Sources of intrinsic rotation in the low flow ordering

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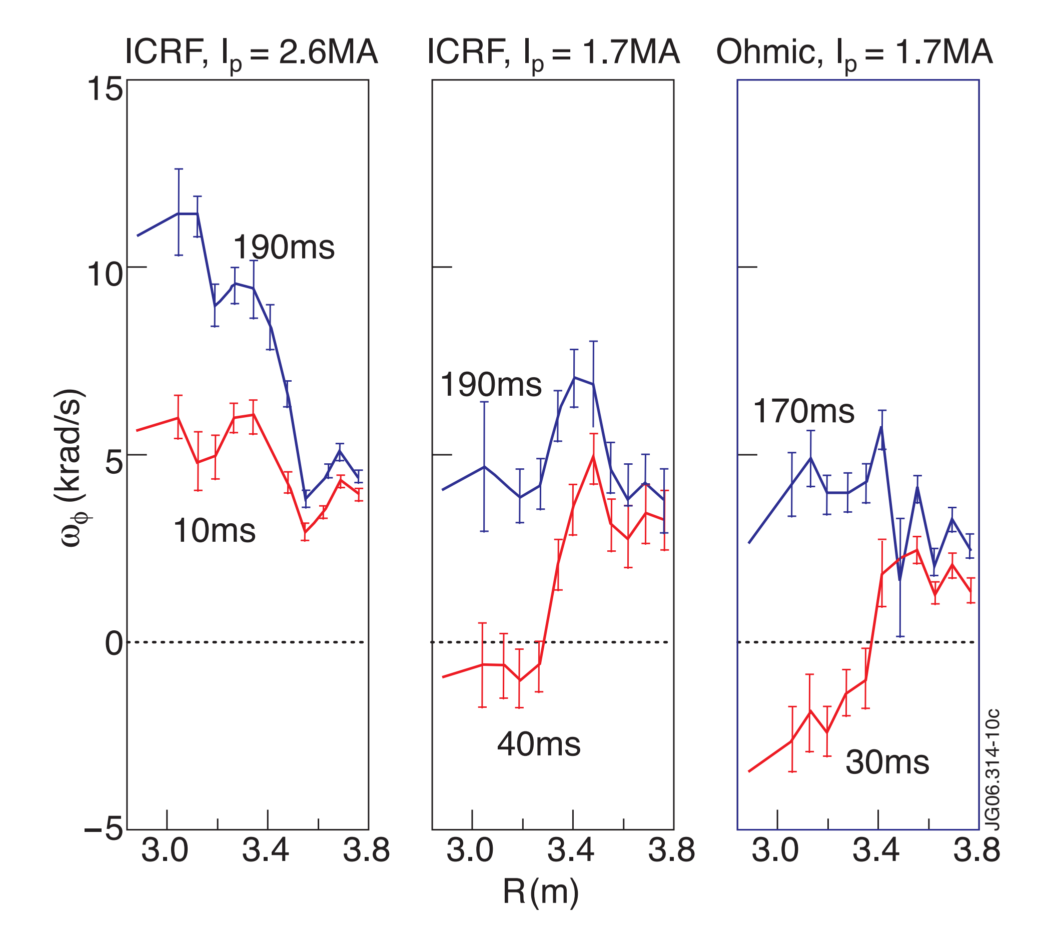
Introduction

- In the absence of obvious momentum input (apart from the edge), tokamaks rotate
- One possible explanation is momentum from the edge being “pinched” into the core
- In this talk, terms that are wrongly neglected explain intrinsic rotation based on gradients of T_i, n_e and T_e, and the sources of heating
- Quantitative predictions still underway

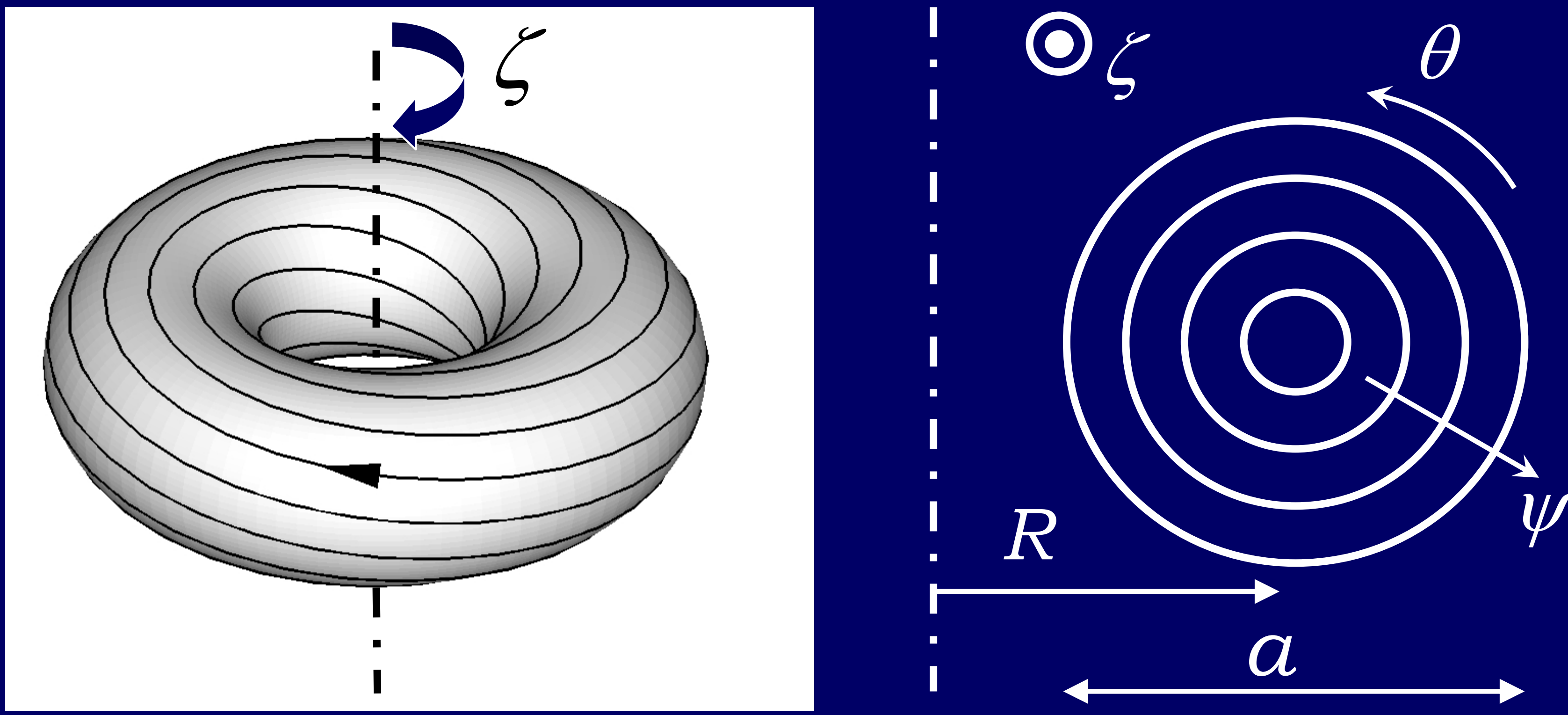
Experimental evidence



Courtesy of M. F. Nave

- Core Ohmic: hollow counter-rotating
- Core ICRF
  - High I_p: peaked, co-rotating
  - Low I_p: hollow, counter-rotating
- Edge: co-rotating independent of scenario

Geometry



- Here, ζ is in the co-current direction
- Flux surface average $\langle \ldots \rangle_\psi = (V')^{-1} \int d\theta d\zeta (\ldots)/\mathbf{B}\cdot\nabla\theta$

Intrinsic rotation

- Assume electrostatics to simplify: $\mathbf{E} = -\nabla\phi$
- Conservation of total toroidal angular momentum
  - $\mathbf{J}\times\mathbf{B}$ force vanishes due to axisymmetry and $n_i = n_e$

$$\frac{\partial}{\partial t}\left\langle n_i M R \mathbf{V}_i \cdot \hat{\zeta} \right\rangle_\psi = -\frac{1}{V'}\frac{\partial}{\partial \psi} V' \left\langle R\hat{\zeta}\cdot \overleftrightarrow{\boldsymbol{\pi}}_i \cdot \nabla\psi \right\rangle_\psi + \frac{1}{c}\left\langle R\hat{\zeta}\cdot\left(\mathbf{J}\times\mathbf{B}\right)\right\rangle_\psi$$

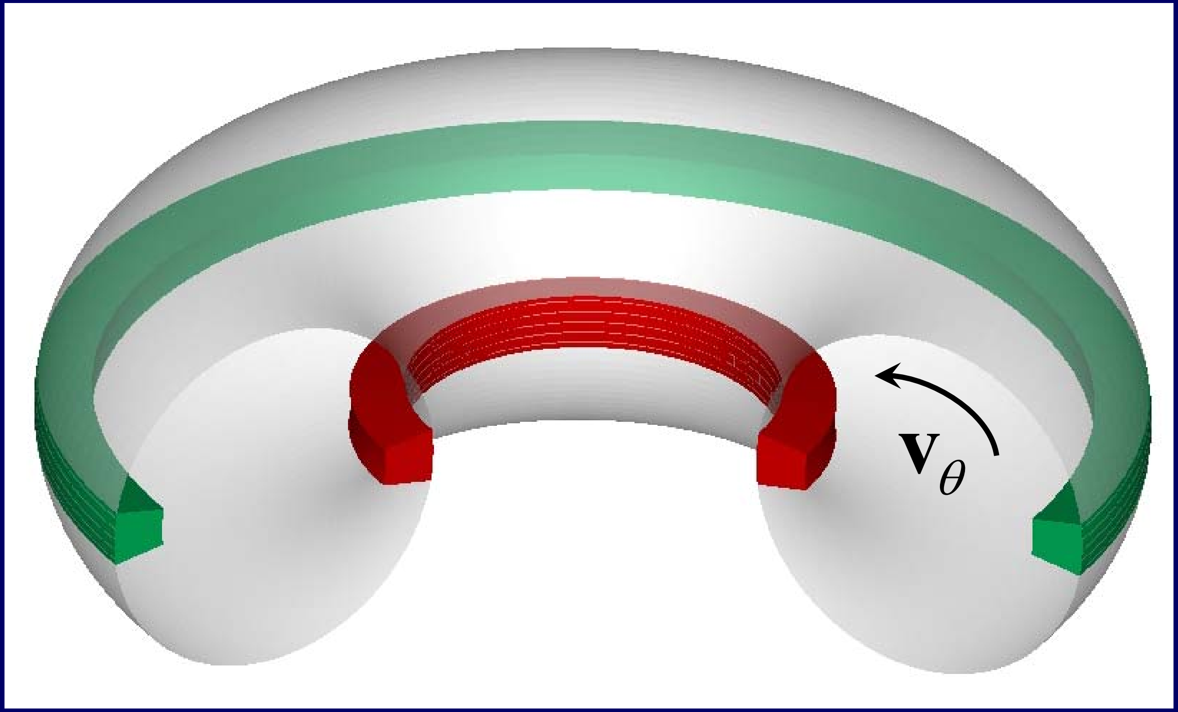
  - Here $\left\langle R\hat{\zeta}\cdot \overleftrightarrow{\boldsymbol{\pi}}_i \cdot \nabla\psi \right\rangle_\psi = \left\langle \int d^3 v\, f_i\, RM \left(\mathbf{v}\cdot\hat{\zeta}\right)\left(\mathbf{v}\cdot\nabla\psi\right)\right\rangle_\psi$

- $\partial/\partial t = 0$, no momentum input $\Rightarrow$ $\left\langle R\hat{\zeta}\cdot \overleftrightarrow{\boldsymbol{\pi}}_i \cdot \nabla\psi \right\rangle_\psi = 0$

High flow ordering

High flow ordering (I)

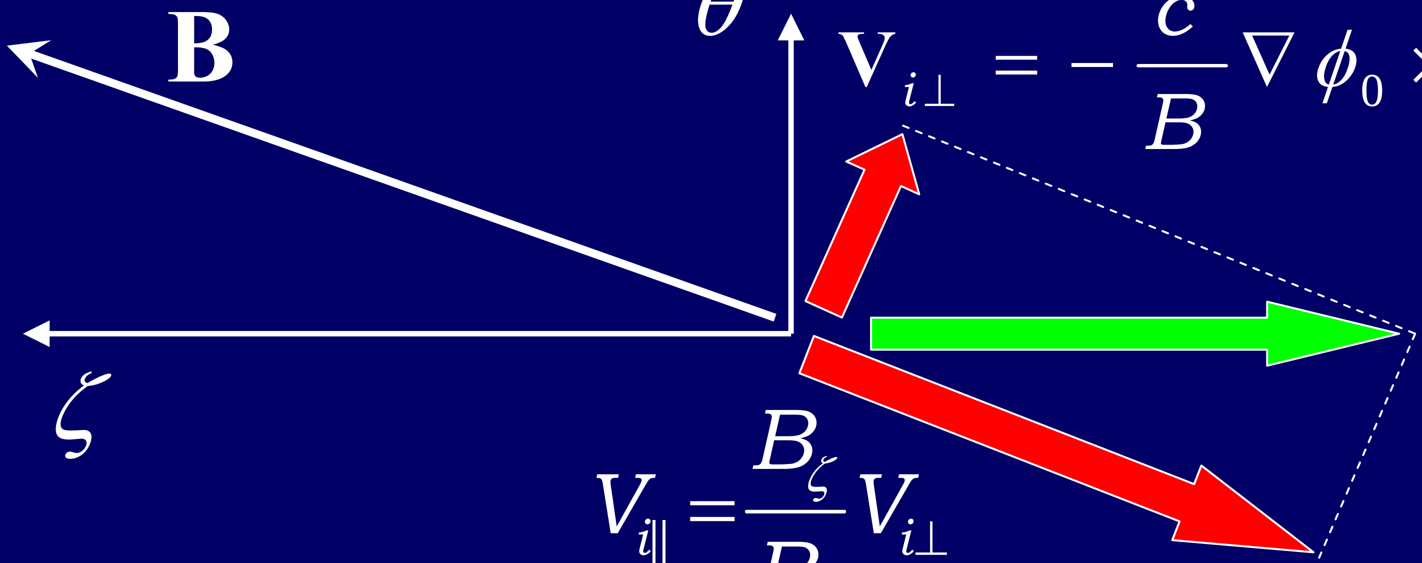
- $\mathbf{E} \sim \frac{1}{c} \upsilon_{ti} B \Rightarrow \mathbf{V}_{i\perp} \cong \frac{c}{B} \mathbf{E} \times \hat{\mathbf{b}} \sim \upsilon_{ti}$
- Parallel velocity



- Poloidal flow
  $\Rightarrow$ compression
  $\Rightarrow$ damping
- Parallel velocity
  $\Rightarrow \mathbf{v}_\theta = 0$

High flow ordering (II)

- To make $\mathbf{v}_\theta = 0$



$$\mathbf{V}_{i\perp} = -\frac{c}{B}\nabla\phi_0 \times \hat{\mathbf{b}}$$

$$V_{i\|} = \frac{B_\zeta}{B_\theta} V_{i\perp}$$

$$\mathbf{V}_i = -cR\hat{\boldsymbol{\zeta}}\frac{\partial \phi_0}{\partial \psi} \equiv \omega_\zeta R\hat{\boldsymbol{\zeta}}$$

δf simulations

- Using $f_i(\mathbf{R}, \varepsilon, \mu, t)$, with $\varepsilon = \upsilon^2/2 + e\phi/M$, $\mu = \upsilon_\perp{}^2/2B$

$$\frac{\partial f_i}{\partial t} + \left(\upsilon_\parallel \hat{\mathbf{b}} + \mathbf{v}_M + \mathbf{v}_E + \ldots\right)\cdot \nabla_{\mathbf{R}} f_i + \left(\frac{e}{M}\frac{\partial\langle\phi\rangle}{\partial t} + \ldots\right)\frac{\partial f_i}{\partial \varepsilon} = \left\langle C\{f_i\}\right\rangle$$

- Since fluctuation << 1, use $f_i = F_i + f_{i1}{}^{\text{tb}}$, $\phi = \phi_0 + \phi_1{}^{\text{tb}}$

$$\frac{\partial f_{i1}^{\text{tb}}}{\partial t} + \left(\upsilon_\parallel \hat{\mathbf{b}} + \mathbf{v}_M + \mathbf{v}_{E0} + \mathbf{v}_{E1}^{\text{tb}}\right)\cdot \nabla_{\mathbf{R}} f_{i1}^{\text{tb}} - \left\langle C^{(\ell)}\left\{f_{i1}^{\text{tb}}\right\}\right\rangle$$

$$= -\mathbf{v}_{E1}^{\text{tb}} \cdot \nabla_{\mathbf{R}} F_i - \frac{e}{M}\frac{\partial\left\langle\phi_1^{\text{tb}}\right\rangle}{\partial t}\frac{\partial F_i}{\partial \varepsilon}$$

  - Similar equation for electrons

- Potential found from $\int d^3\upsilon\, f_{i1}{}^{\text{tb}} - \int d^3\upsilon\, f_{e1}{}^{\text{tb}} + \ldots = 0$

δf flux tube simulations

- Turbulence with $k_\perp \rho_i \sim 1$, equilibrium with $k_\perp \rho_i << 1$

$\Rightarrow \nabla_{\mathbf{R}} F_i$, $\partial F_i / \partial \varepsilon$ = constant in simulation domain

$\Rightarrow$ Fourier analyze $f_{i1}{}^{\mathrm{tb}}$, $f_{e1}{}^{\mathrm{tb}}$ and $\phi_1{}^{\mathrm{tb}}$ in $\mathbf{R}_\perp$

- Most δf flux tubes in the high flow ordering

$$F_i = n_i \left(\frac{M}{2\pi T_i} \right)^{3/2} \exp\left[-\frac{M\left(v_{\|} - V_{i\|}\right)^2 + M v_\perp^2}{2T_i} \right] \cong f_{Mi} \left(1 + \frac{B_\zeta}{B} \frac{M v_{\|} \omega_\zeta R}{T_i} \right)$$

$\Rightarrow f_{i1}{}^{\mathrm{tb}}$, $f_{e1}{}^{\mathrm{tb}}$, $\phi_1{}^{\mathrm{tb}}$ depend on $\omega_\zeta = -c \dfrac{\partial \phi_0}{\partial \psi}$, $\dfrac{\partial \omega_\zeta}{\partial \psi} = -c \dfrac{\partial^2 \phi_0}{\partial \psi^2}$

Momentum transport for high flows

- Using a moment of the full Fokker-Planck equation

$$\left\langle R\hat{\boldsymbol{\zeta}}\cdot\overleftrightarrow{\boldsymbol{\pi}}_i\cdot\nabla\psi\right\rangle_\psi = -\left\langle \frac{c}{B}\left(\nabla\phi_1^{\text{tb}}\times\hat{\mathbf{b}}\right)\cdot\nabla\psi\int d^3v\, f_{i1}^{\text{tb}} RM\left(\mathbf{v}\cdot\hat{\boldsymbol{\zeta}}\right)\right\rangle_\psi$$

- From gyrokinetic equation

$$\left\langle R\hat{\boldsymbol{\zeta}}\cdot\overleftrightarrow{\boldsymbol{\pi}}_i\cdot\nabla\psi\right\rangle_\psi = \Pi\left(\omega_\zeta, \frac{\partial\omega_\zeta}{\partial\psi};\quad \frac{\partial T_i}{\partial\psi}, \frac{\partial n_e}{\partial\psi}, \frac{\partial T_i}{\partial\psi}, \ldots\right)$$

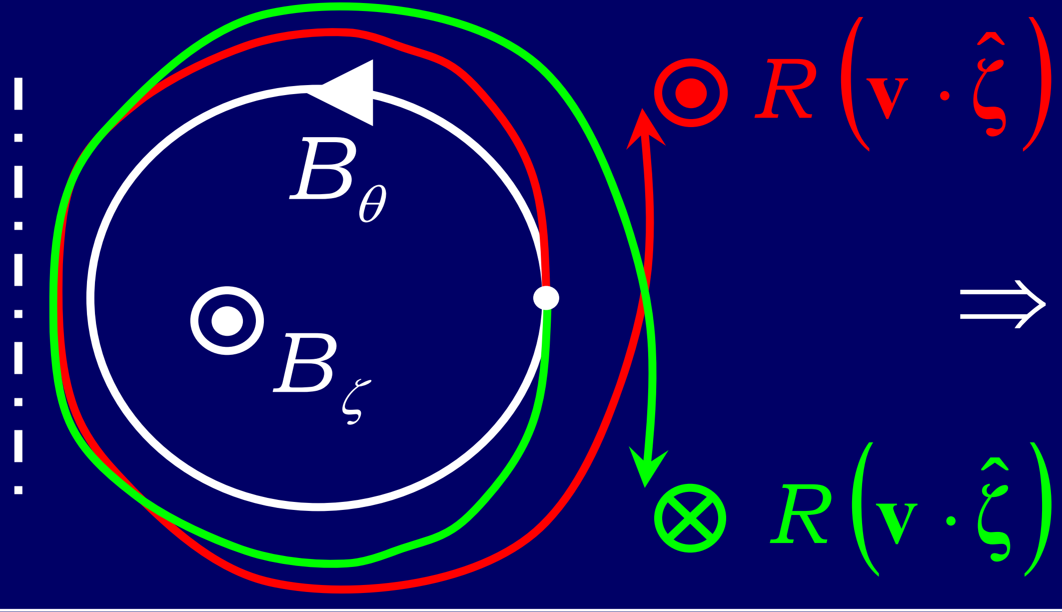
  - Linearize with respect to ω_ζ, $\partial\omega_\zeta/\partial\psi \Rightarrow$ pinch, diffusion

- Pinch from edge cannot explain rotation $\to 0$

$$\left\langle R\hat{\boldsymbol{\zeta}}\cdot\overleftrightarrow{\boldsymbol{\pi}}_i\cdot\nabla\psi\right\rangle \approx P_\zeta\omega_\zeta - \chi_\zeta\frac{\partial\omega_\zeta}{\partial\psi} = 0 \Rightarrow \frac{\partial\omega_\zeta}{\partial\psi} = 0 \text{ for } \omega_\zeta = 0$$

Intrinsic rotation at high flows

- What if $\Pi_0 = \Pi(\omega_\zeta = 0,\ \partial\omega_\zeta/\partial\psi = 0; \ldots) \neq 0$?
  $\Rightarrow$ intrinsic rotation without edge input
- $\Pi_0 = 0$ for up-down symmetry!
  - Not rigorously proven, but observed in codes



$$\Rightarrow \left\langle R\hat{\boldsymbol{\zeta}}\cdot\overleftrightarrow{\boldsymbol{\pi}}_i\cdot\nabla\psi\right\rangle_\psi = 0$$

Low flow ordering

Low flow ordering (I)

- $\mathbf{V}_{i\perp} = \frac{c}{B}\left(\mathbf{E} - \frac{1}{en_i}\nabla p_i\right)\times\hat{\mathbf{b}} \sim \mathbf{v}_M \sim \delta_i \upsilon_{ti} << \upsilon_{ti}$

  with $\delta_i = \rho_i / a << 1$

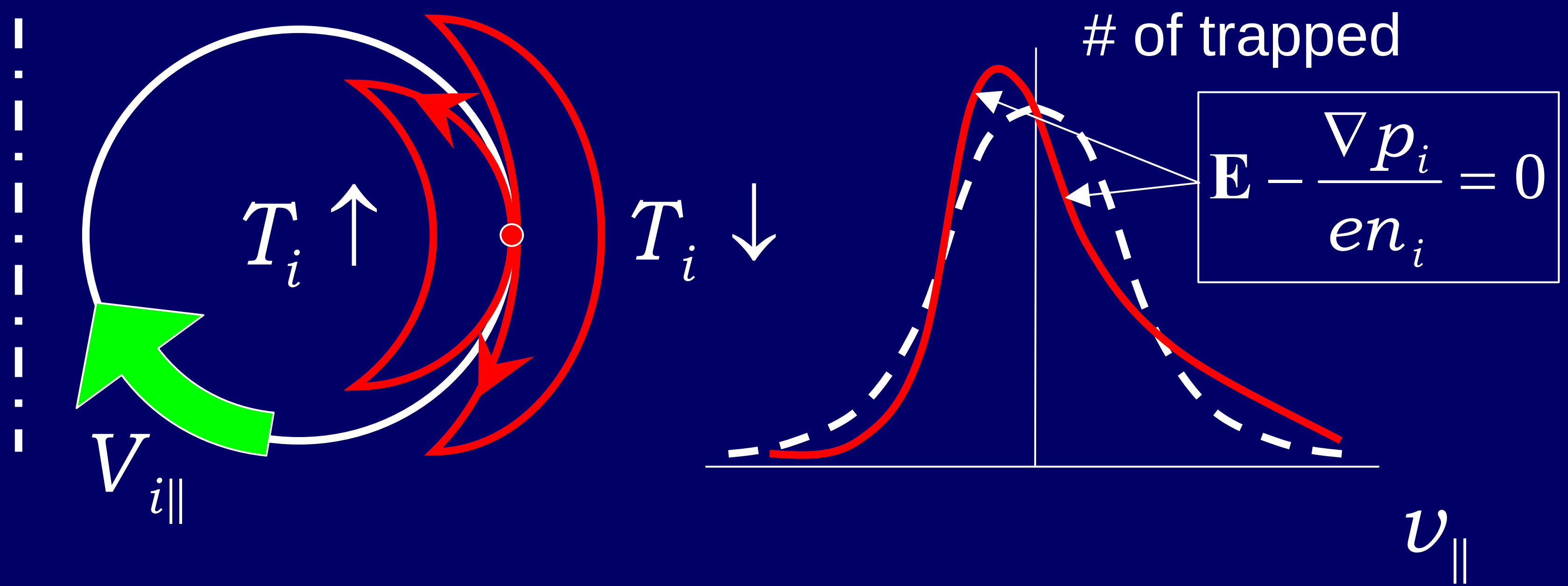
- Velocity responds to compression BUT ∇T_i matters

$$\mathbf{V}_{i\perp} = \frac{c}{B}\hat{\mathbf{b}}\times\left(\nabla\phi + \frac{1}{en_i}\nabla p_i\right) \Rightarrow V_{i\|} = \frac{B_\zeta}{B_\theta}V_{i\perp} + U_\| B$$

$$\mathbf{V}_i = -cR\hat{\zeta}\left(\frac{\partial\phi_0}{\partial\psi} + \frac{1}{en_i}\frac{\partial p_i}{\partial\psi}\right) + \frac{kcRB_\zeta}{\left\langle B^2\right\rangle_\psi}\frac{\partial T_i}{\partial\psi}\mathbf{B}$$

Low flow ordering (II)

- Effect of ∇T_i for $\mathbf{E} - \nabla p_i / en_i = 0$
  - Equivalent to moving to a rotating frame



  - Collision drive towards Maxwellian

Trapped/passing friction ($\nu_{ii}\uparrow$ for $v\downarrow$) $\Rightarrow V_{i\|} \propto \partial T_i / \partial \psi$

Neoclassical first order correction

- To lowest order $f_i = f_{Mi}$ (stationary Maxwellian)
- To obtain the neoclassical piece

$$\upsilon_{\|}\hat{\mathbf{b}}\cdot\nabla F_{i1}^{\text{nc}} - C_{ii}^{(\ell)}\left\{F_{i1}^{\text{nc}}\right\} = -\mathbf{v}_M\cdot\nabla f_{Mi}$$

$$\Rightarrow F_{i1}^{\text{nc}} = \frac{B_\zeta}{B}\frac{M\upsilon_{\|}\omega_\zeta R}{T_i} f_{Mi} + \frac{McRB_\zeta \upsilon_{ti}}{e\sqrt{\left\langle B^2\right\rangle_\psi}}\frac{\partial T_i}{\partial\psi} h_{i1}^{\text{nc}} \sim \frac{B}{B_\theta}\delta_i f_{Mi}$$

$$\text{with}\quad \omega_\zeta = -c\left(\frac{\partial\phi_0}{\partial\psi} + \frac{1}{en_i}\frac{\partial p_i}{\partial\psi}\right)$$

  - $\nabla T_i \Rightarrow$ both parallel flows and parallel heat flows

Momentum transport for low flows

- Using moments of the full Fokker-Planck equation and ignoring neoclassical transport of momentum

$$\left\langle R\hat{\boldsymbol{\zeta}}\cdot\overleftrightarrow{\boldsymbol{\pi}}_i\cdot\nabla\psi\right\rangle_\psi = Mc\left\langle \frac{\partial\phi_1^{\mathrm{tb}}}{\partial\zeta}\int d^3v\, f_{i2}^{\mathrm{tb}} R\left(\mathbf{v}\cdot\hat{\boldsymbol{\zeta}}\right) + \frac{\partial\phi_2^{\mathrm{tb}}}{\partial\zeta}\int d^3v\, f_{i1}^{\mathrm{tb}} R\left(\mathbf{v}\cdot\hat{\boldsymbol{\zeta}}\right)\right\rangle_\psi$$

$$+\frac{Mc}{2Ze}\left\langle R^2\right\rangle_\psi \frac{\partial p_i}{\partial t} + \frac{M^2c^2}{2Ze}\frac{1}{V'}\frac{\partial}{\partial\psi}V'\left\langle \frac{\partial\phi_1^{\mathrm{tb}}}{\partial\zeta}\int d^3v\, f_{i1}^{\mathrm{tb}} R^2\left(\mathbf{v}\cdot\hat{\boldsymbol{\zeta}}\right)^2\right\rangle_\psi$$

$$-\frac{M^2c}{2Ze}\left\langle \int d^3v\, C_{ii}^{(\ell)}\left\{F_{i2}^{\mathrm{tb}}\right\} R^2\left(\mathbf{v}\cdot\hat{\boldsymbol{\zeta}}\right)^2\right\rangle_\psi$$

$\mathbf{E}\times\mathbf{B}$ flow of angular momentum (I)

$$Mc\left\langle \frac{\partial \phi_1^{\text{tb}}}{\partial \zeta}\int d^3v\, f_{i2}^{\text{tb}} R\left(\mathbf{v}\cdot\hat{\zeta}\right) + \frac{\partial \phi_2^{\text{tb}}}{\partial \zeta}\int d^3v\, f_{i1}^{\text{tb}} R\left(\mathbf{v}\cdot\hat{\zeta}\right)\right\rangle_\psi$$

- Problematic because we need f_{i2}^{tb}, f_{e2}^{tb}, ϕ_2^{tb}! Need higher order terms in gyrokinetic equation

$$\frac{\partial f_{i2}^{\text{tb}}}{\partial t} + \left(v_\parallel \hat{\mathbf{b}} + \mathbf{v}_M + \mathbf{v}_{E1}^{\text{tb}}\right)\cdot\nabla_{\mathbf{R}} f_{i2}^{\text{tb}} + \left(\mathbf{v}_{E2}^{\text{tb}} + \dot{\mathbf{R}}^{(2)}\right)\cdot\nabla_{\mathbf{R}} f_{i1}^{\text{tb}} - \left\langle C\{f_i\}\right\rangle^{(2)}$$
$$= -\mathbf{v}_{E1}^{\text{tb}}\cdot\nabla_{\mathbf{R}} F_{i1}^{\text{nc}} - \left(\mathbf{v}_{E2}^{\text{tb}} + \dot{\mathbf{R}}^{(2)}\right)\cdot\nabla_{\mathbf{R}} f_{Mi} + \ldots$$

- But for $B_\theta/B << 1$, $F_{i1}^{\text{nc}} \sim \frac{B}{B_\theta}\delta_i f_{Mi} >> f_{i1}^{\text{tb}} \sim \delta_i f_{Mi}$

$\mathbf{E}\times\mathbf{B}$ flow of angular momentum (II)

- For $B_\theta / B << 1$

$$\frac{\partial f_{i2}^{\text{tb}}}{\partial t} + \left(v_\parallel \hat{\mathbf{b}} + \mathbf{v}_M + \mathbf{v}_{E1}^{\text{tb}}\right)\cdot \nabla_{\mathbf{R}} f_{i2}^{\text{tb}} - \left\langle C^{(\ell)}\left\{f_{i2}^{\text{tb}}\right\}\right\rangle = -\mathbf{v}_{E1}^{\text{tb}} \cdot \nabla_{\mathbf{R}} F_{i1}^{\text{nc}} + \ldots$$

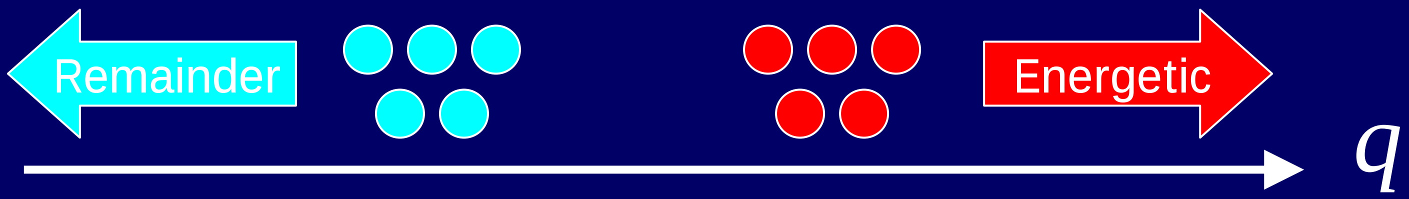
  - Neoclassical parallel flows and heat flows larger than turbulent effects by $\rho_{pi}/\rho_i \sim B/B_\theta >> 1$

- Using $f_{i2}^{\text{tb}} \approx -\int^t d\tau\, \mathbf{v}_{E1} \cdot \nabla_{\mathbf{R}} F_{i1}^{\text{nc}} \propto \omega_\zeta, \frac{\partial \omega_\zeta}{\partial \psi}, \frac{\partial T_i}{\partial \psi}, \frac{\partial^2 T_i}{\partial \psi^2}$

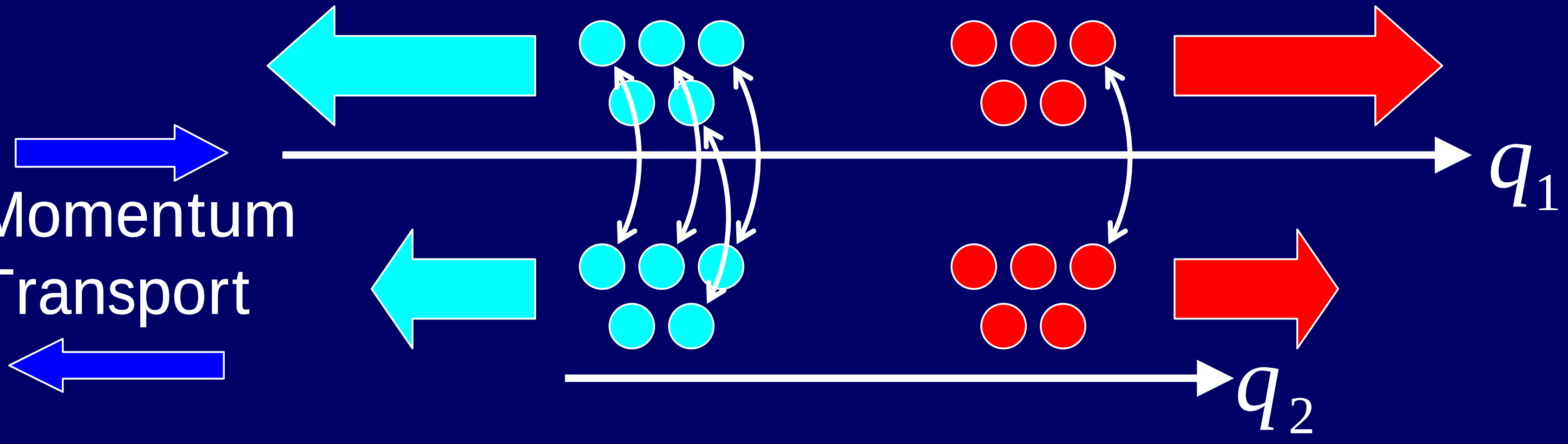
$$Mc\left\langle \frac{\partial \phi_1^{\text{tb}}}{\partial \zeta} \int d^3v\, f_{i2}^{\text{tb}} R\left(\mathbf{v}\cdot\hat{\zeta}\right) + \ldots \right\rangle_\psi \approx P_\zeta \omega_\zeta - \chi_\zeta \frac{\partial \omega_\zeta}{\partial \psi} + K_1 \frac{\partial T_i}{\partial \psi} + L_1 \frac{\partial^2 T_i}{\partial \psi^2}$$

Heat flow and momentum transport

- Parallel heat flow without parallel flow



- ∇(heat flow) ⇒ momentum transport
  - Ex.: collisions more frequent for slow particles



The other terms

$$\frac{Mc}{2Ze}\left\langle R^2\right\rangle_\psi \frac{\partial p_i}{\partial t}+\frac{M^2c^2}{2Ze}\frac{1}{V'}\frac{\partial}{\partial\psi}V'\left\langle\frac{\partial\phi_1^{\text{tb}}}{\partial\zeta}\int d^3\upsilon\, f_{i1}^{\text{tb}}R^2\left(\mathbf{v}\cdot\hat{\zeta}\right)^2\right\rangle_\psi$$

$$-\frac{M^2c}{2Ze}\left\langle\int d^3\upsilon\, C_{ii}^{(\ell)}\left\{F_{i2}^{\text{tb}}\right\}R^2\left(\mathbf{v}\cdot\hat{\zeta}\right)^2\right\rangle_\psi$$

- Caused by changes in width of drift orbits

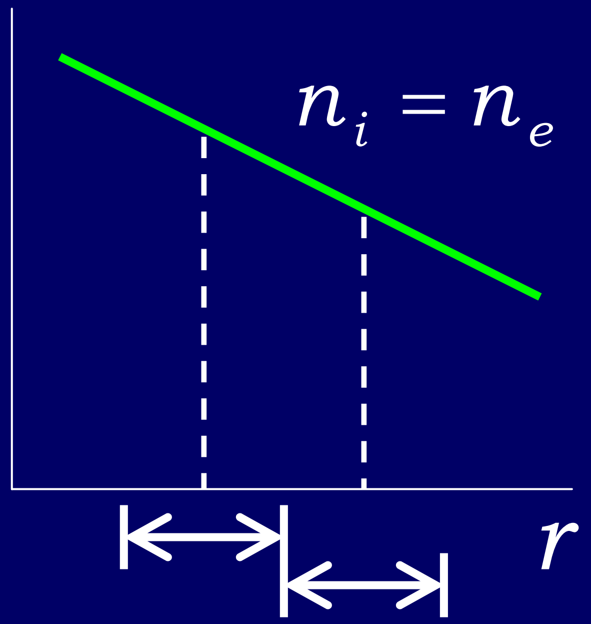
$\psi^* = \psi - \dfrac{Mc}{Ze}R\left(\mathbf{v}\cdot\hat{\zeta}\right) =$ const. gives drift orbit width

$\Rightarrow$ increase in $R^2\left(\mathbf{v}\cdot\hat{\zeta}\right)^2$ widens drift orbits

$\Rightarrow$ modifies $n_i = n_e \Rightarrow$ new ω_ζ

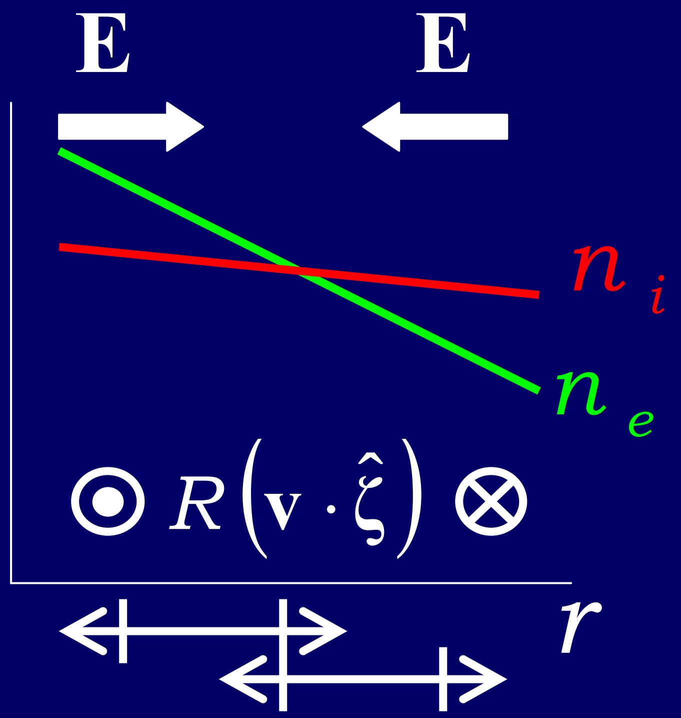
Width of drift orbits

- For increasing $R^2\left(\mathbf{v}\cdot\hat{\zeta}\right)^2$



$\Rightarrow$

$R^2\left(\mathbf{v}\cdot\hat{\zeta}\right)^2 \uparrow$



GK polarization $\Rightarrow \mathbf{E} \Rightarrow$ neo polarization $\Rightarrow \Delta\omega_\zeta$

Turbulent transport of $R^2 v_\zeta^2$

$$\frac{M^2c^2}{2Ze}\frac{1}{V'}\frac{\partial}{\partial\psi}V'\left\langle\frac{\partial\phi_1^{\text{tb}}}{\partial\zeta}\int d^3v\, f_{i1}^{\text{tb}}R^2\left(\mathbf{v}\cdot\hat{\boldsymbol{\zeta}}\right)^2\right\rangle_\psi$$

- From ITG and TEM turbulence, f_{i1}^{tb} and ϕ_1^{tb} depend on

$$\frac{\partial T_i}{\partial\psi},\frac{\partial n_e}{\partial\psi},\frac{\partial T_e}{\partial\psi}$$

- $$\frac{M^2c^2}{2Ze}\frac{1}{V'}\frac{\partial}{\partial\psi}V'\left\langle\frac{\partial\phi_1^{\text{tb}}}{\partial\zeta}\int d^3v\, f_{i1}^{\text{tb}}R^2\left(\mathbf{v}\cdot\hat{\boldsymbol{\zeta}}\right)^2\right\rangle_\psi \approx K_2\frac{\partial T_i}{\partial\psi}+L_2\frac{\partial^2T_i}{\partial\psi^2}$$
$$+M_2\frac{\partial n_e}{\partial\psi}+N_2\frac{\partial^2n_e}{\partial\psi^2}+R_2\frac{\partial T_e}{\partial\psi}+Z_2\frac{\partial^2T_e}{\partial\psi^2}$$

Collisional-turbulent term

$$-\frac{M^2c}{2Ze}\left\langle \int d^3v\, C^{(\ell)}\left\{F_{i2}^{\text{tb}}\right\} R^2\left(\mathbf{v}\cdot\hat{\zeta}\right)^2 \right\rangle_\psi$$

- Collisions drive incoming turbulent flux towards f_{Mi} $\Rightarrow$ modify $R^2\left(\mathbf{v}\cdot\hat{\zeta}\right)^2$ in the process

$$v_\parallel \hat{\mathbf{b}}\cdot\nabla F_{i2}^{\text{tb}} - C^{(\ell)}\left\{F_{i2}^{\text{tb}}\right\} = -\frac{\partial f_{Mi}}{\partial t} + S + \frac{1}{J_{GK}}\nabla\cdot\left(J_{GK}\left\langle f_{i1}^{\text{tb}}\mathbf{v}_{E1}^{\text{tb}}\right\rangle_{\text{T}}\right)$$
$$+\frac{Ze}{M}\frac{1}{J_{GK}}\frac{\partial}{\partial\varepsilon}\left(J_{GK}\left\langle f_{i1}^{\text{tb}}\frac{\partial\left\langle\phi_1^{\text{tb}}\right\rangle}{\partial t}\right\rangle_{\text{T}}\right)$$

- Depends on gradients, but also on heating source!

Intrinsic rotation

- Adding all the contributions

$$\left\langle R\hat{\boldsymbol{\zeta}}\cdot\overset{\leftrightarrow}{\boldsymbol{\pi}}_i\cdot\nabla\psi\right\rangle_\psi \approx P_\zeta\omega_\zeta-\chi_\zeta\frac{\partial\omega_\zeta}{\partial\psi}+L\frac{\partial T_i}{\partial\psi}+K\frac{\partial^2 T_i}{\partial\psi^2}+M\frac{\partial n_e}{\partial\psi}$$

$$+N\frac{\partial^2 n_e}{\partial\psi^2}+R\frac{\partial T_e}{\partial\psi}+Z\frac{\partial^2 T_e}{\partial\psi^2}+\text{heating sources}=0$$

- In order of magnitude comparable to observations

$$\frac{\partial V_i}{\partial r}\sim\frac{c}{eB_\theta}\frac{\partial^2 T}{\partial r^2}$$

Possible transport barriers?

- Is it possible to produce a transport barrier by increasing $\partial^2 T/\partial r^2$?

- Need $\dfrac{\partial V_i}{\partial r} \sim \gamma \sim \dfrac{\upsilon_{ti}}{\sqrt{RL_T}}$

- Then,

$$\frac{\partial V_i}{\partial r} \sim \frac{c}{eB_\theta}\frac{\partial^2 T}{\partial r^2} \Rightarrow \frac{\upsilon_{ti}}{\sqrt{RL_T}} \sim \frac{\rho_i \upsilon_{ti}}{L_T^2} \Rightarrow L_T \sim \rho_i^{2/3} R^{1/3}$$

In JET, it requires $L_T \sim 30\rho_i \sim 3$ cm

Conclusions

- New self-consistent model for intrinsic rotation that can explain change of sign in rotation

- Intrinsic rotation depends on gradients of density and temperature of both electron and ions, and on the heating source
  - Predicts transport barrier for $L_T \sim \rho_i^{2/3} R^{1/3}$

- Possible sources of intrinsic rotation not studied here are up-down symmetry and ripple