Electric Field Effects in a Tokamak Pedestal

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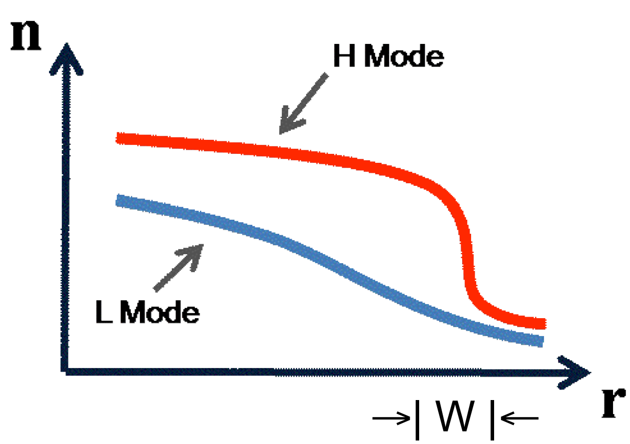
Motivation: pedestal = transport barrier



- Higher energy content
- Larger energy confinement time

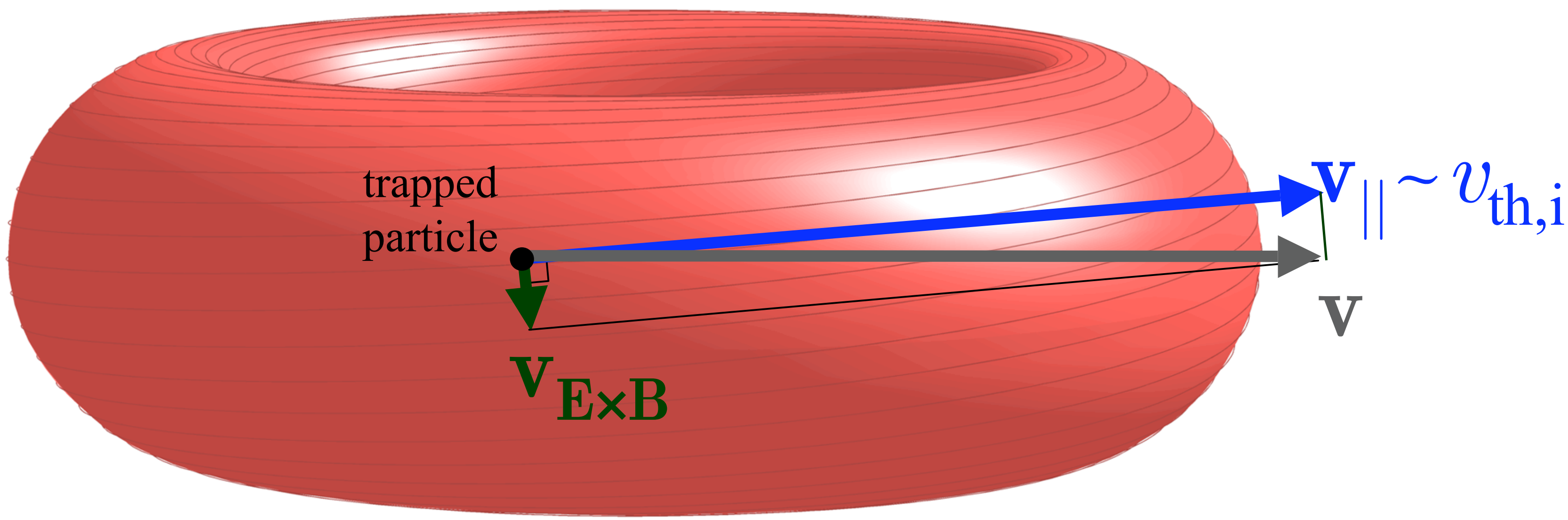
Existence of the pedestal associated with decreased transport and turbulence

Density pedestal results in strong radial electric field and electrostatically confined banana regime ions

Pedestal width $w \sim \rho_{pol}$

Particle orbits in pedestal

Strong radial electric field: $(v_{\|}\vec{n} + \vec{v}_E)\cdot\nabla\theta \approx (v_{\|} + cI\Phi_0'/B)\vec{n}\cdot\nabla\theta$



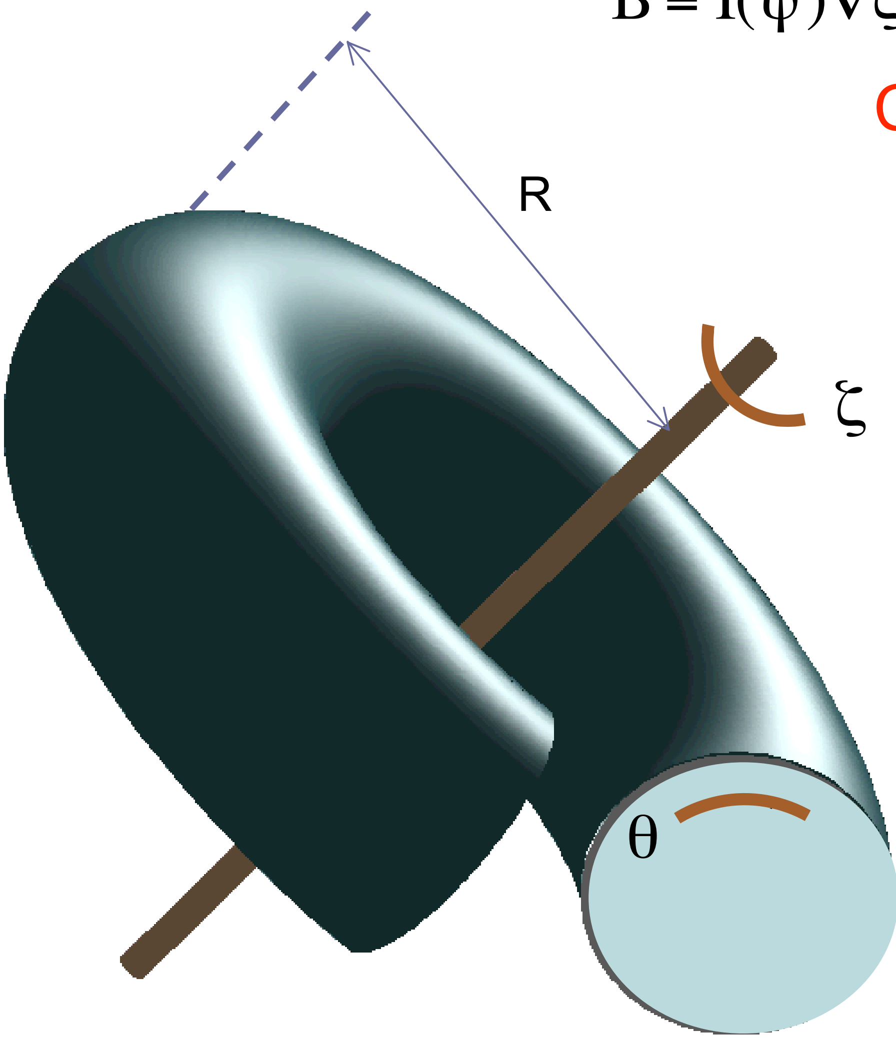
ExB drift ~ $v_i\rho/w$ ~ $v_i\rho/\rho_{pol}$ << $v_{||}$, but geometry makes it comparable to poloidal projection of $v_{||}$

Overview of Pedestal Topics

- Version of gyrokinetics useful in pedestal: convenient for pedestal widths w ~ ion poloidal gyroradius ρ_{pol}
- Neoclassical ion heat flux and ion flow in pedestal retaining finite radial electric field effects: must treat finite drift orbit effects

Gyrokinetic variables

$$\vec{B} = I(\psi)\nabla\zeta + \nabla\zeta \times \nabla\psi = B\vec{n}$$



Canonical angular momentum

$$\psi_* = \psi - (Mc/Ze)R^2\nabla\zeta \cdot \vec{v}$$
$$= \psi + \Omega^{-1}\vec{v} \times \vec{n} \cdot \nabla\psi - Iv_{||}/\Omega$$

gyration $(\rho/a)\psi$ drift $(\rho_{pol}/a)\psi$

Toroidal angle ζ_*

Poloidal angle θ_*

Total energy E

Magnetic moment μ

Gyrophase φ

Axisymmetric gyrokinetic equation

Axisymmetric ($\partial/\partial\zeta = 0$) gyrokinetic equation

$$\frac{\partial\langle f\rangle}{\partial t} + \langle\frac{d\theta_*}{dt}\rangle\frac{\partial\langle f\rangle}{\partial\theta_*} = \langle C\{f\}\rangle - \frac{Ze}{M}\frac{\partial\langle\Phi\rangle}{\partial t}\frac{\partial\langle f\rangle}{\partial E}$$

Steady state ($\partial/\partial t = 0$) to leading order in ρ_{pol}:
transit averaging in banana regime

$$\overline{\langle C\{f_*\}\rangle} = 0$$

where $\overline{Q} = \oint d\tau Q / \oint d\tau$ with $d\tau = d\theta_* / \langle\dot{\theta}_*\rangle$

Are there non-Maxwellian solutions in pedestal?

Entropy production analysis: no!

G. Kagan, P.J. Catto, Plasma Phys. Controlled Fusion 50, 085010 (2008)

Pedestal ion temperature variation

In the banana regime $\partial f_* / \partial \theta_* = 0$ so $f_*(\psi_*, E, \mu)$

The only Maxwellian possible is

$$f_* = \eta\left(\frac{M}{2\pi T}\right)^{3/2} \exp\left(-\frac{Ze\Phi}{T} + \frac{M\omega^2 R^2}{2T} - \frac{Ze\omega\psi}{cT}\right) \exp\left[-\frac{M(\vec{v} - \omega R\vec{\zeta})^2}{2T}\right]$$

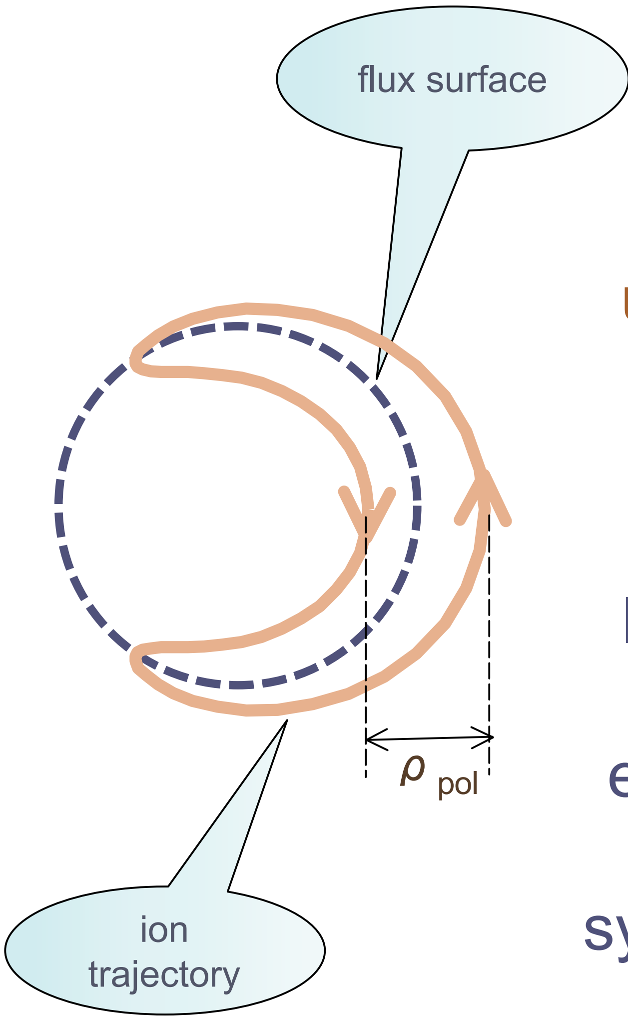
where η, ω, and T are constants & n is Maxwell-Boltzmann with

$$\omega = -c[\partial\Phi/\partial\psi + (T/Zen)\partial n/\partial\psi]$$

Non-isothermal modifications can only enter to next order in the B_p/B expansion

T, η, ω must vary slowly compared to ρ_{pol}

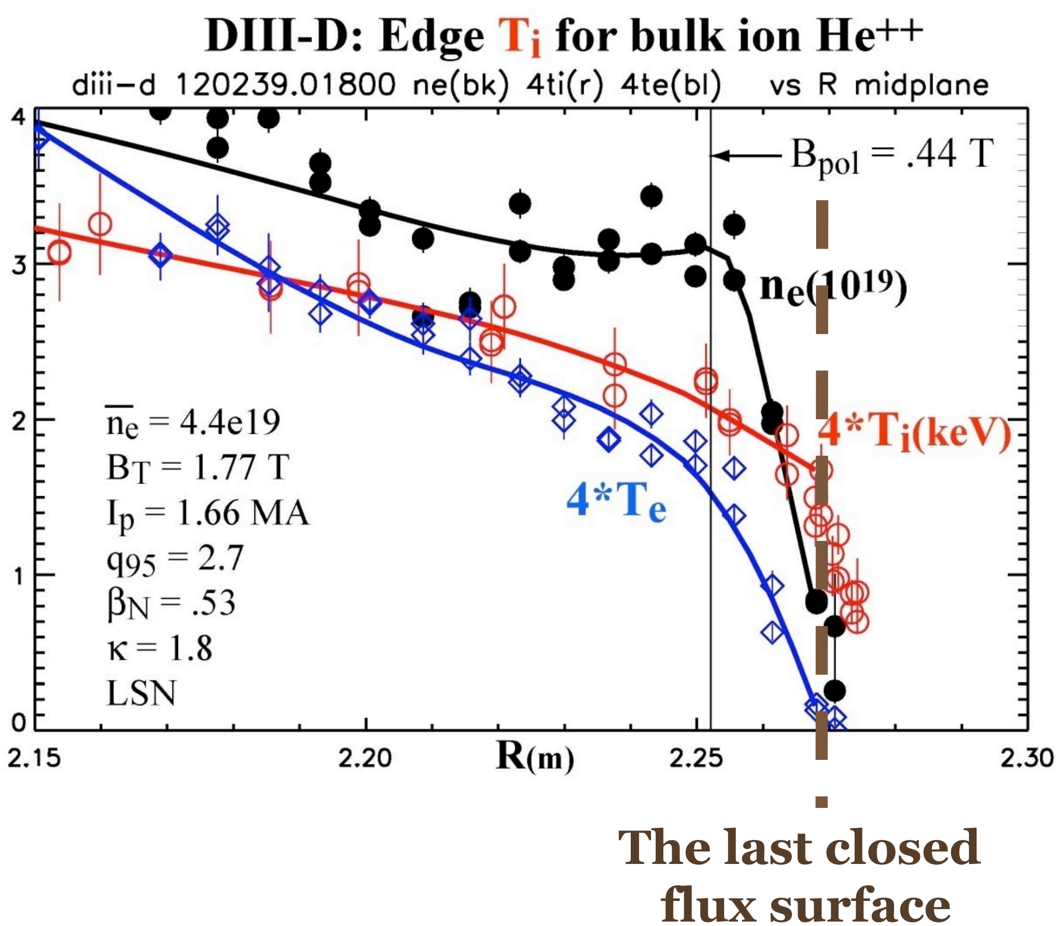
Physical interpretation



In the core gradients are so weak ion departures from a flux surface are unimportant - can consider any given flux surface a closed system

In the pedestal, gradients are as large as $1/\rho_{pol}$ so drift departures affect the equilibration of neighboring flux surfaces - the entire pedestal region is a closed system (rather than individual flux surfaces)

The T_i gradient for the bulk ion in a DIII-D ECH H-mode pedestal is small relative to gradients in electron temperature and density



- The thermal ion full banana width is computed to be $2\rho_\theta$ = 10 mm for He^{++} at the top of the density pedestal.
- The smooth spline fits to the data (solid lines) end at the LCFS as computed by EFIT. Note the clear break in slope for T_i beyond the LCFS.
- In a nominally identical companion discharge we measured T_i for the minor C^{6+} impurity constituent. The T_i profile for C^{6+} has a very similar slope to that for He^{++}, but is ~ 150 eV greater in this region, probably because this discharge had an increase in β_N of ~ 10% compared with the one shown here.

DIII-D NATIONAL FUSION FACILITY

c.f. deGrassie, Groebner, and Burrell, PoP vol 13, 112507-1 (2006).

Pedestal pressure balance

Radial ion pressure balance using $\vec{V}_i = \omega_i R\vec{\zeta} + u_i\vec{B}$ gives

$$\omega_i \approx -c[\partial\Phi/\partial\psi + (T_i/Zen)\partial n/\partial\psi]$$

subsonic pedestal ($w \sim \rho_{pol}$) $\omega_i/[(T_i/en)\partial n/\partial\psi] \sim \omega_i R/v_i << 1$ ⇒ $\frac{\partial\Phi}{\partial\psi} = -\frac{T_i}{Zen}\frac{\partial n}{\partial\psi} > 0$

pedestal electric field inward for subsonic ion flow

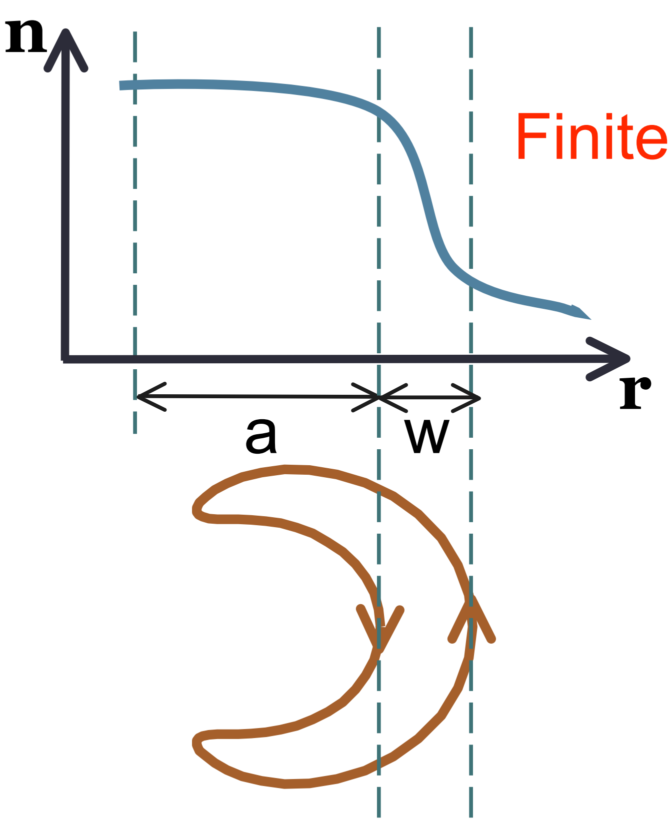
Radial electron pressure balance: $\vec{V}_e = \omega_e R\vec{\zeta} + u_e\vec{B}$

$$\omega_e = -c[\partial\Phi/\partial\psi - (en)^{-1}\partial p_e/\partial\psi]$$

Electron pressure gradient adds to radial electric field making $\omega_e R \sim v_i$ so that $J_{ped} \sim env_i$ & co-current

Thus, the electric field balancing the $1/\rho_{pol}$ density gradient requires a stationary ion Maxwellian & large *electron* flow

Pedestal orderings & ExB drift effects



Drift departure ρ_{pol} is of order pedestal width w

Finite drift orbits effects enter in leading order

Estimating $Ze\nabla\Phi \sim T/\rho_{pol}$ we note

$$\vec{v}_E \cdot \nabla\theta \sim v_{||}\vec{n}\cdot\nabla\theta$$

where $\vec{v}_E$ is the ExB drift velocity

Poloidal streaming ~ ExB

Orbit localization from $\varepsilon << 1$

Decouple neoclassical & classical by assuming $\rho_{pol} >> \rho$

Ion motion for $\varepsilon = a/R << 1$

Assume a quadratic potential well and expand about ψ_*-Iu/Ω

$$\Phi = \Phi_* + \frac{Iv_{||}}{\Omega}\Phi'_* + \frac{I^2 v_{||}^2}{2\Omega^2}\Phi''_*$$

using E, μ and ψ_* invariance while keeping Φ' find

$$\frac{1}{2}S(v_{||}+u_*)^2 + \mu B - \frac{1}{2}Su_*^2 \approx \text{constant}$$

orbit squeezing
$S = 1+cI^2\Phi''/B\Omega$

magnetic
dipole energy

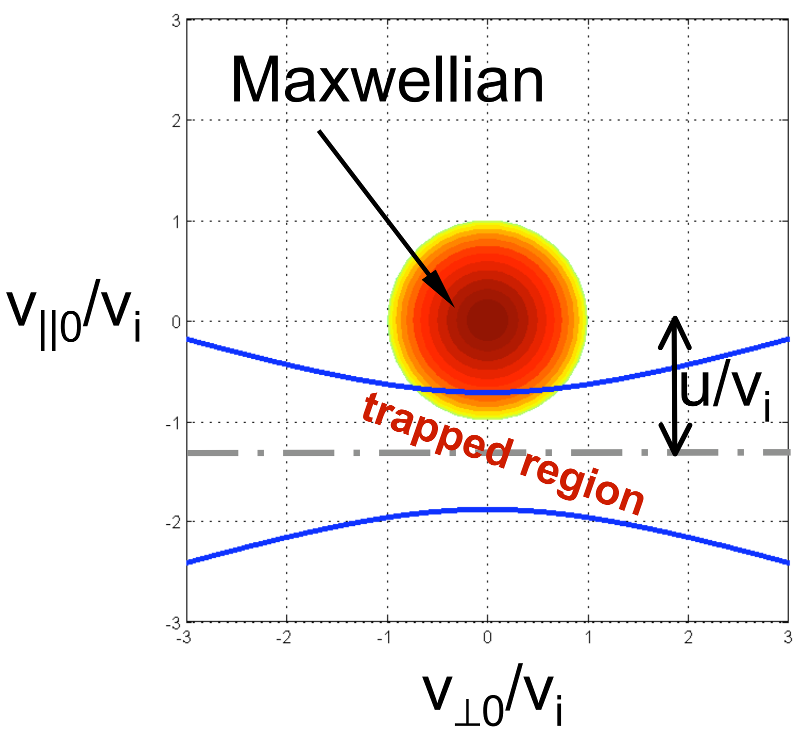
ExB energy
$u_* = cI\Phi'_*/SB$

S>0 (S<0) trapped particles outboard (inboard)
Denote equatorial plane crossing by "0" then

$$v_{||}+u_* \approx (v_{||0}+u_0)\sqrt{1-\kappa^2\sin^2(\theta/2)}\,,\quad \kappa^2 \approx \frac{4\varepsilon(\mu B_0+u_{*0}^2)}{S(v_{||0}+u_{*0})^2} \approx 4\varepsilon S\frac{(\mu B_0+u_0^2)}{(v_{||0}+u_0)^2}$$

with $\kappa^2 = 1$ the trapped-passing boundary & u= cIΦ'/B

Trapped particle fraction



ExB drift:

i) Increases effective potential well depth: μ = 0 trapped by Φ poloidal variation at fixed ψ_*

ii) Shifts the axis of symmetry of the trapped particle region - fewer trapped!

Trapped fraction decays exponentially if $u = cI\Phi'/B > v_i$

Neoclassical and polarization phenomena strongly modified

Recall $u \approx (\rho_{pol}/\rho)v_E >> v_E$ so particle dynamics qualitatively changed by a subsonic ExB drift

Neoclassical ion heat flux & parallel flow

Gradient T drive only

$$\overline{C_1\{g + f_M \frac{Iv_{||}Mv^2}{2\Omega T^2}\frac{\partial T}{\partial \psi}\}} = 0$$

Need a model for the collision operator - must keep energy scatter as well as pitch angle scatter

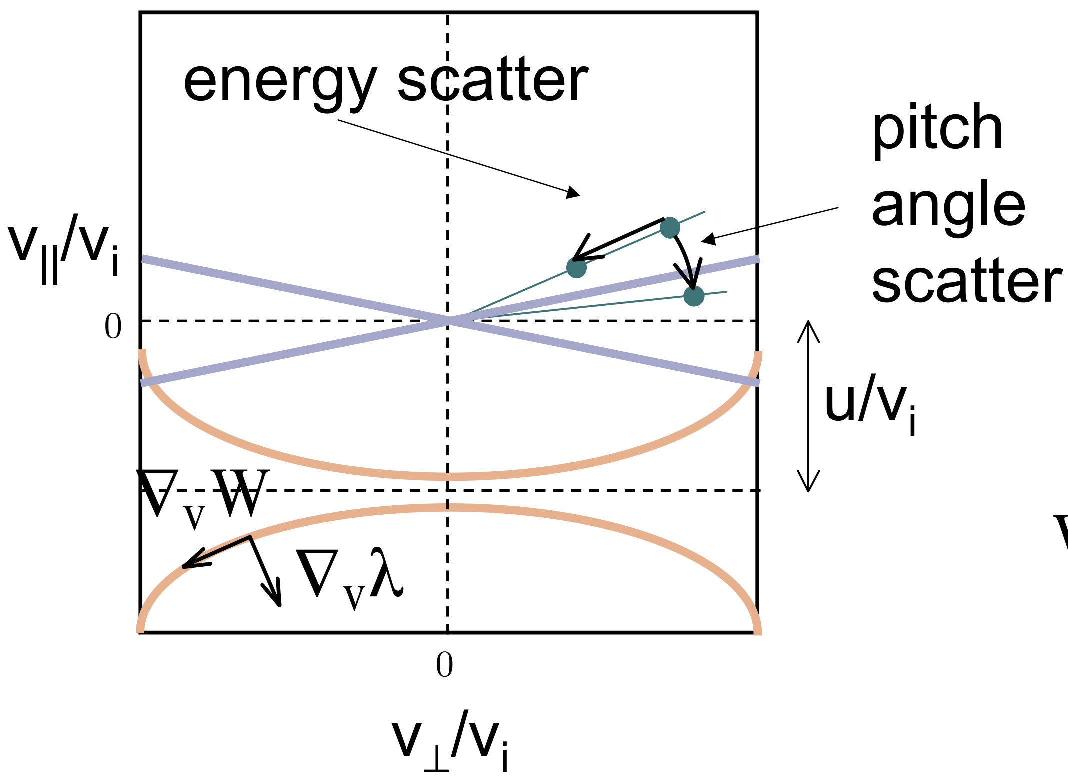
⇩

Solve for *g*

⇩

Calculate quantities of interest by taking moments of the distribution function

Collisions in the pedestal



Convenient variables are λ and W:

$$\lambda = \frac{\mu B_0 + u_0^2}{W} = \frac{\kappa^2}{\kappa^2 + 2\varepsilon}$$

$$W(1 - \lambda B / B_0) = \frac{1}{2} S(v_{||} + u)^2$$

Pitch-angle scattering is not sufficient to retain transitions across the trapped-passing boundary!

Kagan & Catto, to appear in PPCF

Neoclassical parallel ion flow

Localized portion g - h higher order in ε

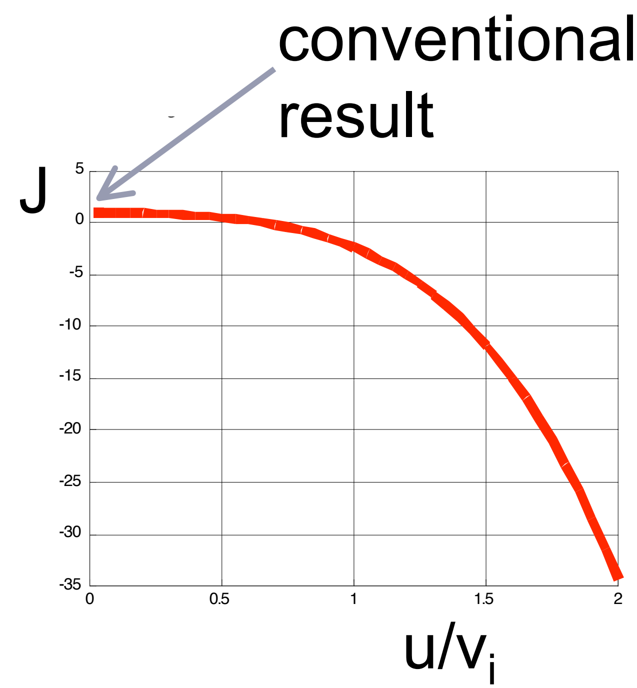
$$V_{\parallel i} = -\frac{cI}{B}\left(\frac{\partial \Phi}{\partial \psi} + \frac{1}{Zen}\frac{\partial p_i}{\partial \psi}\right) - \frac{7cI}{6ZeB_0}\frac{\partial T_i}{\partial \psi} J(u/v_i)$$

No orbit squeezing effect

J changes to Pfirsch-Schluter sign at $u/v_i \sim 0.6$

May help explain C-Mod flow measurements in pedestal

More pedestal bootstrap current



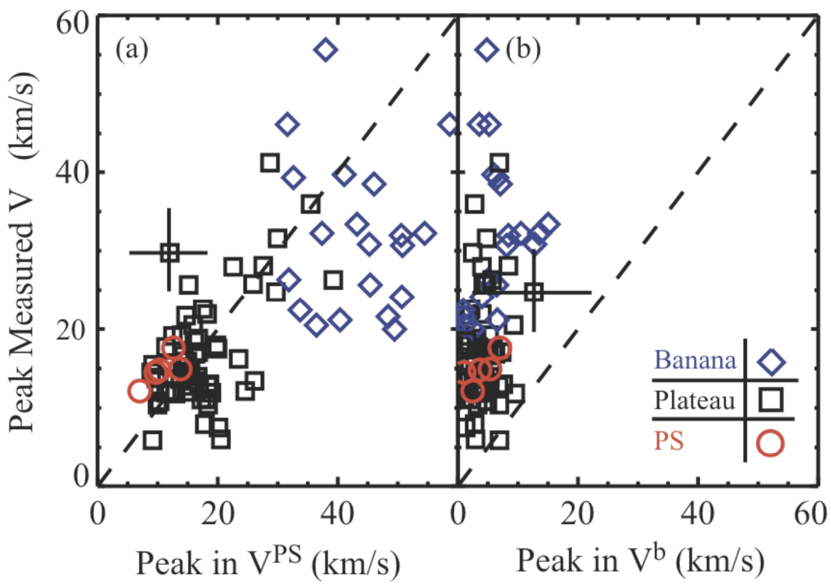
Pedestal impurity flow

Change in poloidal ion flow alters impurity flow

For Pfirsch-Schluter impurities & banana ions:

$$V_z^{pol} = V_i^{pol} - \frac{cIB_{pol}}{eB^2}\left(\frac{1}{n_i}\frac{\partial p_p}{\partial \psi} - \frac{1}{Zn_z}\frac{\partial p_z}{\partial \psi}\right) \quad \& \quad V_i^{pol} \approx -\frac{7cIB_{pol}}{6eB_0^2}\frac{\partial T_i}{\partial \psi}J\left(\frac{u}{v_i}\right)$$

C-Mod pedestal flow:



Pfirsch-Schluter: ~ agree

Banana: problem - need E_r

Marr *et al* to appear PPCF

Pedestal ion heat flux

Modified ion flow :

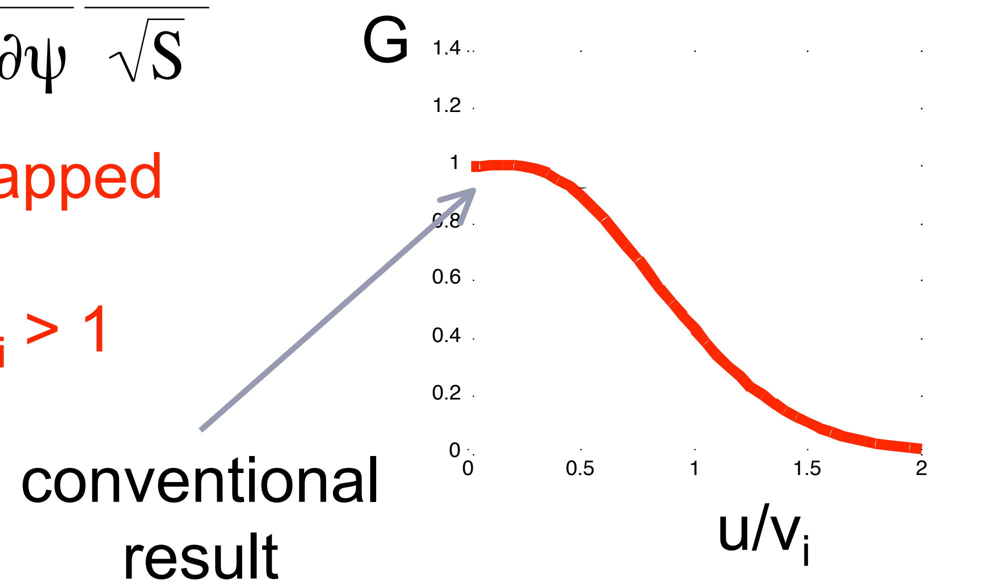
$$\langle \vec{q}\cdot\nabla\psi\rangle = -\frac{McIT}{Ze}\left\langle \int d^3v\left(\frac{Mv^2}{2T}-\frac{5}{2}\right)\frac{v_{\parallel}}{B}C_1\{g-h\}\right\rangle$$

Evaluating:

$$\langle \vec{q}\cdot\nabla\psi\rangle = -1.35\sqrt{\varepsilon}n\nu_i\frac{I^2T_i}{M\Omega^2}\frac{\partial T_i}{\partial\psi}\frac{G(u)}{\sqrt{S}}$$

Radial ion heat flux and trapped population become exponentially small for $u/v_i > 1$

Ion heat flux more sensitive to Φ' than Φ''



Summary

- Pedestal ions nearly isothermal ($\rho_{pol} \nabla T_i \ll 1$): subsonic ions electrostatically confined + magnetically confined electrons
- Banana regime ion heat flux reduced & poloidal ion flow can change sign in the pedestal due to Φ' as in C-Mo