# Nonlocal gyrokinetic simulations with GENE

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# Motivation

- **Why present a global code in group 1?**

  → new possibilities for coupling with transport solvers (benchmarks, speed-up?)

  → access to meso-scale investigations

- **What are the differences from a computational point of view?**

- **Boundary conditions?**

- **Which types of “operations” are available?**

# GENE – basic equations

- GENE solves the gyrokinetic Vlasov-Maxwell system of eqs.

## Gyrokinetic Vlasov equation

$$\boxed{\frac{\partial f_\sigma}{\partial t} + \dot{\mathbf{X}} \cdot \nabla f_\sigma + \dot{\mu}\frac{\partial f_\sigma}{\partial \mu} + \dot{v}_\parallel \frac{\partial f_\sigma}{\partial v_\parallel} = 0}$$

with gyrocenter position **X**

$$\begin{aligned}
\dot{\mathbf{X}} &= v_\parallel \mathbf{b}_0 + \frac{B_0}{B_{0\parallel}^*}\left(\frac{v_\parallel}{B_0}\bar{\mathbf{B}}_{1\perp} + \mathbf{v}_\perp\right) \\
\mathbf{v}_\perp &\equiv \frac{c}{B_0^2}\bar{\mathbf{E}}_1 \times \mathbf{B}_0 + \frac{\mu}{m_\sigma \Omega_\sigma}\mathbf{b}_0 \times \nabla B_0 \\
&\quad + \frac{v_\parallel^2}{\Omega_\sigma}(\nabla \times \mathbf{b})_\perp
\end{aligned}$$

parallel velocity $v_\parallel$

$$\dot{v}_\parallel = \frac{\dot{\mathbf{X}}}{m_\sigma v_\parallel} \cdot \left(q_\sigma \bar{\mathbf{E}}_1 - \mu \nabla B_0\right)$$

and magnetic moment μ

$$\dot{\mu} = 0$$

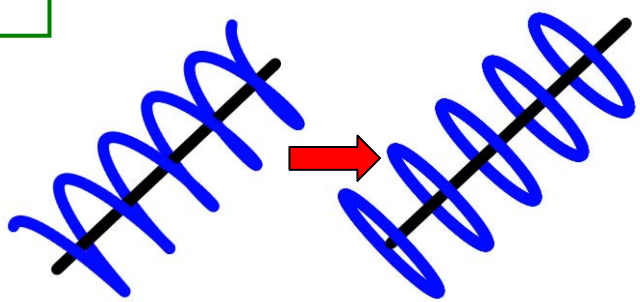
## Poisson equation

$$\boxed{\left\{-\frac{1}{4\pi}\nabla_\perp^2 + \sum_\sigma n_{0\sigma}\frac{q_\sigma^2}{T_{0\sigma}}\left[1 - \frac{B_0}{T_{0\sigma}}\int \mathcal{G}^2\, \mathrm{e}^{-\frac{\mu B_0}{T_{0\sigma}}}\,\mathrm{d}\mu\right]\right\}\phi_1 = \sum_\sigma q_\sigma \bar{n}_{1\sigma}}$$

with gyroaverage operator $\mathcal{G}$

## Ampère’s law

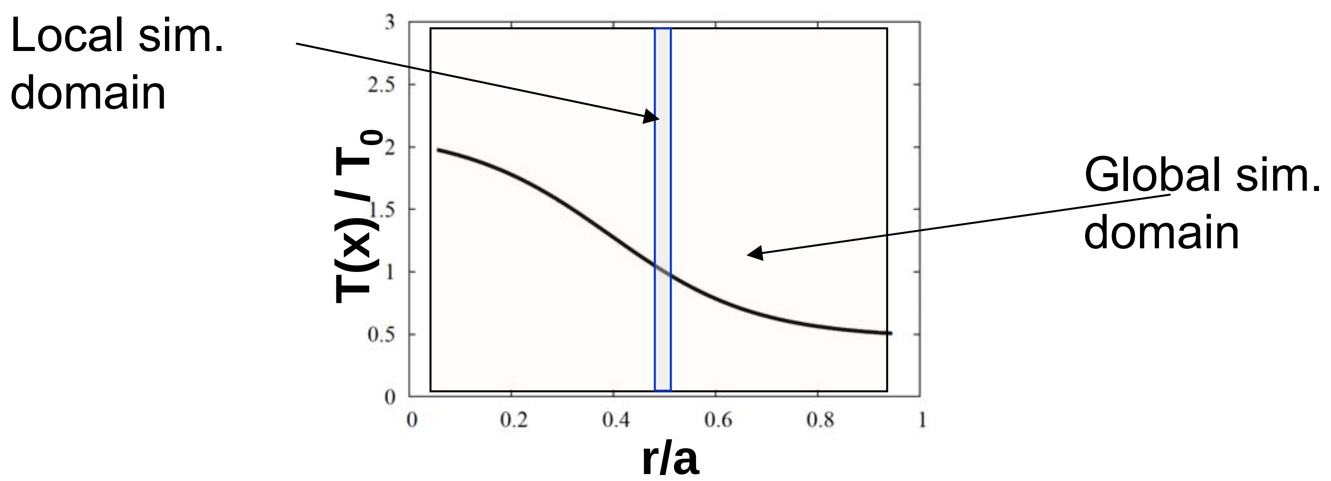
$$\boxed{-\nabla_\perp^2 A_{1\parallel} = \frac{4\pi}{c} j_{1\parallel}}$$



Gyrokinetics: reduced description (“charged rings”)

# Local vs. global GENE

- **Local** in the radial direction if the gyroradius << machine size
  - Simulation domain **small** compared to machine size; thus, *constant* temperatures/densities and *fixed* gradients
  - Periodic boundary conditions; allows application of *spectral methods*




- **Global:**
  - Consider full temperature & density profiles; radially varying metric
  - Boundary conditions: e.g., Dirichlet or v. Neumann boundary
  - Effectively, a complete rewrite of the core code parts

# Local vs. global – numerical point of view

- **Local:**
  - Derivatives: $\frac{\partial f}{\partial x} \rightarrow \mathrm{i} k_x f(k_x)$
  - Gyroaverage and field solver operators can be given analytically, e.g.

$$\langle \phi(\mathbf{x}+\mathbf{r}) \rangle = \sum_{\mathbf{k}_\perp} J_0(k_\perp \rho) \phi(\mathbf{k}_\perp, z) \mathrm{e}^{\mathrm{i}\mathbf{k}_\perp \mathbf{x}}$$

- **Global:**
  - Derivatives: finite differences, 4th order centered
  - Gyroaverage: Interpolation required (FEM), here: local polynomial base

$$\langle \phi_1(\mathbf{X}+\mathbf{r}) \rangle = \sum_{k_y} \mathrm{e}^{\mathrm{i}k_y Y} \mathcal{G}(X, k_y, z, \mu) \cdot \phi_1(X, k_y, z)$$

defined on coarse grid

with gyromatrix

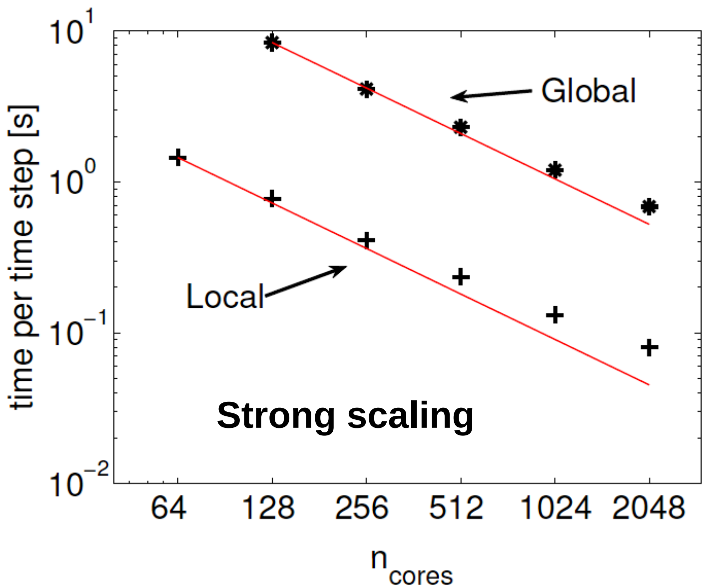
$$\mathcal{G}_{in}(x, k_y, z, \mu) = \frac{1}{2\pi} \int_0^{2\pi} \boxed{\Lambda_n(x_{(i)} - r^1(\theta))} \, \mathrm{e}^{-\mathrm{i}k_y r^2(\theta)} \mathrm{d}\theta$$

local polynomial base (coarse grid values can directly be extracted)

# Local vs. global – numerical point of view II

- Resolution requirements significantly higher
  - in radial direction (larger box)
  - in binormal direction (depends on safety factor profile)
  - in both velocity space directions since structure scale with thermal velocity
    - alternative grids are currently investigated

- All in all: Highly efficient and massively parallelized code needed

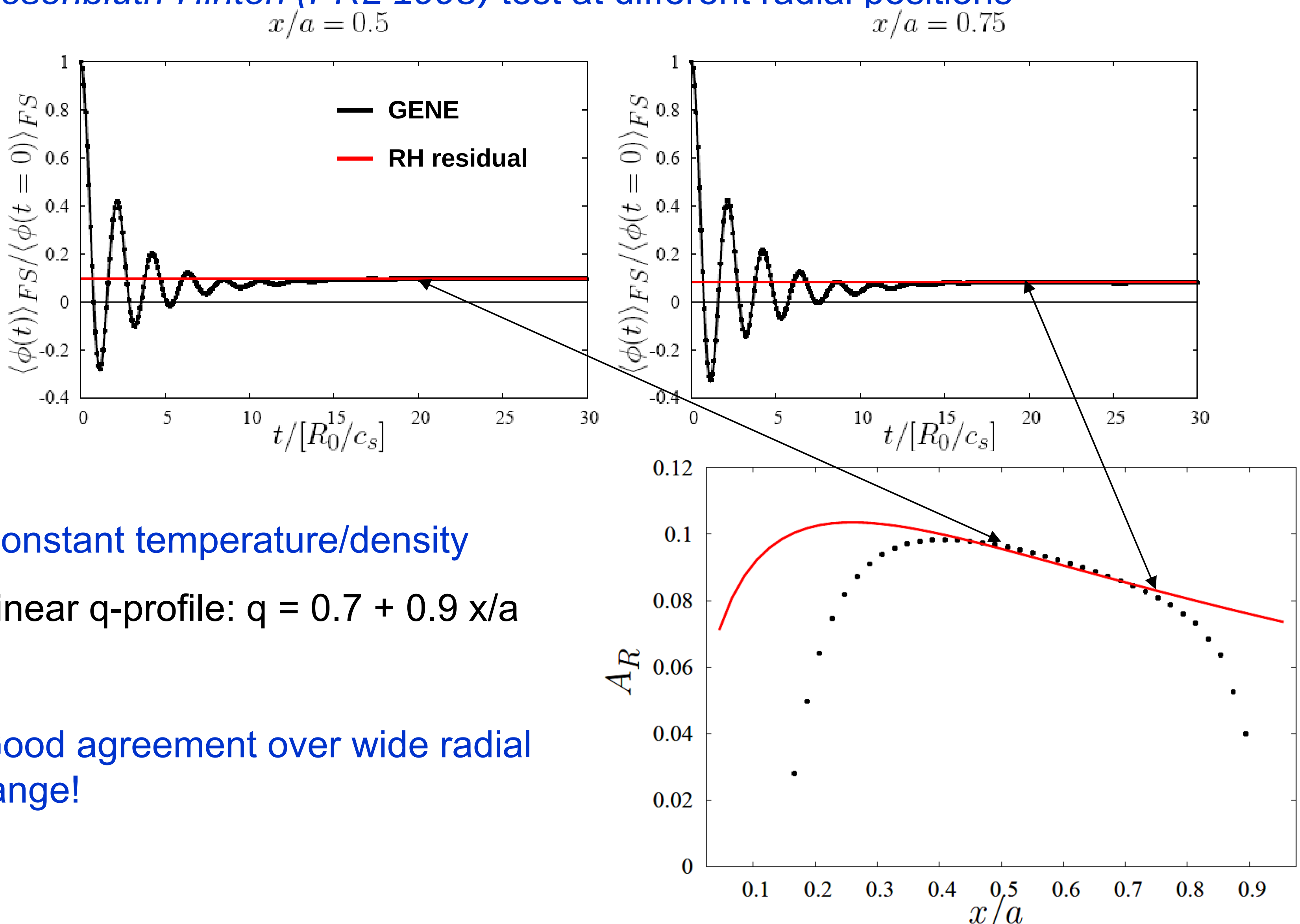



Some of the features/improvements:

- Parallelization in radial direction
- Strip mining
- Arakawa scheme for nonlinearity
- Automatic parallelization

# Code verification

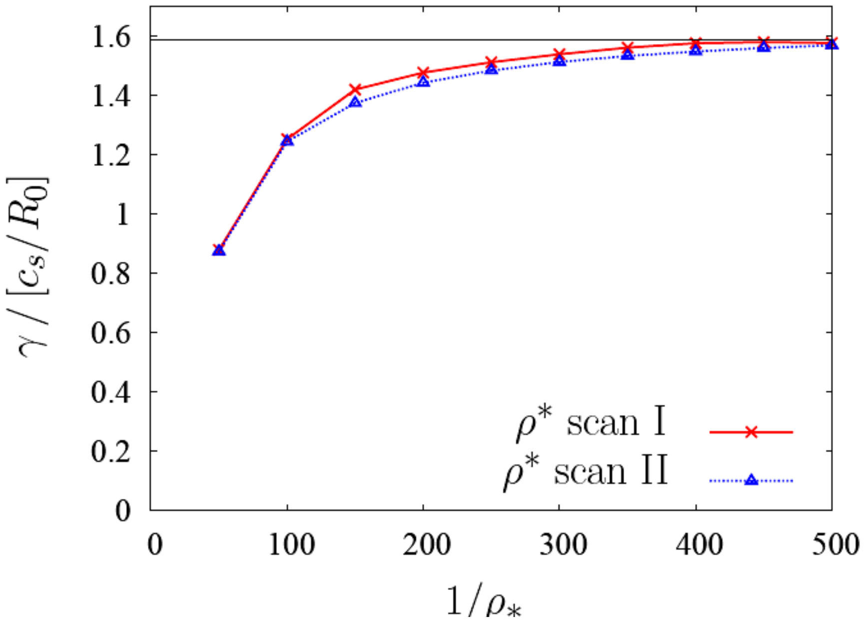
- *Rosenbluth-Hinton (PRL 1998)* test at different radial positions

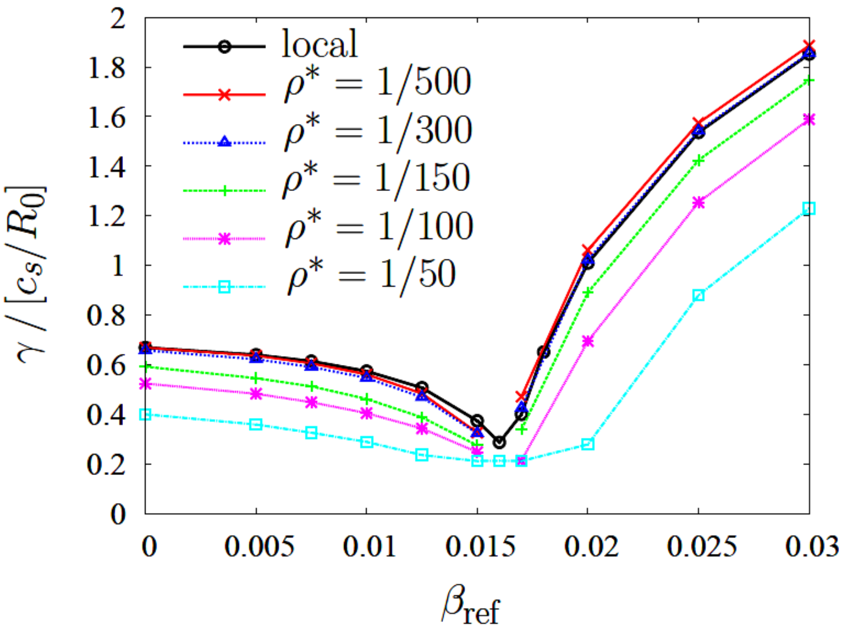



- Constant temperature/density
- Linear q-profile: q = 0.7 + 0.9 x/a
- Good agreement over wide radial range!

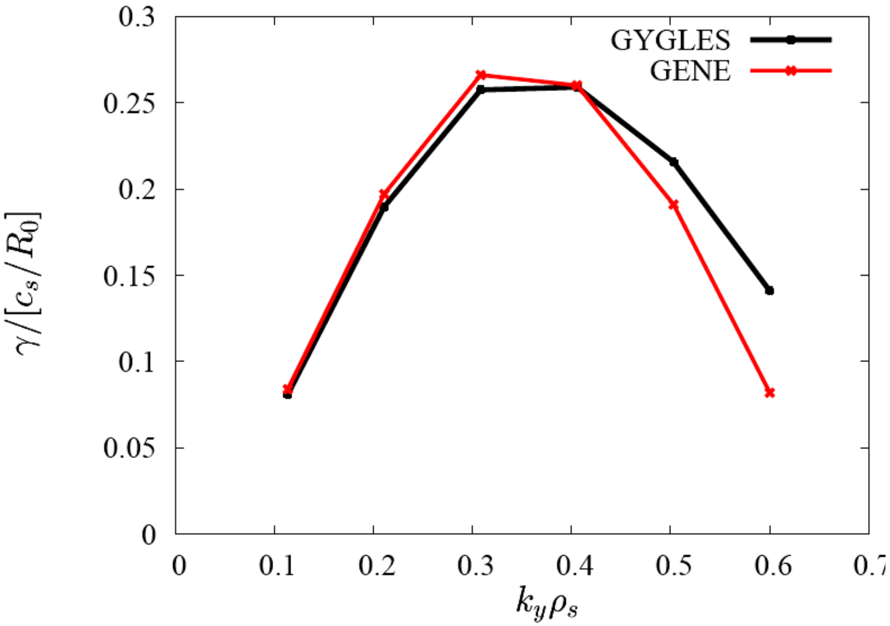
# Code verification (cont'd)

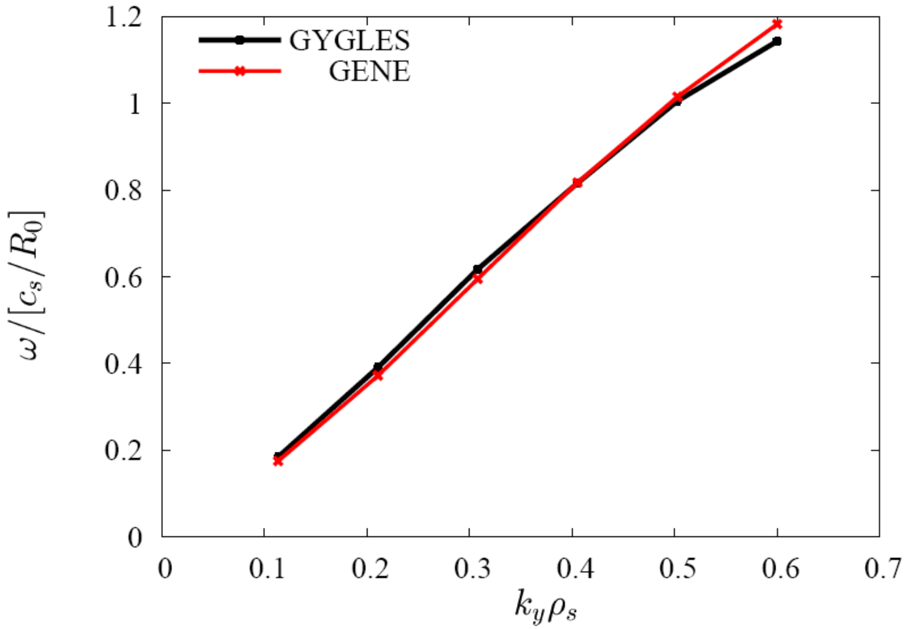
- Local limit tests
  - decrease $\rho^*=\rho_s/a$
  - fix box width with respect to
    - gyroradius (scan I)
    - minor radius (scan II)
  - peaked gradient profiles and 4th order polynomial for safety factor
  - two cases shown:
    - electromagnetic ($\beta$=2.5%) test case at ky~0.28;
    - $\beta$-scan at same wave number



# Linear GYGLES-GENE benchmark




- Cyclone-like, adiabatic electrons
- Peaked temperature and density gradient profiles

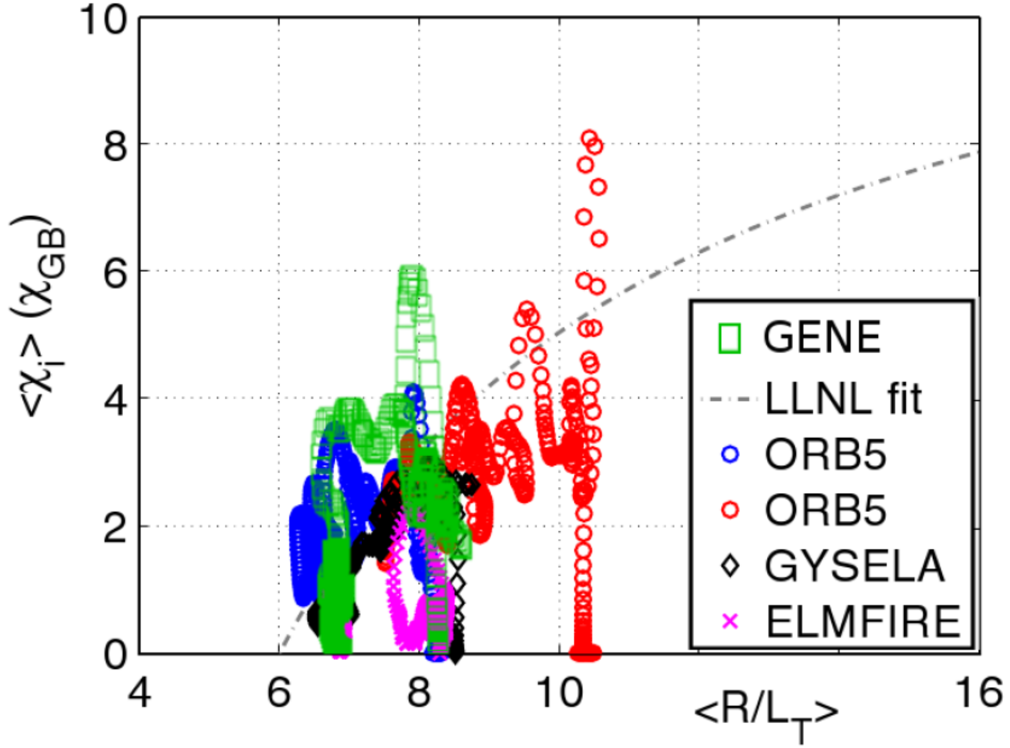
$$T_{i,e} = T_0 \exp\left[\kappa_T\, a/R_0\, \Delta T \tanh\left(\frac{(x-x_0)/a}{\Delta T}\right)\right] \qquad n_{i,e} = n_0 \exp\left[\kappa_n\, a/R_0\, \Delta n \tanh\left(\frac{(x-x_0)/a}{\Delta n}\right)\right]$$

with $\kappa_T = 6.96, \kappa_n = 2.23, \Delta n = \Delta T = 0.3,$
$a/R_0 = 0.36, x_0 = 0.5\,a,$ and $\rho^* = \rho_s/a = 1/180.$

- quadratic q profile: $q(x/a) = 0.85 + 2.4\,(x/a)^2$

# ITM benchmark case *[Falchetto et al., PPCF ’08]*

- Nonlinear ITG turbulence, adiabatic electrons
- Cyclone-like parameters with flat gradient profiles
- Relaxing profiles




The volume averaged heat fluxes and gradients are well within the ITM benchmark range

# Nonlinear, quasi-stationary ITG-Sims

Contained GENE/ORB5 results;
Please refer to X. Lapillonne's PhD thesis
(to be published soon), CRPP, EPFL,
Lausanne, 2010

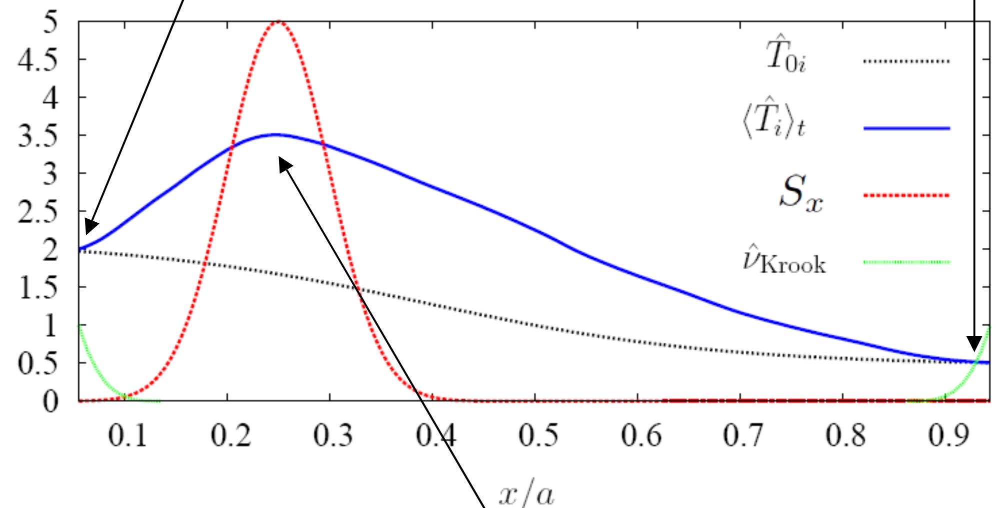
# Applications of the Krook-type heat source

Contained GENE/ORB5 results;
Please refer to X. Lapillonne's PhD thesis
(to be published soon), CRPP, EPFL,
Lausanne, 2010

# Localized sources/sinks

- **Krook operator**
  - damp fluctuations at simulation box edges to be consistent with, e.g., Dirichlet boundary conditions
  - $$\frac{\mathrm{d}f_1}{\mathrm{d}t} = -\nu_{\mathrm{krook}}(x) f_1$$

Krook term fixes temperature and density at boundaries



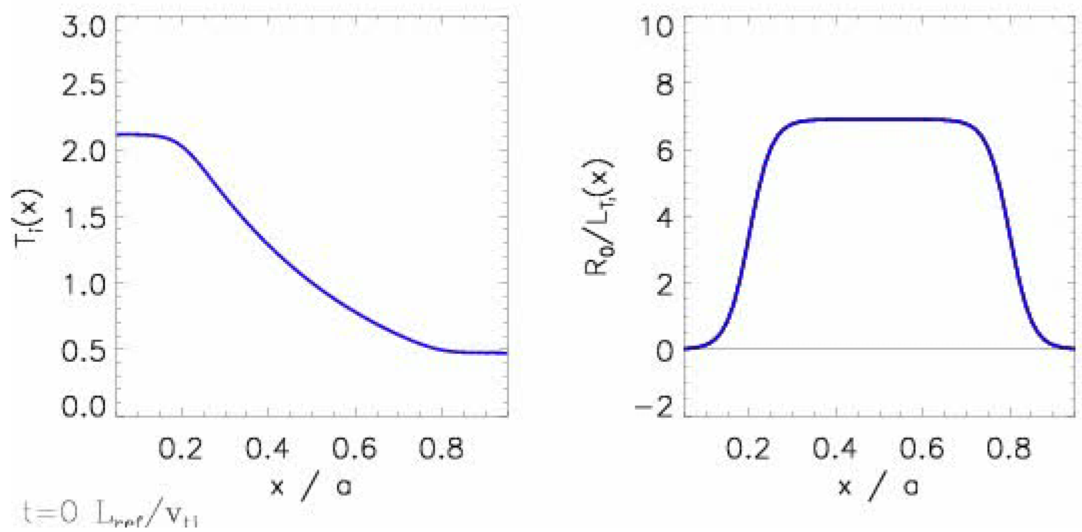

Temperature increase due to heat source

- **Heat source:**
  - Model (cmp. GYSELA, *[Grandgirard et al., PPCF '07])*

$$\frac{\mathrm{d}f_1}{\mathrm{d}t} = S_0 \cdot \underbrace{\frac{S_x}{\langle S_x \rangle_V}}_{\text{radial profile}} \cdot \underbrace{\frac{2}{3}\frac{1}{\hat{p}_{0\sigma}} \left( \frac{\hat{v}_\parallel^2 + \hat{\mu}\hat{B}_0}{T_{0\sigma}(x)/T_{0\sigma}(x_0)} \right)}_{\text{energy distribution (no particle/momentum source)}}$$

amplitude (total power injection)

# Localized sources/sinks + von Neumann b.c.

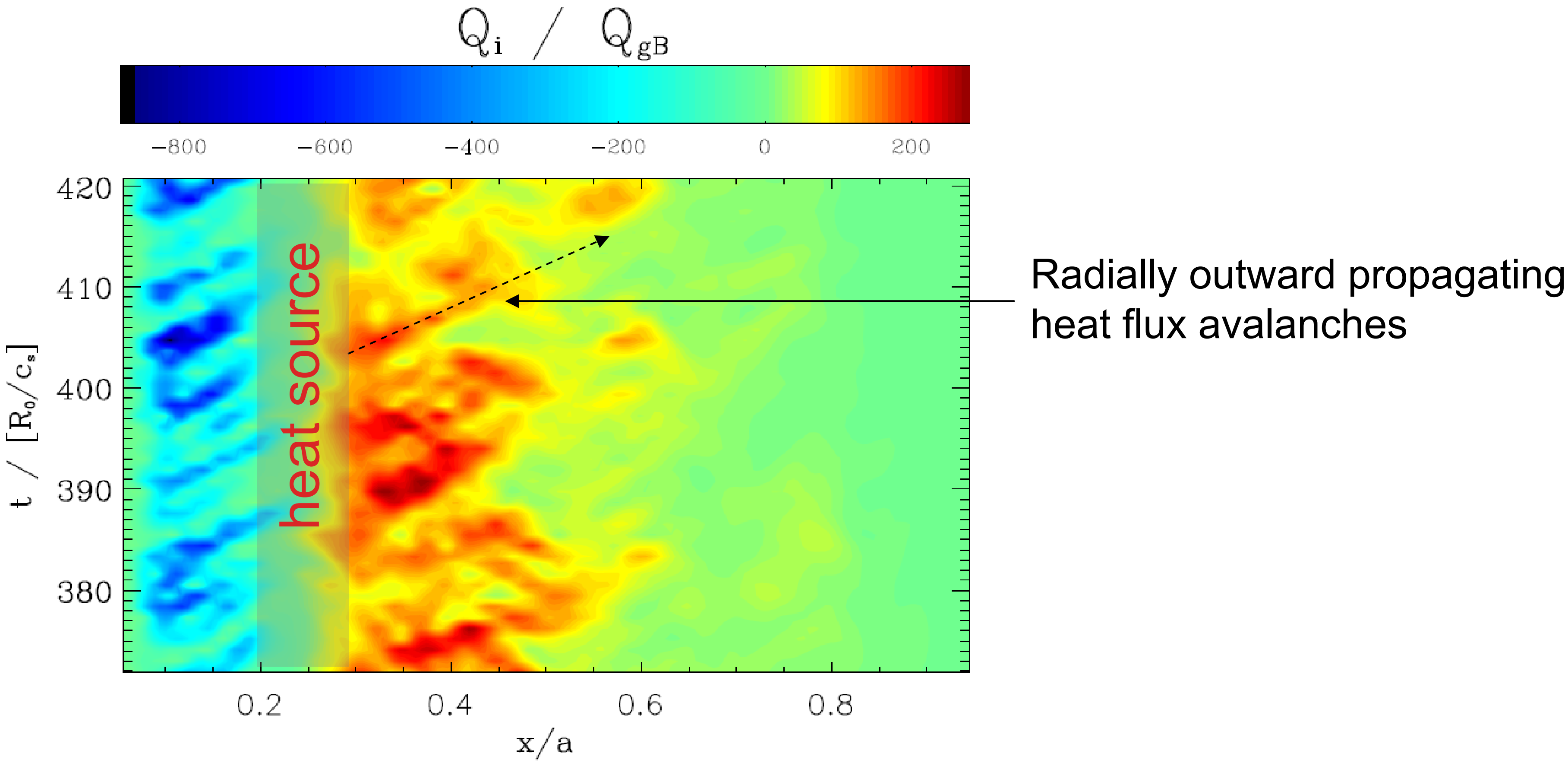



- Possible benchmark for Transport solver coupled to local/nonlocal gyrokinetic code; what about long-time behavior?

- Gradient driven ↔ Flux driven simulations

# Signatures of nonlocal effects

- **Simulation with heat source:**




  - Radially outward propagating heat flux amplitudes
  - To which extend is the transport scaling affected?

# Conclusion

# Summary and Outlook

- **GENE has been extended to a nonlocal code**
- **Various new possibilities:**
  - **replace flux tubes by annulus computations and couple to transport solvers**
  - **Investigate meso-scale dynamics**
  - **flux-driven simulations?**