Theory and simulations of tokamak plasmas with transonic poloidal flows and the MHD edge pedestal

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The relevant sound speed for transonic poloidal flow is the poloidal sound speed

\[ C_s = \sqrt{\gamma P / \rho} \]

\[ C_{s\theta} = C_s \frac{B_\theta}{B} \]

• Sound waves propagate along B-field lines with speed

• To propagate in the poloidal direction, the sound waves must go the long way \( \longrightarrow \) effective speed
The Magnetic Field Creates a De Laval Nozzle for the Poloidal Flow.

- In ideal MHD, the *frozen in* condition holds: plasma cannot flow across magnetic surfaces.
- Due to toroidal geometry, in a tokamak the cross section between any two nested magnetic surfaces varies with the poloidal angle.
- For the poloidal flow, nested magnetic surfaces act as a de Laval nozzle.
Due to the Geometry, Shocks (Time-Dependent) or Discontinuities/Pedestals (Steady-State) Form in the Plasma.

- The relevant velocity is the poloidal sound speed \( C_{sp} = C_s B_p / B \).
- Due to periodicity:
  - If the flow becomes supersonic at the nozzle throat, a shock will form.
  - At steady state shocks are not allowed, and the flow can be sonic only at the nozzle throat.
  - At steady state, tangential discontinuities will remain between the subsonic and supersonic region.
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Equilibria with transonic poloidal velocity profiles are radially discontinuous

- Transonic equilibria are characterized by a poloidal flow varying from subsonic to supersonic with respect to the poloidal sound speed $C_{s\theta} = (B_{\theta}/B_0)(\gamma P/\rho)^{1/2}$.

- $V$, $P$ and $\rho$ are discontinuous at the transonic surface -> large velocity shear.

MHD equilibrium equations

\[ \nabla \cdot (\rho \vec{v}) = 0 \]

\[ \rho \vec{v} \cdot \nabla \vec{v} = -\nabla P + \vec{J} \times \vec{B} \]

\[ \frac{1}{\gamma} \nabla \cdot p^\gamma \vec{v} = 0 \]

\[ \nabla \times (\vec{v} \times \vec{B}) = 0 \]

\[ \nabla \cdot \vec{B} = 0 \]
MHD equilibria with flow require the solution of the Bernoulli and Grad-Shafranov equations

\[
\rho = D(\psi) / X
\]

\[
p = P(\psi) / X^\gamma
\]

\[
v_\phi \approx C_s(\psi)[M_\phi(\psi) + (X - 1)M_\theta(\psi)]
\]

\[
C_s(\psi) = \sqrt{\gamma P(\psi) / D(\psi)}
\]

\[
B_\phi R = F(\psi)(1 + O(\varepsilon^2 X))
\]

\[
v_\theta = XC_s(\psi)M_\theta(\psi)B_\theta / B
\]

\[
|\nabla \psi| \Delta^* \psi + e_\psi \cdot \nabla (R^2 p + B_\phi^2 R^2 / 2) = O(\varepsilon)
\]

Grad-Shafranov equation
\[
(\beta \sim \varepsilon^2, \ \varepsilon<<1)
\]

Bernoulli equation
\[
\tilde{R}^{-2} M_\phi^2(\psi) X^{\gamma+1} - \left\{ \frac{2}{\gamma - 1} + M_\phi^2(\psi) + \left[ M_\phi(\psi) - M_\phi(\psi) \right](\tilde{R}^2 - 1) \right\} X^{\gamma-1} + \frac{2}{\gamma - 1} = 0
\]

\[
\tilde{R} = 1 + \varepsilon \cos \theta
\]

\[
X = X(\psi, \theta)
\]

P(\psi), D(\psi), F(\psi), M_\phi(\psi) and M_\theta(\psi) are free functions.
Poloidal Mach number $M_p$ vs free function $M_\theta(\Psi)$

$$M_p = X(\psi, \theta)^{\gamma+1/2} M_\theta(\psi)$$

$$M_p = \frac{V_\theta}{C_{s\theta}}$$
The poloidal Mach number profile is radially discontinuous for transonic poloidal flow profiles.
2D MHD SIMULATIONS OF AN INITIALLY STATIC PLASMA SET IN MOTION BY A SOURCE OF POLOIDAL MOMENTUM
SIM2D Simulations Are Based on the MHD Model.

We solve the standard resistive-MHD model hyperbolic system of time-dependent equations:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad \text{(Continuity)}
\]

\[
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} - \mathbf{B} \mathbf{B} + P \mathbf{I}) = \mathbf{S}_{\text{mom}}, \quad \text{(Momentum)}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{J}), \quad \text{(Faraday’s Law)}
\]

\[
\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot [(\mathcal{E} + P) \mathbf{v} - \mathbf{B} (\mathbf{v} \cdot \mathbf{B}) + \eta \mathbf{J} \times \mathbf{B}] = \mathbf{v} \cdot \mathbf{S}_{\text{mom}}, \quad \text{(Energy)}
\]

\[
P \equiv p + \frac{B^2}{2}, \quad \mathcal{E} = \frac{p}{\gamma - 1} + \rho \frac{V^2}{2} + \frac{B^2}{2}. \quad \text{(Definitions)}
\]

The equations are written in conservative form to ensure conservation of physical quantities.
The “Halo” Region is Treated as a Resistive Plasma.

- Resistive-MHD equations are used to separate plasma and halo region.
- The main plasma region is \( \sim \) ideal.
- The edge and halo region have non-negligible resistivity.

- Spitzer resistivity is used in the rest of this presentation.
- An artificially high halo resistivity was also tried, with \( \eta_{\text{halo}} / \eta_{\text{plasma}} = 10^8 \).
- Notice that the focus is on the main (ideal) plasma region.
Initial Poloidal Sound Speed Is Low.

- A typical free-boundary DIII-D equilibrium used in SIM2D simulations is shown on the right.
- Initial poloidal sound speed is small at the plasma edge.
- The boundary of the computational domain corresponds to a superconductive wall.
- Outflow boundary conditions \( (v \sim C_s) \) are used at the wall.
A shock is observed at the transonic surface.
The shock travels in the poloidal direction from the outboard to the inboard part of the plasma.
The shock vanishes at the inner midplane, where the flow is sonic.
The shock is an MHD feature.
A Quasi-Steady-State MHD Pedestal Is Found In Fixed-Boundary Time-Dependent Simulations.

- Time-dependent simulations only reach an approximate steady state.
- At near-steady state, the density pedestal structure is clearly visible.
- The properties of the pedestal (e.g., poloidal angle height dependence) are in qualitative agreement with equilibrium calculations.
Fixed-Boundary Simulations Are Consistent with Equilibrium Calculations.

- After the initial transient, a discontinuity forms in e.g. density and velocity profiles.
- The discontinuity is qualitatively consistent with equilibrium calculations.
- This discontinuity is a tangential discontinuity, **NOT** a shock.
Evolution of the MHD pedestal in free boundary simulations
Details of the MHD pedestal
Similar pedestal can be obtained from the equilibrium solution.
In Free-Boundary Simulations, Mach Number and Velocity Develop Discontinuous Profiles.

- A static DIII-D equilibrium is perturbed with a (smooth) poloidal momentum source.
- Velocity and Mach number quickly develop a discontinuous profile across the transonic surface.
- As predicted by theory, profiles are smooth on the inboard side, sharply discontinuous on the outboard side of the plasma (notice the $M_p = 1$ line!).
Density Profiles Develop a Pedestal Structure.

- When the flow becomes supersonic \((V_\theta > C_{sp})\) a shock forms and travels in the poloidal direction.
- After the shock formation, the density profile steepens.
- A shock-less, quasi-steady state condition is reached, with a density pedestal (sharp density gradient) at the transonic surface.
Main ion perpendicular edge velocity on C-MOD

Conclusions

• Theory, equilibrium solutions and 2D free-boundary MHD simulations show the formation of a pedestal when the poloidal velocity exceeds the poloidal sound speed

• At the pedestal, a velocity radial-shear layer forms where the poloidal velocity jumps from subsonic to supersonic

• Such a layer is NOT a shock but a contact discontinuity
Bibliography

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