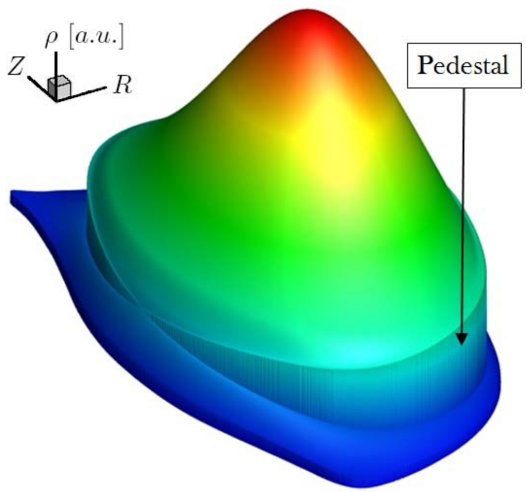
Theory and simulations of tokamak plasmas with transonic poloidal flows and the MHD edge pedestal



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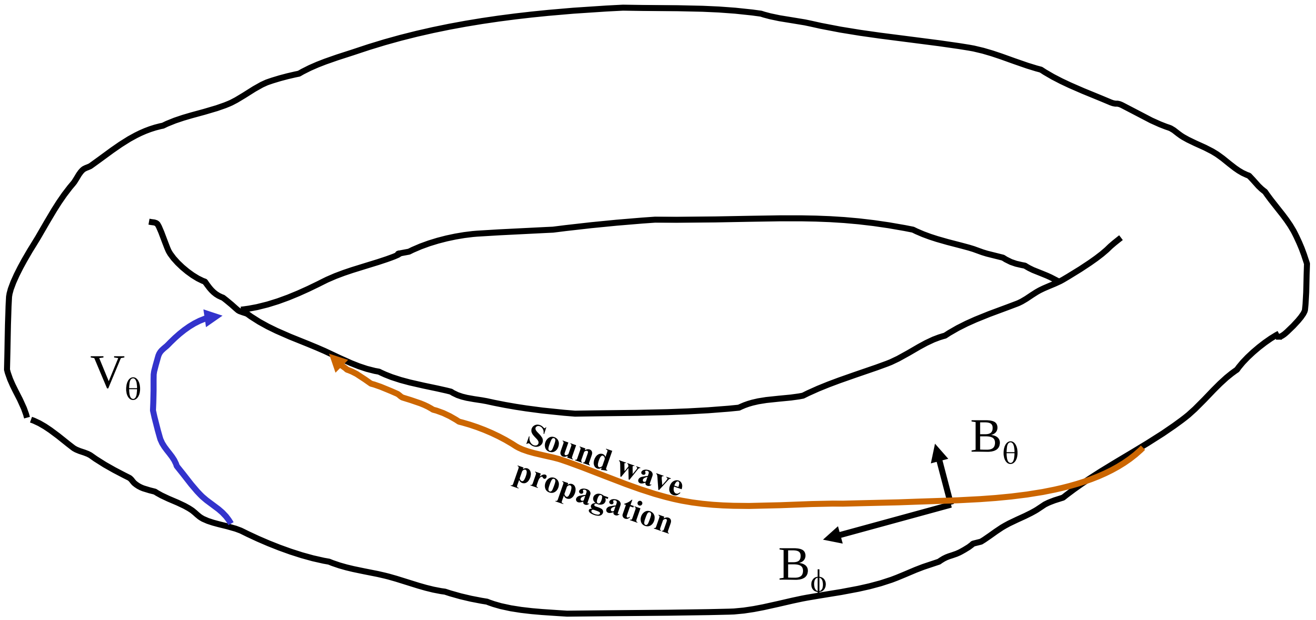
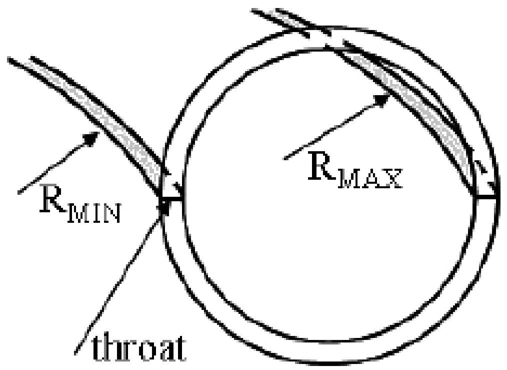
The relevant sound speed for transonic poloidal flow is the poloidal sound speed

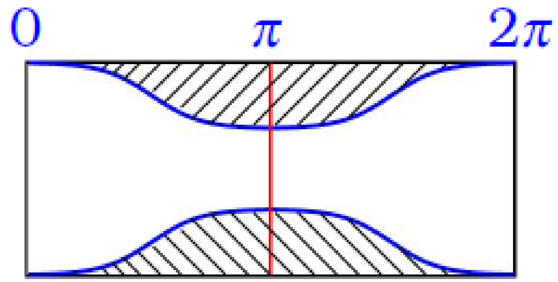


- Sound waves propagate along B-field lines with speed $C_s = \sqrt{\gamma P / \rho}$
- To propagate in the poloidal direction, the sound waves must go the long way -----------> effective speed $C_{s\theta} = C_s \dfrac{B_\theta}{B}$

The Magnetic Field Creates a De Laval Nozzle for the Poloidal Flow.

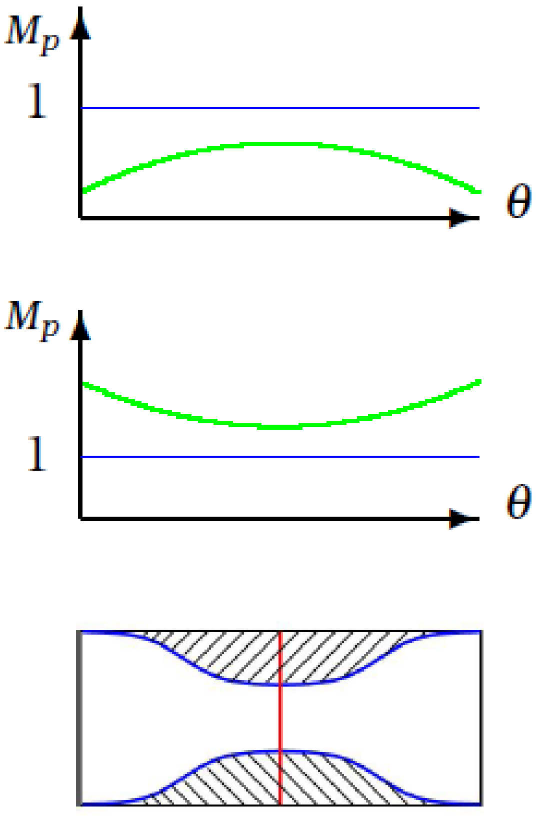
- In ideal MHD, the *frozen in* condition holds: plasma cannot flow across magnetic surfaces.
- Due to toroidal geometry, in a tokamak the cross section between any two nested magnetic surfaces varies with the poloidal angle.
- For the poloidal flow, nested magnetic surfaces act as a de Laval nozzle.





Due to the Geometry, Shocks (Time-Dependent) or Discontinuities/Pedestals (Steady-State) Form in the Plasma.

- The relevant velocity is the poloidal sound speed $C_{sp} = C_s B_p / B$.
- Due to periodicity:
    - If the flow becomes supersonic at the nozzle throat, a shock will form.
    - At steady state shocks are not allowed, and the flow can be sonic only at the nozzle throat.
    - At steady state, tangential discontinuities will remain between the subsonic and supersonic region.



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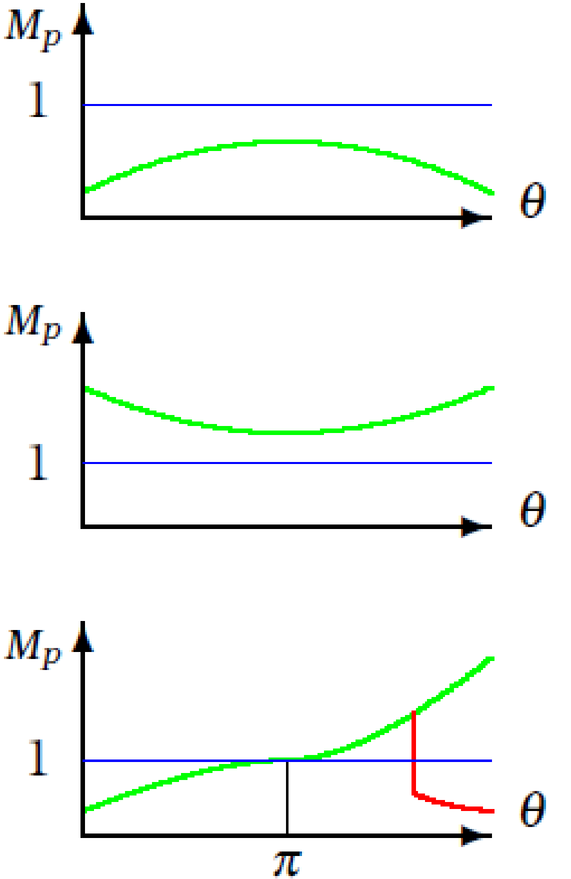

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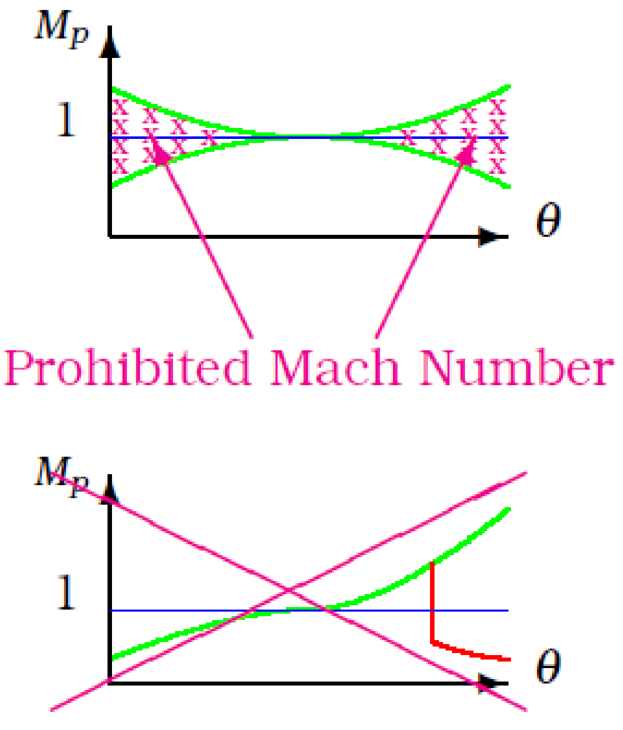
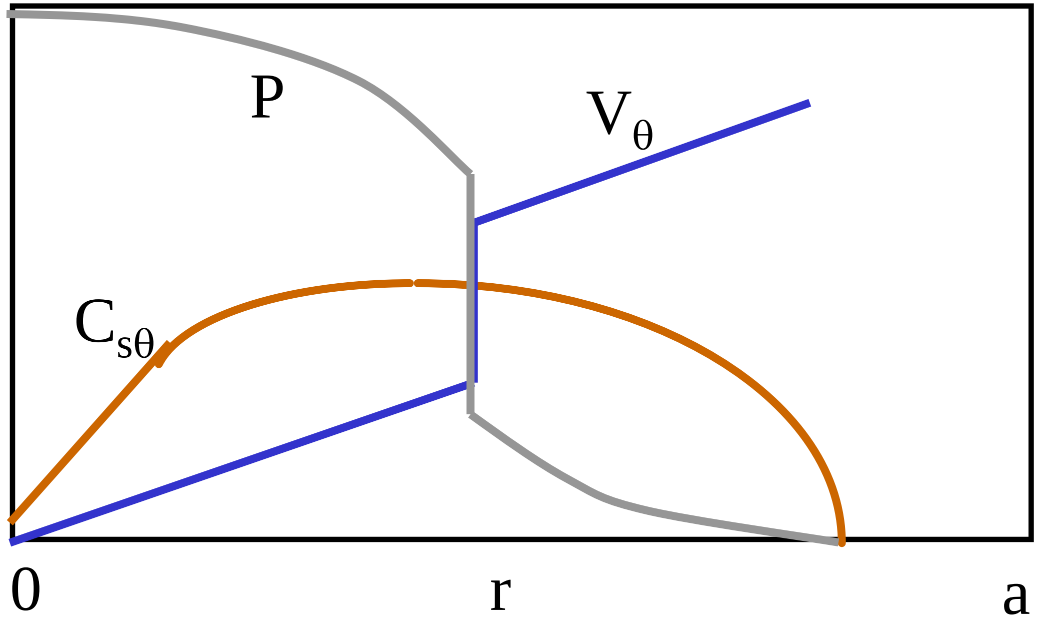


Equilibria with transonic polodial velocity profiles are radially discontinuous

- Transonic equilibria are characterized by a poloidal flow varying from subsonic to supersonic with respect to the poloidal sound speed $C_{s\theta}=(B_\theta/B_0)(\gamma P/\rho)^{1/2}$

- <u>V, P and ρ are discontinuous at the transonic surface -> large velocity shear</u>



_Betti and Freidberg, Phys. Plasmas Vol. 7, p. 2439 (2000)

MHD equilibrium equations

$$\nabla \bullet (\rho \vec{\mathrm{v}}) = 0$$

$$\rho \vec{\mathrm{v}} \cdot \nabla \vec{\mathrm{v}} = -\nabla P + \vec{J} \times \vec{B}$$

$$\nabla \bullet p^{\frac{1}{\gamma}} \vec{\mathrm{v}} = 0$$

$$\nabla \times (\vec{\mathrm{v}} \times \vec{B}) = 0$$

$$\nabla \bullet \vec{B} = 0$$

MHD equilibria with flow require the solution of the Bernoulli and Grad-Shafranov equations

$$\rho = D(\psi)/X$$

$$p = P(\psi)/X^{\gamma}$$

$$v_\phi \cong C_s(\psi)[M_\phi(\psi) + (X-1)M_\vartheta(\psi)]$$

$$C_s(\psi) = \sqrt{\gamma P(\psi)/D(\psi)}$$

$$B_\phi R = F(\psi)(1 + O(\in^2 X))$$

$$v_\theta = XC_s(\psi)M_\theta(\psi)B_\theta / B$$

$$|\nabla\psi|\Delta^*\psi + e_\psi \bullet \nabla(R^2 p + B_\phi^2 R^2/2) = O(\in)$$

⟵ **Grad-Shafranov equation ($\beta \sim \in^2$, $\in \ll 1$)**

Bernoulli equation

$$\hat{R}^{-2} M_\vartheta^2(\psi) X^{\gamma+1} - \left\{ \frac{2}{\gamma-1} + M_\vartheta^2(\psi) + \left[M_\vartheta(\psi) - M_\phi(\psi)\right]\left(\hat{R}^2 - 1\right)\right\} X^{\gamma-1} + \frac{2}{\gamma-1} = 0$$

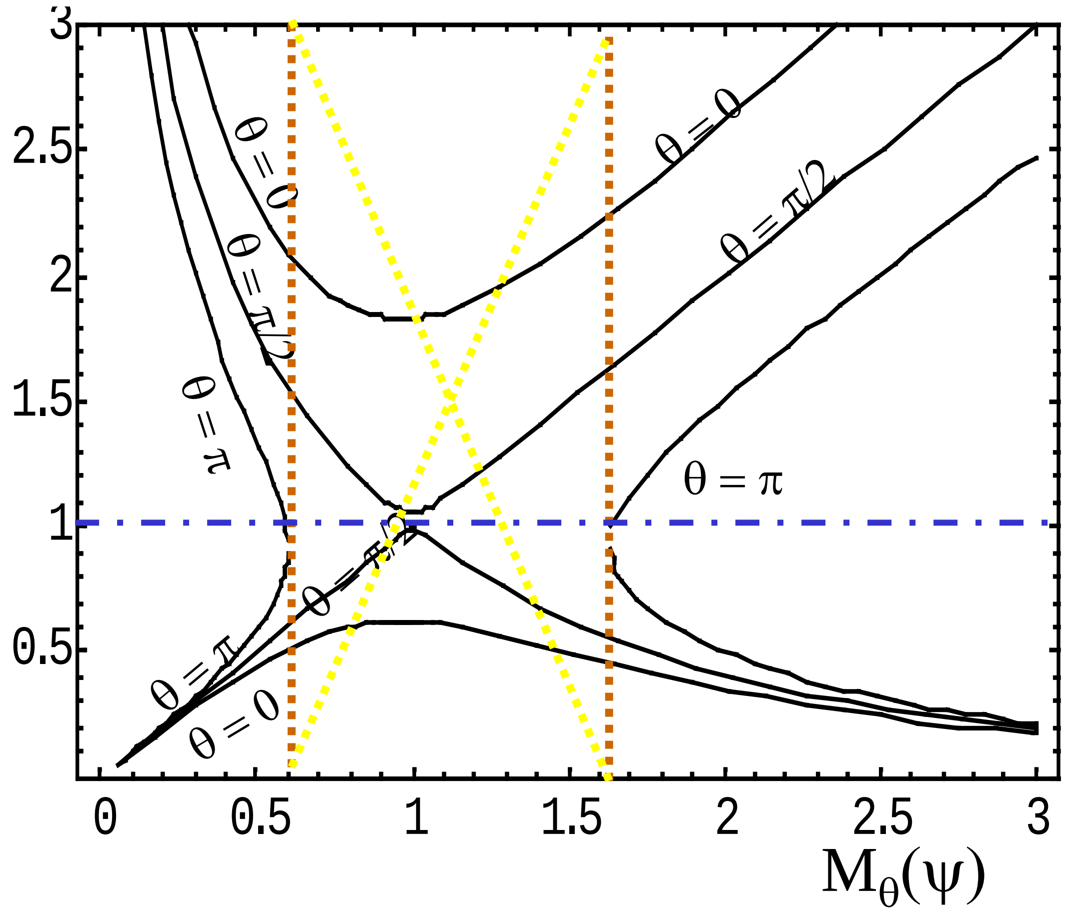
$$\hat{R} = 1 + \in\cos\vartheta \qquad X = X(\psi, \vartheta)$$

P(ψ), D(ψ), F(ψ), M_ϕ(ψ) and M_θ(ψ) are free functions.

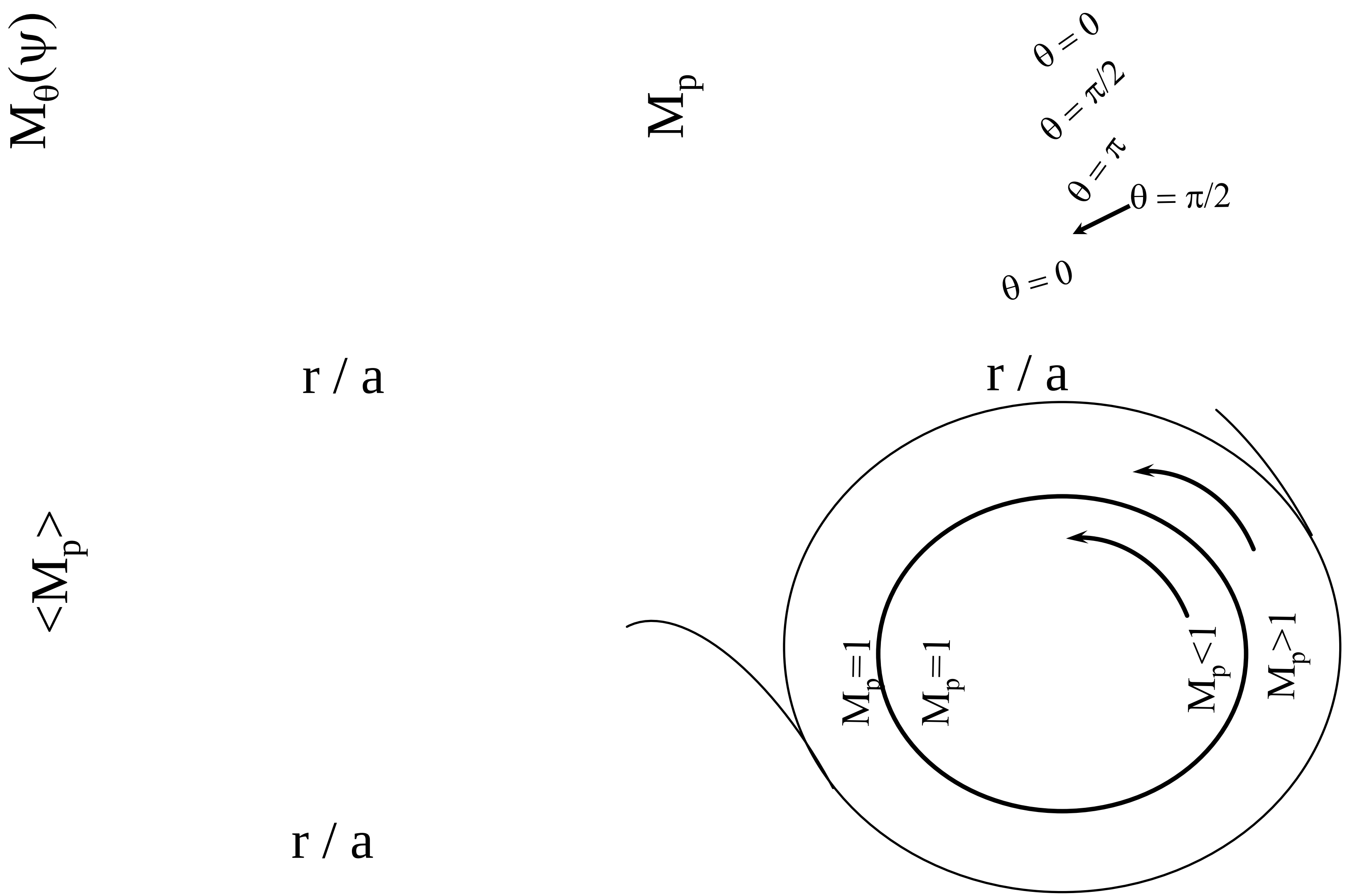
Poloidal Mach number M_p vs free function $M_\theta(\Psi)$

$$M_p = X(\psi,\theta)^{(\gamma+1)/2} M_\theta(\psi)$$

$$M_p = \frac{V_\theta}{C_{s\theta}}$$



The poloidal Mach number profile is radially discontinuous for transonic poloidal flow profiles.



2D MHD SIMULATIONS OF AN INITIALLY STATIC PLASMA SET IN MOTION BY A SOURCE OF POLOIDAL MOMENTUM

SIM2D Simulations Are Based on the MHD Model.

We solve the standard resistive-MHD model *hyperbolic* system of time-dependent equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{V}) = 0, \qquad \text{(Continuity)}$$

$$\frac{\partial \rho \underline{V}}{\partial t} + \nabla \cdot (\rho \underline{V}\,\underline{V} - \underline{B}\,\underline{B} + P \underline{\underline{I}}) = \underline{S}_{mom}, \qquad \text{(Momentum)}$$

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{V} \times \underline{B} - \eta \underline{J}), \qquad \text{(Faraday's Law)}$$

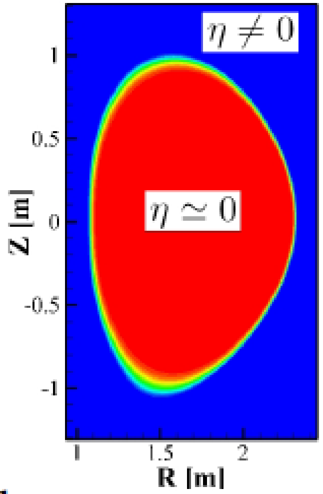
$$\frac{\partial \mathscr{E}}{\partial t} + \nabla \cdot [(\mathscr{E} + P)\underline{V} - \underline{B}(\underline{V} \cdot \underline{B}) + \eta \underline{J} \times \underline{B}] = \underline{V} \cdot \underline{S}_{mom}. \qquad \text{(Energy)}$$

$$P \equiv p + \frac{B^2}{2}, \qquad \mathscr{E} = \frac{p}{\gamma - 1} + \rho \frac{V^2}{2} + \frac{B^2}{2}. \qquad \text{(Definitions)}$$

The equations are written in conservative form to ensure conservation of physical quantities.

The "Halo" Region is Treated as a Resistive Plasma.

- Resistive-MHD equations are used to separate plasma and halo region.
- The main plasma region is ~ideal.
- The edge and halo region have non-negligible resistivity.

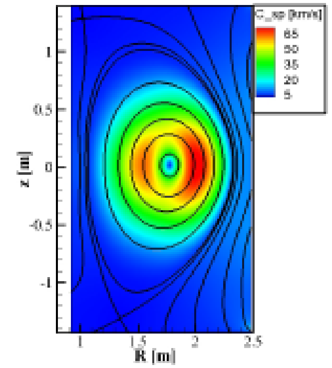


- Spitzer resistivity is used in the rest of this presentation.
- An artificially high halo resistivity was also tried, with $\eta_{halo}/\eta_{plasma} = 10^8$.
- Notice that the focus is on the main (ideal) plasma region.

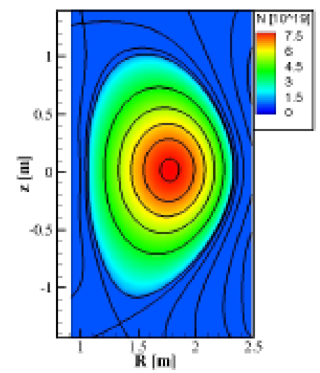
PPPL THEORY

Initial Poloidal Sound Speed Is Low.

- A typical free-boundary DIII-D equilibrium used in SIM2D simulations is shown on the right.
- Initial poloidal sound speed is small at the plasma edge.
- The boundary of the computational domain corresponds to a superconductive wall.
- Outflow boundary conditions ($v \sim C_s$) are used at the wall.

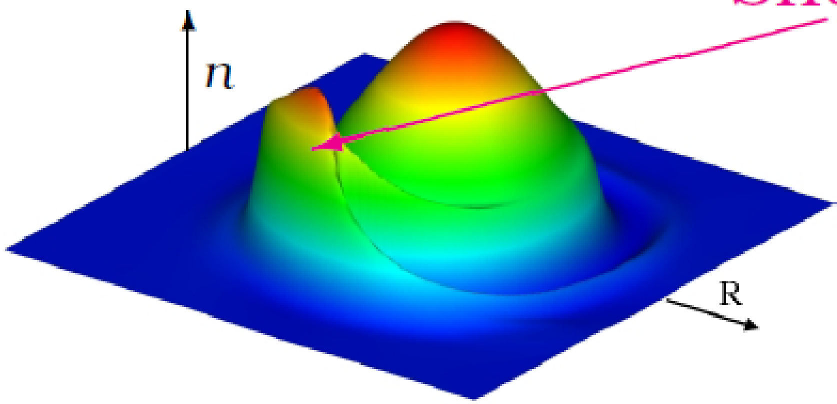


Poloidal sound speed [km/s] (colormap) and magnetic surfaces (lines)

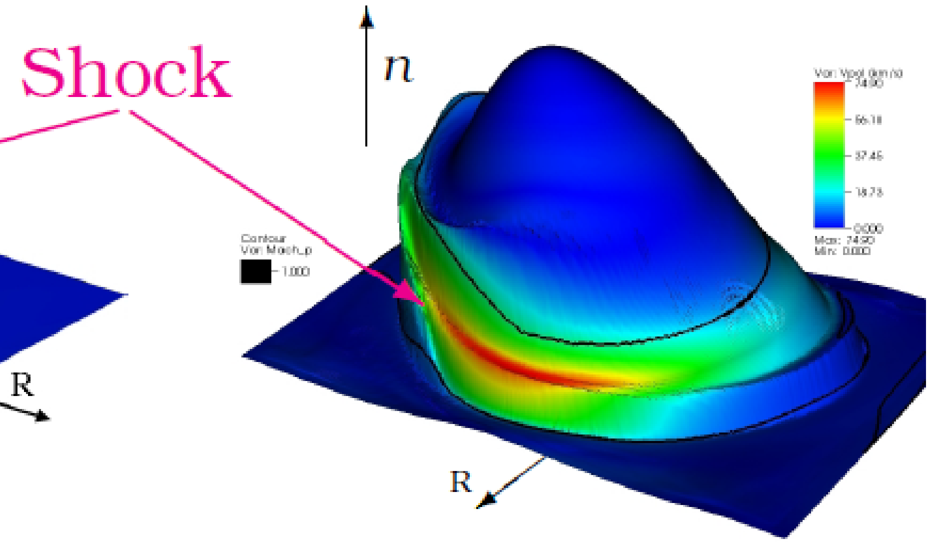


Density [$10^{19} m^{-3}$] (colormap) and magnetic surfaces (lines)

Initial Transient Is Consistent With Theory.



Fixed-boundary, ideal simulation.



Free-boundary DIII-D simulation.

- A shock is observed at the transonic surface.
- The shock travels in the poloidal direction from the outboard to the inboard part of the plasma.
- The shock vanishes at the inner midplane, where the flow is sonic.
- The shock is an MHD feature.

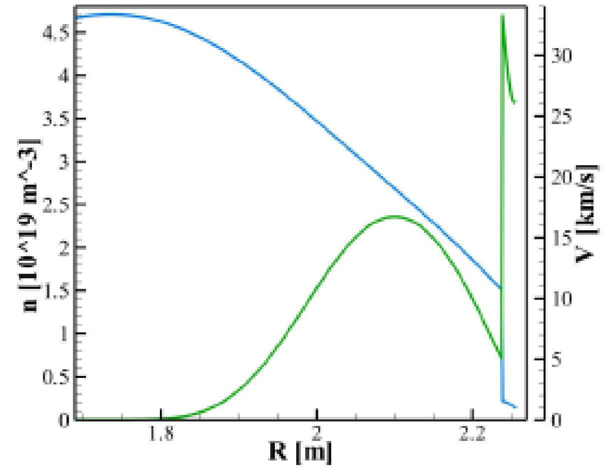
A Quasi-Steady-State MHD Pedestal Is Found In Fixed-Boundary Time-Dependent Simulations.

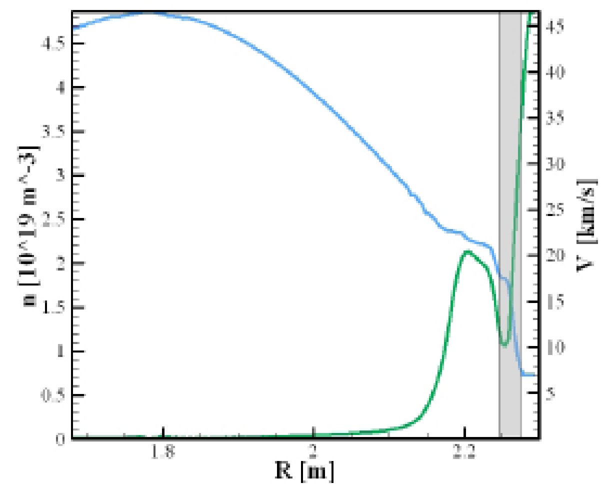
- Time-dependent simulations only reach an *approximate* steady state.
- At near-steady state, the density pedestal structure is clearly visible.
- The properties of the pedestal (e.g., poloidal angle height dependence) are in qualitative agreement with equilibrium calculations.

Transonic

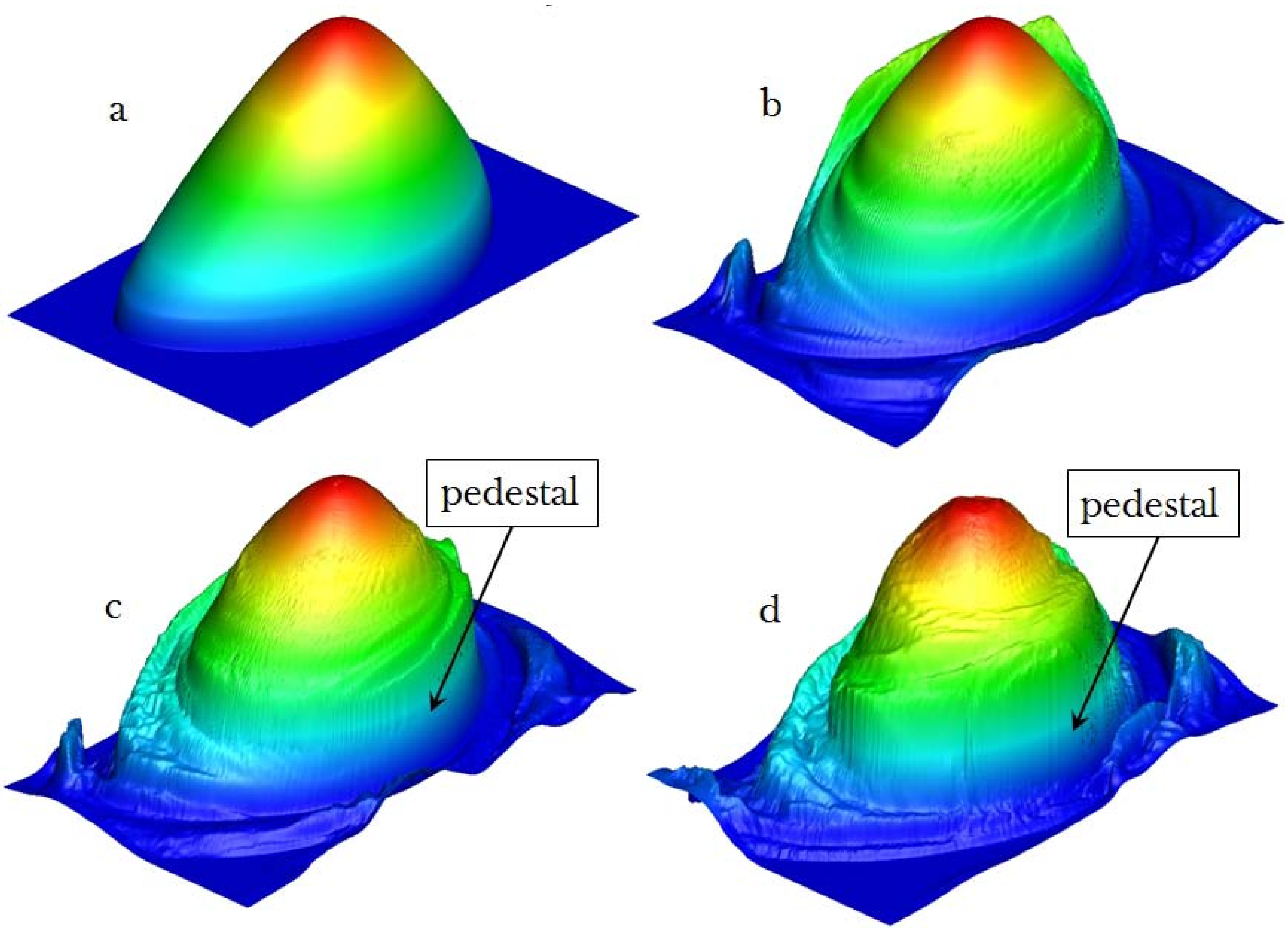
Fixed-Boundary Simulations Are Consistent with Equilibrium Calculations.

- After the initial transient, a discontinuity forms in e.g. density and velocity profiles.
- The discontinuity is qualitatively consistent with equilibrium calculations.
- This discontinuity is a tangential discontinuity, NOT a shock.

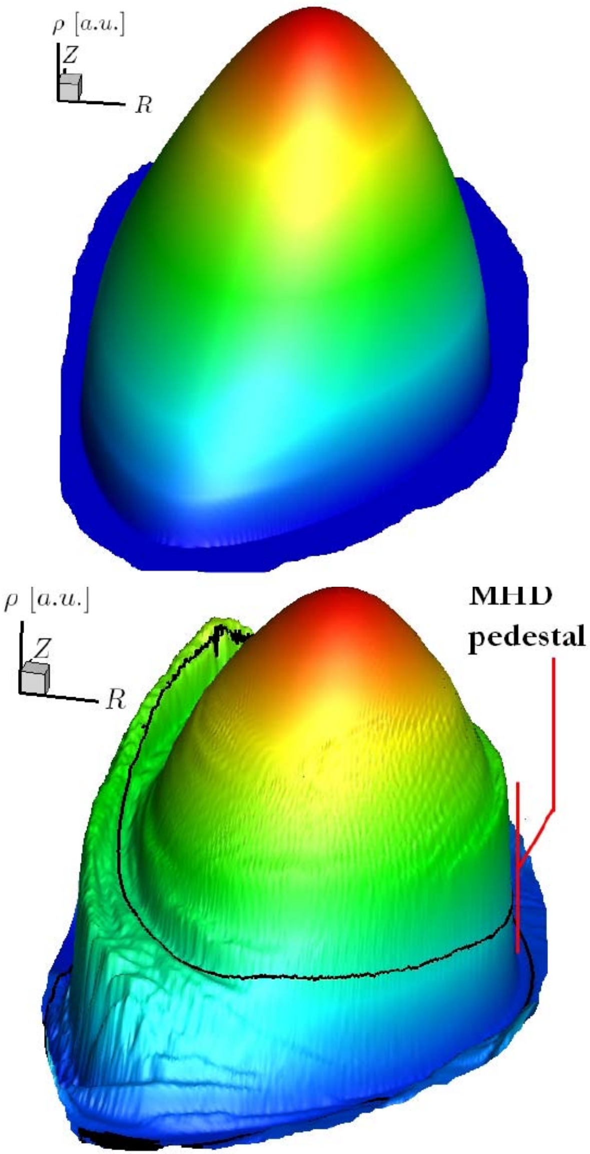


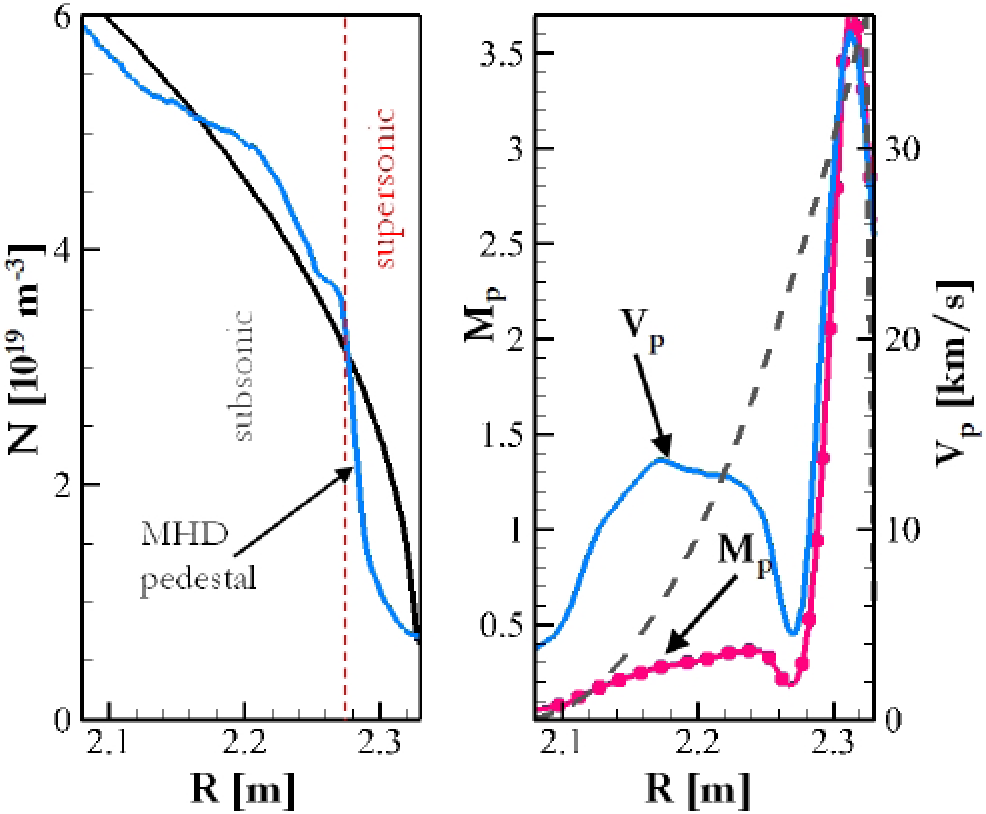


Evolution of the MHD pedestal in free boundary simulations

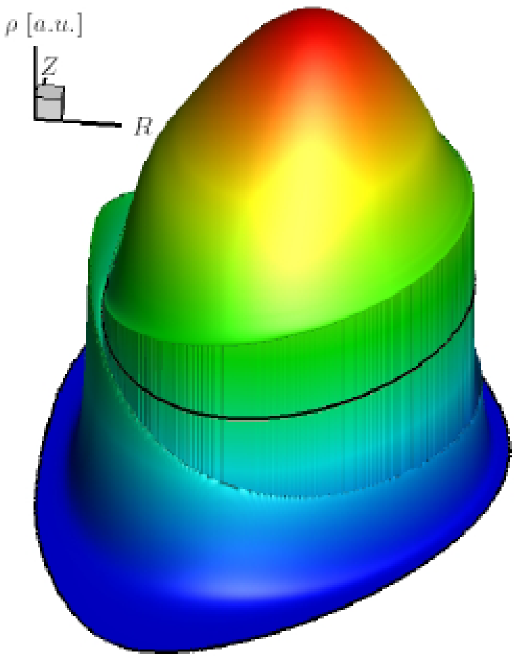


Details of the MHD pedestal

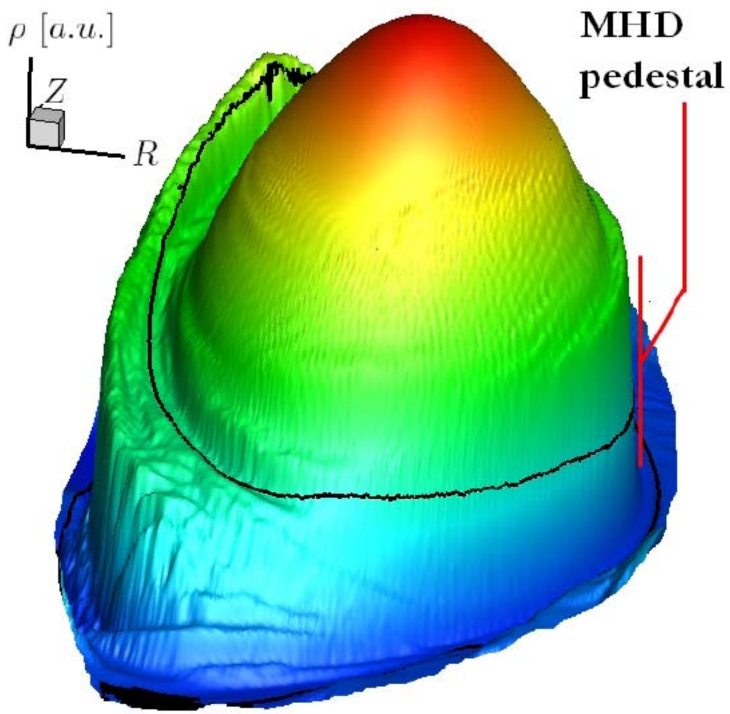




Similar pedestal can be obtained from the equilibrium solution



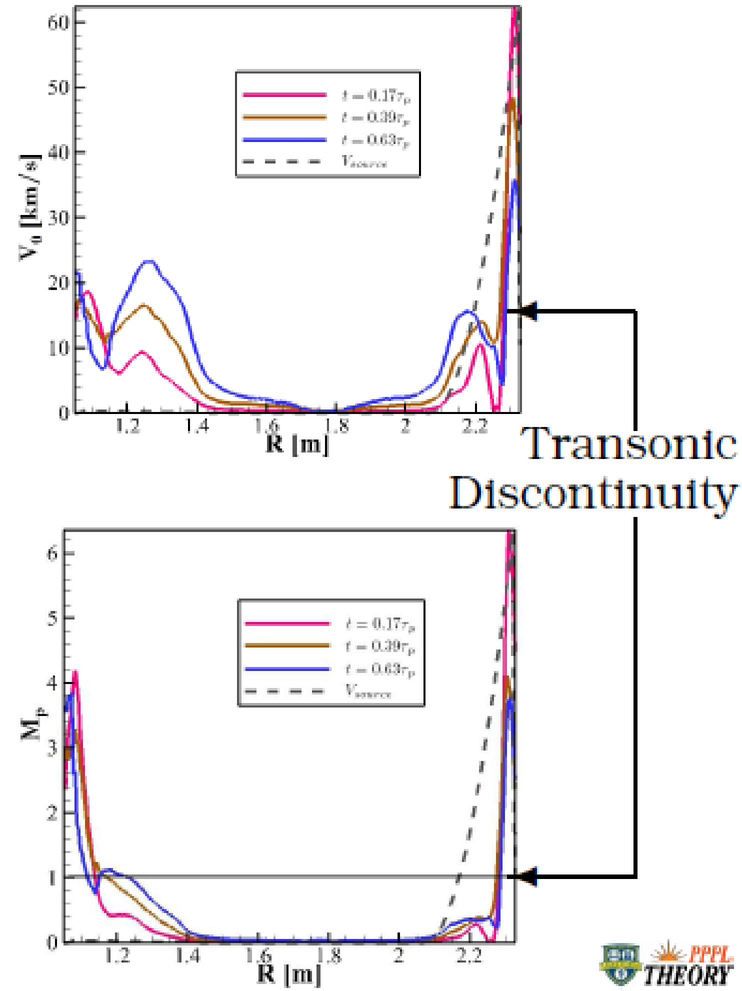
Transonic FLOW equilibrium.



Free Boundary

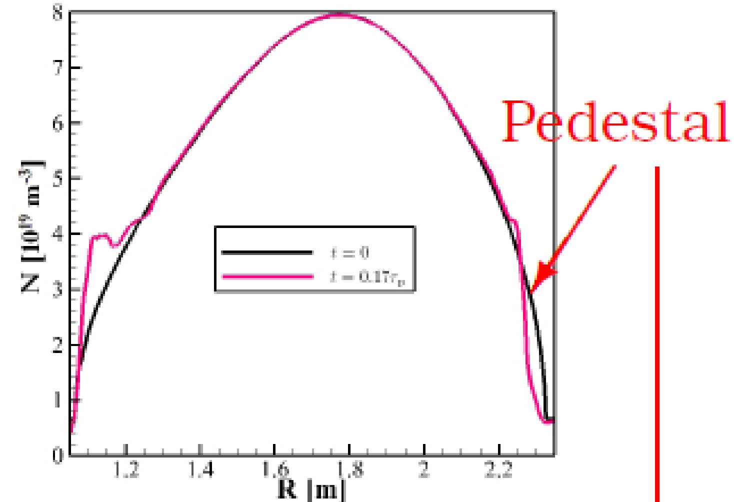
In Free-Boundary Simulations, Mach Number and Velocity Develop Discontinuous Profiles.

- A static DIII-D equilibrium is perturbed with a (smooth) poloidal momentum source.
- Velocity and Mach number quickly develop a discontinuous profile across the transonic surface.
- As predicted by theory, profiles are smooth on the inboard side, sharply discontinuous on the outboard side of the plasma (notice the $M_p = 1$ line!).

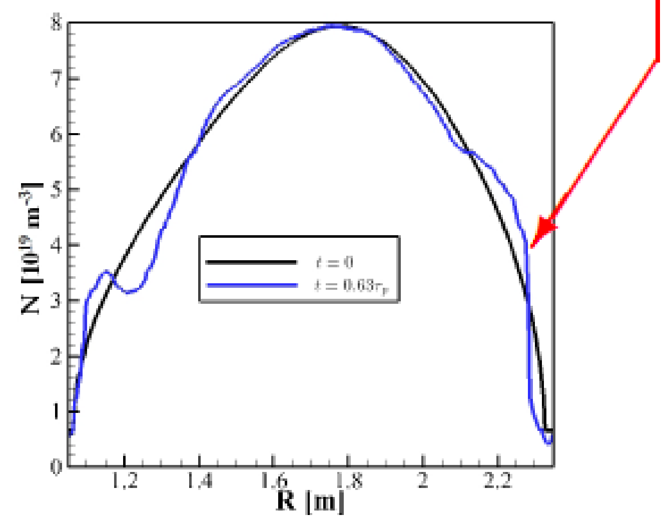


Density Profiles Develop a Pedestal Structure.

- When the flow becomes supersonic ($V_\theta > C_{sp}$) a shock forms and travels in the poloidal direction.
- After the shock formation, the density profile steepens.
- A shock-less, quasi-steady state condition is reached, with a density pedestal (sharp density gradient) at the transonic surface.

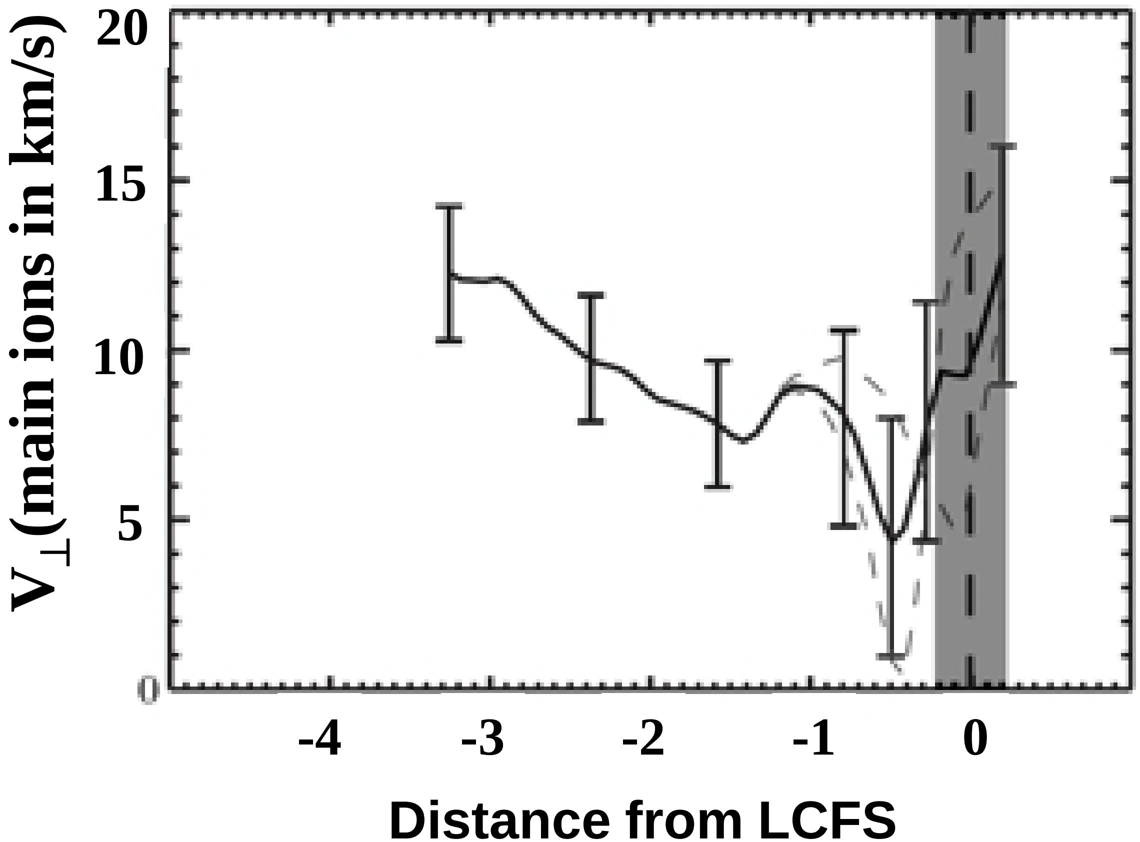


Density profile shortly after the shock formation.



Density profile after the shock has disappeared

Main ion perpendicular edge velocity on C-MOD



R. McDermott et al, Phys. Plasmas 16, 056103 (2009)

Conclusions

• Theory, equilibrium solutions and 2D free-boundary MHD simulations show the formation of a pedestal when the poloidal velocity exceeds the poloidal sound speed

•At the pedestal, a velocity radial-shear layer forms where the poloidal velocity jumps from subsonic to supersonic

• Such a layer is NOT a shock but a contact discontuinity

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5. L. Guazzotto, R. Betti *A magnetohydrodynamic mechanism for pedestal formation*, submitted for publication.