Advances in the study of intrinsic rotation with flux tube gyrokinetics

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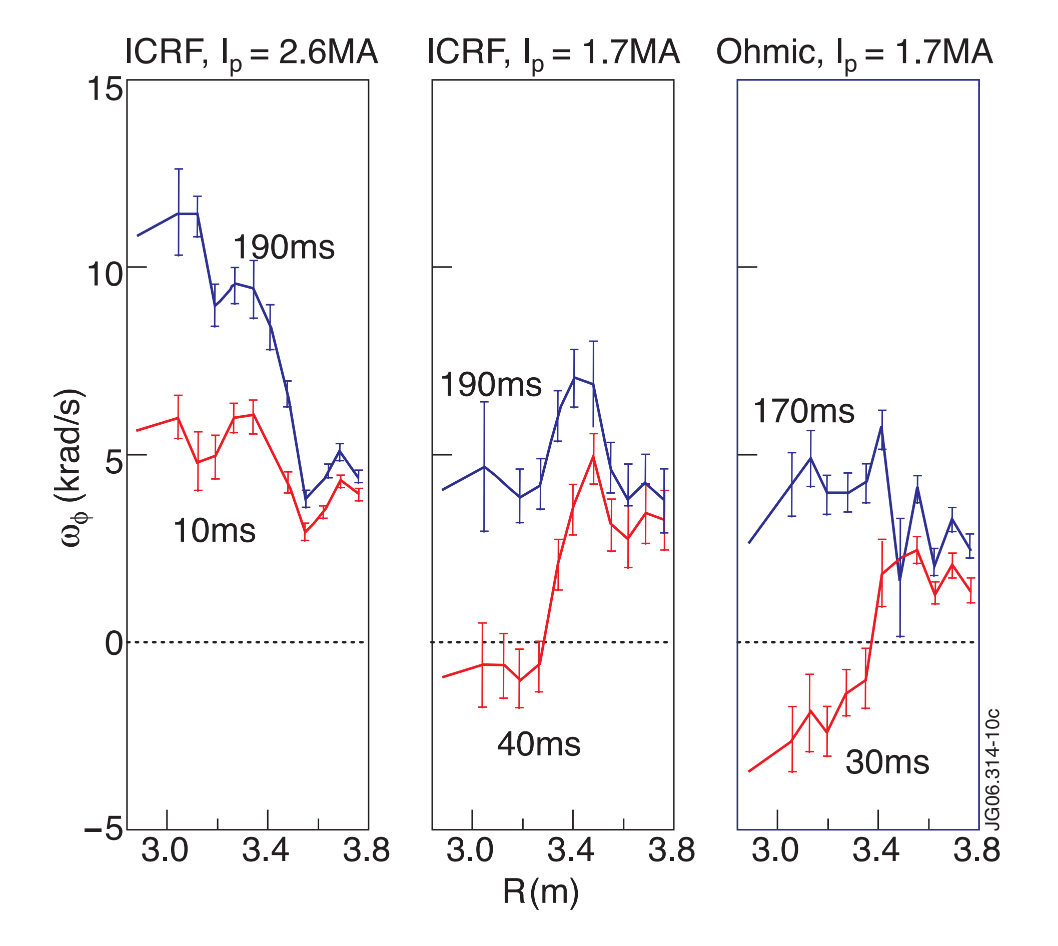
Introduction

- In the absence of obvious momentum input (apart from the edge), tokamak plasmas rotate
- Last year, presented low flow terms that can explain intrinsic rotation
- This talk:
  - Show that low flow ordering is needed: symmetry of transport of momentum in tokamaks
  - Improved ordering that gives new contributions: new scaling of turbulence with B_θ/B

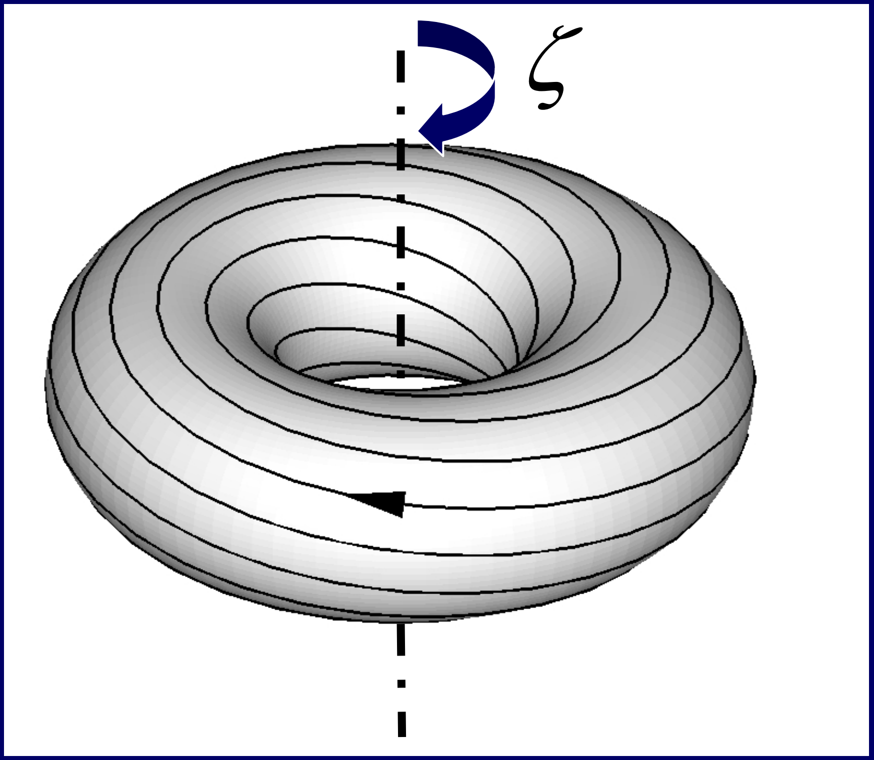
Experimental evidence

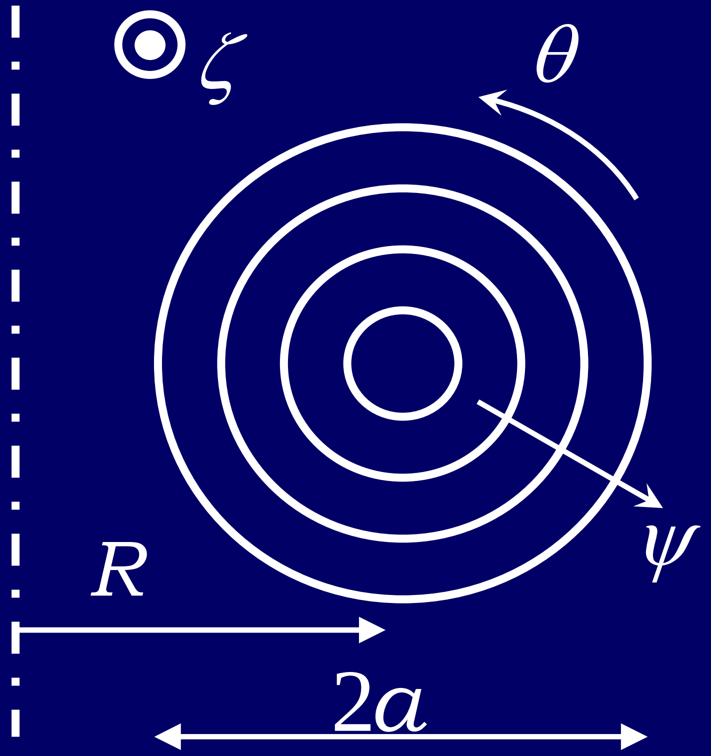


Courtesy of M. F. Nave

- Core Ohmic: hollow counter-rotating
- Core ICRF
  - High I_p: peaked, co-rotating
  - Low I_p: hollow, counter-rotating
- Edge: co-rotating independent of scenario

Geometry





- Here, ζ is in the co-current direction
- Flux surface average $\langle \ldots \rangle_\psi = (V')^{-1} \int d\theta d\zeta (\ldots)/\mathbf{B}\cdot\nabla\theta$

Intrinsic rotation

- Assume electrostatics to simplify: $\mathbf{E} = -\nabla\phi$

- Conservation of total toroidal angular momentum
  - $\mathbf{J}\times\mathbf{B}$ force vanishes due to axisymmetry and $n_i = n_e$

$$\frac{\partial}{\partial t}\left\langle n_i M R \mathbf{V}_i \cdot \hat{\zeta} \right\rangle_\psi = -\frac{1}{V'}\frac{\partial}{\partial \psi}\left(V'\,\Pi\right) + \cancel{\frac{1}{c}\left\langle R\hat{\zeta}\cdot\left(\mathbf{J}\times\mathbf{B}\right)\right\rangle_\psi}$$

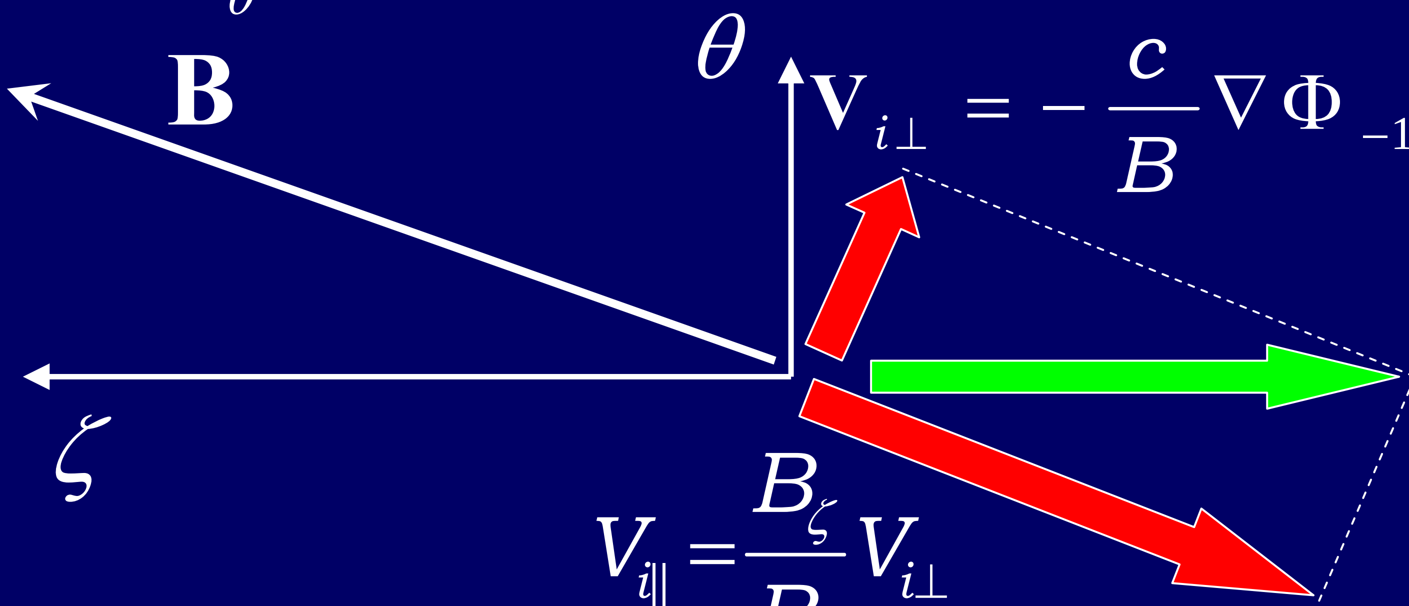
  - Here $\Pi = \left\langle \int d^3 v\, f_i\, RM \left(\mathbf{v}\cdot\hat{\zeta}\right)\left(\mathbf{v}\cdot\nabla\psi\right)\right\rangle_\psi$

- $\partial/\partial t = 0$, no momentum input $\Rightarrow \boxed{\Pi = 0}$

High flow ordering

High flow ordering

- To make $\mathbf{v}_\theta = 0$



$$\mathbf{V}_{i\perp} = -\frac{c}{B}\nabla\Phi_{-1}\times\hat{\mathbf{b}}$$

$$V_{i\|} = \frac{B_\zeta}{B_\theta} V_{i\perp}$$

$$\boxed{\mathbf{V}_i = -cR\hat{\boldsymbol{\zeta}}\frac{\partial\Phi_{-1}}{\partial\psi} \equiv \Omega_\zeta R\hat{\boldsymbol{\zeta}} \sim \upsilon_{ti}}$$

δf simulations

- Since fluctuation << 1, use $f_i = F_i + f_{i1}{}^{tb}$, $\phi = \phi_0 + \phi_1{}^{tb}$
- Variables $\varepsilon = \dfrac{\upsilon^2}{2} + \dfrac{e\phi^{tb}}{M} + \dfrac{e\Phi_0}{M} - \dfrac{R^2\Omega_\zeta^2}{2}, \mu = \dfrac{\upsilon_\perp^2}{2B}$

$$\frac{Df_{i1}^{\,tb}}{Dt} + \left(\upsilon_\parallel \hat{\mathbf{b}} + \mathbf{v}_M + \mathbf{v}_{E0} + \mathbf{v}_{co} + \mathbf{v}_{cf} + \mathbf{v}_{E1}^{tb}\right)\cdot \nabla_{\mathbf{R}} f_{i1}^{\,tb} - \left\langle C^{(\ell)}\left\{f_{i1}^{\,tb}\right\}\right\rangle$$

$$= -\mathbf{v}_{E1}^{tb} \cdot \nabla_{\mathbf{R}} f_{Mi} - \left[\frac{e}{M}\frac{\partial\left\langle \phi_1^{tb}\right\rangle}{\partial t} - \left(\frac{I\upsilon_\parallel}{B} + R^2\Omega_\zeta\right)\mathbf{v}_{E1}^{tb} \cdot \nabla_{\mathbf{R}}\Omega_\zeta\right]\frac{\partial f_{Mi}}{\partial \varepsilon}$$

  - Similar equation for electrons
- Potential found from $\int d^3\upsilon\, f_{i1}{}^{tb} - \int d^3\upsilon\, f_{e1}{}^{tb} + \ldots = 0$

Momentum transport for high flows

- Turbulence with $k_\perp \rho_i \sim 1$, equilibrium with $k_\perp \rho_i << 1$

$\Rightarrow \nabla_{\mathbf{R}} f_{Mi}$, $\partial f_{Mi}/\partial \varepsilon$ = constant in computational domain

$\Rightarrow$ Fourier analyze f_{i1}^{tb}, f_{e1}^{tb} and ϕ_1^{tb} in $\mathbf{R}_\perp$

- Momentum transport

$$\Pi = -\left\langle \frac{c}{B}\left(\nabla \phi_1^{tb} \times \hat{\mathbf{b}}\right) \cdot \nabla \psi \int d^3 v\, f_{i1}^{tb} R M \left(\mathbf{v} \cdot \hat{\zeta}\right) \right\rangle_\psi$$

$$= \Pi\left(\Omega_\zeta, \frac{\partial \Omega_\zeta}{\partial \psi}; \quad \frac{\partial T_i}{\partial \psi}, \frac{\partial n_e}{\partial \psi}, \frac{\partial T_i}{\partial \psi}, \ldots \right)$$

$$\approx P_\zeta \Omega_\zeta - \chi_\zeta \frac{\partial \Omega_\zeta}{\partial \psi} + \Pi_{ud}\left(\frac{\partial T_i}{\partial \psi}, \frac{\partial n_e}{\partial \psi}, \frac{\partial T_i}{\partial \psi} \right)$$

Symmetry of momentum transport

- In up-down symmetric tokamak

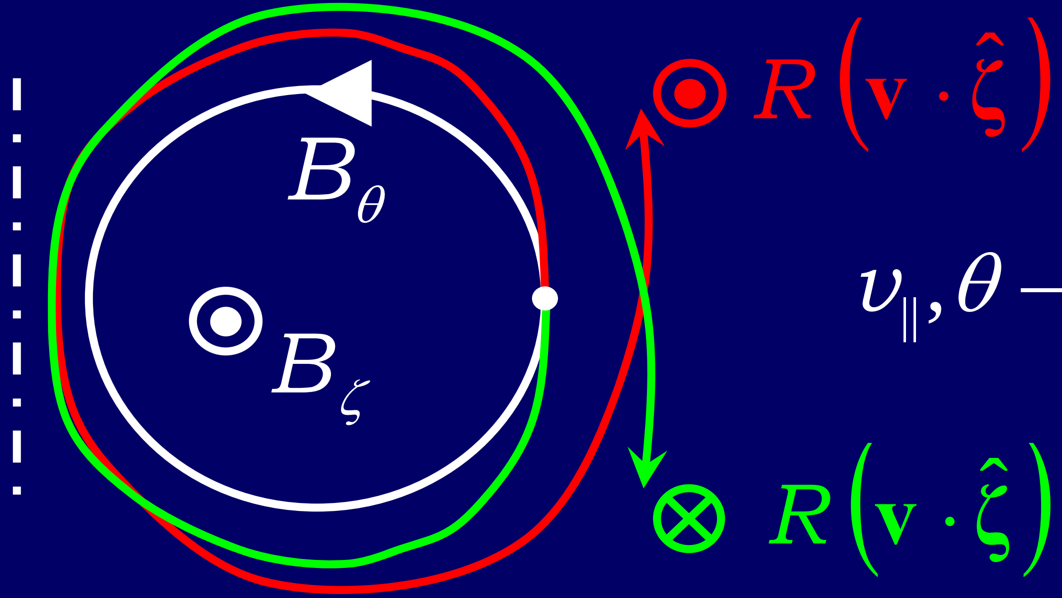
$$\Omega_\zeta, \frac{\partial \Omega_\zeta}{\partial \psi}, \theta, k_\psi, v_\parallel \rightarrow -\Omega_\zeta, -\frac{\partial \Omega_\zeta}{\partial \psi}, -\theta, -k_\psi, -v_\parallel$$

$$f_{i1}^{tb}, f_{e1}^{tb}, \phi_1^{tb} \rightarrow -f_{i1}^{tb}, -f_{e1}^{tb}, -\phi_1^{tb}$$

- Thus, $\Omega_\zeta, \frac{\partial \Omega_\zeta}{\partial \psi} \rightarrow -\Omega_\zeta, -\frac{\partial \Omega_\zeta}{\partial \psi} \Rightarrow \Pi \rightarrow -\Pi$
- In addition, $\Omega_\zeta = 0 = \frac{\partial \Omega_\zeta}{\partial \psi} \Rightarrow \Pi = 0$
- Hence, for subsonic $\Pi \approx P_\zeta \Omega_\zeta - \chi_\zeta \frac{\partial \Omega_\zeta}{\partial \psi}$

Physical interpretation

- Parallel motion of particles



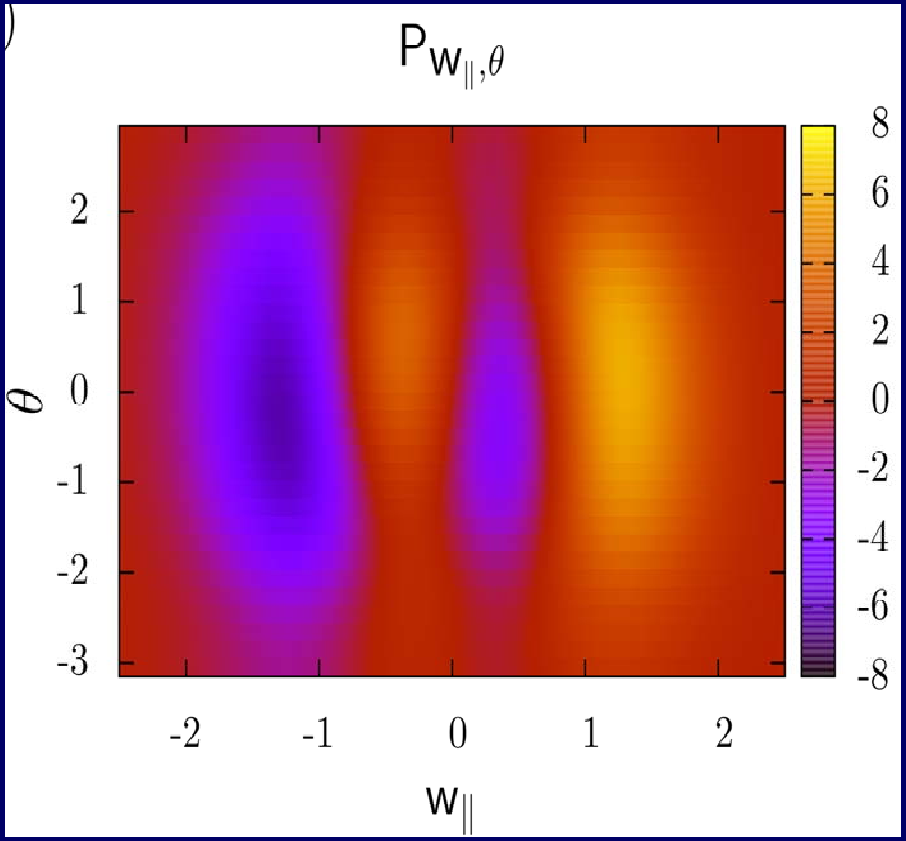
$$v_\parallel,\theta \rightarrow -v_\parallel,-\theta \Rightarrow \Pi = 0$$

- Similar picture for perpendicular diamagnetic drift

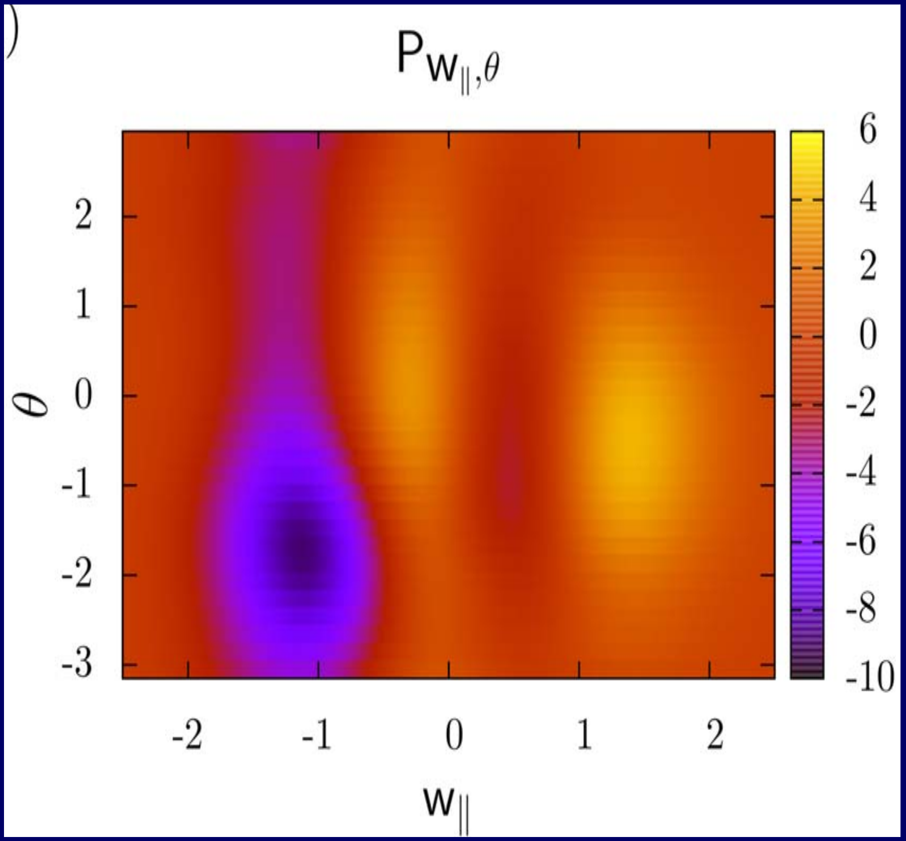
Changes sign due to $k_\psi \rightarrow -k_\psi$

Numerical evidence (I)

$$\frac{\partial \Omega_\zeta}{\partial \psi} = 0$$

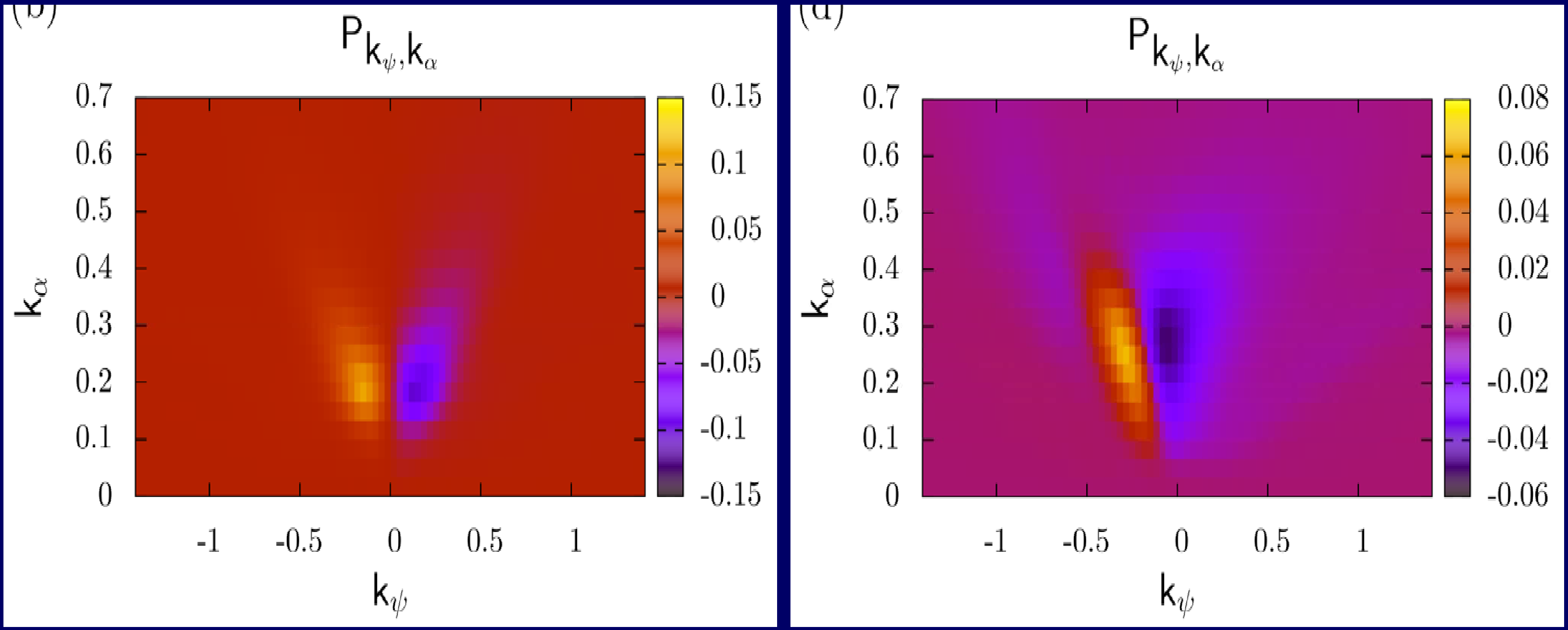


$$\frac{\partial \Omega_\zeta}{\partial \psi} = 0.2$$



Numerical evidence (II)

$$\frac{\partial \Omega_\zeta}{\partial \psi} = 0 \qquad\qquad \frac{\partial \Omega_\zeta}{\partial \psi} = 0.2$$



Intrinsic rotation with high flow

- In the high flow ordering, only up-down asymmetry can produce intrinsic rotation with enough structure

$$P_\zeta \Omega_\zeta - \chi_\zeta \frac{\partial \Omega_\zeta}{\partial \psi} = 0 \Rightarrow \Omega_\zeta \propto \exp\left(\int \frac{P_\zeta}{\chi_\zeta} d\psi \right)$$

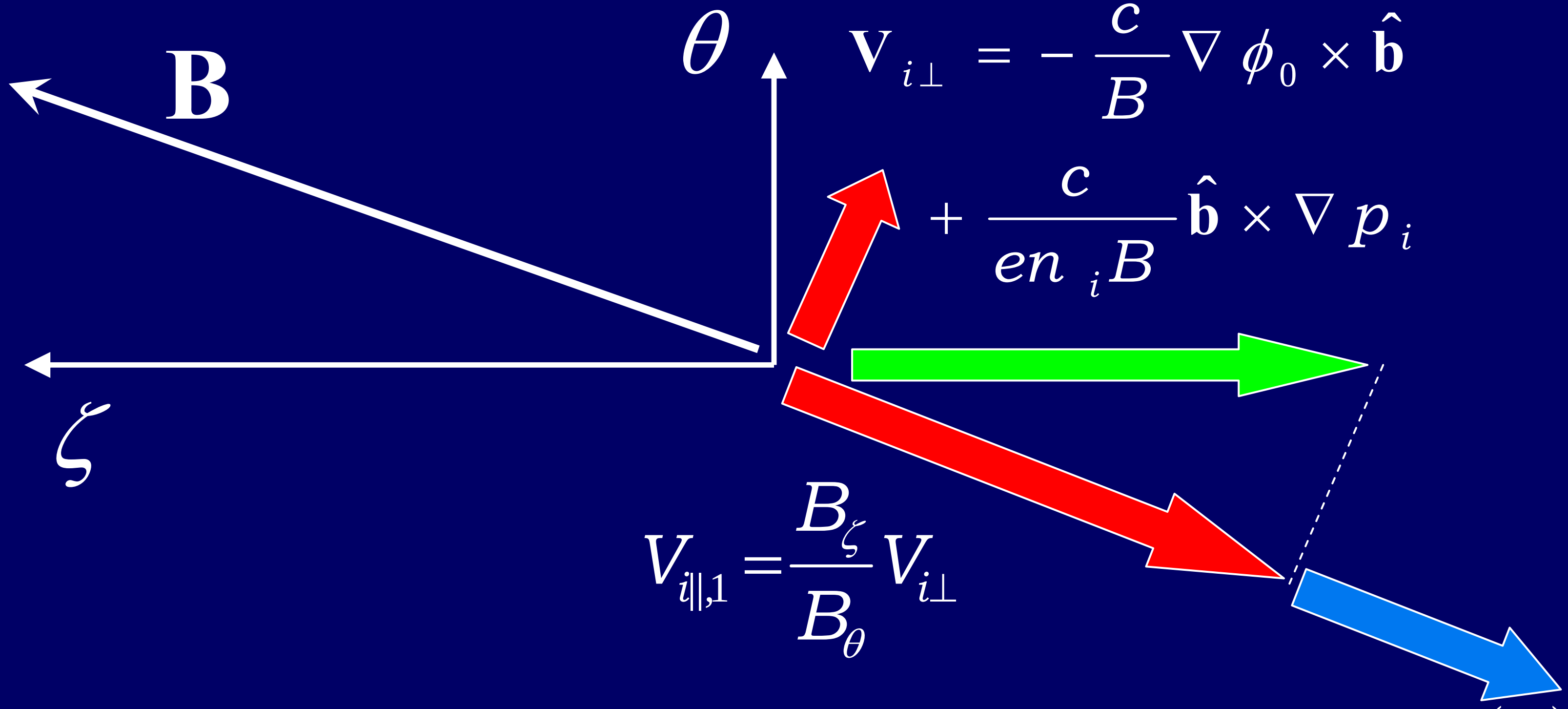
- Up-down asymmetry only strong near edge

- Up-down asymmetry seems unable to explain sign changes in rotation and dependences with heating
  - Still active area of research

- Need to continue to next order

Low flow ordering

Low flow ordering



$$\mathbf{V}_{i\perp} = -\frac{c}{B}\nabla\phi_0 \times \hat{\mathbf{b}} + \frac{c}{en_iB}\hat{\mathbf{b}} \times \nabla p_i$$

$$V_{i\|,1} = \frac{B_\zeta}{B_\theta} V_{i\perp}$$

$$V_{i\|,2} = BK(\psi) \propto \frac{\partial T_i}{\partial \psi}$$

$$\boxed{\mathbf{V}_i = \Omega_\zeta R\hat{\boldsymbol{\zeta}} + \mathbf{B}K(\psi) \sim \frac{\rho_i}{a} v_{ti}}$$

Changes in momentum flux

- Need to calculate Π to an order higher in $\delta_i = \rho_i / a$

$$\Pi \sim D_{gB} \frac{\partial}{\partial r}(n_i MR \underbrace{V_i}_{\delta_i v_{ti}}) \left| \nabla \psi \right| << \Pi \text{ at high flow}$$

- Two different contributions
  - New terms in the expression for Π
  - New terms in the gyrokinetic equation

Turbulent momentum transport

- Using moments of the full Fokker-Planck equation and ignoring neoclassical transport of momentum

$$\Pi = -M\left\langle \frac{c}{B}\left(\nabla\phi^{tb}\times\hat{\mathbf{b}}\right)\cdot\nabla\psi\int d^3v\, f_i^{tb} R\left(\mathbf{v}\cdot\hat{\boldsymbol{\zeta}}\right)\right\rangle_\psi$$

$$\boxed{\begin{aligned}&+\frac{Mc}{2e}\left\langle R^2\right\rangle_\psi \frac{\partial p_i}{\partial t} + M\left\langle \frac{cI}{B}\hat{\mathbf{b}}\cdot\nabla\phi^{tb}\int d^3v\, f_i^{tb} R\left(\mathbf{v}\cdot\hat{\boldsymbol{\zeta}}\right)\right\rangle_\psi \\ &-\frac{M^2c}{2e}\frac{1}{V'}\frac{\partial}{\partial\psi}V'\left\langle \frac{c}{B}\left(\nabla\phi^{tb}\times\hat{\mathbf{b}}\right)\cdot\nabla\psi\int d^3v\, f_i^{tb} R^2\left(\mathbf{v}\cdot\hat{\boldsymbol{\zeta}}\right)^2\right\rangle_\psi \\ &-\frac{M^2c}{2e}\left\langle \int d^3v\, C_{ii}^{(\ell)}\left\{F_{i2}^{tb}\right\} R^2\left(\mathbf{v}\cdot\hat{\boldsymbol{\zeta}}\right)^2\right\rangle_\psi\end{aligned}}$$

New terms

$$-\frac{M^2c}{2e}\frac{1}{V'}\frac{\partial}{\partial\psi}V'\left\langle\frac{c}{B}\left(\nabla\phi^{tb}\times\hat{\mathbf{b}}\right)\cdot\nabla\psi\int d^3v\, f_i^{tb}R^2\left(\mathbf{v}\cdot\hat{\zeta}\right)^2\right\rangle_\psi$$

$$+\frac{Mc}{2e}\left\langle R^2\right\rangle_\psi\frac{\partial p_i}{\partial t}+\ldots\sim\frac{d}{dt}\left[R^2\left(\mathbf{v}\cdot\hat{\zeta}\right)^2\right]$$

- Caused by changes in width of drift orbits

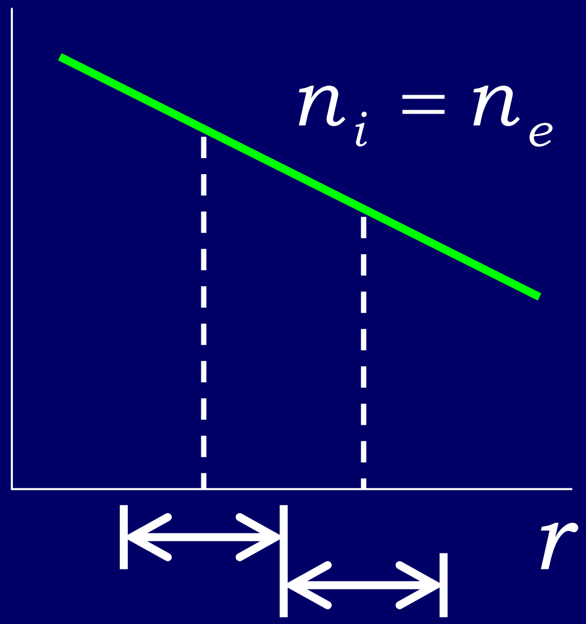
$\psi^* = \psi - \frac{Mc}{e}R\left(\mathbf{v}\cdot\hat{\zeta}\right) =$ const. gives drift orbit width

$\Rightarrow$ increase in $R^2\left(\mathbf{v}\cdot\hat{\zeta}\right)^2$ widens drift orbits

$\Rightarrow$ modifies $n_i = n_e \Rightarrow$ new Ω_ζ

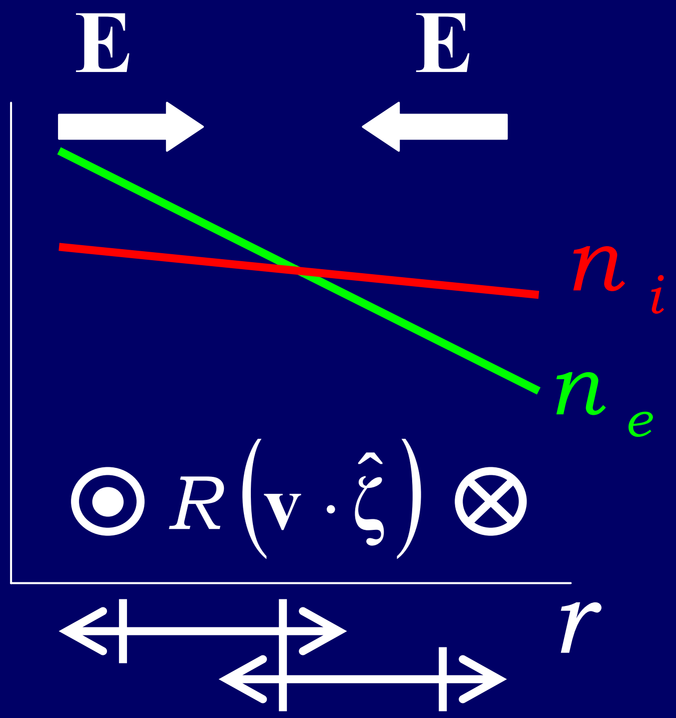
Width of drift orbits

- For increasing $R^2\left(\mathbf{v}\cdot\hat{\zeta}\right)^2$



$$\Rightarrow$$

$$R^2\left(\mathbf{v}\cdot\hat{\zeta}\right)^2 \uparrow$$



GK polarization $\Rightarrow \mathbf{E} \Rightarrow$ neo polarization $\Rightarrow \Delta\Omega_\zeta$

Gyrokinetic equation

- To lowest order, due to symmetry

$$-M\left\langle \frac{c}{B}\left(\nabla\phi_1^{tb}\times\hat{\mathbf{b}}\right)\cdot\nabla\psi\int d^3v\, f_{i1}^{tb} R\left(\mathbf{v}\cdot\hat{\zeta}\right)\right\rangle_\psi = 0$$

- To next order

$$-M\left\langle \frac{c}{B}\left(\nabla\phi_1^{tb}\times\hat{\mathbf{b}}\right)\cdot\nabla\psi\int d^3v\, f_{i2}^{tb} R\left(\mathbf{v}\cdot\hat{\zeta}\right)\right.$$

$$\left. + \frac{c}{B}\left(\nabla\phi_2^{tb}\times\hat{\mathbf{b}}\right)\cdot\nabla\psi\int d^3v\, f_{i1}^{tb} R\left(\mathbf{v}\cdot\hat{\zeta}\right)\right\rangle_\psi$$

- Problematic because we need f_{i2}^{tb}, f_{e2}^{tb}, ϕ_2^{tb}! Need higher order terms in gyrokinetic equation

Higher order gyrokinetic equation

- Flux tube equation to second order

$$
\begin{aligned}
&\frac{\partial f_{i2}^{tb}}{\partial t} + v_{\|}\hat{\mathbf{b}}\cdot\nabla\theta\frac{\partial f_{i2}^{tb}}{\partial\theta} + \left(\mathbf{v}_M + \mathbf{v}_{E1}^{tb}\right)\cdot\nabla_{\mathbf{R}\perp} f_{i2}^{tb} + \mathbf{v}_{E2}^{tb}\cdot\nabla_{\mathbf{R}\perp} f_{i1}^{tb} - \left\langle C^{(\ell)}\left\{f_{i2}^{tb}\right\}\right\rangle \\
&\quad + \mathbf{v}_M\cdot\nabla\theta\frac{\partial f_{i1}^{tb}}{\partial\theta} - \frac{c}{B}\frac{\partial\left\langle\phi_1^{tb}\right\rangle}{\partial\theta}\left(\nabla\theta\times\hat{\mathbf{b}}\right)\cdot\nabla_{\mathbf{R}} f_{Mi} + \frac{ef_{Mi}}{T_i}\mathbf{v}_M\cdot\nabla\theta\frac{\partial\left\langle\phi_1^{tb}\right\rangle}{\partial\theta} \\
&\quad - \frac{c}{B}\frac{\partial\left\langle\phi_1^{tb}\right\rangle}{\partial\theta}\left(\nabla\theta\times\hat{\mathbf{b}}\right)\cdot\nabla_{\mathbf{R}} f_{i1}^{tb} - \frac{c}{B}\frac{\partial f_{i1}^{tb}}{\partial\theta}\left(\nabla_{\mathbf{R}}\left\langle\phi_1^{tb}\right\rangle\times\hat{\mathbf{b}}\right)\cdot\nabla\theta \\
&\quad + \frac{e}{M}\frac{\partial\left\langle\phi_1^{tb}\right\rangle}{\partial t}\frac{\partial f_{i1}^{tb}}{\partial E} + \mathbf{v}_{E1}^{tb}\cdot\nabla_{\mathbf{R}} F_{i1}^{nc} + \frac{e}{M}\frac{\partial\left\langle\phi_1^{tb}\right\rangle}{\partial t}\frac{\partial F_{i1}^{nc}}{\partial E} \\
&\quad + \text{radial profile variation} \\
&+ \dot{\mathbf{R}}^{(2)}\cdot\nabla_{\mathbf{R}\perp} f_{i1}^{tb} + \dot{E}^{(2)}\frac{\partial f_{i1}^{tb}}{\partial E} - \left\langle C\left\{f_i\right\}\right\rangle^{(2)} + \dot{\mathbf{R}}^{(2)}\cdot\nabla_{\mathbf{R}\perp} f_{Mi} + \dot{E}^{(2)}\frac{\partial f_{Mi}}{\partial E} = 0
\end{aligned}
$$

Higher order gyrokinetic equation

- Flux tube equation to second order
  - Neoclassical equilibrium effect on turbulence
    - Flow and heat flow within flux surface
  - Slowly varying envelope of the turbulence in the poloidal direction
  - Parallel nonlinearity
  - Radial profile variation
  - Higher order corrections to gyrokinetic equation

First proposed simplification

- For $B_\theta/B << 1$ & turbulence NOT scaling with B_θ/B

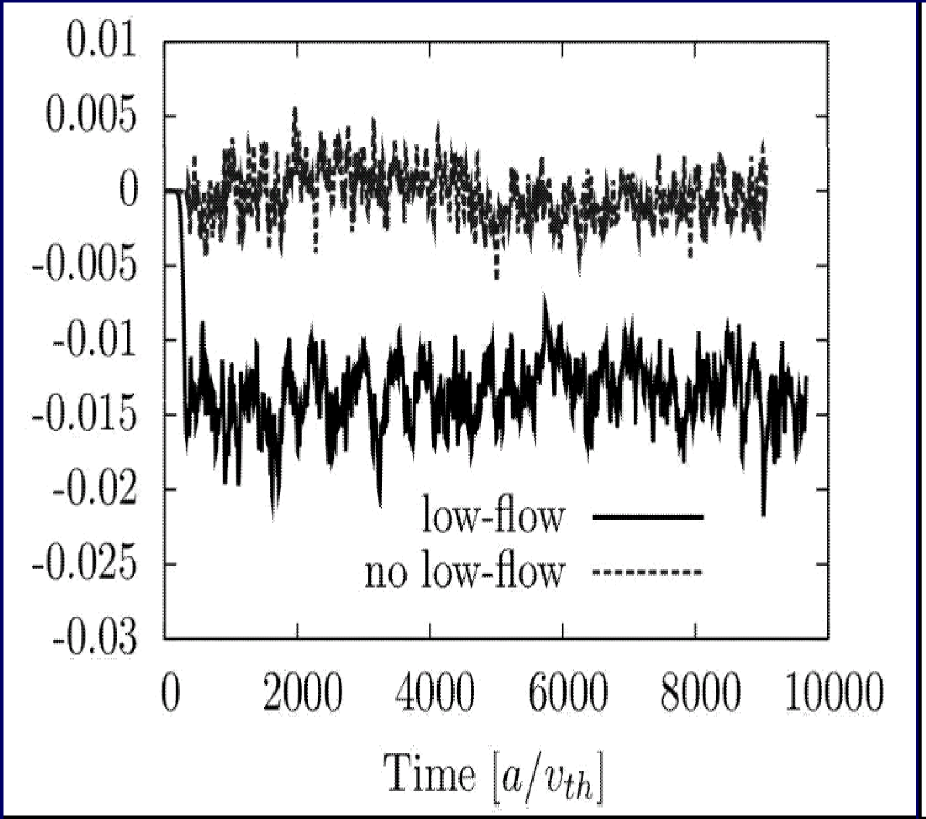
$$\frac{\partial f_{i2}^{\text{tb}}}{\partial t} + \left(v_\parallel \hat{\mathbf{b}} + \mathbf{v}_M + \mathbf{v}_{E1}^{\text{tb}}\right)\cdot \nabla_{\mathbf{R}} f_{i2}^{\text{tb}} - \left\langle C^{(\ell)}\left\{f_{i2}^{\text{tb}}\right\}\right\rangle = -\mathbf{v}_{E1}^{\text{tb}} \cdot \nabla_{\mathbf{R}} F_{i1}^{\text{nc}} + \ldots$$

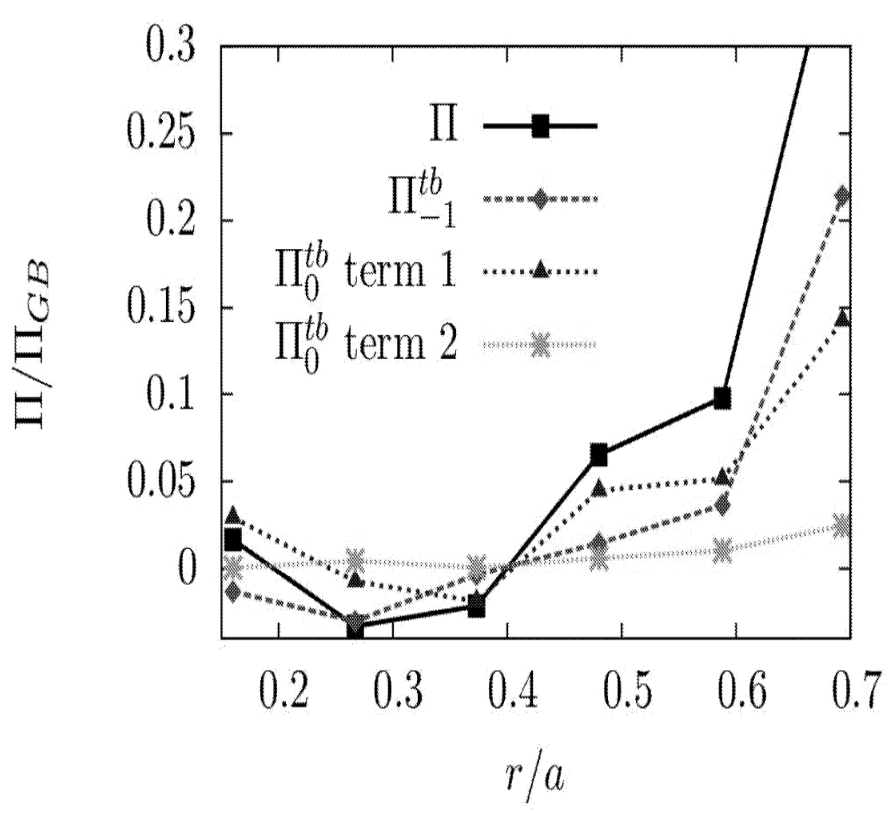
  - Neoclassical parallel flows and heat flows larger than turbulent effects by $\rho_{pi}/\rho_i \sim B/B_\theta >> 1$

- Using $f_{i2}^{\text{tb}} \approx -\int^t d\tau\, \mathbf{v}_{E1} \cdot \nabla_{\mathbf{R}} F_{i1}^{\text{nc}} \propto \Omega_\zeta, \frac{\partial \Omega_\zeta}{\partial \psi}, \frac{\partial T_i}{\partial \psi}, \frac{\partial^2 T_i}{\partial \psi^2}$

$$\Pi \approx P_\zeta \Omega_\zeta - \chi_\zeta \frac{\partial \Omega_\zeta}{\partial \psi} + K_1 \frac{\partial T_i}{\partial \psi} + L_1 \frac{\partial^2 T_i}{\partial \psi^2}$$

Numerical results





Scaling of turbulence with B_θ / B

- Important terms in gyrokinetic equation

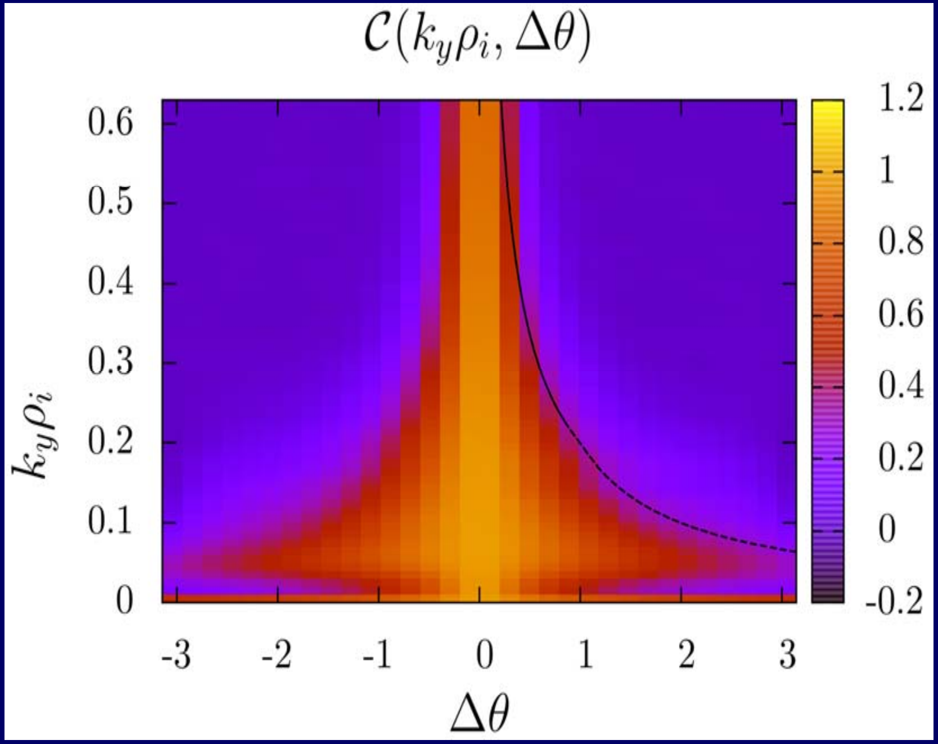
$$\ldots + v_\parallel \hat{\mathbf{b}} \cdot \nabla\theta \frac{\partial f_{i1}^{tb}}{\partial\theta} + \mathbf{v}_{E1}^{tb} \cdot \nabla_{\mathbf{R}} f_{i1}^{tb} + \mathbf{v}_{E1}^{tb} \cdot \nabla_{\mathbf{R}} f_{Mi} = 0$$

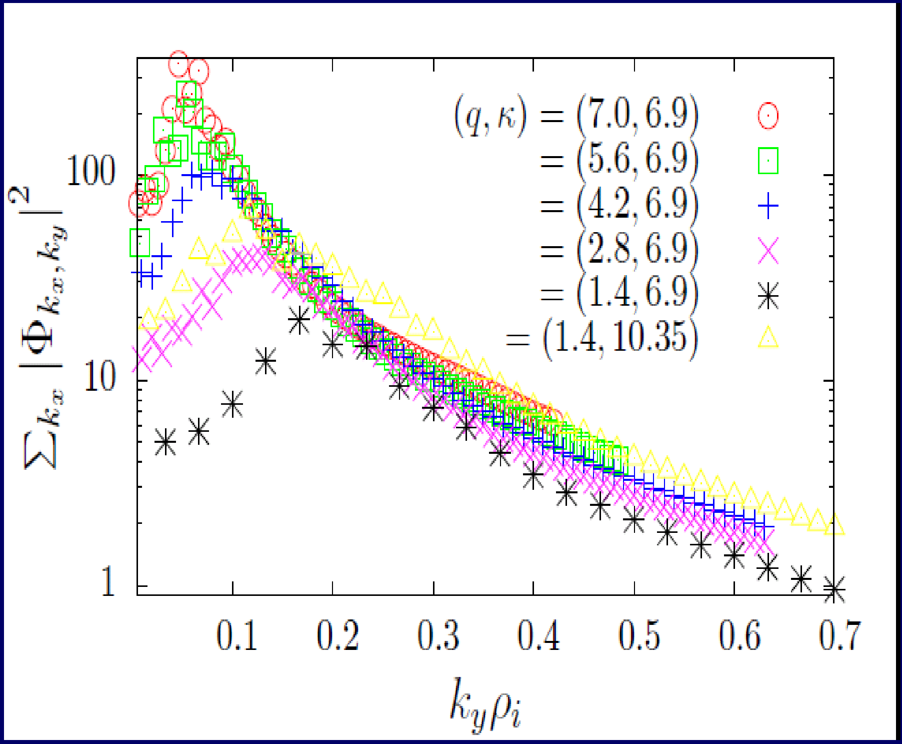
$$\frac{qR\Delta\theta}{a} \sim \frac{\ell_\perp}{\rho_i} \qquad \frac{e\phi_1^{tb}}{T_e} \sim \frac{f_{i1}^{tb}}{f_{Mi}} \sim \frac{f_{e1}^{tb}}{f_{Me}} \sim \frac{\ell_\perp}{a}$$

- Maximum $\Delta\theta \sim 1$

$$\frac{e\phi_1^{tb}}{T_e} \sim \frac{f_{i1}^{tb}}{f_{Mi}} \sim \frac{f_{e1}^{tb}}{f_{Me}} \sim \frac{B}{B_\theta}\frac{\rho_i}{a}, \quad \ell_\perp \sim \frac{B}{B_\theta}\rho_i$$

Numerical scalings





When this B_θ / B scaling works…

- Need to consider
  - Slowly varying envelope of the turbulence in the poloidal direction
  - Parallel nonlinearity
  - Radial profile variation
- Higher order corrections to gyrokinetic equation can be neglected
- Still studying under what circumstances this scaling works

Intrinsic rotation

- Adding all the contributions

$$\Pi \approx P_\zeta \Omega_\zeta - \chi_\zeta \frac{\partial \Omega_\zeta}{\partial \psi} + L\frac{\partial T_i}{\partial \psi} + K\frac{\partial^2 T_i}{\partial \psi^2} + M\frac{\partial n_e}{\partial \psi} + N\frac{\partial^2 n_e}{\partial \psi^2}$$
$$+ R\frac{\partial T_e}{\partial \psi} + Z\frac{\partial^2 T_e}{\partial \psi^2} + \text{heating sources} = 0$$

- In order of magnitude comparable to observations

$$\frac{\partial V_i}{\partial r} \sim \frac{c}{eB_\theta}\frac{\partial^2 T}{\partial r^2}$$

Conclusions

- New self-consistent model for intrinsic rotation that can explain change of sign in rotation
- Intrinsic rotation depends on gradients of density and temperature of both electron and ions, and on the heating source
- Possible sources of intrinsic rotation not studied here are up-down symmetry and ripple