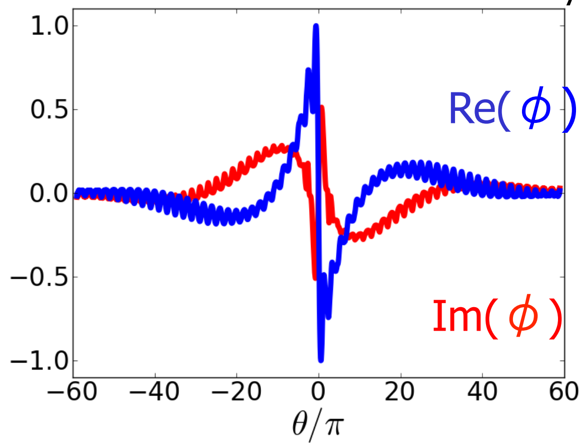
Far along field line condition is violated $k_\perp^2 = k_y^2(1 + \frac{z^2}{l_s^2})$

Dissipation Scale

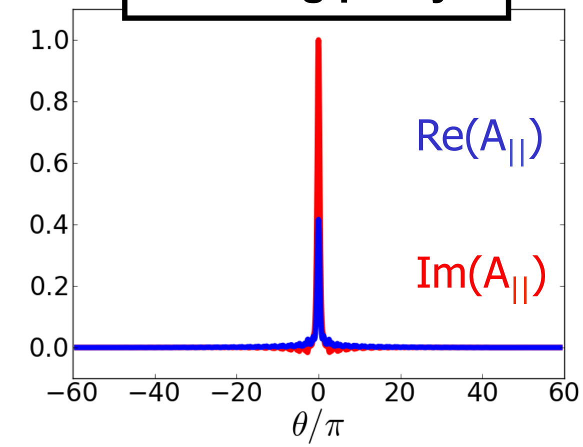
$$\frac{z}{l_s} = \left(\frac{1}{k_y\rho_i}\right)\left(\frac{\rho_i}{\rho_e}\right)\left(\frac{\omega}{\nu_e}\right)^{1/2}$$

Scale at which field slips through plasma.

Microtearing modes (k_y=0.12)



Tearing parity

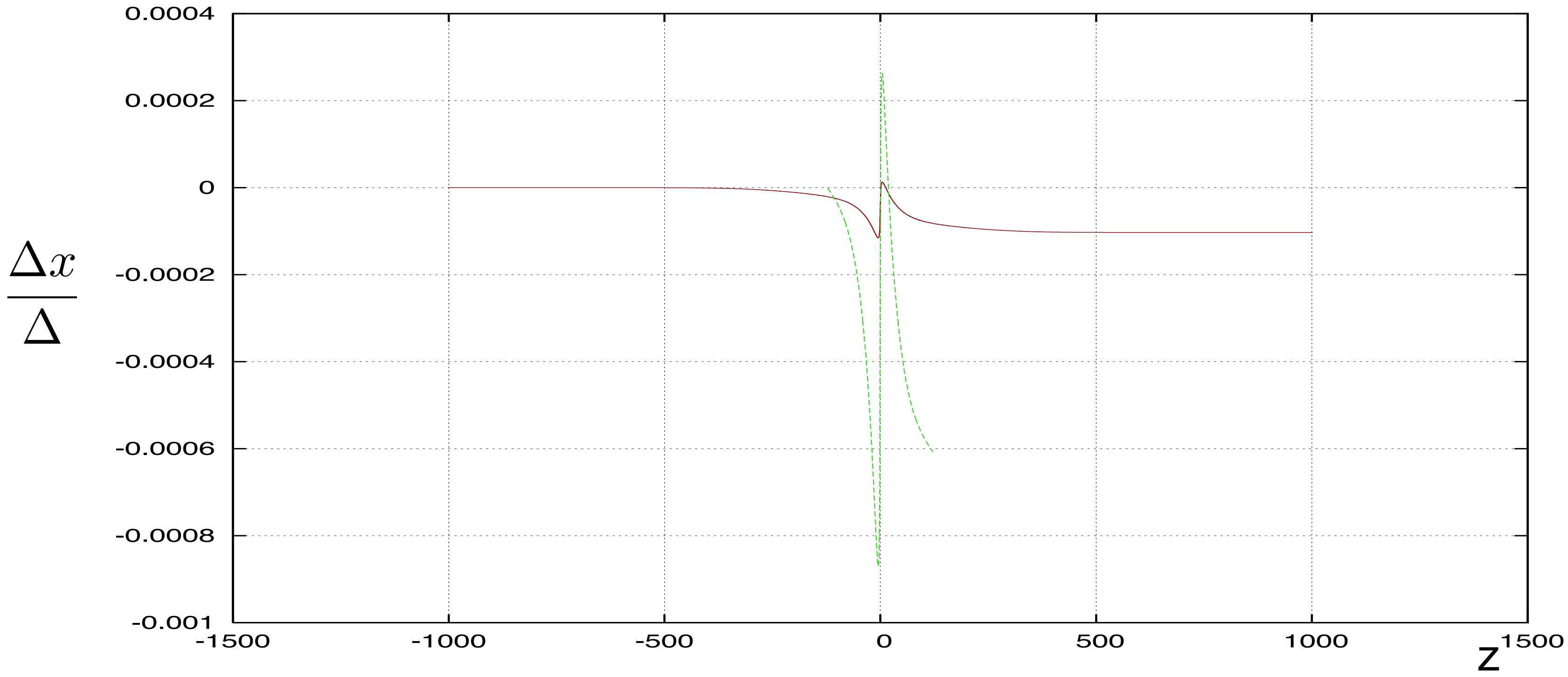


Field Lines -- steps

$$\int_0^z \frac{\delta B_x}{B} dz = \Delta x$$

Normalise to the typical EXB displacement -- $v_{EXB} / \omega = \Delta$

Microtearing



Field Lines -- steps

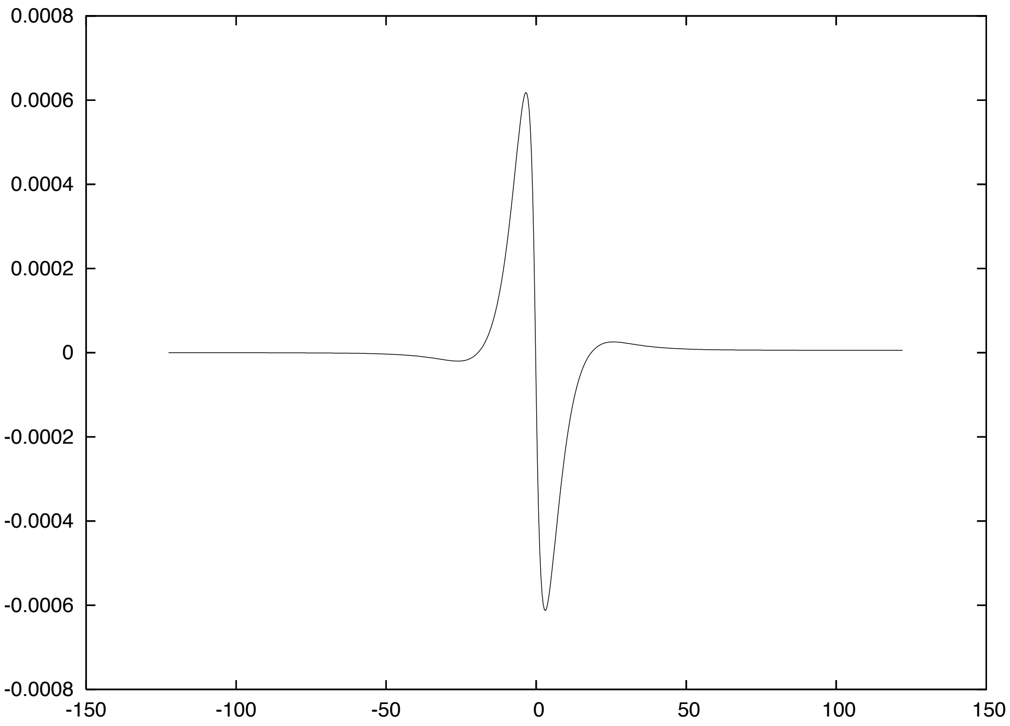
$$\int_0^z \frac{\delta B_x}{B} dz = \Delta x$$

Interchange ITG
No step of course

Normalise to the typical EXB displacement -- $v_{EXB}/\omega = \Delta$

ITG

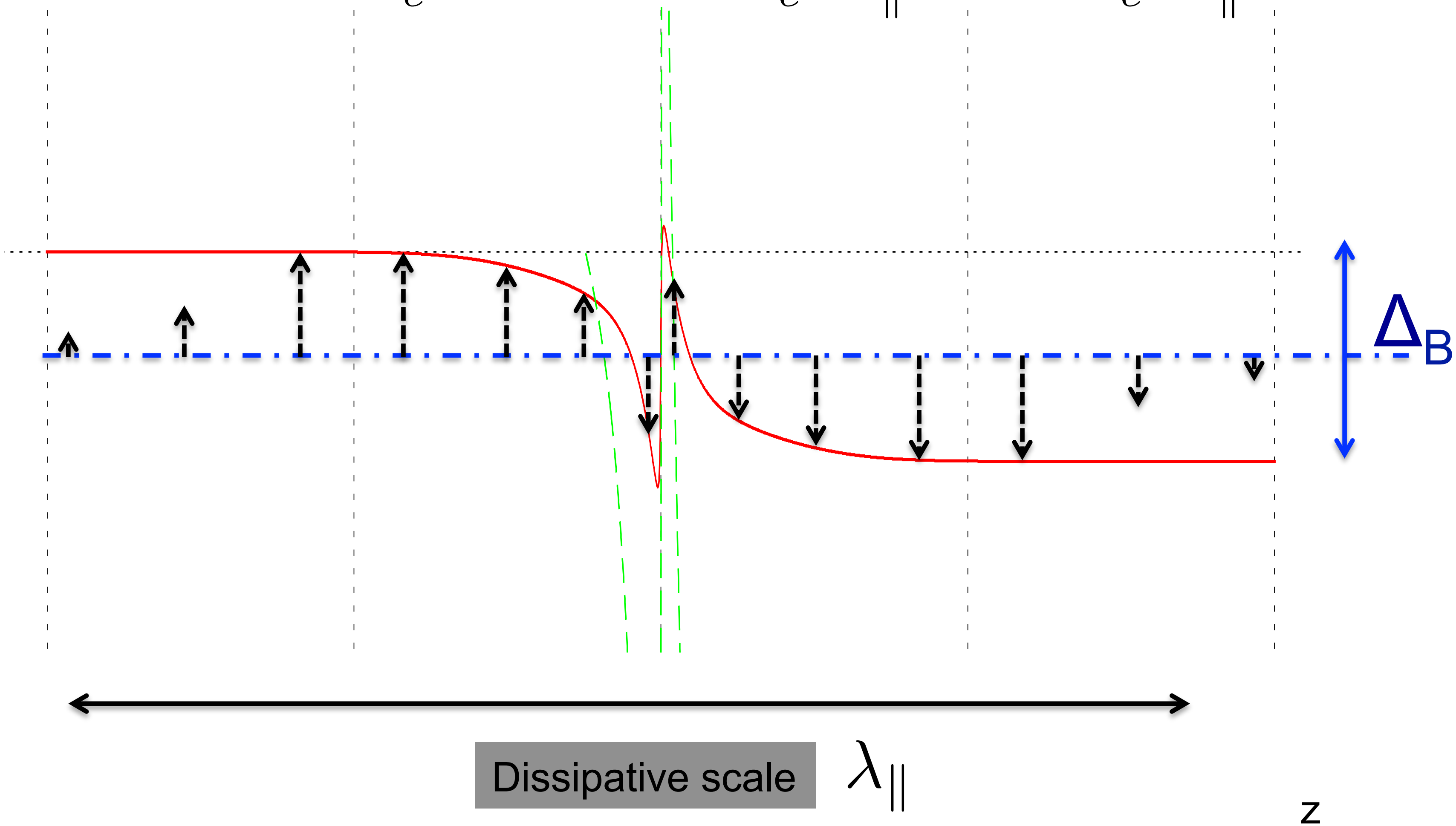
$$\frac{\Delta x}{\Delta}$$



z

Field Lines -- steps

$$q_{\|} = \kappa_{\|}\nabla_{\|}T \sim n_0 \frac{v_{the}^2}{\nu_e}\nabla_{\|}T \sim n_0 \frac{v_{the}^2}{\nu_e}\frac{\delta T}{\lambda_{\|}} \sim n_0 \frac{v_{the}^2}{\nu_e}\frac{1}{\lambda_{\|}}\Delta_B|\nabla T|$$



Field Lines -- steps

Radial heat flux
From B perturbation:

$$q_r = q_{\|}\frac{\delta B_r}{B} = n_0 T\frac{\Delta_B^2}{L_T}\frac{v_{the}}{l_s}(\frac{\omega}{\nu_e})^{1/2}$$

Radial heat flux
electrostatic:

$$q_r = n_0 T\frac{\Delta^2}{L_T}\omega$$

$$\Delta_B \sim \beta_e \Delta \quad ?$$

$$q_r = n_0 T\beta_e^2\frac{\Delta^2}{L_T}\frac{v_{the}}{l_s}(\frac{\omega}{\nu_e})^{1/2}$$

Conclusion

At betas of interest for efficient fusion we could have large Transport…

- But perhaps it just makes transport stiffer but critical gradient is higher?

- Clearly it is not settled.