Small scales turbulence and LES modelling with GENE

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outline

Sub grid and modelling

Free energy balance

Hyperdiffusion as a first model

Dynamic procedure

On sub grid modelling

Applying a filter to gyrokinetics equations solved in GENE leads to:

$$\partial_t \overline{\delta g} = \mathcal{Z} + \mathcal{L}[\overline{\delta g}] + \mathcal{N}[\overline{\delta g}, \overline{\delta g}] + \mathcal{T}[\delta g, \overline{\delta g}]$$

With the sub grid term:

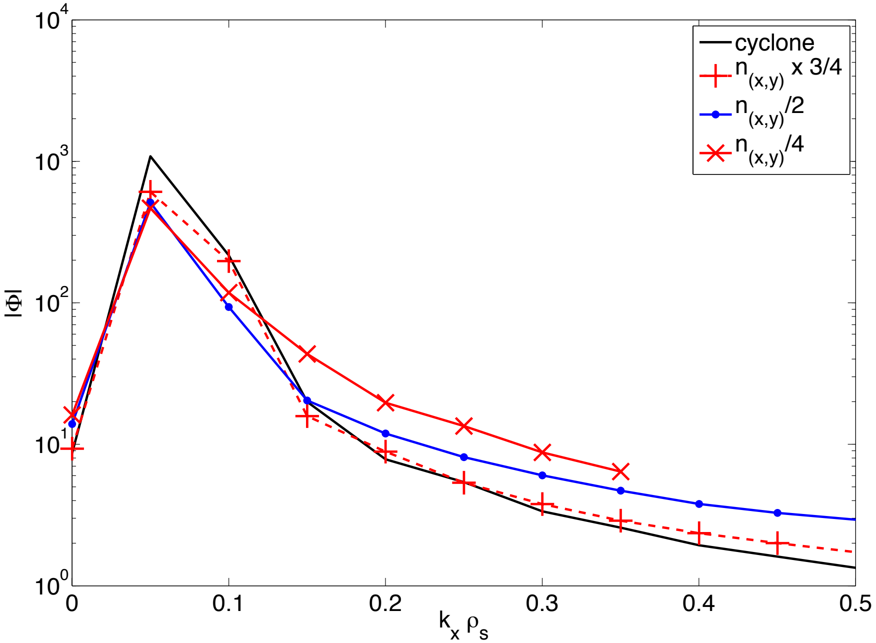
$$\mathcal{T}[\delta g, \overline{\delta g}] = \overline{\mathcal{N}[\delta g, \delta g]} - \mathcal{N}[\overline{\delta g}, \overline{\delta g}]$$

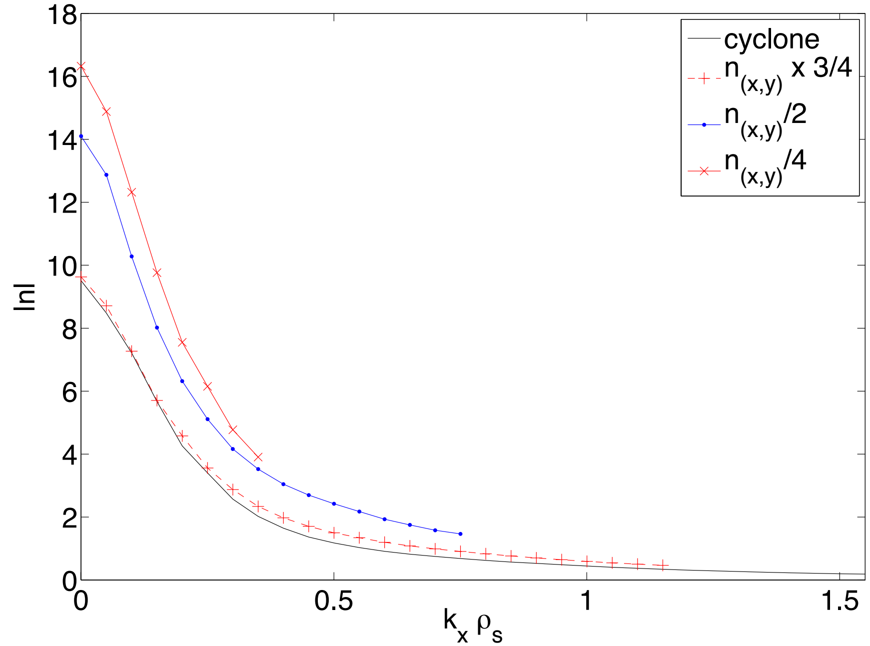
To gain in resolution and numerical effort, this term has to be modelled and analysed.

On sub grid modelling

First analysis: under resolved study

Principle: decrease resolution without model, see what is lost:

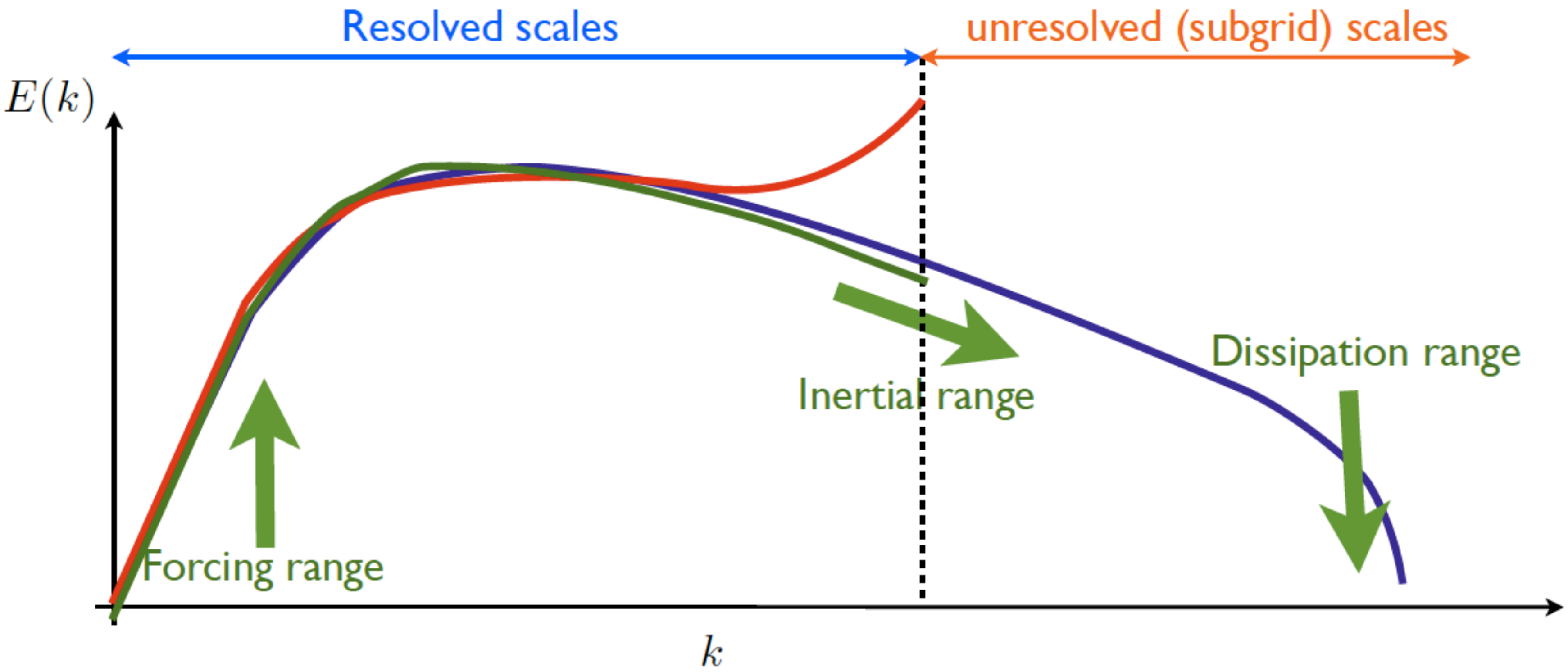




CBC case with decreasing perpendicular resolution
electrostatic potential (left) and density (right) along kx (radial): crucial need of a model!

On sub grid modelling

Basic idea: sub grid must insert some dissipations



A good candidate for quantifying these dissipation processes is the free energy

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Free energy balance

In local version of GENE, free energy balance can be expressed:

$$\partial_t \left(\mathcal{F} + \mathcal{W}^{\phi}\right) = \mathcal{G}_0 + \mathcal{G} + \mathcal{H}_z + \mathcal{H}_{v_\parallel}$$

By applying the free energy operator:

$$\Omega. = \sum_s \sum_k \left\langle \int d\mu dv_\parallel \pi B_0 n_{0j} T_{0j} \left[\frac{g_{-kj}}{F_{0j}} + \frac{q_j \chi_{-kj}}{T_{0j}} \right] . \right\rangle$$

to Vlasov equation.

NB: the drive is a L.C of eq. gradients and fluxes
NBB: dissipation in collisionless regime ensured by hyper-diffusion operators

Free energy balance – terms (adiabatic electrons)

$$\partial_t \left(\mathcal{F} + \mathcal{W}^\phi\right) = \mathcal{G}_0 + \mathcal{G} + \mathcal{H}_z + \mathcal{H}_{v_\parallel}$$

$$\mathcal{F} = \sum_j \sum_k \left\langle \int d\mu dv_\parallel \pi B_0 n_{0j} T_{0j} \frac{|\delta f_{kj}|^2}{F_{0j}} \right\rangle$$

TdS: Entropy

$$\mathcal{W}^\phi = \frac{q_e^2 n_{e0}}{T_{e0}} \sum_k \left(\left\langle |\phi_k|^2 \right\rangle - | \left\langle \phi_k \right\rangle |^2 \right) + \sum_i \frac{q_{i0}^2 n_{i0}}{T_{i0}} \sum_k \left\langle [1 - \Gamma_0(b_{kj})] \, |\phi_k|^2 \right\rangle$$

Electrostatic

$$\mathcal{G} = \Omega. - \left[\omega_{nj} + \left(v_\parallel^2 + \mu B_0 - \frac{3}{2}\right) \omega_{Tj}\right] F_{0j} i k_y \chi_{kj}$$

Gradients:
Free energy
sources

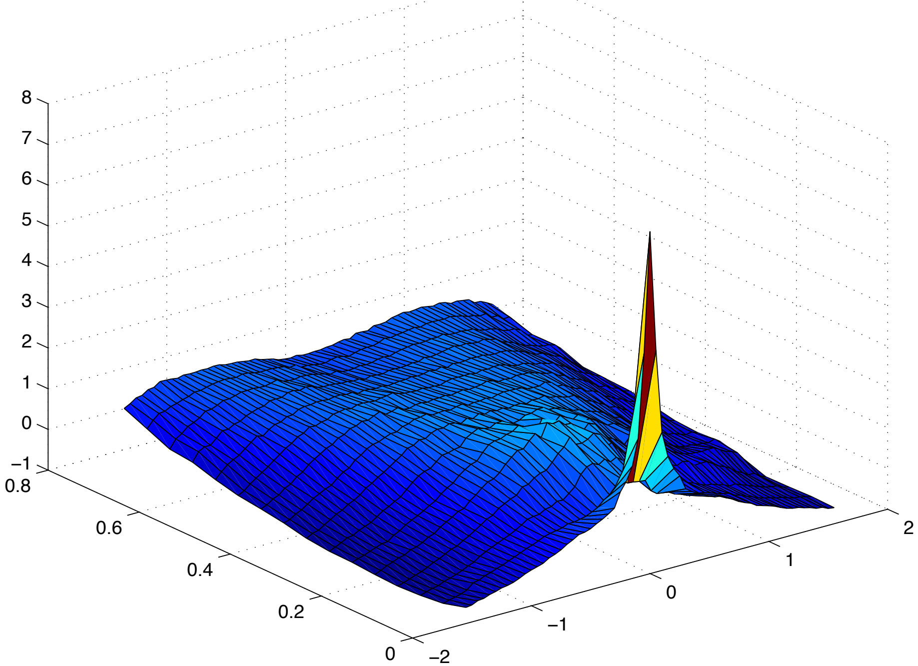
$$\mathcal{H}_z = \Omega. h_z \partial_z^4 f_{kj}$$

$$\mathcal{H}_{v_\parallel} = \Omega. h_{v_\parallel} \partial_{v_\parallel}^4 f_{kj}$$

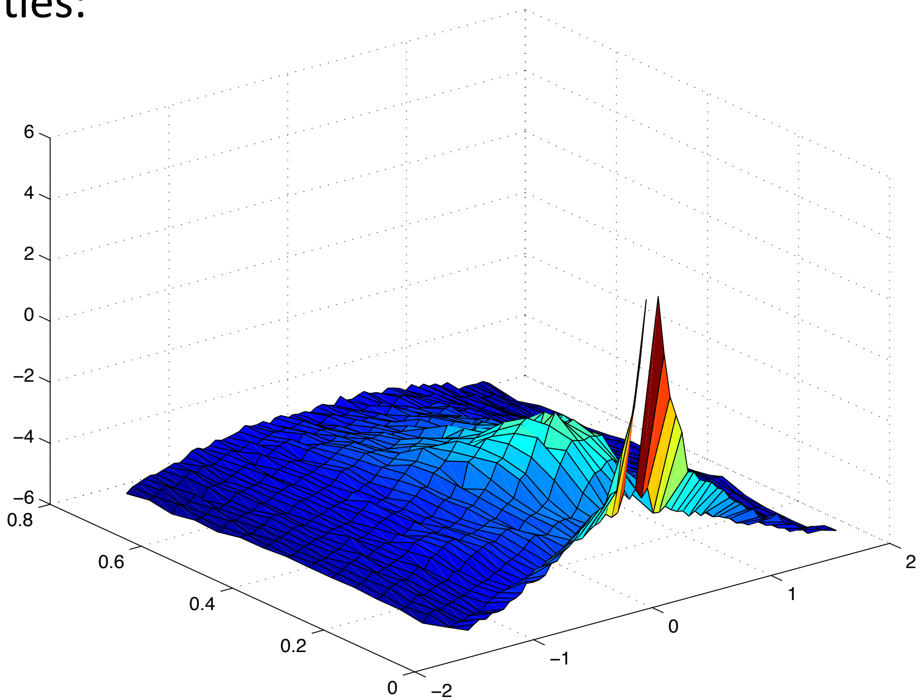
Hyperdiffusions:
Free energy sinks

Free energy balance – terms: lhs

Perp spectra of time averaged quantities:



Entropy



electrostatic energy

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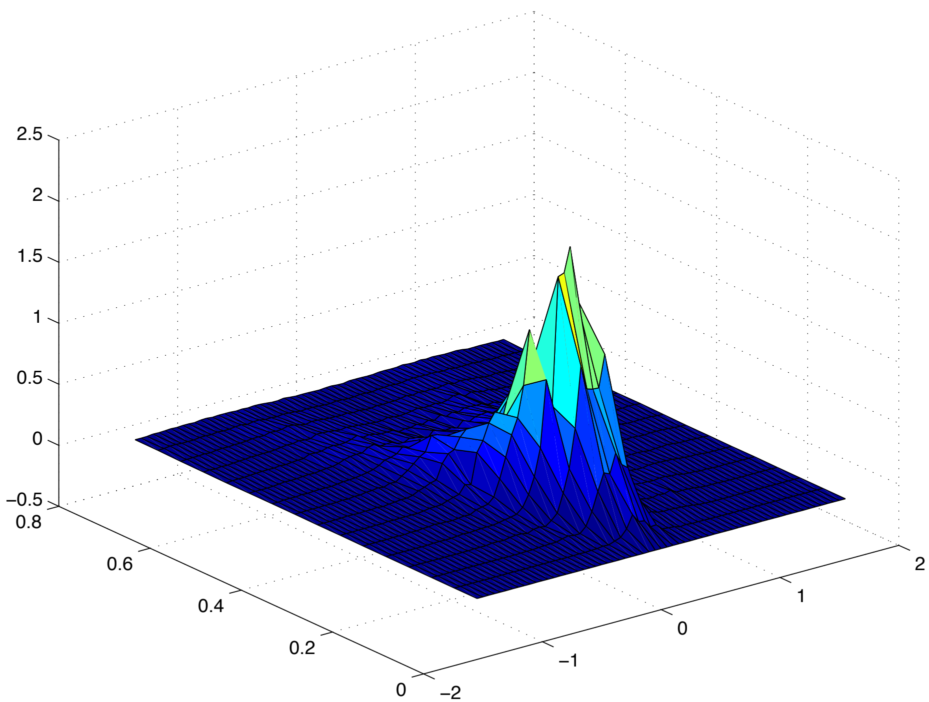
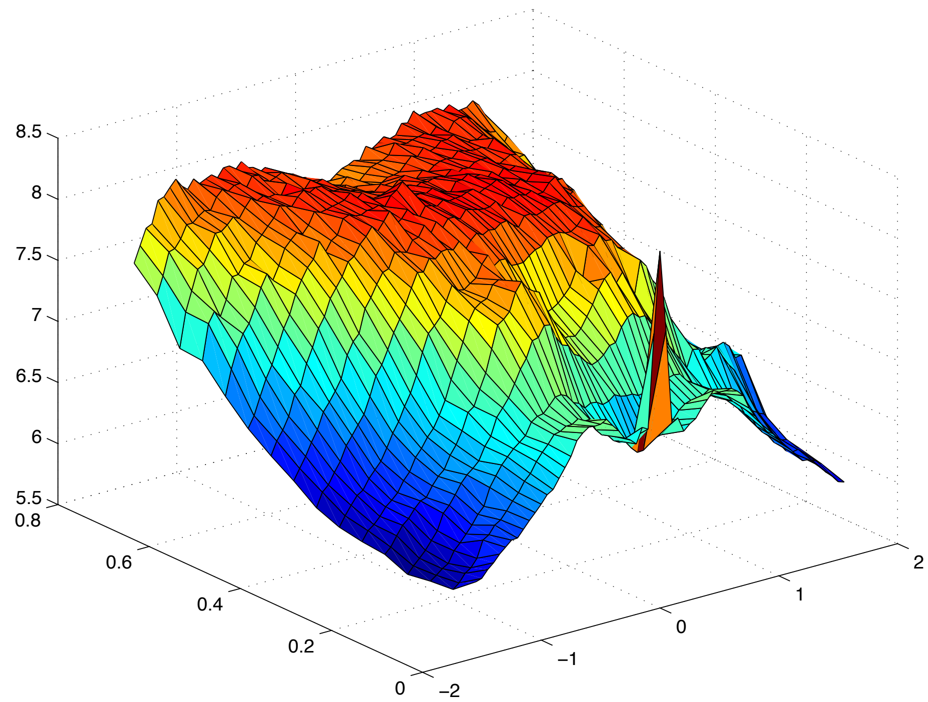
Free energy balance – terms: rhs



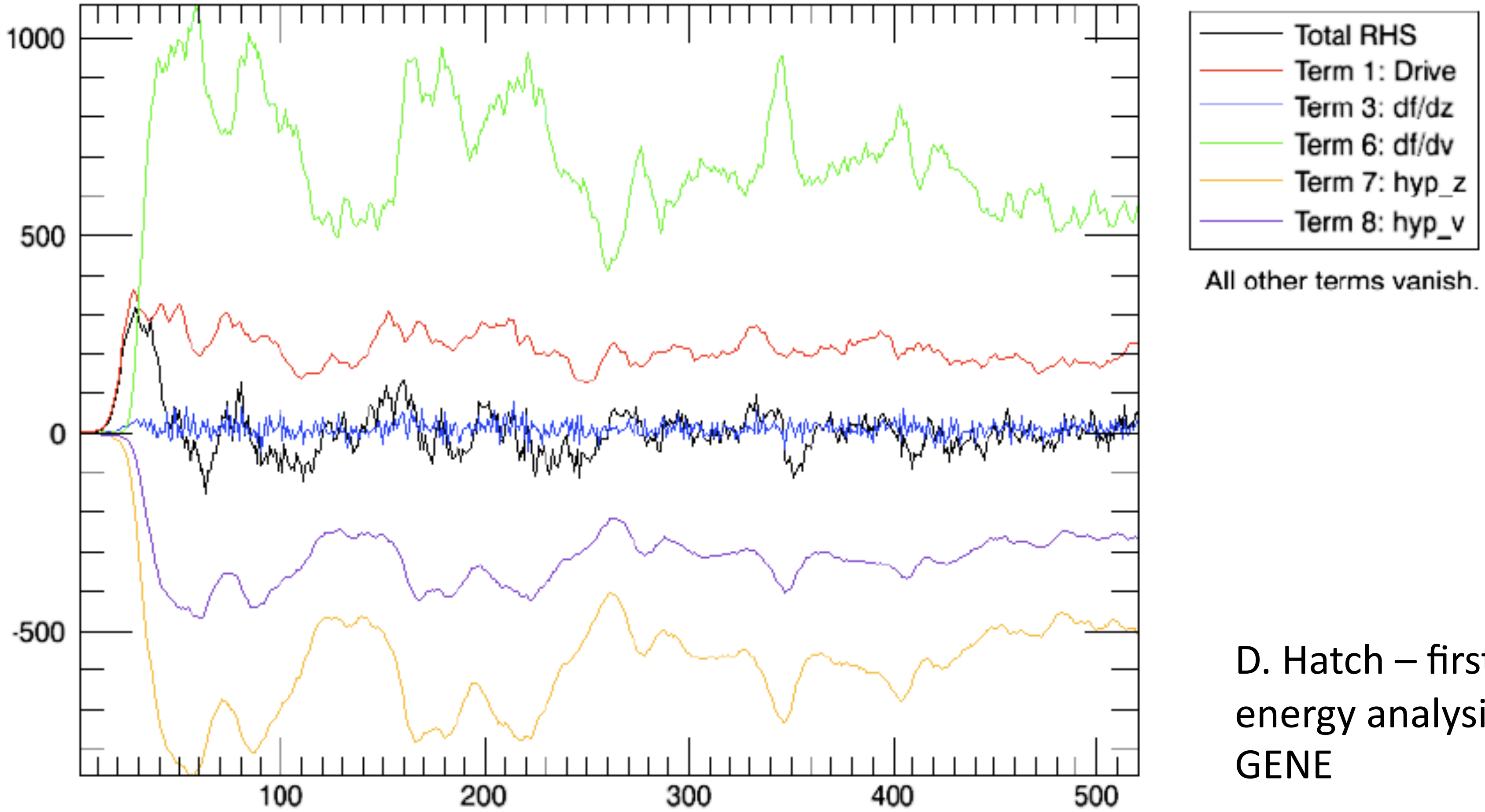
Fluxes drive (eq. gradients)



velocity hyperdiffusion (log scale)

Free energy balance – conservation in time

Formally, the parallel derivatives terms should cancel under v-z integration – Not always verified numerically:



All other terms vanish.

D. Hatch – first free energy analysis with GENE

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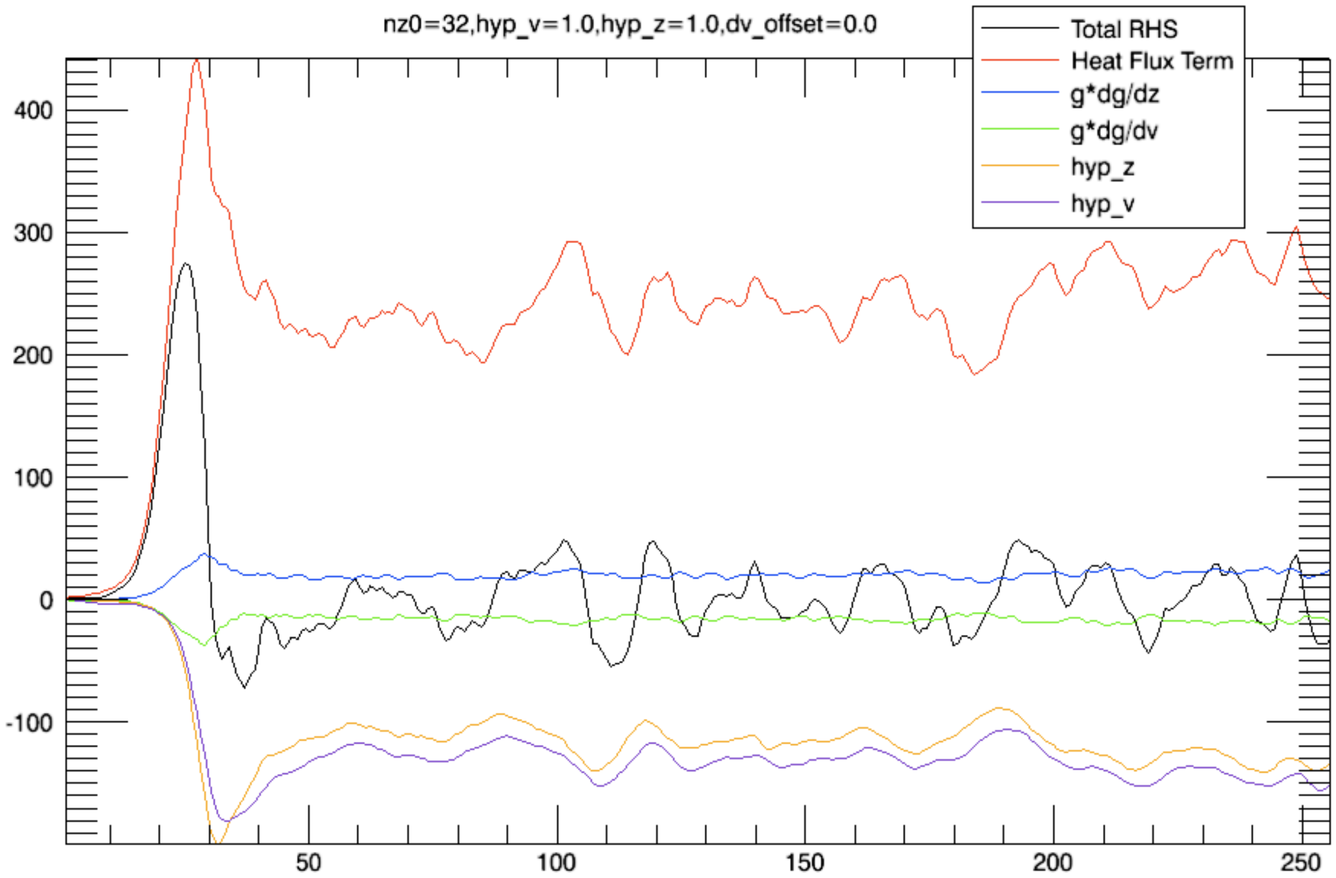
Free energy balance – conservation in time

modified velocity boundaries (based on H_0 trajectories)



Source: G

Parallel derivatives: Very closed to 0

Sinks: hyp terms

Free energy balance - nonlinearity

Nonlinear term has no action in the total entropy balance (thanks to PB)

Is responsible of the mode to mode transfer:

$$\mathcal{N}[g_{kj}] = \sum_{k'_\perp} (k'_x k_y - k_x k'_y)\chi_j(k'_\perp) g_j(k_\perp - k'_\perp)$$

Free energy balance - nonlinearity

The free energy balance depends on resolution

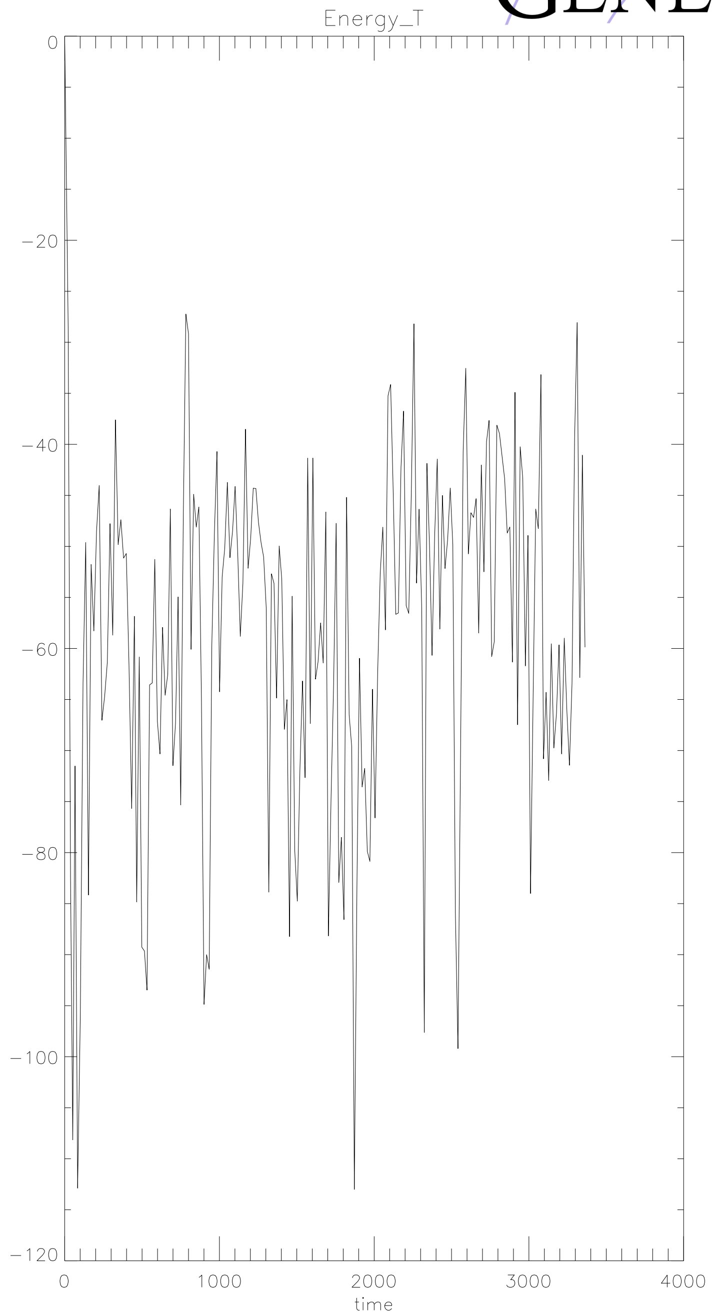
$$\partial_t \left(\overline{\mathcal{F}} + \overline{\mathcal{W}^\phi} \right) =$$

$$\overline{\mathcal{G}_0} + \overline{\mathcal{G}} + \overline{\mathcal{H}_z} + \overline{\mathcal{H}_{v_\parallel}} + \overline{\Omega}.\overline{\mathcal{T}[\delta g_{kj}, \delta g_{kj}]}$$

The filtering introduces the sub grid term T must act as a dissipation of the resolved scales:

$$\overline{\Omega}.\overline{\mathcal{T}}[\delta g_{kj}, \delta g_{kj}] < 0$$

Verified numerically:



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Dynamic procedure

Hyper-diffusion as model

In collisionless regime, dissipation is ensured by hyper-diffusion terms.

Such terms depends on the numerics, in GENE:

$$H_{\{z,v_\parallel\}} = -i^n \varepsilon_c \left(\frac{\Delta\{z, v_\parallel\}}{2} \right)^n \partial^n_{\{z,v_\parallel\}} \delta f$$

Practically, prefactor is a fixed parameter in most of cases

NB: a detailed study of these terms can be found in M. Pueschel Thesis

Hyper-diffusion as model

Analyze correlation between hyp's and sub grids:

$$\mathcal{C} = \frac{\langle \mathcal{M} | \mathcal{T} \rangle}{\sqrt{\langle \mathcal{M}^2 \rangle \langle \mathcal{T}^2 \rangle}}$$

With hyp's as model:

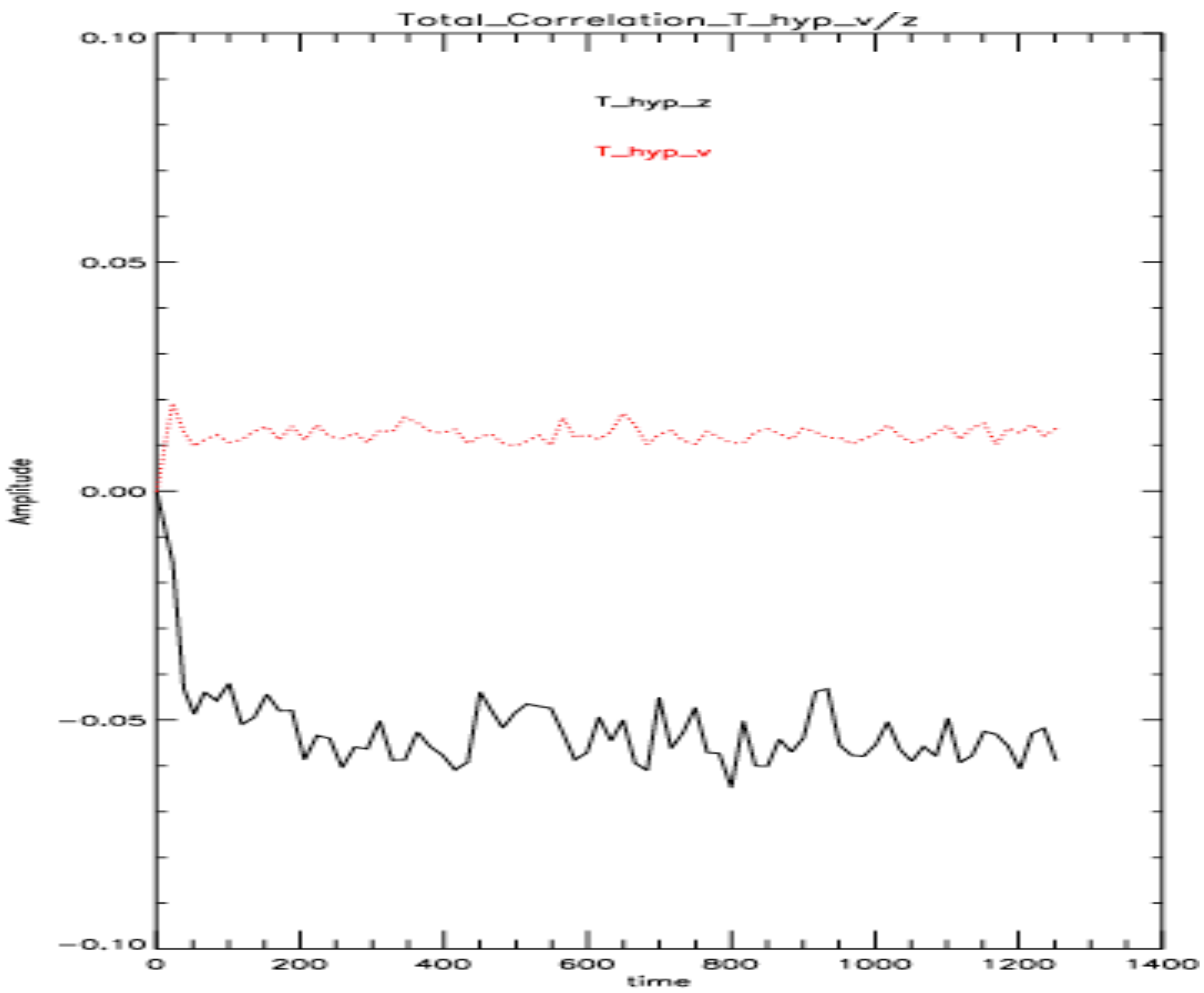
$$\mathcal{M} = h_z \partial_z^4 \overline{\delta f} + h_v \partial_v^4 \overline{\delta f}$$

$$\mathcal{T} = \overline{\mathcal{N}[\delta g, \delta g]} - \mathcal{N}[\overline{\delta g}, \overline{\delta g}]$$

Hyper-diffusion as model

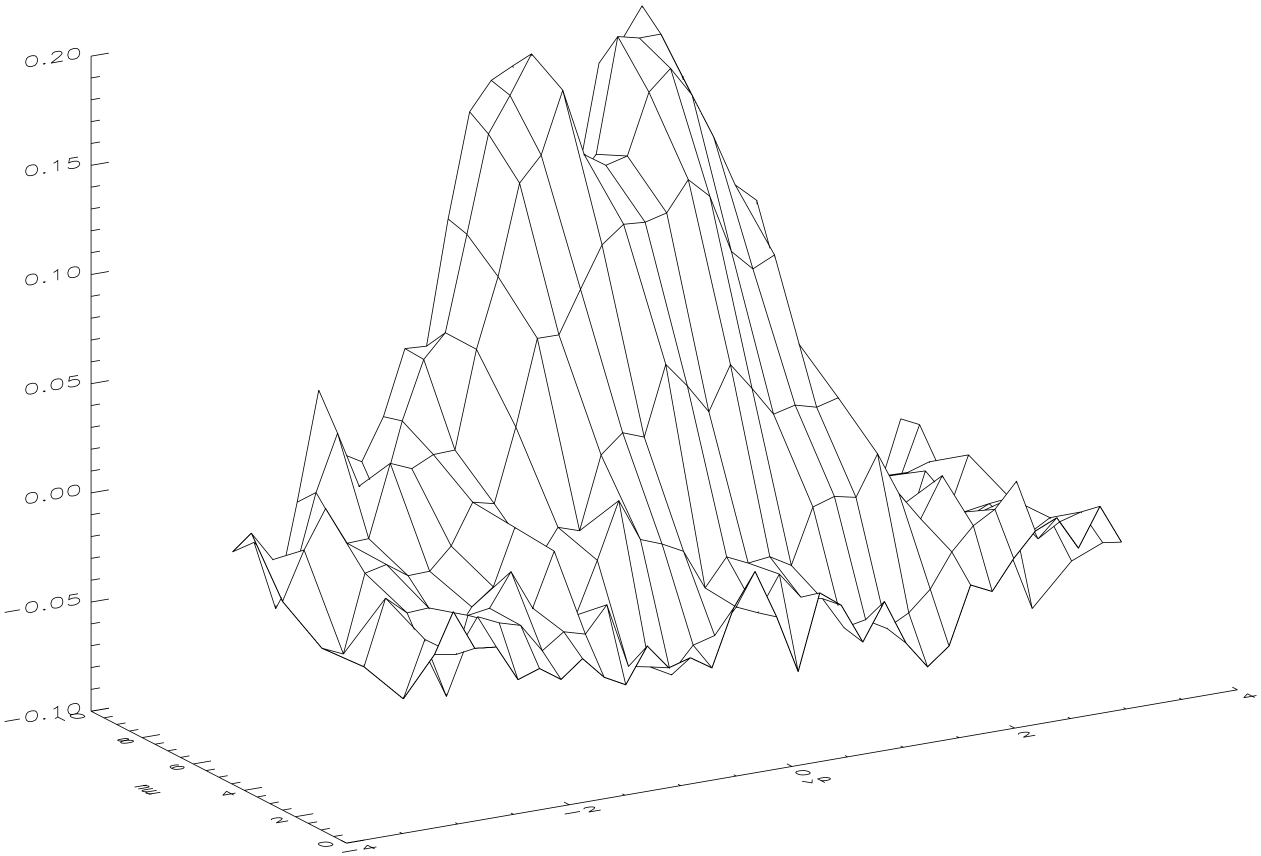
Correlation betw T and hyp's during time

Reach stable but low level



Hyper-diffusion as model

Velocity plane correlations



Good correlations at mid domains in velocity plane

Hyper-diffusion as model

Free energy: velocity hyperdiffusion act on f/F_0

$$H_{v_\parallel} = -c_{v_\parallel} \partial^4_{v_\parallel} \frac{\delta f}{F_0}$$

Action: damps free energy till perturbed df reaches a constant ratio of equilibrium

$$\partial_t < \frac{|\delta f|^2}{F_0} > \propto -2c_{v_\parallel} < \left| \partial^2_{v_\parallel} \frac{\delta f}{F_0} \right|^2 >$$

$$\partial_t < \delta f > \propto -c_{v_\parallel} < \partial^4_{v_\parallel} \frac{\delta f}{F_0} >$$

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Dynamic procedure

consider a simulation with model: $\mathcal{T} \approx c\, m(\overline{\delta g}, \overline{\Delta})$

$$\partial_t \overline{\delta g}_{\overline{\Delta}} = \mathcal{L}_{\overline{\Delta}}[\overline{\delta g}_{\overline{\Delta}}] + \mathcal{N}_{\overline{\Delta}}[\overline{\delta g}_{\overline{\Delta}}, \overline{\delta g}_{\overline{\Delta}}] + c\, m_{\overline{\Delta}}(\overline{\delta g}_{\overline{\Delta}}, \overline{\Delta}) \quad (1)$$

introduce an intermediate scale, keep same resolution, complete with zeroes:

$$\partial_t \widehat{\delta g}_{\overline{\Delta}} = \mathcal{L}_{\overline{\Delta}}[\widehat{\delta g}_{\overline{\Delta}}] + \mathcal{N}_{\overline{\Delta}}[\widehat{\delta g}_{\overline{\Delta}}, \widehat{\delta g}_{\overline{\Delta}}] + c\, m_{\overline{\Delta}}(\widehat{\delta g}_{\overline{\Delta}}, \widehat{\Delta})$$

filter eq(1) at intermediate scale:

$$\widehat{\overline{X}} = \widehat{X} \qquad \partial_t \widehat{\delta g}_{\overline{\Delta}} = \mathcal{L}_{\overline{\Delta}}[\widehat{\delta g}_{\overline{\Delta}}] + \mathcal{N}_{\overline{\Delta}}[\widehat{\overline{\delta g}_{\overline{\Delta}}, \overline{\delta g}_{\overline{\Delta}}}] + c\, m_{\overline{\Delta}}(\widehat{\overline{\delta g}_{\overline{\Delta}}, \overline{\Delta}})$$

minimization leads to the value of c:

$$c = \frac{\left\langle \mathcal{M} \mathcal{T}^{\hat{\Delta}}_{\overline{\Delta}} \right\rangle}{\langle \mathcal{M}\mathcal{M} \rangle} \qquad \mathcal{M} = m_{\overline{\Delta}}(\widehat{\delta g}, \hat{\Delta}) - m_{\overline{\Delta}}(\overline{\delta g}, \overline{\Delta})$$

$$\mathcal{T}^{\hat{\Delta}}_{\overline{\Delta}} = \mathcal{N}_{\overline{\Delta}}[\widehat{\overline{\delta g}_{\overline{\Delta}}, \overline{\delta g}_{\overline{\Delta}}}] - \mathcal{N}_{\overline{\Delta}}[\widehat{\delta g}_{\overline{\Delta}}, \widehat{\delta g}_{\overline{\Delta}}]$$

Dynamic procedure