The effect of shielded RMP fields on ballooning stability

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ELM control is essential for large Tokamaks.

- ELMs generally lead to abrupt loss of 5-10% of stored energy.
- For ITER, heat flux to the divertor would be prohibitively large.
- Presence of ELMs agrees very well with P-B stability boundary.
- Characterizing transport in the H-mode pedestal is still an open issue – even without RMP fields.

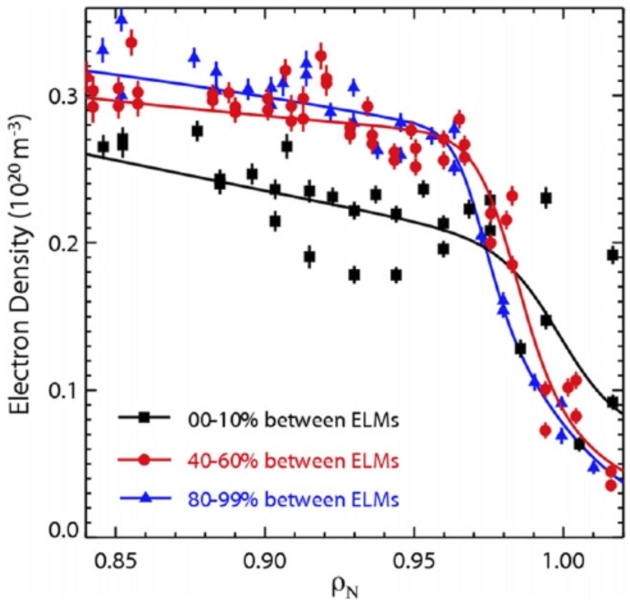
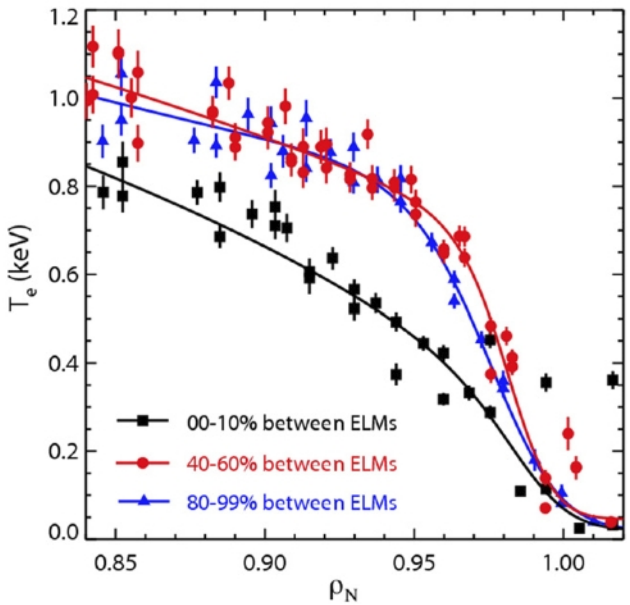


Figure 1. Fit of experimental electron density measured by Thomson scattering.



[Groebner et al., Nuclear Fusion 2011.]

3D RMP fields have suppressed and mitigated ELMs in DIII-D.

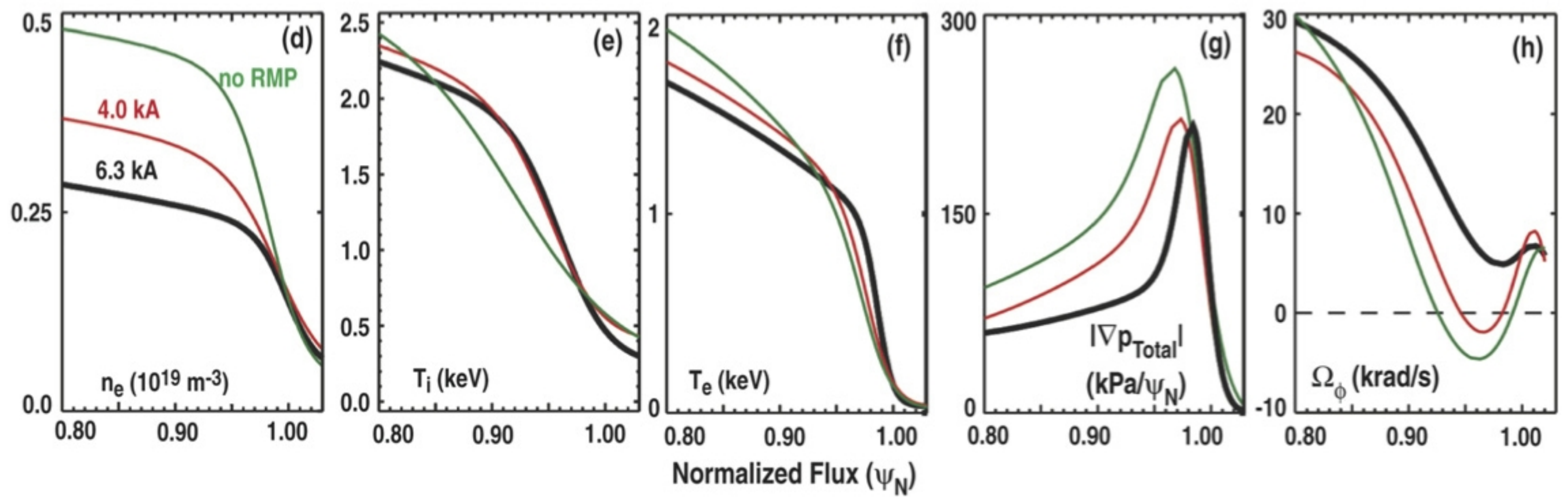


Figure 6. Lower divertor D_α signals showing the ELM characteristics in similar ISS plasmas with $n = 3$ I-coil currents of (*a*) 6.3 kA, (*b*) 4.0 kA and (*c*) 0 kA. Pedestal profiles showing the (*d*) density, (*e*) ion temperature, (*f*) electron temperature, (*g*) absolute value of the total pressure gradient and (*h*) C^{6+} toroidal rotation for the 3 I-coil currents shown in (*a*), (*b*) and (*c*) where black, red and green correspond to 6.3 kA, 4.0 kA and 0 kA, respectively.

- ITER similar shape (delta = 0.53, nu* < 0.2) experiments.
- Key features: “density pump-out”, sensitivity to edge q value.
- Initial explanation: multiple islands overlap, break surfaces, increase transport.
- Could a different mechanism be at work?

[T. Evans et al., Nuclear Fusion 2008]

Theory suggests that rotational screening is operative.

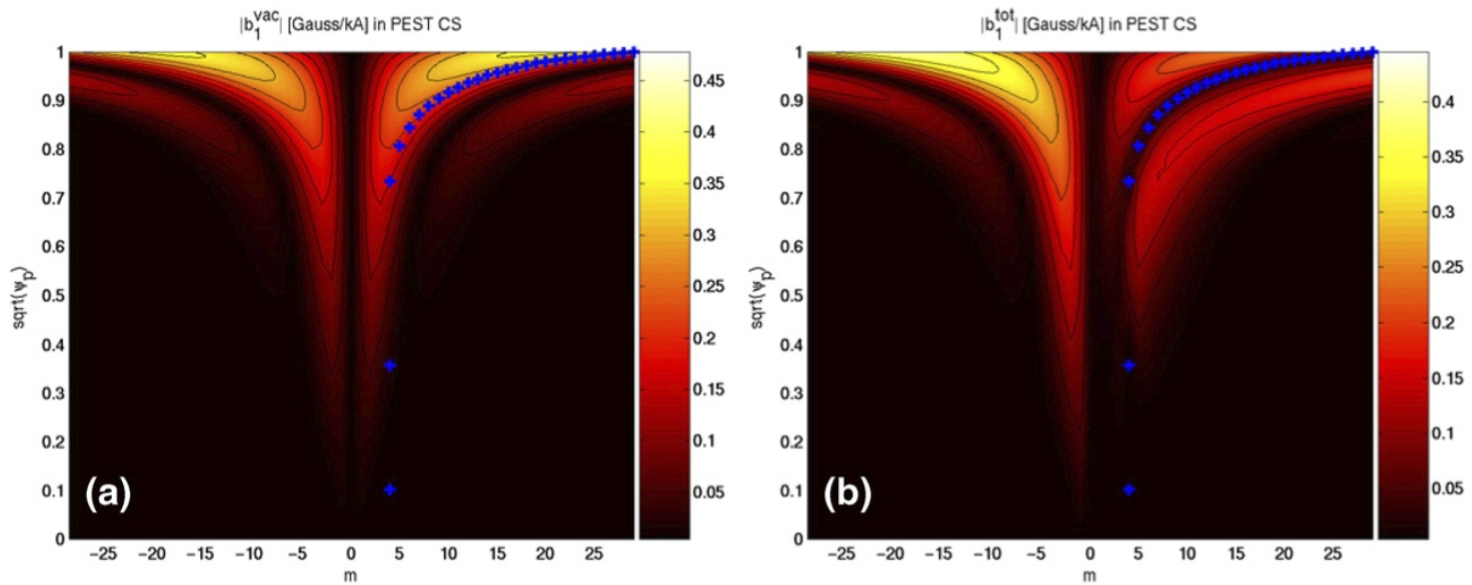
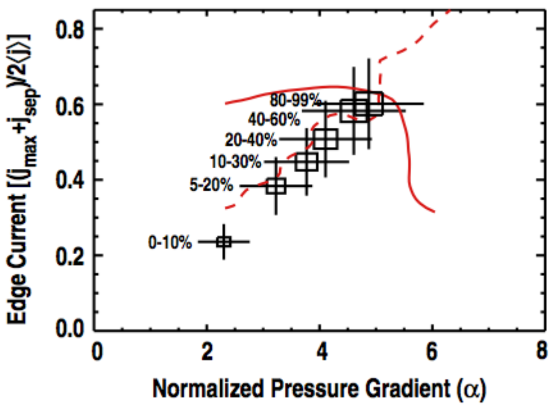


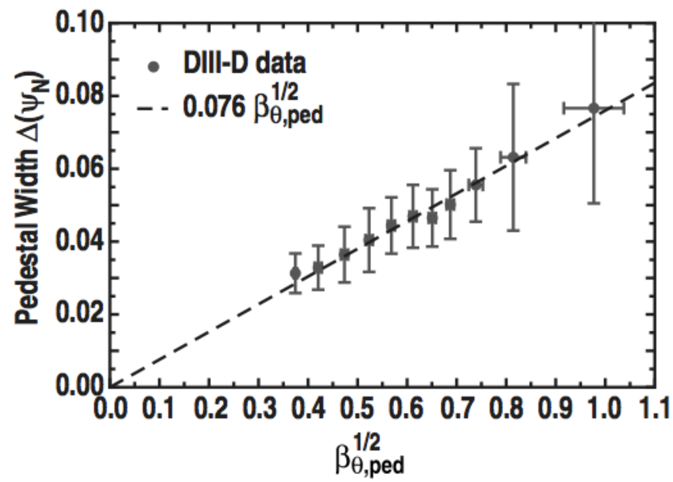
Figure 3. Comparison of the poloidal spectra in the full plasma region, between (*a*) the vacuum field and (*b*) the total field including the plasma response, computed by MARS-F for shot 20333 with odd parity of the coil current. The symbols '+' indicate the location of $q = m/n$ rational surfaces.

- MARS-F Calculations: Resistive MHD with toroidal flow.
- Resonant component of radial B field shielded by order of magnitude or more.
- Non-resonant components penetrate, can be amplified by plasma response.

[Y. Liu et al., Nuclear Fusion 2011.]

KBM turbulence may limit the pedestal pressure gradient.

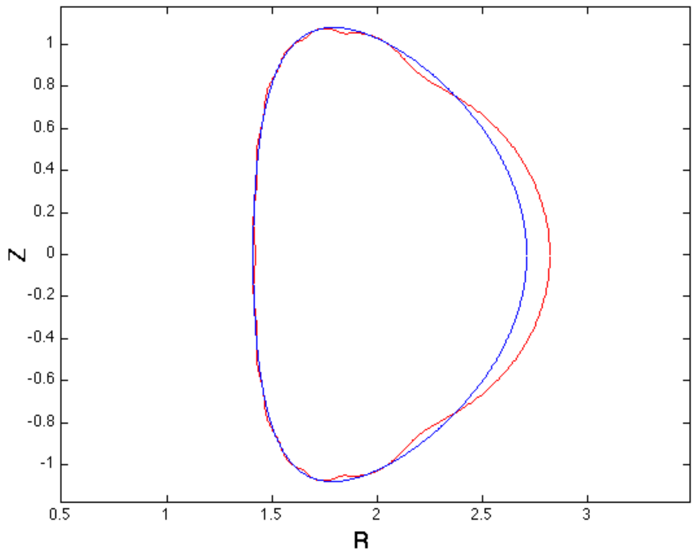


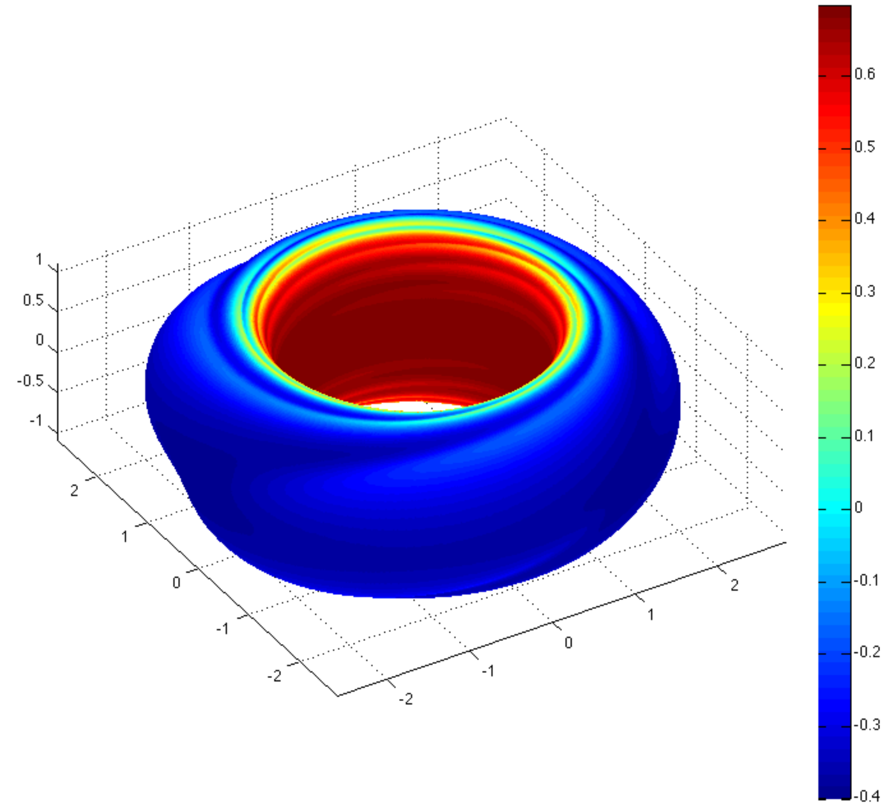


- EPED1 predicts the pedestal width with a simple MHD proxy for KBM onset.
- Gyrokinetic simulations find KBMs are dominant in region of steep pressure gradient. (Dickinson et al., PPCF 2011)

[Groebner et al, NF 2011, Snyder et al, PoP 2009

Can the non-resonant fields affect transport or stability?





Local 3D Equilibrium model [C.C. Hegna, Physics of Plasmas, 2000].

- One would not expect very small RMPs to affect ballooning stability.
- Local model allows us to study the effect of small 3D flux surface deformation in a careful way.

Let's assume there exists some 3-D MHD equilibrium.

- Consider the inverse coordinate mapping, with general straight field line coordinates:

$$\vec{X}(\psi, \Theta, \zeta) = [R, \phi, Z]$$

- For example:

$$\begin{aligned} R &= R_0 + \rho\cos\left(\Theta + \arcsin(\delta)\sin(\Theta)\right) \\ \phi &= -\zeta \\ Z &= \kappa\rho\sin(\Theta) \end{aligned}$$

- The Jacobian of this transformation is then:

$$\sqrt{g} = \frac{\partial \vec{x}}{\partial \psi} \cdot \frac{\partial \vec{x}}{\partial \Theta} \times \frac{\partial \vec{x}}{\partial \zeta}$$

From this we can construct the equilibrium quantities.

- The magnetic field and plasma currents are given by:

$$\vec{B} = \left(\frac{\partial \vec{x}}{\partial \zeta} + \iota \frac{\partial \vec{x}}{\partial \Theta} \right) \frac{1}{\sqrt{g}}$$

$$\mu_o \vec{J} \cdot \nabla \Phi^k = \epsilon_{ijk} \frac{1}{\sqrt{g}} \frac{\partial}{\partial \Phi_i} \frac{g_{\zeta j} + \iota g_{\theta j}}{\sqrt{g}}$$

- The geometric properties of field lines are given by:

$$(\hat{b} \cdot \nabla)\hat{b} = \kappa_n \hat{n} + \kappa_g \hat{b} \times \hat{n}$$

$$(\hat{b} \cdot \nabla)\hat{n} = -\kappa_n \hat{b} + \tau_n \hat{b} \times \hat{n}$$

$$(\hat{b} \cdot \nabla)\hat{b} \times \hat{n} = -\tau_n \hat{n} - \kappa_g \hat{b}$$

- Choice of the flux surface geometry also sets the following metric elements:

We avoid the issues of global 3D equilibrium theory.

- Local stability calculations only require info at once surface. Let's pick one and expand in the radial coordinate:

$$\vec{x}(\psi, \Theta, \zeta) = \vec{x}(\psi_0, \Theta, \zeta) + (\psi - \psi_0)\frac{\partial \vec{x}}{\partial \psi}(\psi_0, \Theta, \zeta) + \ldots.$$

We are free to chose
the flux surface shape.

Set by 3D Ideal MHD
equilibrium theory.

- We can calculate the Jacobian by using a basic property of MHD equilibrium:

$$\hat{n} \cdot \vec{J} = 0 \implies \frac{\partial}{\partial \Theta}\frac{g_{\zeta\zeta} + \iota g_{\zeta\Theta}}{\sqrt{g}} = \frac{\partial}{\partial \zeta}\frac{g_{\Theta\zeta} + \iota g_{\Theta\Theta}}{\sqrt{g}}$$

Using this theory we can add RMP fields to the Miller Eq.

- For the parametrization of flux surface shape:

$$R = R(\Theta) + \sum_i \gamma_i cos(M\Theta - N\zeta)$$

$$Z = Z(\Theta) + \sum_i \gamma_i sin(M\Theta - N\zeta)$$

- This already specifies all of the geometric properties. Then, the partial differential equation for the jacobian must be solved:

$$\frac{\partial}{\partial\Theta}\frac{g_{\zeta\zeta} + \iota g_{\zeta\Theta}}{\sqrt{g}} = \frac{\partial}{\partial\zeta}\frac{g_{\Theta\zeta} + \iota g_{\Theta\Theta}}{\sqrt{g}}$$

- This determines the magnetic field strength on the surface, and now the MHD equilibrium solution is uniquely specified in a radially local area in the vicinity of the surface.

The Miller Equilibrium uses 7 shaping parameters.

- The familiar triangularity and elongation (and also aspect ratio):

$$R = R_0 + \rho cos\left(\theta + (sin^{-1}\delta)sin\theta\right)$$
$$Z = \kappa\rho sin\theta$$

- The poloidal magnetic field is also specified – here 4 more parameters are used:

$$B_p = \frac{\delta_r \psi \kappa^{-1} R^{-1}\left[sin^2(\theta + xsin\theta)(1 + xcos\theta)^2 + \kappa^2 cos^2\theta\right]}{cos(xsin\theta) + \delta_r R_0 cos\theta + [s_\kappa - s_\delta cos(\theta) + (1 + s_\kappa)xcos\theta]sin\theta}$$

- To use 3D local eq machinery and add RMP fields we need to convert to the proper straight field line coordinate:

$$\frac{\partial\Theta}{\partial\theta} = \left|\frac{\partial\vec{x}}{\partial\theta}\right| \frac{f}{qR^2B_p}$$

3D perturbations with a broad poloidal mode spectrum are used.

$$R = R(\Theta) + \sum_i \gamma_i cos(M\Theta - N\zeta)$$

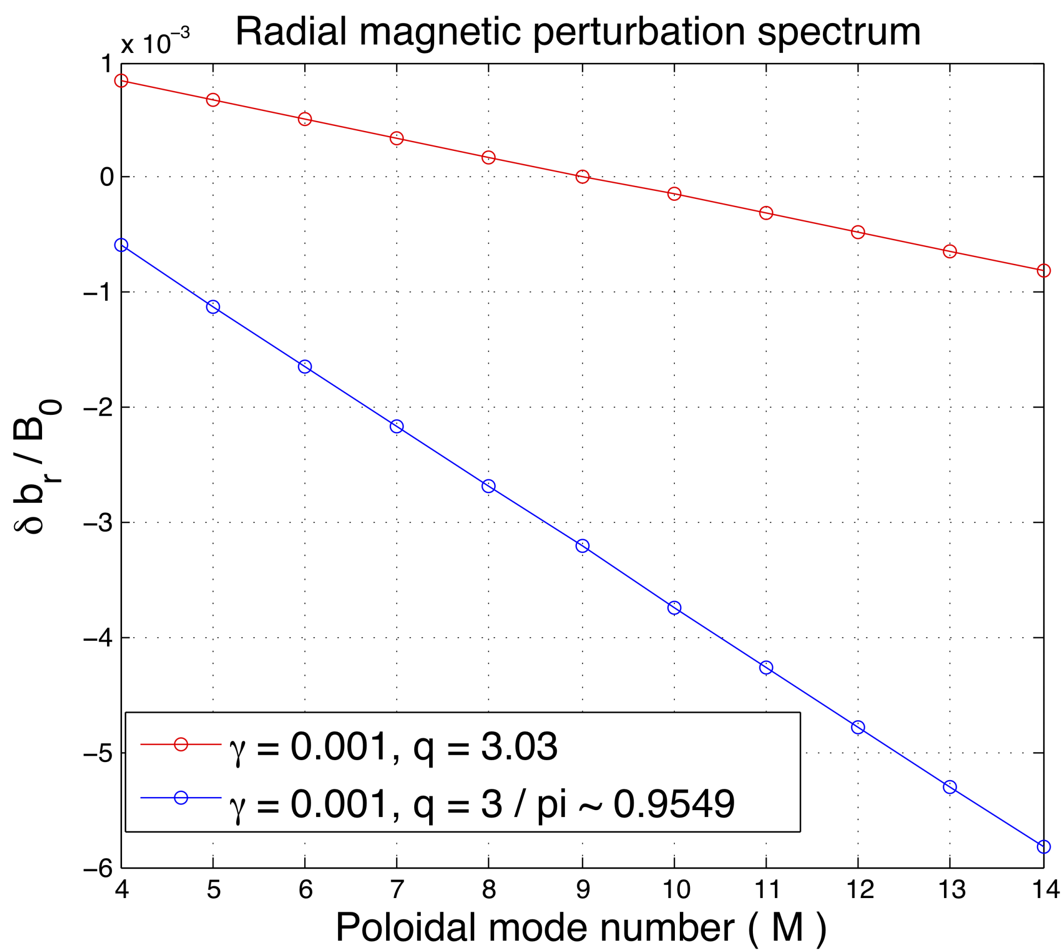
$$Z = Z(\Theta) + \sum_i \gamma_i sin(M\Theta - N\zeta)$$

- Axisymmetric shaping:

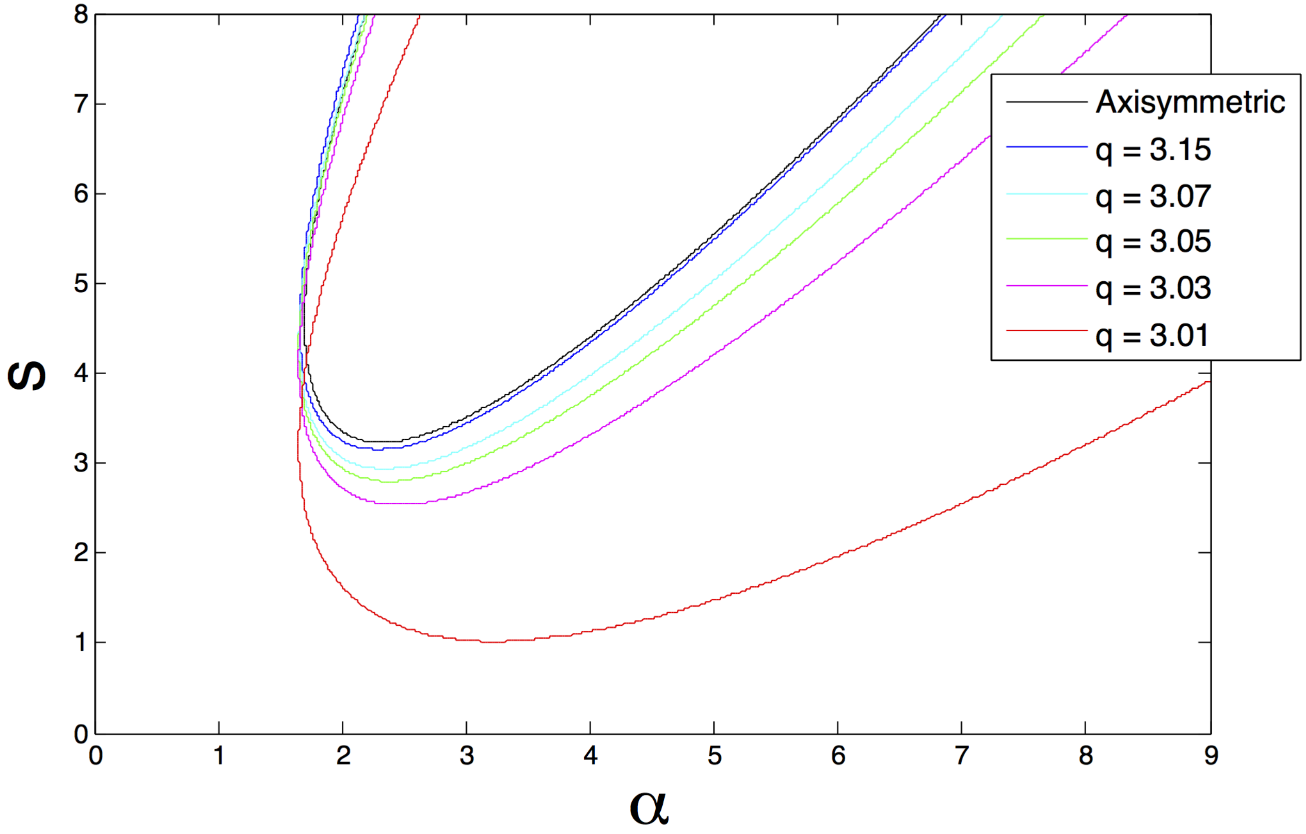
$$A = R_0/\rho = 3.17$$
$$\delta = 0.416$$
$$\kappa = 1.66$$
$$s_\kappa = 0.70$$
$$s_\delta = 1.37$$
$$\delta_r R_0 = -0.354$$

- N = 3

  M = 4 → 14



There is a strong effect on stability near resonant surfaces.



Let's examine the ballooning equation.

- How much is each term in the ballooning equation modified by the 3-D distortion to the flux surface shape?

$$\frac{\partial}{\partial\eta}[\frac{B^2}{|\nabla\psi|^2}(1+\Lambda^2)\frac{\partial\xi}{\partial\eta}] + (\sqrt{g})^2\frac{B^2}{|\nabla\psi|}2\mu_o p'(\kappa_n + \Lambda\kappa_g)\xi = -\omega^2\rho(\sqrt{g})^2\frac{B^2}{|\nabla\psi|^2}(1+\Lambda^2)\xi \quad (12)$$

~ 4e-3

~6!

~ 4e-3

~ 1e-2

~ 1e-2

- The local magnetic shear:

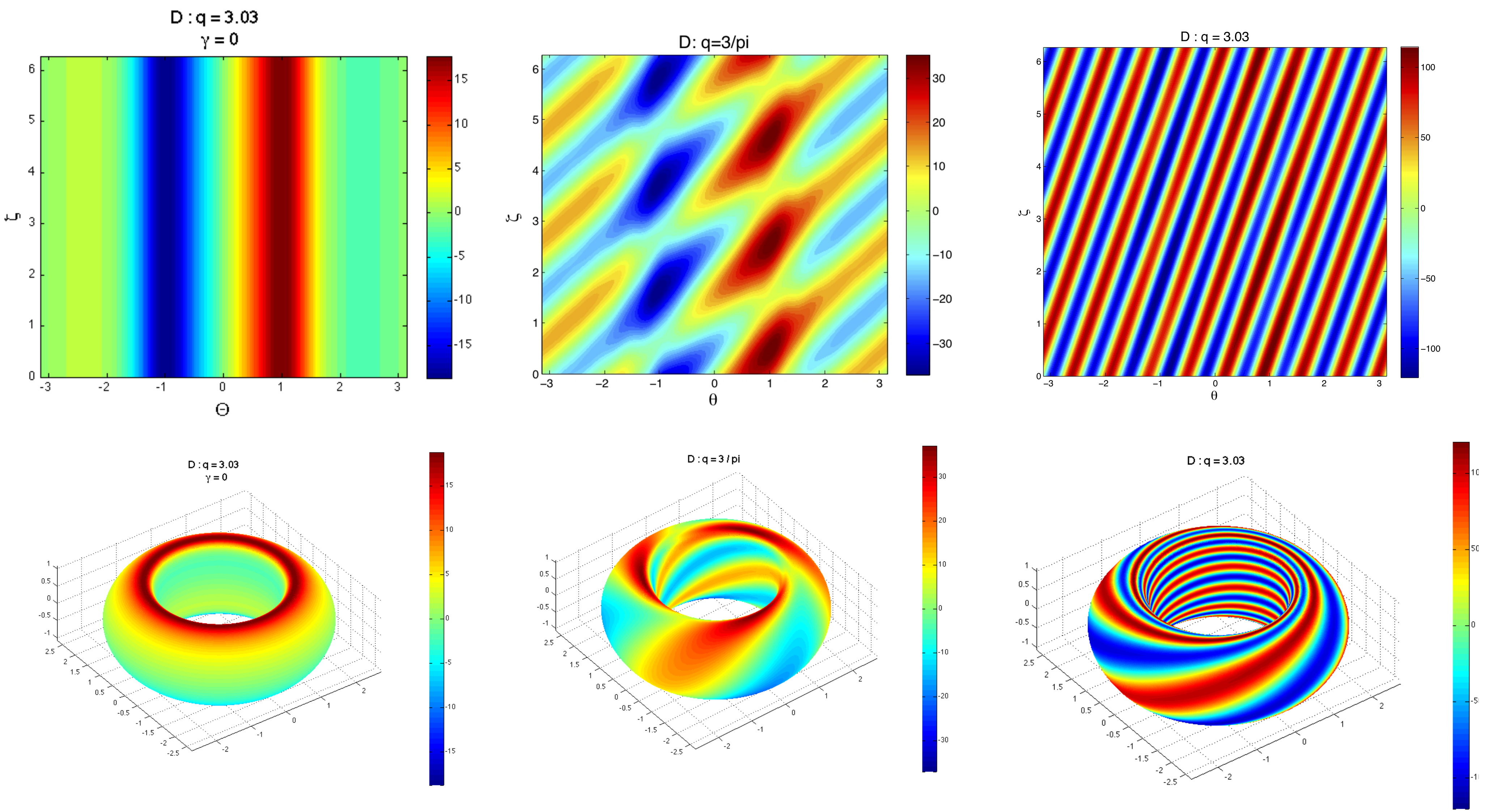
$$s = \frac{|\nabla\psi|^2}{B^2}(\vec{B}\cdot\nabla)[\iota'\zeta + D]$$

3D modulation encapsulated in the quantity D

- The integrated local magnetic shear:

$$\Lambda = \frac{|\nabla\psi|^2}{B}\int_{\eta_k}^{\eta} d\eta\left[\iota' + \frac{\partial}{\partial\eta}D\right]$$

The integrated local shear is strongly modulated.



Resonant, helical Pfirsch-Schlüter currents modulate the shear.

- D represents the local variation of the integrated magnetic shear, and is governed by the following magnetic differential equation:

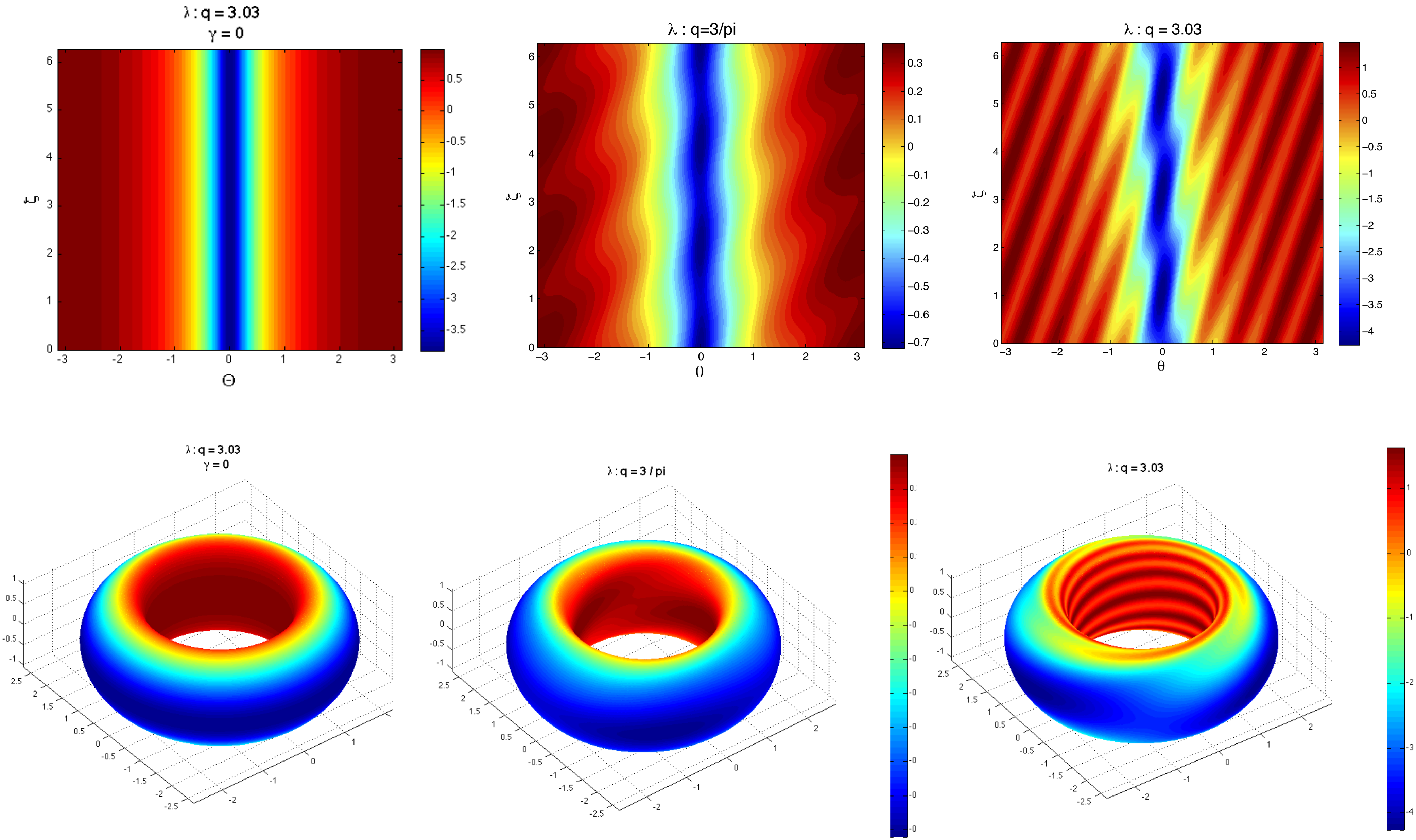
$$(\vec{B}\cdot\nabla)D = \iota'\left(\frac{B^2}{|\nabla\psi|^2}\frac{1}{\left\langle\frac{B^2}{|\nabla\psi|^2}\right\rangle} - \frac{1}{\sqrt{g}}\right) + p'\left(\frac{B^2}{|\nabla\psi|^2}\lambda - \frac{B^2}{|\nabla\psi|^2}\frac{\left\langle\frac{B^2}{|\nabla\psi|^2}\lambda\right\rangle}{\left\langle\frac{B^2}{|\nabla\psi|^2}\right\rangle}\right) + 2\left(\frac{B^2}{|\nabla\psi|^2}\frac{\left\langle\frac{B^2}{|\nabla\psi|^2}\tau_n\right\rangle}{\left\langle\frac{B^2}{|\nabla\psi|^2}\right\rangle} - \frac{B^2}{|\nabla\psi|^2}\tau_n\right)$$

- Another magnetic differential equation governs the Pfirsch-Schlüter currents:

$$(\vec{B}\cdot\nabla)\lambda = 2\mu_0\kappa_g\frac{|\nabla\psi|}{B} \qquad \lambda_{mn} \sim \frac{(RHS)_{mn}}{q - m/n}$$

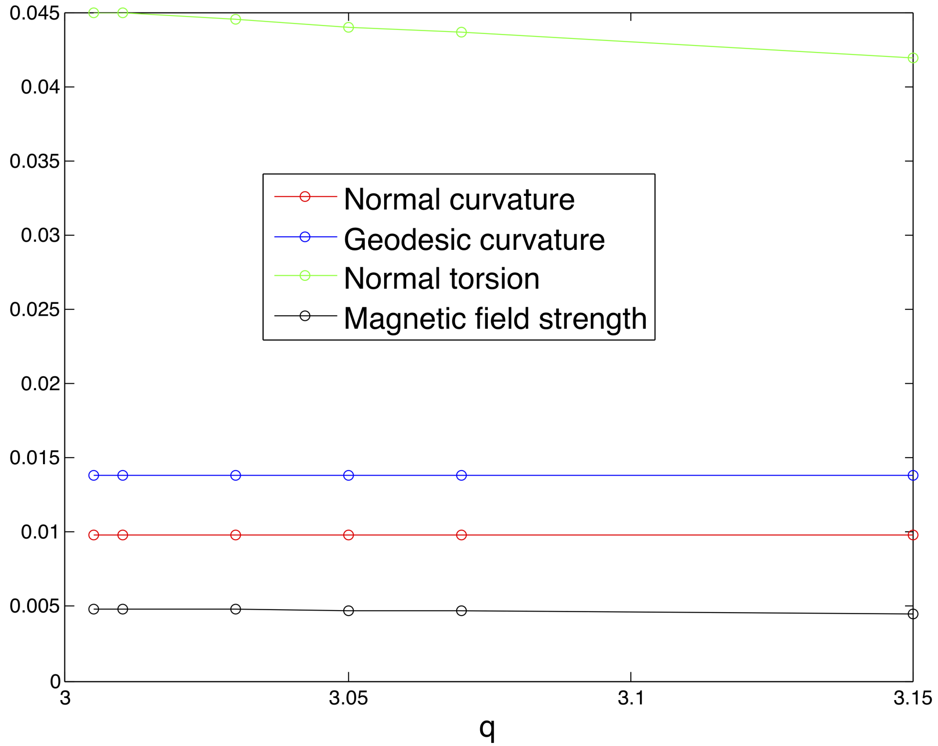
When the right hand side has non-zero 3D components, this can drive resonant Pfirsch-Schlüter currents.

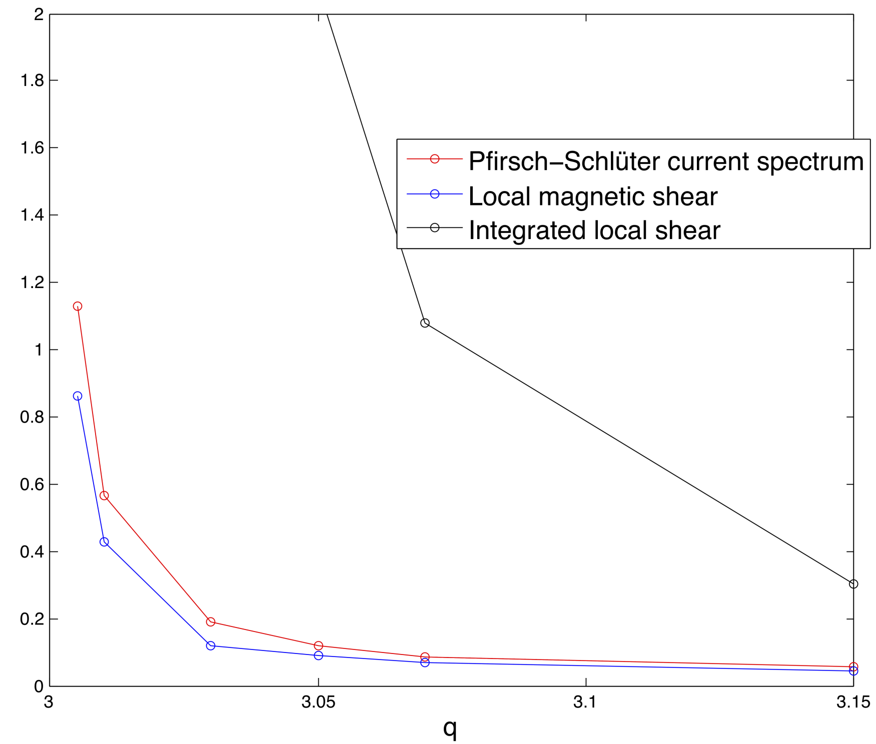
The Pfirsch-Schlüter currents are also quite helical.



How sensitive is this effect to the q value?

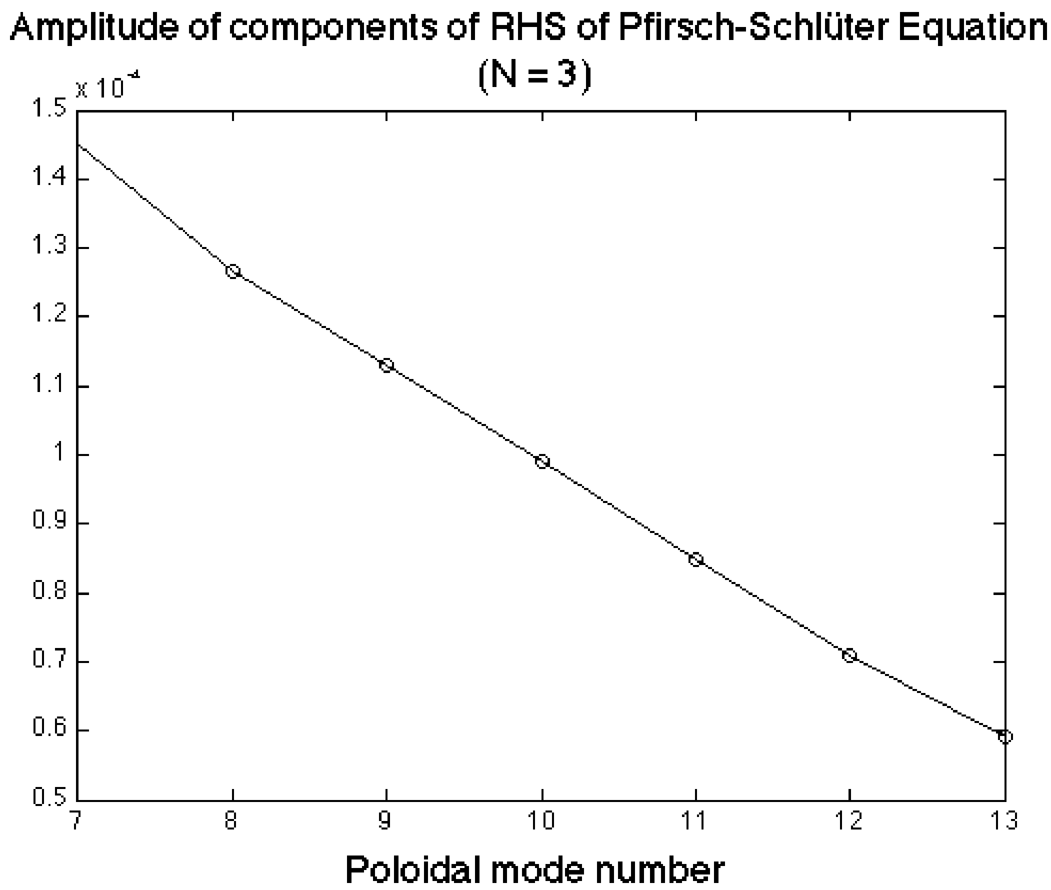
Relative change between quantities from 2D -> 3D





This effect is strongest near low order rational surfaces.

$$(\vec{B} \cdot \nabla)\lambda = 2\mu_0 \kappa_g \frac{|\nabla\psi|}{B}$$



- Onset of KBMs at lower pressure gradient could halt the inward progress of the pedestal at a low order rational surface.

There are two aspects which require further study.

- Equilibrium:

  - What happens at the rational surface?

  - How much shielding / amplification occurs?

- KBM physics:

  - How well does ideal MHD ballooning predict KBM onset in 3D?

  - What happens with more realistic shaping?

  - How does turbulence change from 2D → 3D?

  - What is the nature of the particle and heat transport? Can the density pump-out be explained?

What happens at the rational surface?

- "Kinetic shielding of magnetic islands in 3D" C.C. Hegna, PPCF 2011
- Kinetic shielding of helical Pfirsch-Schlüter currents driving the island (generally weak.)
- 3D deformation of B → net bounce-averaged radial drifts → additional currents within the island that inhibit island growth

  (low collisionality, ~nu regime)
- Island width given by:

$$|q_o' w|^2 = \sum_{mn} C_{mn}\delta_{mn}(1 - \underset{\uparrow}{\mathcal{R}_{mn}}) - C_{m_o n_o} w \epsilon' \underset{\uparrow}{\mathcal{R}_w},$$

Small "shielding" response

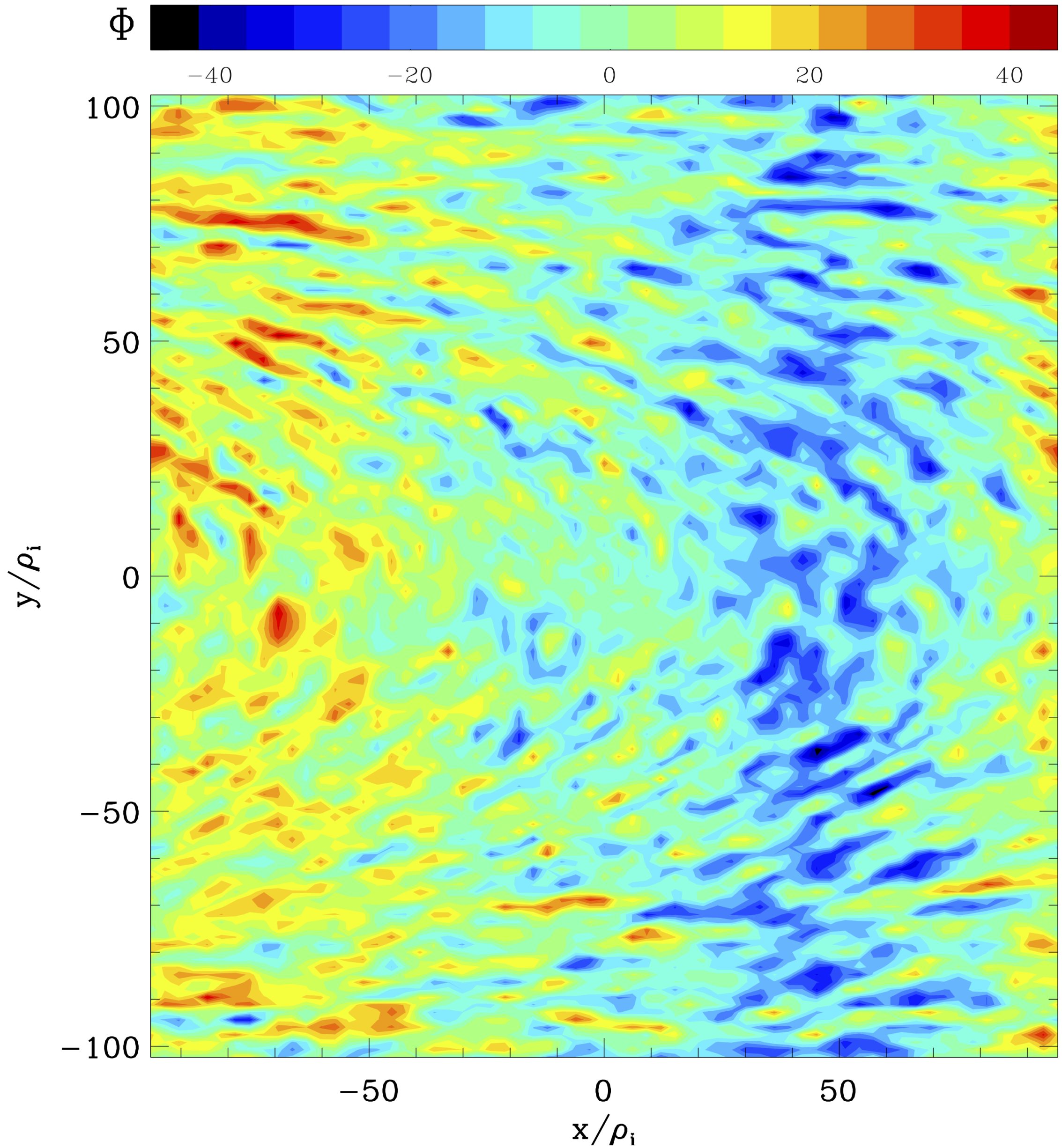
From the island-producing currents

$$w_k = \frac{\sum_{mn} C_{mn}\delta_{mn}}{C_{m_o n_o}\epsilon' \mathcal{R}_w}.$$

- "Tools that employ the MHD model to describe breakup of magnetic surfaces in 3D equilibria may be unduly pessimistic. High-temperature stellarators are more resilient to flux surface integrity than theoretical predictions using conventional MHD models would imply."

We will examine 3D KBMs with a new version of GENE.

- "Poloidally global" version of GENE
- Essentially the same code as the radially global version (i.e. T. Görler et al., JCP 2011)
- "Heavy lifting" done by F. Merz.
- Many thanks to D. Told, T. Görler as well.



Nonlinear ITG-ae in W7-X

In summary...

- The 3D local equilibrium model provides a clever way to study the effect of 3D perturbations when rotational screening is operative.
- 3D inhomogeneity in the magnetic field spectrum is a source for resonant, helical Pfirsch-Schlüter currents.
- The ideal MHD ballooning stability boundary is lowered near rational surfaces, in particular low order surfaces.
- This could provide a mechanism for explaining the effect of RMP fields on plasma profiles when rotational screening precludes stochasticity.