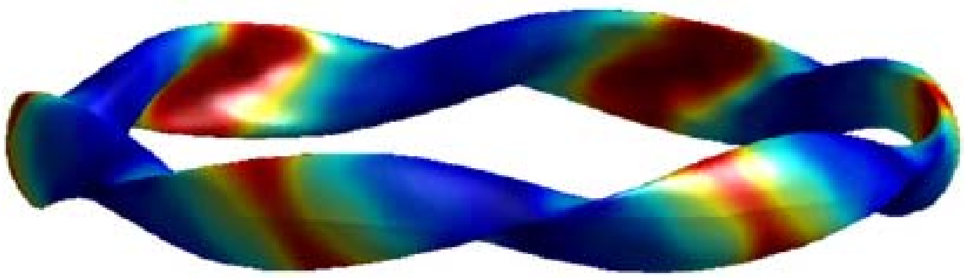
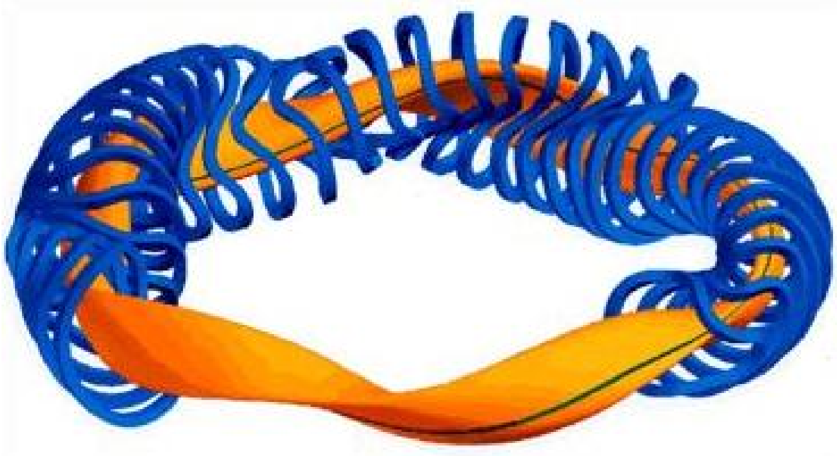
Omnigenous stellarators





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Apr 4, 2011 – Vienna workshop

Outline

1. Introduction to stellarator confinement

- Omnigenity and quasisymmetry
- Currents

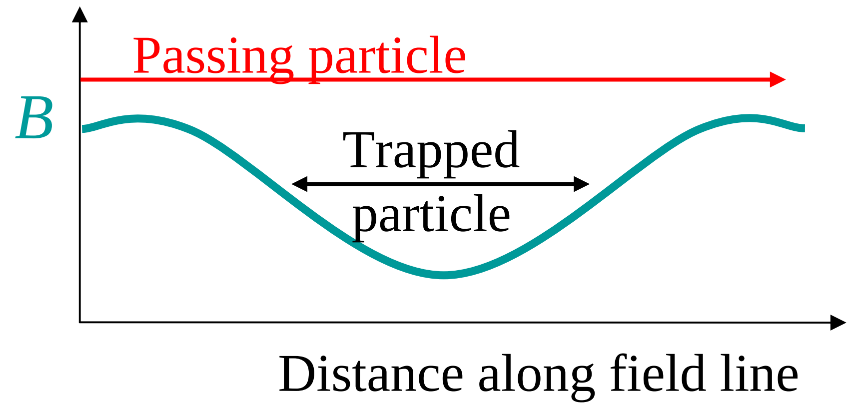
2. Neoclassical theory of omnigenous stellarators

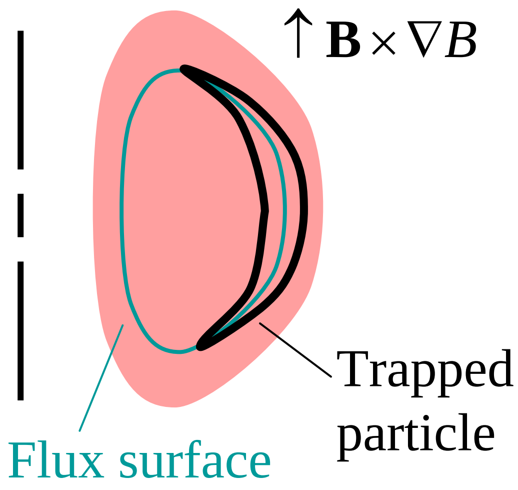
- Geometric properties
- Currents & flows
- Radial electric field

1. Introduction to stellarator confinement

Single-particle confinement

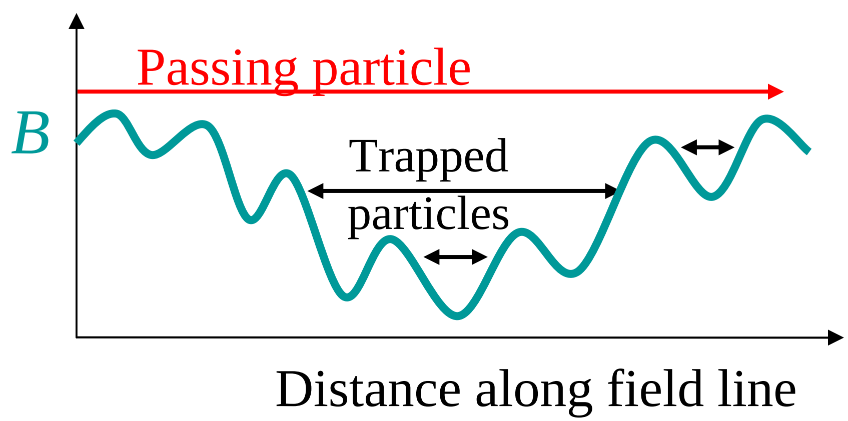
Tokamak

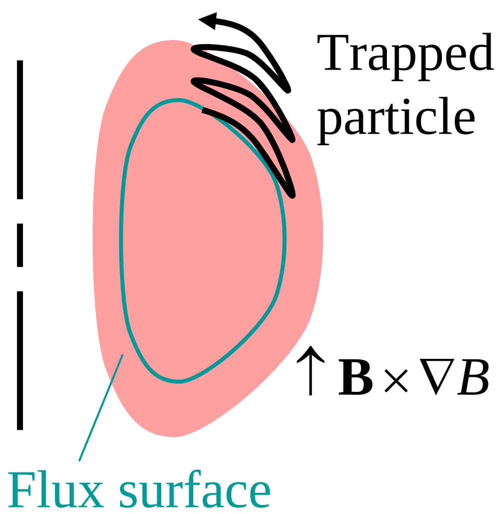




Tamm's theorem: "In the absence of turbulence and collisions, all particles in a tokamak are confined."

Non-optimized stellarator

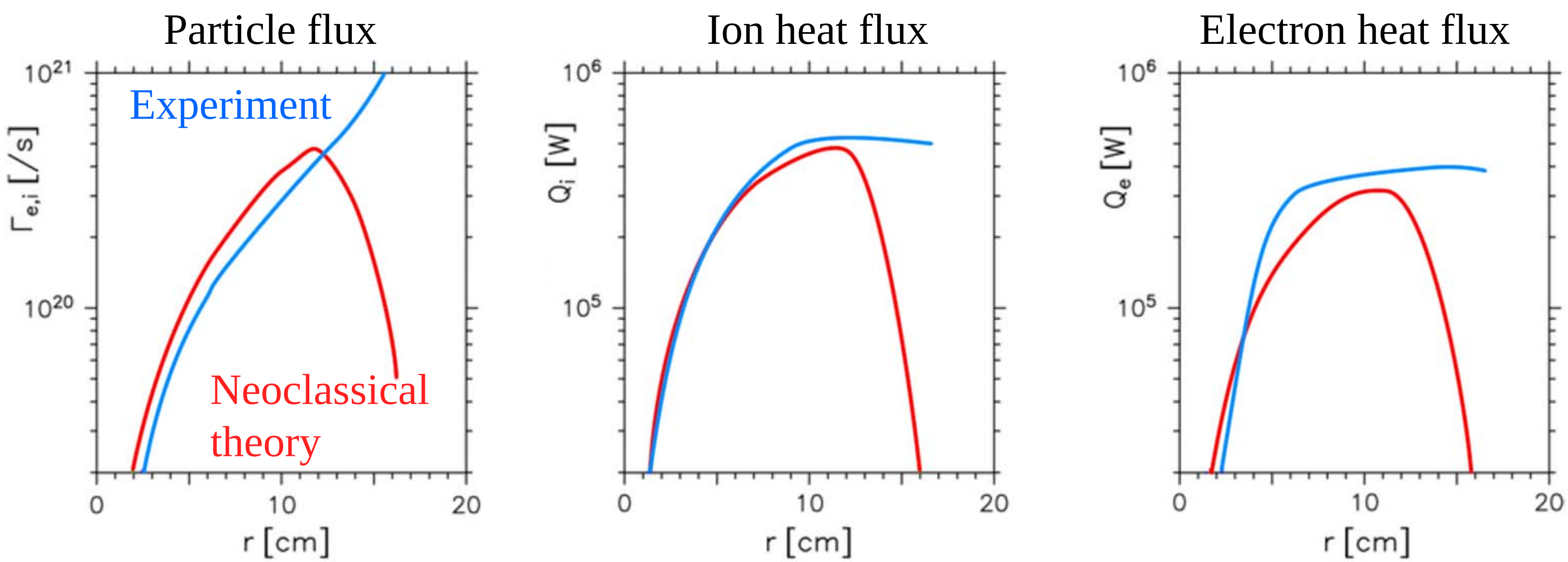




Neoclassical transport matters

• Neoclassical radial transport can dominate turbulent transport in stellarator cores.

e.g. W7-AS stellarator: M. Hirsch, *PPCF* **50**, 053001 (2008)



• Neoclassical radial transport of α particles can cause them to hit wall before thermalizing, damaging wall and reducing self-heating.

• Turbulence should have little impact on neoclassical flows & current.

The ideal stellarator: omnigenity

Definition: A field is “omnigenous” if the radial guiding-center drift averages over a bounce to zero for all trapped particles:

$$0 = \Delta\psi \text{ per bounce} = \oint_{\text{bounce}} \left(\mathbf{v}_d \cdot \nabla\psi\right) dt \; .$$

In a general field it can be shown that $\Delta\psi = \dfrac{mc}{Ze}\dfrac{\partial J}{\partial\alpha}$

where $J = \oint \upsilon_{\|} \, d\ell =$ longitudinal adiabatic invariant

and $\alpha = \theta - \left(\zeta / q\right) =$ field line label.

So an equivalent definition of omnigenity is $\partial J / \partial\alpha = 0$,

i.e. J is a flux function.

One way to achieve omnigenity is *quasisymmetry* …

Motivation for quasisymmetry

In axisymmetry, $\underset{\sim L^2 B}{\psi_*} = \psi - G\dfrac{\upsilon_\parallel}{\underset{\sim \rho L B}{\Omega}}$ is conserved during drift motion,

where $G(\psi) = RB_{\text{tor}}$ is a flux function.

ψ_* conserved $\Rightarrow$ a particle's ψ can't vary much $\Rightarrow$ good confinement

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<u>Proof:</u>

$$\frac{d\psi_*}{dt} = \left(\upsilon_\parallel \mathbf{b} + \mathbf{v}_d\right)\cdot\nabla\psi_*$$

$$= \cancel{\mathbf{b}\cdot\nabla\psi} \;+\; \underbrace{\upsilon_\parallel \mathbf{b}\cdot\nabla\left(-\frac{G\upsilon_\parallel}{\Omega}\right)}_{\frac{\upsilon_\parallel^2+\upsilon_\perp^2/2}{\Omega B} G\mathbf{b}\cdot\nabla B} \;+\; \underbrace{\mathbf{v}_d\cdot\nabla\psi}_{\frac{\upsilon_\parallel^2+\upsilon_\perp^2/2}{\Omega B}\mathbf{b}\times\nabla B\cdot\nabla\psi} \;+\; \cancel{\mathbf{v}_d\cdot\nabla\left(-\frac{G\upsilon_\parallel}{\Omega}\right)}$$

And in axisymmetry, $G\mathbf{b}\cdot\nabla B = -\mathbf{b}\times\nabla B\cdot\nabla\psi$ $\quad\Rightarrow\quad \dfrac{d\psi_*}{dt} = 0.$ Q.E.D.

Quasisymmetry

The only place we needed axisymmetry in that proof was to say

$$\frac{\mathbf{b}\times\nabla B\cdot\nabla\psi}{\mathbf{b}\cdot\nabla B} \text{ is a flux function.}$$

Suppose we had a stellarator in which $\dfrac{\mathbf{b}\times\nabla B\cdot\nabla\psi}{\mathbf{b}\cdot\nabla B}$ was some flux function $Y(\psi)$:

then $\psi_* = \psi + Y\dfrac{v_\parallel}{\Omega}$ will be conserved during drift motion!

ψ_* conserved $\Rightarrow$ a particle's ψ can't vary much $\Rightarrow$ good confinement

Definition of quasisymmetry:

$$\frac{\mathbf{b}\times\nabla B\cdot\nabla\psi}{\mathbf{b}\cdot\nabla B} = Y(\psi) \quad\Leftrightarrow\quad B = B(\psi,\ M\theta - N\zeta) \text{ in Boozer coordinates}$$

Boozer isomorphism

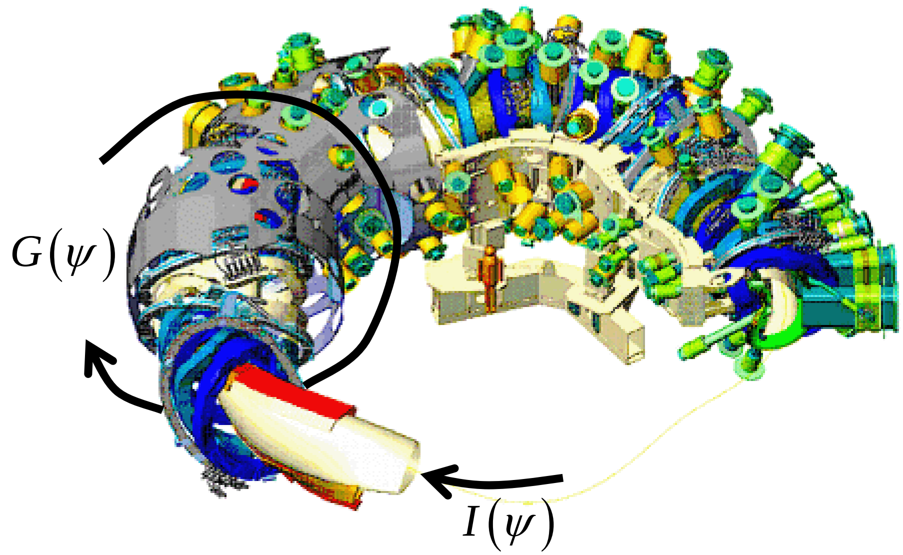
Neoclassical formulae for quasisymmetric fields and axisymmetric fields are nearly identical. E.g., bootstrap current:

Tokamak: $$\left\langle j_{\|}B\right\rangle = -2.4\sqrt{\varepsilon}c\left(\frac{dp_i}{d\psi}+\frac{dp_e}{d\psi}-0.74n_e\frac{dT_e}{d\psi}-1.17n_e\frac{dT_i}{d\psi}\right)G$$

QS stellarator: $$\left\langle j_{\|}B\right\rangle = -2.4\sqrt{\varepsilon}c\left(\frac{dp_i}{d\psi}+\frac{dp_e}{d\psi}-0.74n_e\frac{dT_e}{d\psi}-1.17n_e\frac{dT_i}{d\psi}\right)\frac{MG+NI}{M-qN}$$

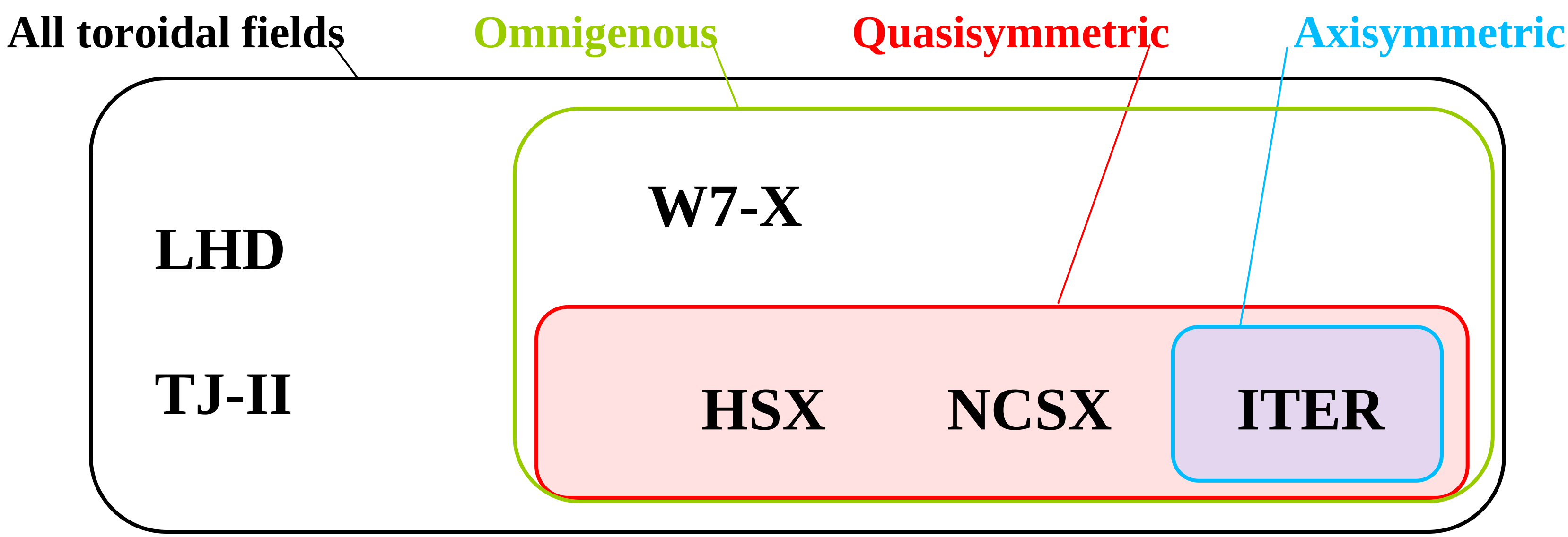
where $B = B(\psi,\, M\theta - N\zeta)$,

$I(\psi)$ and $G(\psi)$ are currents:



All axisymmetric fields are quasisymmetric, all quasisymmetric fields are omnigenous,

but the converses are not true*. *Cary & Shasharina, PoP (1997)*



Quasisymmetry is restrictive and no better than omnigenity for reducing neoclassical transport,

so is the stellarator community too focused on quasisymmetry?

Quasisymmetric fields **may** have reduced instability & turbulent transport:

- Flow speeds comparable to $\upsilon_{\mathrm{th,i}}$ are only permitted in quasisymmetric fields. (*Helander, PoP 2007*)

- Flows are clamped to a small neoclassical value except in quasisymmetry, so quasisymmetric fields permit larger flow shear. (*Helander & Simakov, PRL 2008*)

Currents in tokamaks & stellarators

- $\mathbf{j}\times\mathbf{B}=\nabla p \quad \Rightarrow \quad \mathbf{j}_\perp = B^{-2}\mathbf{B}\times\nabla p,$ but $j_{||}$ is harder to calculate.
- $$j_{||} = j_{\text{Ohmic}} + j_{\text{RF current drive}} + j_{\text{NBI}} + \underbrace{j_{\text{Pfirsch-Schlüter}} + j_{\text{bootstrap}}}_{\propto \frac{dn}{dr},\ \frac{dT}{dr}}$$
- In tokamaks you want big $j_{\text{bootstrap}}$ to extend pulse length.
- In stellarators, you want small $j_{\text{bootstrap}}$ and $j_{\text{Pfirsch-Schlüter}}$
  - to minimize drive for instability
  - to make equilibrium insensitive to pressure (maintaining optimization)
  - to avoid an equilibrium β limit

W7-X is designed to have minimum $|j_{||}|$.

Currents in tokamaks & stellarators

General stellarator:

$$j_{\|} = j_{\|}^{PS} + 1.64 \frac{1}{f_c} \left[\langle g_2 \rangle - \frac{3\langle B^2 \rangle}{4B_{\max}^2} \int_0^1 \frac{\langle g_4 \rangle}{\langle g_1 \rangle} \lambda \, d\lambda \right] \frac{B}{\langle B^2 \rangle} \left[\frac{dp_e}{d\psi} + \frac{dp_i}{d\psi} - 0.74 n \frac{dT_e}{d\psi} - 1.17 n \frac{dT_i}{d\psi} \right]$$

where $j_{\|}^{PS}$ is defined by $\mathbf{B}\cdot\nabla\left(\frac{j_{\|}^{PS}}{B}\right) = -\left(\frac{dp_e}{d\psi} + \frac{dp_i}{d\psi}\right)\mathbf{B}\times\nabla\psi\cdot\nabla\left(\frac{1}{B^2}\right)$ and $\langle j_{\|}^{PS} B \rangle = 0$,

$$g_1 = \sqrt{1 - \lambda B / B_{\max}}, \qquad f_c = \frac{3\langle B^2 \rangle}{4B_{\max}^2} \int_0^1 \frac{\lambda \, d\lambda}{\langle g_1 \rangle},$$

g_2 is defined by $\mathbf{B}\cdot\nabla\left(\frac{g_2}{B^2}\right) = \mathbf{B}\times\nabla\psi\cdot\nabla\left(\frac{1}{B^2}\right)$ and $g_2 = 0$ at $B = B_{\max}$,

g_4 is defined by $\mathbf{B}\cdot\nabla\left(\frac{g_4}{g_1}\right) = \mathbf{B}\times\nabla\psi\cdot\nabla\left(\frac{1}{g_1}\right)$ and $g_4 = 0$ at $B = B_{\max}$.

(i.e., complicated)

Tokamak:

$$j_{\|} = \frac{RB_\phi}{B} \left[\frac{B^2}{\langle B^2 \rangle} - 1 \right] \left[\frac{dp_e}{d\psi} + \frac{dp_i}{d\psi} \right] + 2.4 \frac{\sqrt{\varepsilon} RB_\phi B}{\langle B^2 \rangle} \left[\frac{dp_e}{d\psi} + \frac{dp_i}{d\psi} - 0.74 n \frac{dT_e}{d\psi} - 1.17 n \frac{dT_i}{d\psi} \right]$$

2. Neoclassical theory of omnigenous stellarators

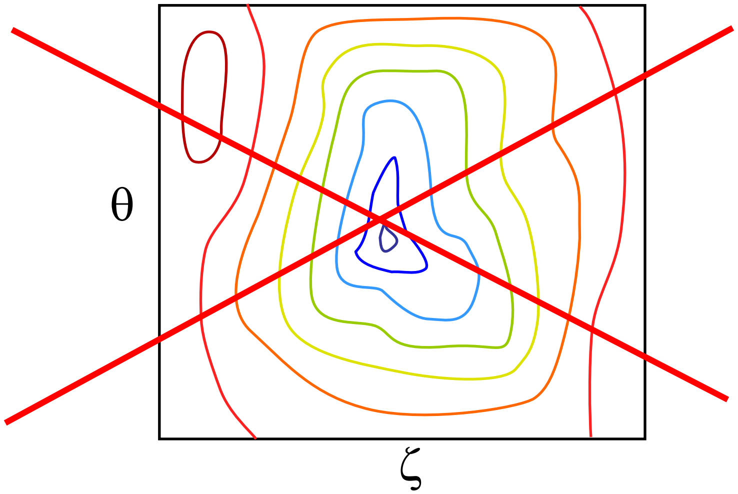
Omnigenity

$\overline{\mathbf{v}_d \cdot \nabla \psi} = 0$ puts strong constraints on **B**, e.g.: (after some algebra…)

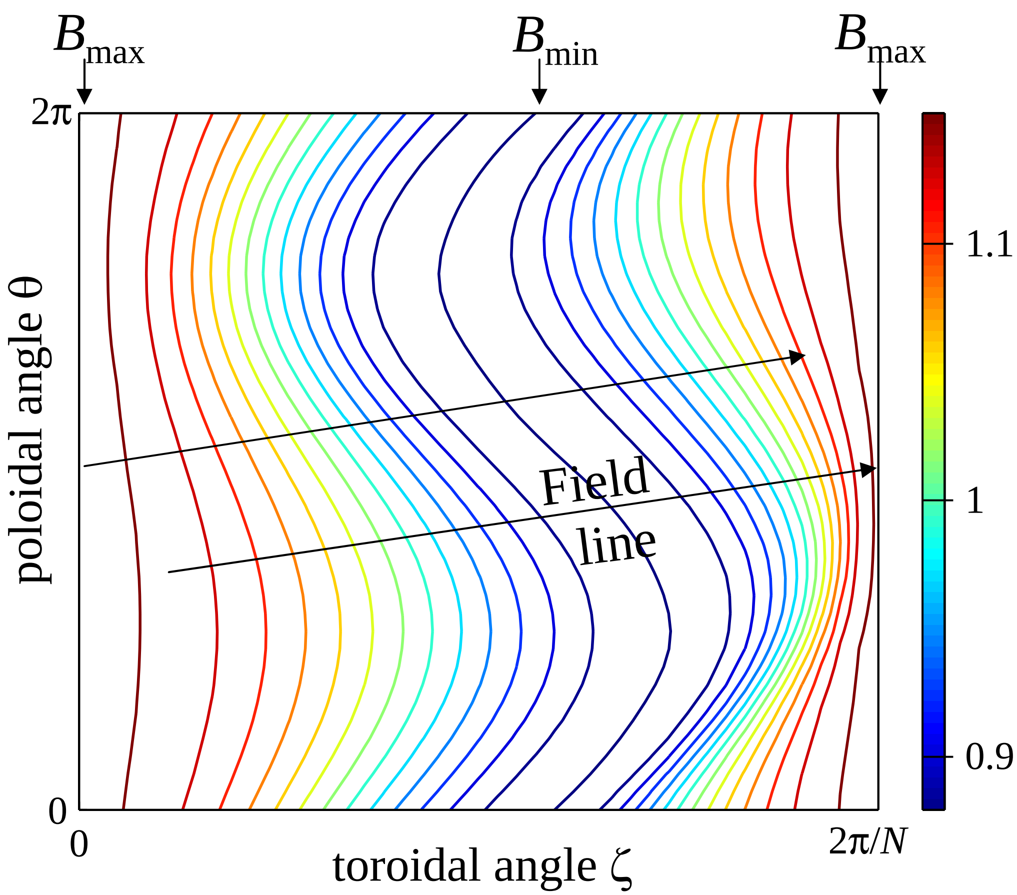
- All B contours link the torus toroidally, poloidally, or both.

- Each time a field line transits the torus, B_{max} and B_{min} are the same.

- B_{max} is a straight line in the θ–ζ plane.



Contours of B



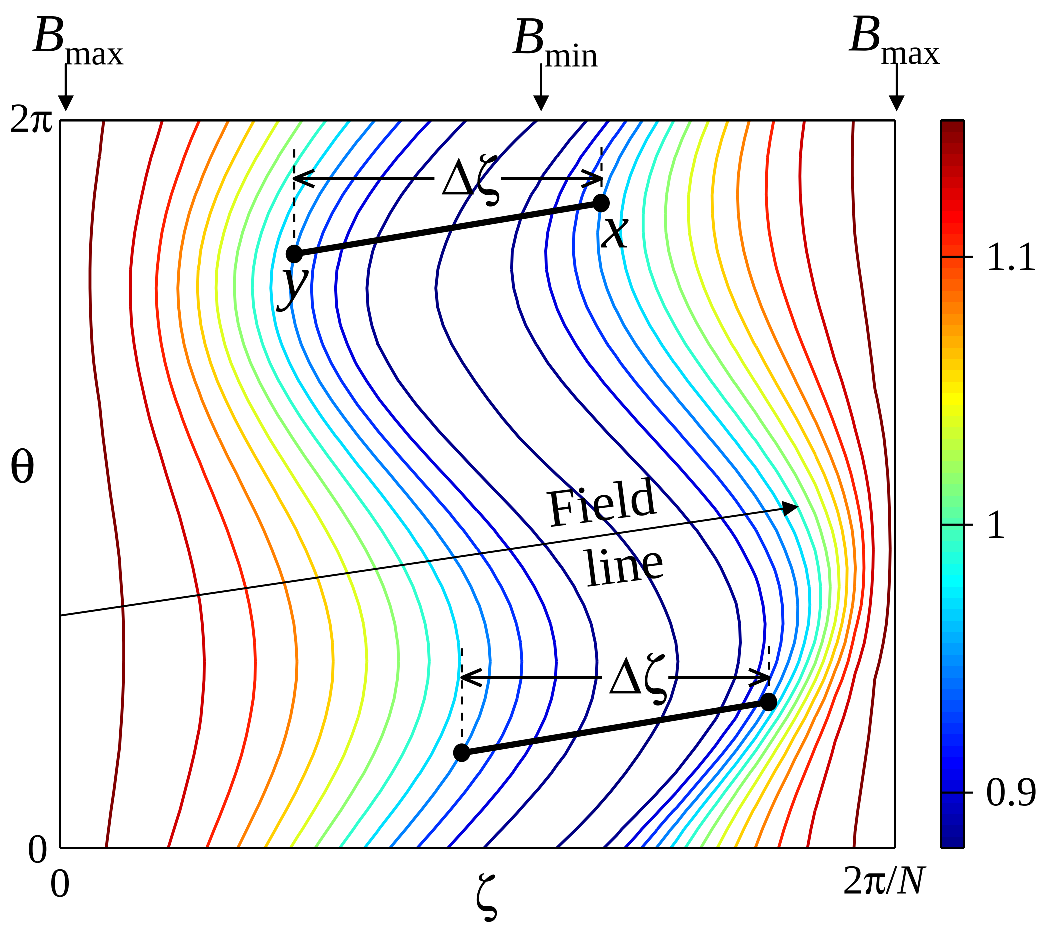
Omnigenity

$\overline{\mathbf{v}_d \cdot \nabla \psi} = 0$ puts strong constraints on **B**, e.g.: (after some algebra…)

- $\Delta\zeta$ between the two points with same B on a field line is independent of field line.

(*Cary & Shasharina, PoP 1997*)

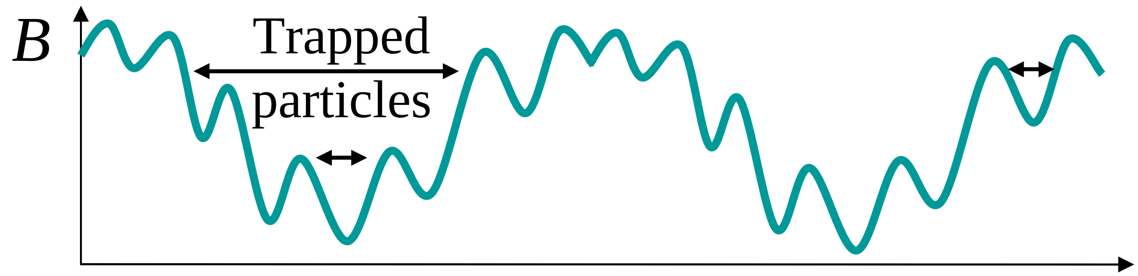
<u>Contours of *B*</u>



Another perspective

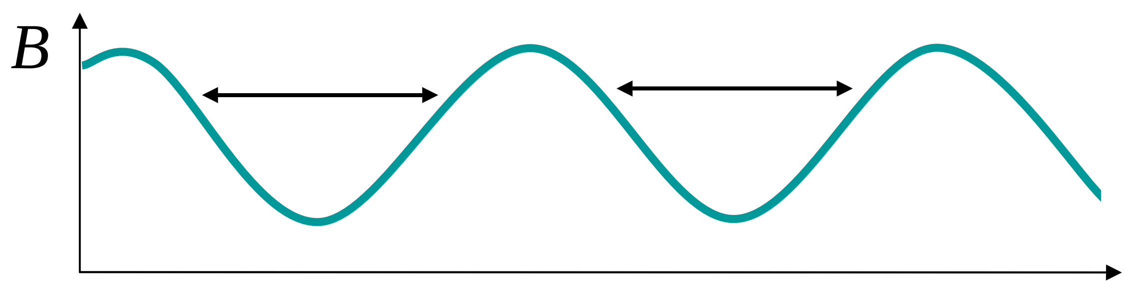
Non-optimized stellarator:

$$\overline{\mathbf{v}_d \cdot \nabla \psi} \neq 0$$



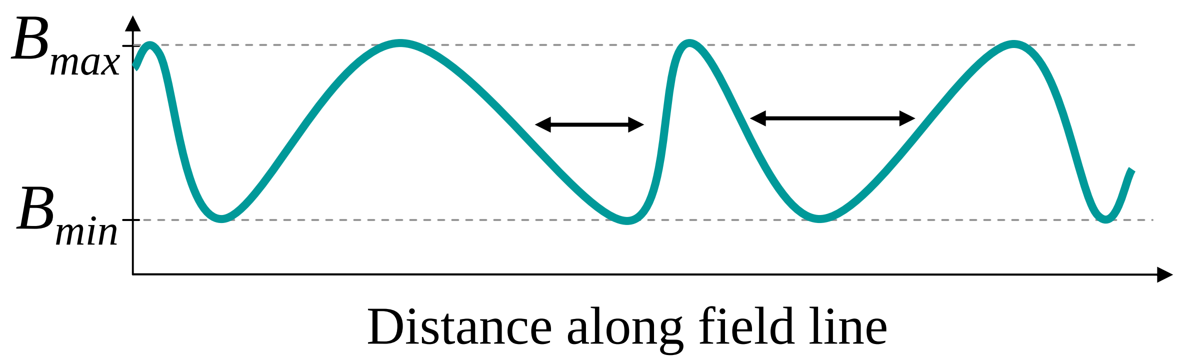
Tokamak or quasisymmetric stellarator:

$$\overline{\mathbf{v}_d \cdot \nabla \psi} = 0$$



Omnigenous stellarator:

$$\overline{\mathbf{v}_d \cdot \nabla \psi} = 0$$



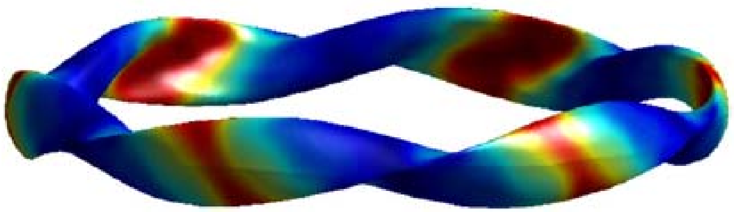
Omnigenity

• Define M and N: all constant-B contours close after linking the torus M times toroidally and N times poloidally.

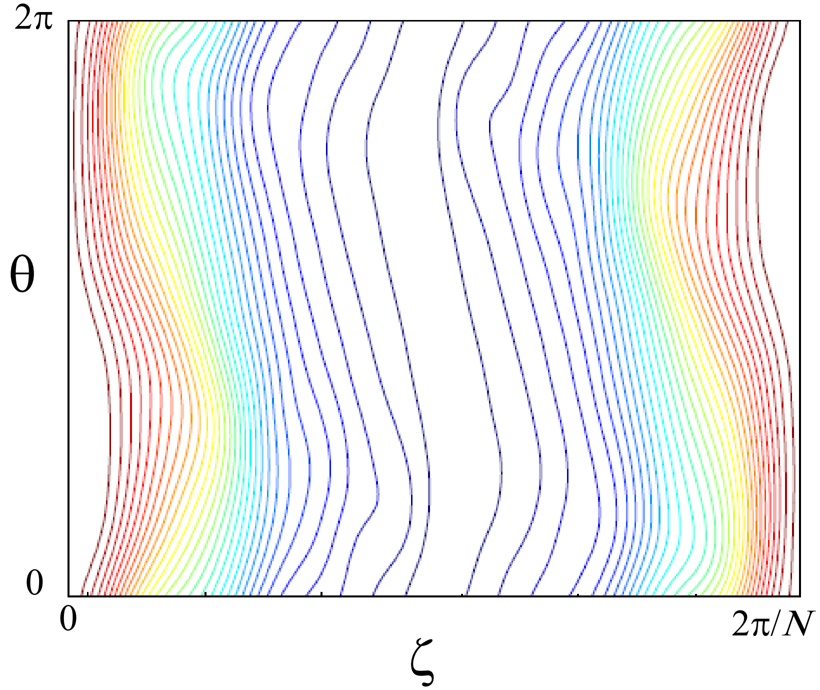
• In quasisymmetric limit, then B=$B(M\theta – N\zeta)$.

• W7-X can be approximately omnigenous with M=0 & N=1. ("quasi-isodynamic")

• Recently, other equilibria have been designed which are even closer to being omnigenous.



Contours of B



Subbotin *et al.*, Nucl. Fusion **46**, 921 (2006)

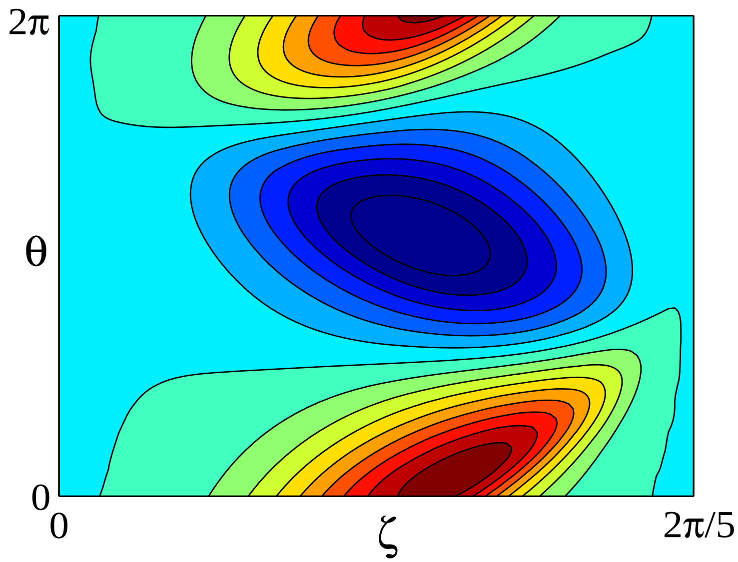
Current in omnigenous plasmas

For any collisionality,

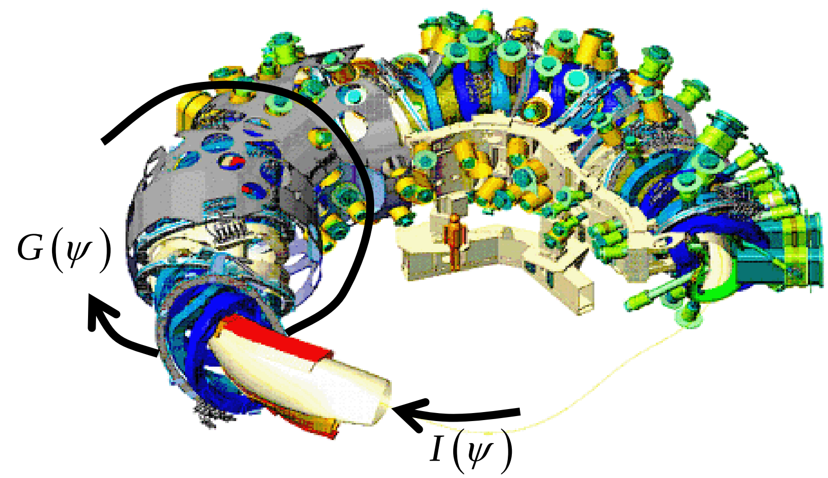
$$j_\parallel = \frac{\langle j_\parallel B\rangle B}{\langle B^2\rangle} + j_\parallel^{PS} \quad \text{where} \quad j_\parallel^{PS} = \frac{c}{B(N-\iota M)}\frac{dp}{d\psi}\left[\left(1-\frac{B^2}{\langle B^2\rangle}\right)(NI+MG)+W\right]$$

is the Pfirsch-Schlüter current,

$$W = 2B^2(G+\iota I)\int^{\zeta}\frac{d\zeta'}{B'^3}\left[N\left(\frac{\partial B'}{\partial\theta}\right)+M\left(\frac{\partial B'}{\partial\zeta}\right)\right],$$ and



$I(\psi)$ and $G(\psi)$ are the toroidal & poloidal currents.



Current in omnigenous plasmas

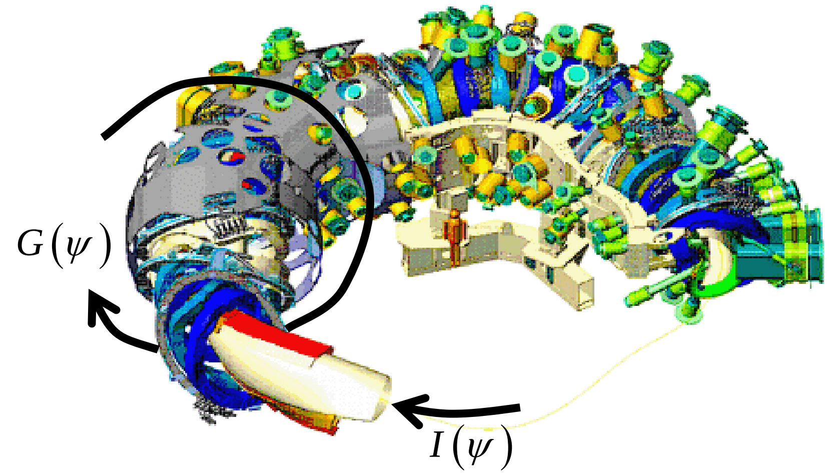
$$j_{\|} = \frac{\left\langle j_{\|} B \right\rangle B}{\left\langle B^2 \right\rangle} + j_{\|}^{PS} \quad \text{where} \quad j_{\|}^{PS} = \frac{c}{B\left(N - \iota M\right)} \frac{dp}{d\psi} \left[\left(1 - \frac{B^2}{\left\langle B^2 \right\rangle} \right) \left(NI + MG \right) + W \right]$$

and for low collisionality, the bootstrap current is identical to j_{bs} in a quasisymmetric stellarator:

$$\left\langle j_{\|} B \right\rangle = 1.64 f_t c \left(\frac{NI + MG}{N - \iota M} \right) \left(\frac{dp_e}{d\psi} + \frac{dp_i}{d\psi} - 0.74 n_e \frac{dT_e}{d\psi} - 1.17 n_e \frac{dT_i}{d\psi} \right)$$

= Tokamak result with $G \to -\iota\left(NI + MG\right)/\left(N - \iota M\right)$

$I(\psi)$ and $G(\psi)$ are the toroidal & poloidal currents.



In M=0 omnigenous fields, j_{bs} vanishes

Subbotin *et al*, NF **46**, 921 (2006), Helander & Nührenberg, PPCF **51**, 055004 (2009).

Toroidal current inside a flux surface = $\frac{c}{2} I(\psi) = \int^{\psi} \mathbf{j} \cdot d^2\mathbf{r}.$

$$\Rightarrow \qquad \frac{dI}{d\psi} = -4\pi \frac{I}{\langle B^2 \rangle} \frac{dp}{d\psi} + \frac{4\pi}{c\langle B^2 \rangle} \langle j_{\|} B \rangle$$

From last page: $\langle j_{\|} B \rangle \propto (NI + MG).$

So if B contours close poloidally ($M = 0$) rather than toroidally or helically,

$$\frac{dI}{d\psi} = (...)I.$$

Initial condition: $I(\psi = 0) = 0.$

$\Rightarrow$ Self-consistent current profile is $I = 0$ with $\langle j_{\|} B \rangle = 0.$

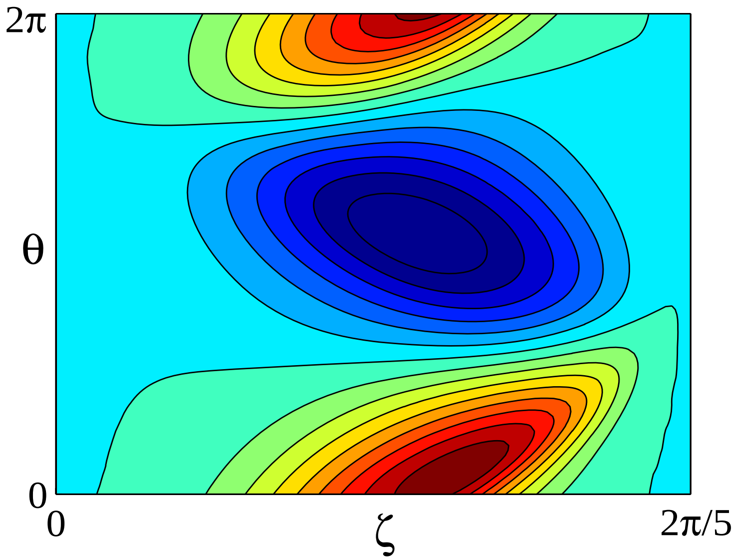
$\Rightarrow$ $M = 0$ is the best choice for minimizing $j_{\|}$.

Flow in omnigenous plasmas

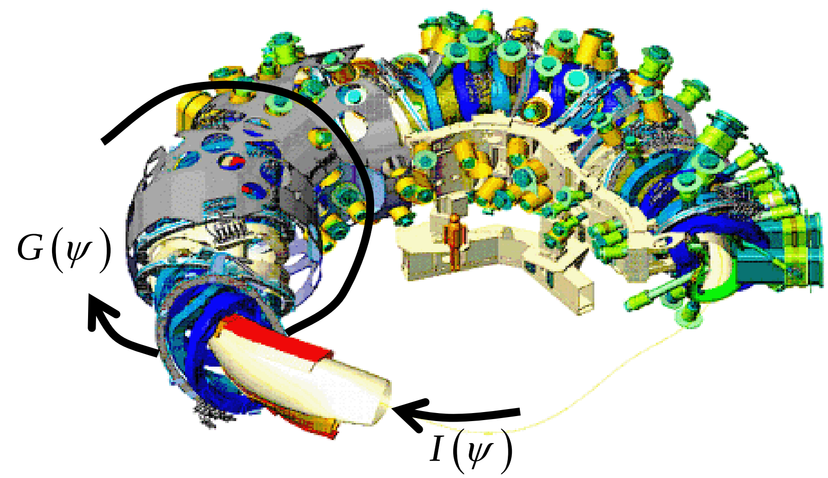
$$V_{\|} = -1.17\frac{cB}{Ze\langle B^2\rangle}\frac{dT}{d\psi}\frac{(NI+MG)}{(N-\iota M)} + \frac{c}{B}\left(\frac{d\Phi}{d\psi}+\frac{1}{Zen}\frac{dp}{d\psi}\right)\frac{(NI+MG+W)}{(N-\iota M)}$$

Tokamak result with $G \rightarrow -\iota(NI+MG)/(N-\iota M)$

$$W = 2B^2(G+\iota I)\int^{\zeta}\frac{d\zeta'}{B'^3}\left[N\left(\frac{\partial B'}{\partial\theta}\right)+M\left(\frac{\partial B'}{\partial\zeta}\right)\right],$$ and



$I(\psi)$ and $G(\psi)$ are the toroidal & poloidal currents.



Solving for E_r

Non-quasisymmetric stellarators:

- Radial fluxes of ions & electrons would be different, unless E_r is just right.

$\Rightarrow$ You can solve for E_r.

Tokamaks & perfect quasisymmetry:

- Radial fluxes of ions and electrons are always equal, regardless of E_r ("intrinsic ambipolarity")

(*Helander & Simakov, PRL 2008*)

$\Rightarrow$ You cannot solve for E_r.

Solving for E_r

Non-quasisymmetric stellarators:

- Radial fluxes of ions & electrons would be different, unless E_r is just right.

$\Rightarrow$ You can solve for E_r.

Omnigenous plasmas:

$$\langle \mathbf{j}\cdot\nabla\psi\rangle = \left(Zen_i \frac{d\Phi}{d\psi} + T_i \frac{dn_i}{d\psi} - 0.17 n_i \frac{dT_i}{d\psi} \right)\Delta$$

where $\Delta \propto \left\langle \text{departure from quasisymmetry}^2 \right\rangle$

Universal result: if Δ is nonzero,

$$\boxed{\mathbf{E} = \frac{1}{Ze}\left[\frac{T_i}{n_i}\nabla n_i - 0.17\nabla T_i \right]}$$

(Totally independent of the details of **B**!)

Recap

- Neoclassical transport is particularly important in stellarators because of unconfined orbits.

- Modern stellarators are optimized to minimize radial neoclassical transport.

- The ideal is “omnigenity”: average radial drift = 0.

- In stellarators it is desirable to have small $j_{\text{bootstrap}}$ and $j_{\text{Pfirsch-Schlüter}}$.

- In the omnigenous limit, expressions for the neoclassical flow and current simplify dramatically.

- Unlike quasisymmetric stellarators, in omnigenous devices you can solve for E_r explicitly, and the form is independent of the **B** geometry.

For details, see Landreman and Catto, PPCF **53**, 035106 (2011).