Gyrokinetic Large Eddy Simulations

A. Bañón Navarro[1], P. Morel[1], M. Albrecht-Marc[1], D. Carati[1], F. Merz[2], T. Görler[2], and F. Jenko[2]

[1]Laboratoire de Physique Statistique et des Plasmas
Université Libre de Bruxelles

[2]Max-Planck-Institut für PlasmaPhysik
Garching bei München

Wolfgang Pauli Institute, Vienna: Gyrokinetics for ITER II

Outline

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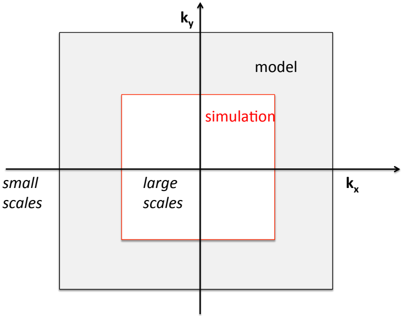
Local GK simulations: Perpendicular directions (respect to B) are spectral

Direct Numerical Simulation (DNS): resolves all active scales



Large Eddy Simulation (LES): resolves large scales + models smallest ones

objective: decrease numerical effort

GK DNS can go up to $N_x \times N_y \times N_z \times N_{v_\parallel} \times N_\mu = 768 \times 384 \times 16 \times 32 \times 8$

up to $100\,000\,CPUh$[1], and even more (Trinity).

[1] T. Görler, *et al*, *Phys. Rev. Lett.*, **100** (2008).

gyrokinetic equation

Distribution function $f(k_x, k_y, z, v_\parallel, v_\perp, t)$ evolution:

$$\partial_t f = \underbrace{L[f]}_{\substack{\text{linear drive} \\ \textit{local in } k_\perp}} + \underbrace{N[f,f]}_{\substack{\text{nonlinear } E\times B \text{ advection} \\ \textit{couples } k_\perp \textit{ to } k'_\perp \neq k_\perp}} - \underbrace{D[f]}_{\substack{\text{dissipations} \\ \textit{local in } k_\perp}}$$

Applying a Fourier cutoff filter $\overline{\cdots}$ in perp. plane $(k_x,\ k_y)$ to remove the smallest scales from the distribution leads to a closure problem:

$$\partial_t \overline{f} = L[\overline{f}] + N[\overline{f}, \overline{f}] - D[\overline{f}] - T$$

Sub-grid term: $T = N[\overline{f}, \overline{f}] - \overline{N[f,f]}$

Describes the effect of the under-resolved
scales on the largest scales

Sub-grid term: $T = N[\overline{f},\overline{f}] - \overline{N[f,f]}$

contains under-resolved information f

$\Rightarrow$ Comparisons: free energy diagnostics $\mathcal{E} \propto f^2$.

filtered GK equation:

$$\underbrace{\partial_t \overline{f}}_{\propto \overline{f}} = \underbrace{L[\overline{f}]}_{\propto \overline{f}} + \underbrace{N[\overline{f},\overline{f}]}_{\propto \overline{f}^2} - \underbrace{D[\overline{f}]}_{\propto \overline{f}} - \underbrace{T}_{\propto f^2}$$

filtered free energy equation:

$$\partial_t \underbrace{\mathcal{E}_{\overline{f}}}_{\propto \overline{f}^2} = \underbrace{\mathcal{G}_{\overline{f}}}_{\propto \overline{f}^2} + 0 - \underbrace{\mathcal{D}_{\overline{f}}}_{\propto \overline{f}^2} - \underbrace{\mathcal{T}_{\overline{T}}}_{\propto \overline{f} f^2}$$

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Sub-grid term: $T = N[\overline{f}, \overline{f}] - \overline{N[f,f]}$

contains under-resolved information f

$\Rightarrow$ Can we simply ignore the small scales ?

Free energy spectral density $\mathcal{E}^{k_x}$:

Reference *DNS* with $128 N_x \times 64 N_y$

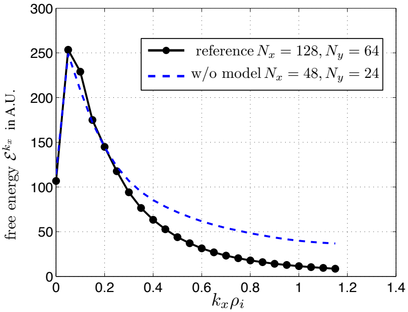
$$\partial_t \mathcal{E} = \mathcal{G} - \mathcal{D}$$

vs

Reduced *DNS* with $48 N_x \times 24 N_y$:

$$\partial_t \mathcal{E}_{\overline{f}} = \mathcal{G}_{\overline{f}} - \mathcal{D}_{\overline{f}} - \underbrace{\mathcal{T}_{\overline{T}}}_{\text{neglect}}$$

free energy accumulated at smallest scales



$\Rightarrow$ No, even if the filter does not remove many scales

Sub-grid term: $T = N[\overline{f}, \overline{f}] - \overline{N[f, f]}$

contains under-resolved information f

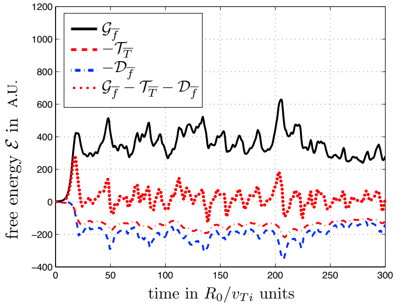
$\Rightarrow$ What is the role of the sub-grid term ?

consider DNS $128 N_x \times 64 N_y$
$+$ test filter $\overline{\Delta}_\perp$

consider resolved free energy:

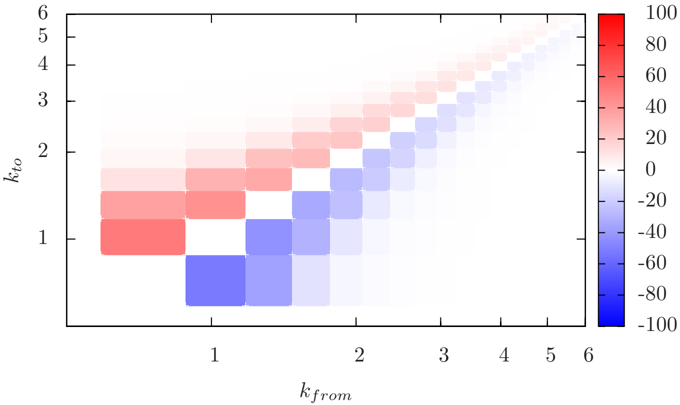
$$\partial_t \mathcal{E}_{\overline{f}} = \mathcal{G}_{\overline{f}} - \mathcal{D}_{\overline{f}} - \mathcal{T}_{\overline{T}}$$

with $\mathcal{T}_{\overline{T}}$ the sub-grid contribution



$\Rightarrow$ Sub-grid term dissipate free energy

Agreement with nonlinear free energy transfers studies[2]



Free energy is subject to a (strongly) local, forward cascade

[2]A. Bañón Navarro, *et al.* *Phys. Rev. Lett.*, **106**, 055001 (2011).

Model approximates sub-grids, depending only on *resolved* scales:

$$T \approx M[\overline{f}]$$

Model dissipates free energy

- perpendicular hyper-diffusions[3] $\propto k_\perp^4$:

$$M[c_\perp, \overline{f}] = c_\perp k_\perp^4 \overline{f}$$

- Free energy contribution:

$$\mathcal{T}_{\overline{T}} \approx \mathcal{M}$$

  Satisfies:

$$-\mathcal{M} < 0$$

[3] S. A. Smith and G. W. Hammett, *Phys. Plasmas*, **4** (1997).

Outline

Model: $$M[c_\perp, \overline{f}] = c_\perp k_\perp^4 \overline{f}$$

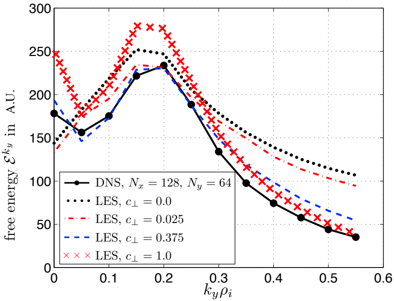
Unknown free parameter: $c_\perp$

Free energy spectra vs $c_\perp$:

k_y spectrum

Cyclone Base Case (ITG)

- ⋆ $c_\perp$ too small
  - $\Rightarrow$ not enough dissipation
- ⋆ $c_\perp$ too strong
  - $\Rightarrow$ overestimates injection
- ⋆ $c_\perp = 0.375$ good agreement
  - $\rightarrow$ "plateau" for $c_\perp \in [0.25, 0.625]$



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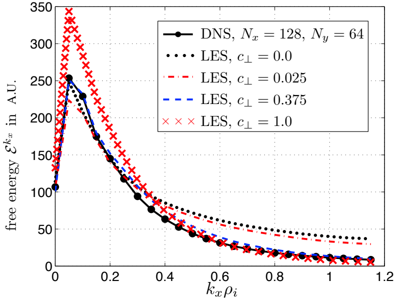
k_x spectrum



Numerical cost $\sim DNS/30$

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1. Introduction to GyroLES
   - General principles
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4. Future work
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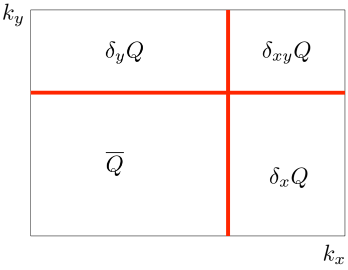
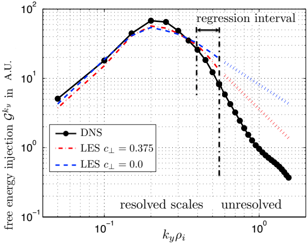
Global quantities (Q) are truncated: heat flux, ...



- ⋆ In a LES, only the resolved part of Q is directly accessible $\rightarrow \overline{Q}$
- ⋆ The unresolved part of Q has to be estimated $\rightarrow \delta Q$
- ⋆ Q can be approximated by decaying power laws in the LES spectra:

$$Q^{k_x} \approx A_x\, k_x^{-\alpha_x} \qquad Q^{k_y} \approx A_y\, k_y^{-\alpha_y}$$

$\rightarrow \delta_{xy} Q << \delta_x Q, \delta_y Q$



$$Q \approx \overline{Q} + \delta_x Q + \delta_y Q \approx \overline{Q} + \sum_{|k_x|>K_x^{\text{LES}}}^{K_x^{\text{DNS}}} A_x\, k_x^{-\alpha_x} + \sum_{|k_y|>K_y^{\text{LES}}}^{K_y^{\text{DNS}}} A_y\, k_y^{-\alpha_y}$$

$$\mathcal{G}^{\text{LES}} = 1.11\,\mathcal{G}^{\text{DNS}} \qquad \mathcal{G}^{\text{No Model}} = 1.38\,\mathcal{G}^{\text{DNS}}$$

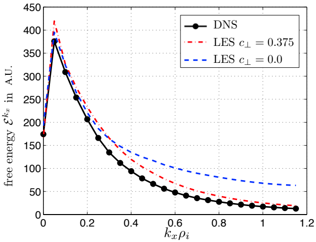
$$\mathcal{E}^{\text{LES}} = 1.10\,\mathcal{E}^{\text{DNS}} \qquad \mathcal{E}^{\text{No Model}} = 2.1\,\mathcal{E}^{\text{DNS}}$$

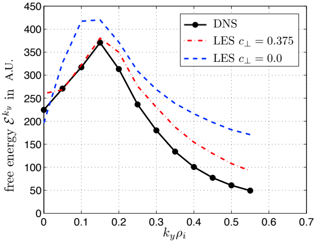
$\Rightarrow$ Good agreement with model

Outline

Robustness of $c_\perp = 0.375$?

Comparison between GyroLES and DNS - $\omega_{Ti} = 8.0$.



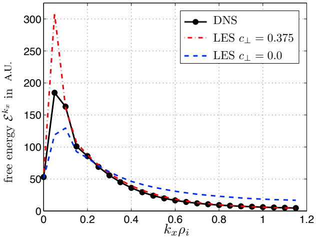


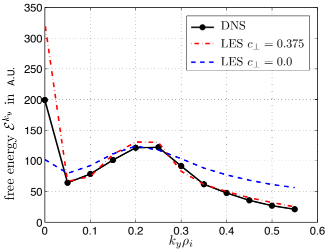
- ⋆ strong turbulence: good agreement
- ⋆ slightly overestimate of the free energy at small scales

  $\rightarrow$ $c_\perp$ should be increased a little

Robustness of $c_\perp = 0.375$?

Comparison between GyroLES and DNS - $\omega_{Ti} = 6.0$.





- ⋆ weak turbulence: less satisfactory agreement

  → $c_\perp$ should be decreased

develop alternative methods: dynamic calibration

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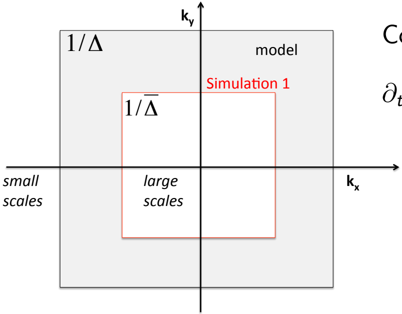
model coefficient has to vary when varying external parameters

$\Rightarrow$ Calibrate model parameters **dynamically**

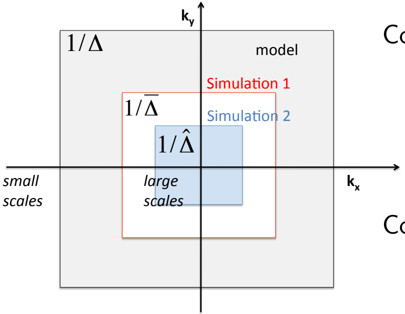


Consider filtered simulation ($\overline{\Delta}$):

$$\partial_t \overline{f} = L[\overline{f}] + N[\overline{J_0\phi}, \overline{f}] - M[c, \overline{\Delta}, \overline{f}]$$

model coefficient has to vary when varying external parameters

$\Rightarrow$ Calibrate model parameters **dynamically**



Consider filtered simulation ($\overline{\Delta}$):

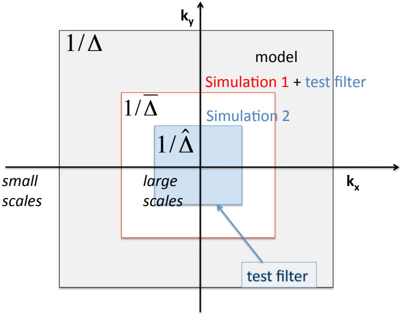
$$\partial_t \overline{f} = L[\overline{f}] + N[\overline{J_0\phi}, \overline{f}] - M[c, \overline{\Delta}, \overline{f}]$$

Consider filtered simulation ($\widehat{\Delta}$):

$$\partial_t \widehat{f} = L[\widehat{f}] + N[\widehat{J_0\phi}, \widehat{f}] - M[c, \widehat{\Delta}, \widehat{f}]$$

model coefficient has to vary when varying external parameters

$\Rightarrow$ Calibrate model parameters **dynamically**



Consider filtered simulation ($\overline{\Delta}$):

$$\partial_t \overline{f} = L[\overline{f}] + N[\overline{J_0\phi}, \overline{f}] - M[c, \overline{\Delta}, \overline{f}]$$

$\Rightarrow$ Apply a test-filter $\widehat{\cdots}$, ($\widehat{\overline{\cdots}} = \widehat{\cdots}$):

$$\partial_t \widehat{f} = L[\widehat{f}] + N[\widehat{J_0\phi}, \widehat{f}] - T_{\overline{\Delta},\widehat{\Delta}} - \widehat{M[c, \overline{\Delta}, \overline{f}]}$$

Consider filtered simulation ($\widehat{\Delta}$):

$$\partial_t \widehat{f} = L[\widehat{f}] + N[\widehat{J_0\phi}, \widehat{f}] - M[c, \widehat{\Delta}, \widehat{f}]$$

model coefficient has to vary when varying external parameters

$\Rightarrow$ Calibrate model parameters **dynamically**

Minimize difference between GyroLES with $\widehat{\Delta}$ and GyroLES with $\overline{\Delta}$ and test filter:

$$\int d\overline{\Lambda}\, \partial_c \left(M[c, \widehat{\Delta}, \widehat{f}] - \widehat{M[c, \overline{\Delta}, \overline{f}]} - T_{\overline{\Delta}, \widehat{\Delta}} \right)^2 \approx 0$$

$\Rightarrow$ optimized free parameter c

$\Rightarrow$ can be done for many parameters

$$\begin{aligned} M[c_\perp, \Delta, f] &= c_\perp \Delta^a k_\perp^n f \\ M[c_x, c_y, \Delta, f] &= c_x \Delta^a k_x^n f + c_y \Delta^a k_y^n f \Rightarrow \text{Anisotropy} \end{aligned}$$

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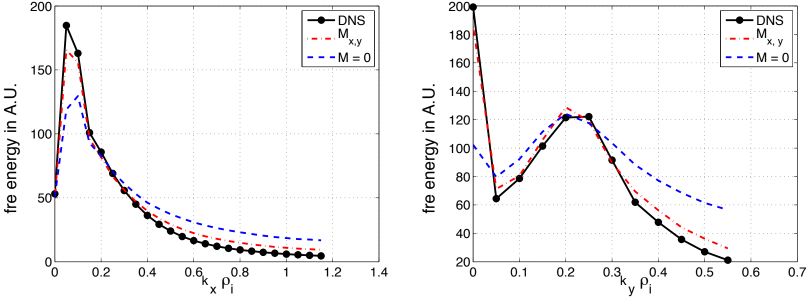
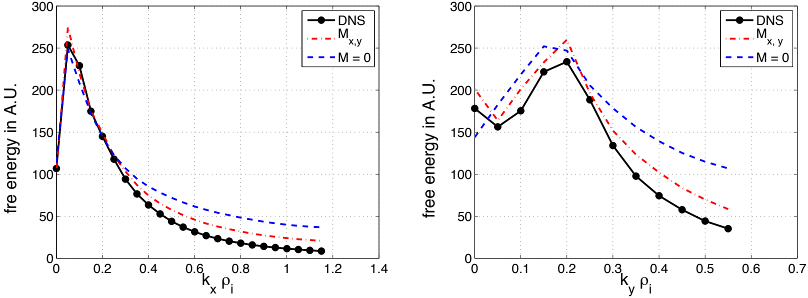


⋆ weak turbulence: good agreement

Comparison between GyroLES and DNS - $\omega_{Ti} = 8.0$.



- ⋆ strong turbulence: good agreement
- ⋆ slightly overestimate of the free energy at small scales

Numerical cost $\sim DNS/20$

Future work

- Test dynamic procedure for different set of parameters: magnetic shear $\hat{s}$, safety factor q ... (analyses in progress)
- Studies with ETG driven turbulence
- Studies with two kinetic species
- Implementation of more sophisticated models

Summary

- Model is needed even if resolution is not decreased dramatically
- Analysis of DNS shows that models have to dissipate free energy
- A simple model $M = c_\perp k_\perp^4 f$ has been successfully tested
- $c_\perp \simeq 0.375$ has been calibrated by trial and error for a given set of parameters [4]
- Dynamic calibration of the amplitude of the model has been successfully tested for some parameters.

[4] P. Morel, *et al*, submitted to Physics of Plasmas

Thank you